PLATO Hare and Hounds Main-Sequence Model Fitting

Aim:
To test how well one can recover the properties of model "observed" stars by astro-seismology, searching for fits in a data base of models; particularly M, R, age.

Input data from Hares: 6 "observed" stars: L, Teff, [Fe/H], $\Delta\nu$, ν_{max} and frequencies (MESA models ADIPLS frequencies - Birmingham)

Model set: 3 million models L, Teff, Fe/H, $\Delta\nu$, ν_{max} , M, R, age, Z, Y, α_{MLT}, ov_{core} , <rho>, logg, frequencies (MESA models ADIPLS frequencies - Aarhus)

6 Hounds: Basu (Yale), Nsamba (Porto), Ong (Yale), Reese (Paris), Roxburgh (London, Bham), Silva-Aguirre (Aarhus), using their preferred method(s)

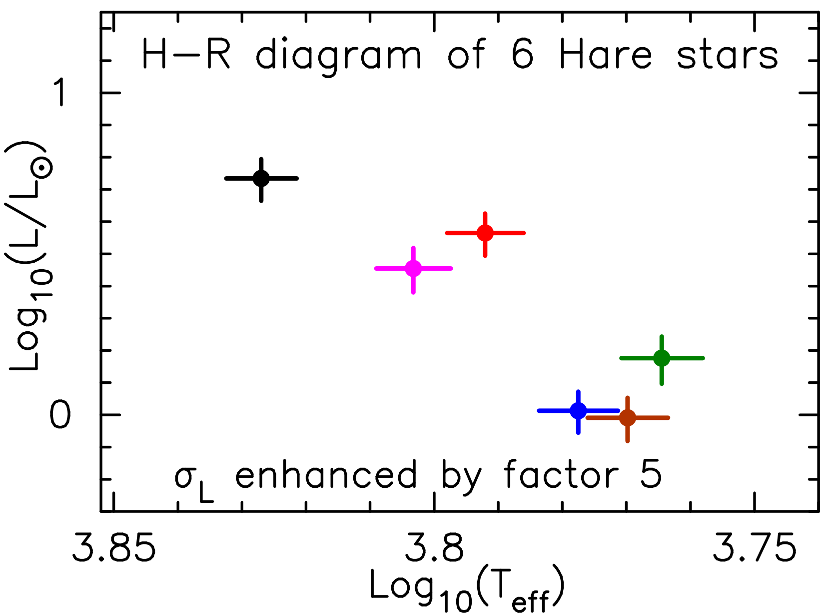
Hound output: mean, st dev, 16, 50, 84 percentiles of M, R, age, Z, Y, α, ov, $<\rho>$, logg

Ian Roxburgh QMUL & UoB

Hare input data: 6 stars (Ball, Chapman, Bham)

Input data from Hares: L, Teff, [Fe/H], $\Delta\nu$, ν_{max} and frequencies –all with uncertainties. [MESA models ADIPLS frequencies, Ball, Chaplin (Bham)]

Parameters not exactly those of any model in Hounds model set. Realization of errors, some function of ν added to model frequencies. Example "Gerald"



$L/L_\odot$=1.50±0.05 T_{eff}=5814±85 [Fe/H]= 0.03±0.09

$\Delta\nu = 106.3 \pm 2.1$ $\nu_{max} = 2207 \pm 108$

n	$\nu_{n,0}$	$e\nu_{n,0}$	$\nu_{n,1}$	$e\nu_{n,1}$	$\nu_{n,2}$	$e\nu_{n,2}$
14	0.00	0.00	1650.16	0.29	0.00	0.00
16	1807.47	0.15	1854.06	0.11	1904.16	0.19
17	1910.07	0.11	1957.02	0.09	2008.10	0.17
18	2013.11	0.10	2060.37	0.09	2111.20	0.17
19	2116.48	0.11	2164.05	0.09	2214.53	0.19
20	2219.29	0.12	2267.16	0.11	2318.34	0.25
21	2322.75	0.16	2370.64	0.16	2421.70	0.38
22	2425.89	0.24	2474.73	0.27	2527.13	0.72
23	2529.59	0.46	2579.56	0.58	0.00	0.00

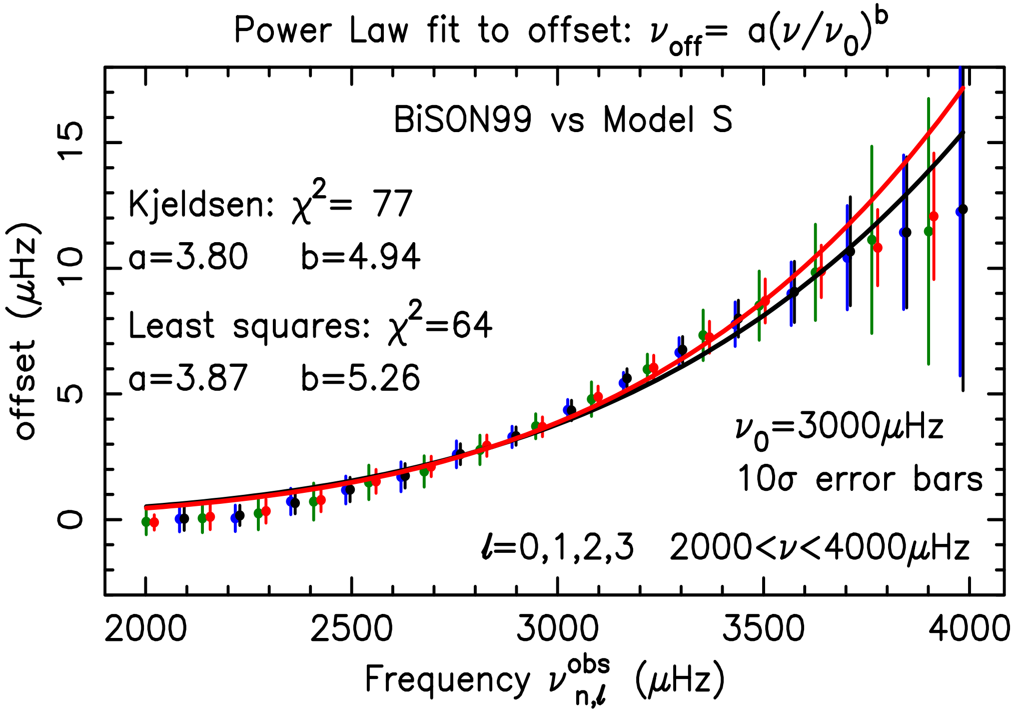
Model fitting and Surface “correction”

Poor understanding of physics in the outer surface layers implies poor modelling of surface layers

Attempts to compensate for this by adding a surface correction to model frequencies

Or
Surface layer independent techniques
Separation ratios , Epsilon phase matching

Model fitting and Surface "correction"



Kjeldsen et al: Empirical model.
Offset between observations and solar model S (JCD) fitted by a power law $F=a\,\nu^b$
For other stars use single power law but determine a [b] to give best fit of model $\nu^m+F(\nu^m)$ to observed ν^{obs}

Ball&Gizon: Theoretical model
$F_{nl}(\nu_{nl}) = A_3\,\nu^3_{nl}/I_{nl} + [A_{-1}/(\nu_{nl}\,I_{nl})]$
I_{nl} mode inertia
Determine A_3, $[A_{-1}]$ to get best fit of model $\nu_{nl} + F_{nl}(\nu_{nl})$ to observed ν^{obs}
(other models exist)

Alternatives: $F(\nu)$ as linear combination of set of basis functions (eg $\Sigma a_k \nu^k$)
Surface layer independent phase matching;
separation ratios r_{10}, r_{02}, epsilon fitting ε_{nl},

No way of empirically testing these correction laws !

χ^2 fits of observed and model data

$$\chi_s^2 = \sum \left(\frac{L^o - L^m}{\sigma_L^o} \right)^2 + \ldots \quad L, T_{eff}, [\mathrm{Fe/H}] \;\; (\Delta_\nu, \nu_{max} + \ldots$$

$$\chi_\nu^2 = \sum_{n,\ell} \left(\frac{\nu_{n\ell}^m + F_{n\ell}(\nu_{n\ell}^m) - \nu_{n\ell}^o}{\sigma_{n\ell}^o} \right)^2 \quad \text{minimise wrt F}$$

$$\chi_0^2 = \left(\frac{\nu_{k0}^m - \nu_{k0}^o}{\sigma_{k0}^o} \right)^2 \quad \chi_1^2 = \left(\frac{\nu_{k1}^m - \nu_{k1}^o}{\sigma_{k1}^o} \right)^2 \quad \ldots \quad k = n_{min}$$

$$\chi_r^2 = \sum_{n,\ell} \left(\frac{r_{n\ell}^m - r_{n\ell}^o}{s_{n\ell}^r} \right)^2 \quad eg \quad r_{n2} = \frac{\nu_{n,0} - \nu_{n-1,2}}{\nu_{n,1} - \nu_{n-1,1}}$$

$$\chi_\epsilon^2 = \sum_{n,\ell} \left(\frac{\epsilon_\ell^o(\nu_{n\ell}^o) - \epsilon_\ell^m(\nu_{n\ell}^o) - E(\nu_{n\ell}^o)}{s_{n\ell}^\epsilon} \right)^2 \quad \epsilon_\ell(\nu_{n\ell}) = \frac{\nu_{n\ell}}{\Delta} - n - \frac{\ell}{2}$$

Comparison sets

Basu (own code) $\chi^2_s + \chi^2_\nu + (\chi^2_0 + \chi^2_1 + \ldots)/10000$ B&G corr

Nasamba (AIMS code) $\chi^2_s + \chi^2_\nu / N_\nu$ B&G corr

Ong (own code) $\chi^2_s + \chi^2_\nu / N_\nu + \chi^2_\nu / N_\nu \ldots$ B&G corr

Reese (AIMS code) $\chi^2_s + 3\chi^2_\nu / N_\nu$ (+many others) B&G corr

Roxburgh (own code) $\chi^2_s + 3\chi^2_\varepsilon / N_\varepsilon$ (+many others) No corr

Silva-Aguirre (BASTA code) $\chi^2_s + \chi^2_\nu / N_\nu$ (+others) B&G corr

N number of degrees of freedom $\exp - \chi^2/2$ → PDF

Results of fits

Names of stars suppressed as still in use for "glitch" fitting

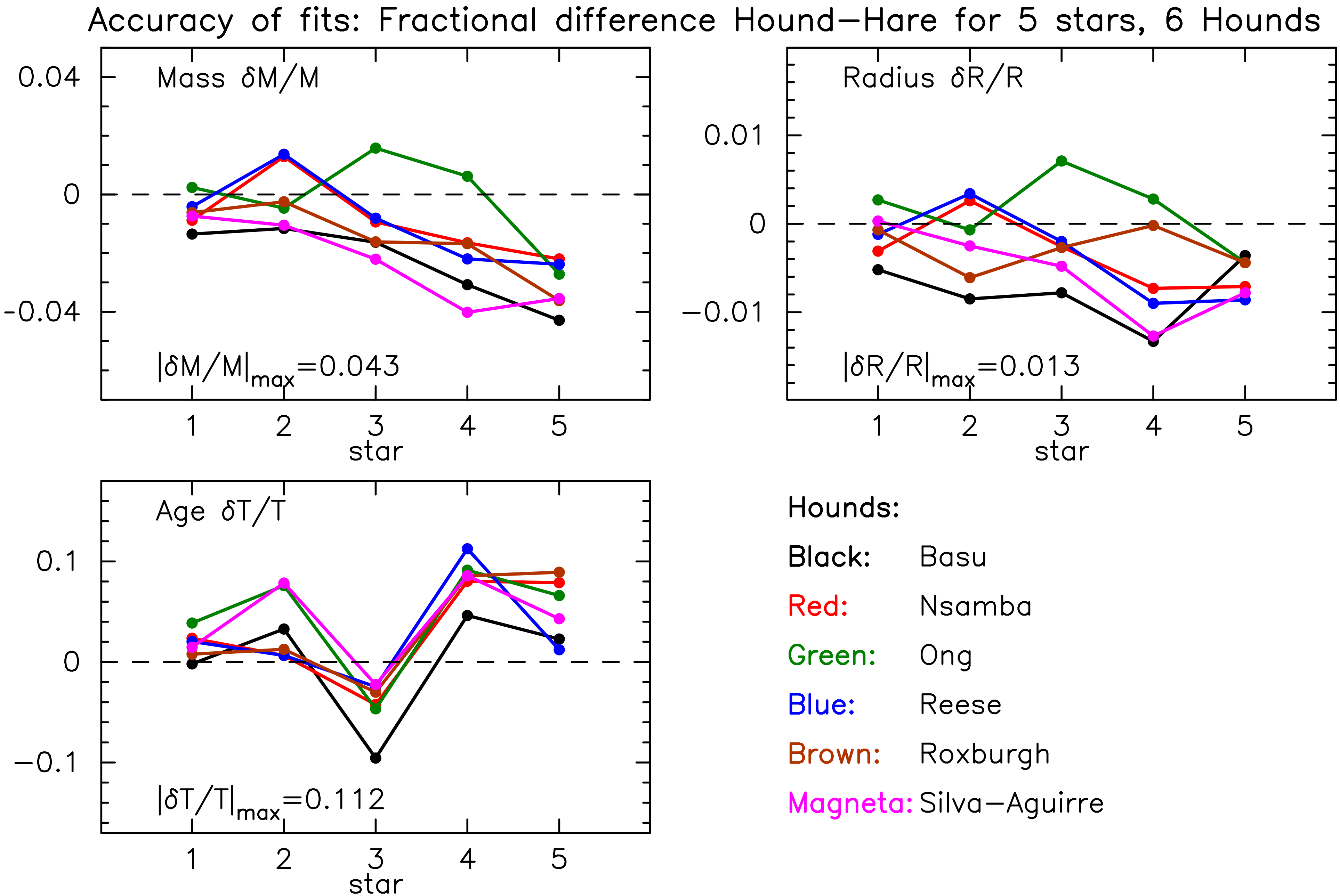
Results for 5 Hare stars. One star excluded from report – all fitters encountered problems dealing with the Hare data for this star (we understand why)

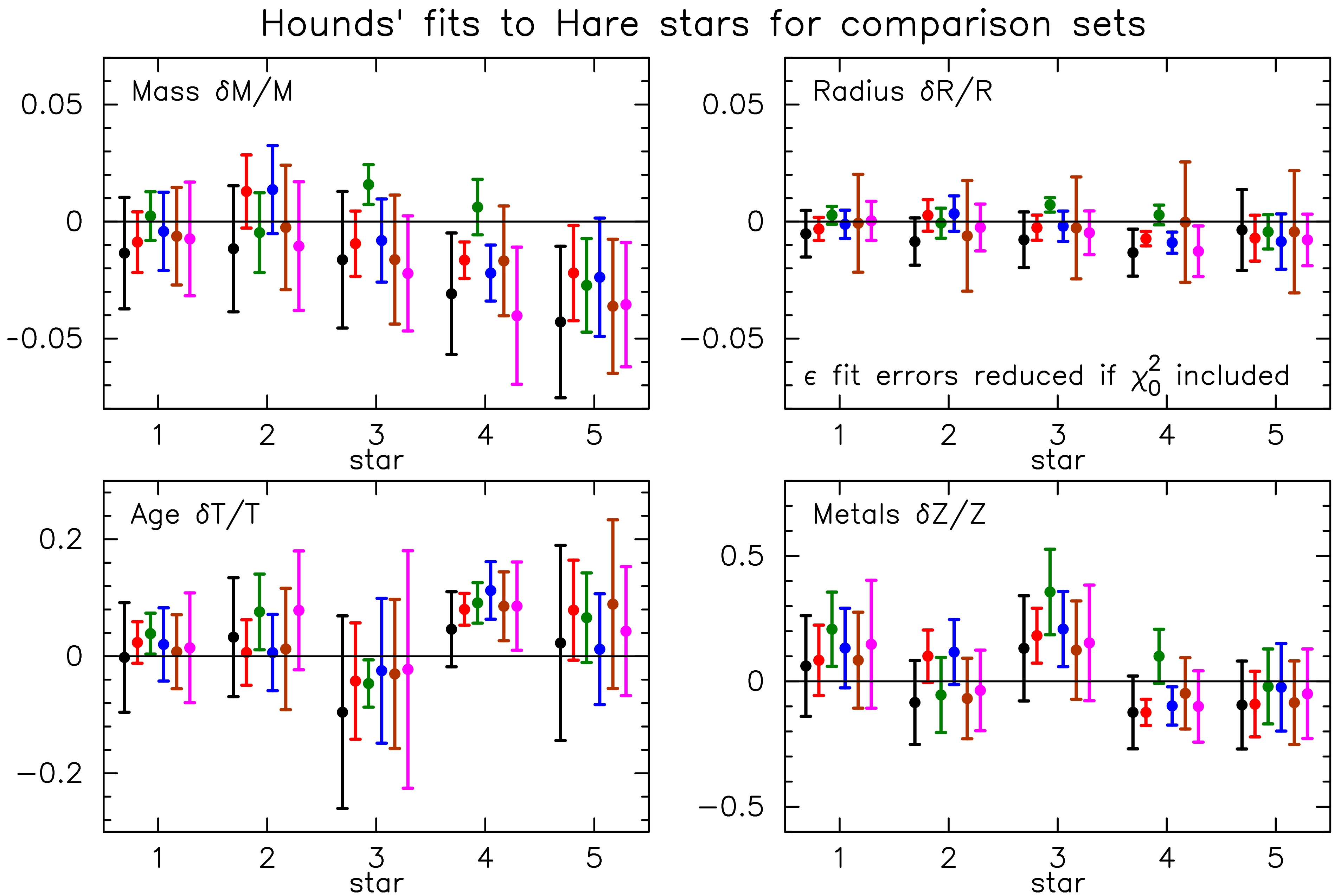
Fits to Mass, Radius, Age, Z etc to 5 stars by 6 fitters

