Asteroseismic measurements of stellar differential rotation and asphericity

- **Observables: frequency splittings, *a*-coefficients, seismic travel times**
- **Differential rotation**
- **Asphericities**
- **Unsteady perturbations, e.g. due to a single active region**

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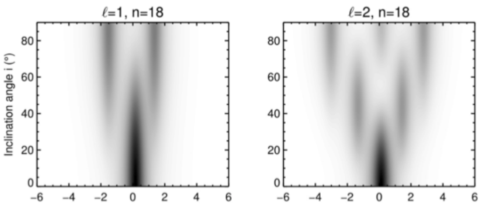
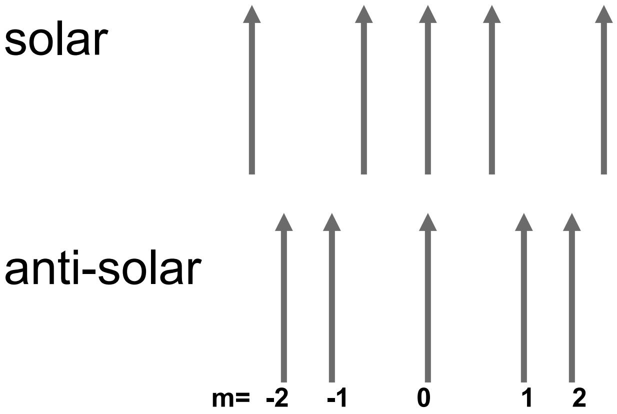
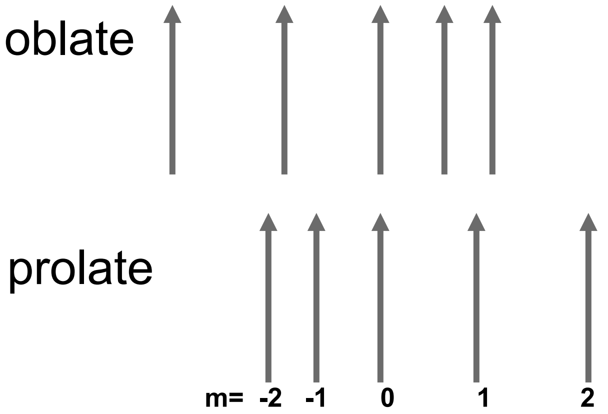
l=1 or 2 (3 if we are lucky)
Star inclination 30 deg < i < 70 deg



At fixed n and l, a multiplet consists of $2l+1$ modes with frequencies ν_{nlm}, where $-l \le m \le l$

	differential rotation	asphericity (centrifugal distortion, magnetic activity)
mode frequencies	ν_{nlm}	ν_{nlm}
frequency splittings	$S_{nlm} = (\nu_{nlm} - \nu_{nl,-m})/2m$	$s_{nlm} = (\nu_{nlm} + \nu_{nl,-m})/2 - \nu_{nl0}$
a coefficients	odd: $a_{2k+1}(n,l)$	even: $a_{2k}(n,l)$
seismic travel times	not easy	yes



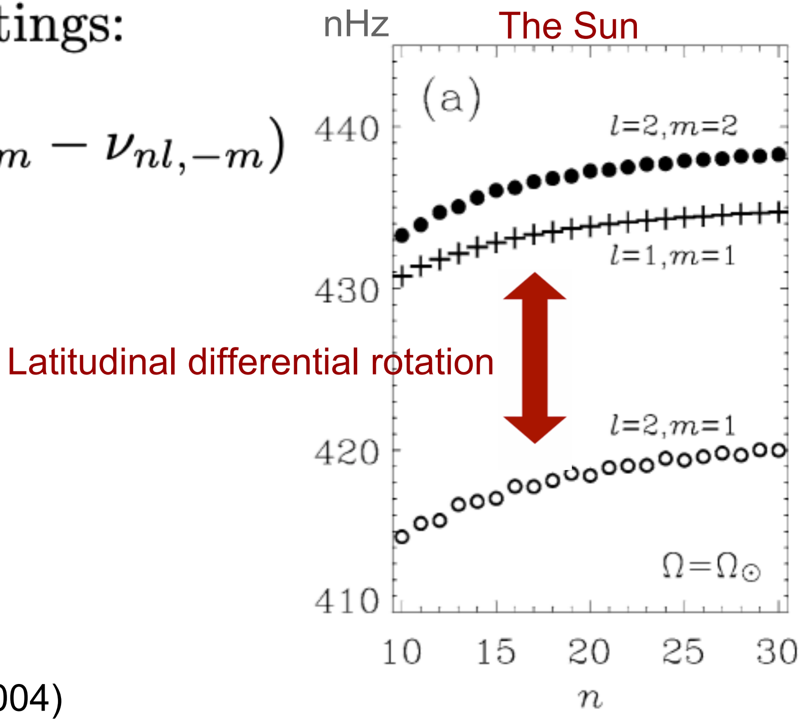


Mode frequencies (weak rotation):

$$\nu_{nlm} = \nu_{nl} + m \iint K_{nl|m|}(r,\theta)\Omega(r,\theta)\ r dr d\theta + \text{centrifugal term}(|m|)$$

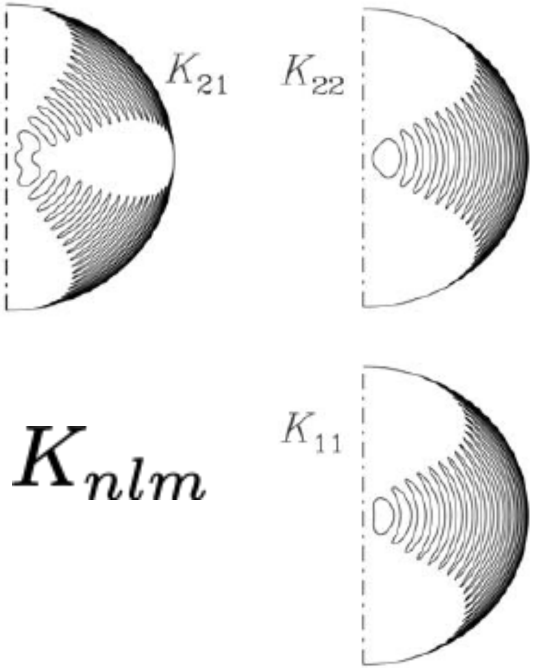
Frequency splittings:

$$S_{nlm} = \frac{1}{2m}(\nu_{nlm} - \nu_{nl,-m})$$



Sectoral modes

$S_{n22} \approx S_{n11}$



Gizon & Solanki (2004)

a-coefficients

$$\nu_{nlm} = \nu_{ln} + \sum_{j=1}^{2l+1} a_j(n,l)\mathcal{P}_j^{(l)}(m).$$

The $\mathcal{P}_j^{(l)}(m)$ are polynomials of degree j that are uniquely determined by the orthogonality condition $\sum_m \mathcal{P}_j^{(l)}(m)\mathcal{P}_k^{(l)}(m) = 0$ for $j \neq k$, and the normalization $\mathcal{P}_j^{(l)}(l) = l$ (Ritzwoller & Lavely 1991; Schou, JCD, Thompson 1994). . The frequency perturbations due to the linear effect of rotation are encoded in the odd coefficients,

$$S_{n11} = a_1(1,n),$$

$$S_{n2m} = a_1(2,n) + \frac{1}{3}(5m^2 - 17)a_3(2,n).$$

Ritzwoller & Lavely (1991) showed that the inversion of the a-coefficients is made easy by parametrizing the angular velocity as follows:

$$\Omega(r,\theta) = \Omega_0(r) + \sum_{j=1} \Omega_j(r) W_j(\theta),$$

where $W_j(\theta) = P^1_{2j+1}(\cos\theta)/\sin\theta$. This decomposition is used routinely in helioseismology. The first two terms are

$$W_1(\theta) = \frac{3}{2}(5\cos^2\theta - 1)$$
$$W_2(\theta) = \frac{15}{8}(21\cos^4\theta - 14\cos^2\theta + 1)$$

$$\boxed{\text{The advantage of this expansion is that each } \Omega_j(r) \text{ can be determined from the coefficients } a_{2j+1}(n,l) \text{ alone.}}$$

