# Doing Gravitational Wave Astronomy with one or more neutron stars

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http://astro-gr.org pau@ice.cat What the lectures are going to be

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  I will not talk about binaries from the point of view of stellar evolution

# Outline

 $\checkmark \ \ \textit{Very} \ \text{succinct introduction to GWs}$ 

- √ Very succinct introduction to GWs
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- Main physical concepts of fundamental dynamics
- √ Applications to the formation of NS binaries, and mixed binaries

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- $\checkmark$  NS with supermassive black holes

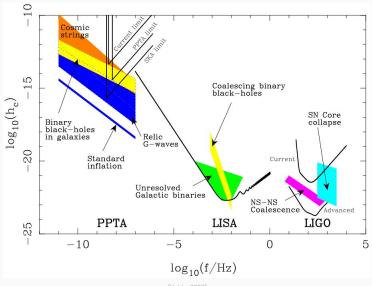
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- √ What are SMBHs and IMBHs?

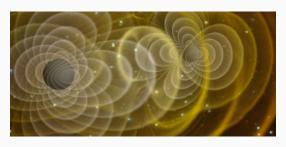
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- ✓ What are SMBHs and IMBHs?
- √ Prospects of GW Astronomy thanks to neutron stars

The different windows in GW Astronomy

# Ground-based and space-borne detectors



#### **Gravitational Waves**



[Henze, NASA]

- Predicted in 1918 by Albert Einstein
- ▶ Electromagnetic waves produced by accelerated charged particles
- Gravitational Waves produced by accelerated masses

# What are they good for?



[ESO/L. Calçada]

▷ Slow decay: Propagate to very long distances



[Film: Jungle and galaxy, win+j, win+0]

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- Slow decay: Propagate to very long distances
- Difficult to detect: Almost do not interact with matter
- Pristine probe: They contain very detailed information about space and time



[Film: Jungle and galaxy, win+j, win+0]

stellar dynamics

How to form binaries of neutron stars, and mixed binaries, with

## The equation

#### The Almighty Equation

$$\ddot{r}_i = -G\sum_{j=i,\,j\neq i}^{j=N} m_j\,\frac{\left(r_i-r_j\right)}{\left|r_i-r_j\right|^3}$$



- r<sub>i</sub> position vector of jth star at t, m its mass, G a constant
- · Recognize it?
- Good approximation to solve
  - ✓ solar system ...300 yrs later we also do
  - √ star clusters
  - √ whole galaxies as well as
  - √ clusters of galaxies
- · Not bad for a single equation

#### Gravity is weird

- Gravity = attractive long-range force
- ▶ Electromagnetism too, but positive and negative charges tend to screen each other
- Short-range forces (gas pressure) only important on small scales (interior stars)
- Stellar dynamics is <u>simple</u> (but not easy), contrary to plasma astrophysics, radiative transfer, or nuclear astrophysics (complex and not easy)
- ▶ If you care about GWs: GR

## Stellar dynamics

Stellar dynamics := studying the consequences of "The Equation" in astrophysical contexts

- → Historically: Planets, celestial mechanics
- Solar system is a very regular system
- Planets move in orbits close to the ecliptic
- All revolve in the same direction
- Orbits are well-separated
- No close encounters take place
- Not true for stars in the galactic plane, or in globular clusters
- ▶ Very irregular systems: Computer needed
- ▷ Still: (semi-) analytical approaches important

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- √ Collisionless = the heat flow due to pairwise interactions of stars is neglected

# What kind of system?



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- Some systems in nature can be approximated well
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- 2011: 157 Milky Way globular clusters
- One of the main topics of the lectures



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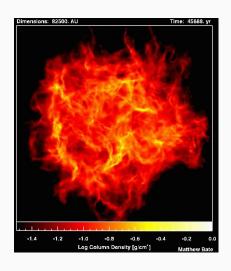
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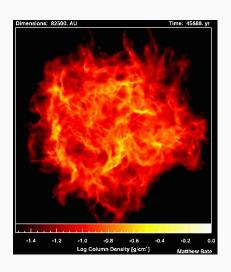


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- Factor 10<sup>6</sup> higher than in our neighbourhood

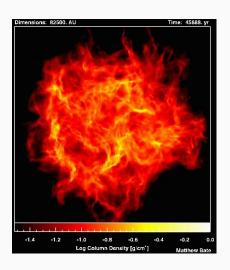






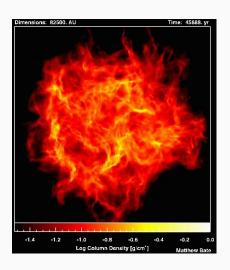
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- ▷ Simulation gas cloud





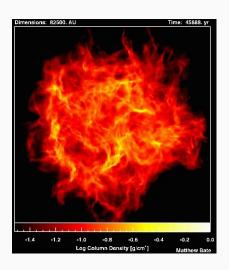
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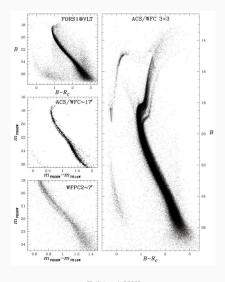
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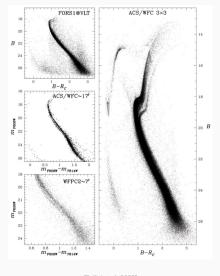
#### Free parameters

"I remember my friend Johnny von Neumann used to say, with four parameters I can fit an elephant, and with five I can make him wiqqle his trunk." *Enrico Fermi* 

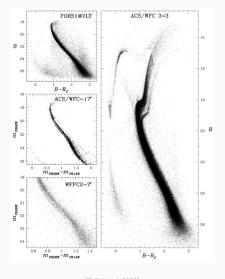
- The formation of GCs is not well-understood [Brodie & Strader 2006]
- Some clues: gas-rich merging galaxies contain large numbers of young massive star clusters [Schweizer 1987; Whitmore & Schweizer 1995]
- Physical processes related to star formation are very complex
- Single or multiple generations over a period of several 10<sup>8</sup> years?



