Results of Hare & Hound exercises for subgiants

PLATO STESCI Workshop – Barcelona – 11/2019

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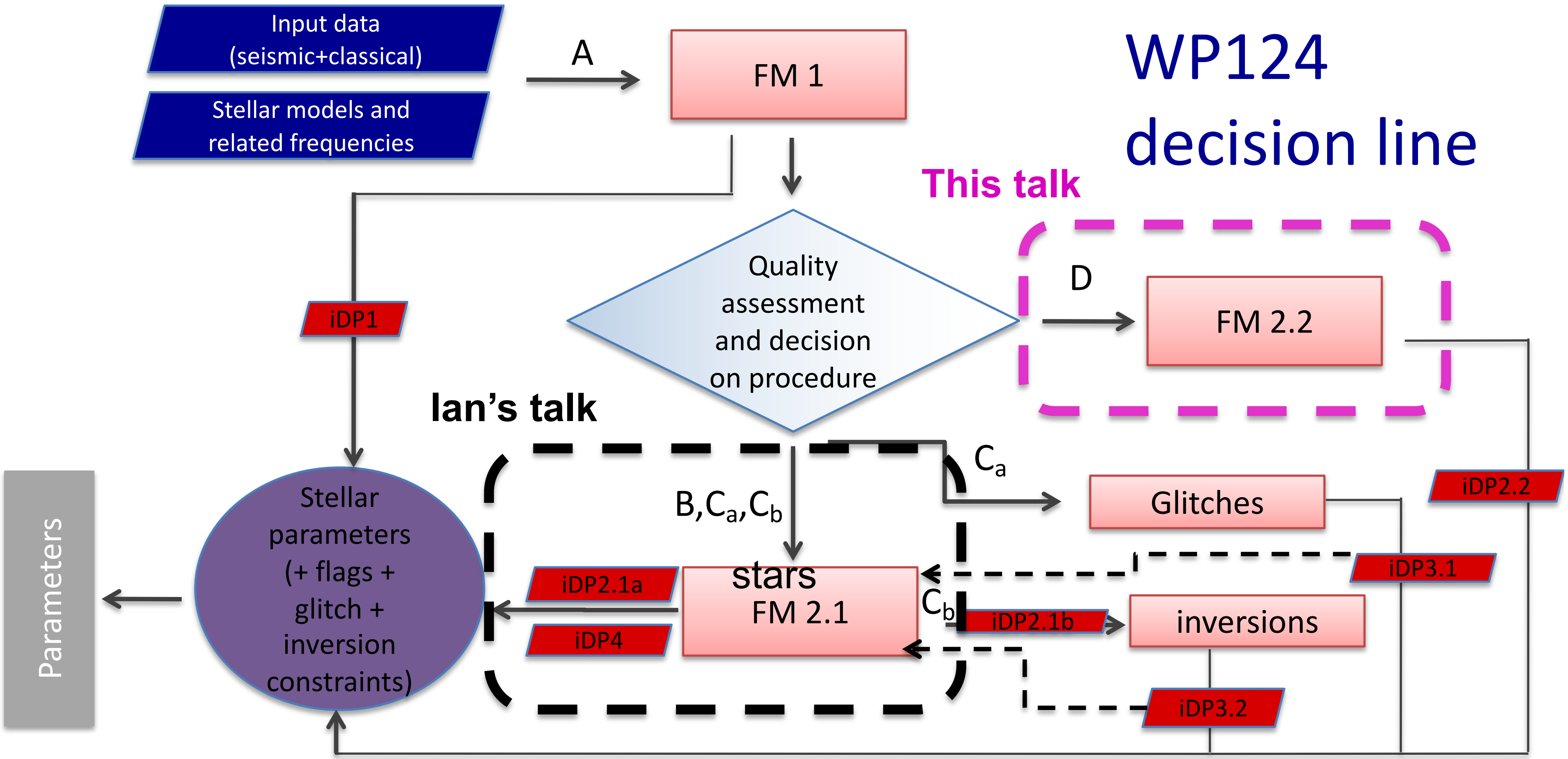
irap astrophysique & planétologie



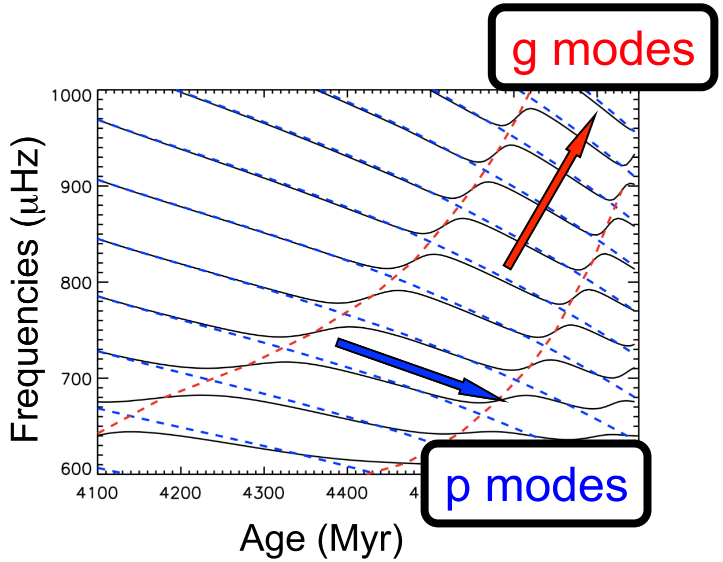
A: all stars with at least global seismic observations; B: all stars with individual modes frequencies; Ca: stars with no. radial orders > [tbd] / obs length > [tbd] for glitch analysis: C_b: stars with no. of frequencies > [tbd] for inversion; D: star with mixed modes.

FM 1: Forward modelling (FM) based on global seismic observations; FM2.1: FM based on individual frequencies + glitch and/or inversion constraints when available (with no mixed modes); FM 2.2: FM for stars showing mixed modes.

iDP1: [M, R, Age] from FM1 for all stars with seismic data; iDP2.1a: [M, R, Age] from FM2.1 for subsets B and C without glitch or inversion constraints; iDP2.1b: reference models for inversions; iDP2.2: [M, R, Age] from FM2.2 for subset D; iDP3.1: Glitch parameters; iDP3.2: products from inversions + flags; iDP4: [M, R, Age] from FM 2.1 including glitch and/or inversion constraints.

Mixed modes

- Mixed modes in subgiant stars



- Interest of mixed modes:
  - They are sensitive to the core structure
  - Their amplitudes are much larger then those of pure g modes
- Challenge of mixed modes:
  - Frequencies vary on very short timescale => traditional seismic optimization techniques need to be adapted
- Goal of the H&H exercises:
  - Chose an optimization procedure (not the models or the input physics)

Preparation of hares

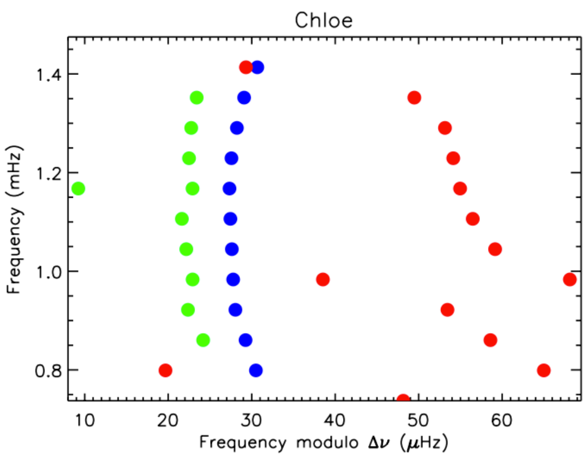
- Performed in Birmingham (W. Ball, B. Chaplin)

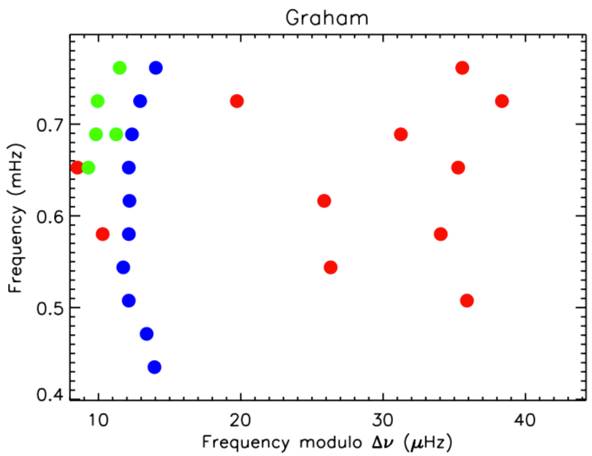
- Hares computed with
  - MESA evolution code (r10108)
  - Mode frequencies computed with ADIPLS