In practice, with disk-integrated data, it is too ambitious to want to measure all coefficients $a_1(1,n)$, $a_1(2,n)$, and $a_3(2,n)$. Instead we may fit two parameters:

$$a_1 := a_1(1,n)$$
$$a_3 := a_3(2,n)$$

The additional constraint $S_{n11} = S_{n22}$ prompts us to parametrize the frequencies as follows:

Recommended parametrization of rotational splitting for sun-like stars

$$\nu_{n1m} = \nu_{n1} + m(a_1 + a_3) + \text{asphericity term}$$
$$\nu_{n2m} = \nu_{n2} + ma_1 + \frac{1}{3}(5m^3 - 17m)a_3 + \text{asphericity term}$$

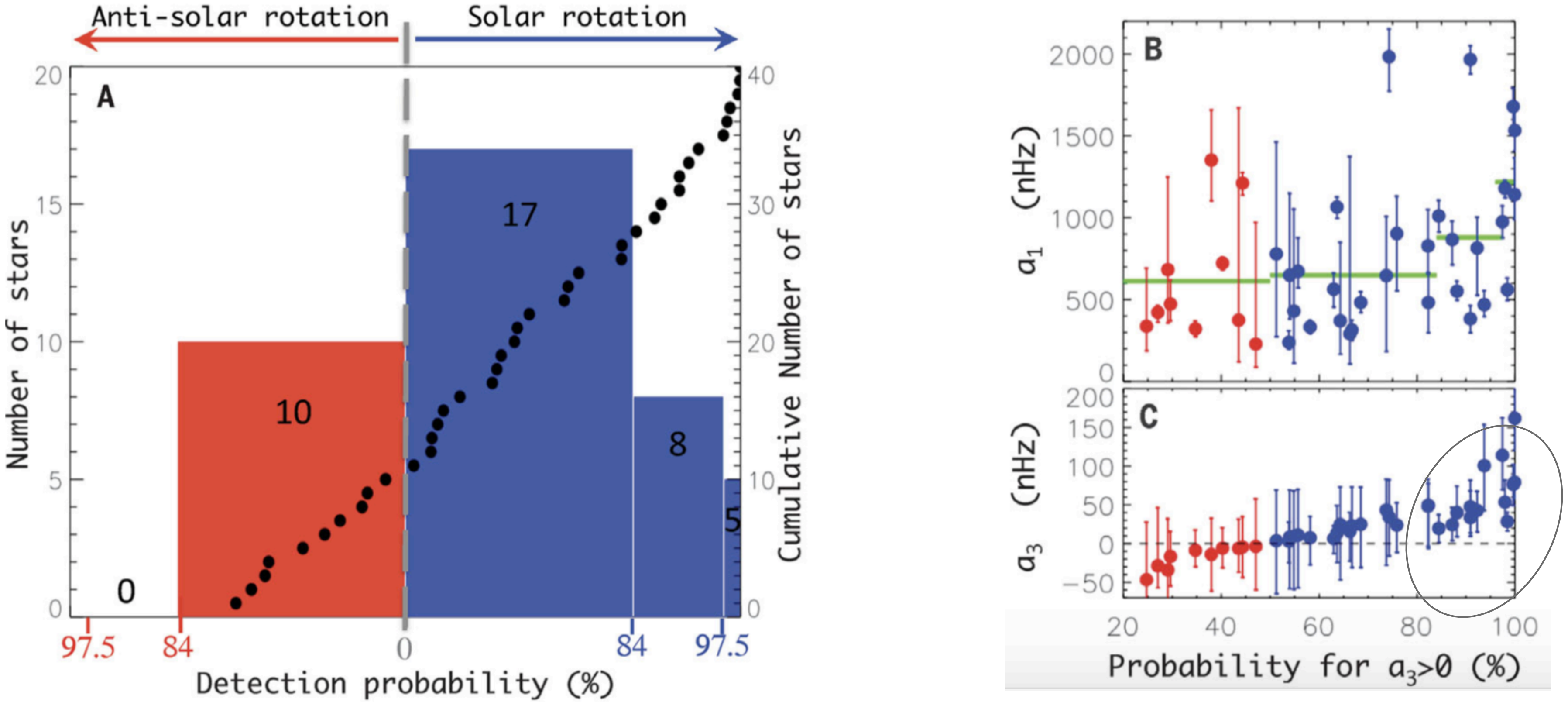
Gizon & Solanki (2004)

With only two independent parameters we can constrain models of the form

$$\Omega(r,\theta) = \begin{cases} \Omega_0 + \Omega_1 W_1(\theta) & r_c \leq r \leq R \\ \Omega_0 & 0 \leq r < r_c \end{cases}$$

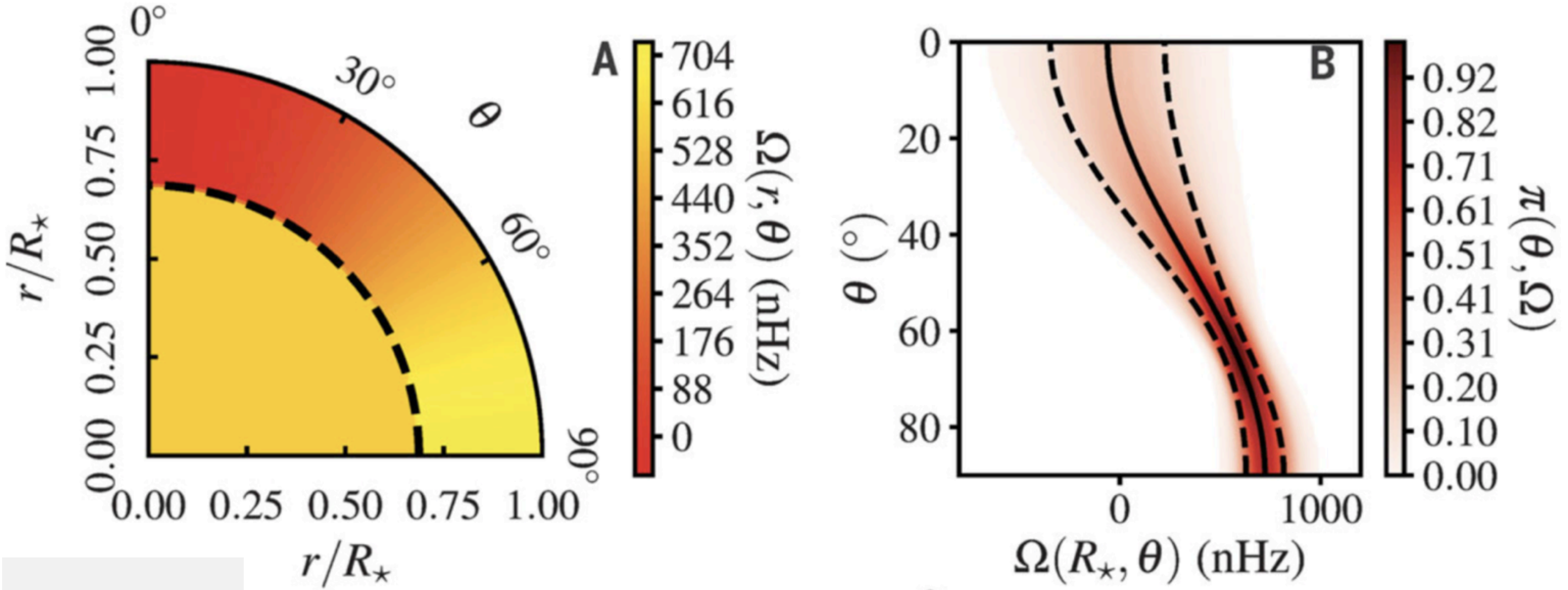
assuming we have an estimate of the depth of the convection zone, r_c. The inversion is trivial, there is a one-to-one relationship between Ω_0 and a_1 and between Ω_1 and a_3 (see Gizon & Solanki 2004).