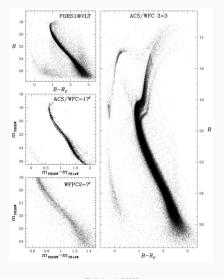
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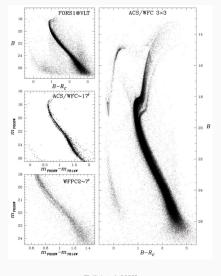


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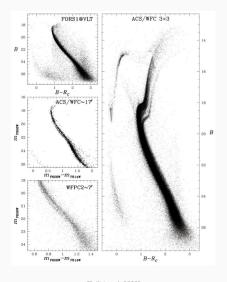


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- Single, simple stellar populations challenged
- Increasing number of photometric observations: multiple stellar populations in Galactic globular clusters
- Split of different evolutionary sequences

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 $N_{
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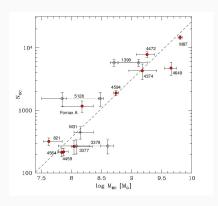
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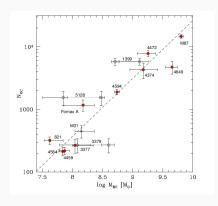
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- N<sub>GC</sub> ∝ total luminosity of the galaxy's spheroidal component[Harris & van den Bergh 1981]
- $S_N \equiv N_{\rm GC} \times 10^{0.4(M_V+15)}$
- $S_N$  the specific globular cluster frequency := # of GCs per unit absolute visual magnitude  $M_V = -15$



Number of GS vs. mass of central  $M_{ullet}$  for 13 giant elliptical, lenticular and early-type spiral galaxies [Burkert & Tremaine 2010]

■ Correlation between M<sub>•</sub> and # of GCs in elliptical and lenticular galaxies

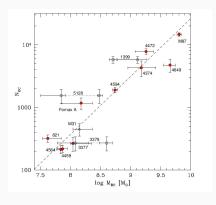


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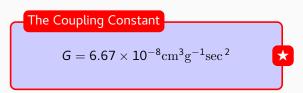
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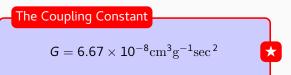
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- $M_{\bullet} = 1.7 \, 10^5 \, N_{\rm GC}^{1.08 \pm 0.04} \, M_{\odot}$

# Thermodynamics



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- One single coupling constant
- No decoupling of scales
- ▷ Only freedom: # bodies, N

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- ▶ Rate of exponential divergence of nearby trajectories

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Interior of plasma ball (credit: Ruy Lestrade)

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- What about thermodynamics?

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- ★ No, we cannot! But let's do it anyway...

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- ★ We cannot reach an asymptotic thermodynamic limit
- ★ (Equilibrium thermodynamics ruled out)

Cheating ... This is physics, after all

The *formal* inability to apply traditional thermodynamic concepts does not seriously hinder us from thinking and working with them.



Thinking NGC 6752 out of the box



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# Cheating with thermodynamics



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- Place it in a colder box: It heats up without limits
- Want to cool it down? Place it in an even hotter one
- "stars act like donkeys slowing down when pulled forwards and speeding up when held back."

[Lynden-Bell and Kalnajs (1972)]

## The power of binaries



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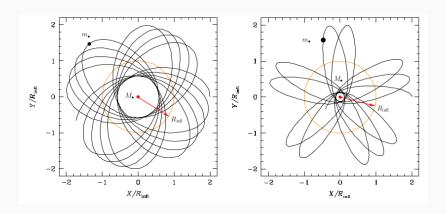
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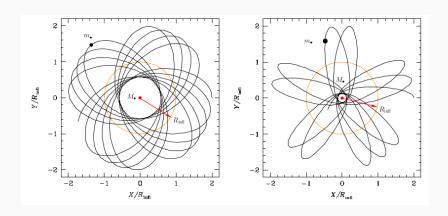
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- $E_{\text{bin}}$  arbitrarily large: Unlimited amount of +E to the rest of the system

