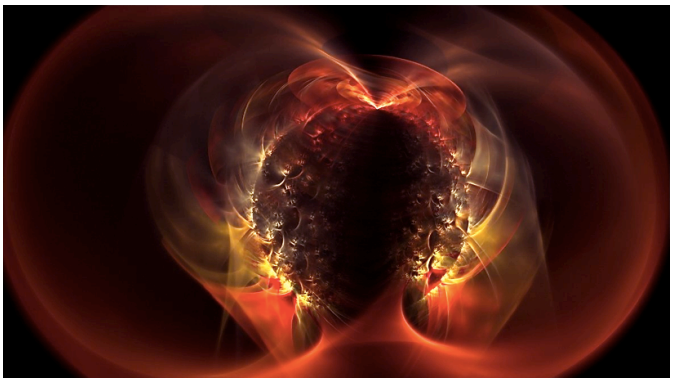
Neutron Star thermal and magnetic evolution.

Jose A. Pons
University of Alicante, Spain

1st ICE Summer School, Barcelona, July 2017

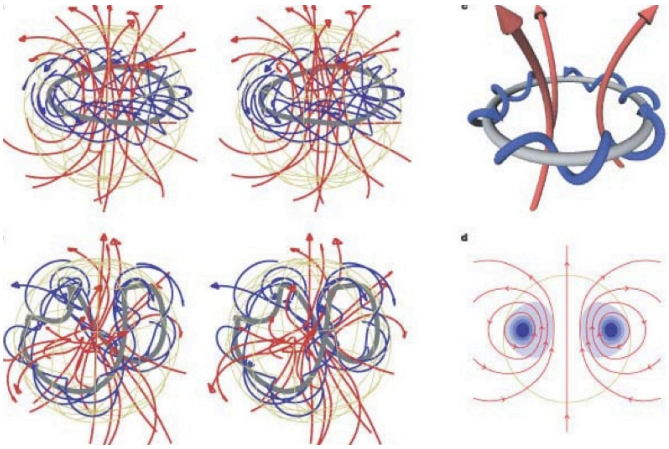


Outline

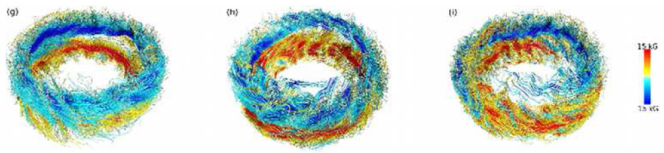
- Lecture 2: Magnetic field evolution.
  1. Magnetic fields in neutron stars: origin and topology ?
  2. Magnetic field diffusion and Joule heating.
  3. The magnetic field in the crust: the Hall term in the induction equation.
  4. Some open problems related to B field evolution: Outbursts, CCOs, maximum period of isolated NSs.

The initial conditions.

A neutron star is born hot and liquid (melting T $\sim 10^{10}$ K). Let us believe that somehow a strong magnetic field has been created (how ??)
Hydrodynamics is appropriate, and if a strong magnetic field is present we can use MHD.
Stable MHD solutions are complex and require a toroidal component.
MHD equilibrium must be established in a few dynamical timescales (seconds), still during the PNS phase, so what is the topology of the B field in MHD equilibrium ?



[Braithwaite & Spruit 2004, 2006]



[Nelson+ 2013]

Magnetic equilibria.

In axial symmetry, the magnetic field can be expressed in terms of the *poloidal* P and *toroidal* T components:

$$\vec{B} = \nabla P \times \nabla \phi + T \nabla \phi$$

where $\nabla \phi = \frac{1}{\varpi} \vec{e}_\phi$, and $\varpi \equiv r \sin \theta$ is the cylindrical radial coordinate. The current in terms of these two scalar function is:

$$\frac{4\pi \vec{J}}{c} = \nabla \times \vec{B} = -\Delta_{GS} P \nabla \phi + \nabla T \times \nabla \phi$$

where the Grad-Shafranov elliptic operator, $\Delta_{GS} = \varpi^2 \nabla \cdot (\varpi^{-2} \nabla)$, is a sort of modified Laplacian. The Lorentz force is then:

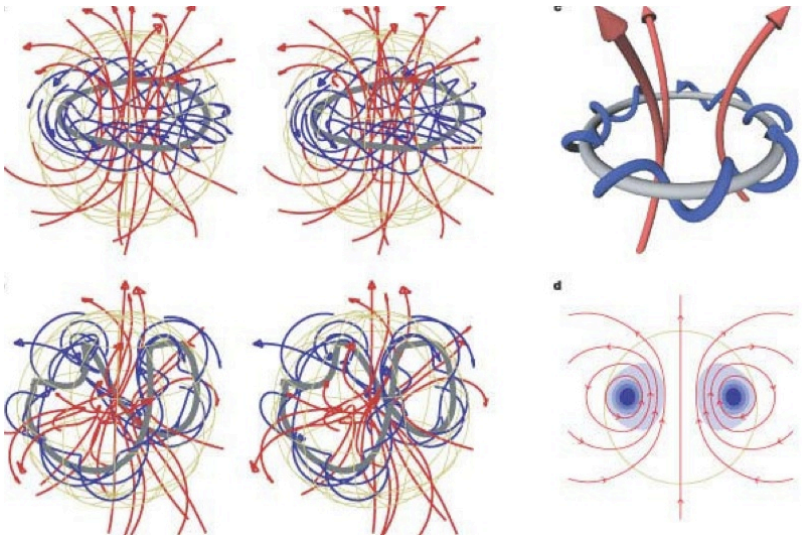
$$4\pi \vec{f} = (\nabla \times \vec{B}) \times \vec{B} = -\frac{\Delta_{GS} P + T(P) T'(P)}{\varpi^2} \nabla P.$$

Magnetic equilibria.

A necessary condition for the existence of MHD equilibria in barotropic stars is that T is only a function of P. In addition, $\vec{f}/\rho$ must be the gradient of an undetermined function $F(P)$, and the MHD equilibria equation can be written as:

$$\Delta_{GS} P + T(P)T'(P) = \rho\varpi^2 F(P).$$

The problem is that $F(P)$ must be provided (as well as $T(P)$), and it is COMPLETELY FREE, except in very simple cases (e.g. force-free $F(P) = 0$).



Magnetic field evolution: solve Maxwell equations.

$$\vec{\nabla} \cdot \vec{E} = 4\pi n_e$$
$$\frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \vec{\nabla} \times (e^{\nu}\vec{B}) - \frac{4\pi}{c} e^{\nu}\vec{J}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times (e^{\nu}\vec{E})$$
$$\vec{E} = \hat{R}\vec{J}$$

To simplify the problem:
1) We can safely neglect the displacement current, so $4\pi\vec{J} = ce^{-\nu}\vec{\nabla} \times (e^{\nu}\vec{B})$, and the evolution is governed by the induction equation.
2) The resistivity tensor $\hat{R}$ can be decomposed in symmetric and anti-symmetric parts. The symmetric part, for non-quantizing fields, is isotropic and determined by the scalar $\sigma_{\parallel}$, the electric conductivity in the direction of the MF. In a one component plasma, the electric field is then:

$$\vec{E} = \frac{\vec{J}}{\sigma_{\parallel}} + \frac{1}{n_e ec}\vec{B} \times \vec{J}$$

where n_e is the electron number density and e is the elementary charge.