Differential rotation: *a*3-coefficients --- Sun-like stars



Benomar et al. (2018), following Gizon & Solanki (2004) parametrization

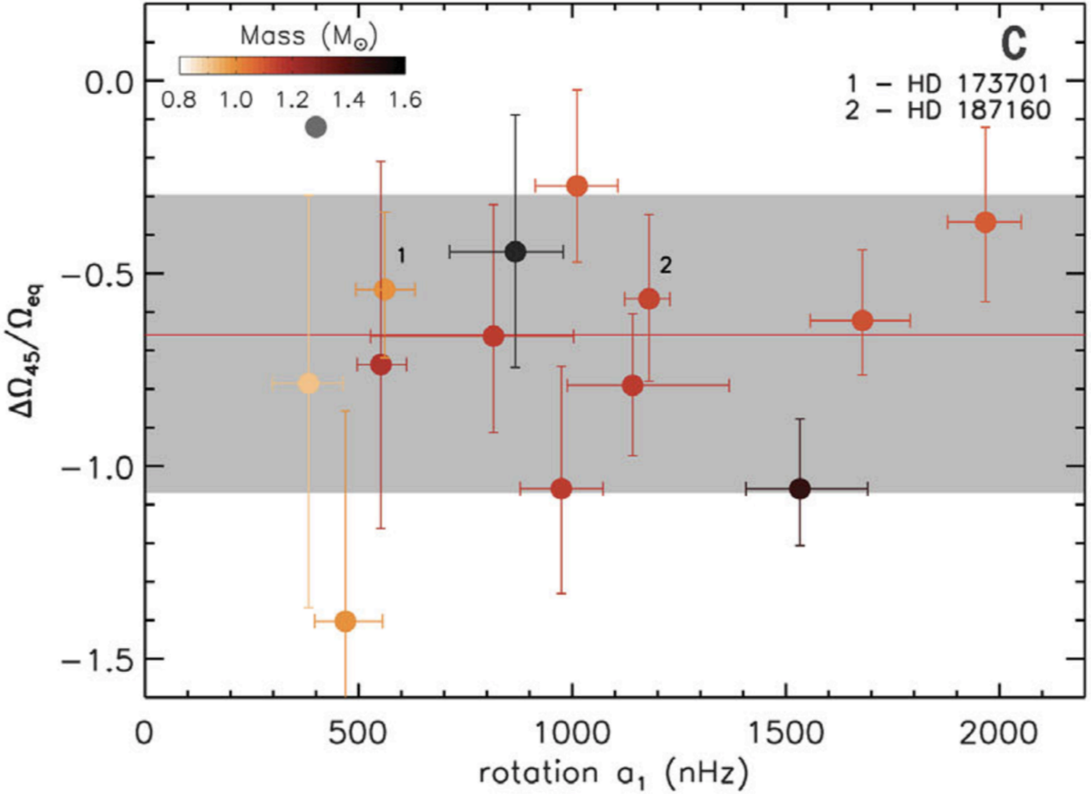
Differential rotation: *a*3-coefficients --- Sun-like stars



HD 173701

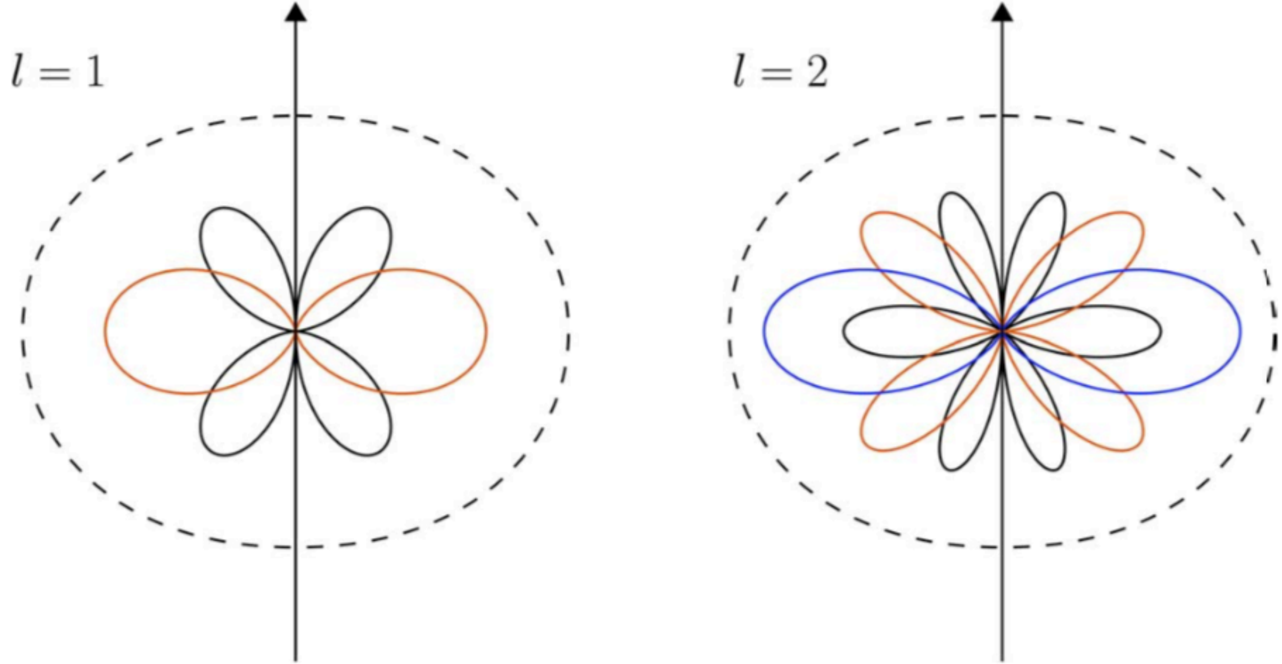
Benomar et al. (2018)

Differential rotation: *a*3-coefficients --- Sun-like stars



Benomar et al. (2018)

Measurements of stellar asphericity



Polar plots of the kinetic energy density $E_{lm}(\theta) = c_{lm} \, [P_l^m(\cos\theta)]^2 \sin\theta,$

Gizon et al. (2017)