Comparison set: preferred results from each Hound

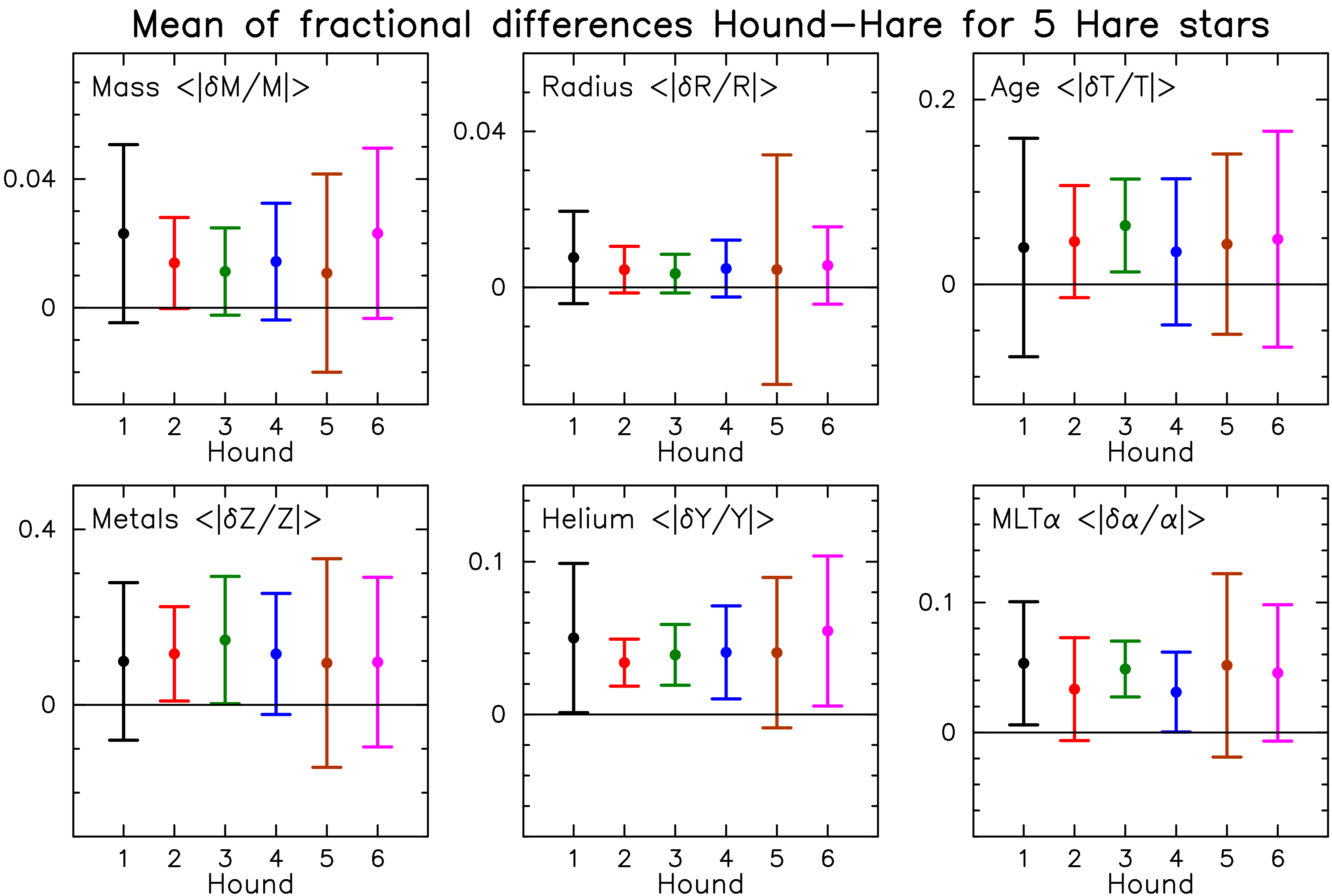
Different fitting techniques using frequencies, ratios, epsilons, different weights global parameters χ_s to frequencies χ_ν , ratios χ_r epsilons χ_ε ; reduced lengths of data sets

What have we learnt ?


Accuracy of fits: Fractional difference Hound−Hare for 5 stars, 6 Hounds
Mass δM/M
0.04
0
−0.04
|δM/M|max=0.043
1
2
3
4
5
star
Radius δR/R
0.01
0
−0.01
|δR/R|max=0.013
1
2
3
4
5
star
Age δT/T
0.1
0
−0.1
|δT/T|max=0.112
1
2
3
4
5
star
Hounds:
Black: Basu
Red: Nsamba
Green: Ong
Blue: Reese
Brown: Roxburgh
Magneta: Silva−Aguirre



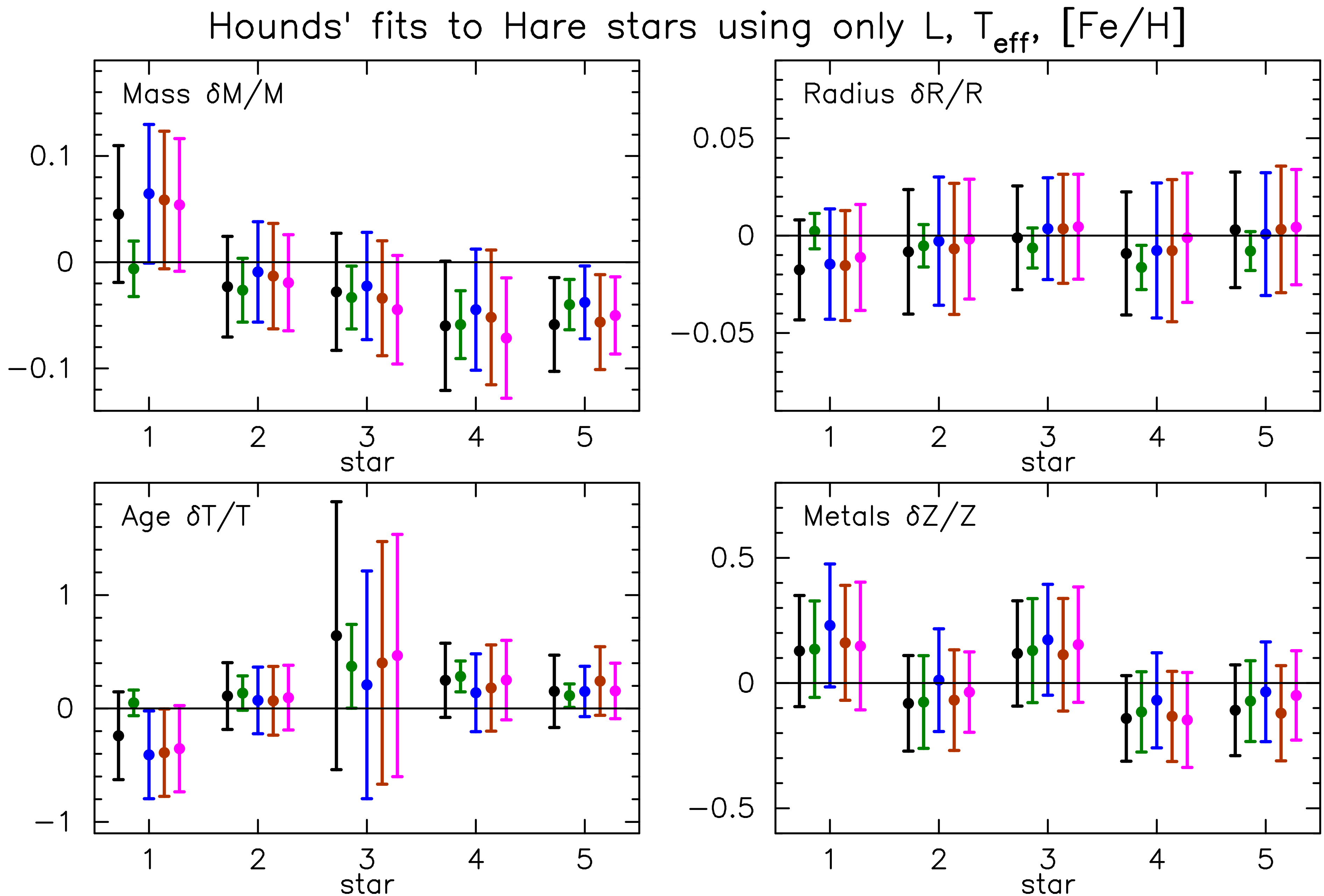
Hounds' fits to Hare stars for comparison sets
Mass δM/M
0.05
0
−0.05
1
2
3
4
5
star
Radius δR/R
ϵ fit errors reduced if χ_0^2 included
Age δT/T
0.2
−0.2
Metals δZ/Z
0.5
−0.5



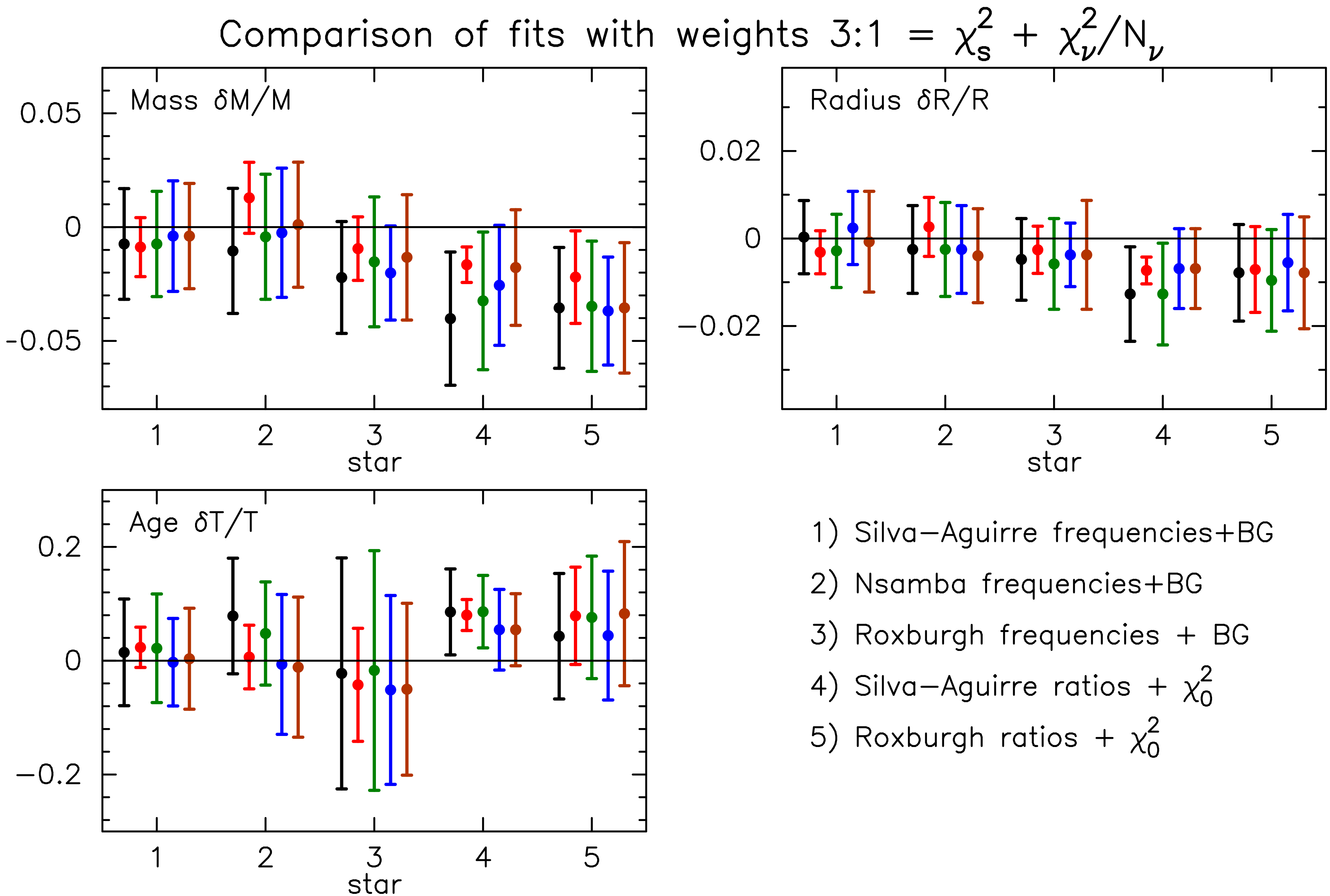
Mean of fractional differences Hound–Hare for 5 Hare stars
Mass <|δM/M|>
Radius <|δR/R|>
Age <|δT/T|>
Metals <|δZ/Z|>
Helium <|δY/Y|>
MLTα <|δα/α|>
0.04
0
0.2
0.4
0.1
1 2 3 4 5 6
Hound

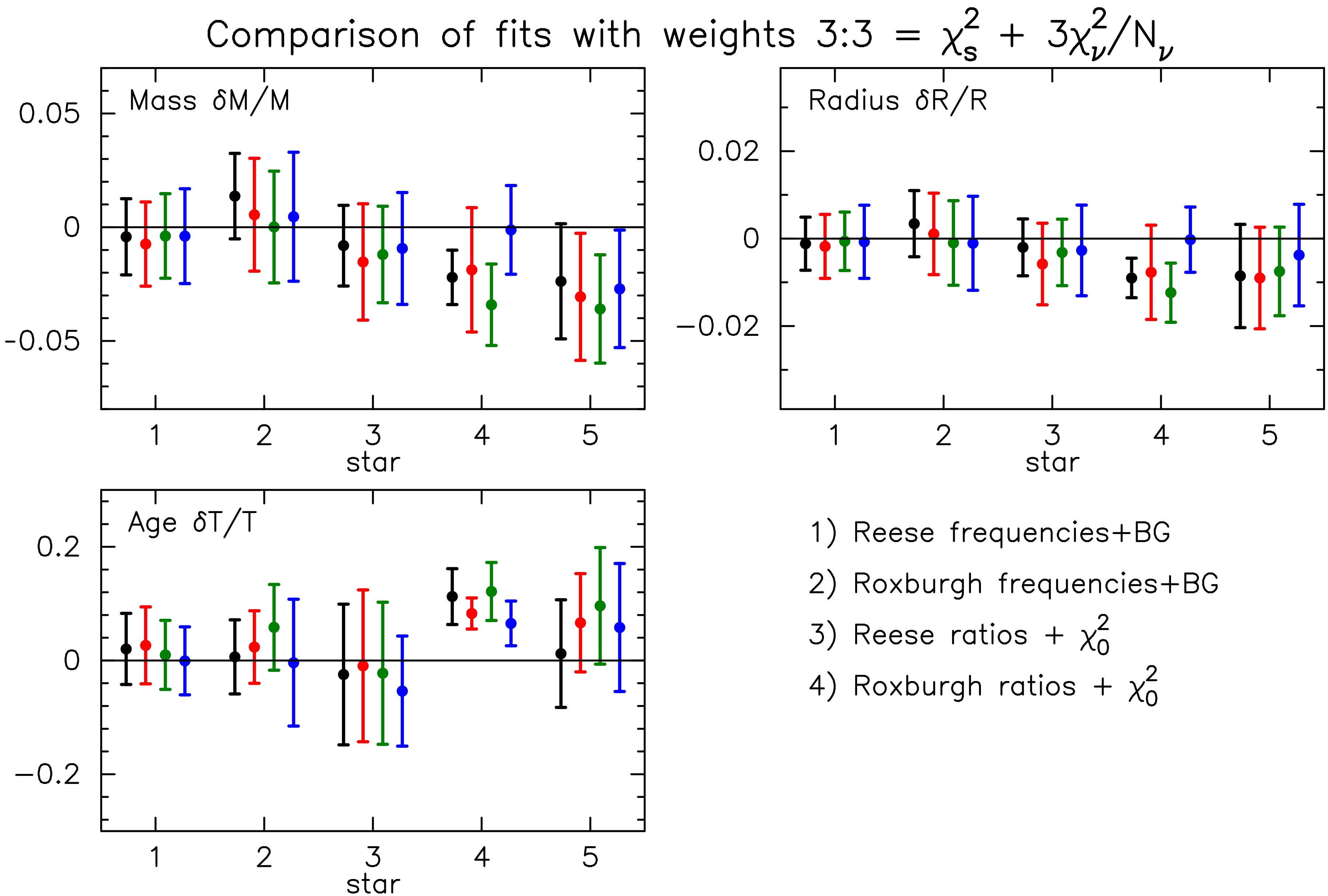
Comparison of different weights $\chi^2_s : \chi^2_\nu$

