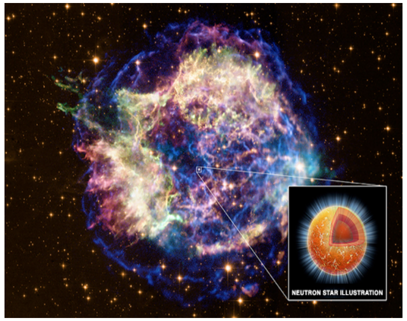

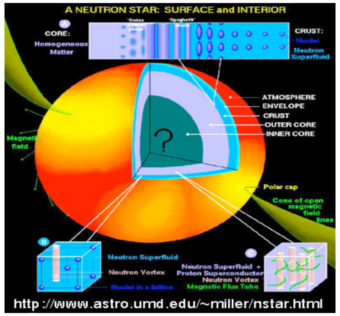

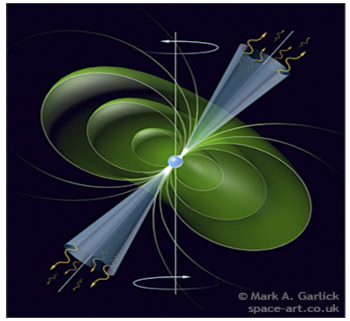

The Equation of State for Neutron Stars

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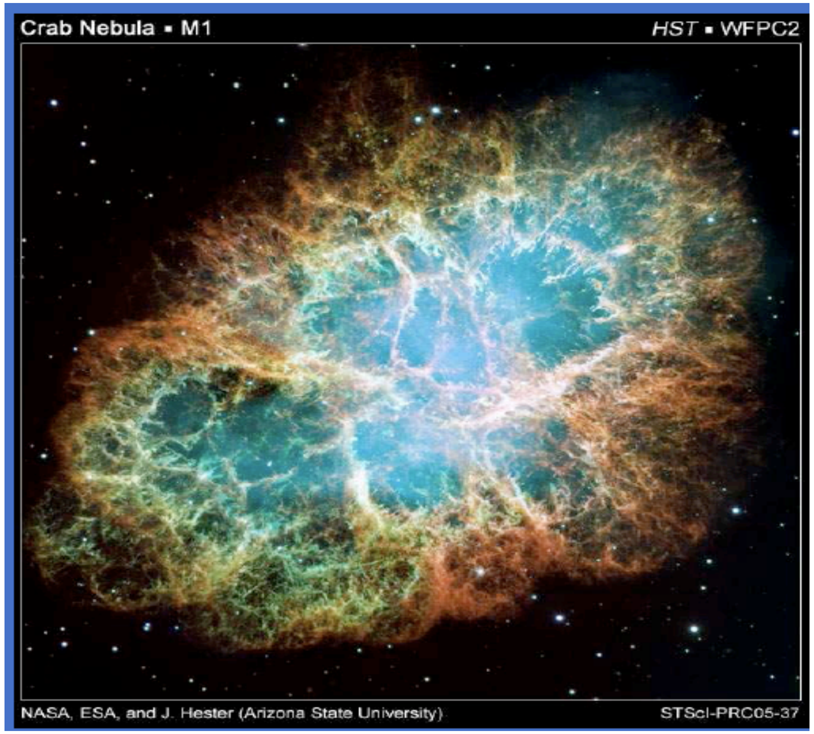
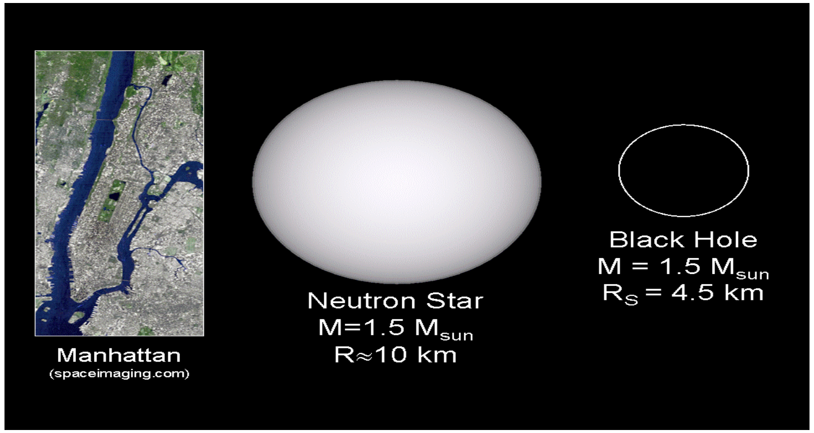
Outline

- What is a Neutron Star?
- Bulk properties: Masses and Radius
- Internal structure and composition: the Core
- Equation of State of dense matter: free Fermi gas
- Baryonic matter in the core
- Structure Equations for neutron stars
- Mass-Radius relation
- Summary

Neutron Star (I)





- first observations by the Chinese in 1054 A.D. and prediction by Landau after discovery of neutron by Chadwick in 1932

- produced in core collapse supernova explosions

- usually refer to compact objects with $M \approx 1\text{-}2\ M_{\odot}$ and $R \approx 10\text{-}12$ Km

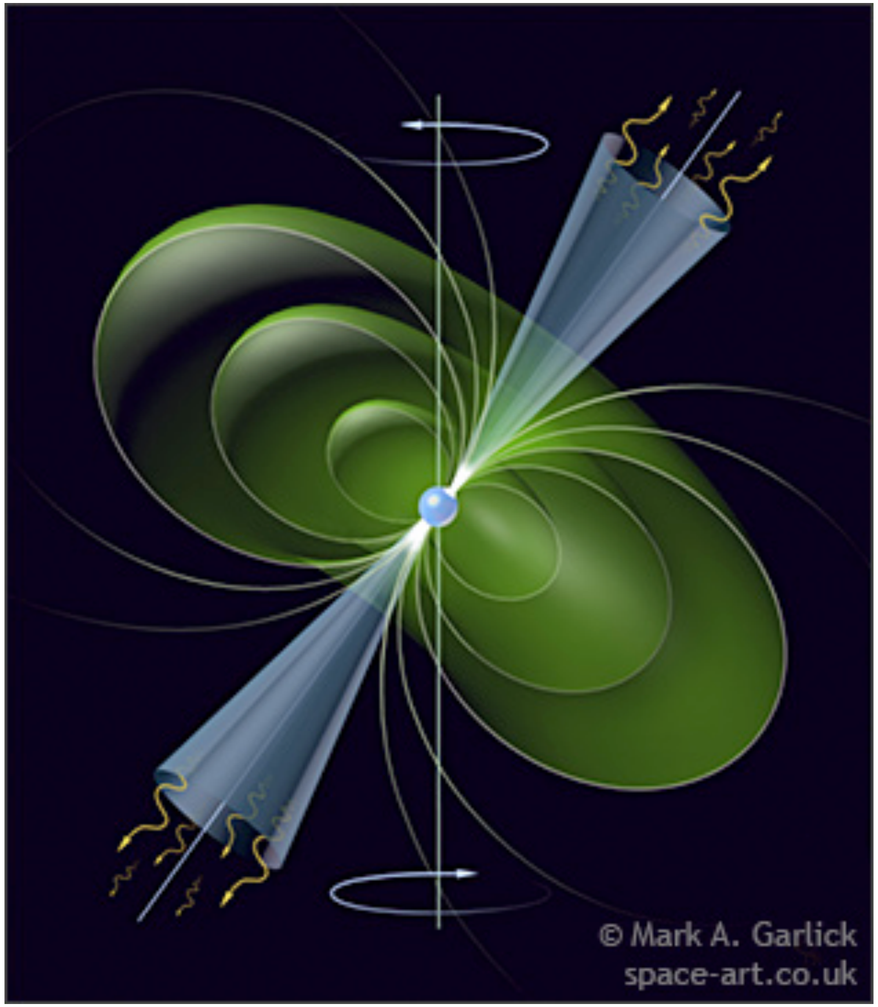
- extreme densities up to 5-10 times nuclear density ρ_0
($\rho_0 = 0.16\ \mathrm{fm}^{-3} \Rightarrow n_0 = 3 \cdot 10^{14}\ \mathrm{g/cm}^3$)
$n_{Universe} \sim 10^{-30}\ \mathrm{g/cm}^3$
$n_{Sun} \sim 1.4\ \mathrm{g/cm}^3$
$n_{Earth} \sim 5.5\ \mathrm{g/cm}^3$

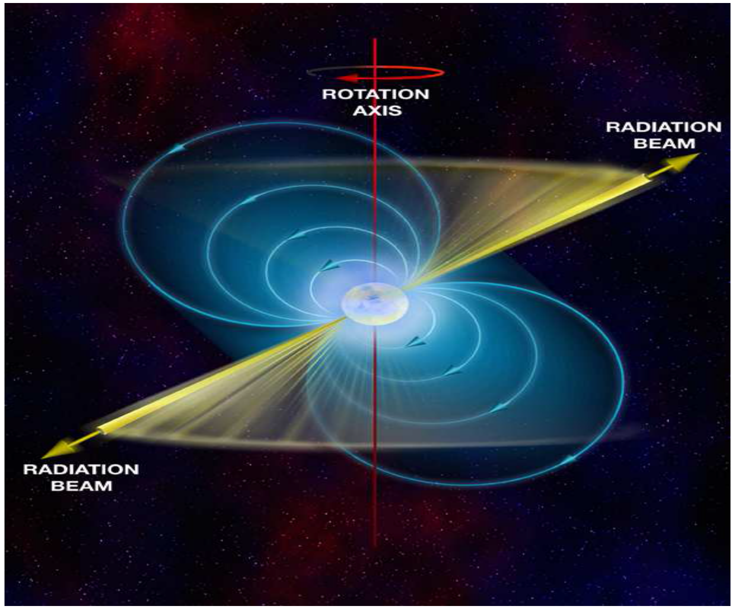
Neutron Star (II)

• usually observed as pulsars with rotational periods from milliseconds to seconds

• magnetic field : B ~ $10^{8..16}$ G

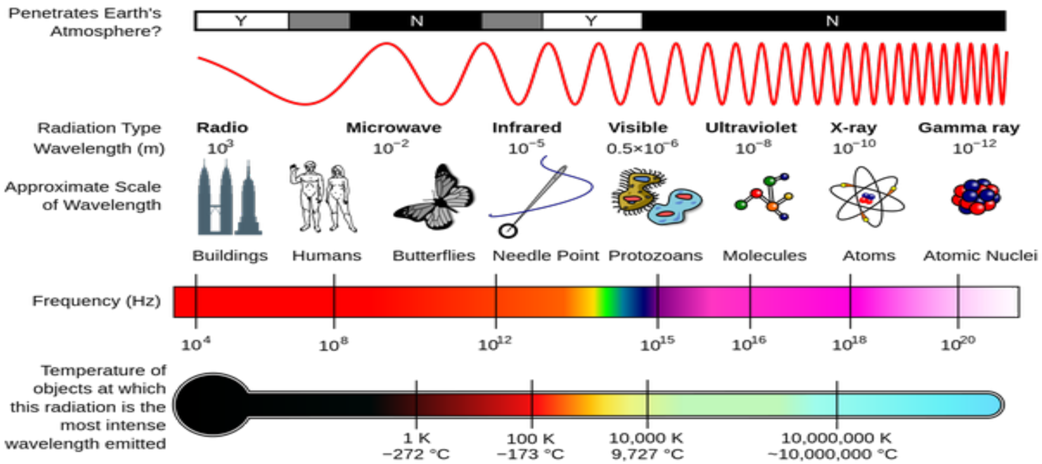
• temperature: T ~ $10^{6\ldots11}$ K (1 eV=10^4 K)

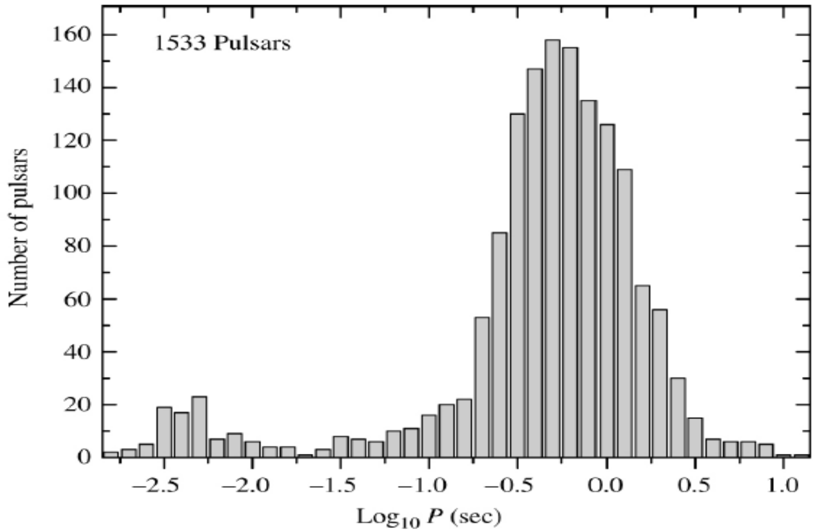




Pulsars are magnetized rotating neutron stars emitting a highly focused beam of electromagnetic radiation oriented long the magnetic axis.
The misalignment between the magnetic axis and the spin axis leads to a lighthouse effect: from Earth we see pulses

Since 1967, ~ 2500 pulsars have been discovered.
http://www.atnf.csiro.au/research/pulsar/psrcat/



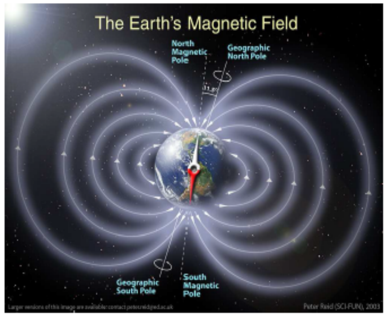


Most of them have been detected as radio pulsars, but also some observed in X-rays and an increasingly large number detected in gamma rays.