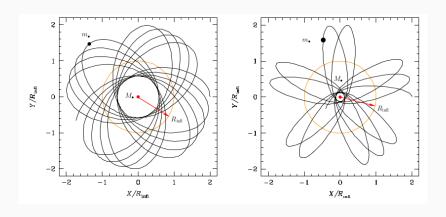
# **Potentials**



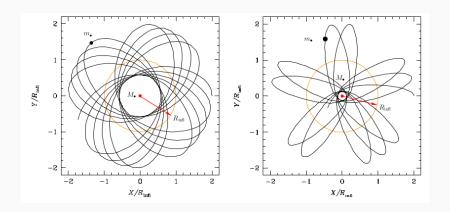
■ Two orbits differing in eccentricity



■ Two orbits differing in eccentricity Rosettes characterised by E and J



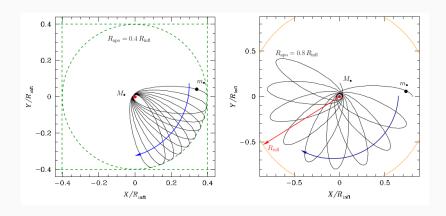
- Two orbits differing in eccentricity Rosettes characterised by *E* and *J*
- Smooth background potential: Orbital elements kept



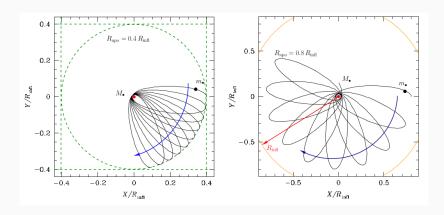
- Two orbits differing in eccentricity Rosettes characterised by *E* and *J*
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  J conserved: Fixed periapsis, cannot come arbitrarily close to MBH:

  Need perturbations

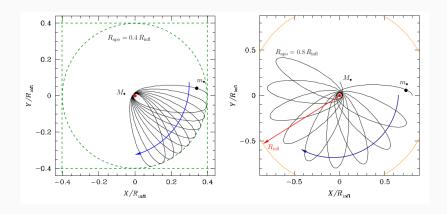


■ Orbit within R<sub>infl</sub>

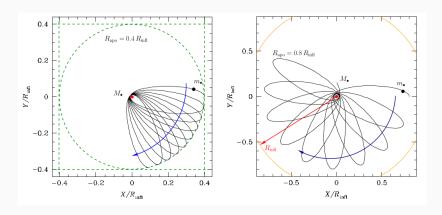


# $\blacksquare$ Orbit within $R_{\mathrm{infl}}$

Precession of the orbits

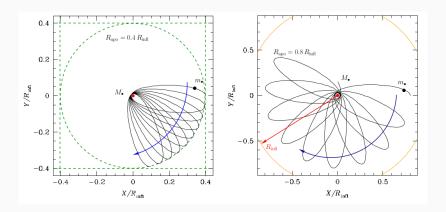


- Orbit within R<sub>infl</sub>
  - Precession of the orbits
- Perihelion retard, counterclockwise



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m orb}$$



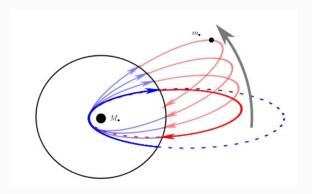
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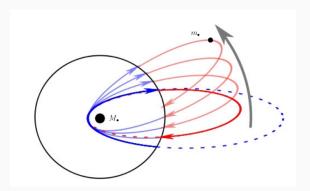
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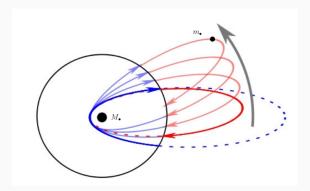
Result of having not a point but an extended mass distribution



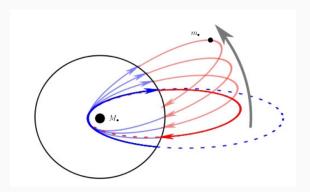
■ The star feels more mass far away than closer to the centre



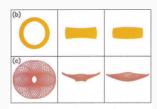
- The star feels more mass far away than closer to the centre
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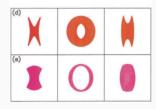
- The star feels more mass far away than closer to the centre
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- The object goes back to the centre faster



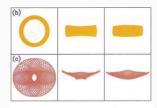
- The star feels more mass far away than closer to the centre
- When crossing the sphere, the trajectory abruptly changes and becomes a smaller ellipse
- The object goes back to the centre faster
- The orbit precesses in the opposite direction to the orbital one

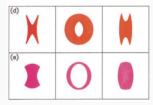


■ Most general case: Triaxial potential



Centrophobic orbits - Never reach centre

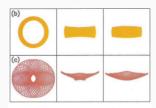


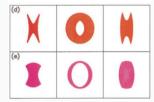


Centrophobic orbits - Never reach centre

[Poon & Merritt 2001]

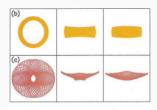
Most general case: Triaxial potential Some symmetry but neither sph. nor axial-symm.

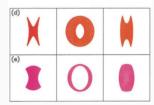




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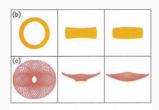
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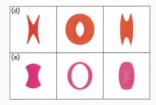




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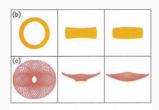
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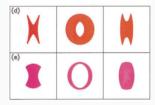




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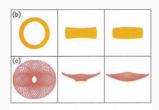


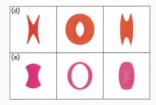
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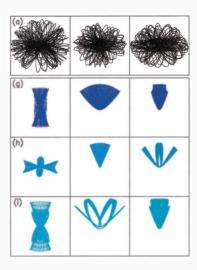




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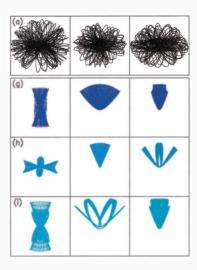
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- Shape of the potential close to the MBH?