3:0	χ^2_s	no frequencies
3:1	$\chi^2_s + \chi^2_\nu/N$	
3:3	$\chi^2_s + 3\,\chi^2_\nu/N$	
3:N	$\chi^2_s + \chi^2_\nu$	
0:N	χ^2_ν	frequencies only

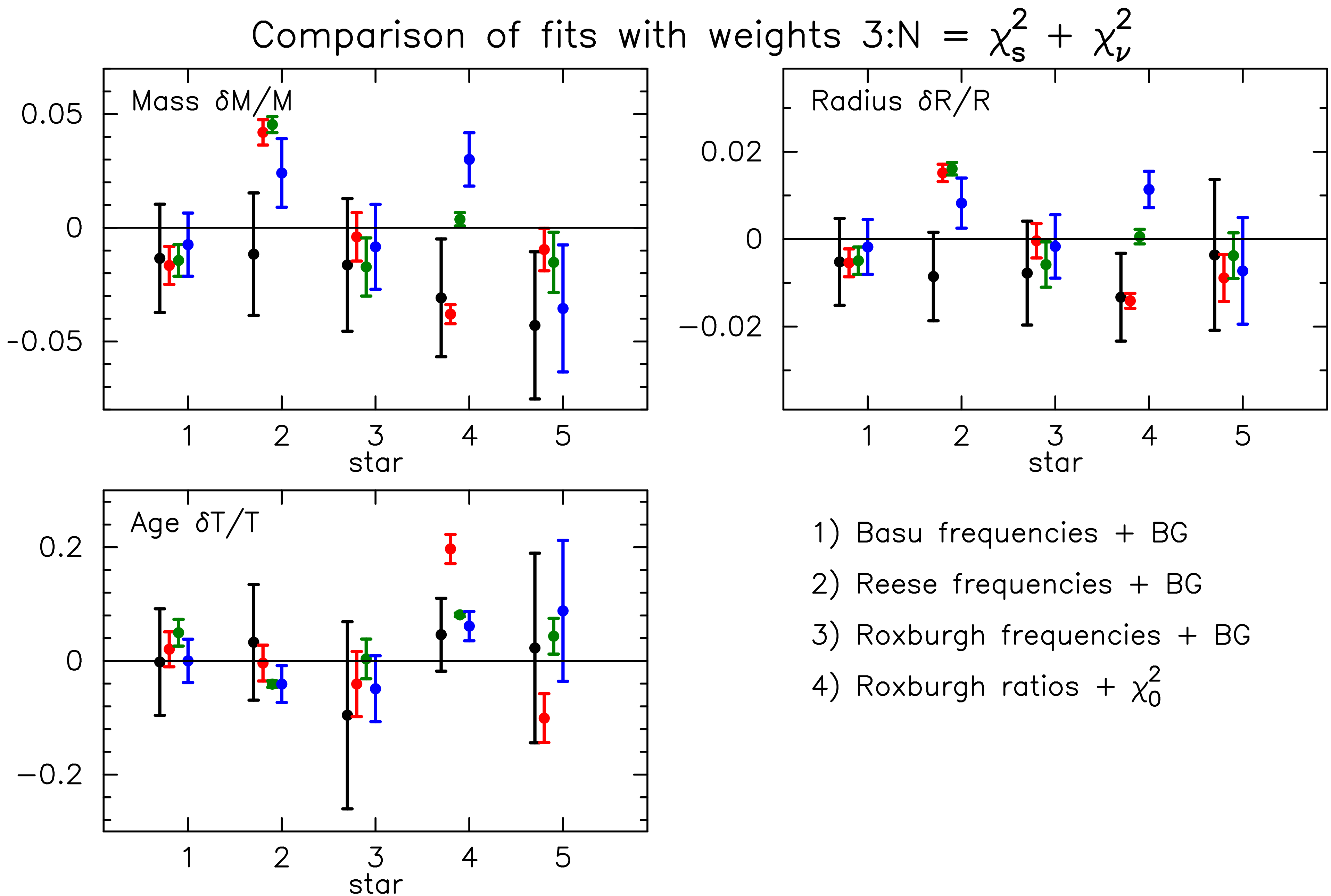


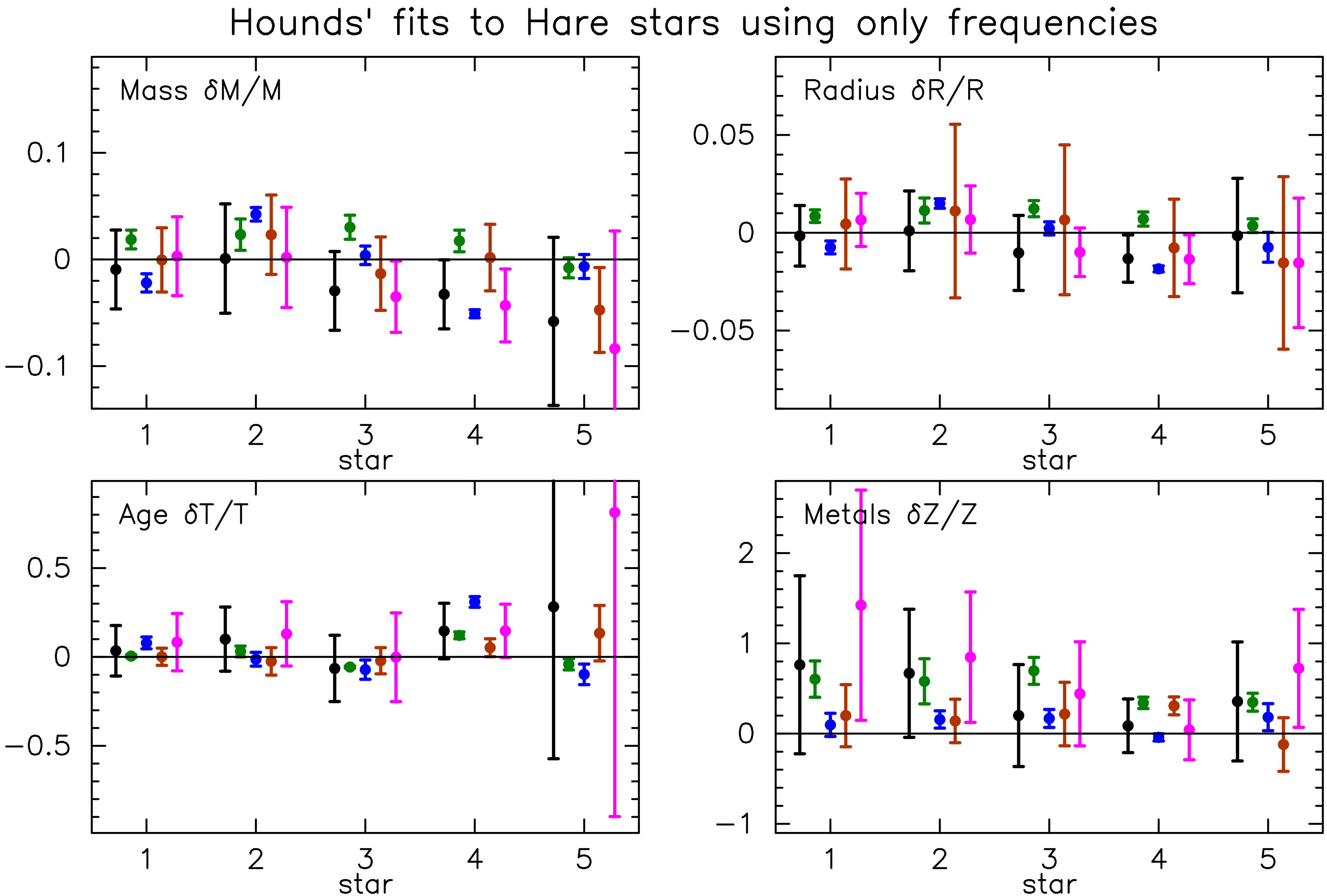
Hounds' fits to Hare stars using only L, T_{eff}, [Fe/H]
Mass δM/M
0.1
0
−0.1
1
2
3
4
5
star
Radius δR/R
0.05
0
−0.05
1
2
3
4
5
star
Age δT/T
1
0
−1
1
2
3
4
5
star
Metals δZ/Z
0.5
0
−0.5
1
2
3
4
5
star

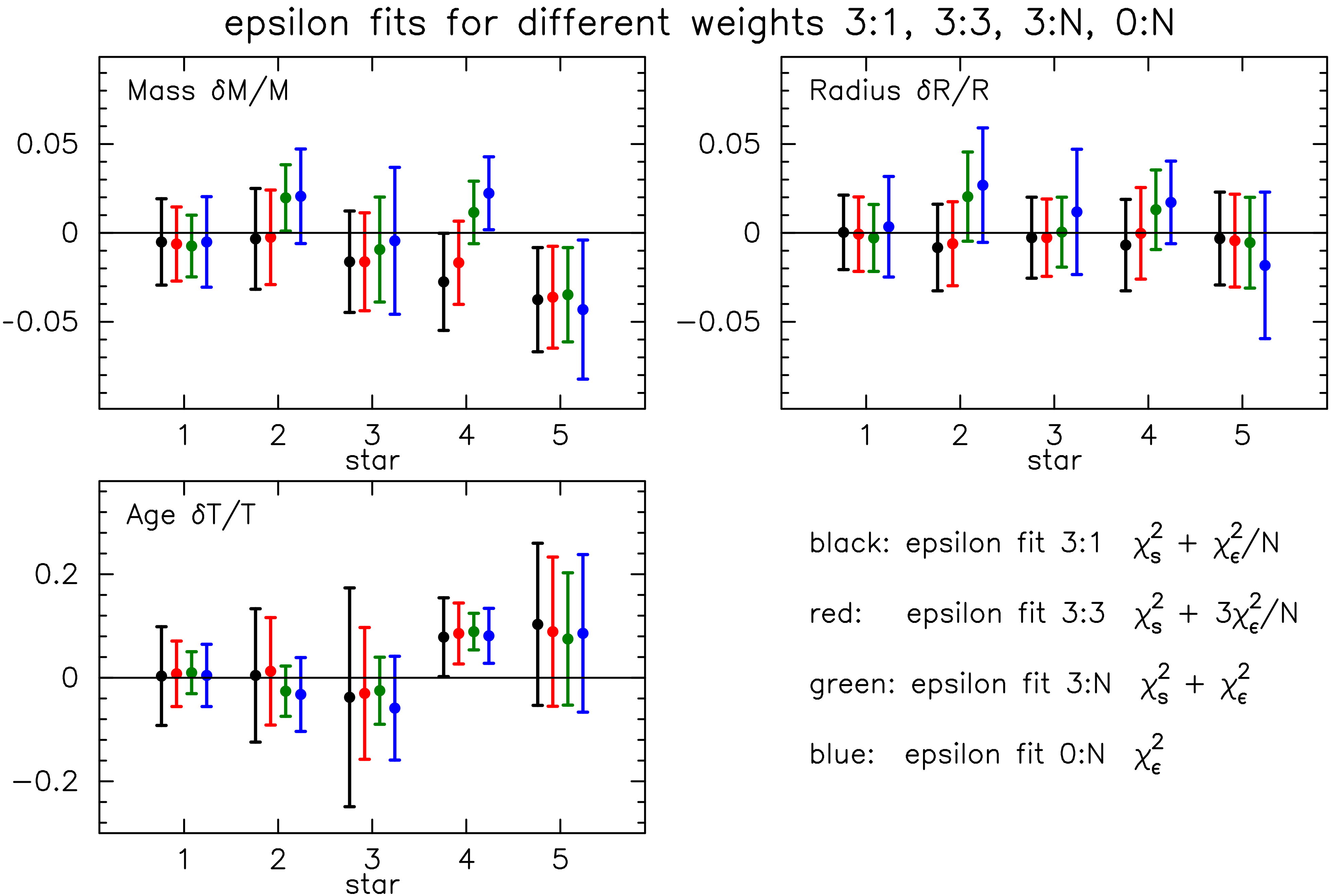

Comparison of fits with weights 3:1 = $\chi_s^2 + \chi_\nu^2/N_\nu$
Mass δM/M
0.05
0
−0.05
1
2
3
4
5
star
Radius δR/R
0.02
0
−0.02
1
2
3
4
5
star
Age δT/T
0.2
0
−0.2
1
2
3
4
5
star
1) Silva−Aguirre frequencies+BG
2) Nsamba frequencies+BG
3) Roxburgh frequencies + BG
4) Silva−Aguirre ratios + χ_0^2
5) Roxburgh ratios + χ_0^2



Comparison of fits with weights 3:3 = $\chi^2_s + 3\chi^2_\nu/N_\nu$
Mass $\delta M/M$
0.05
0
−0.05
1
2
3
4
5
star
Radius $\delta R/R$
0.02
0
−0.02
Age $\delta T/T$
0.2
−0.2
1) Reese frequencies+BG
2) Roxburgh frequencies+BG
3) Reese ratios + χ^2_0
4) Roxburgh ratios + χ^2_0


Comparison of fits with weights 3:N = $\chi_s^2 + \chi_\nu^2$
Mass δM/M
0.05
0
−0.05
1
2
3
4
5
star
Radius δR/R
0.02
0
−0.02
Age δT/T
0.2
−0.2
1) Basu frequencies + BG
2) Reese frequencies + BG
3) Roxburgh frequencies + BG
4) Roxburgh ratios + χ_0^2


Hounds' fits to Hare stars using only frequencies
Mass δM/M
Radius δR/R
Age δT/T
Metals δZ/Z
star



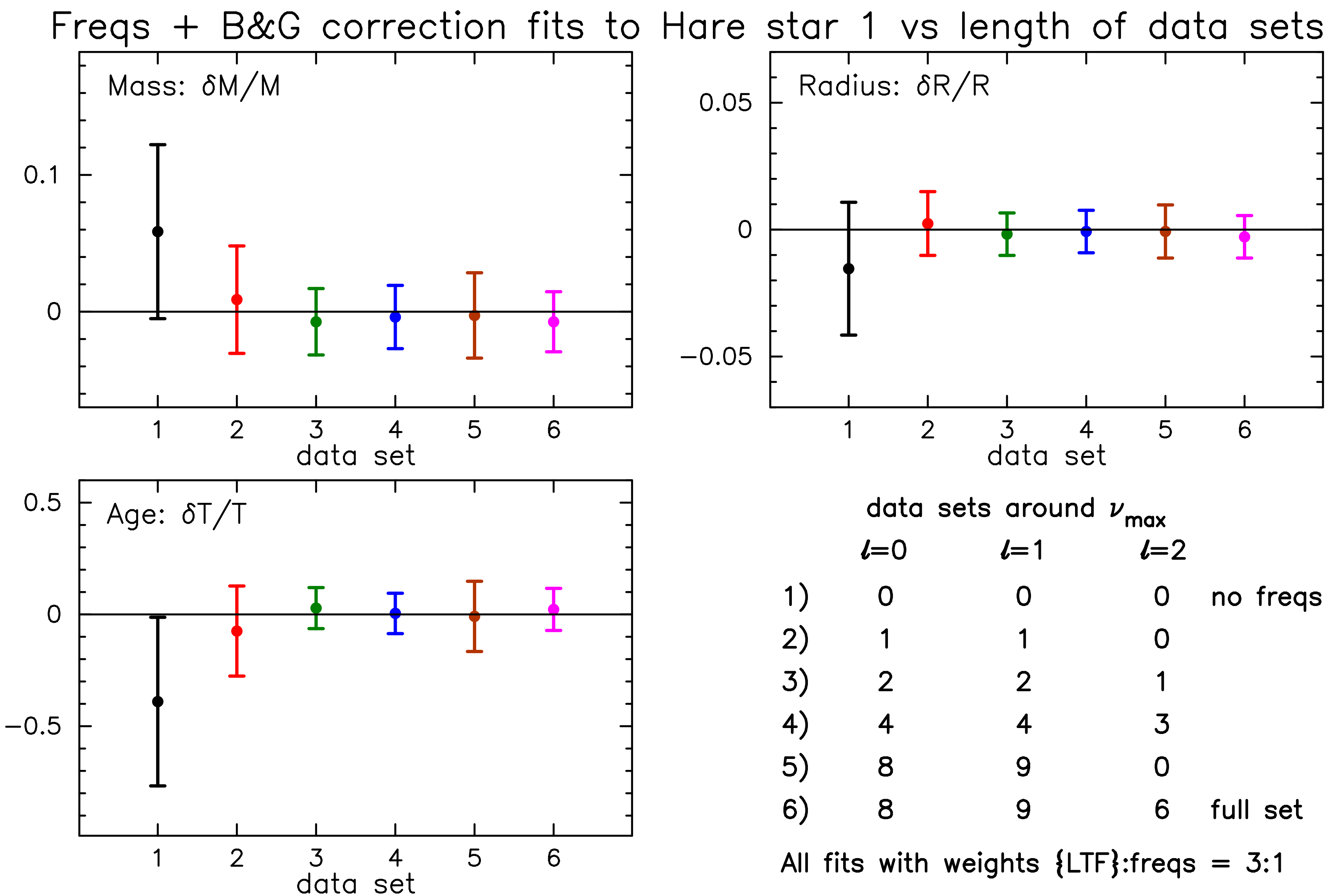
epsilon fits for different weights 3:1, 3:3, 3:N, 0:N
Mass δM/M
Radius δR/R
Age δT/T
0.05
0
−0.05
0.2
−0.2
1
2
3
4
5
star
black: epsilon fit 3:1 $\chi_s^2 + \chi_\epsilon^2/N$
red: epsilon fit 3:3 $\chi_s^2 + 3\chi_\epsilon^2/N$
green: epsilon fit 3:N $\chi_s^2 + \chi_\epsilon^2$
blue: epsilon fit 0:N χ_ϵ^2