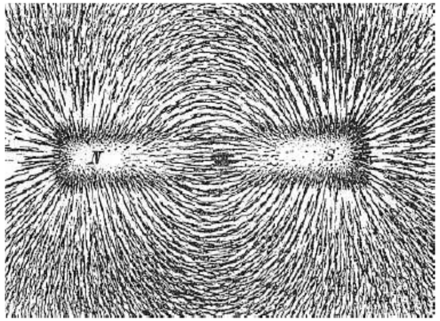
Their period P ranges from 1.396 ms for PSRJ1748−2446ad up to 8.5 s for PSR J2144−3933.

Magnetic fields

Earth $B \simeq 0.3 - 0.6$ G



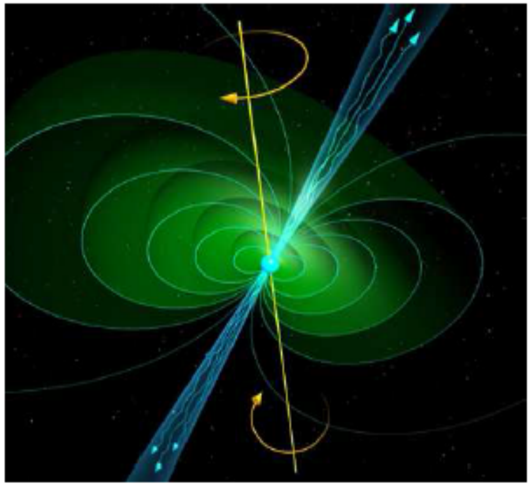
magnet $B \sim 10^3 - 10^4$ G



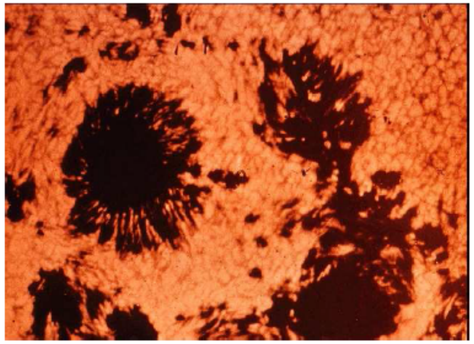
lab $B \simeq 4.5 \times 10^5$ G

strongest continuous field (Florida State University, USA)

Pulsars $B \sim 10^{12}$ G



Sun spot $B \sim 10^5$ G



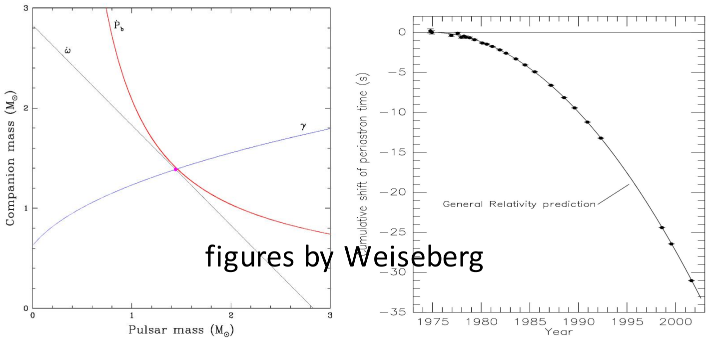
lab $B \simeq 2.8 \times 10^7$ G

strongest pulsed field (VNIIEF, Sarov, Russia)

Bulk properties: Masses

- > 2000 pulsars known

- best determined masses:
Hulse-Taylor pulsar
M=1.4414 ± 0.0002 $M_\odot$
Hulse-Taylor Nobel Prize 94

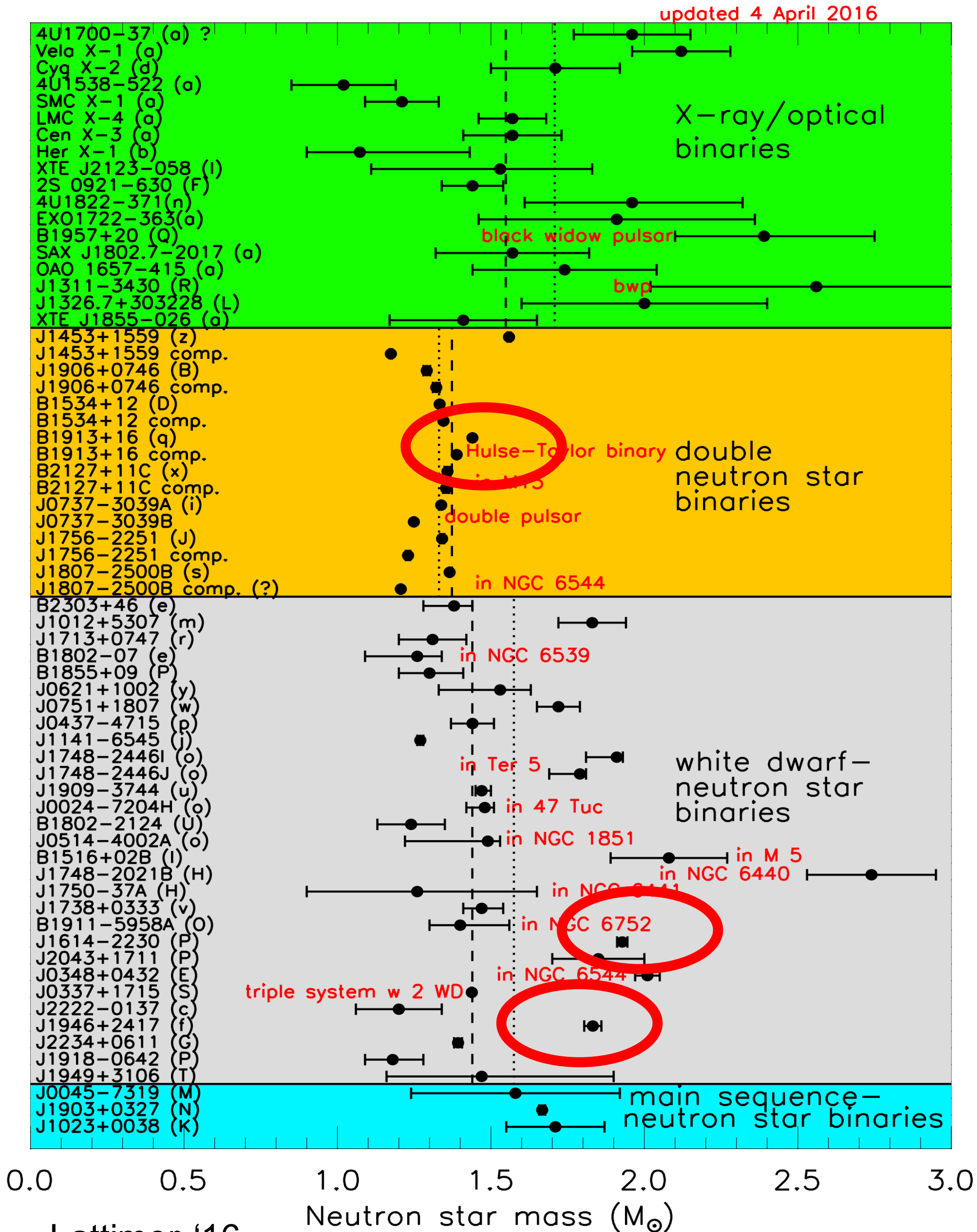


figures by Weiseberg

- PSR J1614-2230[1]
M=(1.97 ± 0.04) $M_\odot$;
PSR J0348+0432[2]
M=(2.01 ± 0.04) $M_\odot$

[1]Demorest et al '10;

[2]Antoniadis et al '13



Lattimer '16

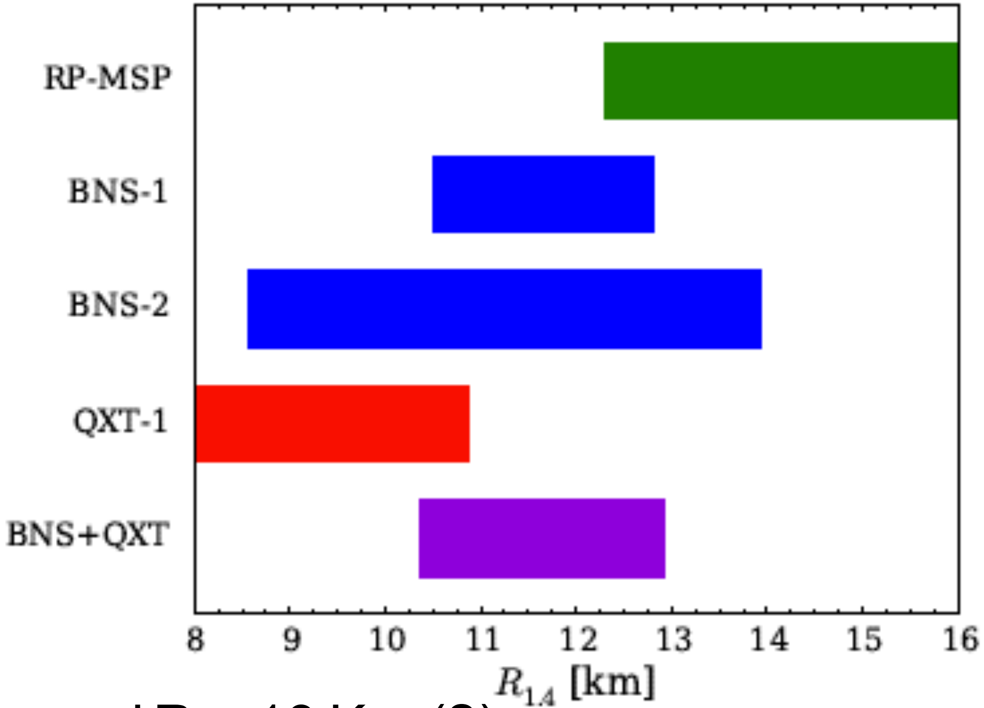
Bulk properties: Radius

analysis of X-ray spectra from neutron star (NS) atmosphere:

• RP-MSP: X-ray emission from radio millisecond pulsars
• BNS: X-burst from accreting NSs
• QXT: quiescent thermal emission of accreting NSs

theory + pulsar observations:
$R_{1.4M}$~11-13 Km Lattimer and Prakash '16

Fortin et al '15:
- RP-MSP: Bodganov '13
- BNS-1: Nattila et al '16
- BNS-2: Guver & Ozel '13
- QXT-1: Guillot & Rutledge '14
- BNS+QXT: Steiner et al '13