Stellar asphericity: centrifugal distortion

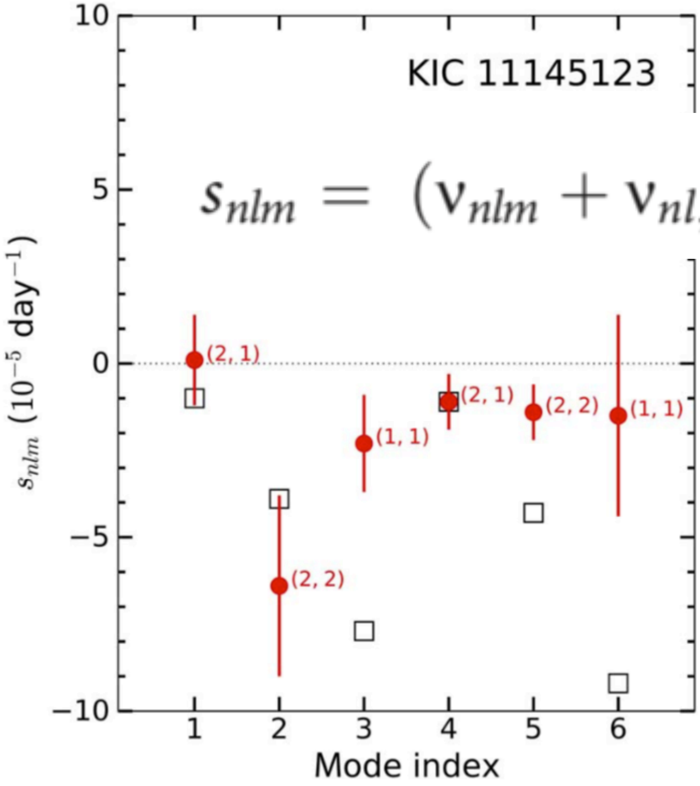
$$\nu_{nlm} \approx \nu_{nl} + \; m\Omega/(2\pi) + \varepsilon\nu_{nl}\Delta_{ln}Q_{2lm}$$

The effect is proportional to the ratio of the centrifugal to the gravitational forces $\varepsilon = \Omega^2 R^3/(GM)$

$$Q_{2lm} = \int_0^{\pi} \mathrm{d}\theta \, P_2(\cos\theta) E_{lm}(\theta) = [l(l+1) - 3m^2](2l-1)^{-1}(2l+3)^{-1}$$

$$\Delta_{nl} = \; 4/3\int_0^R \mathrm{d}r (r/R)^3 \xi_{nl}^2(r)\rho r^2$$

Example: long lived acoustic oscillations



Gizon et al. (2017)

$$s_{nlm}^{\mathrm{rot}} = -\varepsilon m^3 \nu_{nl} \Delta_{nl} (2l-1)^{-1} (2l+3)^{-1}$$

$$s_{nlm} = (\nu_{nlm} + \nu_{nl,-m})/2 - \nu_{nl0}, \text{ for } 1 \le m \le l$$

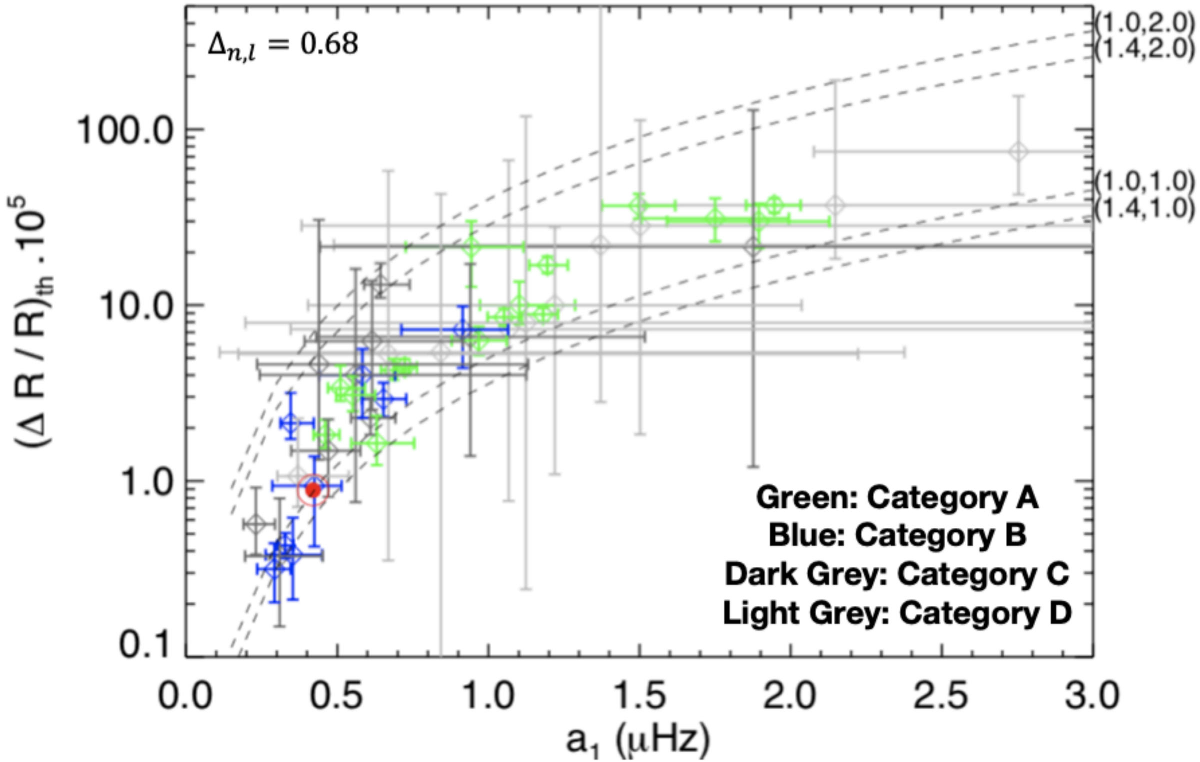
$$\varepsilon = \Omega^2 R^3/(GM) \qquad \varepsilon = 1.0 \times 10^{-5}$$

The star is measured to be oblate.
But the flattening is smaller than that due to centrifugal distortion alone.
→ activity at low latitudes?

Note that sensitivity here is several orders of magnitude higher than optical interferometry.

Asphericity: solar-like stars

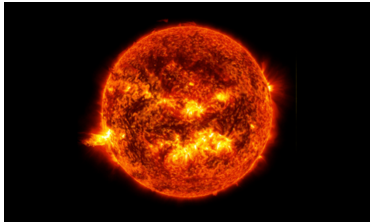
$$(\Delta R/R)^{\mathrm{rot}} = \varepsilon/2$$



