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Transport in neutron stars



see recent review: A. Schmitt and P. Shternin, *Astrophys. Space Sci. Libr.* 457, 455 (2018)

- other reviews:
- N. Chamel and P. Haensel, *Living Reviews in Relativity* 11, 10 (2008) ([crust](#))
 - D. Page and S. Reddy, in "Neutron Star Crust", 1201.5602 ([inner crust](#))
 - D. G. Yakovlev *et al.*, *Phys. Rep.* 354, 1 (2001) ([neutrino emissivity](#))
 - A. Sedrakian, *Prog. Part. Nucl. Phys.* 58, 168 (2007) ([many-body techniques](#))
 - M. G. Alford *et al.*, *Rev. Mod. Phys.* 80, 1455 (2008) ([quark matter](#))
 - A. Y. Potekhin, J. A. Pons, and D. Page, *Space Sci. Rev.* 191, 239 (2015) ([cooling](#))

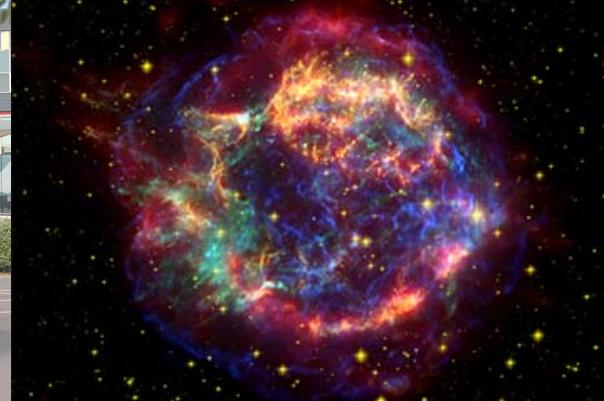


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Transport in neutron stars



- What is transport? Why are transport properties interesting?
- Specific examples: shear viscosity & bulk viscosity of dense matter in neutron star cores
- Open questions and future directions

General remarks

”Transport”: transfer of conserved quantities
(energy, momentum, particle number, electric charge, ...)

from one region to another due to non-equilibrium
(temperature gradient, non-uniform chemical composition, ...)

- transport properties usually computed from particle collisions:
energy & momentum exchange or flavor-changing processes
(strong coupling: quasiparticle picture not valid)
- transport at small T typically dominated by fermions
(electrons, muons, neutrinos, neutrons, protons, hyperons, quarks)
- if fermion contribution suppressed (e.g., Cooper pairing), bosons may dominate
(pions, kaons, lattice phonons, superfluid phonons)

What do we learn from calculating transport properties of neutron star matter?

- Phenomenology of neutron stars
 - see next page
- Theoretical aspects
 - analogies/differences to heavy-ion transport ("hot QCD vs. dense QCD")
 - conceptual questions of (general) relativistic hydrodynamics
(e.g., second order hydro)
 - strong coupling effects:
 - of practical relevance (e.g., modified Urca rate) and
 - of theoretical interest (beyond quasiparticle picture?
compare to AdS/CFT, see heavy ions?)
 - superfluid (multifluid) transport,
transport in the presence of vortices (quantum turbulence?)

Transport and phenomenology

Phenomenon	Transport properties
oscillatory modes (<i>r</i> -modes)	shear & bulk viscosity
pulsar glitches	superfluid transport (vortex pinning)
thermal radiation	heat transport in outermost layers
cooling	neutrino emissivity, heat conductivity
magnetic field evolution	magnetohydrodynamics electrical & thermal conductivities
crust disruption (accretion, magnetar flares)	transport properties of the crust nuclear reactions ("deep crustal heating")
core-collapse supernovae	neutrino transport, neutrino-nucleus reactions
neutron star mergers	high-temperature transport (viscous) magnetohydrodynamics

Basic transport coefficients (page 1/2)