Some conclusions:
✓ marginally consistent analyses, favored R < 13 Km (?)
✓ future X-ray telescopes (NICER, eXTP) with precision for M-R of ~ 5%
✓ GW signals from NS mergers with precision for R of ~1 km
Bauswein and Janka '12; Lackey and Wade '15

adapted from Fortin's talk
@ NewCompstar Annual Meeting '16

Internal structure and composition: the Core

A. Watts et al. ‘15

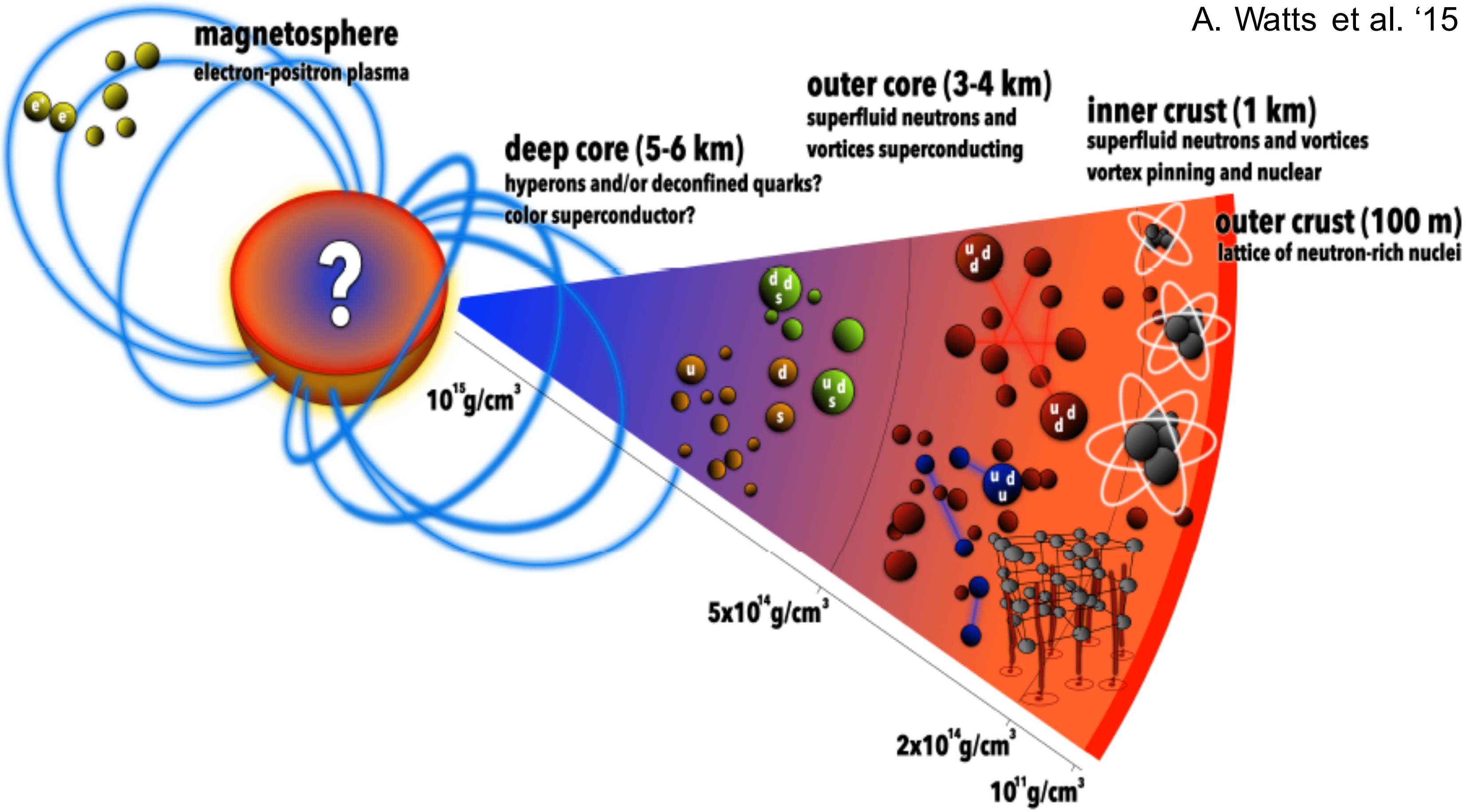


Figure 1: Schematic structure of a NS. The outer layer is a solid ionic crust supported by electron degeneracy pressure. Neutrons begin to leak out of nuclei at densities $\sim 4\times10^{11}$ g/cm^3 (the neutron drip line, which separates inner and outer crust), where neutron degeneracy also starts to play a role. At densities $\sim 2\times10^{14}$ g/cm^3, the crust-core boundary, nuclei dissolve completely. In the core, densities may reach up to ten times the nuclear saturation density $\rho_{\rm sat} = 2.8\times10^{14}$ g/cm^3 (the density in normal atomic nuclei).

- Atmosphere

few tens of cm, $\rho \leq 10^4$ g/cm^3 made of atoms

- Outer crust or envelope

few hundred m's, $\rho = 10^4$-$4 \cdot 10^{11}$ g/cm^3 made of free e^- and lattice of nuclei

- Inner crust

1-2 km, $\rho = 4 \cdot 10^{11}$-10^{14} g/cm^3 made of free e^-, neutrons and neutron-rich atomic nuclei

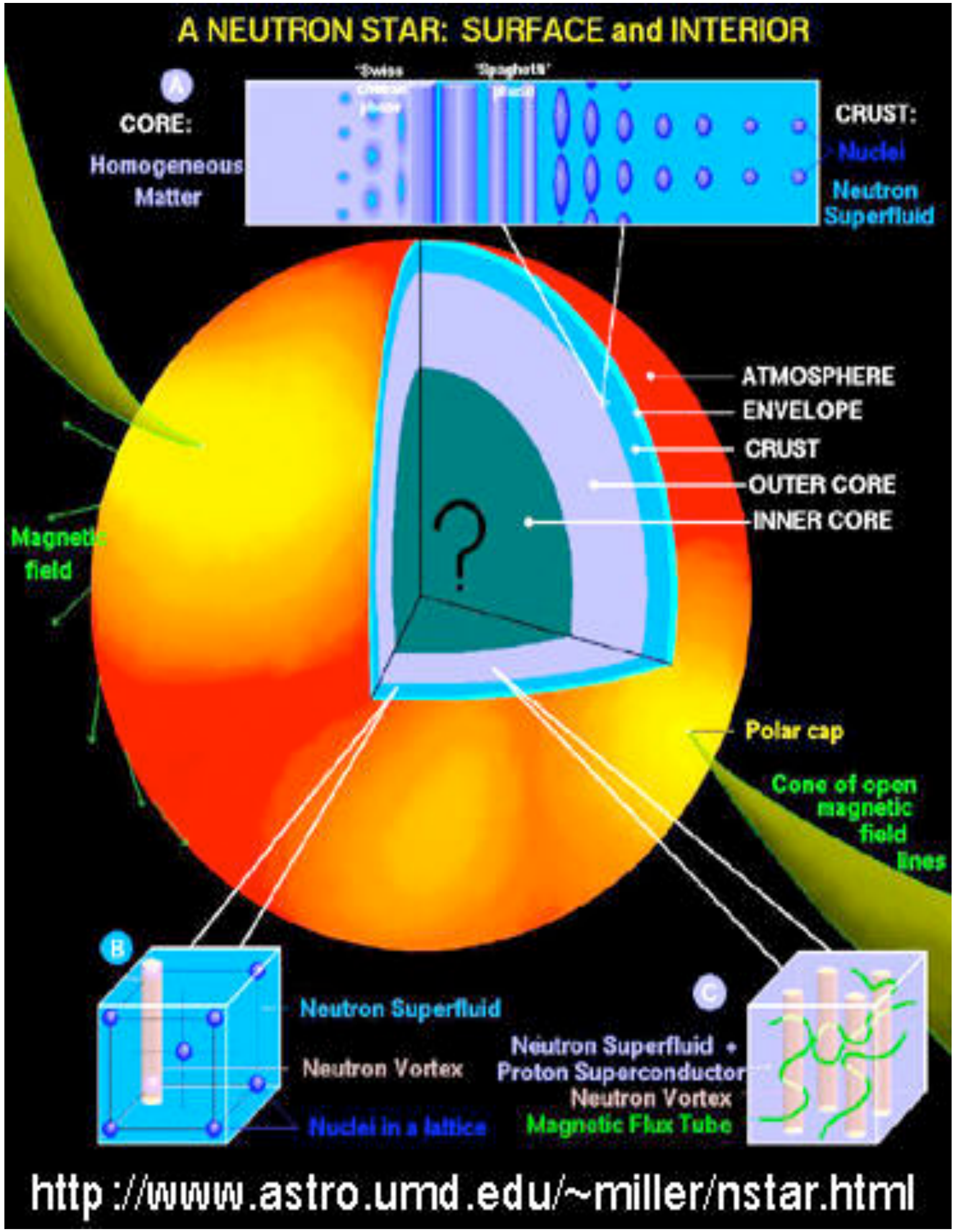
$\sim\rho_0/2$: uniform fluid of n,p,e^-

- Outer core

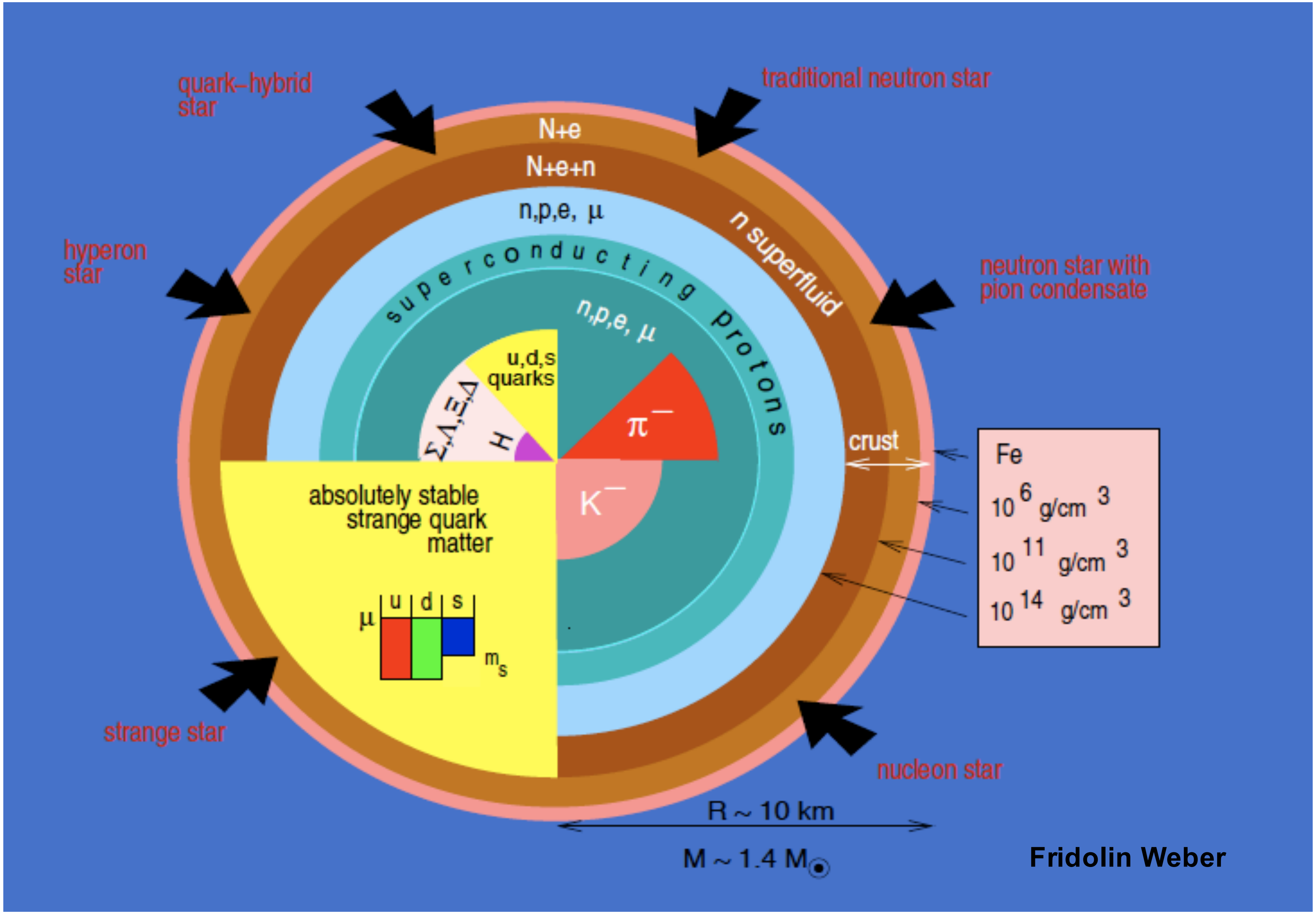
$\rho_0/2$-$2\rho_0$ is a soup of n,e^-,μ and possible neutron $^3P_2$ superfluid or proton $^1S_0$ superconductor

- Inner core (?)