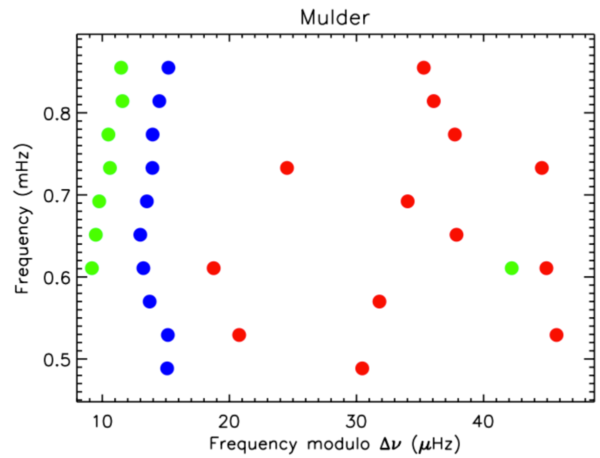
- Simplified input physics
  - Fixed mixing length parameter α_{MLT} = 1.84
  - Eddington grey atmosphere
  - Grevesse & Sauval (1998) solar mixture
  - OP opacities completed by Ferguson et al. (2005) at low T
  - Enrichment law ($Y = 1.4 \times Z + 0.248$)
  - No gravitational settling or radiative levitation
  - No convective core overshooting, rotational mixing…

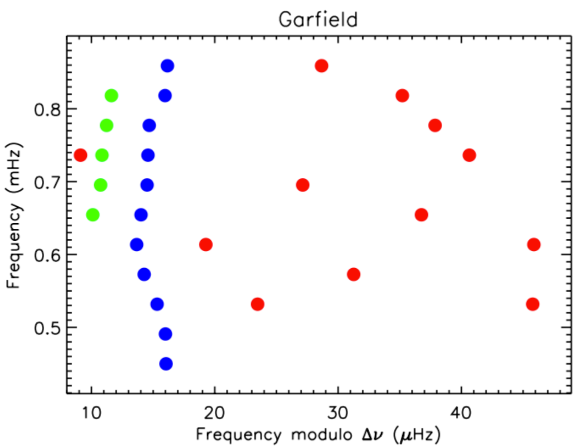
Preparation of hares

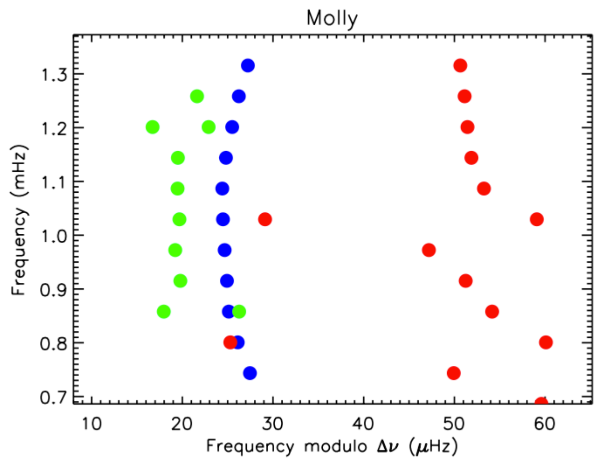
- Echelle diagrams of hares (l=0, l=1, l=2)

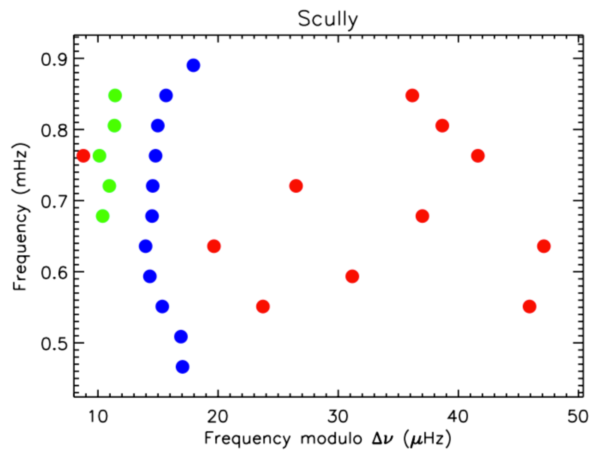












Preparation of hares

- Characteristics of hares

Star	Xini	Yini	Zini	M/Msun	t/Gyr	R/Rsun
Chloe	0.6883	0.2852	0.02655	1.0360	10.9866	1.7219
Garfield	0.7236	0.2646	0.01183	1.2565	4.0201	2.4032
Graham	0.7067	0.2744	0.01886	1.4366	2.8892	2.7272
Molly	0.6796	0.2902	0.03017	1.0882	9.1537	1.8365
Mulder	0.7132	0.2706	0.01616	1.3018	3.8441	2.4338
Scully	0.7022	0.2871	0.01077	1.1583	4.5294	2.2872

Radiative core
Convective core

Preparation of hares

- For three hares, slightly modified physics
  - **Molly**: includes gravitational settling
  - **Mulder**: VAL-C model for the atmosphere instead of Eddington grey
  - **Scully**: enhanced initial He abundance compared to enrichment law (+ 0.01)

- Simulation of near-surface effects
  - Cubic term from Ball & Gizon (2014) $$\delta\nu = \frac{a_3}{\mathcal{I}}\left(\frac{\nu}{\nu_{\mathrm{ac}}}\right)^3$$
  - Coefficients obtained from linear interpolation of the results of Ball & Gizon (2017) for Kepler low-luminosity red giants + the Sun

- Addition of normally-distributed random noise to classical and seismic observables (with expected errors for PLATO data, ranging from ~ 0.02 to ~ 0.4 μHz)

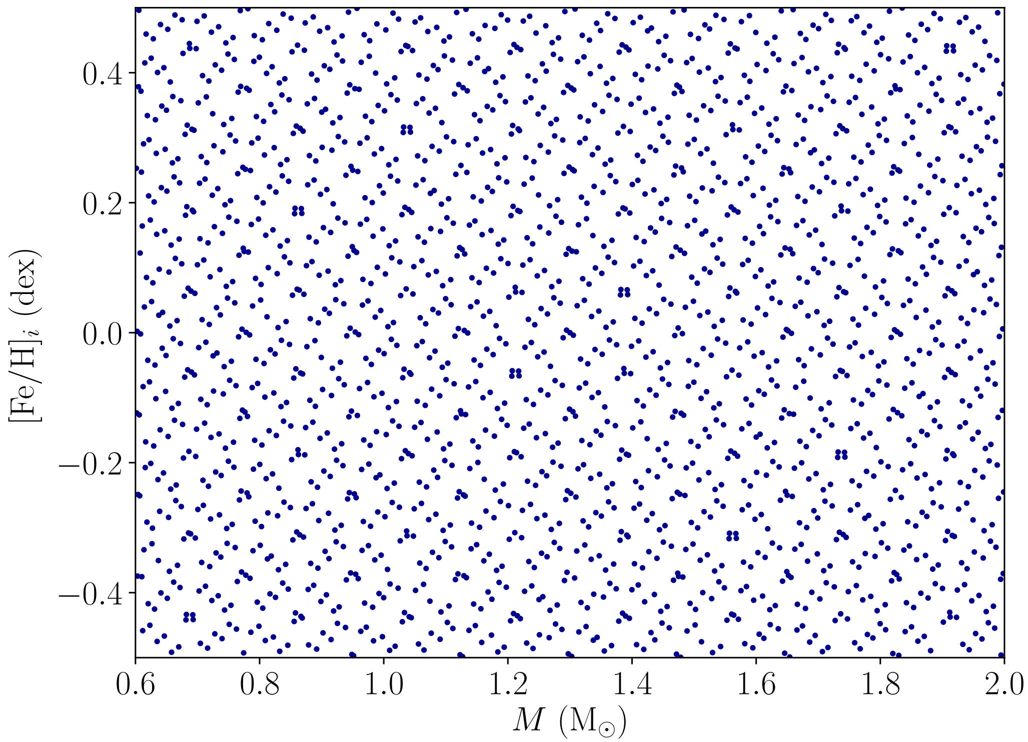
Hounds

- Same evolution & oscillation codes as those used to generate the hares (MESA + ADIPLS)
- Same input physics as the (regular) hares
- Two different approaches
  - **Grid modeling** using the pre-calculated grid (BASTA, AIMS)
  - Calculation of **models on-the-fly** (Levenberg-Marquardt algorithm)
- In principle the “true” model can be found (at least for the regular hares), so that we expect $E(\chi^2_{\mathrm{red}}) = 1$, where

$$\chi^2_{\mathrm{red}} = \frac{1}{N-p} \sum_{i=1}^{N} \frac{\left(x_i^{\mathrm{obs}} - x_i^{\mathrm{mod}}\right)^2}{\sigma_i^2}$$