Minimum data

Fitting Hare stars with reduced data sets

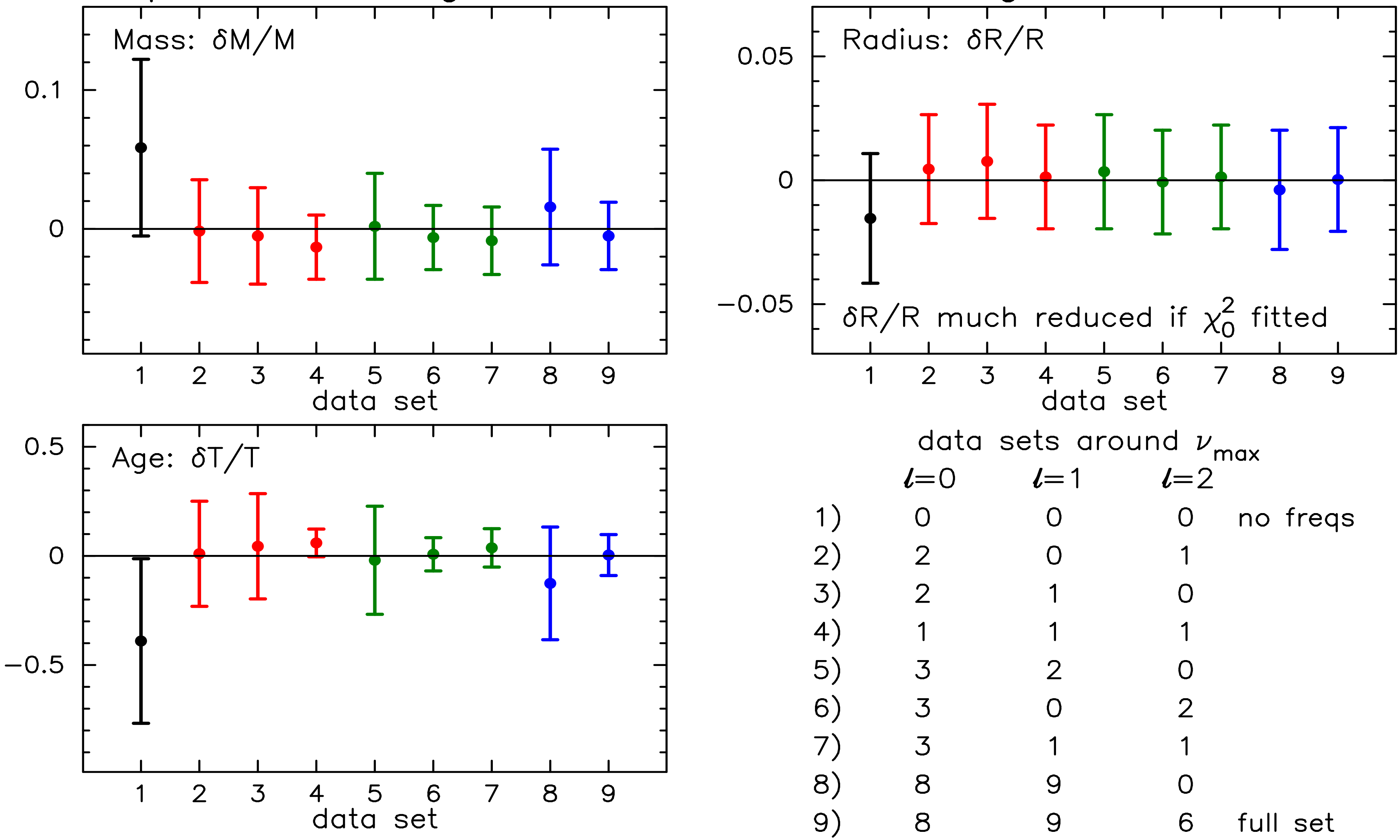
1) Frequencies + BG offset

2) Epsilons and Ratios



Freqs + B&G correction fits to Hare star 1 vs length of data sets
Mass: δM/M
0.1
0
1 2 3 4 5 6
data set
Radius: δR/R
0.05
0
−0.05
1 2 3 4 5 6
data set
Age: δT/T
0.5
0
−0.5
1 2 3 4 5 6
data set
data sets around ν max
l=0 l=1 l=2
1) 0 0 0 no freqs
2) 1 1 0
3) 2 2 1
4) 4 4 3
5) 8 9 0
6) 8 9 6 full set
All fits with weights {LTF}:freqs = 3:1

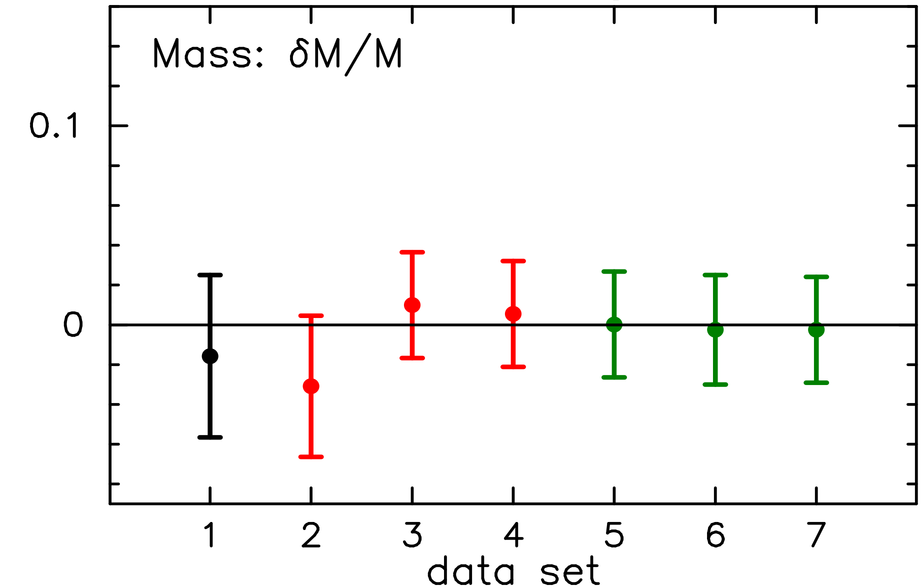
epsilon matching: fits to Hare star 1 vs length of data sets

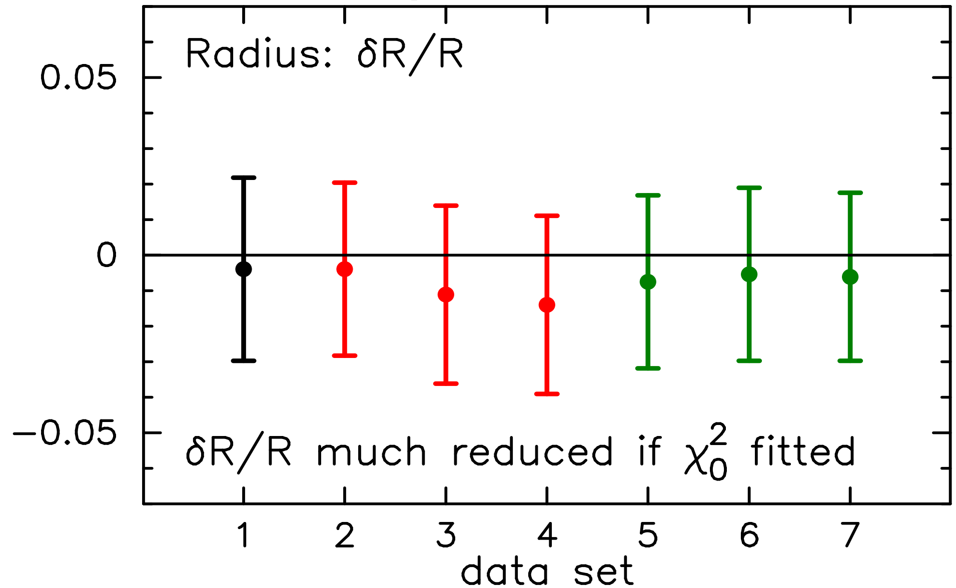


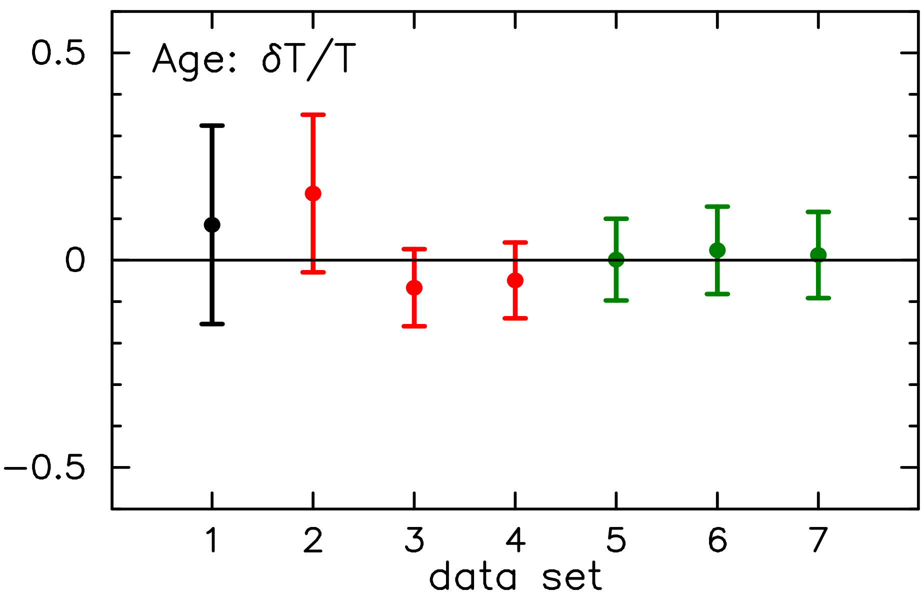
data sets around ν_{max}

	l=0	l=1	l=2	
1)	0	0	0	no freqs
2)	2	0	1	
3)	2	1	0	
4)	1	1	1	
5)	3	2	0	
6)	3	0	2	
7)	3	1	1	
8)	8	9	0	
9)	8	9	6	full set

epsilon matching: fits to Hare star 2 vs length of data sets







data sets around ν_{max}

	l=0	l=1	l=2	
1)	0	0	0	no freqs
2)	3	1	2	
3)	3	3	3	
4)	4	4	4	
5)	5	5	5	
6)	6	6	6	
7)	11	12	10	full set

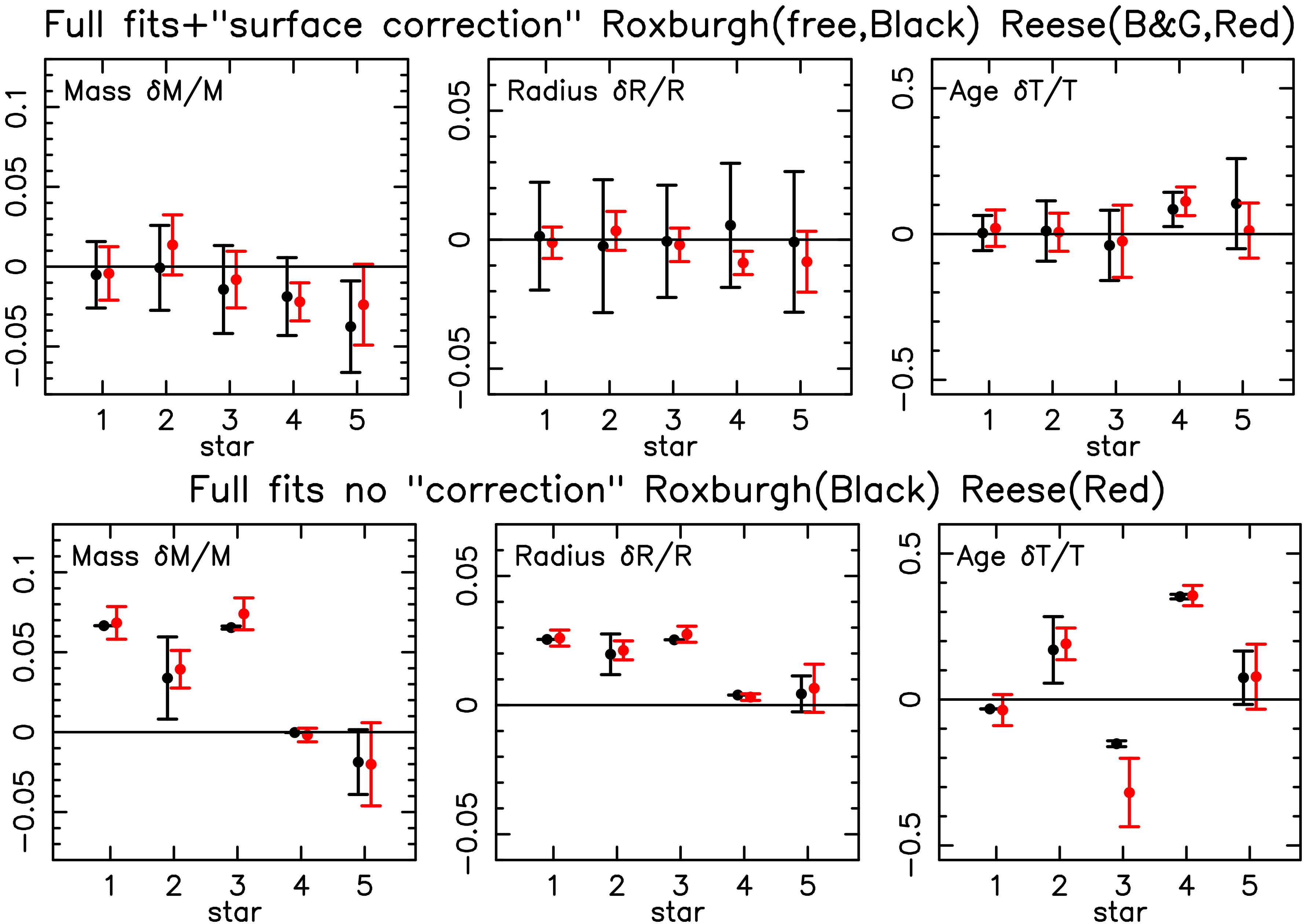
Inferences from results

What have we learnt ??

Not much!! Hare models and Hound models from same codes – MESA&ADIPLS - expect to find good fits !

Frequency fitting

Frequency fits with a Ball&Gizon (or other) "surface correction" and weights 3:1, and 3:3 are good. Fits with weights 3:N, no frequencies, or frequencies alone not good enough. Fits with no surface correction are poor



Full fits+"surface correction" Roxburgh(free,Black) Reese(B&G,Red)
Mass δM/M
Radius δR/R
Age δT/T
star
Full fits no "correction" Roxburgh(Black) Reese(Red)
Mass δM/M
Radius δR/R
Age δT/T
star

Inferences from results

Frequency fitting

Frequency fits with a Ball&Gizon (or other) "surface correction" and weights 3:1, and 3:3 are good. Fits with weights 3:N ,no frequencies, or frequencies alone not good enough. Fits with no surface correction are poor

But one cannot conclude that the Ball & Gizon" correction" is correct

Hare models had added surface corrections similar to the solar offset
The Hounds used a scaled Ball & Gizon correction similar to the solar offset.
They should get a good fit !

With a totally different offset for Hare models (eg a constant) Hounds with B&G would not have good fit; their "basis functions" not capable of fitting a constant But a Kjeldsen like correction $a\nu^b$ could fit this offset with b=0.

We have no empirical knowledge on the shape or magnitude of the frequency offset for stars - may be - maybe not - like the sun vs solar model

Surface layer independent fitting

"almost blind" to uncertainties in surface layer structure and does as well as frequency fits for mass and age not for radius (unless add χ^2_0 to fit.)
Robust for 3:N, 0:N weightings.