2-10 ρ_0 with unknown interior made of hadronic, exotic or deconfined matter



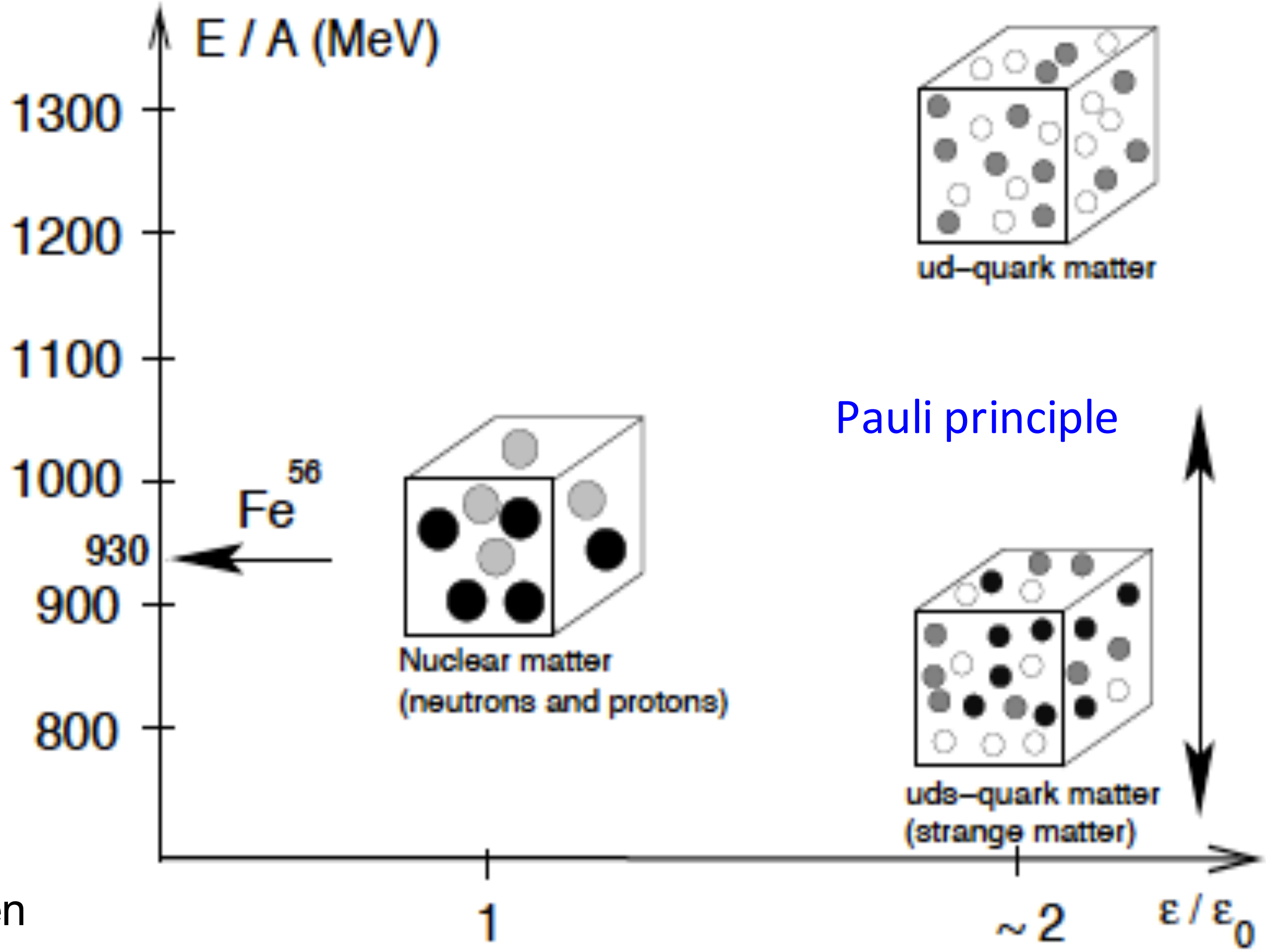
The Core of a Neutron Star



Strange Quark Matter Hypothesis

working hypothesis: strange quark matter constitutes the ground state of strong interaction rather than ^{56}Fe

Cristina Manuel's lecture



Bodmer,Witten
& Terazawa

Equation of State of dense matter: free Fermi gas

The Equation of State (EoS) is
the relation between the mass/energy density and pressure: P(n), P(E)

Extreme Conditions in the stellar interior!
* Atoms are ionized
* Particles can be degenerate and ultrarelativistic
* Pressure due to radiation can be significant

As a first approximation, let's consider the simplest thermodynamical system, the **ideal gas**.

The ideal gas consists of a large number of particles occupying quantum states whose energy is not affected by the interaction between the particles.

Depending on the conditions of the stellar interior the gas can be:

Classical or **Quantum (degenerate)**

and the energies of the particles can be:

Non-relativistic or **Relativistic**

Ideal Gas

Classical vs quantum (degenerate)

Distribution of the particles in the quantum states, $f(e_p)$

$$f(e_p) = \frac{1}{\mathrm{e}^{\frac{e_p - \mu}{kT}} \pm 1}$$

$$e_p^2 = m^2 c^4 + p^2 c^2$$

+1 ➔ Fermi-Dirac distribution

- 1 ➔ Bose-Einstein distribution

A gas is classical when the average occupation of any quantum state is small

$$f(e_p) = \frac{1}{\mathrm{e}^{\frac{e_p - \mu}{kT}} \pm 1} \simeq \mathrm{e}^{-\frac{e_p - \mu}{kT}} \ll 1$$

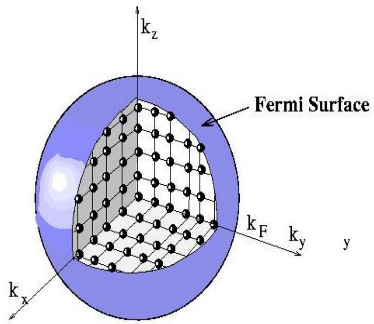
μ decreases or T increases

Relativistic vs non relativistic

Non-relativistic: $e_p = mc^2 + \frac{p^2}{2m} \quad v_p = \frac{p}{m}$

Ultra-relativistic: $e_p = pc \quad v_p = c$

General formulae



Number of (wave vector) states with magnitude between k and k+dk $g_s \frac{V}{(2\pi)^3} 4\pi k^2 dk$

Broglie relation (p= ħ/λ=hk) , number of momentum states between p and p+dp: $g(p)dp = g_s \frac{V}{h^3} 4\pi p^2 dp$

degeneracy factor

Internal energy: $$E = \int_0^\infty e_p f(e_p) g(p) dp$$

Number of particles: $$N = \int_0^\infty f(e_p) g(p) dp$$

Pressure:

dE=T dS- P dV + μ dN

$$P = -\left(\frac{\partial E}{\partial V}\right)_{S,N} = -\int_0^\infty \frac{de_p}{dV} f(e_p)\, g(p)\, dp$$

$$\frac{de_p}{dV} = \frac{de_p}{dp}\frac{dp}{dV}$$

$$\frac{de_p}{dp} = \frac{pc^2}{e_p} = v_p$$ (definition)

$$p \propto V^{-1/3} \rightarrow \frac{dp}{dV} = -\frac{p}{3V}$$

➔ $$P = \frac{1}{3V}\int_0^\infty p v_p f(e_p) g(p) dp = \frac{N}{3V}\langle p v_p \rangle$$

$$P = \frac{1}{3V}\int_0^{\infty} p v_p f(e_p) g(p) dp = \frac{N}{3V}\langle p v_p \rangle$$

Two kinematical limits:

Non-relativistic: $e_p = mc^2 + \frac{p^2}{2m} \quad v_p = \frac{p}{m}$

$$P = \frac{2N}{3V}\langle \frac{p^2}{2m} \rangle = \frac{2}{3}$$ of kinetic energy density

Ultra-relativistic: $e_p = pc \quad v_p = c$

$$P = \frac{N}{3V}\langle pc \rangle = \frac{1}{3}$$ of kinetic energy density

Pressure is “smaller” in ultrarelativistic gases!!

Up to here, the expressions found are applicable to an ideal gas in its most general form: for a dilute classical gas or for a dense gas where quantum effects are important

Ideal Classical (Diluted) Gas $f(e_p) = \frac{1}{e^{\frac{e_p-\mu}{kT}} \pm 1} \simeq e^{-\frac{e_p-\mu}{kT}} \ll 1$

$$P = \frac{kT}{V} e^{\frac{\mu}{kT}} \int_0^\infty e^{-\frac{e_p}{kT}} g_s \frac{V}{h^3} 4\pi p^2 dp$$

(integrating by parts)

$$N = e^{\frac{\mu}{kT}} \int_0^\infty e^{-\frac{e_p}{kT}} g_s \frac{V}{h^3} 4\pi p^2 dp$$

$$P = \frac{N}{V} kT = \rho kT$$

Comparing expressions for P

non-relativistic: $\langle \frac{p^2}{2m} \rangle = \frac{3}{2} kT$

ultra-relativistic: $\langle pc \rangle = 3kT$

(average kinetic energy per particle)

A more intuitive way of determining if a gas is classical (diluted) is by checking whether the interparticle separation is much larger that the typical de Broglie wave length λ=h/p, which is determined by

non-relativistic: $$\boxed{\lambda = \frac{h}{p} \sim \frac{h}{\sqrt{mkT}} \to \rho \ll \left(\frac{mkT}{h^2}\right)^{3/2}}$$

ultra-relativistic: $$\lambda = \frac{h}{p} \sim \frac{hc}{kT} \to \rho \ll \left(\frac{kT}{hc}\right)^3$$

Gases formed by particles with mass, such as electrons and ions, can behave classically or quantum mechanically, depending on their density

Free Fermi Gas

When the concentration of fermions becomes large, the interparticle separation may be comparable with the de Broglie wavelength.
In particular, the high density requirement for a non-relativistic quantum gas

$$\rho \gg \left(\frac{mkT}{h^2}\right)^{3/2}$$

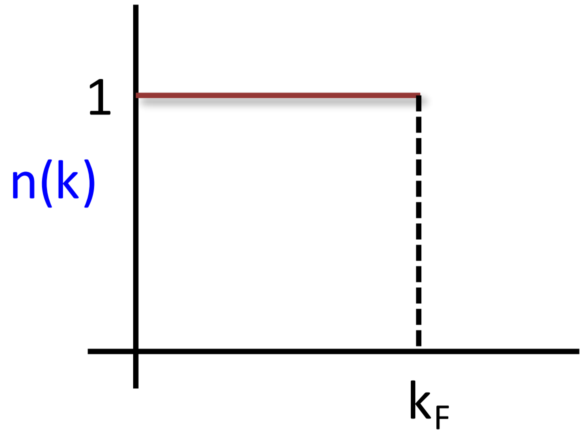
can be viewed as a low temperature requirement $kT \ll \frac{h^2 \rho^{2/3}}{m}$

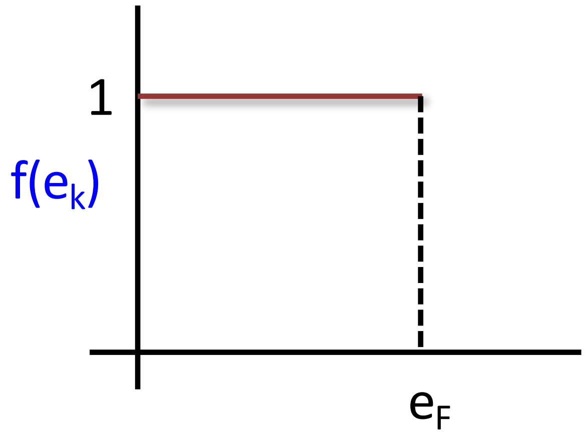
Therefore a quantum gas is a cold gas but the *coldness* is set by the density.
Temperatures of 10^6 K can be low if the densities are very high!

Let's consider then a **cold gas of fermions** (electrons or neutrons at T=0)

The ground state is referred to as the free Fermi sea
The fermions have fallen into quantum states with the lowest possible energy ➔ Pauli principle!
They are distributed so that each quantum state is occupied up to a certain energy (Fermi level, e_F) and quantum states above this level are empty.