Grid of models

- Grid of models calculated in Aarhus (K. Verma, V. Silva-Aguirre)
  - 2000 tracks
  - $0.6 < M/M_{\odot} < 2.0$
  - $-0.5 <$ [Fe/H] (dex) < 0.5



Optimizations with BASTA

- BASTA (Bayesian Stellar Algorithm, **Serenelli et al. 2013, Silva Aguirre et al. 2015**), computations by V. Silva Aguirre

Víctor Silva Aguirre

- Asteroseismology
  - 5 types of scaling relations, individual frequencies, 2 surface corrections, frequency ratios, frequency glitches, mixed modes in subgiants, period spacing in red giants
- Classical observables
  - Teff, [Fe/H], [M/H], [alpha/Fe], Parallax, luminosity, > 100 photometric bands
- 4 types of fit
  1. Using $\Delta\nu$, ν_{max}
  2. Using $\Delta\nu$, ν_{max} + luminosity
  3. Using individual freq
  4. Using freq + luminosity

Optimizations with AIMS

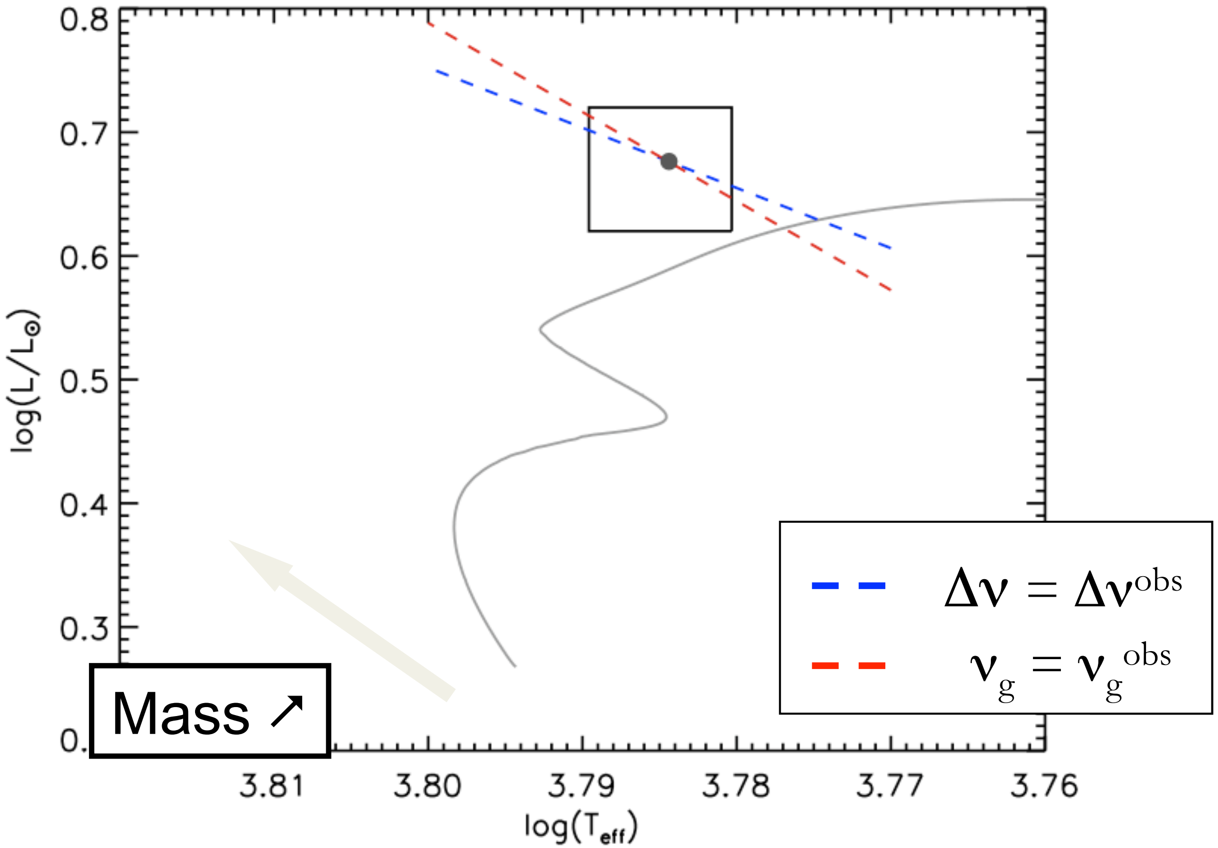
- AIMS (Asteroseismic Inference on a Massive Scale, **Rendle et al. 2019**), computations by B. Nsamba
- Bayesian statistics and MCMC approach
- Interpolations on a pre-calculated grid (between evolutionary tracks / along evolutionary tracks)
- Posterior PDFs for parameters are generated
- Correction for near-surface effects
  - Two-term correction from Ball & Gizon (2014) (cubic term + inverse term)
- 2 types of fit
  1. Using all mode frequencies
  2. Using only radial modes

On-the-fly modeling

- Optimization using the Levenberg-Marquardt algorithm (computations by A. Noll)
- Levenberg-Marquardt algorithm: mix between gradient-descent method (robustness to initial conditions) and Newton method (fast convergence)
- Errors on recovered parameters obtained using the inverse of the Hessian matrix
- Classical Levenberg-Marquardt algorithm impractical for subgiants with mixed modes

On-the-fly modeling

- *For a given physics*, the knowledge of $\Delta\nu$ and ν_g imposes one and one only mass **(Deheuvels & Michel 2011)**



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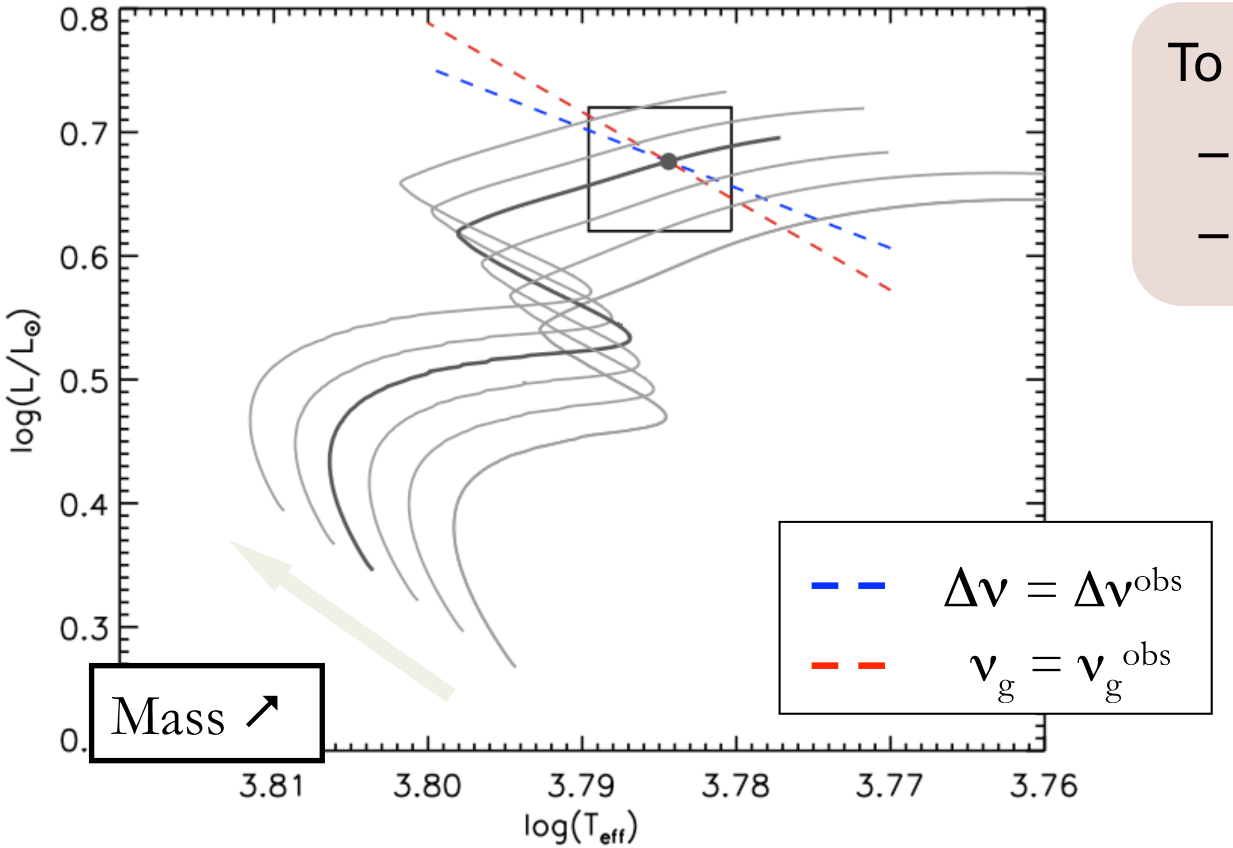
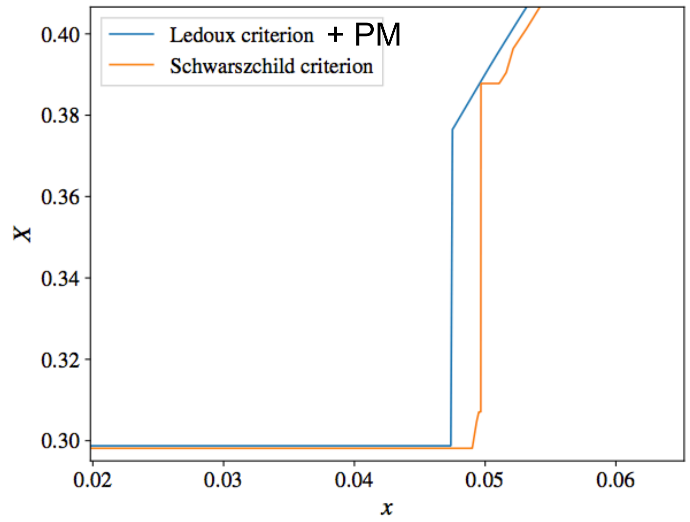