Magnetic field evolution: multicomponent systems.

- Momentum equations of neutron, proton and electron plasma:

$$\cancel{\partial_t \vec{v}_\mathrm{p} + \vec{v}_\mathrm{p} \cdot \nabla \vec{v}_\mathrm{p}} - \nabla \mu_\mathrm{p} - m_\mathrm{p}^* \nabla \Phi + e\left(\vec{E} + \frac{\vec{v}_\mathrm{p}}{c} \times \vec{B}\right) = \frac{m_\mathrm{p}^* \vec{w}_\mathrm{pn}}{\tau_\mathrm{pn}} + \frac{m_\mathrm{p}^* \vec{w}_\mathrm{pe}}{\tau_\mathrm{pe}}$$

$$\cancel{\partial_t \vec{v}_\mathrm{e} + \vec{v}_\mathrm{e} \cdot \nabla \vec{v}_\mathrm{e}} - \nabla \mu_\mathrm{e} - m_\mathrm{e}^* \nabla \Phi - e\left(\vec{E} + \frac{\vec{v}_\mathrm{e}}{c} \times \vec{B}\right) = \cancel{\frac{m_\mathrm{e}^* \vec{w}_\mathrm{en}}{\tau_\mathrm{en}}} + \frac{m_\mathrm{e}^* \vec{w}_\mathrm{ep}}{\tau_\mathrm{ep}}$$

$$\cancel{\partial_t \vec{v}_\mathrm{n} + \vec{v}_\mathrm{n} \cdot \nabla \vec{v}_\mathrm{n}} - \nabla \mu_\mathrm{n} - m_\mathrm{n}^* \nabla \Phi = \frac{m_\mathrm{n}^* \vec{w}_\mathrm{np}}{\tau_\mathrm{np}} + \cancel{\frac{m_\mathrm{n}^* \vec{w}_\mathrm{ne}}{\tau_\mathrm{ne}}}$$

Relative velocity: $\vec{\omega}_\mathrm{xy} = \vec{v}_\mathrm{x} - \vec{v}_\mathrm{y}$

Conservation of total momentum → $\dfrac{n_\mathrm{x} m_\mathrm{x}^\star}{\tau_\mathrm{xy}} = \dfrac{n_\mathrm{y} m_\mathrm{y}^\star}{\tau_\mathrm{yx}}$

Scattering rate: $\dfrac{1}{\tau_\mathrm{pn}} = 4.7 \times 10^{16} T_8^2 \left(\dfrac{\rho}{\rho_{nuc}}\right)^{-1/3} \mathrm{s}^{-1}$

$$\frac{1}{\tau_\mathrm{en}} \approx \frac{10^{-4}}{\tau_\mathrm{pn}}$$

Magnetic field evolution: generalized Ohm's law.

By manipulating the multicomponent hydro equations (and under SEVERAL approximations), one can factor out the Electric field in terms of the current, and other magnetic field dependent quantities. The most general form is:

(Goldereich & Reisenegger 1992)

- Electric field

$$\boldsymbol{E} = \frac{\boldsymbol{j}}{\sigma} + \frac{1}{e n_c c}\boldsymbol{j} \times \boldsymbol{B} - \frac{1}{c}\boldsymbol{v}_{amb} \times \boldsymbol{B} + \ldots$$

Ohmic term Hall drift Ambipolar diffusion

- From the momentum equations:

$$\frac{\boldsymbol{f}_{\mathrm{B}}}{n_{\mathrm{c}}} - \nabla(\Delta\mu) \simeq \frac{\boldsymbol{v}_{amb}}{\tau_{\mathrm{pn}}}$$

where $\Delta\mu \equiv \mu_{\mathrm{p}} + \mu_{\mathrm{e}} - \mu_{\mathrm{n}}$

- **High temperature** $\sim 10^9\,\mathrm{K}$ $\Delta\mu \simeq 0$
- **Low temperature** $\sim 10^8\,\mathrm{K}$ $\Delta\mu$ **is relevant**

Magnetic field evolution: timescales.

- **Magnetic field evolution in the core (Goldereich & Reisenegger 1992)**
  - **Ambipolar diffusion** $t_{amb} \sim 10^3 \frac{T_8^2 L_5^2}{B_{15}^2} \mathrm{yr}$
  - **Ohmic dissipation** $t_{ohm} \sim 10^{11} \frac{L_5^2}{T_8^2} \mathrm{yr}$
  - **Hall drift** $t_{hall} \sim 5 \times 10^5 \frac{L_5^2}{B_{15}} \mathrm{yr}$

A word of caution: do NOT assume that timescales give you the right *evolution time*, they strongly depend on the GEOMETRY of the field, which changes very quickly if timescales are short.

Magnetic field evolution

CRUST
The solid crust can be considered a simplified version of a Hall plasma: ions have very restricted mobility and only electrons can move freely through the lattice, carrying currents and heat: the proper equations are Hall MHD. If ions are strictly fixed in the lattice, the limit is known as EMHD (electron MHD).

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \vec{\nabla} \times (e^{\nu} \vec{B}) - \left[\frac{c e^{-\nu}}{4\pi e n_e} \vec{\nabla} \times (e^{\nu} \vec{B}) \right] \times (e^{\nu} \vec{B}) \right\}$$

$\eta = \dfrac{c^2}{4\pi\sigma}$ is the magnetic diffusivity, and the electron fluid velocity is

$$v_e = -\frac{\vec{J}}{e n_e} = -\frac{c e^{-\nu}}{4\pi e n_e} \vec{\nabla} \times (e^{\nu} \vec{B})$$

In the limit of small deformations, the metric is still spherically symmetric. Relativistic corrections included with the $e^{-\nu}$ factor.

CORE
Not clear how much flux penetrates into the core, and what is the evolution of a SC fluid (fluxoids drift and interact with vortices? magnetic buoyancy?).