- start from Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{u} \cdot \frac{\partial f}{\partial \mathbf{x}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = - \int_{\mathbf{p}_1} \int_{\mathbf{p}'} \int_{\mathbf{p}'_1} W(\mathbf{p}, \mathbf{p}_1; \mathbf{p}', \mathbf{p}'_1) [f f_1 (1-f') (1-f'_1) - (1-f) (1-f_1) f' f'_1]$$

- multiply by $1, \mathbf{p}, \frac{p^2}{2m}$ and integrate over $\mathbf{p} \rightarrow$ hydrodynamic eqs

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad \text{continuity equation}$$

$$\frac{\partial s}{\partial t} + \nabla \cdot \left(s \mathbf{v} + \frac{\mathbf{j}_T}{T} \right) = - \frac{\pi_{ji} \partial_j v_i + \mathbf{j}_T \cdot \nabla T / T}{T} \quad \text{entropy production}$$

$$\frac{\partial v_i}{\partial t} + (\mathbf{v} \cdot \nabla) v_i = - \frac{\partial_i P}{\rho} + \frac{F_i}{m} - \frac{\partial_j \pi_{ji}}{\rho} \quad \text{Navier-Stokes equation}$$

- dissipative terms

$$\begin{aligned} \mathbf{j}_T &= -\kappa \nabla T \\ \pi_{ij} &= -\eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \nabla \cdot \mathbf{v} \right) - \zeta \delta_{ij} \nabla \cdot \mathbf{v} \end{aligned}$$

with thermal conductivity κ , shear viscosity η , bulk viscosity ζ

Basic transport coefficients (page 2/2)

- more generally:

add dissipative terms systematically order by order in derivatives

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \textcolor{red}{q^\mu u^\nu + q^\nu u^\mu + \tau^{\mu\nu}} + \dots, \quad j^\mu = n u^\mu + \textcolor{red}{J^\mu} + \dots$$

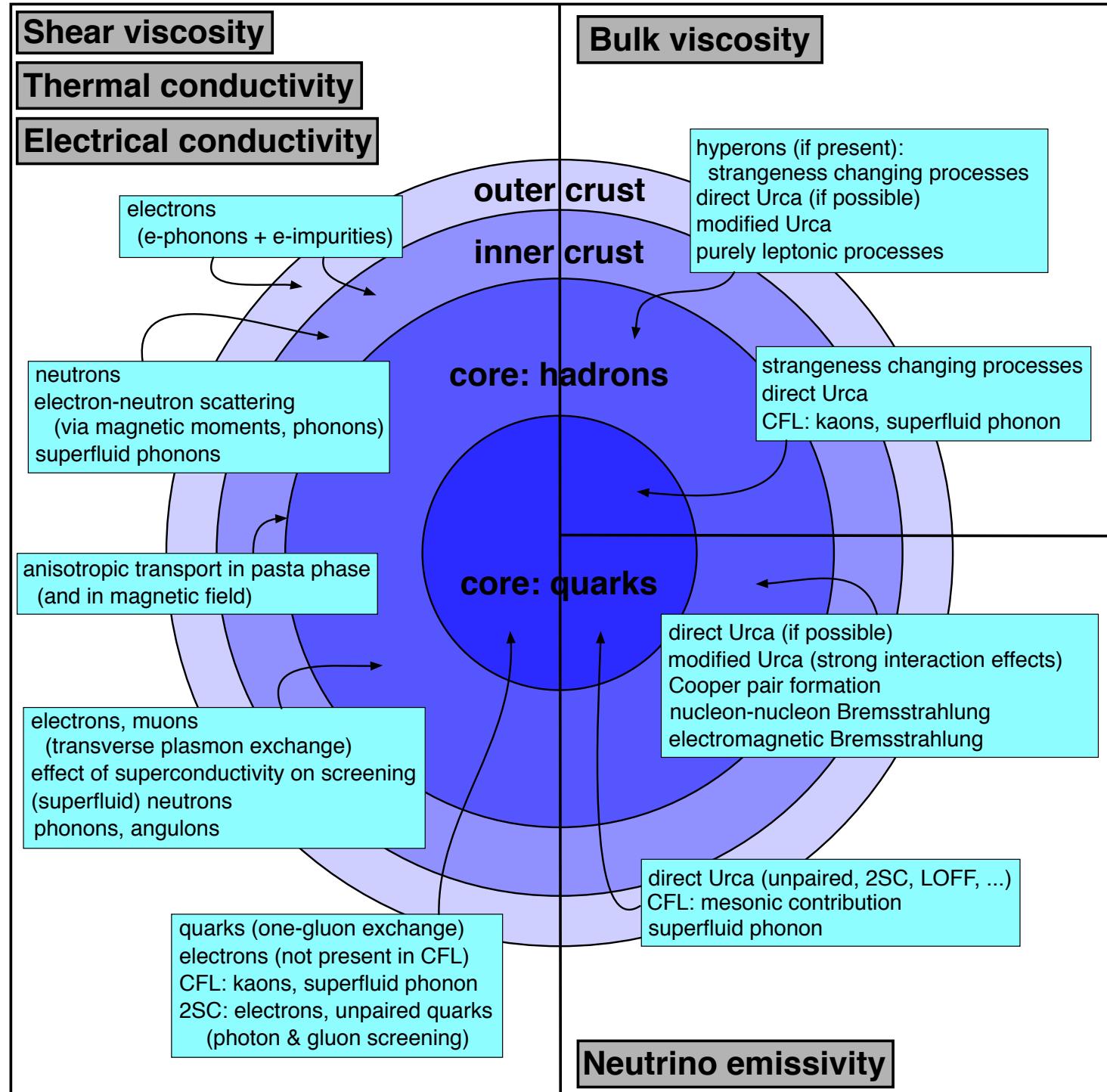
where $\textcolor{red}{q^\mu, \tau^{\mu\nu}, J^\mu \sim \mathcal{O}(\partial)}$, and consider conservation eqs.