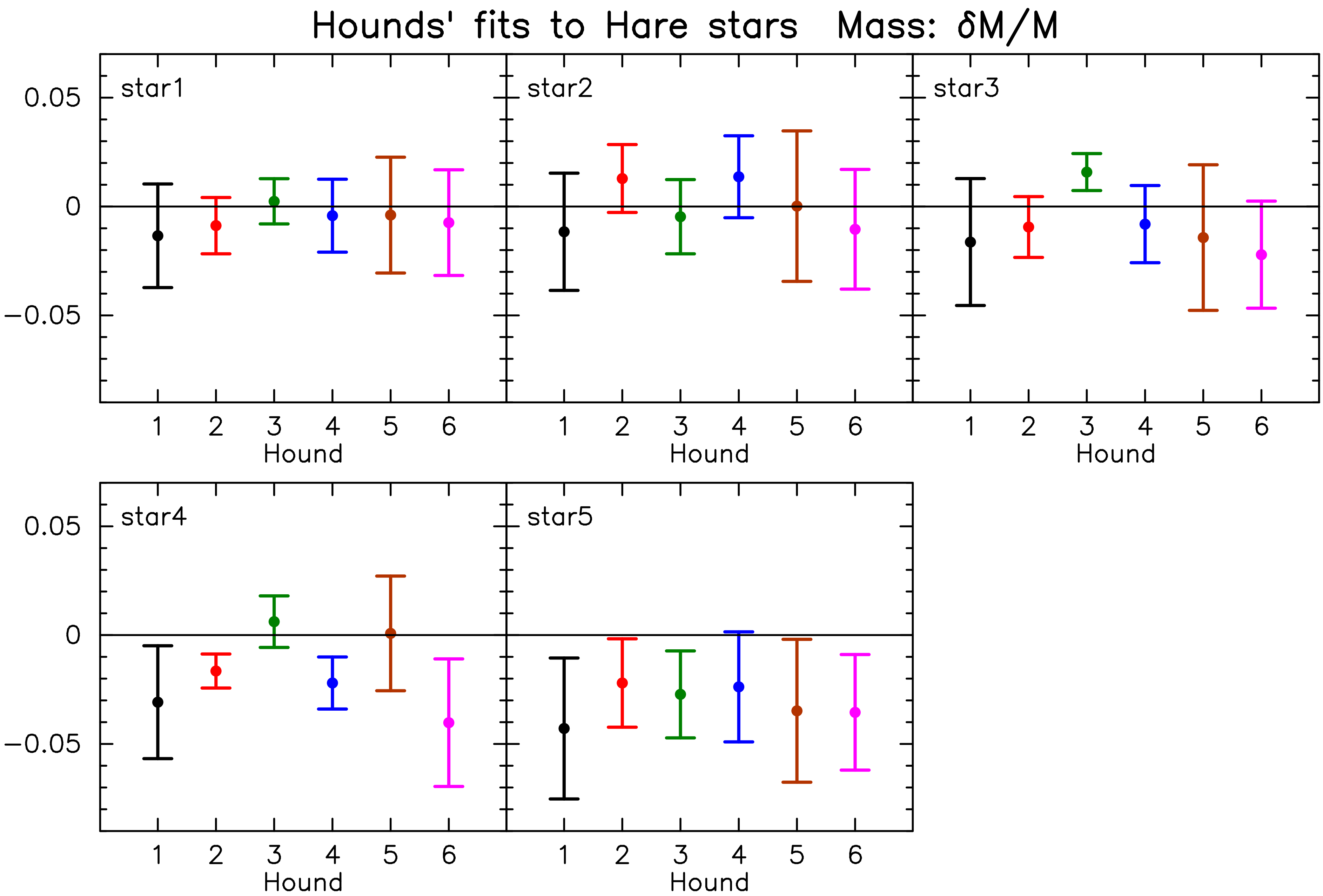
Future work

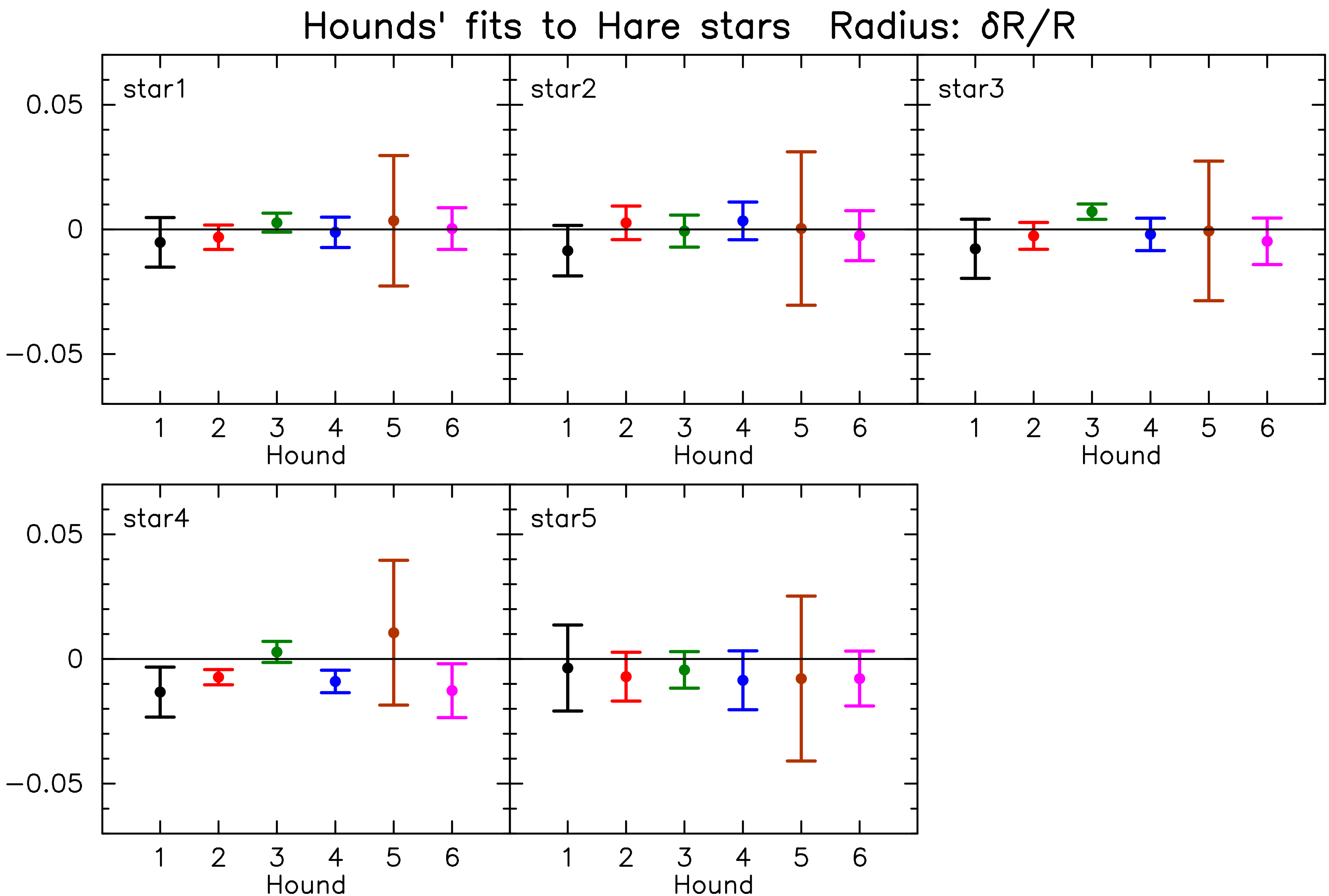
Need to have a new experiment where the "corrections" added to Hare frequencies are not similar to the solar offset.

Need to explore further the quality of fits using smaller data sets and just some average properties of poor quality data

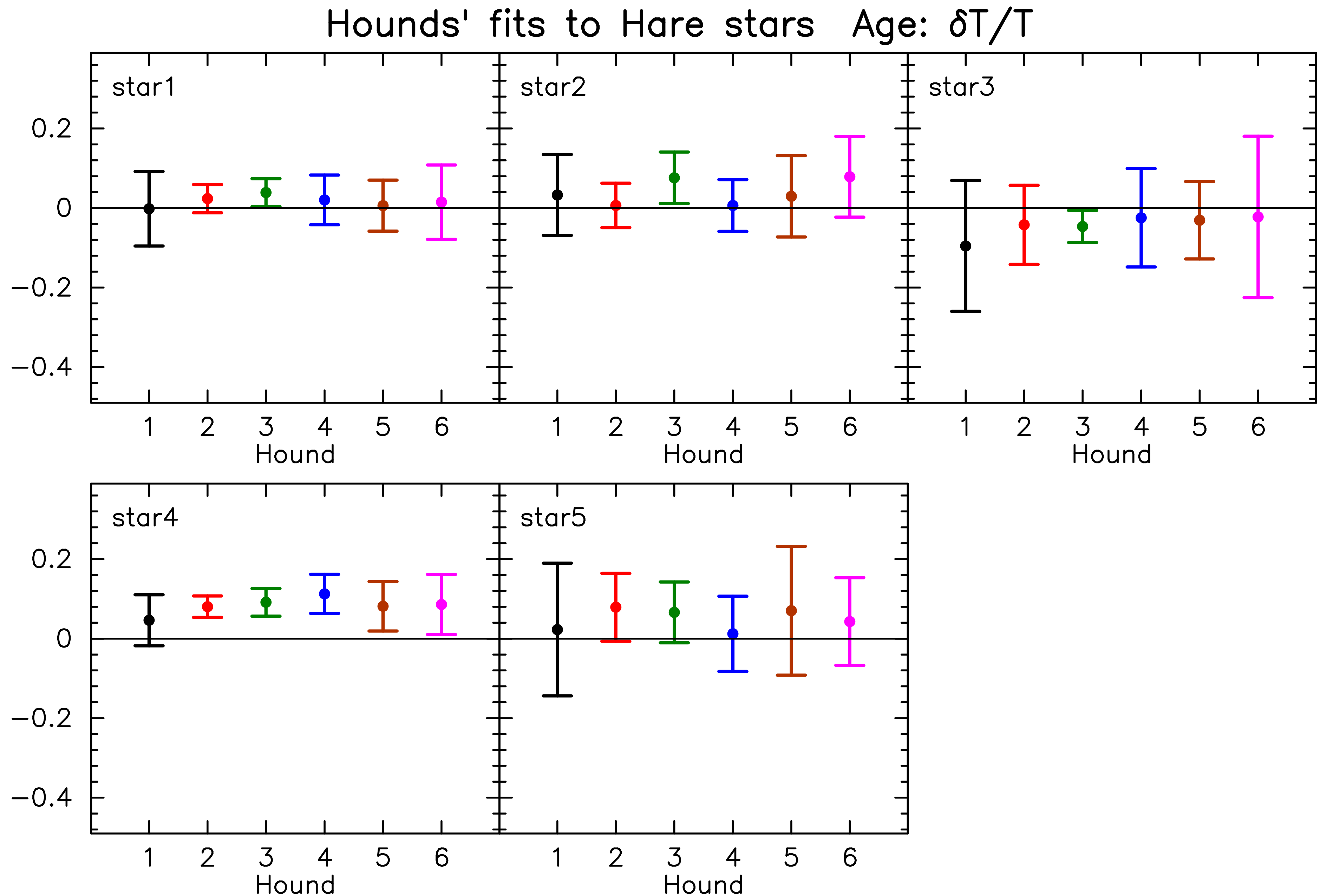
We have some problems to sort out over fitting ratios including the large separation in the fit.

The End

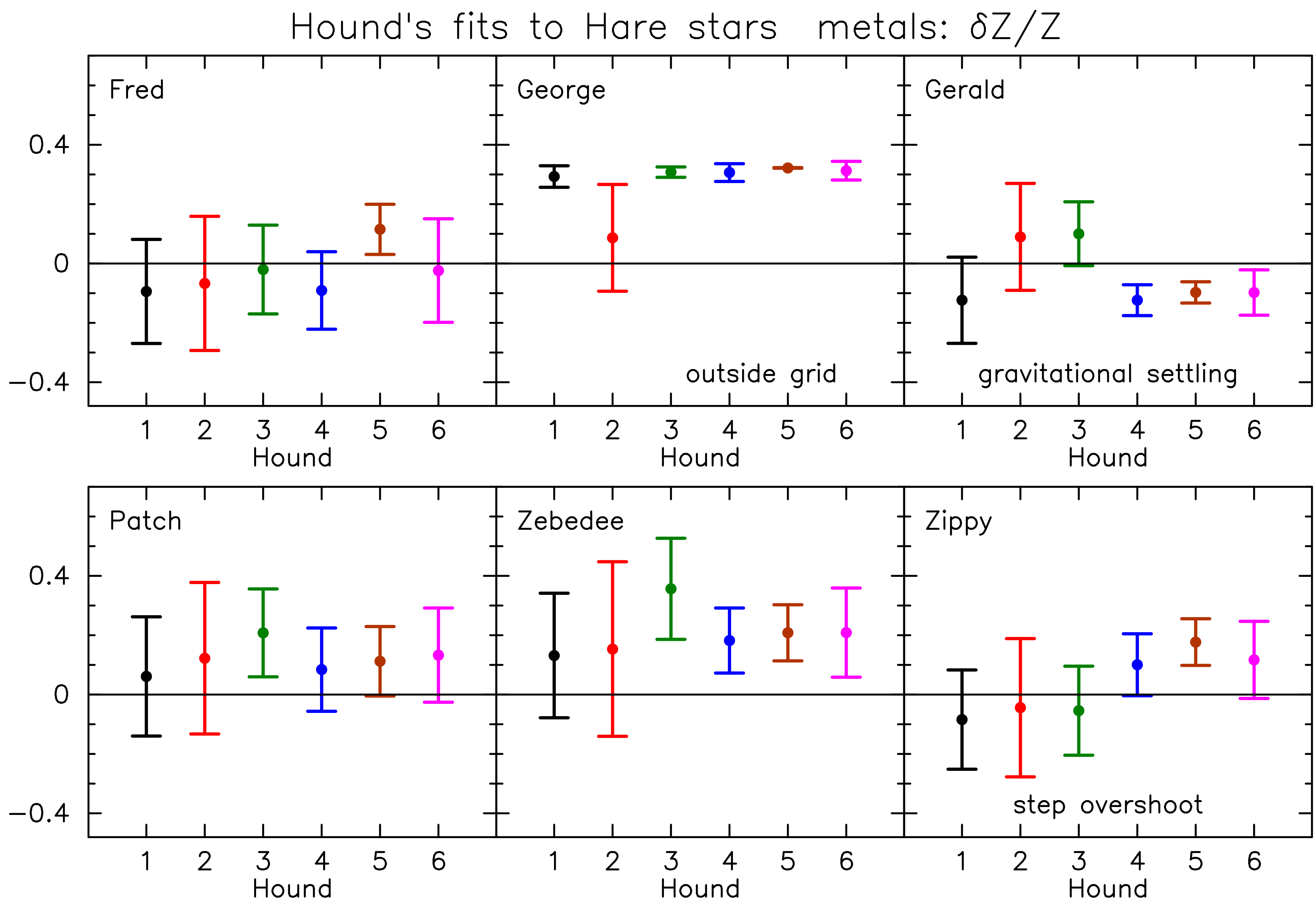

Hounds' fits to Hare stars Mass: δM/M
star1
star2
star3
star4
star5
0.05
0
−0.05
1 2 3 4 5 6
Hound



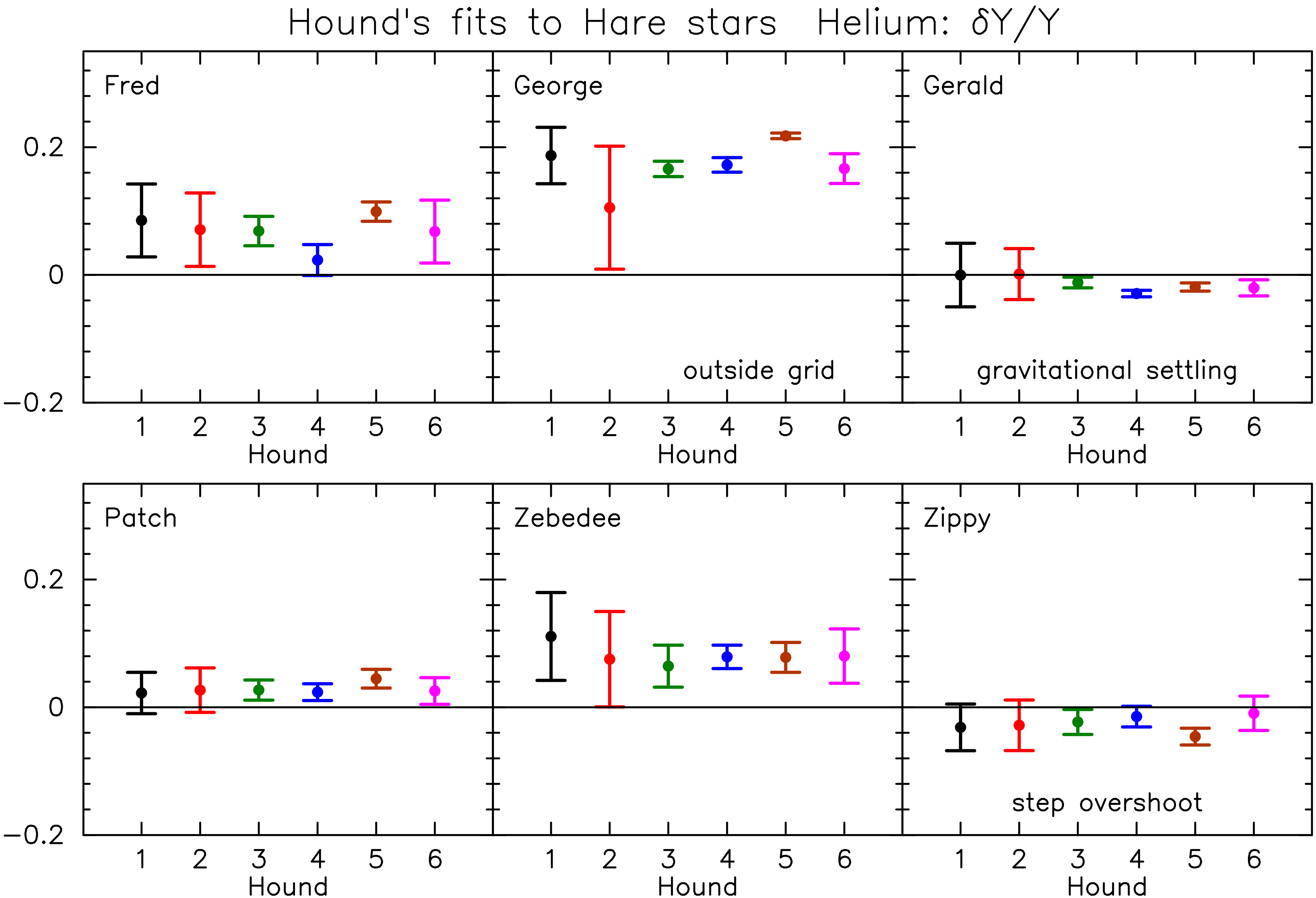
Hounds' fits to Hare stars Radius: δR/R
star1
star2
star3
star4
star5
0.05
0
−0.05
1
2
3
4
5
6
Hound