This distribution is the T=0 limit of the Fermi-Dirac distribution (setting $\mu=e_F$)





e_F (the energy of the most energetic particle) is called the Fermi energy and k_F is the Fermi wave-number ($p_F = \hbar k_F$ is the Fermi momentum): $e_F=e(p_F)$

Relation between the Fermi momentum and the density:

$$N = g_s \sum_{\vec{k}} \theta(k_F - |\vec{k}|) = g_s \frac{V}{(2\pi)^3} \int d^3k \theta(k_F - |\vec{k}|) = g_s \frac{V}{(2\pi)^3} \frac{4}{3} \pi k_F^3$$

$$\rightarrow \frac{N}{V} = \rho = g_s \frac{k_F^3}{6\pi^2} \qquad k_F = \left(\frac{6\pi^2\rho}{g_s}\right)^{1/3} \rightarrow \lambda_F = \frac{2\pi}{k_F} = 2\pi \left(\frac{g_s}{6\pi^2\rho}\right)^{1/3}$$

Note that de Broglie wave-length of the most energetic particle is comparable with $\rho^{-1/3}$, that is of the order of the average distance between fermions

Equation of state for a non-relativistic free Fermi gas

Non relativistic case: $p_F c = \hbar k_F c << m c^2$

$$k_F = \left(\frac{6\pi^2\rho}{g_s}\right)^{1/3} \ll \frac{mc}{\hbar} \to \rho \ll \frac{g_s}{6\pi^2} 8\pi^3 \left(\frac{mc}{h}\right)^3$$

$\frac{h}{mc}$ (neutrons) ~ 10^{-16} m
(electrons) ~ 10^{-12} m

neutrons are still not relativistic when electrons are!

The Compton wavelength of a particle is equivalent to the wavelength of a photon whose energy is the same as the rest-mass energy of the particle

Internal energy:

$$e_p = mc^2 + \frac{p^2}{2m}$$

$$E = \int_0^{k_F} \left(mc^2 + \frac{\hbar^2 k^2}{2m}\right) g_s \frac{V}{(2\pi)^3} 4\pi k^2 dk = Nmc^2 + g_s \frac{V}{(2\pi)^3} \frac{4}{3}\pi k_F^3 \frac{\hbar^2 k_F^2}{2m} \frac{3}{5}$$

$$= N\left(mc^2 + \frac{3}{5}\frac{\hbar^2 k_F^2}{2m}\right)$$

$E = Ne(\rho)$ with

$$\boxed{e(\rho) = mc^2 + \frac{3}{5}\frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g_s}\right)^{2/3} \rho^{2/3}}$$

From internal energy, we can obtain the pressure and chemical potential

$$P = -\left(\frac{\partial E}{\partial V}\right)_N = -\left(\frac{\partial(E/N)}{\partial(V/N)}\right)_N = -\left(\frac{\partial e(\rho)}{\partial(1/\rho)}\right)_N = \rho^2\frac{\partial e(\rho)}{\partial\rho}$$

$$\mu = \left(\frac{\partial E}{\partial N}\right)_V = \left(\frac{\partial N e(\rho)}{\partial N}\right)_V = e(\rho) + N\frac{\partial e(\rho)}{\partial\rho}\left(\frac{\partial\rho}{\partial N}\right)_V = e(\rho) + \rho\frac{\partial e(\rho)}{\partial\rho} \rightarrow \mu = e(\rho) + \frac{P(\rho)}{\rho}$$

so we finally obtain for **internal energy and pressure**

$$\boxed{\begin{aligned} e(\rho) &= mc^2 + \frac{3}{5}\frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g_s}\right)^{2/3}\rho^{2/3} \\ P &= \rho^2\frac{\partial e(\rho)}{\partial\rho} = \frac{2}{5}\frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g_s}\right)^{2/3}\rho^{5/3} = C_{nr}\rho^{5/3} \end{aligned}}$$

$$\mu = mc^2 + \frac{3}{5}\frac{\hbar^2 k_F^2}{2m} + \frac{2}{5}\frac{\hbar^2 k_F^2}{2m} = e(k_F)$$

For ultrarelativistic free Fermi gas, the EoS becomes less stiff (do it!)

$$e(\rho) = \frac{3}{4}\hbar c\left(\frac{6\pi^2}{g_s}\right)^{1/3}\rho^{1/3}$$

$$P = \rho^2\frac{\partial e(\rho)}{\partial\rho} = \frac{1}{4}\hbar c\left(\frac{6\pi^2}{g_s}\right)^{1/3}\rho^{4/3} = C_{ur}\rho^{4/3}$$

Baryonic matter in the core

A Fermi gas model for only neutrons inside neutron stars is unrealistic:

1. real neutron star consists not just of neutrons, but contains a small fraction of protons and electrons - to inhibit the neutrons from decaying into protons and electrons by their weak interactions!

2. the Fermi gas model ignores nuclear interactions, which give important contributions to the energy density

3. more exotic degrees of freedom are expected, in particular hyperons, due to the high value of density at the center and the small mass difference between nucleons and hyperons

Hyperon	Quarks	$I(J^P)$	Mass (MeV)
Λ	uds	$0(1/2^+)$	1115
Σ^+	uus	$1(1/2^+)$	1189
Σ^0	uds	$1(1/2^+)$	1193
Σ^-	dds	$1(1/2^+)$	1197
Ξ^0	uss	$1/2(1/2^+)$	1315
Ξ^-	dss	$1/2(1/2^+)$	1321
Ω^-	sss	$0(3/2^+)$	1672

1. Neutrons, protons and electrons in β-equilibrium

The composition of neutron star matter is found by demanding equilibrium against weak interaction processes (β-stability). Therefore, the reaction for the decay of a free neutron: $n \rightarrow p\, e^-\, \nu_e$

(responsible for the free neutron lifetime of 15 minutes) is halted in neutron star matter by the presence of protons and electrons. Protons and neutrons in their lowest levels of their corresponding Fermi seas are occupied and the reaction is Pauli blocked.
In this regime the decay reaction is equilibrated with the electron capture one: $n \rightarrow p\, e^-\, \nu_e$ $\quad p\, e^- \rightarrow n\, \nu_e$

-This equilibrium can be expressed in terms of the chemical potentials.
Since the mean free path of the ν_e is >> 10 km, they freely escape

$$\mu_n = \mu_p + \mu_e$$

- Charge neutrality is also ensured by demanding $\rho_p = \rho_e$, i.e. $k_{Fp} = k_{Fe}$

$$\rho_p = \rho_e$$

Note that baryon number is conserved too: $\rho = \rho_n + \rho_p$

2. Nuclear interactions

the EoS can only fullfil known properties of nuclear matter and nuclei if nuclear interactions are considered

NN: more than 4000 data for E_{lab}<350 MeV

- saturation density ρ

- binding energy per nucleon $e(\rho)$

- nuclear compressibility $K = 9\partial P/\partial\rho$

- symetry energy

$$\rho_0 = 0.16 \pm 0.02\ \mathrm{fm}^{-3}$$
$$e(\rho_0) = -15.6 \pm 0.2\ \mathrm{MeV}$$
$$K(\rho_0) \approx 220 - 250\ \mathrm{MeV}$$
$$S(\rho_0) = 32.5\ \mathrm{MeV}$$

$$S(\rho) = e(\rho, x_p = 0) - e(\rho, x_p = 1/2)$$

$$x_p = \rho_p/\rho$$

$$e(\rho, x_p) = e(\rho, x_p = 1/2) + \frac{1}{2}\left(\frac{d^2 e}{dx_p^2}(\rho)\right)(x_p - 1/2)^2 + \cdots \simeq e(\rho, x_p = 1/2) + 4S(\rho)(x_p - 1/2)^2$$

- also causality imposes that speed of sound should be smaller than speed of light

$$(v_s/c)^2 = dP/d(e\rho)$$

One may find in the literature many EoS's obtained with different approaches (phenomenologial, microscopic) using various types of interactions (non-relativistic, relativistic, effective theories, meson-exchange)

We employ the simple parametrization of the nucleonic energy per baryon from
H. Heiselberg, M. Hjorth-Jensen, Phys. Reports 328 (2000) 237

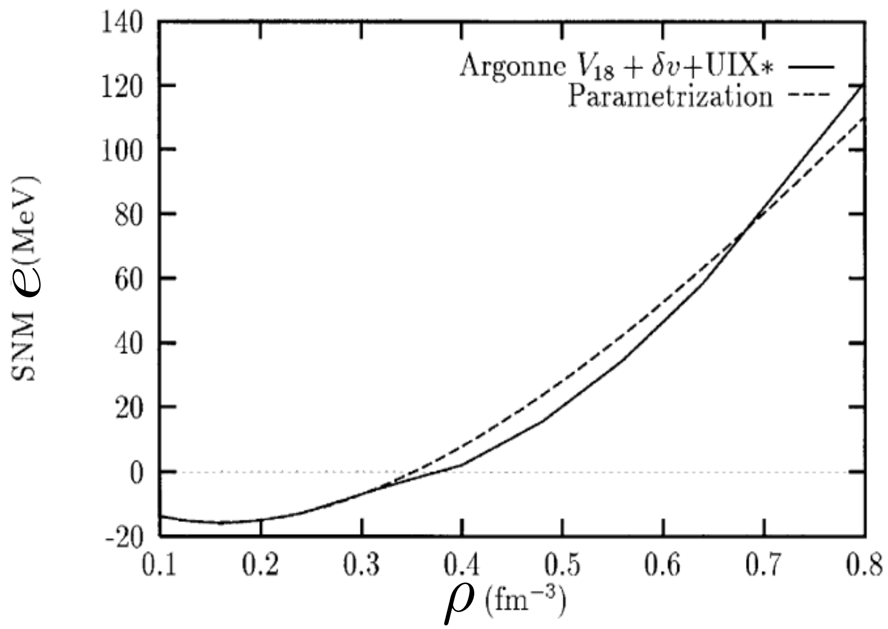
$$e(\rho, x_p) = e_0\, u \frac{u - 2 - \delta}{1 + u\delta} + S_0\, u^\gamma\, (1 - 2x_p)^2$$

$$u = \rho/\rho_0 \qquad x_p = \rho_p/\rho$$

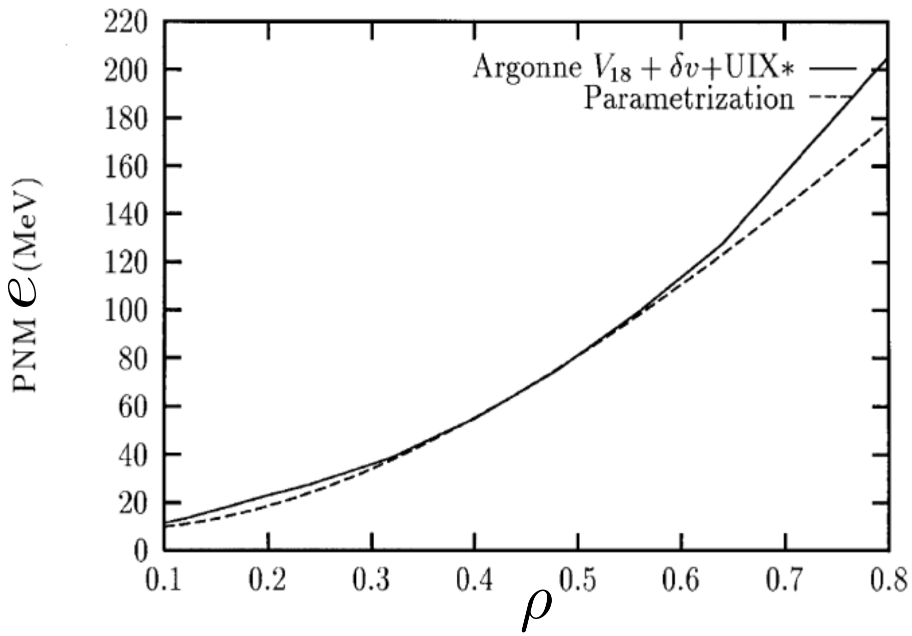
(fitted to reproduce saturation density, binding energy and incompressibility modulus at saturation density)