$$\partial_\mu T^{\mu\nu} = \partial_\mu j^\mu = 0$$

- complications in neutron star context:

- (general) relativistic effects
- external force \rightarrow Lorentz force (magnetic field)
- two-fluid (multi-fluid) transport
(electron-ion in the crust, npe matter in the core)
- superfluid (two-fluid) transport \rightarrow more transport coefficients

- general recipe: compute transport coefficients from some microscopic theory (e.g., Boltzmann eq) and insert into hydro eqs (if sufficiently close to equilibrium)

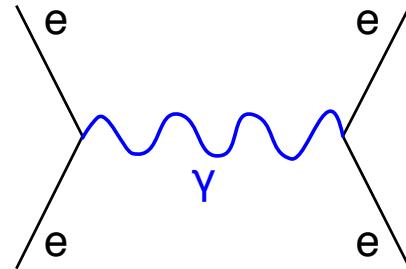


Transport in the core: two examples

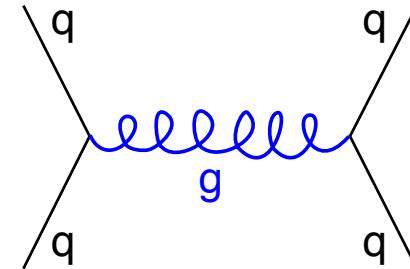
- Shear viscosity η : lepton contribution in nuclear matter
unpaired and 2SC quark matter
- Bulk viscosity ζ : resonance phenomenon
results for nuclear & quark matter

Shear viscosity (page 1/2)

Nuclear matter:
ee (or $\mu\mu$) scattering (also nn)



Quark matter:
qq scattering (also ee)



- need photon (gluon) propagator in medium
photon in dense nuclear matter: S. Stetina, E. Rrapaj and S. Reddy, PRC 97, 045801 (2018)
- scattering dominated by transverse plasmon exchange
H. Heiselberg, G. Baym, C. J. Pethick and J. Popp, NPA 544, 569C (1992)
- transverse plasmons dynamically screened (Landau damping)

$$\eta_{ee} \propto \frac{1}{T^{5/3}} \sim 10^{20} \frac{\text{g}}{\text{cm s}}$$