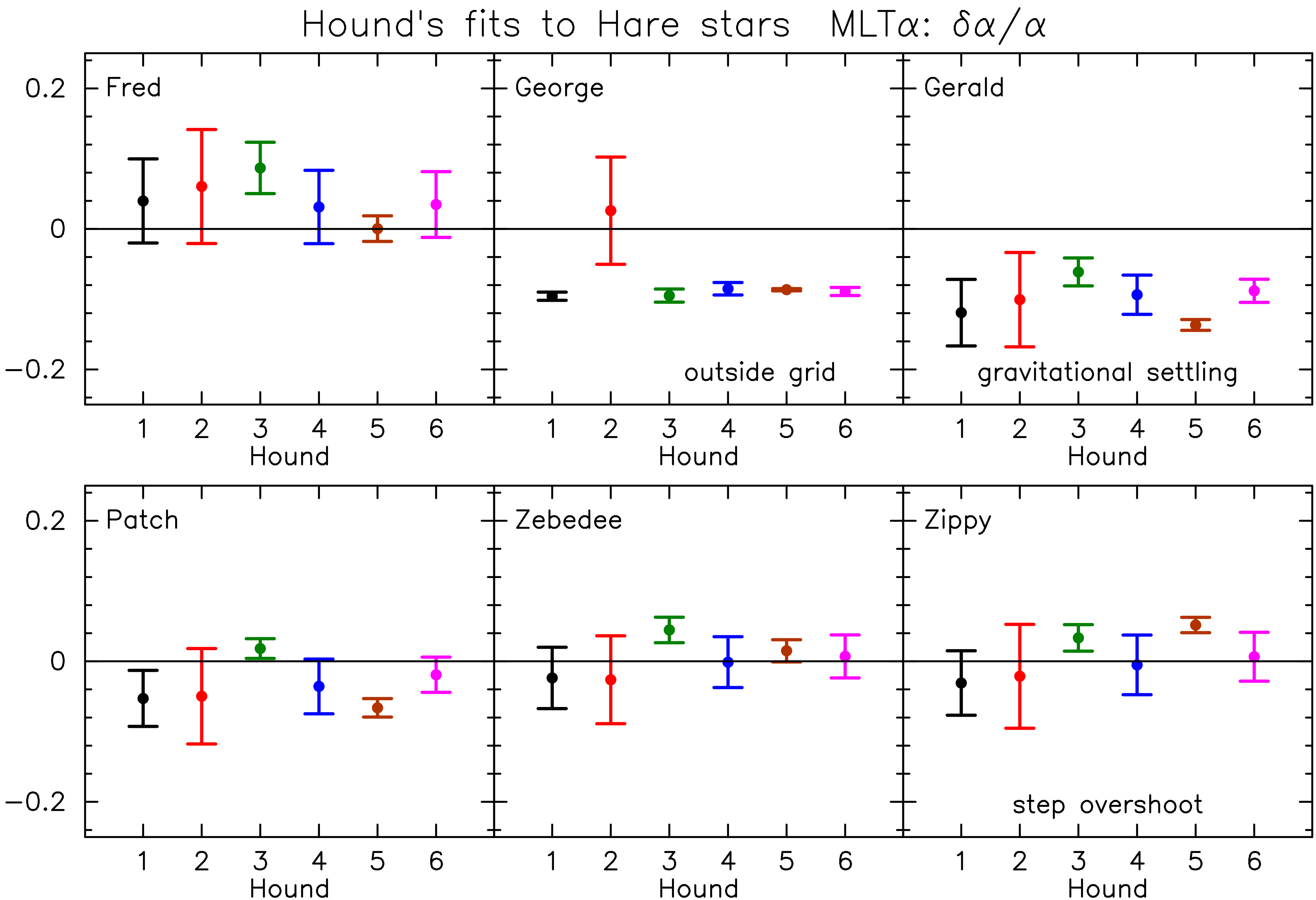


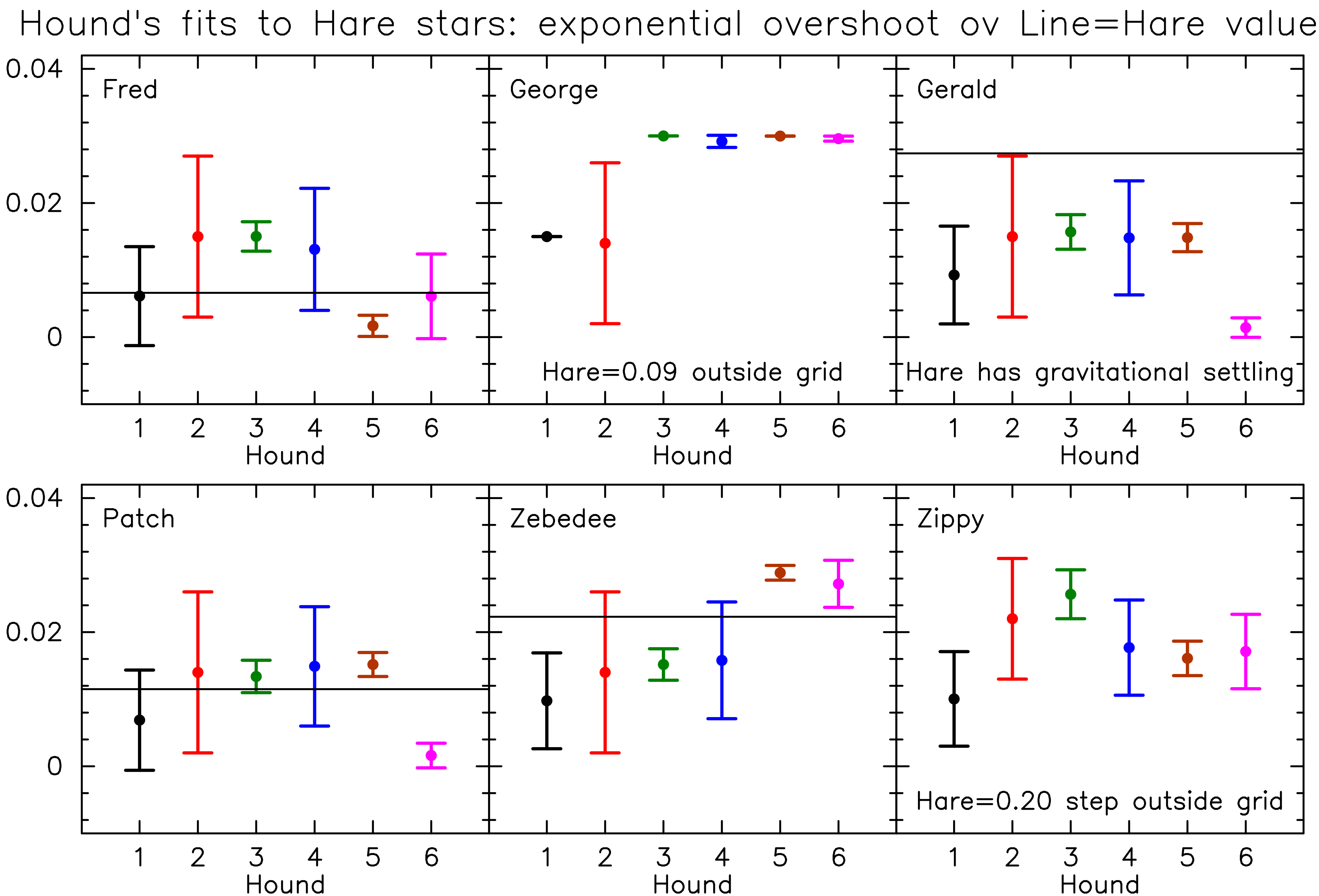
Hounds' fits to Hare stars Age: δT/T
star1
star2
star3
star4
star5
0.2
0
−0.2
−0.4
1 2 3 4 5 6
Hound

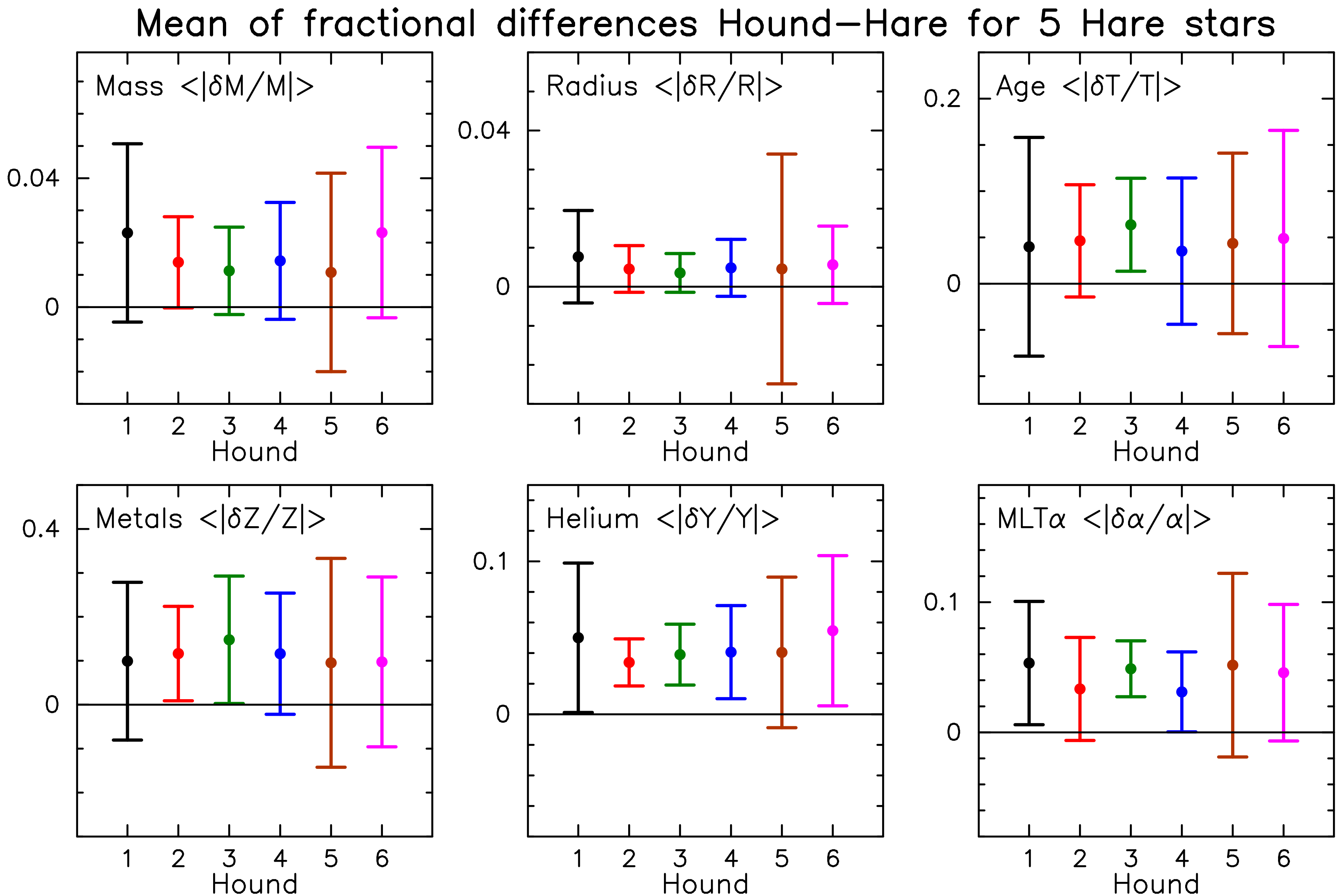


Hound's fits to Hare stars metals: δZ/Z
Fred
George
Gerald
Patch
Zebedee
Zippy
outside grid
gravitational settling
step overshoot
0.4
0
−0.4
1 2 3 4 5 6
Hound


Hound's fits to Hare stars Helium: δY/Y
Fred
George
Gerald
Patch
Zebedee
Zippy
outside grid
gravitational settling
step overshoot
0.2
0
−0.2
1 2 3 4 5 6
Hound


Hound's fits to Hare stars MLTα: δα/α
Fred
George
Gerald
Patch
Zebedee
Zippy
outside grid
gravitational settling
step overshoot
0.2
0
−0.2
1 2 3 4 5 6
Hound

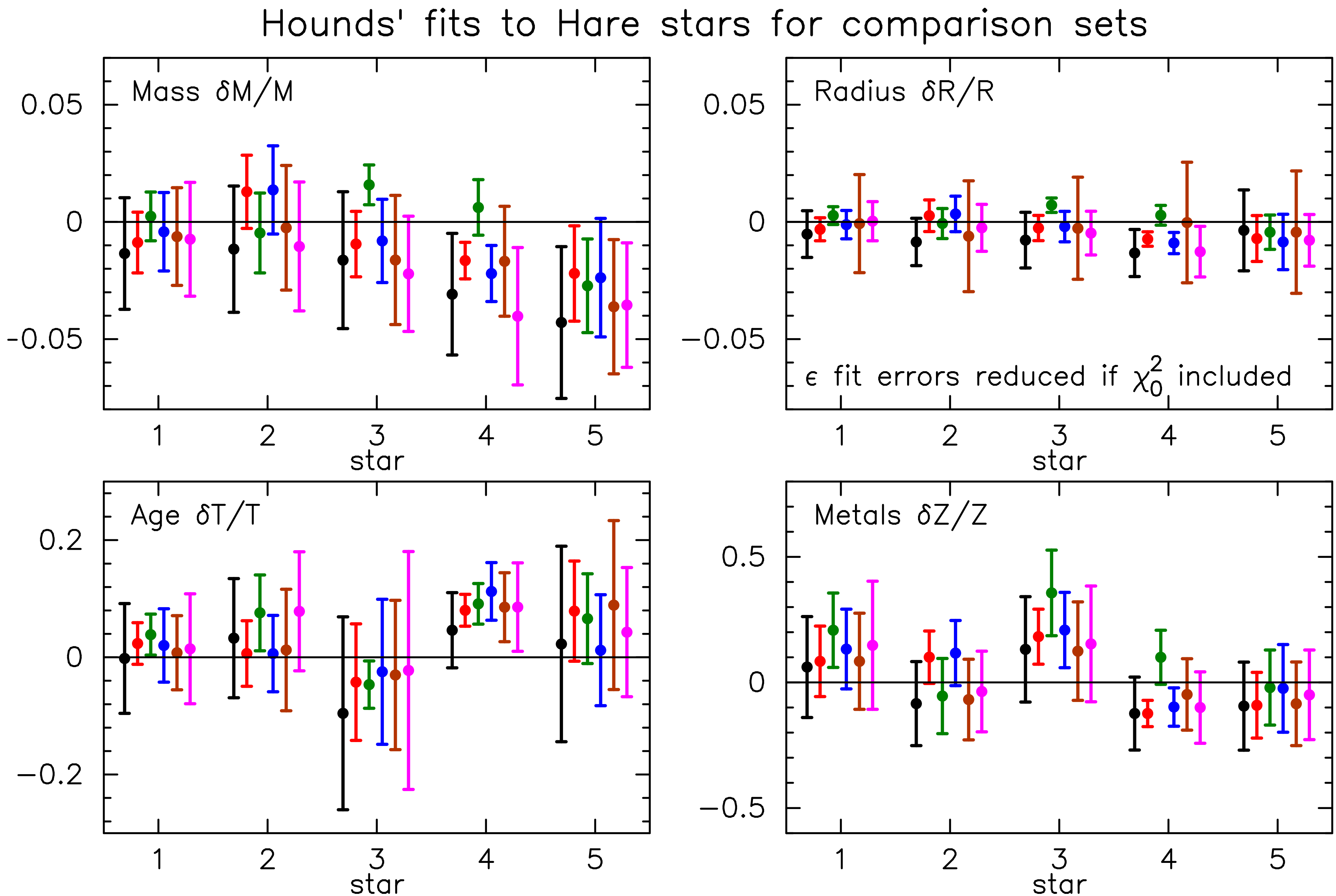

Hound's fits to Hare stars: exponential overshoot ov Line=Hare value
Fred
George
Gerald
Patch
Zebedee
Zippy
Hare=0.09 outside grid
Hare has gravitational settling
Hare=0.20 step outside grid
Hound
0.04
0.02
0
1
2
3
4
5
6