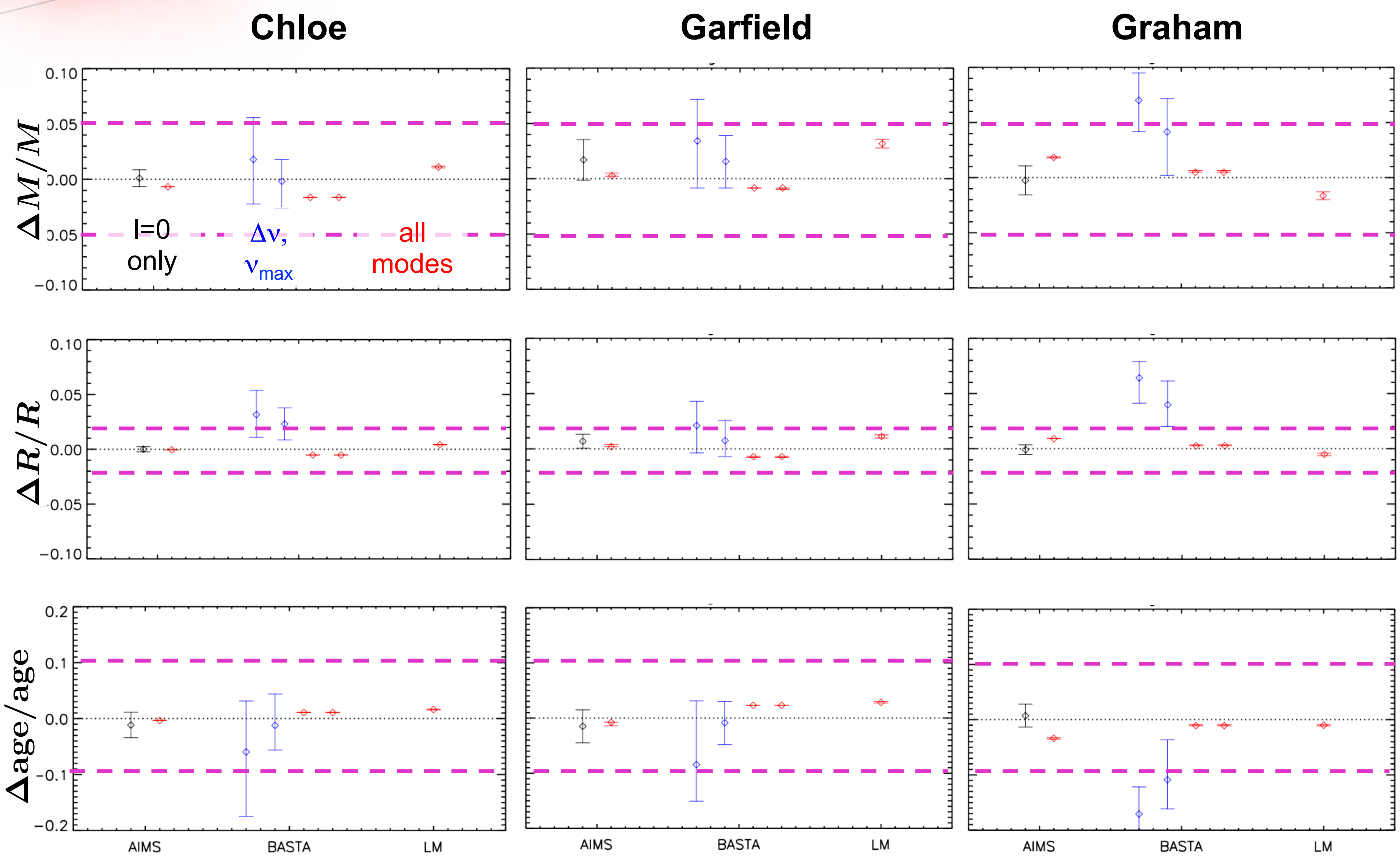
To verify both conditions
- one mass only ($\tilde{M}$)
- one age only ($\tilde{\tau}$)

On-the-fly modeling

- Nested optimization:
  - Global optimization for $(Z/X)_0$, Y_0
  - For each iteration, an optimization is performed using the radial modes and a g-dominated mode to determine the mass and age
- Treatment of the edge of convective cores
  - Schwarzschild criterion in MESA r10108: generates inconsistencies in temperature gradient at core edge + transient convective layers above (Paxton et al. 2018)
  - Problematic for LM method, which requires to calculate derivatives wrt stellar mass
  - We used the "predictive mixing" for stars with convective cores



“Regular” hares



Goodness of fit

- Reduced χ^2

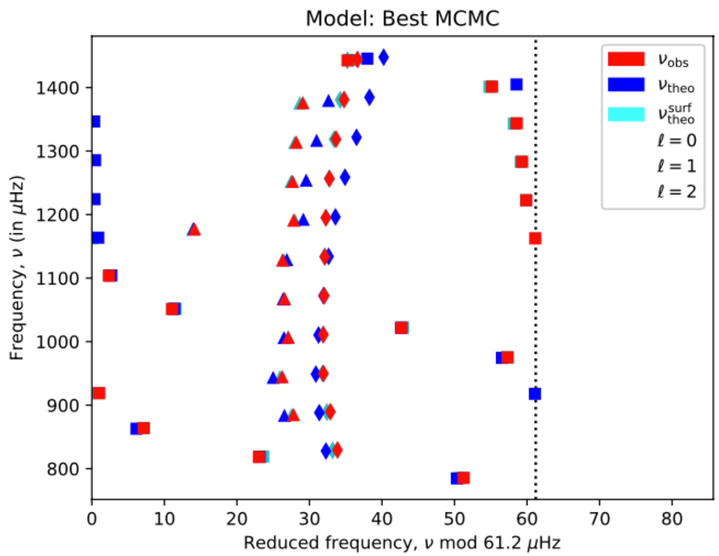
Hare	Physics	χ^2_{red} BASTA[1]	χ^2_{red} AIMS[1]	χ^2_{red} LM[2]
Chloe	reg	60.9	106.4	1.7
Garfield	reg	3.2	6.8	1.6
Graham	reg	9.1	108.6	1.1
Molly	+ grav. sett.	155.1	24.9	15.8
Mulder	VAL-C atm	22.7	74.3	12.7
Scully	Y_0 + 0.01	8.2	8.6	1.6