Ohmic dissipation

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \vec{\nabla} \times \vec{B} \right\}$$

Ohmic dissipation timescale

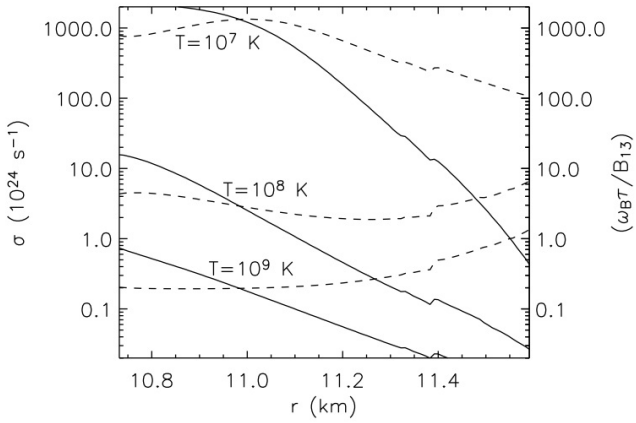
Assume a force-free backgroung field $\vec{\nabla} \times \vec{B} = \mu \vec{B}$ and constant η.

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \mu \vec{B} \right\}$$

$$\frac{\partial \vec{B}}{\partial t} = -\eta \mu^2 \vec{B}$$

$$\vec{B} = \vec{B}(t=0) e^{-t/\tau_{Ohm}}$$

with $\tau_{Ohm} = 1/(\eta \mu^2) \approx L^2/\eta$, where L is the typical scale of the curvature of the magnetic field



The Hall term

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \left[\vec{\nabla} \times \vec{B} - \omega_B \tau_e \left(\vec{\nabla} \times \vec{B} \right) \times \vec{b} \right] \right\}$$

Here $\omega_B \tau_e$ is the "magnetization parameter", where ω_B is the gyro-frequency.
For high temperatures (large resistivity) or weak fields: diffusive regime $\omega_B \tau_e \ll 1$
For low T ($T \lesssim 10^8$ K) or strong fields: hyperbolic regime $\omega_B \tau_e \gg 1$, i.e. Hall activity.

Linear regime: wave modes *[Huba 2005]*

Backgroung field $\vec{B} = B_0 \hat{z}$

- constant n_e, **whistler (or helicon) waves** propagating along field lines
  dispersion relation $\omega = k^2 B / 4\pi e n_e$
  phase velocity $\propto k_z \Rightarrow$ restrictive Courant condition
- $n_e = n_e(x)$, **Hall drift waves** in the $\vec{B} \times \vec{\nabla} n_e$ direction
  dispersion relation $\omega = k_y B_0 / [4\pi e (dn_e/dx)]$
  phase velocity $B_0 / [4\pi e (dn_e/dx)]$

Hall timescale.

$$\tau_{Hall} = \frac{4\pi e n_e}{cB} L^2$$

$$\omega_B \tau_e = \tau_{Ohm} / \tau_{Hall}$$

The Hall term: non-linearity

The evolution equation for a purely toroidal MF, neglecting resistivity, can be cast into the following form:

$$\frac{\partial B_\varphi}{\partial t} + \lambda_r n_e \frac{\partial}{\partial r}\left(\frac{B_\varphi^2}{2n_e}\right) + \lambda_\theta \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{B_\varphi^2}{2}\right) = 0$$

where

$$\lambda_r = -\frac{c}{2\pi e n_e}\frac{\cos\theta}{r\sin\theta}$$
$$\lambda_\theta = -\frac{r^2 c}{4\pi e}\frac{d}{dr}\left(\frac{1}{n_e r^2}\right) .$$

Currents sheets

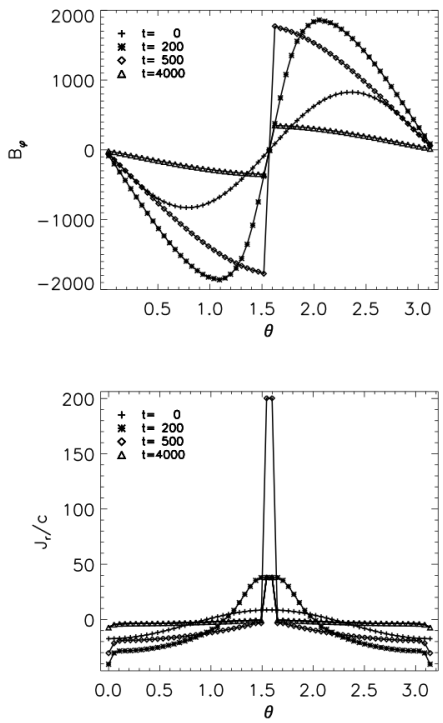
Burgers part in toroidal B_ϕ due to non-linear self-coupling! Impossible to catch it with a spectral method, need for shock-capturing finite-difference methods.

The numerical solutions: some technical details

[Viganò+ 2012, *Comput. Phys. Comm.* 183]

Numerical treatment

1. Finite-difference, divergence-preserving scheme: staggered grid with $\vec{E}$ and $\vec{B}$ components defined in displaced points.
2. Upwind scheme to follow the formation and dissipation of very localized current sheets
3. Cell reconstruction
4. "Semi-implicit" advance implying an alternated advance of B_ϕ and B_{pol}
5. Hyper-resistivity (well below the physical resistivity)
6. Conservation of total (magnetic+dissipated+Poynting flux) energy



Weak field

Intermediate field

Strong field

Core field

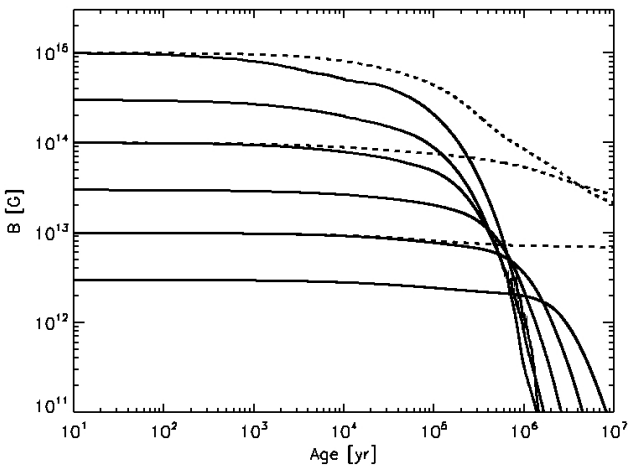
Core+crust field

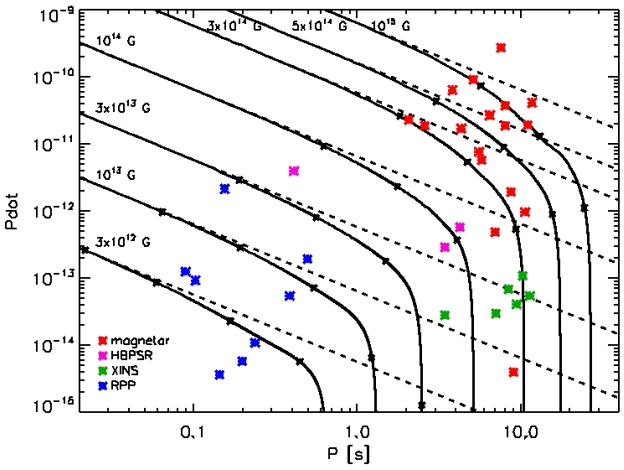
Multipolar field

Toroidal field

Initial topology is quickly reorganized: the Hall term removes the freedom to chose arbitrarily large toroidal fields able to inject very large extra energy (Viganò + 2012, 2013)
Models have more predictive power.