e_0=15.8 MeV, S_0=32 MeV, ϒ=0.6, δ=0.2

Symmetric nuclear matter



Pure neutron matter



β-stable matter with neutrons, protons and electrons

$\mu_n = \mu_p + \mu_e$ $\quad$ $\mu_e = \hbar c(3\pi\rho x_p)^{1/3}$

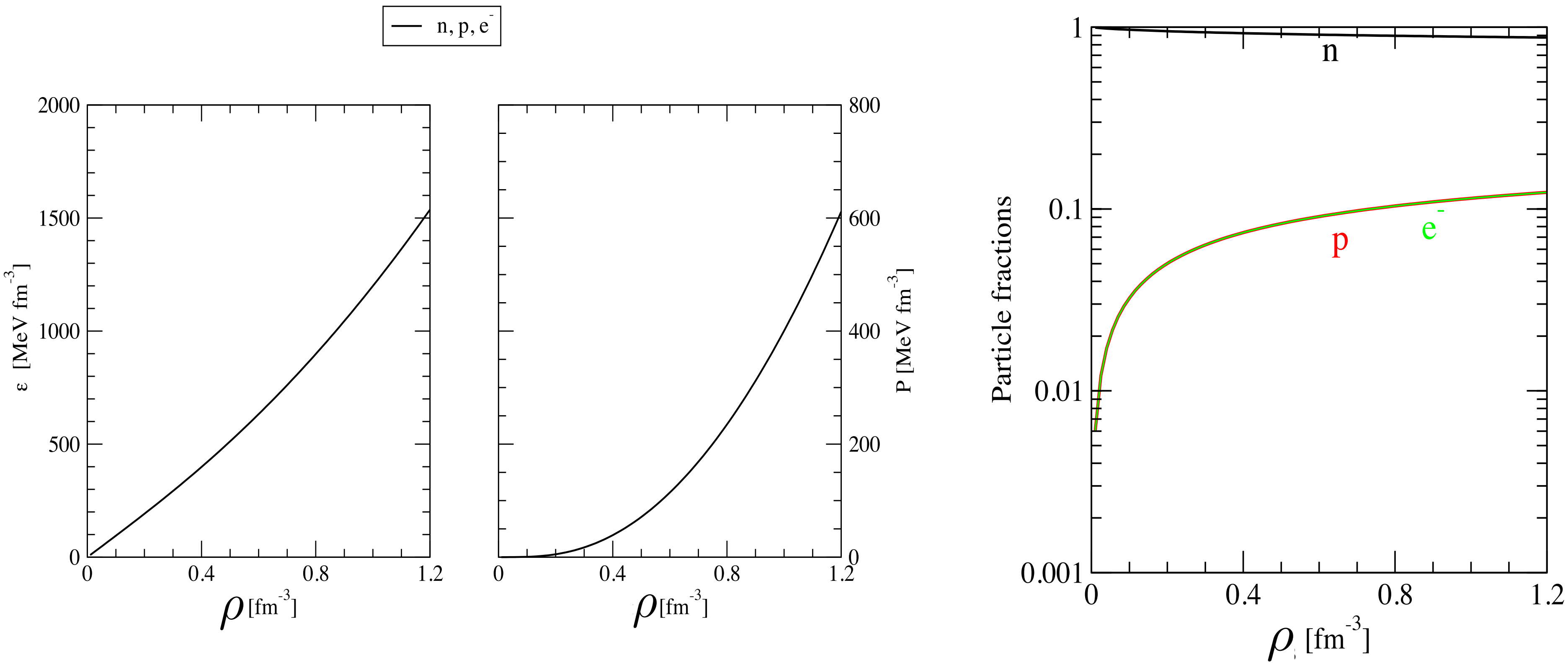
$\rho_p = \rho_e$ $\quad$ (ultrarelativistic)

$$\boxed{\hbar c(3\pi^2\rho x_p)^{1/3} = 4S(\rho)(1-2x_p)^2}$$ (do it!)

This particular model gives and explicit expression for $S(\rho)=S_0\,(\rho/\rho_0)^\gamma$

$$\rho x_p = \frac{(4S_0 u^\gamma(1-2x_p))^3}{3\pi^2}$$

This gives rise to a third degree equation that determines x_p for each $u=\rho/\rho_0$. In fact, there is an analytical solution.



3. Hyperons might be present

Hyperon	Quarks	I(J^P)	Mass (MeV)
Λ	uds	0(1/2$^+$)	1115
Σ^+	uus	1(1/2$^+$)	1189
Σ^0	uds	1(1/2$^+$)	1193
Σ^-	dds	1(1/2$^+$)	1197
Ξ^0	uss	1/2(1/2$^+$)	1315
Ξ^-	dss	1/2(1/2$^+$)	1321
Ω^-	sss	0(3/2$^+$)	1672

First proposed in 1960 by Ambartsumyan & Saakyan

Traditionally neutron stars were modeled by a uniform fluid of neutron rich nuclear matter in equilibrium with respect to weak interactions (β-stable matter)

$$\left.\begin{array}{l} n \rightarrow p + e^- + \bar{\nu}_{e^-} \\ p + e^- \rightarrow n + \nu_{e^-} \end{array}\right\} \longrightarrow \mu_p = \mu_n - \mu_{e^-}$$

but more exotic degrees of freedom are expected, in particular hyperons, due to:

- high value of density at the center and

- the rapid increase of the nucleon chemical potential with density

Hyperons are expected at $\rho \sim (2\text{-}3)\rho_0$

β-stable hyperonic matter

- Equilibrium with respect to weak interactions

$$n \leftrightarrow p\, e^- \, \bar{\nu}_e$$

$$(\mu_n = \mu_p + \mu_{e^-})$$

$$n\, n \leftrightarrow p\, \Sigma^- \quad (\text{or} \;\; e^-\, n \leftrightarrow \Sigma^-\, \nu_e)$$

$$(\mu_{e^-} + \mu_n = \mu_{\Sigma^-})$$

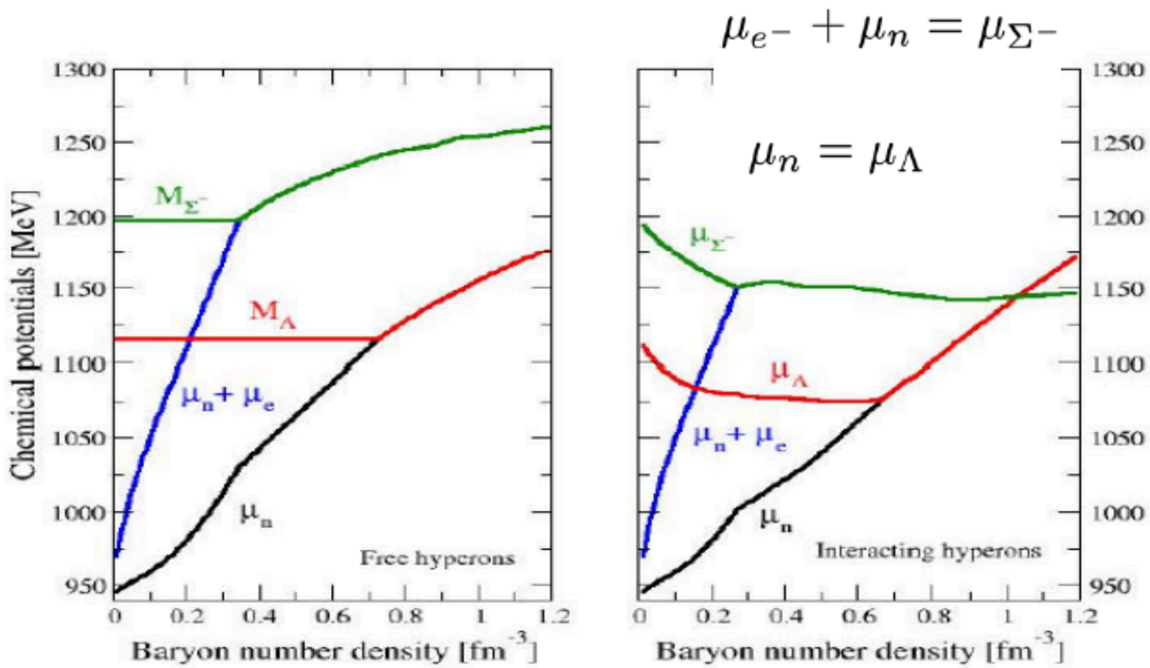
$$n\, n \leftrightarrow n\, \Lambda \quad (\text{or} \;\; e^-\, p \leftrightarrow \Lambda\, \nu_e)$$

$$(\mu_n = \mu_\Lambda)$$

- Charge neutrality

$$n_p + n_{\Sigma^+} = n_{e^-} + n_{\mu^-} + n_{\Sigma^-} + n_{\Xi^-}$$

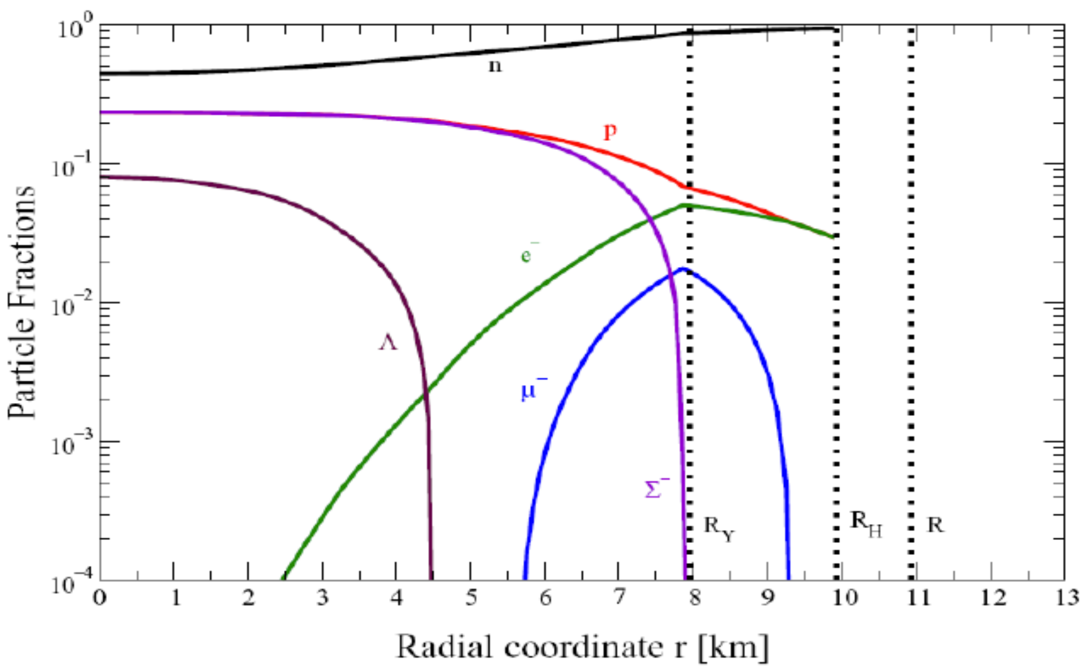
Baryochemical potentials in hyperonic matter: the composition of neutron stars depends on hyperons properties in matter