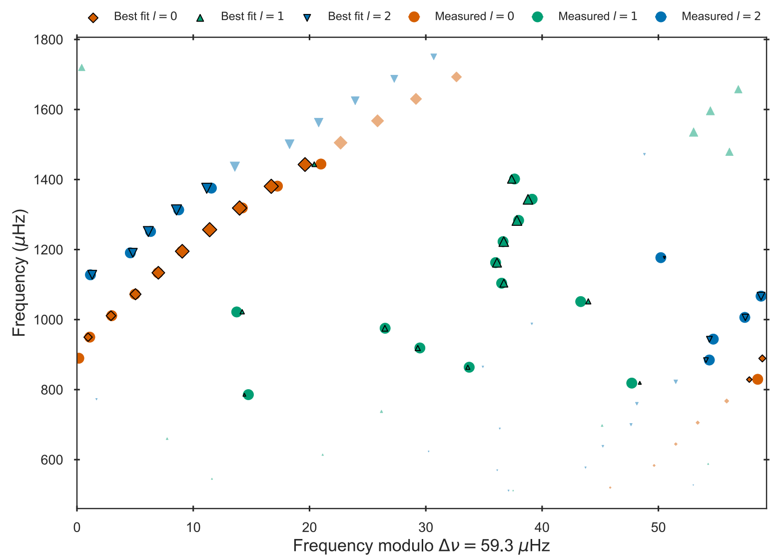
$$(1): \quad \chi^2_{\mathrm{red}} = \frac{1}{N-1}\sum_{i=1}^{N}\frac{\left(\nu_i^{\mathrm{obs}}-\nu_i^{\mathrm{mod}}\right)^2}{\sigma^2_{\nu_i}} + \sum_{j=1}^{M}\frac{\left(X_j^{\mathrm{obs}}-X_j^{\mathrm{mod}}\right)^2}{\sigma^2_{X_j}}$$

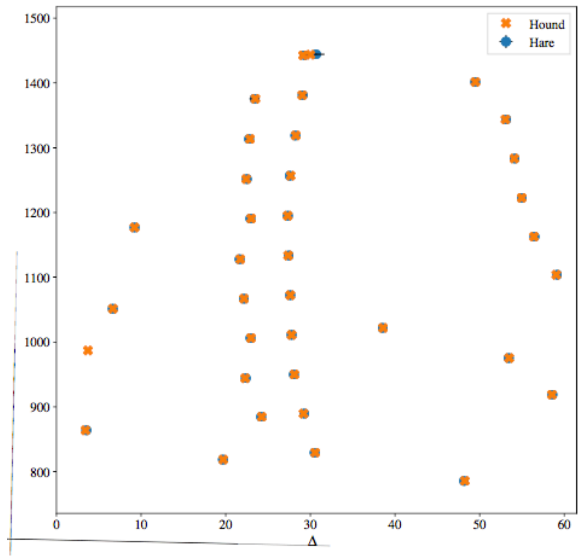
$$(2): \quad \chi^2_{\mathrm{red}} = \frac{1}{N+M-p}\left[\sum_{i=1}^{N}\frac{\left(\nu_i^{\mathrm{obs}}-\nu_i^{\mathrm{mod}}\right)^2}{\sigma^2_{\nu_i}} + \sum_{j=1}^{M}\frac{\left(X_j^{\mathrm{obs}}-X_j^{\mathrm{mod}}\right)^2}{\sigma^2_{X_j}}\right]$$

“Regular” hares

- Echelle diagrams of Chloe







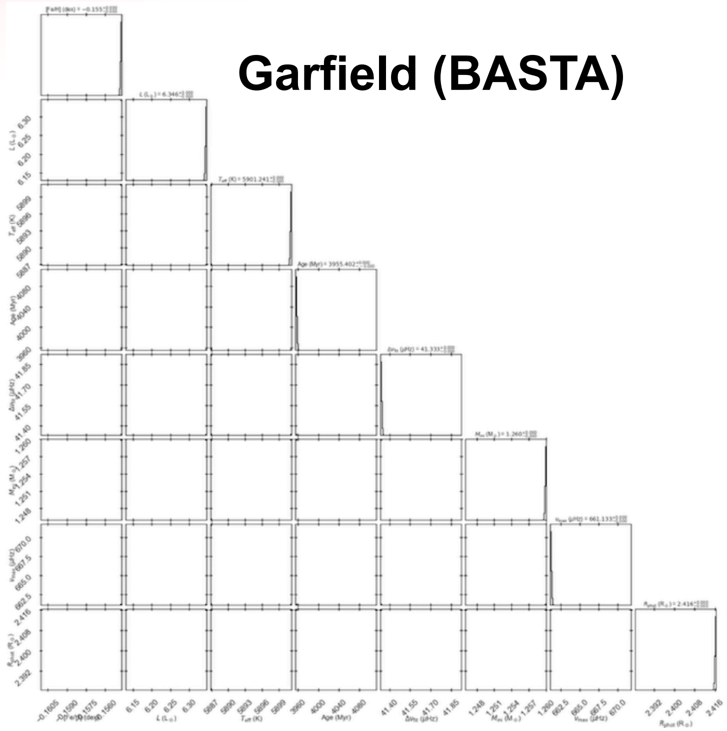
Accuracy

- Why is **grid modeling** not accurate? Possible answers:
  - Could the correction for near-surface effects play a role? (Ball & Gizon 2014 two-term correction for BASTA, AIMS, cubic-term only for the hares)
  - Despite the very fine meshing of the grid, no grid points lie within the valley where $\chi^2_{red} \sim 1$ in parameter space. Algorithm picks closest model to the valley, and error bars can't be correctly evaluated

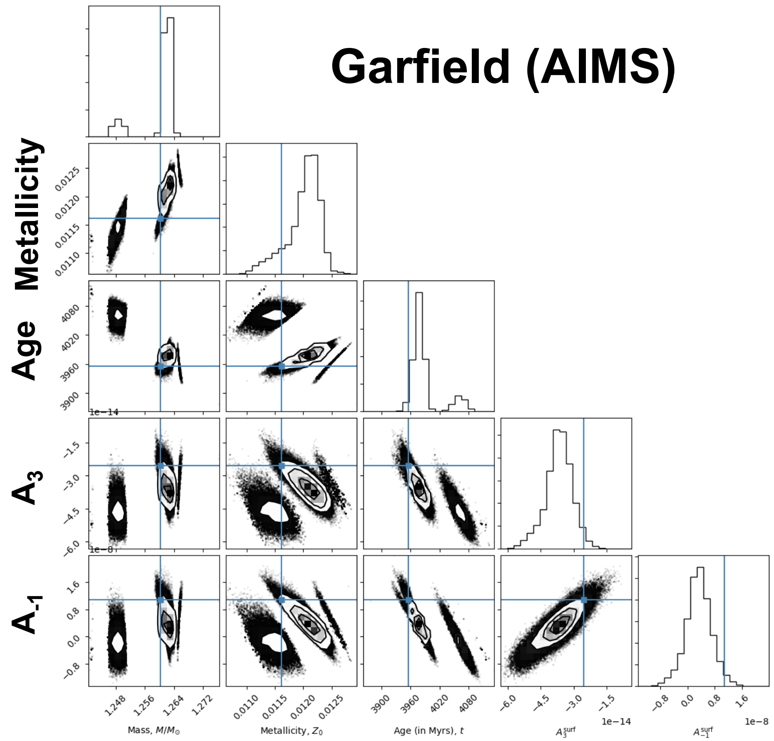
- Why is **on-the-fly modeling** not accurate?
  - Maximal time step during the MS = 10 Myr (1 Myr for the hares). When reducing this parameter, the bias seems to vanish for Chloe (optimization still running…)
  - Different treatment of convective core boundary for stars that have one

PDF from grid modeling

Garfield (BASTA)



Garfield (AIMS)



- Bimodality in solutions
  - Mesh issue?
  - Related to convective core (bimodality important for stars with CC)?