Shternin, Yakovlev, PRD 78, 063006 (2008)

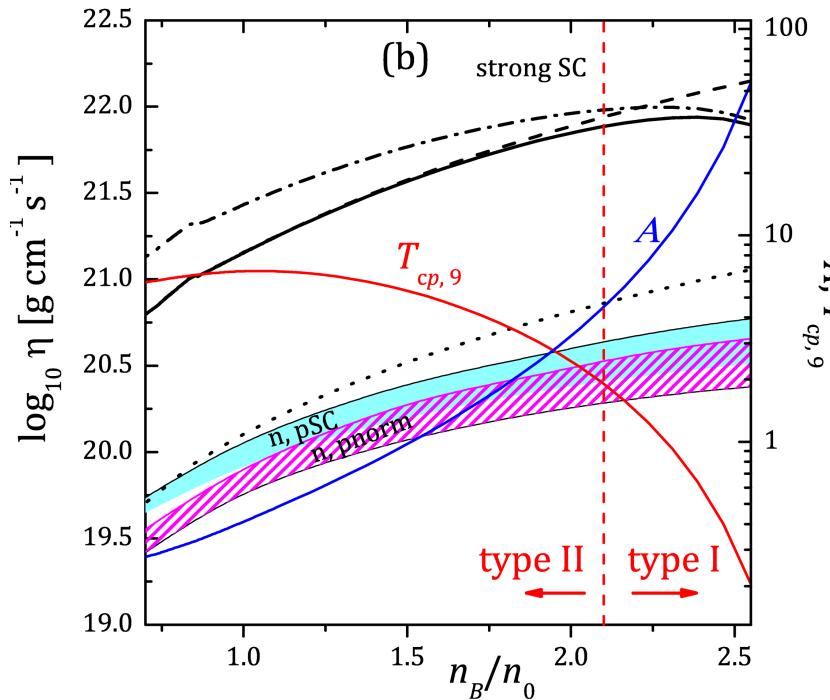
$$\eta_{qq} \propto \frac{1}{T^{5/3}} \sim 10^{15} \frac{\text{g}}{\text{cm s}}$$

Heiselberg, Pethick, PRD 48, 2916 (1993)

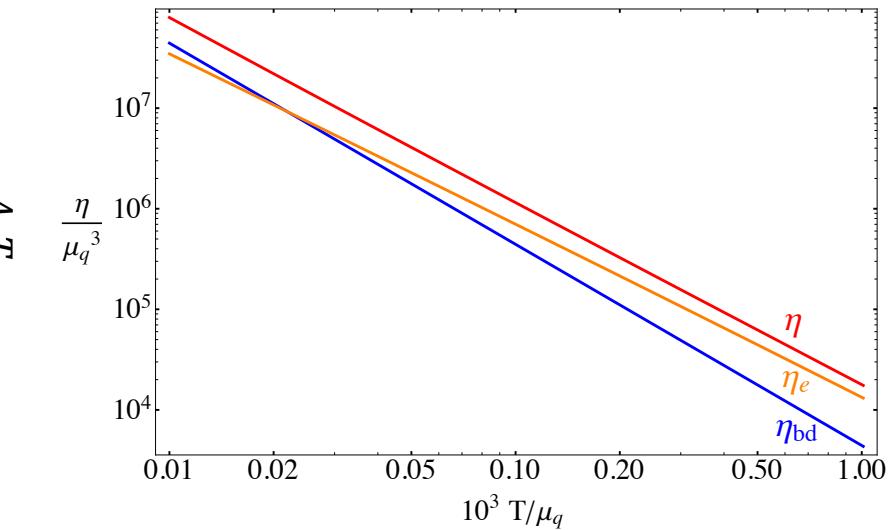
→ different behavior from Fermi liquid $\eta \propto T^{-2}$

Shear viscosity (page 2/2)