Pyramid or box orbits: Regular, and can get very close

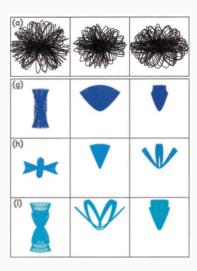
Centrophilic orbits - Get very close centre



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Stochastic orbits: Arbitrarily close

Centrophilic orbits - Get very close centre

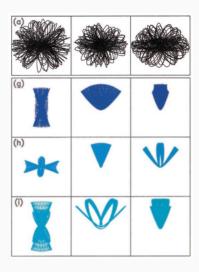


Pyramid or box orbits: Regular, and can get very close

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Probability for an orbit to get within d of MBH?

Centrophilic orbits - Get very close centre



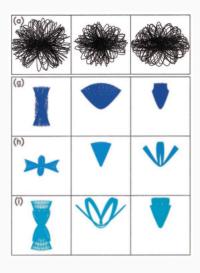
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m p} < d 
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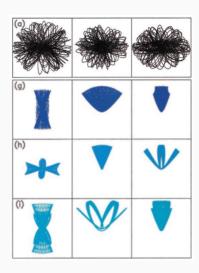
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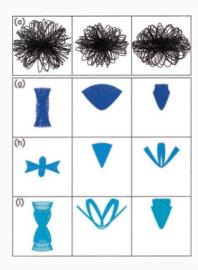
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The focusing of our target influences the projectile

38



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The focusing of our target influences the projectile

■ The projectile is attracted by the target

# Relaxation

■ Back to a spherical system world

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How can we bring stars close to the MBH?

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- Exchange of *E* and *J*

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"Collisional" := Any effect not present in a smooth, static potential including what is known in planetary dynamics as secular effects

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- $a \leftrightarrow E$  and, for a given  $a, e \leftrightarrow J$

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- $\blacksquare$   $a \leftrightarrow E$  and, for a given  $a, e \leftrightarrow J$

Decrease  $J \rightarrow$  increase e

Relaxation (This is not the transition of an atom)

For very dense stellar systems as galactic nuclei, one cannot suppose any longer that stars are moving under the influence of the mean potential generated by all other particles (which is what we call a *collision-less* system, related to the Boltzmann equation).

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**Exchange** of *E* and  $\vec{J}$ : After many tugs  $\star$  forget trajectory

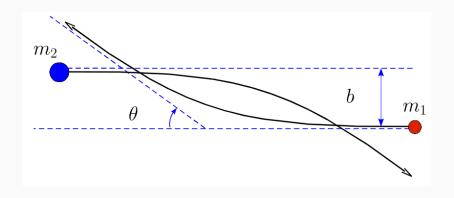
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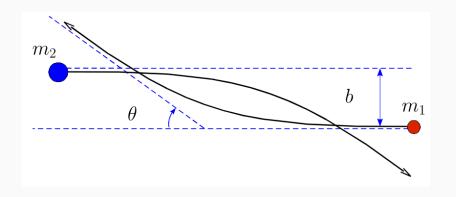
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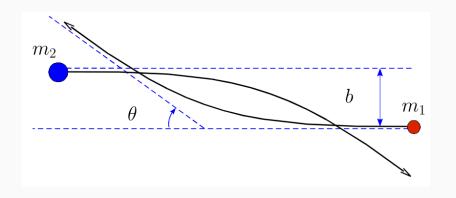
Time-scale:  $T_{\rm relax}$ 



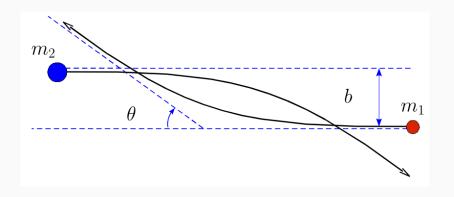
 $\blacksquare$  Two stars, masses  $m_1$  and  $m_2$ 



Two stars, masses  $m_1$  and  $m_2$   $\tan \frac{\theta}{2} = \frac{b_0}{b}$ , with  $b_0 = \frac{G(m_1 + m_2)}{v_{\rm rel}^2}$ 

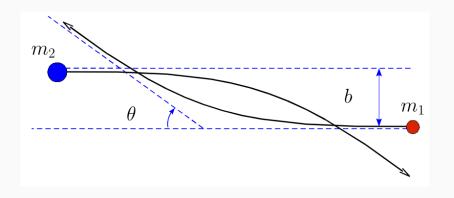


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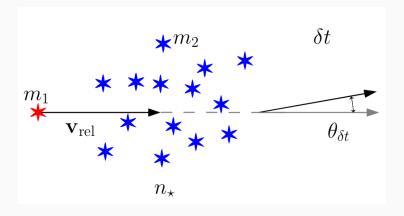


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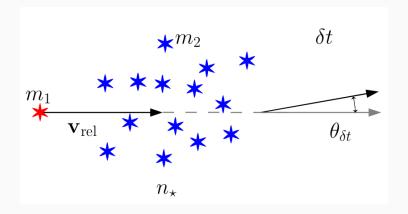
The largest the mass, the stronger the deflection