Summary for "regular" hares

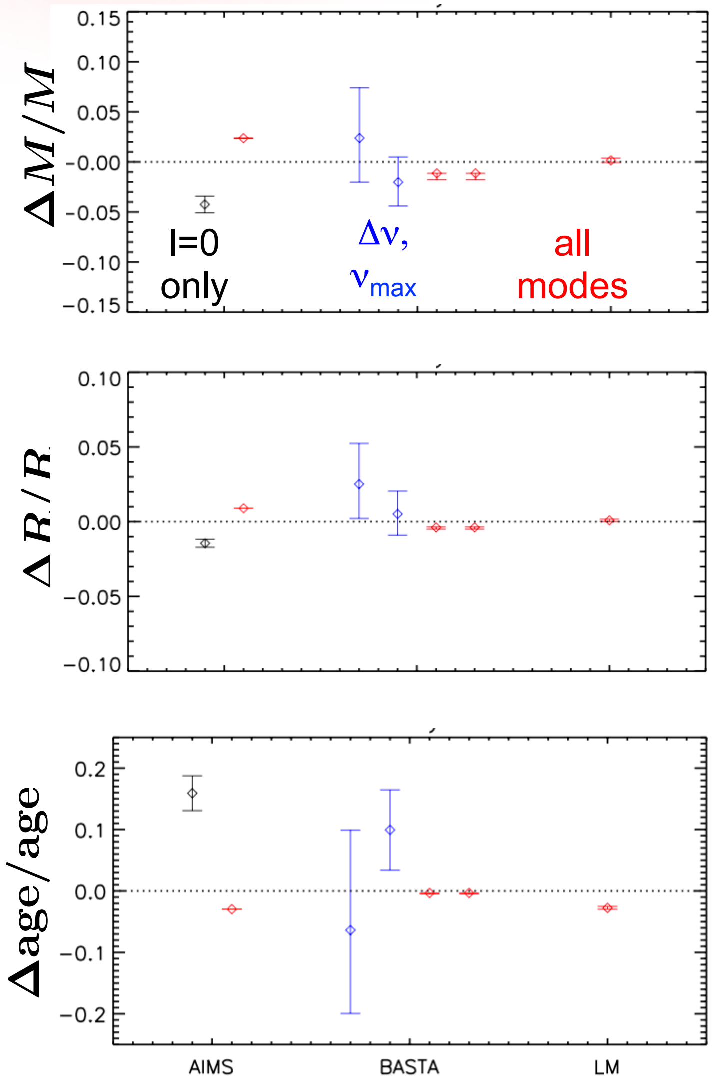
Seismic modeling using **mixed modes**

- Grid modeling
  - Small differences with "true" parameters(< 4% in mass, < 2% in radius, <3% in age), but…
  - … Optimal models show poor statistical agreement with "observations"
  - … Error bars are underestimated
- On-the-fly modeling
  - Accuracy comparable to grid modeling, but not better. Results appear to biased (partly understood)
  - Optimal models are in statistically good agreement with "observations"

Radial modes only:

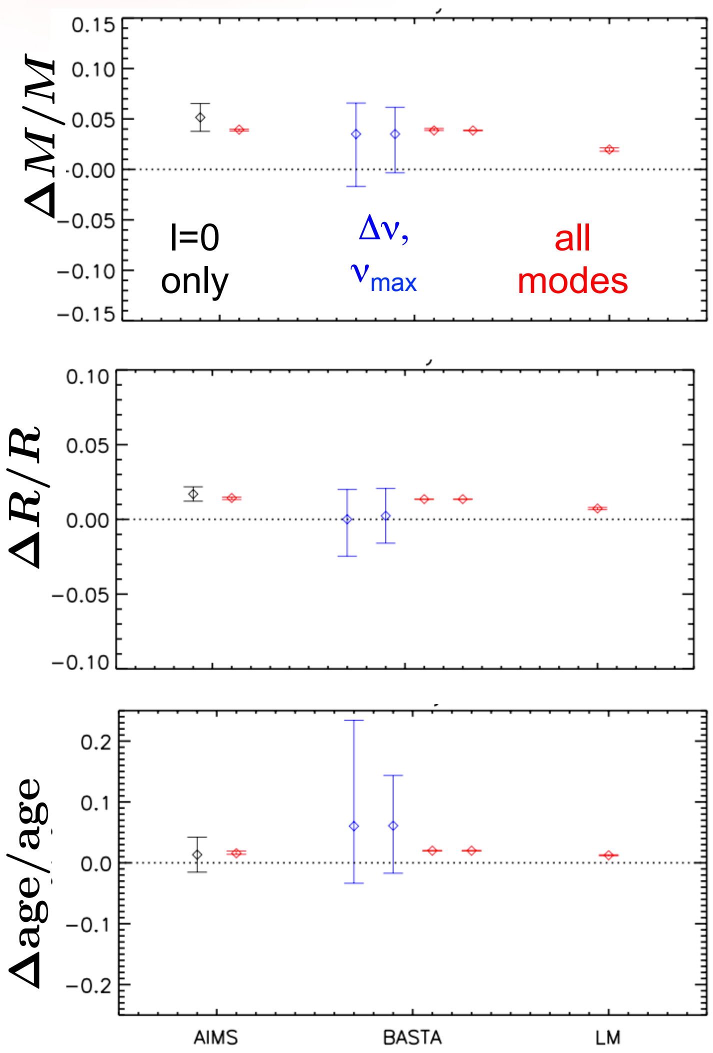
  - Good accuracy, precision < 4% in mass, < 2% in radius, <5% in age, but with same physics as the hares!
  - Performance needs to be evaluated when this is not the case (see "peculiar" hares)

Molly



- Molly (includes gravitational settling)
- With l=0 modes only (AIMS), age is overestimated (well known effect of neglecting diffusion)
- With mixed modes, the bias disappears
  - Best solutions have higher Z/X than "observed" surface value (0.0430 for LM, compared to 0.0302 for obs)
  - Mixed modes are sensitive to the metallicity in the core
  - Solutions with / without microscopic diffusion similar in Deheuvels & Michel (2011)

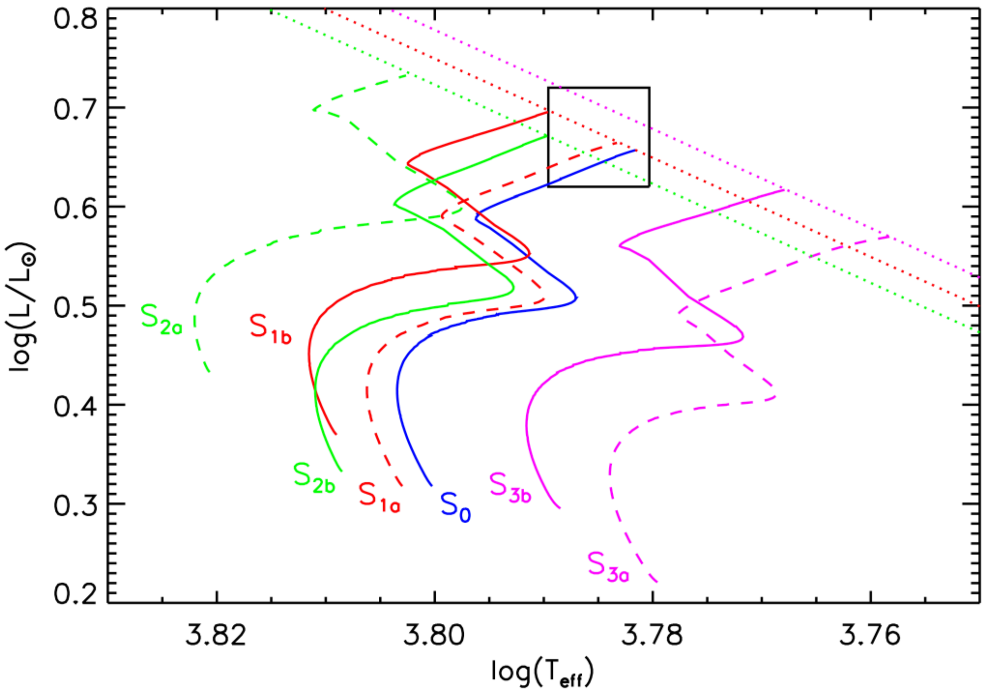
Scully

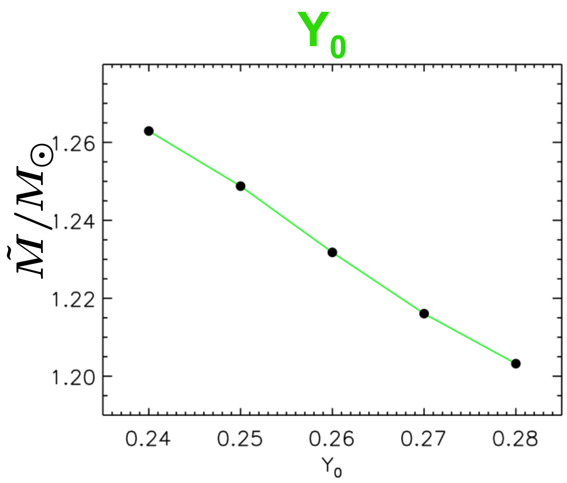


- Scully (helium differs from enrichment law + 0.01)
- Mass and radius overestimated for grid modeling, less the case for on-the-fly modeling (Y_0 is free)
- Age seems unaffected…
  - Robustness of age estimates in subgiants (Li et al. 2019)?