Mean of fractional differences Hound−Hare for 5 Hare stars
Mass <|δM/M|>
Radius <|δR/R|>
Age <|δT/T|>
Metals <|δZ/Z|>
Helium <|δY/Y|>
MLTα <|δα/α|>
0.04
0
0.2
0.4
0.1
1 2 3 4 5 6
Hound

Hound fits: all hounds produced several fits with different input data/weights

Fits chosen for comparison
All fitted L, Teff, [Fe/H] (Spectro)
All except Roxburgh added a"surface correction" (SC) to model frequencies
(Roxburgh surface layer independent)

Basu (own code) $\chi s + \chi\nu$
Nasamba (AIMS code) $\chi s + \chi\nu/N$
Ong (own code)) $\chi s + \chi\nu/N + \chi\varepsilon/N + \ldots$
Reese (AIMS code) $\chi s + 3\chi\nu/N$ (+many others)
Roxburgh (own code) $\chi s + 3\chi\varepsilon/N$ (+many others)
Silva-Aguirre (BASTA code) $\chi s + \chi\nu/N$ (+others)


Hounds' fits to Hare stars for comparison sets
Mass δM/M
0.05
0
−0.05
1
2
3
4
5
star
Radius δR/R
ϵ fit errors reduced if χ^2_0 included
Age δT/T
0.2
−0.2
Metals δZ/Z
0.5
−0.5

Hound fits: all hounds produced several fits with different input data/weights

Fits chosen for comparison
All fitted L, Teff, [Fe/H] (Spectro)
All except Roxburgh added a"surface correction" (SC) to model frequencies
(Roxburgh surface layer independent)

Basu (own code) $\nu(n,l)$ + SC equal weights to L, Teff, [Fe/H] and each $\nu(n,l)$

Nasamba (AIMS code) $\nu(n,l)$ + SC; weights 3:1 spectro : ν_{fit}

Ong (own code) $\nu(n,l)$ +SC + $\langle\varepsilon\rangle$, $\Delta\nu$, ν_{max} ν_{fit}, ε_{fit}
Equal weights to L,Teff, Fe/H, $\langle\varepsilon\rangle$, $\Delta\nu$, ν_{max}, ν_{fit} , ε_{fit}

Reese (AIMS code) $\nu(n,l)$ + SC; equal weights spectro: ν_{fit}

Roxburgh (own code) epsilons; equal weights spectro : ε_{fit}

Silva-Aguirre (BASTA code) $\nu(n,l)$ +SC weights 3:1 spectro: ν_{fit}

Fit to $L, T_{eff}, [Fe/H] \{+\Delta_\nu, \nu_{max}\} \rightarrow \chi^2_s$

1) Fit to Frequencies $\nu_{n\ell}$ + "Surface Correction" $F(\nu)$

$$\chi^2_\nu = \sum_{n,\ell} \left(\frac{\nu^m_{n\ell} + F_{n\ell}(\nu^m_{n\ell}) - \nu^o_{n\ell}}{\sigma^o_{n\ell}} \right)^2 \quad \text{minimise wrt F}$$

2) Surface independent fits: ratios $r_{n\ell}$, epsilons $\epsilon_{n\ell}$, eg

$$\chi^2_r = \sum \left(\frac{r^m_{n\ell} - r^o_{n\ell}}{s^o_{n\ell}} \right)^2 \quad \left\{ + \chi^2_0 = \left(\frac{\nu^m_{k0} - \nu^o_{k0}}{\sigma^o_{k0}} \right)^2 \quad k = n_{min} \right\}$$

Fits chosen for comparison

Basu (own code) $\chi_s + \chi_\nu$

Nasamba (AIMS code) $\chi s + \chi\nu/N$

Ong (own code)) $\chi s + \chi\nu/N + \chi\varepsilon/N$ + …

Reese (AIMS code) $\chi s + 3\chi\nu/N$ (+many others)

Roxburgh (own code) $\chi s + 3\chi\varepsilon/N$ (+many others)

Silva-Aguirre (BASTA code) $\chi s + \chi\nu/N$ (+others)

Fitting "observed" and model data

$$\chi_s^2 = \sum \left(\frac{L^o - L^m}{\sigma^o} \right)^2 + \dots \quad (L, T_{eff}, [Fe/H] + \dots$$

$$\chi_\nu^2 = \sum_{n,\ell} \left(\frac{\nu_{n\ell}^m + F_{n\ell}(\nu_{n\ell}^m) - \nu_{n\ell}^o}{\sigma_{n\ell}^o} \right)^2 \text{ minimise wrt F}$$

$$\chi_r^2 = \sum \left(\frac{r_{n\ell}^m - r_{n\ell}^o}{s_{n\ell}^o} \right)^2 \qquad \chi_0^2 = \left(\frac{\nu_{k0}^m - \nu_{k0}^o}{\sigma_{k0}^o} \right)^2 \; k = n_{min}$$

Basu (own code) $\chi_s + \chi_\nu + \chi_0$

Nasamba (AIMS code) $\chi_s + \chi_\nu / N_\nu$

Ong (own code)) $\chi_s + \chi_\nu / N_\nu + \chi_\varepsilon / N_\varepsilon$ …

Reese (AIMS code) $\chi_s + 3\chi_\nu / N_\nu$ (+many others)

Roxburgh (own code) $\chi_s + 3\, \chi_\nu / N_\varepsilon$ (+many others)

Silva-Aguirre (BASTA code) $\chi_s + \chi_\nu / N_\nu$ (+others)

Fitting "observed" and model data

$$\chi_s^2 = \sum \left(\frac{L^o - L^m}{\sigma_L^o}\right)^2 + \dots \quad (L, T_{eff}, [\mathrm{Fe/H}] + \dots$$

$$\chi_\nu^2 = \sum_{n,\ell} \left(\frac{\nu_{n\ell}^m + F_{n\ell}(\nu_{n\ell}^m) - \nu_{n\ell}^o}{\sigma_{n\ell}^o}\right)^2 \quad \text{minimise wrt F}$$

$$\chi_r^2 = \sum_{n,\ell} \left(\frac{r_{n\ell}^m - r_{n\ell}^o}{s_{n\ell}^o}\right)^2 \quad \chi_0^2 = \left(\frac{\nu_{k0}^m - \nu_{k0}^o}{\sigma_{k0}^o}\right)^2 \; k = n_{min}$$

1) Fit to Frequencies $\nu_{n\ell}$ + "Surface Correction" $F(\nu)$

$$\chi_\nu^2 = \sum_{n,\ell} \left(\frac{\nu_{n\ell}^m + F_{n\ell}(\nu_{n\ell}^m) - \nu_{n\ell}^o}{\sigma_{n\ell}^o}\right)^2 \quad \text{minimise wrt F}$$

2) Surface independent fits: ratios $r_{n\ell}$, epsilons $\epsilon_{n\ell}$, eg

$$\chi_r^2 = \sum \left(\frac{r_{n\ell}^m - r_{n\ell}^o}{s_{n\ell}^o}\right)^2 \quad \left\{+ \chi_0^2 = \left(\frac{\nu_{k0}^m - \nu_{k0}^o}{\sigma_{k0}^o}\right)^2 \; k = n_{min}\right\}$$

Should include epsilons maybe separate slide

Basu (own code) $\chi_s + \chi_\nu + (\chi_0 + \chi_1 + \dots)/10000$ B&G corr

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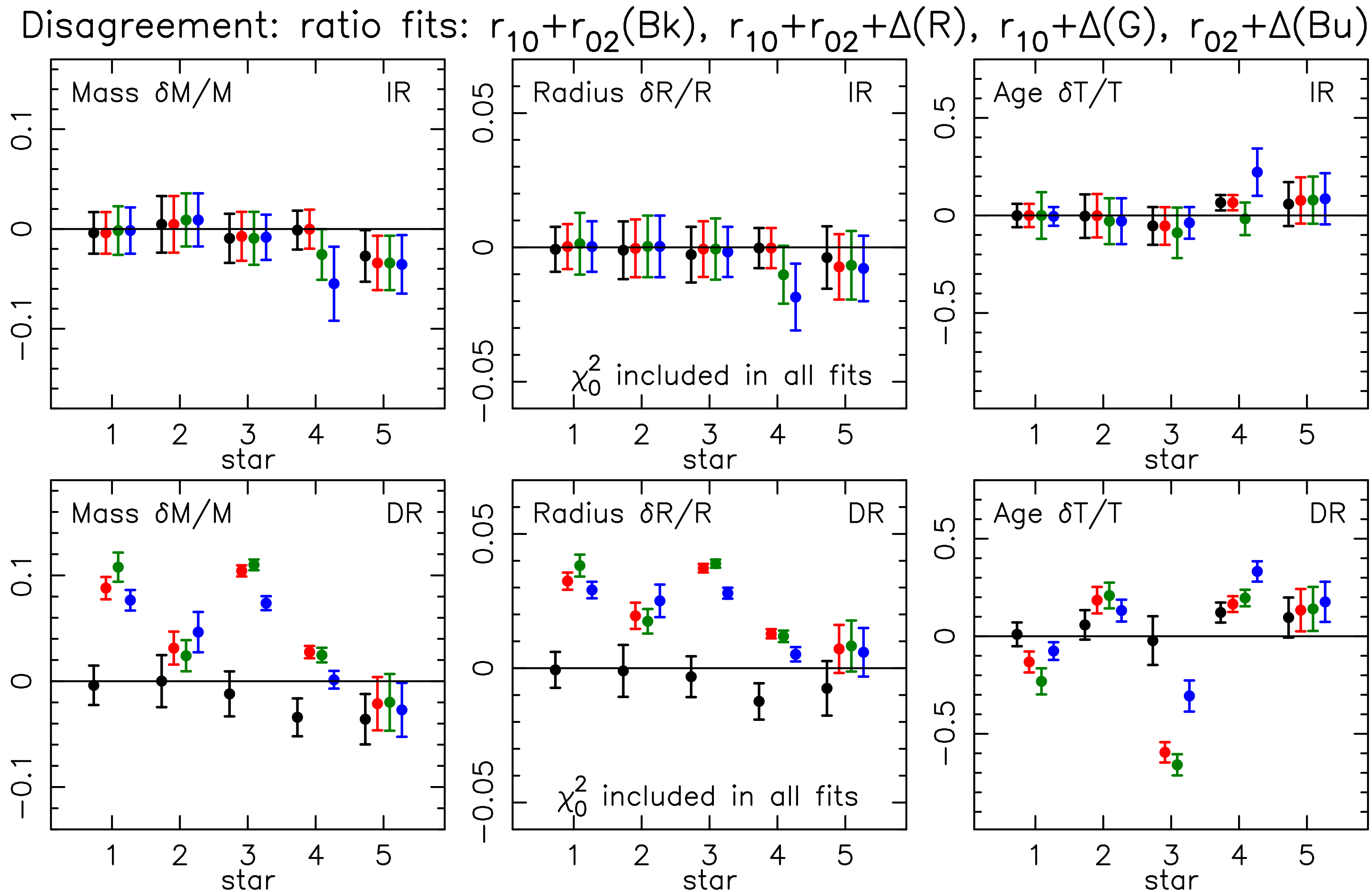
Nasamba (AIMS code) $\chi_s + \chi_\nu / N_\nu$ B&G corr

Ong (own code)) $\chi_s + \chi_\nu / N_\nu + \chi_\varepsilon / N_\varepsilon \dots$ B&G corr

Reese (AIMS code) $\chi_s + 3\chi_\nu / N_\nu$ (+many others) B&G corr

Roxburgh (own code) $\chi_s + 3\,\chi_\varepsilon / N_\varepsilon$ (+many others) No corr

Silva-Aguirre (BASTA code) $\chi_s + \chi_\nu / N_\nu$ (+others) B&G corr



Disagreement: ratio fits: r10+r02(Bk), r10+r02+Δ(R), r10+Δ(G), r02+Δ(Bu)
Mass δM/M
IR
Radius δR/R
IR
χ0² included in all fits
Age δT/T
IR
Mass δM/M
DR
Radius δR/R
DR
χ0² included in all fits
Age δT/T
DR
star

Fitting "observed" and model data

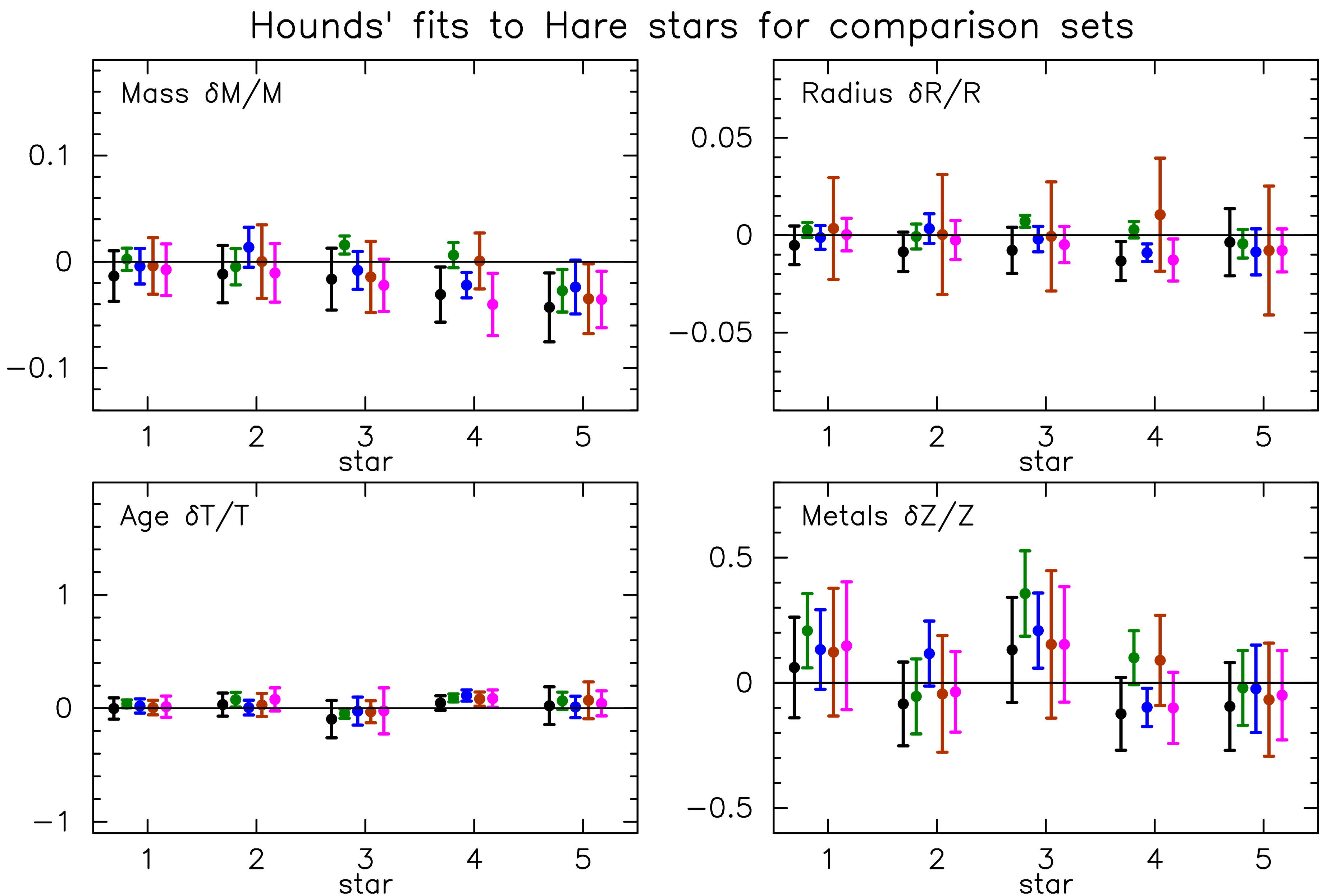
$$\chi_s^2 = \sum \left(\frac{L^o - L^m}{\sigma_L^o} \right)^2 + \ldots \quad L, T_{eff}, [\mathrm{Fe/H}] \; (\Delta_\nu, \nu_{max} + \ldots$$