- Superconductivity affects η , even when scatterers remain unpaired
- Nuclear matter: e scatter via γ
properties of γ changed by $\langle pp \rangle$
- Meissner effect: static magnetic screening (dynamic screening relevant for larger T)
- 2SC quark matter: blue q scatter via g
properties of g changed by red, green $\langle qq \rangle$
- g and γ mix
A. Schmitt *et al.*, PRD 69, 094017 (2004)
- small T : unpaired bd dominates (interacts only via screened gauge bosons)



P. Shternin, PRD 98, 063015 (2018)



M. G. Alford, H. Nishimura, A. Sedrakian,
PRC 90, 055205 (2014)
[$\mu_q^3 \sim 10^{13} \text{ g cm}^{-1} \text{s}^{-1}$]

Bulk viscosity (page 1/3): definition

- bulk viscosity $\zeta(\omega) = \text{dissipative response to compression and expansion}$

$$V(t) = V_0 + \delta V \cos \omega t,$$

$$\zeta(\omega) = 2\langle \dot{E} \rangle \left(\frac{V_0}{\delta V} \right)^2 \frac{1}{\omega^2}$$

with the **dissipated power** in an oscillation period $\tau = 2\pi/\omega$

$$\langle \dot{E} \rangle = -\frac{1}{\tau V_0} \int_0^\tau dt P(t) \frac{dV}{dt} = \frac{B}{C} \gamma \langle \delta \mu(t) \delta V(t) \rangle$$

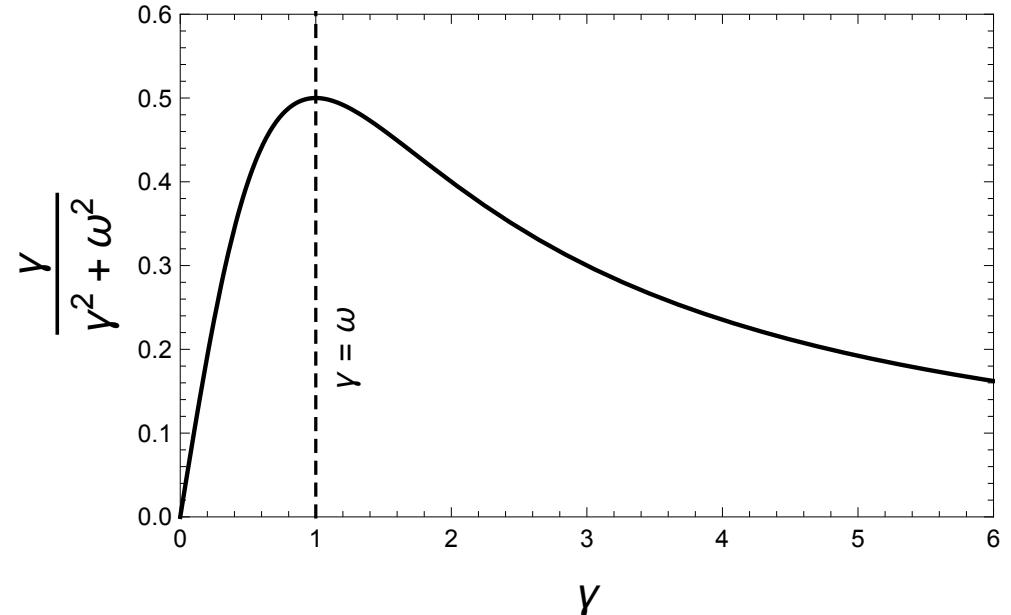
- B, C susceptibilities in equilibrium, and, e.g., $\delta \mu = \mu_p + \mu_e - \mu_n$
- neutrons produced per time and volume in $p+e \rightarrow n+\nu_e$ approximately $\gamma \delta \mu/C$

$$\zeta(\omega) = \frac{B^2}{C} \frac{\gamma}{\gamma^2 + \omega^2}$$

[in above hydro equations, $\zeta = \zeta(\omega = 0)$]