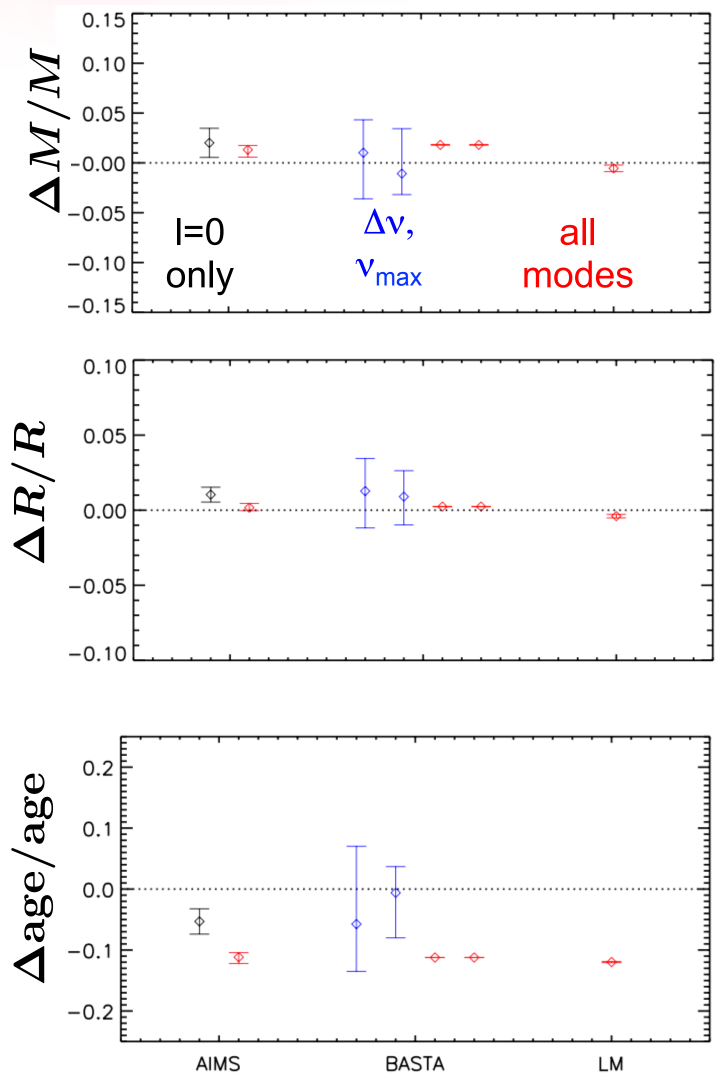
How does $\tilde{M}$ vary with stellar parameters?

- Variations in $\tilde{M}$ for varying **mixing length** parameter α_{conv}, initial **helium** Y_0, and **metallicity** (Deheuvels & Michel 2011)

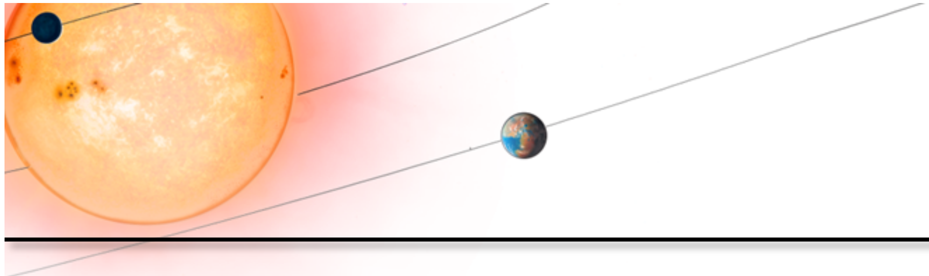




Mulder



- Mulder (VAL-C model for atmosphere)
- Mass and radius seem fine (mass slightly overestimated for BASTA)
- Age underestimated by ~ 10% for all modelers using mixed modes!
  - Effect of a wrong α_{MLT}? (Solar-calibrated α_{MLT} is different with VAL-C) **but** then the mass would be wrong…
  - Effects of near-surface correction?

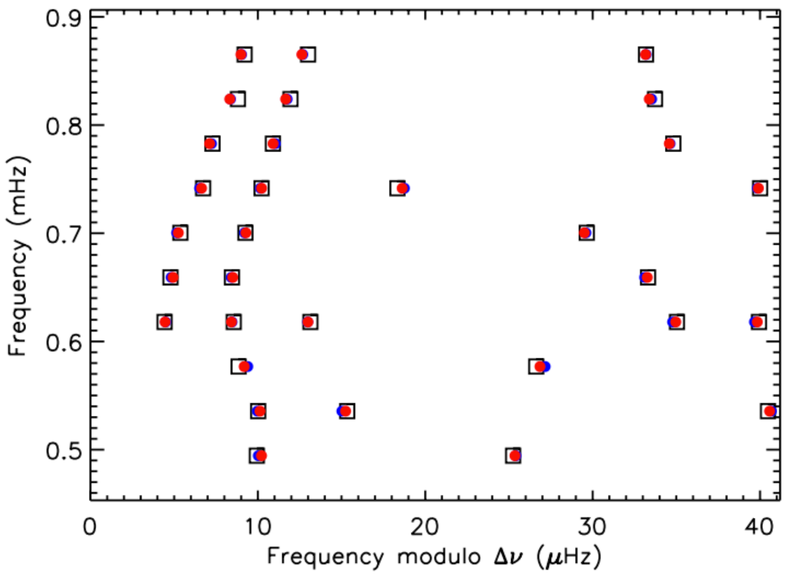


Mulder

- Two tests:
  1. Re-fit Mulder using "observed" mode frequencies without any simulation of near-surface effects (AIMS, LM)

     ⇒ Solutions very similar to original case

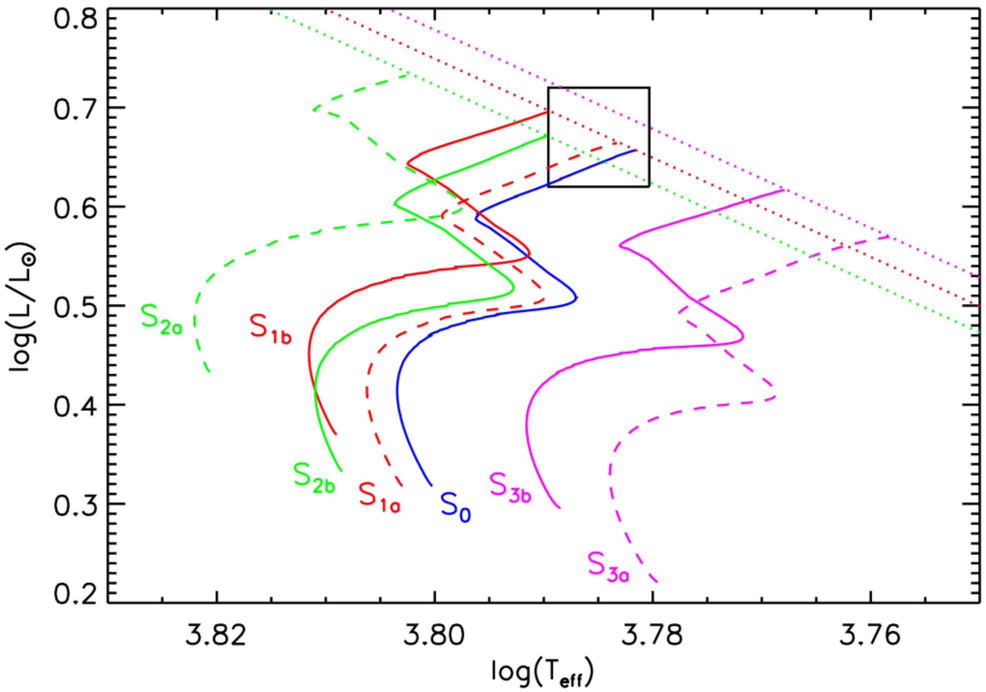
  2. Re-fit Mulder with a free α_{MLT} (LM)

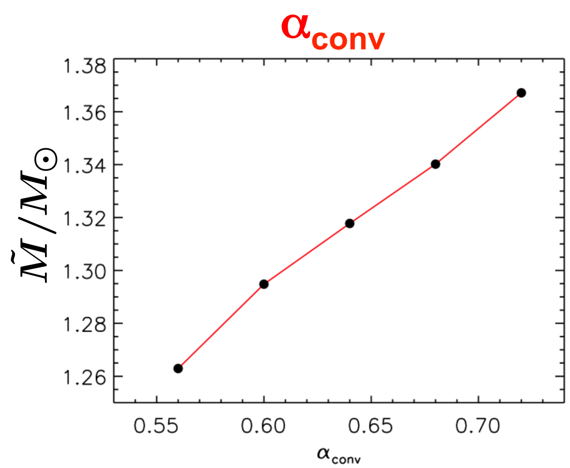


- Solution with α_{MLT} = 1.84 (fixed), χ^2_{red} = 12.7
- Solution with free α_{MLT} : χ^2_{red} = 3.2
  - Age = 3.82 Gyr (true = 3.84)
  - Mass = 1.26 $M_\odot$ (true = 1.30 $M_\odot$)
  - Radius = 2.41 $R_\odot$ (true = 2.43 $R_\odot$)

How does $\tilde{M}$ vary with stellar parameters?