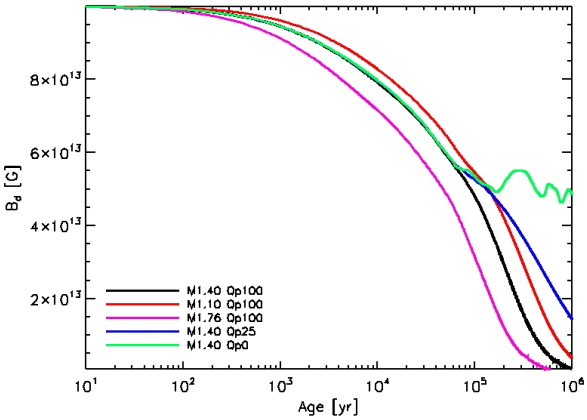
Evolutionary tracks

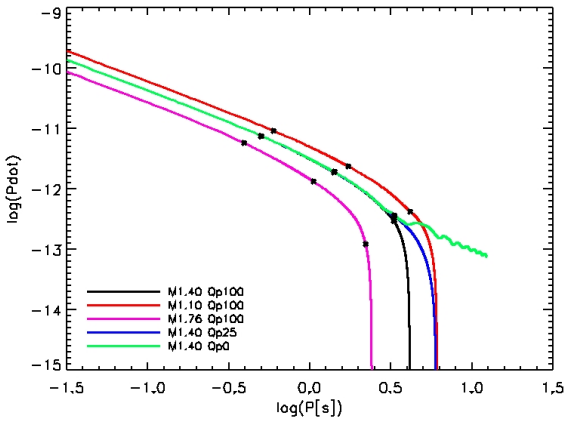


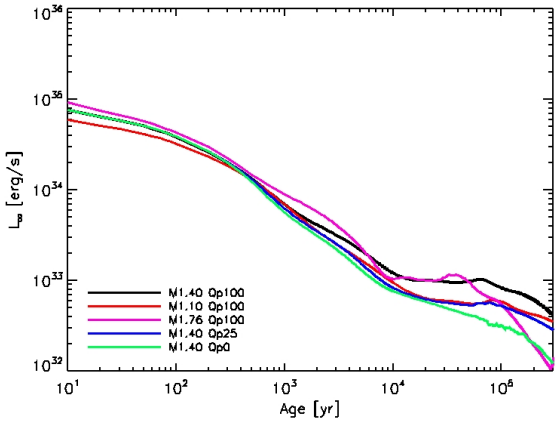


Assuming P_{in}, knowing B_d using spin-down dipole formula $B_d = 6.4 \times 10^{19} G \sqrt{P\dot{P} I_{45}} R_{10}^{-3}$
$\Rightarrow$ evolution of P and $\dot{P}$ (choice of P_{in} is not important).
Dissipation of field results in vertical trajectories. As a consequence, $\tau_c \gg$ real age at late time, even by orders or magnitude.

Varying the microphysics

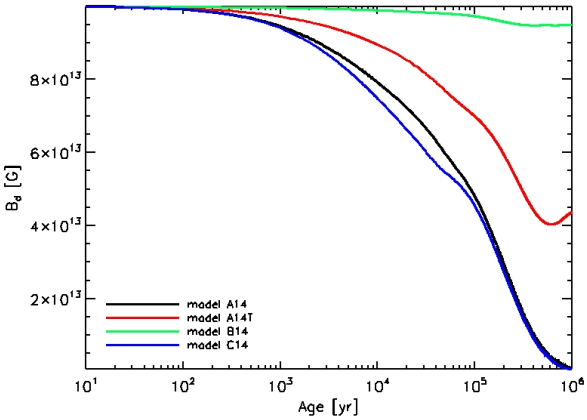


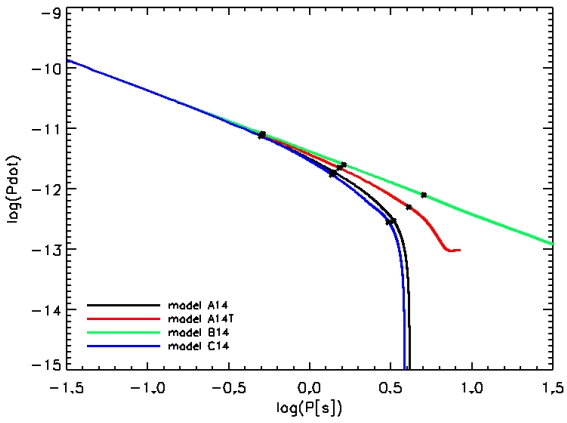


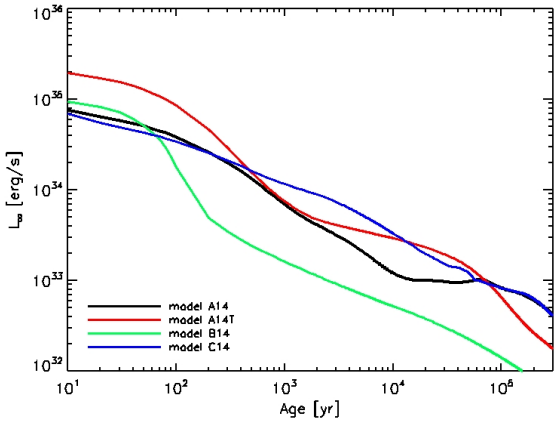


In crustal field models, masses and microphysics have a limited effect due to Joule heating.

Varying the geometry

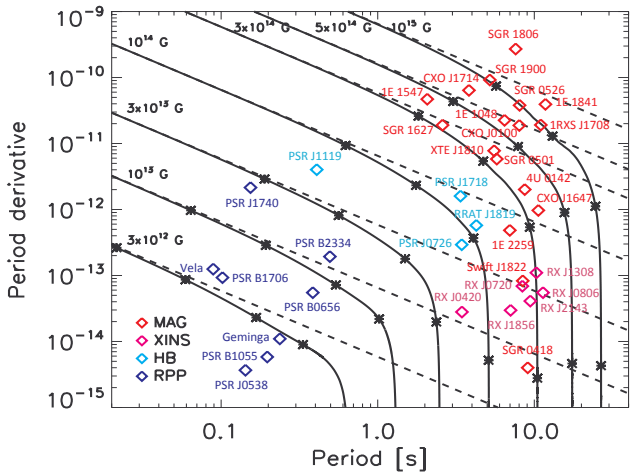


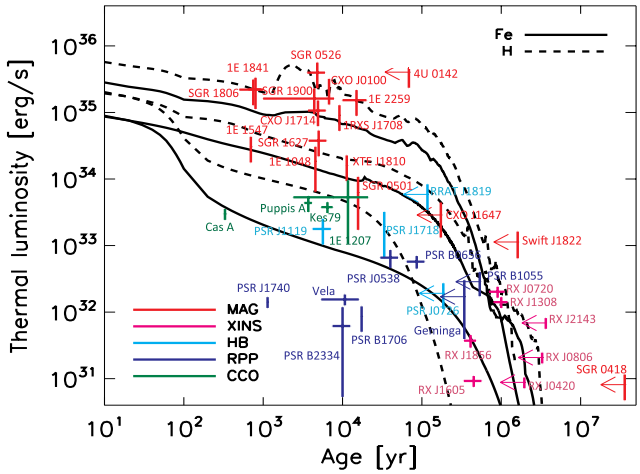




Geometry (currents in the crust or in the core) has a very strong imprint.