Bulk viscosity (page 2/3): resonance phenomenon

- ζ maximal for $\gamma = \omega$

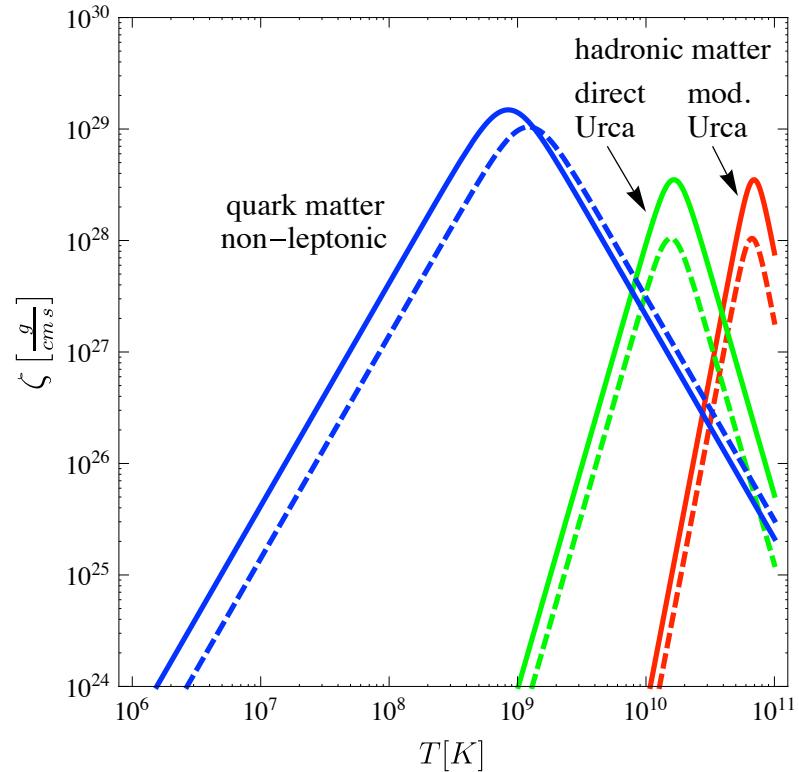


- need microscopic rate γ to be of the order of star oscillations ω
 $\rightarrow \zeta$ dominated by electroweak interactions
- γ is typically monotonically increasing with T
 \rightarrow maximum of ζ at a certain T

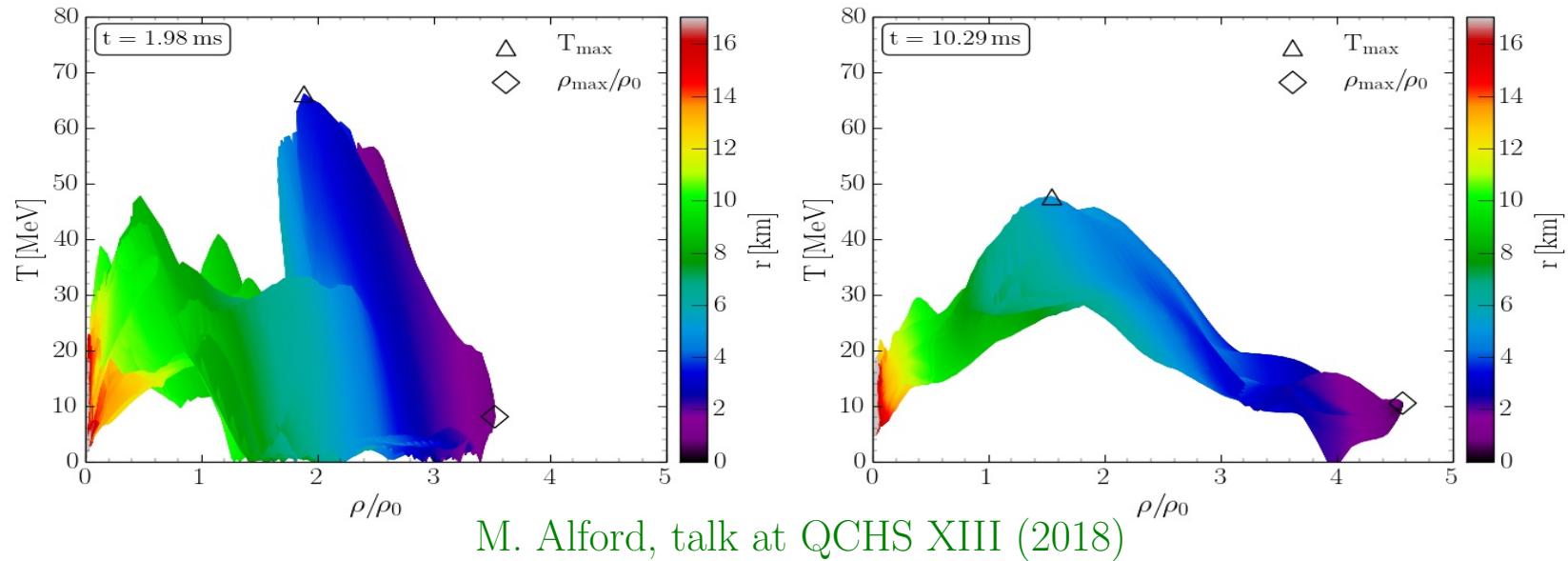
Bulk viscosity (page 3/3): hadronic and quark matter