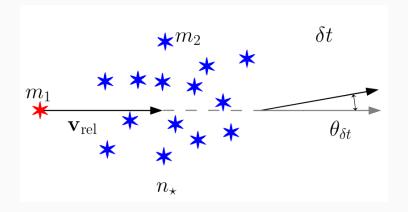
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  - The largest the mass, the stronger the deflection
- Relaxation rate: Integrate over all b



■ Integrate b, keep  $v_{\rm rel}$  and the masses fix



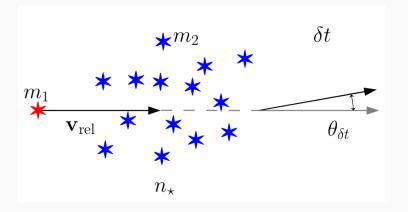
Integrate b, keep  $v_{\rm rel}$  and the masses fix After  $\delta t$  velocity vector has changed direction by  $\theta_{\delta t}$ 



Integrate b, keep  $v_{\rm rel}$  and the masses fix

After  $\delta t$  velocity vector has changed direction by  $\theta_{\delta t}$ 

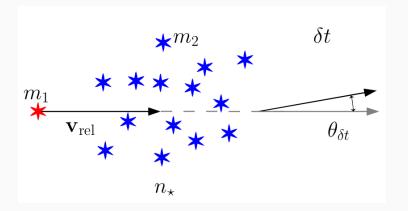
$$\blacksquare$$
  $\langle \theta_{\delta t} \rangle = 0$ 



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■ One star more massive than average

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$$t_{\rm DF} \sim \frac{\langle m \rangle}{m} \, t_{\rm rlx}$$

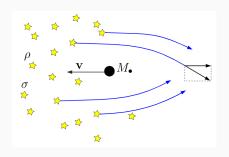
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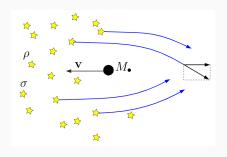
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■ If mass 10 times smaller, timescale also 10 times shorter

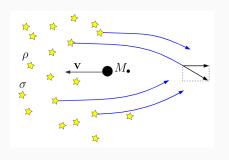


★ Massive intruder: Stellar BH in a homogeneous sea of stars



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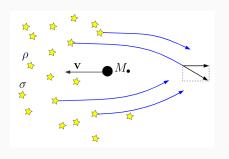
The velocity vector of the stars are rotated after the deflection



★ Massive intruder: Stellar BH in a homogeneous sea of stars

The velocity vector of the stars are rotated after the deflection

★ The projected component in the direction of the deflection is shorter

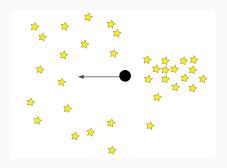


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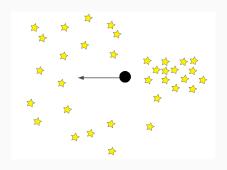
The velocity vector of the stars are rotated after the deflection

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Velocity vector of perturber almost unmodified in direction, cancel out on average

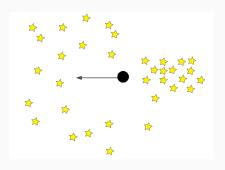


★ Massive perturber accumulates region high density

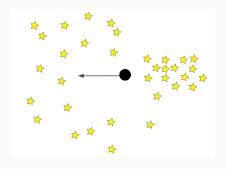


★ Massive perturber accumulates region high density

Conservation of *J*: Perturber feels a drag from that region



- ★ Massive perturber accumulates region high density
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Amplitude decreases

$$ec{a}_{\mathrm{DF}} = -rac{ec{v}}{t_{\mathrm{DF}}} - rac{4\pi\ln\Lambda\,G^2
ho\,M}{v^3}\,\xi(X)ec{v}$$

$$\begin{split} \vec{a}_{\rm DF} &= -\frac{\vec{v}}{t_{\rm DF}} - \frac{4\pi \ln \Lambda G^2 \rho M}{v^3} \, \xi(X) \vec{v} \\ \xi(X) &= {\rm erf}(X) - 2\pi^{-1/2} X e^{-X^2} \end{split}$$

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 $\bigstar$  Plug in velocity from perturber,  $\mathbf{v} \approx \sigma$ 

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 $\bigstar$   $F_{\mathrm{DF}} \propto \mathcal{M}_{ullet}^2$ 

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 $\xi(X) = \mathrm{erf}(X) - 2\pi^{-1/2} X \mathrm{e}^{-X^{2}}$   
 $X = \frac{v}{\sqrt{2}\sigma}$ 

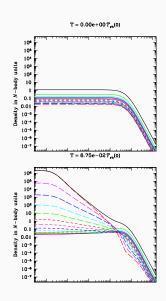
 $\bigstar$  Plug in velocity from perturber,  $\mathbf{v} \approx \sigma$ 

$$t_{\rm DF} \sim \tfrac{m}{M} t_{\rm rlx} \ll t_{\rm rlx}$$

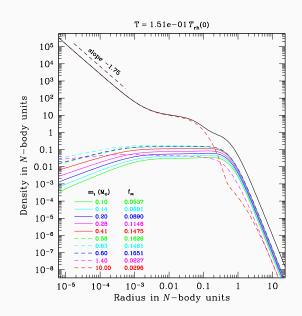
 $\bigstar$   $F_{\mathrm{DF}} \propto \mathcal{M}_{ullet}^2$ 