Data vs. models





Constraining models

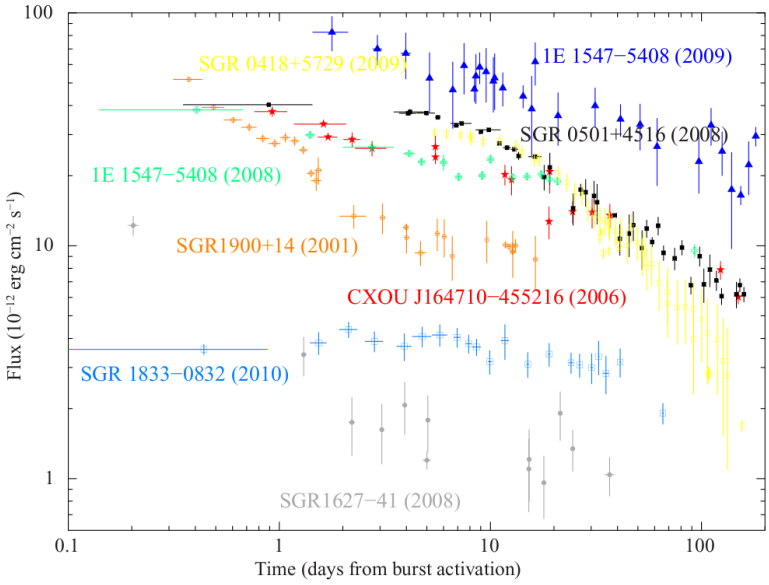
Data requires a large $Q_{imp} \gtrsim 10$ in the inner crust (high electrical resistivity), where most of the currents must be placed.
If $Q_{imp} \lesssim O(1)$, the evolved magnetars would have longer periods, where are they ?

Constraining models

Large Q_{imp}: fast dissipation, brighter early phases, saturation of periods.
Low Q_{imp}: slower dissipation, periods keep increasing at late ages.

Outbursts

Many magnetars are discovered while in outburst (SGR 0418, SGR 1822, SGR 1833, Swift 1834 in the last few years).



[Rea & Esposito 2012]

The flux enhancement can reach a factor 10^3.

Outbursts: the starquake model *[Duncan & Thompson 1992]*

Magnetic stress tensor:

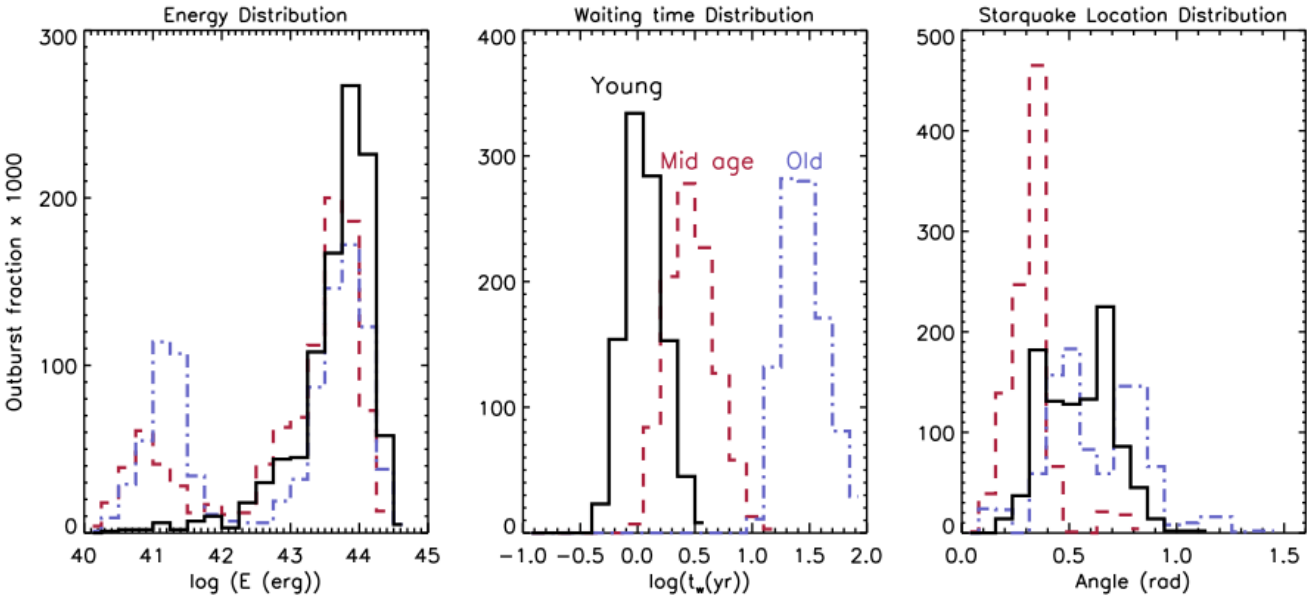
$$M_{ij}(\vec{x},t) = \frac{B_i(\vec{x},t)B_j(\vec{x},t)}{4\pi}$$

Magnetic field evolves in the crust (helicon waves, Hall waves) and dissipates. This changes the stresses balance.
Shear breaking stress

$$\sigma_b^{max} \simeq \left(0.0195 - \frac{1.27}{\Gamma(T(t)) - 71}\right) n_i \frac{Z^2 e^2}{a}$$

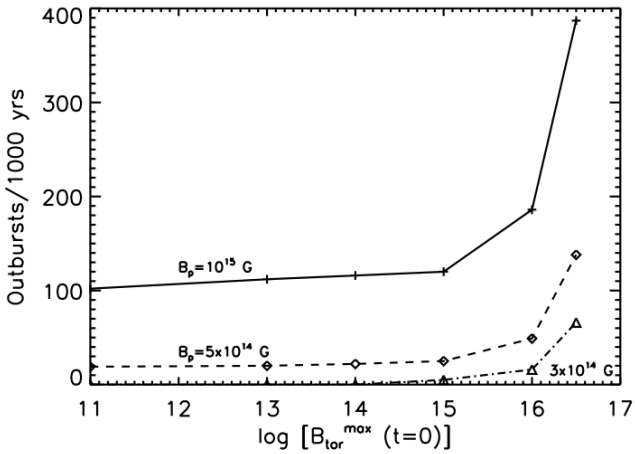
[Chugunov & Horowitz 2010] Breaking strain 100 time larger than usually assumed !!
When $|M_{i,j} - M_{i,j}^{eq}| = \sigma_b(\vec{x},t)$, the stress imbalance exceeds the shear breaking strength of the crust, the crust breaks, and elastic/magnetic energy is released and converted into electromagnetic energy.