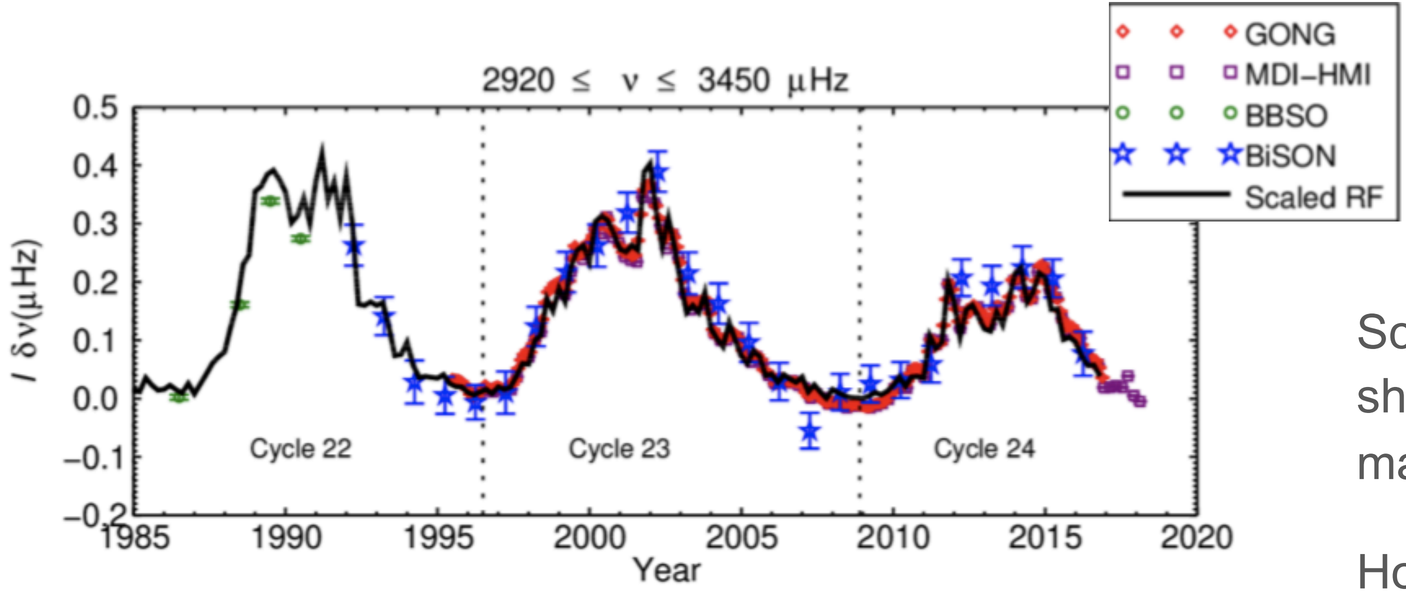
Kepler legacy

Benomar et al.
in preparation

Stellar asphericity: magnetic activity



Surface magnetic activity → local increase in wave speed.

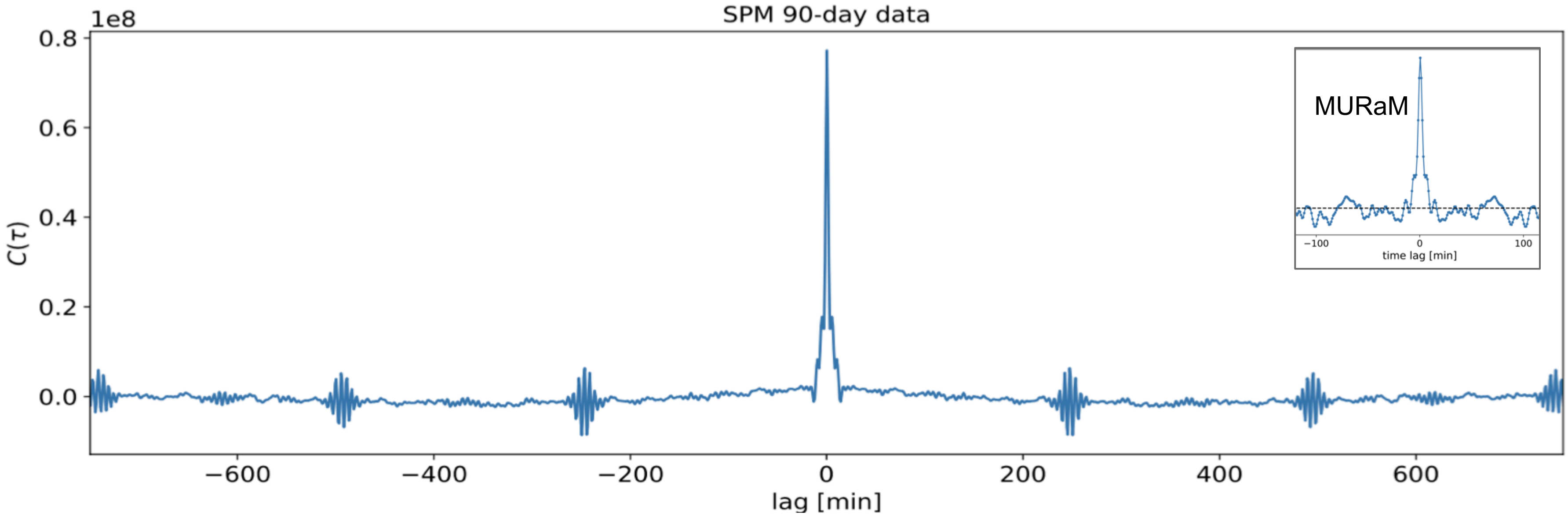


Scaled frequency shifts due to surface magnetic activity

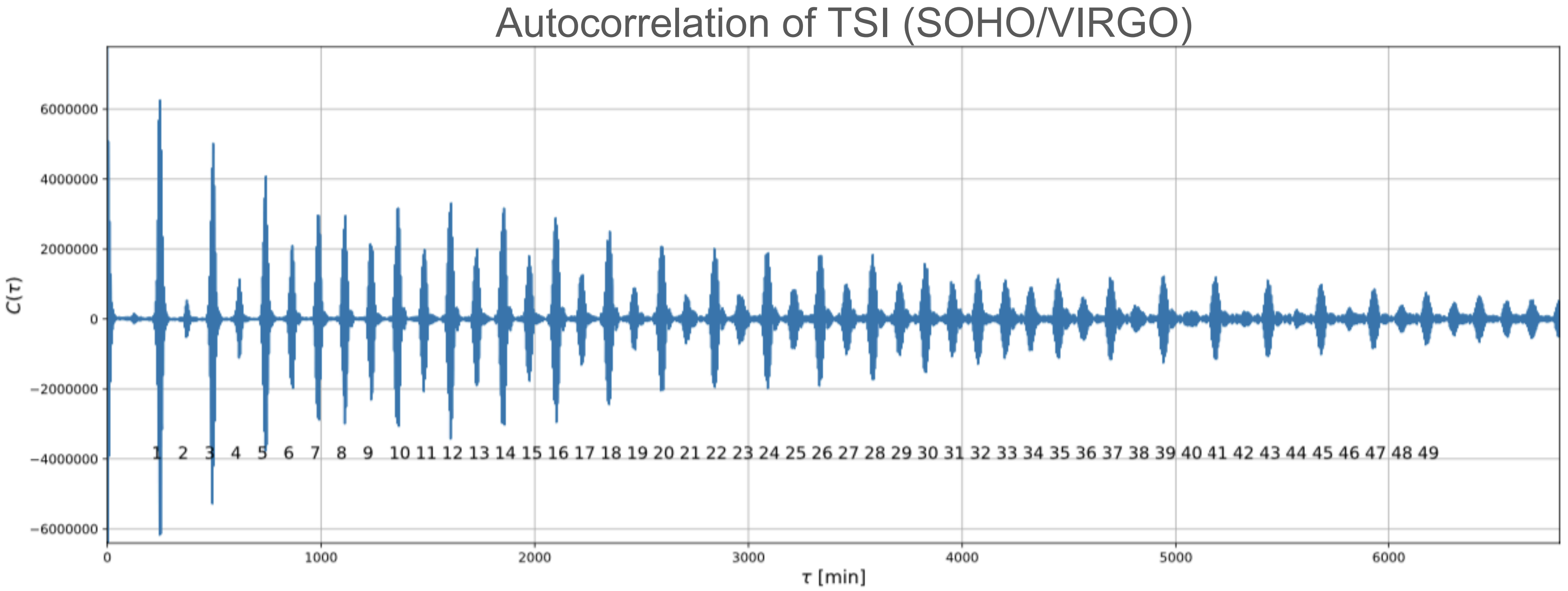
Howe et al. 2018

Asphericity: seismic travel times (Sun)

Autocorrelation of TSI (SOHO/VIRGO)



Asphericity: seismic travel times (Sun)



Asphericity: seismic travel times (Sun)

The measurement and interpretation of travel times is well understood in local helioseismology. See Gizon & Birch (2002, 2004) and Fournier et al. (2014).