The bigger  $\mathcal{M}_{ullet}$ , the bigger DF effects are, in spite of bigger inertia

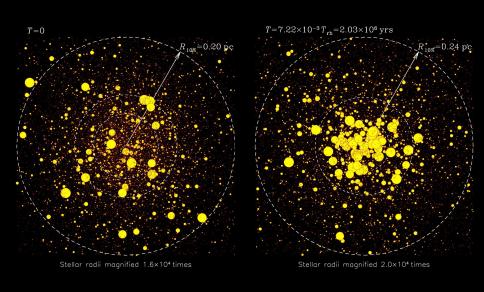
### Mass segregation without a MBH

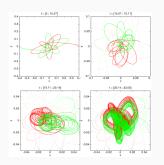


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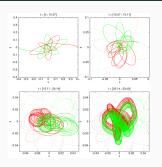


#### Core collapse

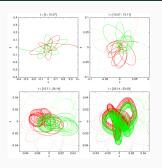




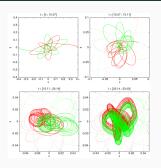
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- They are likely to form a binary with another star which stands a good chance of being another NS
- They form a NS binary and a source of GWs which will merge or not in a Hubble time depending on their orbital properties

# Binaries with massive black holes



 $[Warner\ Bros, Entertainment\ Inc.\ and\ Paramount\ Pictures\ Corporation.\ Author:\ double\ negative,\ http://www.dneg.com]$ 

#### ■ Stellar-mass black holes:



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## ■ Stellar-mass black holes: Formed by the gravitational collapse of a massive star, "collapsars", $m_{\rm bh}/M_{\odot} \in [5, \text{ few tens}[, \text{ everywhere in the galaxy}]$



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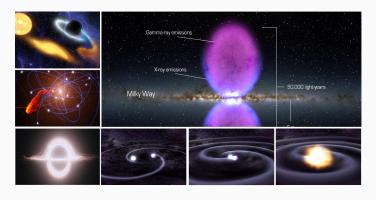
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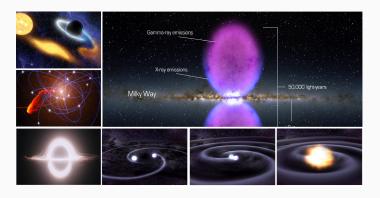
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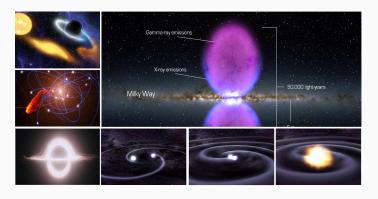
## Black holes: Do they exist?



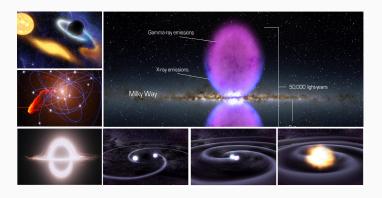
• General Relativity predicts black holes:



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- We can probe the event horizon: We have an excellent probe...

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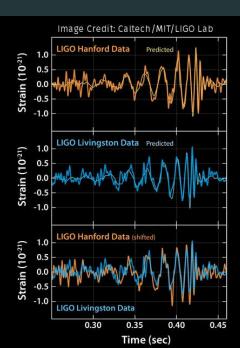
Because we do not have a direct evidence

#### Wait... We do have direct proofs now!

- ✓ General Relative managers to their existence.
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NS as probes of supermassive

black holes



[ESO/M. Kornmesser]

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- How do you create a luminosity of 10<sup>40</sup> W??

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[NASA/JPL-Caltech]

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- Accretion requires a mass between a million and a thousand millions the mass of the Sun
- ➤ To create the typical luminosity of a QSO, the SMBH consumes 10 stars per year
- ▷ The largest one known devours 600 Earths per minute



[NASA/JPL-Caltech/S. Stolovy (SSC/Caltech)]

★ Observations of the Galactic Center reveal a strange fact



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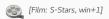
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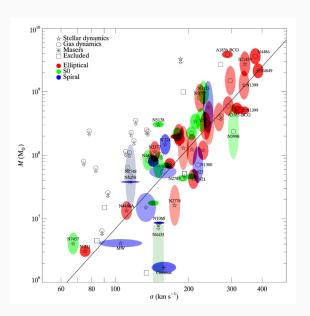


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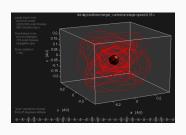
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- $\bigstar$  Within a radius of 22 millions of km, enclosed in  $\sim 1/3$  times the distance to the Sun



#### Correlations



## NS and GWs: A unique probe of MBHs





[Film: Extreme-mass ratio inspiral, S. Drasco, win+2 and Natalia Amaro, win+3]

- imes Stellar mass object spiraling into  $10^4-10^6\,M_{ullet}$
- This range of masses corresponds to relaxed nuclei (!)
- Only compact objects extended stars disrupted early
- $\pmb{x}$  With LISA z  $\sim 1$



[Amaro-Seoane et al 2012ab]