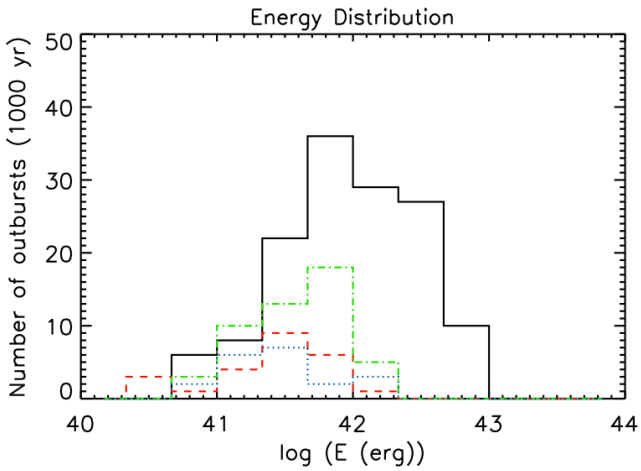
Outbursts: the quake model *[Perna & Pons 2011, Pons & Perna 2011]*



- SGR-like events for young objects are more energetic and more frequent.
- AXP-like events for middle-age objects are less frequent and its location is closer to the pole.
- Objects with strong internal fields are more energetic

Outbursts: the effect of the toroidal field





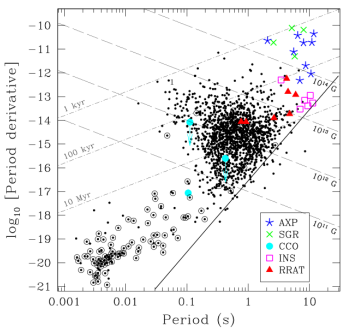
- The outburst rate critically increases when the poloidal field is $B_p > 2 \times 10^{14} G$
- For a fixed poloidal field, the event rate increases when $B_t > B_p$
- low B magnetar-like events for old objects are rare, but still possible.

Central Compact Objects

CCO	SNR	age [kyr]	d [kpc]	P [ms]	$\dot{P}$ [10^{-17} s/s]	f_p %	B_p [10^{10} G]	L_{bol} [10^{33} erg/s]
CXOU J185238.6+004020	Kes 79	5.4-7.5	$\sim$ 7.1	105	0.87	64	3.1	$\sim$ 5.3
1E 1207.45209	G296.5+10.0	3-21	1.3-3.9	424	2.22	9	9.8	0.8-7.6
RX J0822.0-4300	Puppis A	3.3-4.1	1.9-2.5	112	0.93	11	2.9	4.8-8.4
CXOU J232327.9+584842	Cas A	0.33	3.3-3.7	-	-	$<$12	-	4-8.2
CXOU J085201.4-461753	Vela Jr	0.7-1.3	0.3-0.5	-	-	$<$7	-	0.2-0.7
CXOU J160103.1-513353	G330.2+1.0	3-10	$\sim$ 5	-	-	$<$40	-	1-6
XMMU J172054.5-372652	G350.1-0.3	0.9	$\sim$ 4.5	-	-	-	-	$\sim$ 1.1
1WGA J1713.4-3949	G347.3-0.5	1-10	$\sim$ 1.3	-	-	$<$7	-	0.6-15
2XMMi J115836.1-623516	G296.8-0.3	10	$\sim$ 9.6	-	-	-	-	$\sim$ 1.1
XMMU J173203.3-344518	G353.6-0.7	27	$\sim$ 3.2	-	-	$<$ 9	-	7.7-11
CXOU J181852.0-150213	G15.9+0.2	1-3	8-13	-	-	-	-	3-8

[Apadted from Gotthelf, Halpern & Alford 2013 and refs. within them]

[Kaspi 2010]



- Location close to the center of SNRs (like $\sim$ 60% of $<$ 20 kyr radio PSRs *[ATNF catalog]*)
- **No emission in radio/IR/optical, no Pulsar Wind Nebulae**
- **Very low spindown rates $\Rightarrow$ low inferred dipolar field $B_p \lesssim 10^{11}$ G**
- **X-ray: stable flux, thermal-dominated, with small hot blackbody component(s), or large pulsed fraction**

Interpretation of CCOs I: anti-magnetars

[Bonanno+ 2005,2006, Haplern & Gotthelf series]

Neutron stars born with slow rotation ($P_{in} \sim 0.1$ s) at birth.
$\Rightarrow$ inefficient dynamo action: low initial magnetic field

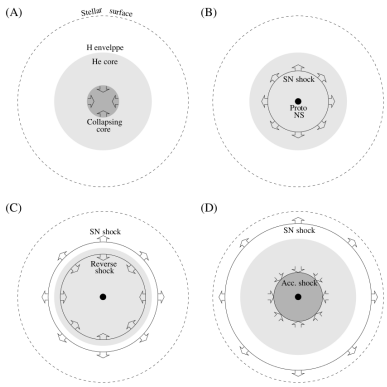
- Timing properties explained in a natural way: very slow evolution in $P - \dot{P}$ diagram
- P_{in} controls initial B and timing evolution

?

- Strong evidences for surface temperature anisotropies (small hot caps in spectra, large f_p)
- Luminosity is quite high
- $P - \dot{P}$ diagram: underpopulated region (pop. synthesis approach)

Interpretation of CCOs: hidden MF scenario

Fallback onto Neutron Star
from SN reverse shock



[image: Bernal+ 2010]

The fallback episode is highly hypercritical.
[Colgate 1971, Blondin 1986, Colpi+ 1996]
Duration: weeks to months.
Total accreted matter: $M_a \sim 10^{-4} - 10^{-1} M_\odot$.
[Chevalier 1989]

Hidden magnetic field scenario
[Young & Chanmugam 1995; Muslimov & Page 1995; Geppert+ 1999; Ho 2011]

1. Neutron stars born with standard P_{in} and B_d.
2. Submergence of MF into inner crust.
3. The field slowly ($10^2 - 10^7$ yr) reemergences back to birth values (damped by Ohmic dissipation).