$$C(t) = \int_0^T I(t')I(t'+t)dt'$$

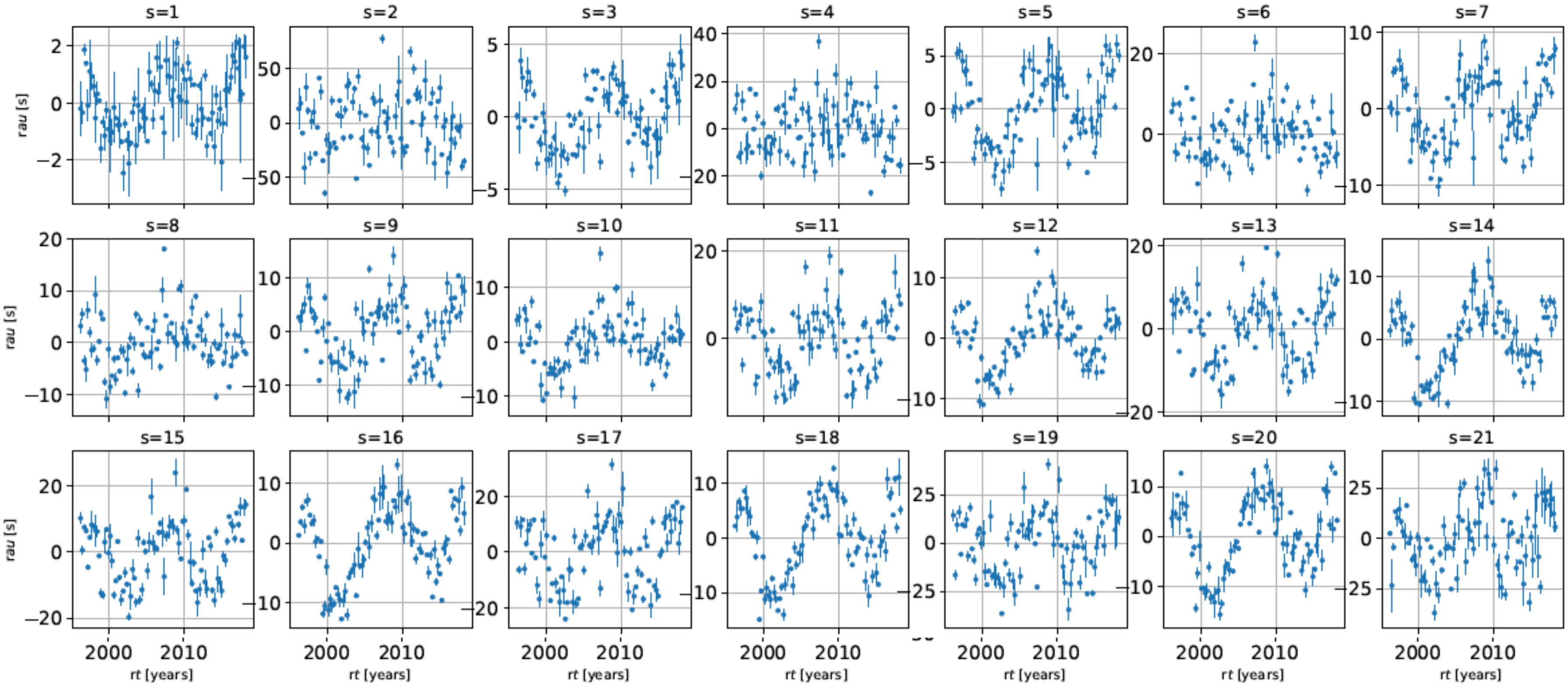
$$\delta\tau_s = \int W_s(t)[C(t) - C_{\mathrm{ref}}(t)]dt$$

$$W_s(t) = -\frac{f_s(t)dC_{\mathrm{ref}}/dt}{\int dt' f_s(t')[dC_{\mathrm{ref}}/dt']^2 dt'}$$

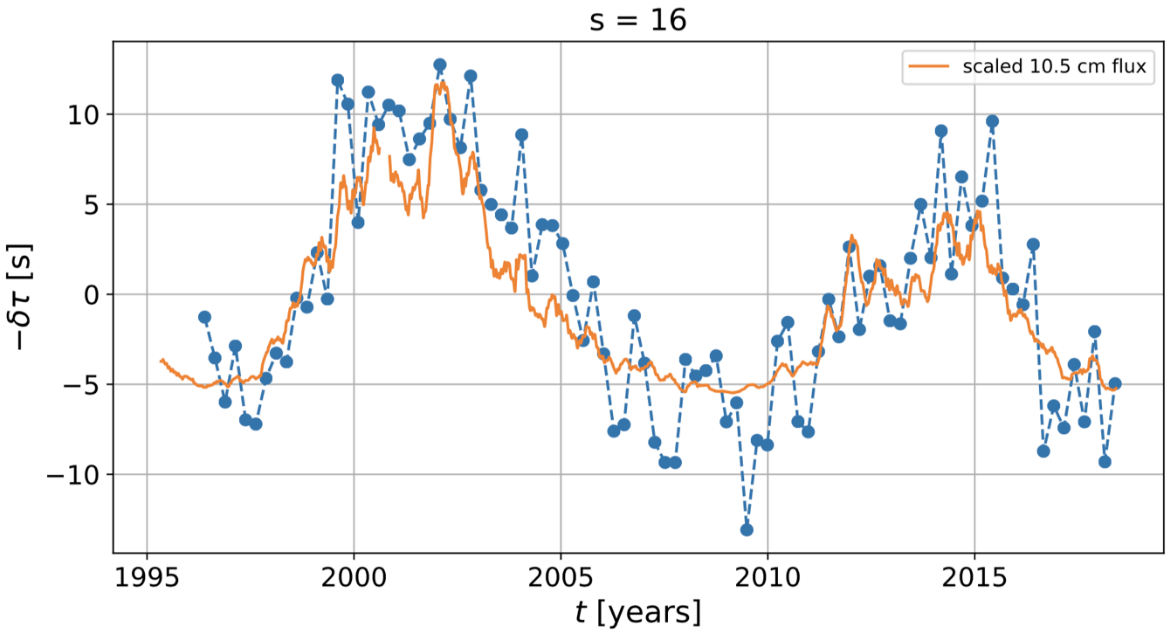
where $f_s(t)$ selects a window around the arrival time for skip s.

Asphericity: seismic travel times (Sun)