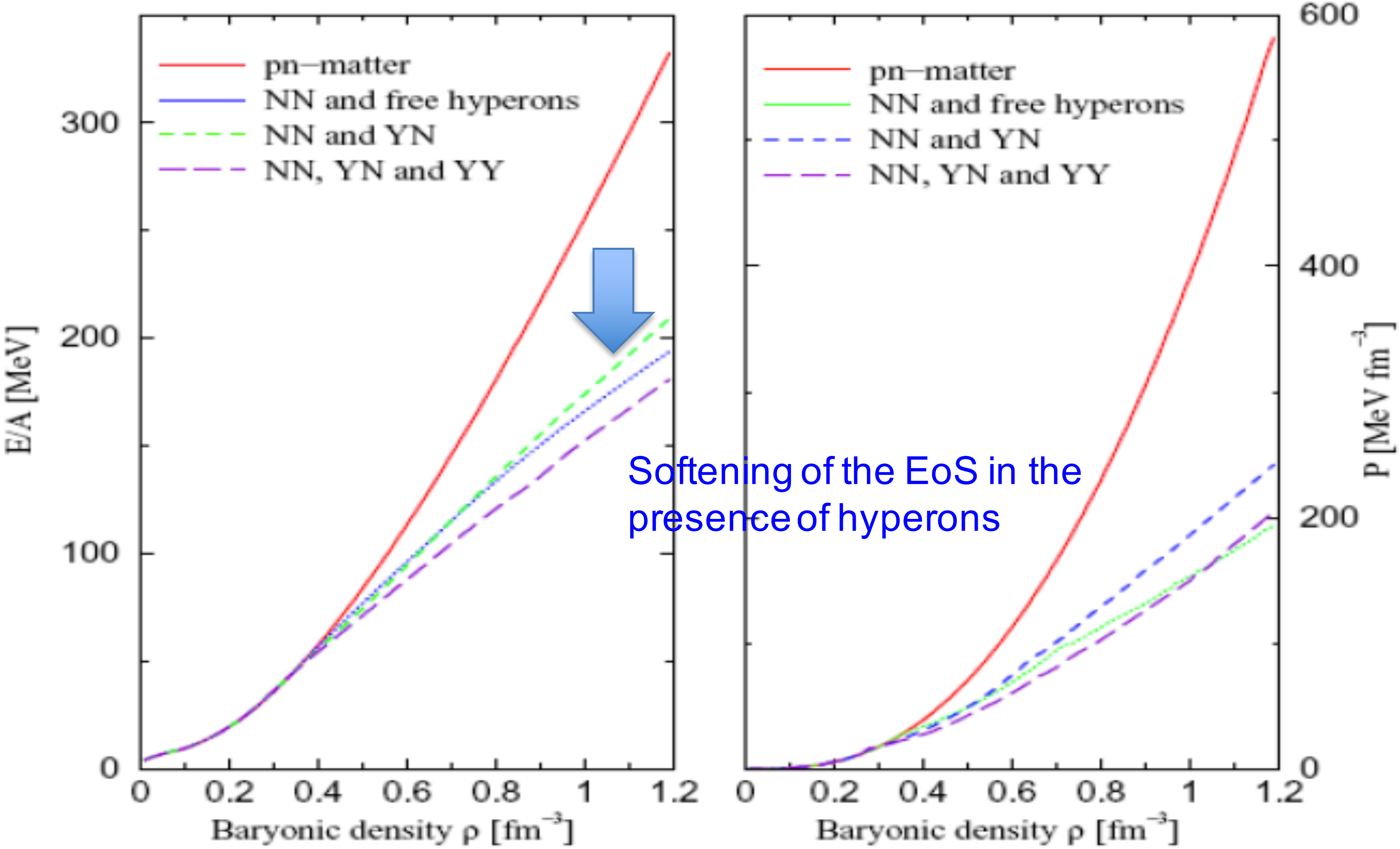
Vidana PhD thesis '01

Profile of a neutron star with hyperons

Vidana, Polls, Ramos, Engvik & Hjorth-Jensen, PRC 62 (2000) 035801



Equation of State of Hyperonic Matter



Vidana, Polls, Ramos, Engvik & Hjorth-Jensen, PRC 62 (2000) 035801

Structure Equations for neutron stars

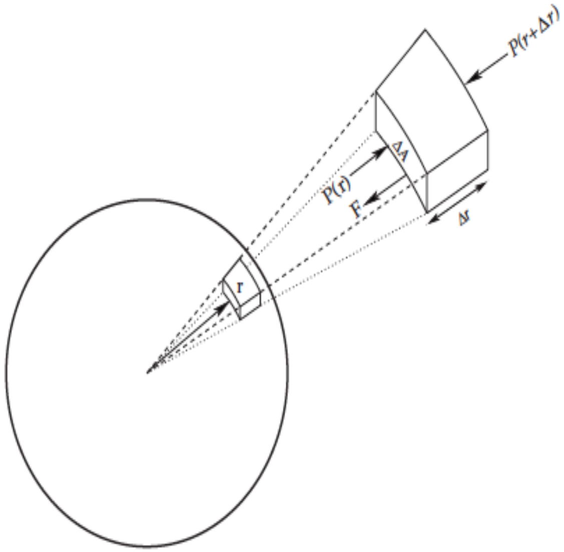


Figure 1. The radial force acting on a small mass element a distance r from the centre of the star.

$$F_r = -\frac{GM(r)\Delta m}{r^2} - P(r+\Delta r)\Delta A + P(r)\Delta A = \Delta m \frac{d^2 r}{dt^2}$$

dividing by
ΔV = ΔA Δr ➔ $$-\frac{GM(r)\rho(r)}{r^2} - \frac{dP}{dr} = \rho(r)\frac{d^2 r}{dt^2} = 0$$

hydrostatic equilibrium $(\ddot{r} = \dot{r} \equiv 0)$

$\rho(r)$: matter density!!!!

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}, \qquad P(r=0) \equiv P_c;$$

$$\frac{dM}{dr} = +4\pi r^2 \rho(r), \qquad M(r=0) \equiv 0,$$

Newtonian formulation

General Relativity Corrections

Since neutron stars have masses M ~1-2 $M_\odot$ and radii R ~ 10-20 Km, the value of the gravitational potential on the neutron star surface is ~ 1

$$\frac{\frac{GM^2}{R}}{Mc^2} \sim 1$$

gravitational binding energy

gravitational mass

with escape velocities of order c/2

$$\frac{1}{2}mv_{sc}^2 - \frac{GMm}{R} = 0 \to v_{sc} \sim c/2$$

Therefore, general relativistic effects become very important!!!

We have to solve Einstein's field equations, $G^{\mu\nu}$, with the energy-density tensor of the stellar matter, $T^{\mu\nu}(\varepsilon,P(\varepsilon))$:

$$G^{\mu\nu} = 8\pi T^{\mu\nu}(\epsilon, P(\epsilon))$$

$$\epsilon = \rho c^2$$

For spherically symmetric non-rotating star, the Einstein's equations reduce to the Tolman-Oppenheimer-Volkoff (TOV) equations:

$$\frac{dP}{dr} = -\frac{Gm\epsilon}{c^2r^2}\left(1+\frac{P}{\epsilon}\right)\left(1+\frac{4\pi r^3 P}{c^2 m}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1}$$

$$\frac{dm}{dr} = \frac{4\pi r^2 \epsilon}{c^2}$$

$$P(r=0) = P(\epsilon_c) \qquad m(r=0) = 0$$

$$P(r=R) = 0 \qquad m(r=R) = M$$

R/M constraints

for $M=1.4\,M_\odot \rightarrow GM/c^2 \sim 2$ km

The radius R of a star with a given mass M must fulfill some constraints coming from:

1) General relativity arguments (Neutron stars are not black holes)

$$R > \frac{2GM}{c^2}$$

2) Compressibility (stability) of matter: $dP/d\rho > 0$ (from TOV equations)

$$R > \frac{9}{4}\frac{GM}{c^2}$$

3) Causality constraint (sound speed must be smaller than the speed of light)

$$R > 2.9\frac{GM}{c^2}$$

4) Rotation must not pull the star apart (the centrifugal force for a particle on the surface cannot exceed the gravitational force)

$$\nu < \nu_K = \frac{1}{2\pi}\sqrt{\frac{GM}{R^3}} \qquad R < \left(\frac{GM}{2\pi}\right)^{1/3}\frac{1}{\nu^{2/3}}$$

"Recipe" for neutron star structure calculation

- Energy density $\epsilon(\rho, x_e, x_p, x_\Lambda, ..); x_i = \dfrac{\rho_i}{\rho}$
- Chemical potentials $\mu_i = \dfrac{\partial \epsilon}{\partial \rho_i}$
- β equilibrium and charge neutrality $\mu_i = b_i \mu_n - q_i \mu_e$, $\sum_i x_i q_i = 0$
- Composition and EoS $x_i(\rho)\ ;\ P(\rho) = \rho^2 \dfrac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$
- TOV equations

$$\frac{dP}{dr} = -\frac{Gm\epsilon}{c^2 r^2}\left(1+\frac{P}{\epsilon}\right)\left(1+\frac{4\pi r^3 P}{c^2 m}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1}$$

$$\frac{dm}{dr} = \frac{4\pi r^2 \epsilon}{c^2} \qquad m(r=0)=0 \quad P(r=0)=P(\epsilon_c)$$

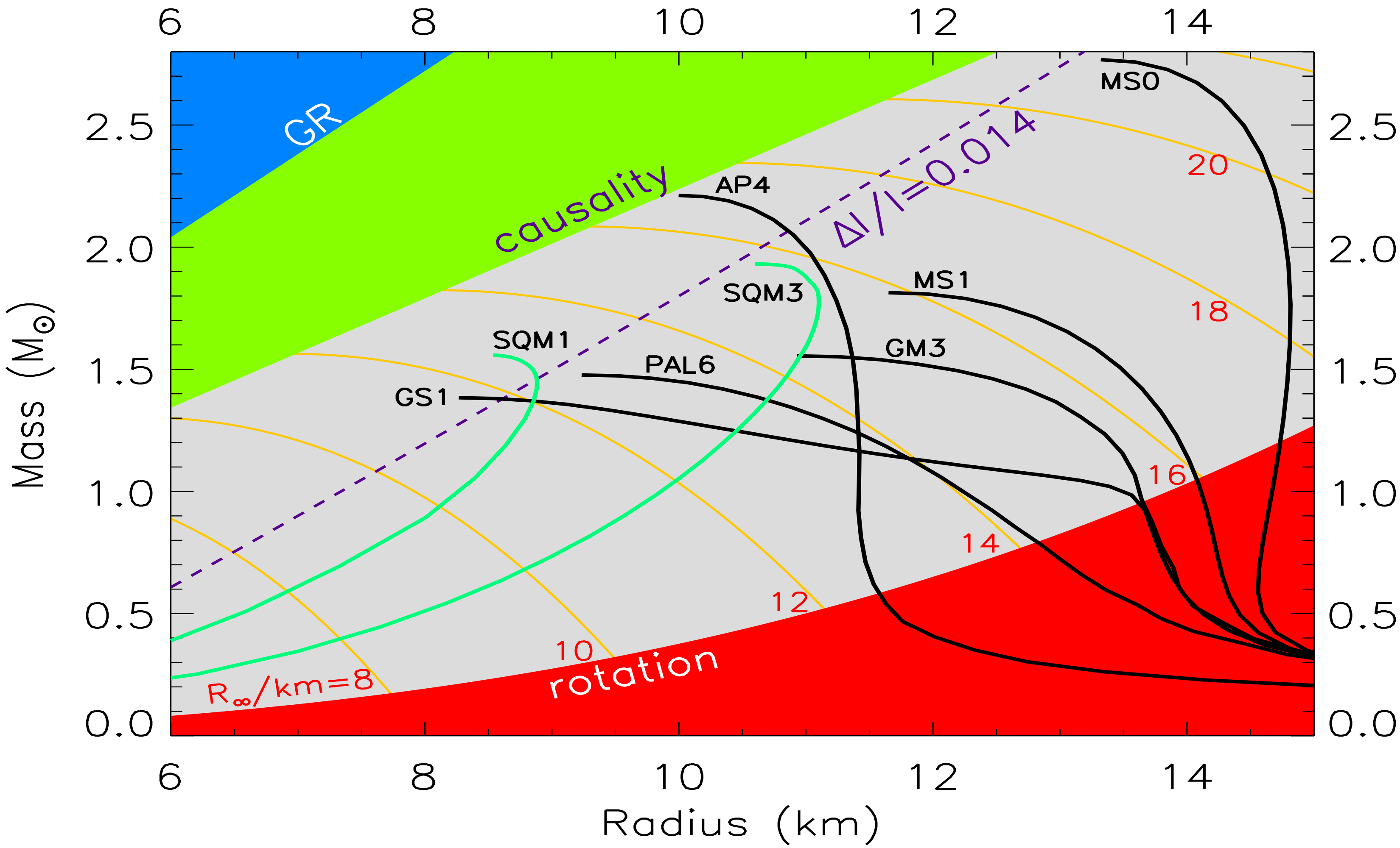
$$m(r=R)=M \qquad P(r=R)=0$$

- Structure of the neutron star $\rho(r), M(R), ..$

Shulze@Compstar07

Mass-Radius relation

M-R diagram for various EoS, showing also constrained areas



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Demorest et al. , Nature 467 (2010) 1081

PSR J1614−2230

doi:10.1038/nature09466

A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]

Neutron stars are composed of the densest form of matter known to exist in our Universe, the composition and properties of which are still theoretically uncertain. Measurements of the masses or radii of these objects can strongly constrain the neutron star matter equation of state and rule out theoretical models of their composition[1,2]. The observed range of neutron star masses, however, has hitherto been too narrow to rule out many predictions of 'exotic' non-nucleonic components[3–6]. The Shapiro delay is a general-relativistic increase in light travel time through the curved space-time near a massive body[7]. For highly inclined (nearly edge-on) binary millisecond radio pulsar systems, this effect allows us to infer the masses of both the neutron star and its binary companion to high precision[8,9]. Here we present radio timing observations of the binary millisecond pulsar J1614-2230[10,11] that show a strong Shapiro delay signature. We calculate the pulsar mass to be $\boxed{(1.97 \pm 0.04) M_{\odot}}$, which rules out almost all currently proposed[2–5] hyperon or boson condensate equations of state ($M_{\odot}$, solar mass). Quark matter can support a star this massive only if the quarks are strongly interacting and are therefore not 'free' quarks[12].

long-term data set, parameter covariance and dispersion measure variation can be found in Supplementary Information.