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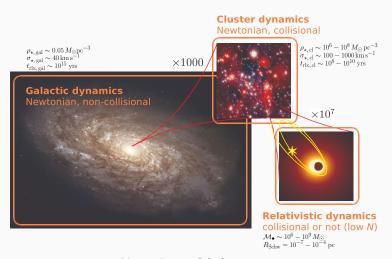
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# A problem of 10 orders of magnitude



Note:  $1pc \sim 3$  light years

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  Output

  Description:

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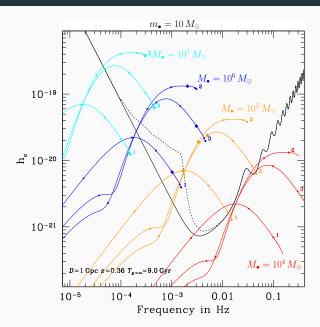


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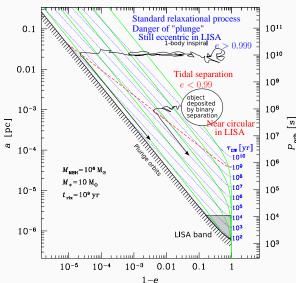
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   We are already making completely new discoveries many years before LISA



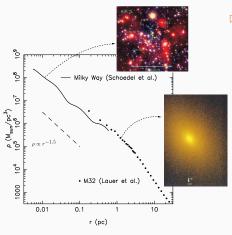
# Range of masses



# Dichotomizing an EMRI

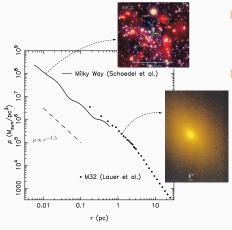


# Do we expect these things to exist?



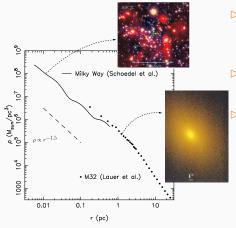
Oth-order question to ask: How many stars? How are they distributed?

[Adapted from Merritt 2006]



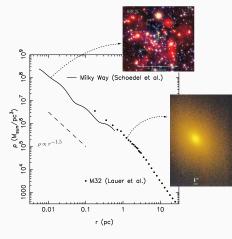
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- ightharpoonup Confirmed later with a detailed kinematic treatment for single-mass [Bahcall & Wolf 1976]:  $\gamma=7/4$  and  $p=\gamma-3/2=1/4$

○ "Only a fool tries the harder problem when he does not understand the simplest special case"

Donald Lynden-Bell (Sec. 4.5 of Lynden-Bell & Wood 1968, MNRAS)

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- More realistic models: Properties of multi-mass systems poorly reproduced by single-mass models
- ightharpoonup Initial Mass Functions  $\in [0.1, \sim 120] M_{\odot}$  to first order by two (well-separated) mass scales:  $\mathcal{O}(1M_{\odot})$  (Main Sequence, White Dwarfs, Neutron Stars) and  $\mathcal{O}(10M_{\odot})$  (Stellar Black Holes)

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- ➤ Two branches for the solution: A "weak" (unrealistic) branch and a "strong" branch

[Hopman & Alexander 2009, Preto & Amaro-Seoane 2010, Amaro-Seoane & Preto 2011

$$\Gamma_{\rm EMRI} = f_{\bullet} \int_{E_{\rm GW}}^{+\infty} dE \; \frac{n(E)}{\ln(J_c(E)/J_{\rm lc}) \; T_{\rm rlx}(E)} \label{eq:emri}$$

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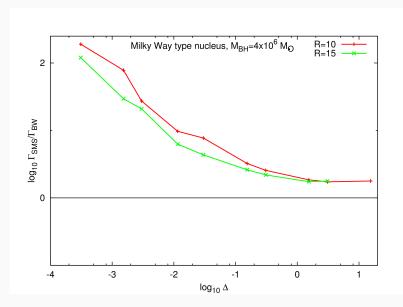
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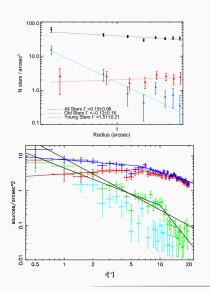
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- $\blacksquare$   $a_{GW}$ , or energy  $E_{GW}$ , for EMRIs is:  $a_{GW} = 0.01r_h$

#### Boost on rates



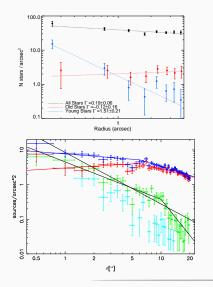
# Cusps in distress



 We can distinguish the young and old population and see a deficit of old stars

[Do et al. 2009, Buchholz et al 2009]



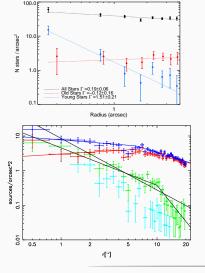


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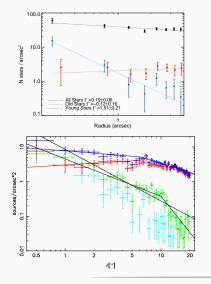
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- ightharpoonup Possibility of a core with  $ho_*$  decreasing, looks like someone carved a hole
- This is old news We know that the problem is not that acute