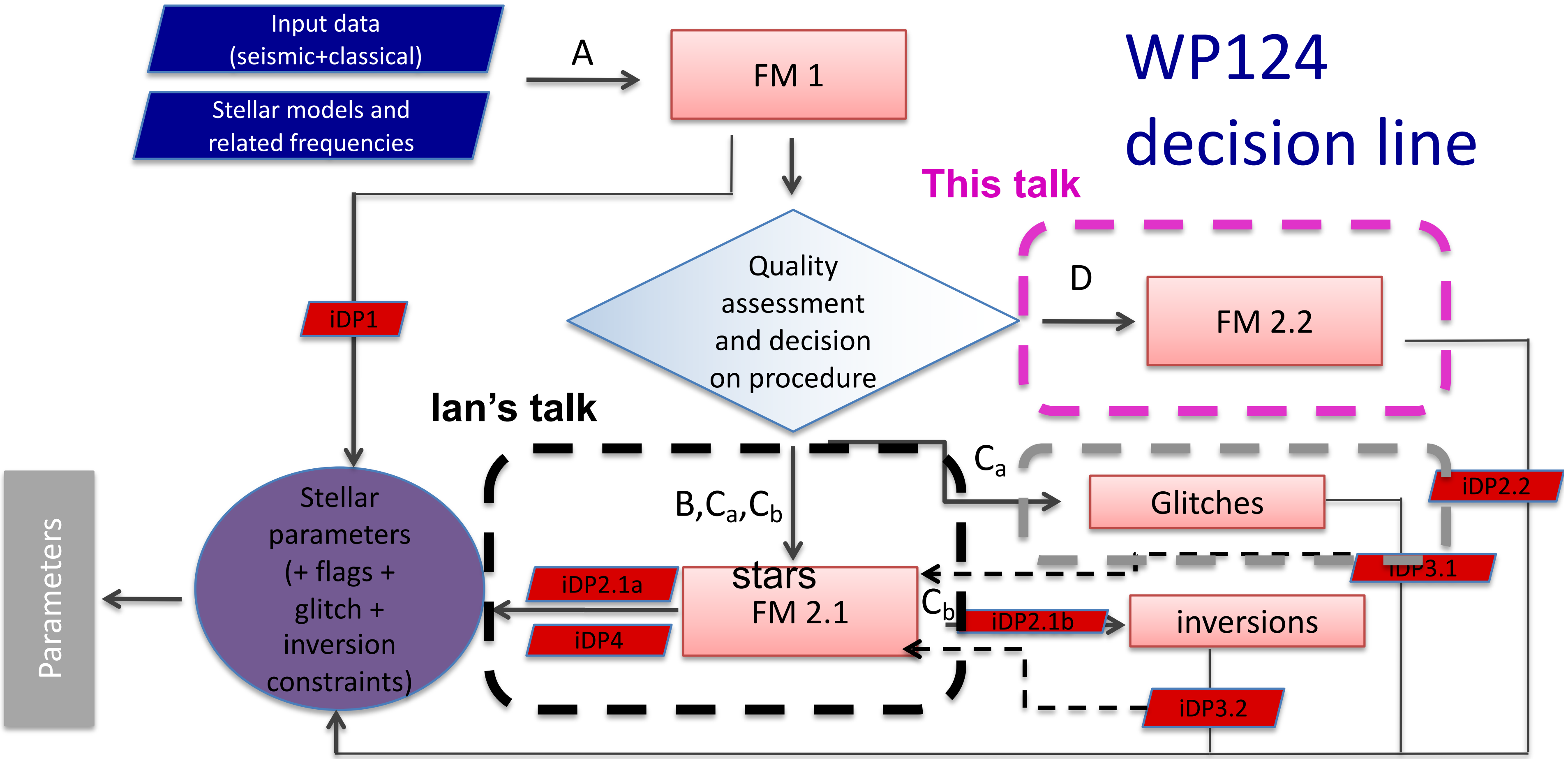
- Variations in $\tilde{M}$ for varying **mixing length** parameter α_{conv}, initial **helium** Y_0, and **metallicity** (Deheuvels & Michel 2011)





Conclusions

- So far, optimization procedures using mixed modes for subgiants (grid modeling, on-the-fly modeling) show biases
  - Work currently performed to understand (solve?) these issues
  - **Grid modeling**: perform interpolation in structure btw grid points?
  - Biases partly understood for **modeling on-the-fly** (max time step), but computation is time-consuming!
- Even when biases are removed, significant work will be needed to evaluate the impact of systematics
- Using radial modes only?
  - A bit frustrating to get rid of constraints from mixed modes…
  - Need to evluate the accuracy reached with a more thorough (Y_0, diffusion, α_{MLT})
  - Add the frequency of one g-dominated mode (weakly affected by near-surface effects, strong dependence on core properties)?



- WP124300 on glitches
  - H&H exercise completed to test how well we recover glitch properties (He ionization zone, base of convective envelope)
  - Second set of tests will start soon to determine how glitch properties can be included in stellar modeling

Calculation of χ^2

- What to choose for χ^2 (not specific to subgiants)?
  - Use regular χ^2, for N seismic observables and M classical observables

$$\chi^2 = \sum_{i=1}^{N} \frac{\left(\nu_i^{\rm obs} - \nu_i^{\rm mod}\right)^2}{\sigma_{\nu_i}^2} + \sum_{j=1}^{M} \frac{\left(X_j^{\rm obs} - X_j^{\rm mod}\right)^2}{\sigma_{X_j}^2}$$

    with an expected value of N+M-p, where p is the nb of free parameters.
    Goodness of fit given by reduced χ^2 with expected value of 1

  - Apply weights to the seismic χ^2, generally justified by the existence of physical correlation between observables

$$\chi^2_{\rm red} = \frac{1}{N-1} \sum_{i=1}^{N} \frac{\left(\nu_i^{\rm obs} - \nu_i^{\rm mod}\right)^2}{\sigma_{\nu_i}^2} + \sum_{j=1}^{M} \frac{\left(X_j^{\rm obs} - X_j^{\rm mod}\right)^2}{\sigma_{X_j}^2}$$

- Impact of physical correlation btw seismic observables
  - Assume that all mode frequencies are determined only by <ρ>

Calculation of χ^2

- Impact of physical correlation btw seismic observables
  - Assume that all mode frequencies are determined **only** by $<\rho>$ (1st order asymptotics is exact), then we have N measurements ρ_i of $<\rho>$

$$\chi^2 = \sum_{i=1}^{N} \frac{(\rho_i - \langle\rho\rangle_{\rm mod})^2}{\sigma^2} + \sum_{j=1}^{M} \frac{\left(X_j^{\rm obs} - X_j^{\rm mod}\right)^2}{\sigma_{X_j}^2}$$

  - If $<\rho>$ is off by 3 σ, $E(\chi^2) = 10*N+M$ (much larger than if $T_{\rm eff}$, or Z/X is off by 3 σ)

  - But in fact, $<\rho>$ is measured much more precisely than $T_{\rm eff}$ of Z/X. The quantity $\mu = \sum_i \rho_i \times (1/N)$ measures $<\rho>$ with an error of $\sigma/\sqrt{N}$

$$\chi^2 = N \times \frac{(\mu - \langle\rho\rangle_{\rm mod})^2}{\sigma^2} + \sum_{j=1}^{M} \frac{\left(X_j^{\rm obs} - X_j^{\rm mod}\right)^2}{\sigma_{X_j}^2}$$

  - Assuming a weight of 1/N for the seismic χ^2 corresponds to assuming that $<\rho>$ is measured with an error of σ instead of $\sigma/\sqrt{N}$

“Peculiar” hares