As shown in Fig. 1, the Shapiro delay was detected in our data with extremely high significance, and must be included to model the arrival times of the radio pulses correctly. However, estimating parameter values and uncertainties can be difficult owing to the high covariance between many orbital timing model terms[14]. Furthermore, the χ^2 surfaces for the Shapiro-derived companion mass (M_2) and inclination angle (i) are often significantly curved or otherwise non-Gaussian[15]. To obtain robust error estimates, we used a Markov chain Monte Carlo (MCMC) approach to explore the post-fit χ^2 space and derive posterior probability distributions for all timing model parameters (Fig. 2). Our final results for the model

Table 1 | Physical parameters for PSR J1614-2230

Parameter	Value
Ecliptic longitude (λ)	245.78827556(5)°
Ecliptic latitude (β)	−1.256744(2)°
Proper motion in λ	9.79(7) mas yr^{-1}
Proper motion in β	−30(3) mas yr^{-1}
Parallax	0.5(6) mas

Anaylsis improved recently: M = 1.928(7) $M_{\odot}$ (Arzoumanian et al. 2015)

Antoniadis et al., Science 340, 6131 (2013)

J0348+0432

‹ Prev | Table of Contents | Next ›

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RESEARCH ARTICLE

A Massive Pulsar in a Compact Relativistic Binary

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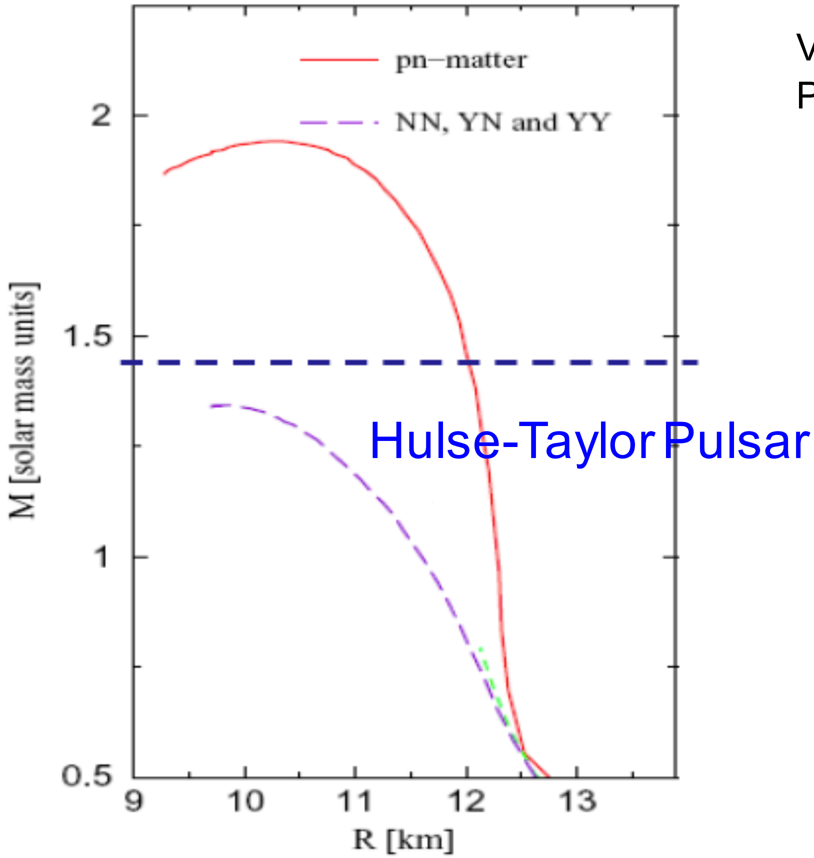
ABSTRACT | STRUCTURED ABSTRACT | EDITOR'S SUMMARY

Many physically motivated extensions to general relativity (GR) predict substantial deviations in the properties of spacetime surrounding massive neutron stars. We report the measurement of a $\boxed{2.01 \pm}$ 0.04 solar mass ($M_\odot$) pulsar in a 2.46-hour orbit with a 0.172 ± 0.003 $M_\odot$ white dwarf. The high pulsar mass and the compact orbit make this system a sensitive laboratory of a previously untested strong-field gravity regime. Thus far, the observed orbital decay agrees with GR, supporting its validity even for the extreme conditions present in the system. The resulting constraints on deviations support the use of GR-based templates for ground-based gravitational wave detectors. Additionally, the system strengthens recent constraints on the properties of dense matter and provides insight to binary stellar astrophysics and pulsar recycling.



Vidana, Polls, Ramos, Engvik & Hjorth-Jensen, PRC 62 (2000) 035801

Scenario	Maximum mass ($M_\odot$)	R (km)
pn-matter	1.9 – 2.2	10.3
NN,NY	1.5 – 1.6	10.2
NN, NY, YY	1.3 – 1.4	10.0

Hadronic model too soft

Hyperons induce a softer EoS than when only nucleons are considered and, hence, a smaller maximum mass that the neutron star can sustain.

EoS is too soft!! Need of extra pressure at high density to compare with observations of 2 M_{sun}

The Hyperon Puzzle

The Hyperon Puzzle

Scarce experimental information:

- data from 40 single and 3 double Λ hypernuclei

- few YN scattering data (~ 50 points) due to difficulties in preparing hyperon beams and no hyperon targets available

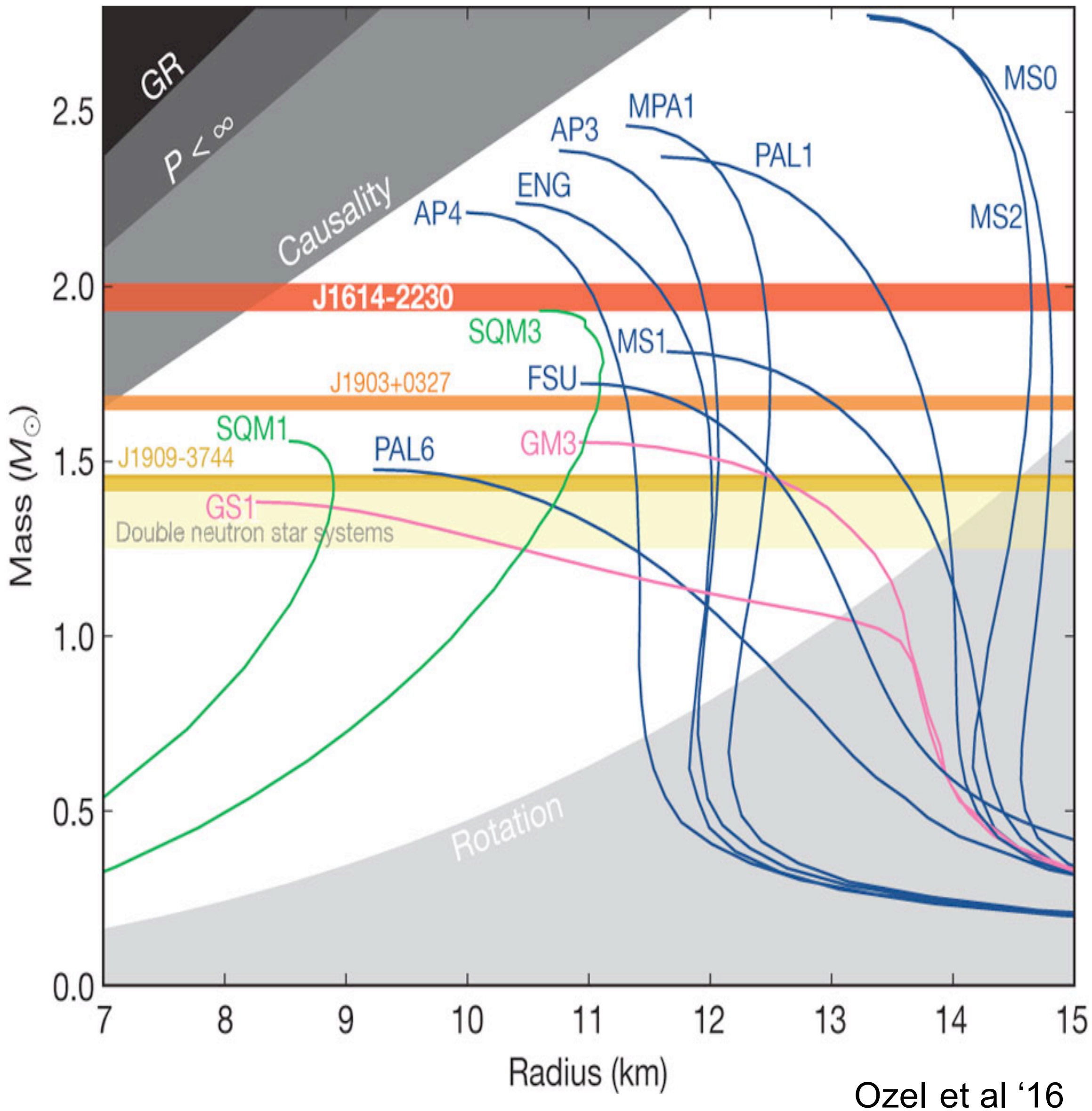
Chatterjee and Vidana '16

The presence of hyperons in neutron stars is energetically probable as density increases. However, it induces a strong softening of the EoS that leads to **maximum neutron star masses < $2M_{sun}$**

Solution?

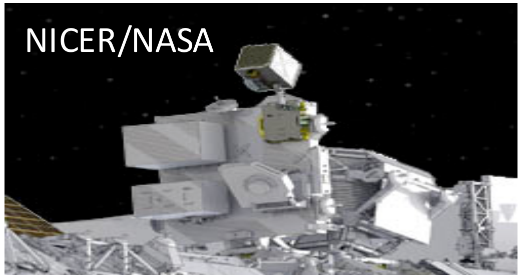
- ➢ stiffer YN and YY interactions
- ➢ hyperonic 3-body forces
- ➢ push of Y onset by Δ or meson condensates
- ➢ quark matter below Y onset

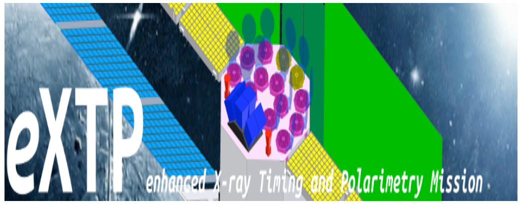
One of the latest Mass-Radius Diagrams

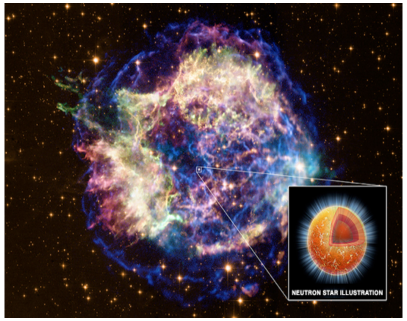


Ozel et al '16

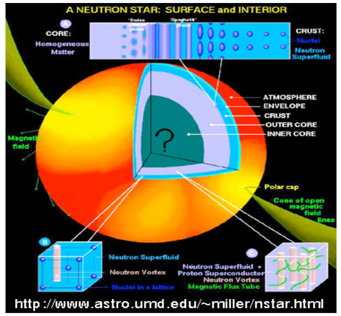
Missions to study the interior of NS (EoS of dense matter)







Summary



- Neutron stars are an excellent laboratory for testing matter under extreme conditions and a lot of effort has been invested in understanding their interior and the EoS of the different phases

- We have shown how to construct the EoS of dense matter, in particular for a free Fermi gas

- We have reviewed the baryonic phase (nucleons and hyperons) in the core of neutron stars

- We have studied the structure (mass-radius relation) of neutron stars for different baryonic EoS

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