# Why would this be a problem?

If indeed there was a hole in the stellar distribution, this could be a problem because of three reasons:

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- ② The Milky Way is template for LISA targets
- ③ If by extrapolation this is typical of many galaxies, cosmic rate drops

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[Baumgardt et al 2006, Portegies Zwart et al 2006]

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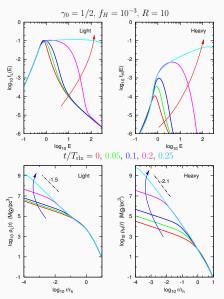
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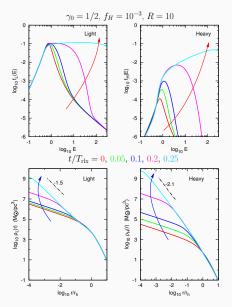
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- ightharpoonup With the correct solution of mass segregation, it's short ... about 1/4 of  $T_{\rm rlx}$





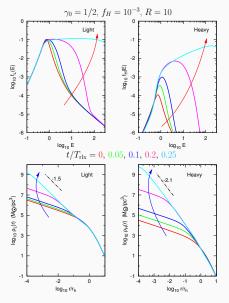
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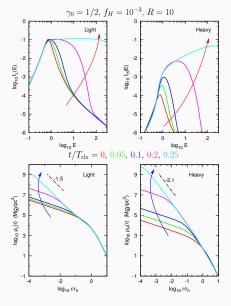
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#### What does this mean for EMRIs?

Stellar cusps may re-grow in less than a T<sub>H</sub> but the existence of cored nuclei still remains a possibility

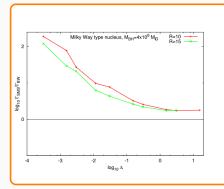
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- The Milky Way nucleus is *not* necessarily the prototype of the nucleus from which e-LISA detections will be more frequent
- We still expect that a substantial fraction of EMRI events will originate from segregated stellar cusps, in particular with our new solution of mass segregation

# Stellar-mass compact objects pile up in galactic nuclei



Stellar-mass compact objects distribute in the galactic nucleus trying to reach an equipartition of energy in such a way that they will dominate in mass density close to the density center of the nucleus.



[PAS et al 2004, Khalisi, PAS & Spurzem 2006, PAS & Preto 2010, Preto & PAS 2010]

# Intermediate-mass black holes



[IMBH in NGC 3783, Credit: ESO/M. Kommesser]

■ We know that supermassive black holes correlate with the host galaxy:

#### **IMBHs**



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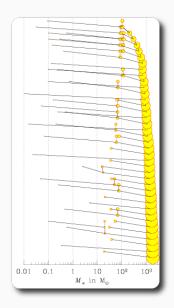
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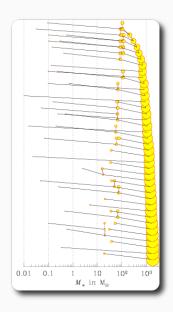


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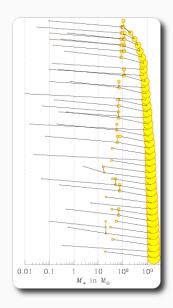


⊳ Follow the growth of a runaway star

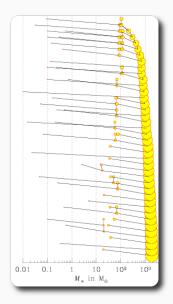


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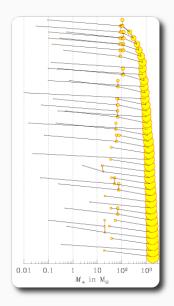
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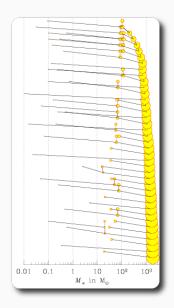
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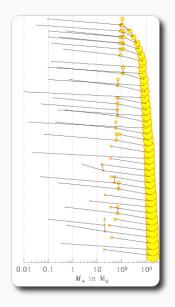
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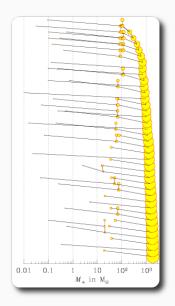
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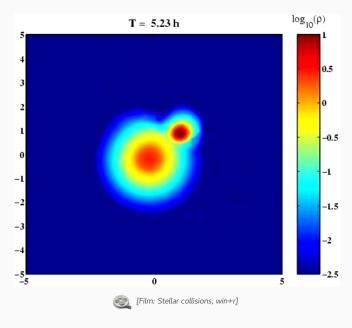
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- We are cheating (Stellar winds, sticky spheres)



# NS as probes of IMBHs with GWs

✓ NS can be captured by IMBHs in a process that we call intermediate-mass ratio inspiral

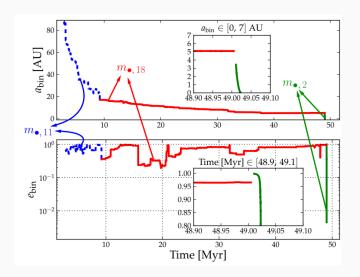
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- ✓ IMBHs are likely to be found at the centres of dense stellar systems and we know that NS should segregate there, too

## Live formation of an IMRI



# Conclusions

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...and a specific torture to you