Solar-cycle dependence of travel times



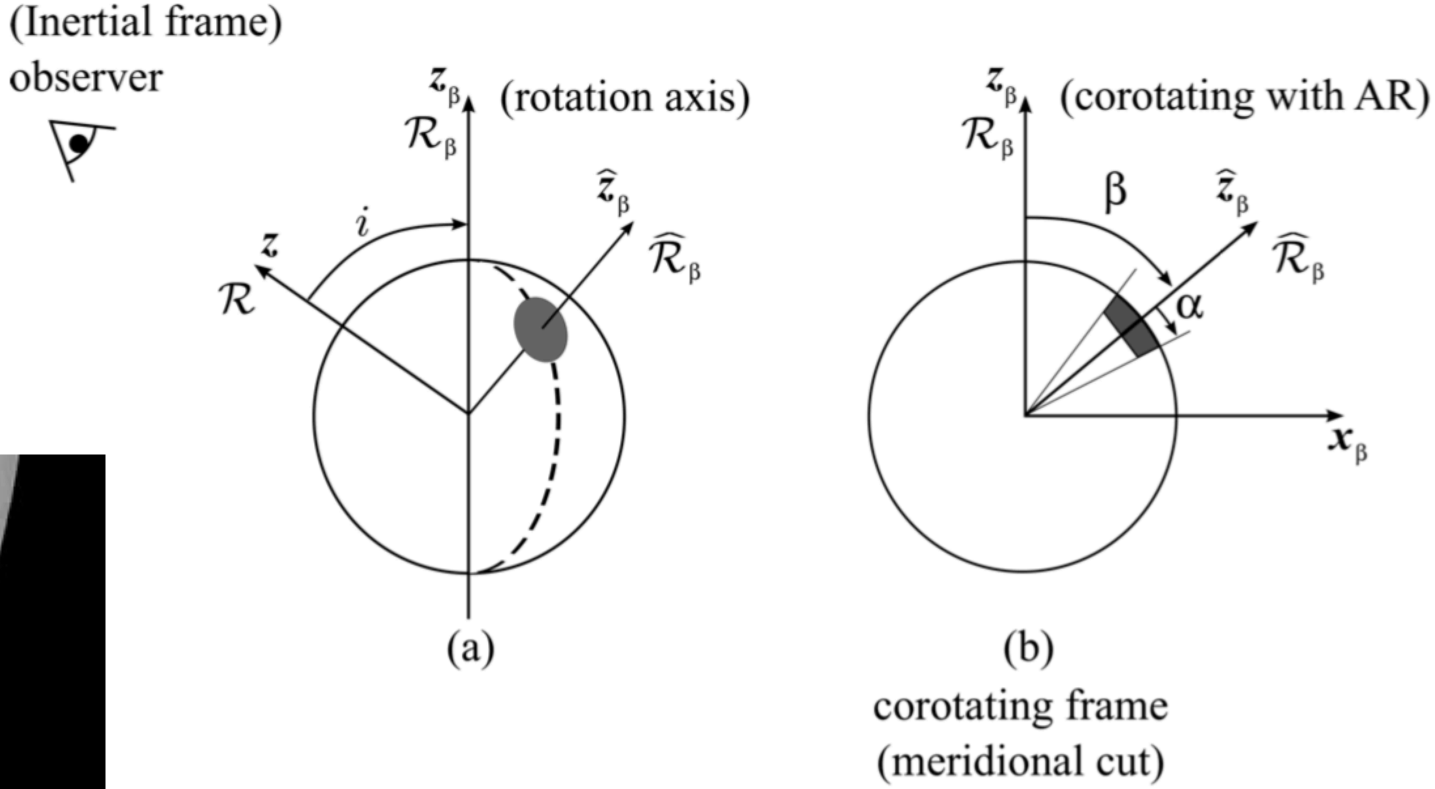
Asphericity: seismic travel times (Sun)

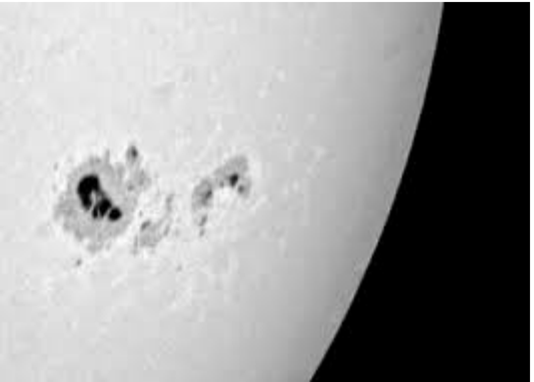


Multiple-skip travel times, taken all together, contain information about the butterfly diagram.

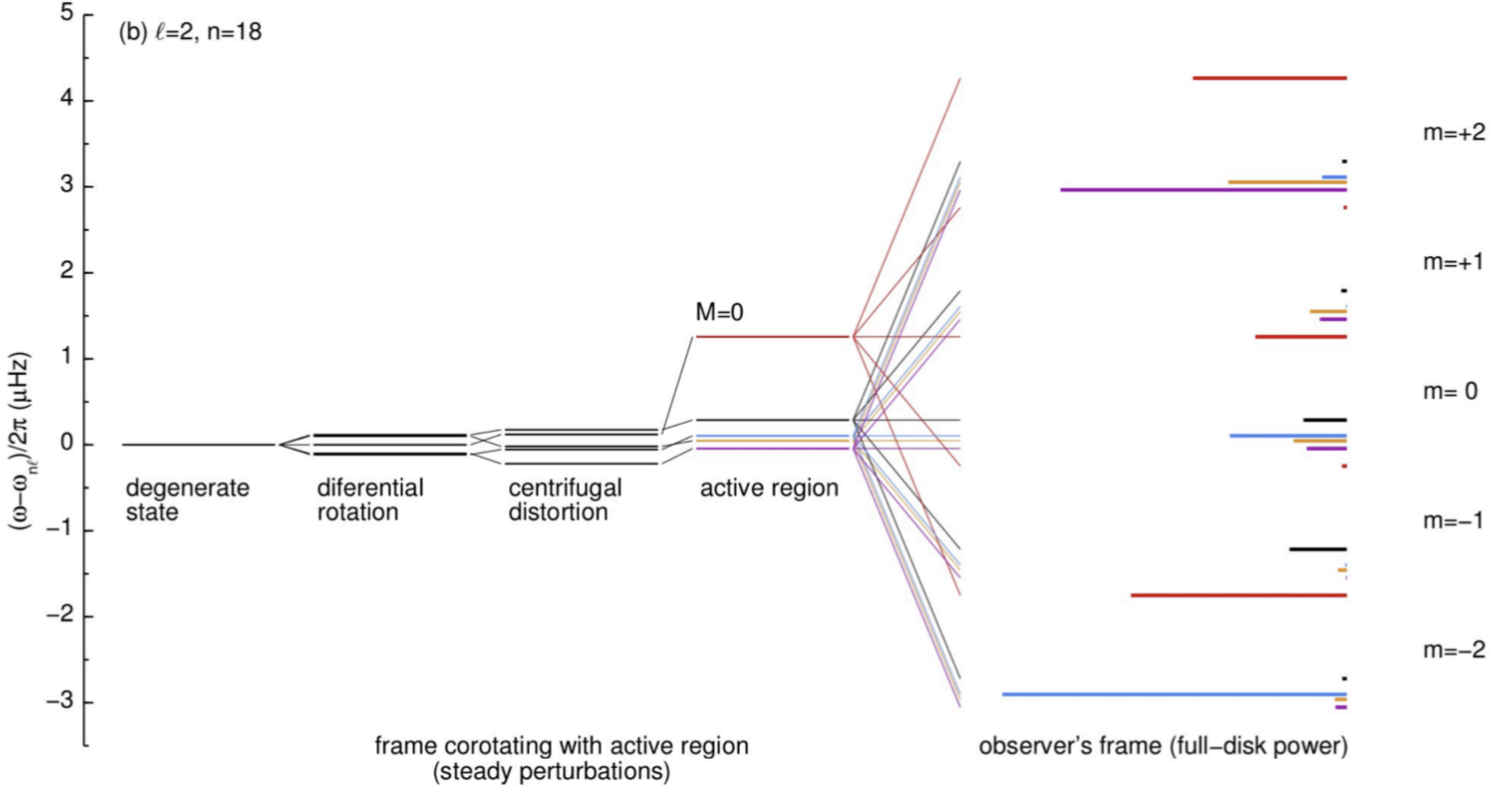
→ Apply methods of *local helioseismology*