Hidden MF scenario: modify the induction equation

The induction equation

In the initial, short fallback episode, we switch on an advective term:

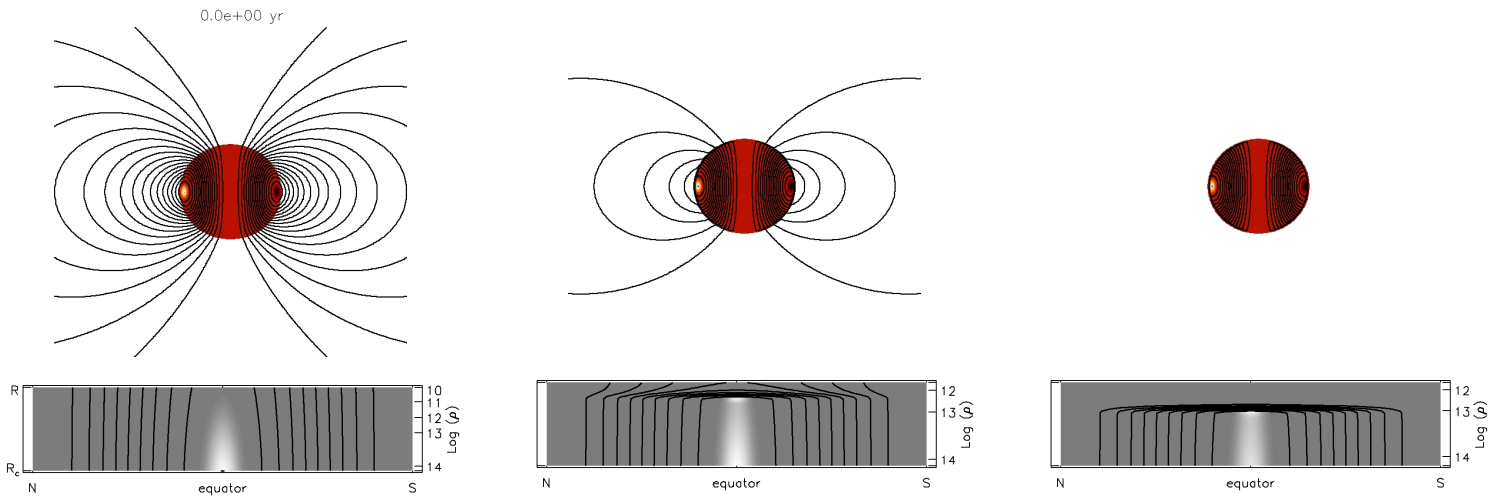
$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \left\{ \eta \vec{\nabla} \times (e^{\nu} \vec{B}) - \left[\vec{v}_a - \frac{e^{-\nu} c}{4\pi e n_e} \vec{\nabla} \times (e^{\nu} \vec{B}) \right] \times (e^{\nu} \vec{B}) \right\} ,$$

Continuity equation $\partial_t \rho + \vec{\nabla} \cdot (\rho \vec{v}_a) = 0$, and $\partial_t \rho = 0$:

$$\vec{v}_a = -\frac{\dot{m}(\theta)}{\rho} e^{-\nu} \hat{r}$$

This is not modeling MHD accretion outside the star *[Bernal+ 2010]*.
We are simply requiring that during the first months of NS life matter piles up on top of the crust and sinks.

Hidden MF scenario: submergence



INITIAL CONFIGURATION | $1 \times 10^{-4} M_\odot$ SPHERICAL | $5 \times 10^{-4} M_\odot$ SPHERICAL

Crustal mass between surface and submergence depth ρ_s is approximately M_a
At ρ_s, MF is compressed and amplified.
Reduction of MF at surface by orders of magnitude (strongly dependent on M_a)

Hidden MF scenario: simulations

$1 \times 10^{-4} M_{\odot}$ SPHERICAL

$5 \times 10^{-4} M_{\odot}$ SPHERICAL

Simulations: accretion flow geometry

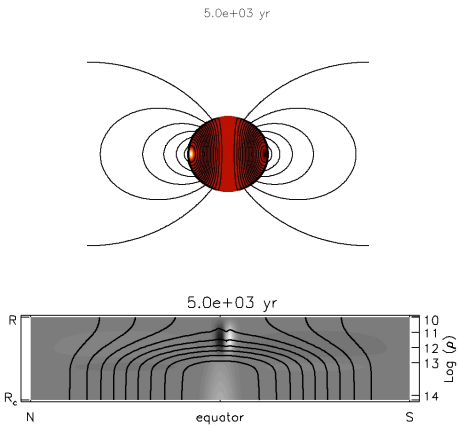
To affect the external field significantly: accretion over a vast fraction of surface

$1 \times 10^{-4} M_{\odot}$ POLAR

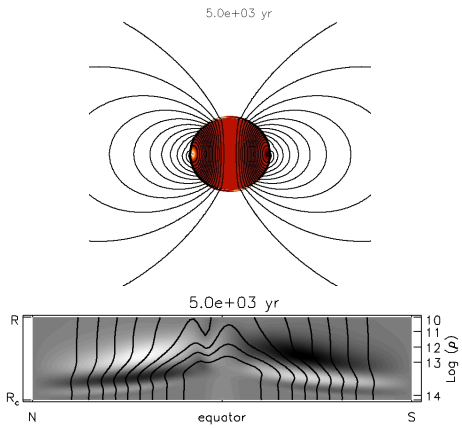
$1 \times 10^{-4} M_{\odot}$ EQUATORIAL

Re-emergence: Hall activity

CONFIGURATION AFTER 5 kyr



$1 \times 10^{-4} M_{\odot}$, $B_{in} = 10^{12}$ G



$1 \times 10^{-4} M_{\odot}$, $B_{in} = 10^{14}$ G

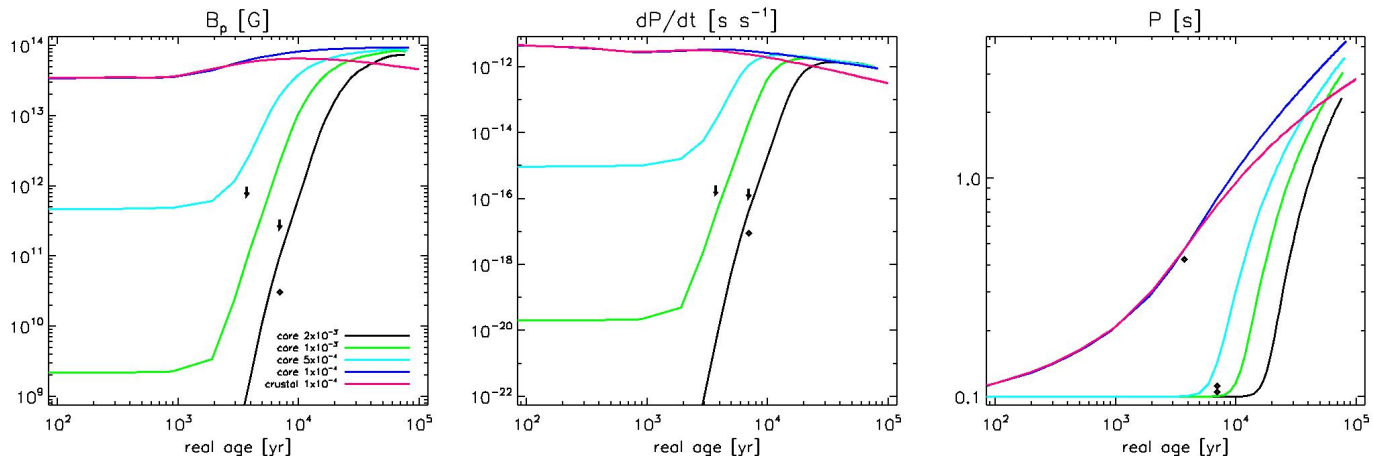
Re-emergence of B_d back to birth values (damped by Ohmic diffusion).
Compression of lines $\Rightarrow$ Hall activity $\Rightarrow$ strong interplay poloidal/toroidal field