hadronic matter	quark matter
direct Urca , $p + e \rightarrow n + \nu_e$ modified Urca	non-leptonic process $u + d \leftrightarrow u + s$
hyperons, e.g.: $n + n \leftrightarrow p + \Sigma^-$	Urca: $u + e \rightarrow d + \nu_e$
nn , pp pairing: rates suppressed	CFL: $K^0 \leftrightarrow \phi + \phi$

- nonleptonic processes:
larger phase space
- uncertainties from strong
interactions in susceptibilities



Open questions/future directions (page 1/3): transport in neutron star mergers



- transport at relatively large $T \rightarrow$ neutrino trapping?
- importance of dissipative coefficients?
 viscosity, heat flow: M. G. Alford *et al.*, PRL 120, 041101 (2018)
 electrical conductivity: A. Harutyunyan *et al.*, Eur. Phys. J. A 54, 191 (2018)

Open questions/future directions (page 2/3): anomalous transport

- anomalous transport for systems with chiral imbalance ($N_R > N_L$)
 → macroscopic manifestation of quantum anomaly ("chiral anomaly")



- chiral magnetohydrodynamics for electrons in core-collapse supernovae due to $p + e_L \rightarrow n + \nu_L$: chiral magnetic instability & inverse energy cascade
 Y. Masada, K. Kotake, T. Takiwaki and N. Yamamoto, PRD 98, 083018 (2018)
 - generate magnetar fields from CME?
 M. Dvornikov and V. B. Semikoz, 1904.05768 [astro-ph.HE]
- chiral hydrodynamics for neutrinos ("chiral vortical effect") in mergers?

Open questions/future directions (page 3/3)

- uncertainties in input from strongly coupled physics
(modified Urca, shear viscosity involving nucleons, ...)
- anisotropic transport: transport in a magnetic field,
transport in "nuclear pasta"
- superfluid transport: vortices, magnetic flux tubes,
nonzero temperature
- cross-disciplinary approaches
 - critical velocities in two-component (super)fluids
A. Haber, A. Schmitt and S. Stetina, PRD 93, 025011 (2016)
cold atoms: M. Delehaye *et al.*, PRL 115, 265303 (2015)
 - simulation of pulsar glitches with cold atoms?
 - explore analogies to superfluid helium (e.g., vortex interfaces)
V. Graber, N. Andersson, and M. Hogg, Int. J. Mod. Phys. D26, 1730015 (2017)
 - transport in heavy-ion collisions
→ future experiments at large μ at FAIR and NICA