Unsteady perturbations: single active region

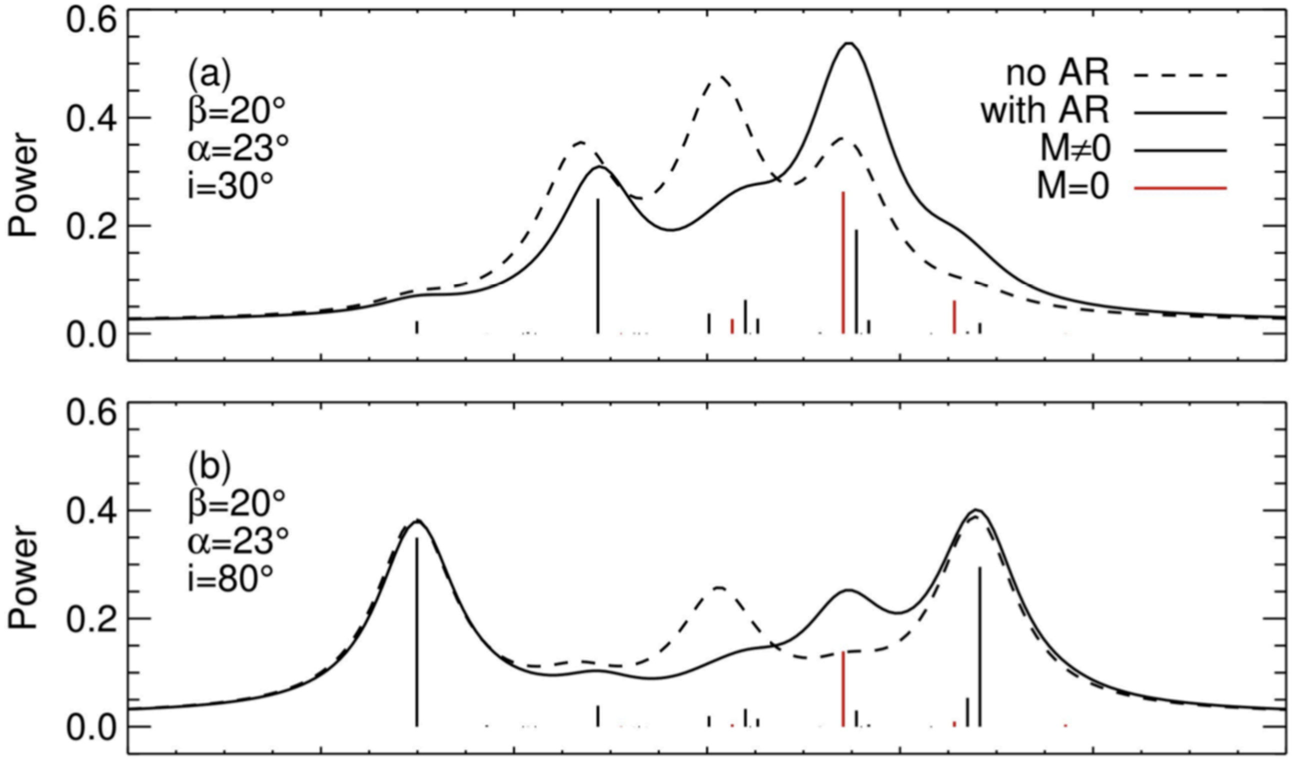




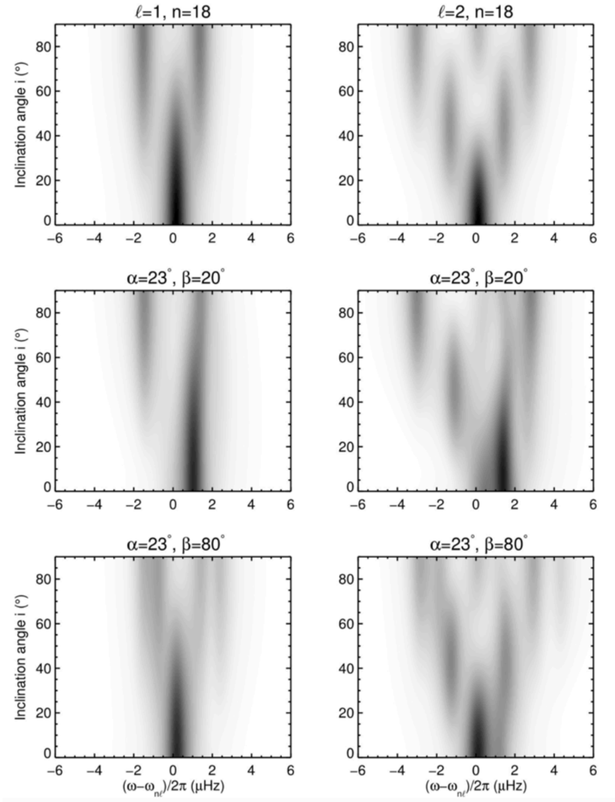
Unsteady perturbations: single active region

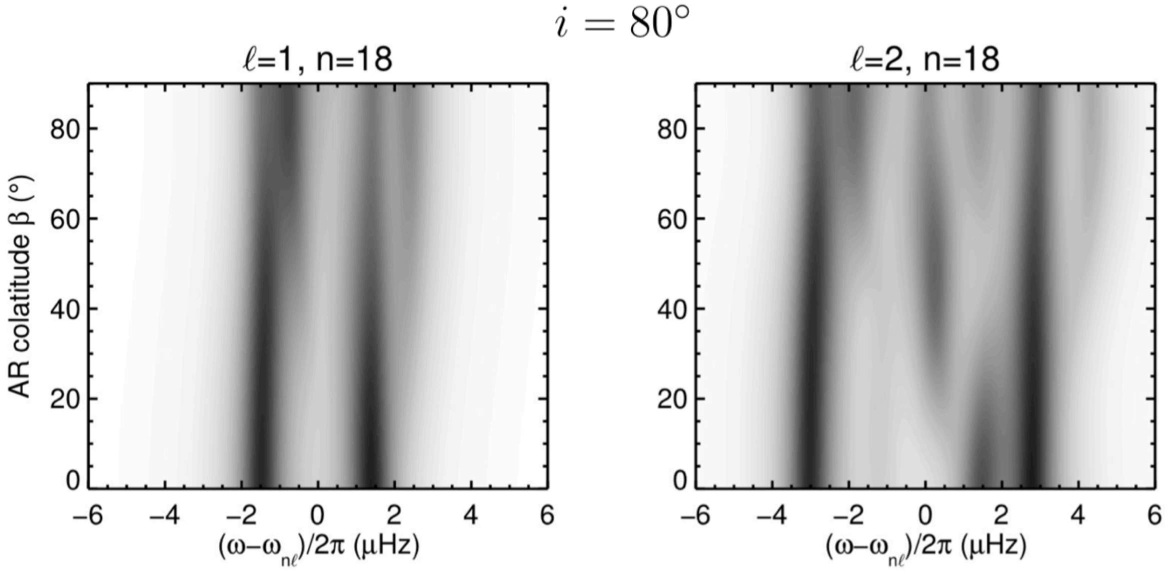


Unsteady perturbations: single active region



Unsteady perturbations: single active region





Summary

- The effects of stellar latitudinal differential rotation, rotational flattening, magnetic activity are measurable in the acoustic spectra of sun-like stars ($i>45$ deg).
- Based on knowledge from helioseismology, the proper way to measure these effects is to fit *a*-coefficients, not individual mode frequencies. (The *a*-coefficients cannot easily be computed from fitted azimuthal frequencies.)
- PLATO development: No need to worry about this immediately, but it may need to be included at some point in the future.
- Warning: It is not ruled out that some p-mode spectra will prove to be very difficult to interpret (cf. unsteady perturbations).