Hidden MF scenario: timing properties

Classical dipole-braking formula

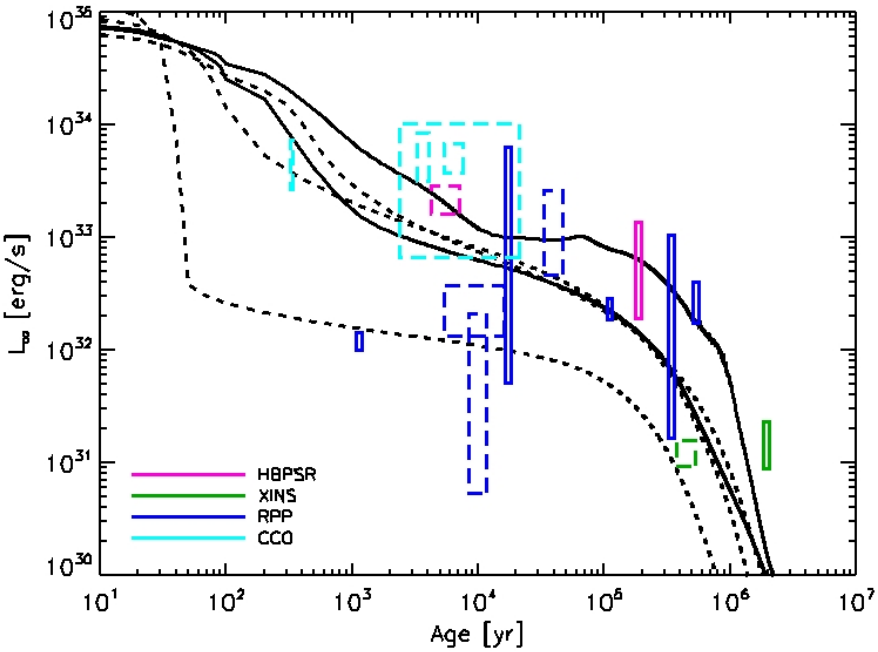
$$B_d = 6.4 \times 10^{19} G \sqrt{P \dot{P} I_{45}} R_{10}^{-3}$$

Assuming P_{in}, knowing $B_d \Rightarrow P$ and $\dot{P}$



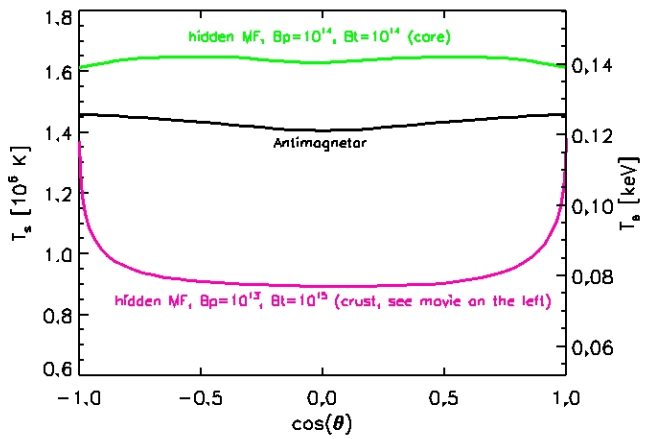
Both anti-magnetar and hidden magnetic field scenario are viable to explain timing properties.

Hidden MF scenario: luminosity



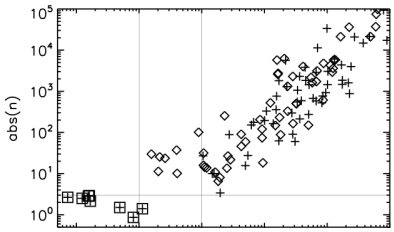
Some CCOs are quite brighter than RPPs
Extra luminosity given by dissipation of the screening currents?

Hidden MF scenario: temperature



(Qualitatively) consistent with thermal features of CCOs: antipodal hotspots of Puppis A [de Luca+ 2012], large pulsed fraction of CCO in kes 79 [Shabaltas & Lai 2012], hot thermal component of 1E 1207.4-5209.
Try to fit individual sources is meaningless, due to the many free parameter we have ($\dot{M}$, B_0 (tor. and pol.), geometry, envelope, P_0)

Data vs. models: braking indices

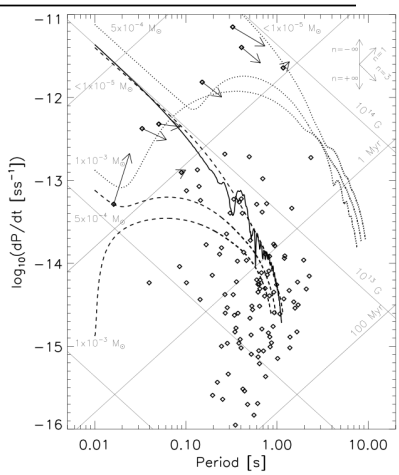


$$n = \frac{\ddot{\nu}\nu}{\dot{\nu}^2} = 2 - \frac{\ddot{P}P}{\dot{P}^2} = 3 - 4\tau_c \frac{\dot{B}_d}{B_d}$$

with the hypothesis of dipole-braking formula.
$n > 3$: decreasing B_d (dissipation)
$n < 3$: increasing B_d (reemerging/oscillating?)

Hobbs+ 2010: 366 pulsars with relatively reliable braking index (observation time > 10 yr, low noise...)

- Hall term provide oscillations of timescales of $10^3 - 10^5$ yr (depending on B and geometry): contribution to the reddest part of timing noise.
- Several young pulsars show $n < 3$ [Espinoza+ 2011]: reemerging field after post-SN submergence?
- High-frequency timing noise is probably due to magnetospheric changes [Lyne+ 2010].

Caveats about observed values of n

Bias towards largest values (more detectable)
Dependence on the chose integration time ($n = n(t)$)
Higher orders ($d^n P/dt^n$) dominating the noise
Glitches are taken into account