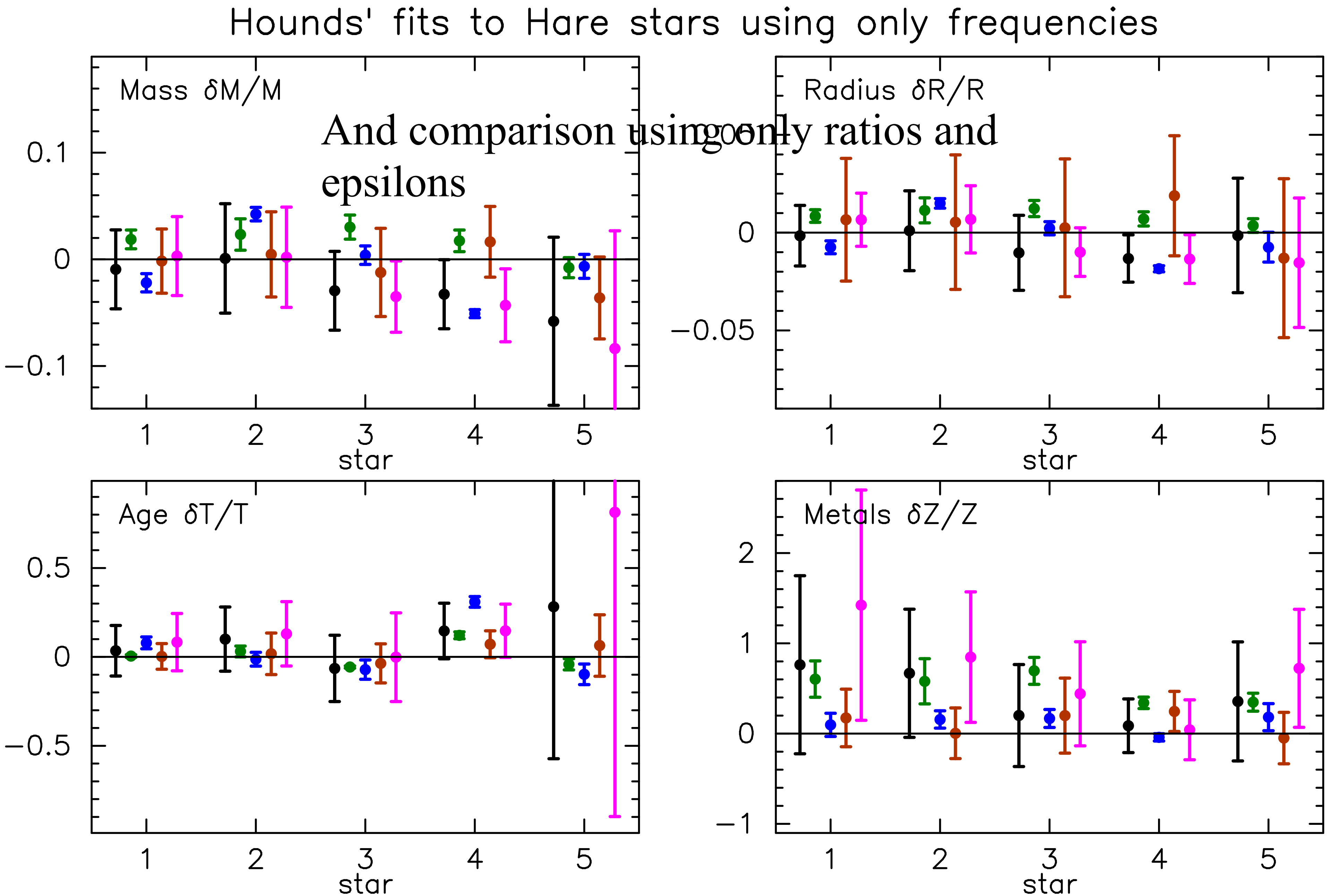
$$\chi_\nu^2 = \sum_{n,\ell} \left(\frac{\nu_{n\ell}^m + F_{n\ell}(\nu_{n\ell}^m) - \nu_{n\ell}^o}{\sigma_{n\ell}^o} \right)^2 \quad \text{minimise wrt F}$$

$$\chi_0^2 = \left(\frac{\nu_{k0}^m - \nu_{k0}^o}{\sigma_{k0}^o} \right)^2 \quad \chi_1^2 = \left(\frac{\nu_{k1}^m - \nu_{k1}^o}{\sigma_{k1}^o} \right)^2 \quad \ldots \quad k = n_{min}$$

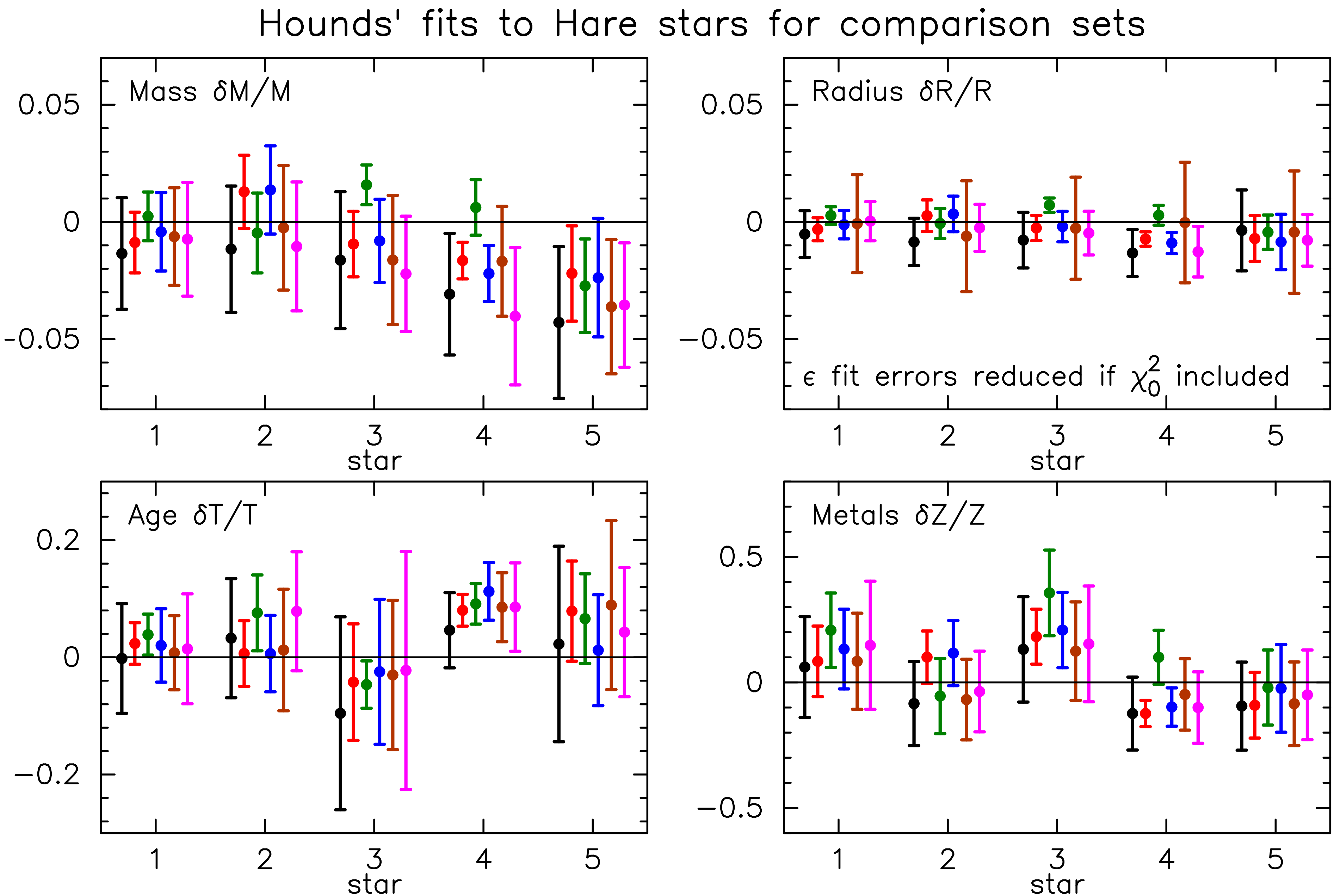
$$\chi_r^2 = \sum_{n,\ell} \left(\frac{r_{n\ell}^m - r_{n\ell}^o}{s_{n\ell}^o} \right)^2 \quad eg \quad r_{n2} = \frac{\nu_{n1,0} - \nu_{n-1,2}}{\nu_{n,1} - \nu_{n-1,1}}$$

$$\chi_\epsilon^2 = \sum_{n,\ell} \left(\frac{\epsilon_\ell^o(\nu_{n\ell}^o) - \epsilon_\ell^m(\nu_{n\ell}^o) - E(\nu_{n\ell}^o)}{s_{n\ell}^\epsilon} \right)^2 \quad \epsilon_\ell(\nu_{n\ell}) = \frac{\nu_{n\ell}}{\Delta} - n - \frac{\ell}{2}$$

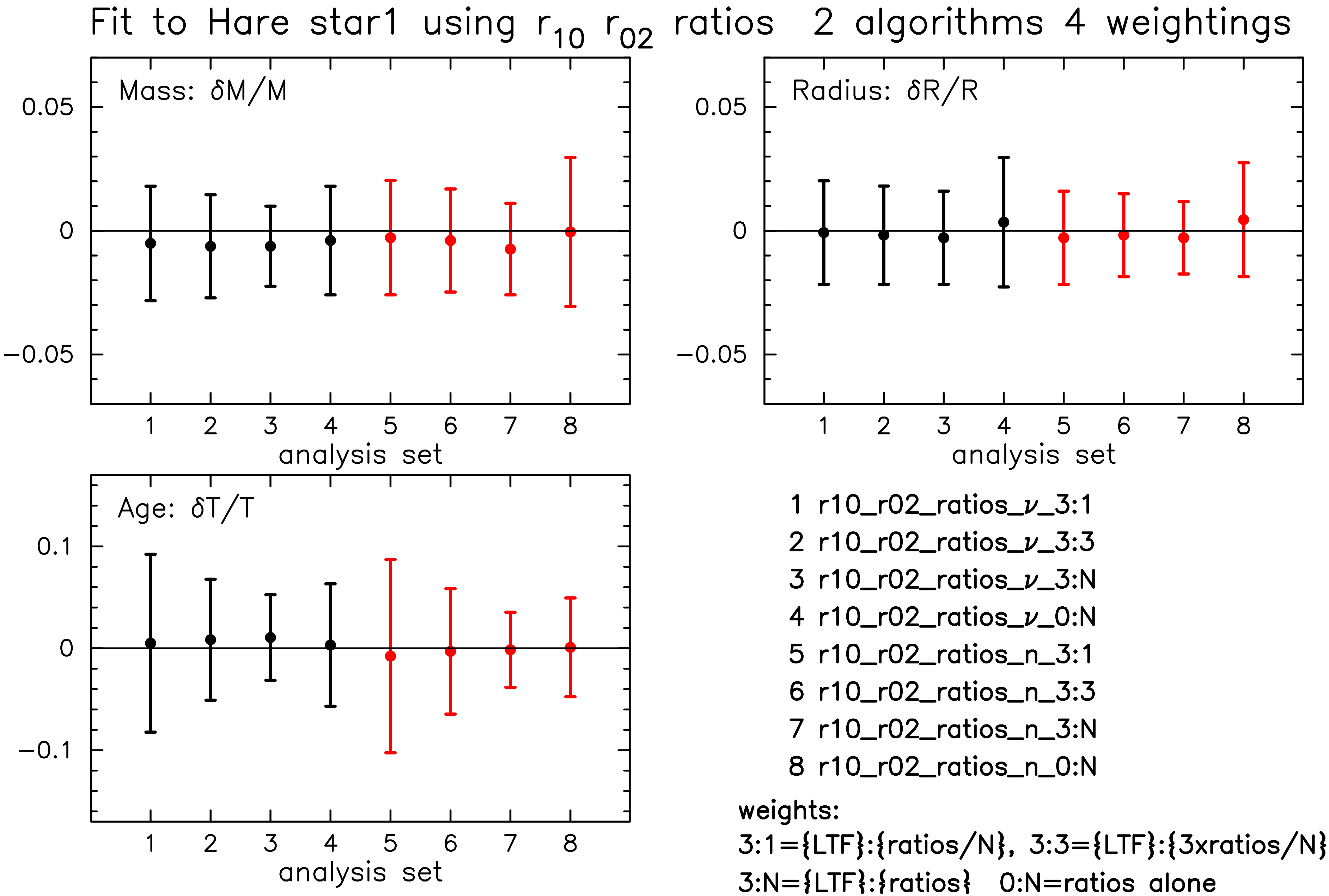

Hounds' fits to Hare stars for comparison sets
Mass δM/M
Radius δR/R
Age δT/T
Metals δZ/Z
0.1
0
−0.1
0.05
−0.05
1
−1
0.5
−0.5
1
2
3
4
5
star



And comparison using only ratios and epsilons



Hounds' fits to Hare stars for comparison sets
Mass δM/M
Radius δR/R
ϵ fit errors reduced if χ_0^2 included
Age δT/T
Metals δZ/Z
0.05
0
−0.05
0.2
−0.2
0.5
−0.5
1 2 3 4 5
star



Fit to Hare star1 using r10 r02 ratios 2 algorithms 4 weightings
Mass: δM/M
Radius: δR/R
Age: δT/T
0.05
0
−0.05
0.1
−0.1
1 2 3 4 5 6 7 8
analysis set
1 r10_r02_ratios_ν_3:1
2 r10_r02_ratios_ν_3:3
3 r10_r02_ratios_ν_3:N
4 r10_r02_ratios_ν_0:N
5 r10_r02_ratios_n_3:1
6 r10_r02_ratios_n_3:3
7 r10_r02_ratios_n_3:N
8 r10_r02_ratios_n_0:N
weights:
3:1={LTF}:{ratios/N}, 3:3={LTF}:{3xratios/N}
3:N={LTF}:{ratios} 0:N=ratios alone

Problem:

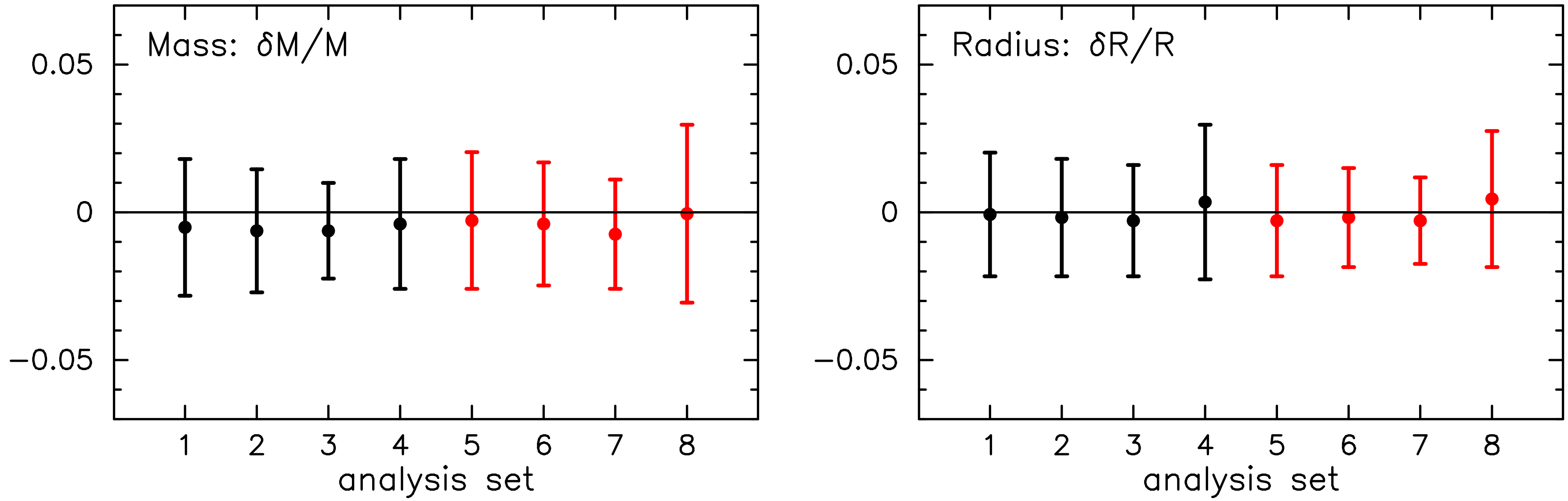
Reese vs Roxburgh

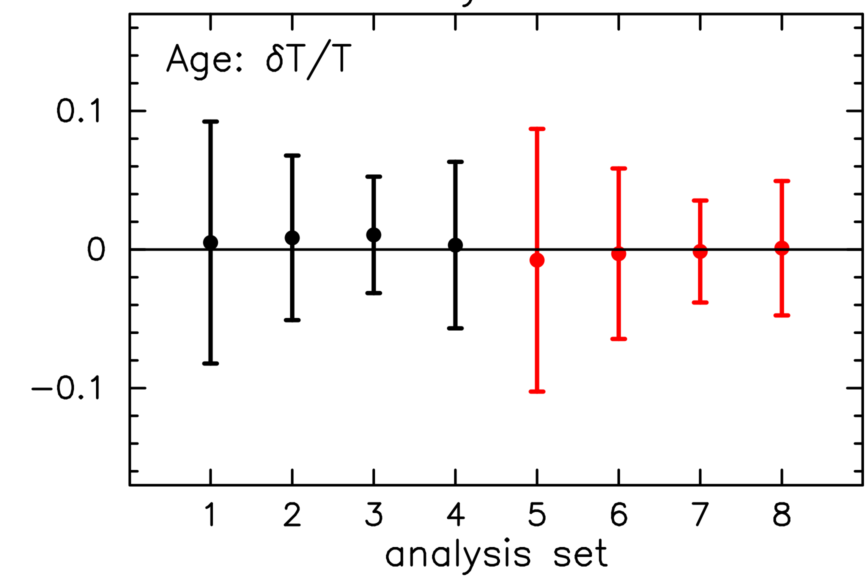
Agree on ratio fits for r_{012} when Δ_ν not added as constraint

Disagree for r_{012} r_{010}, r_{02} when Δ_ν is added as constraint

Not yet resolved

Fit to Hare star1 using r_{10} r_{02} ratios 2 algorithms 4 weightings

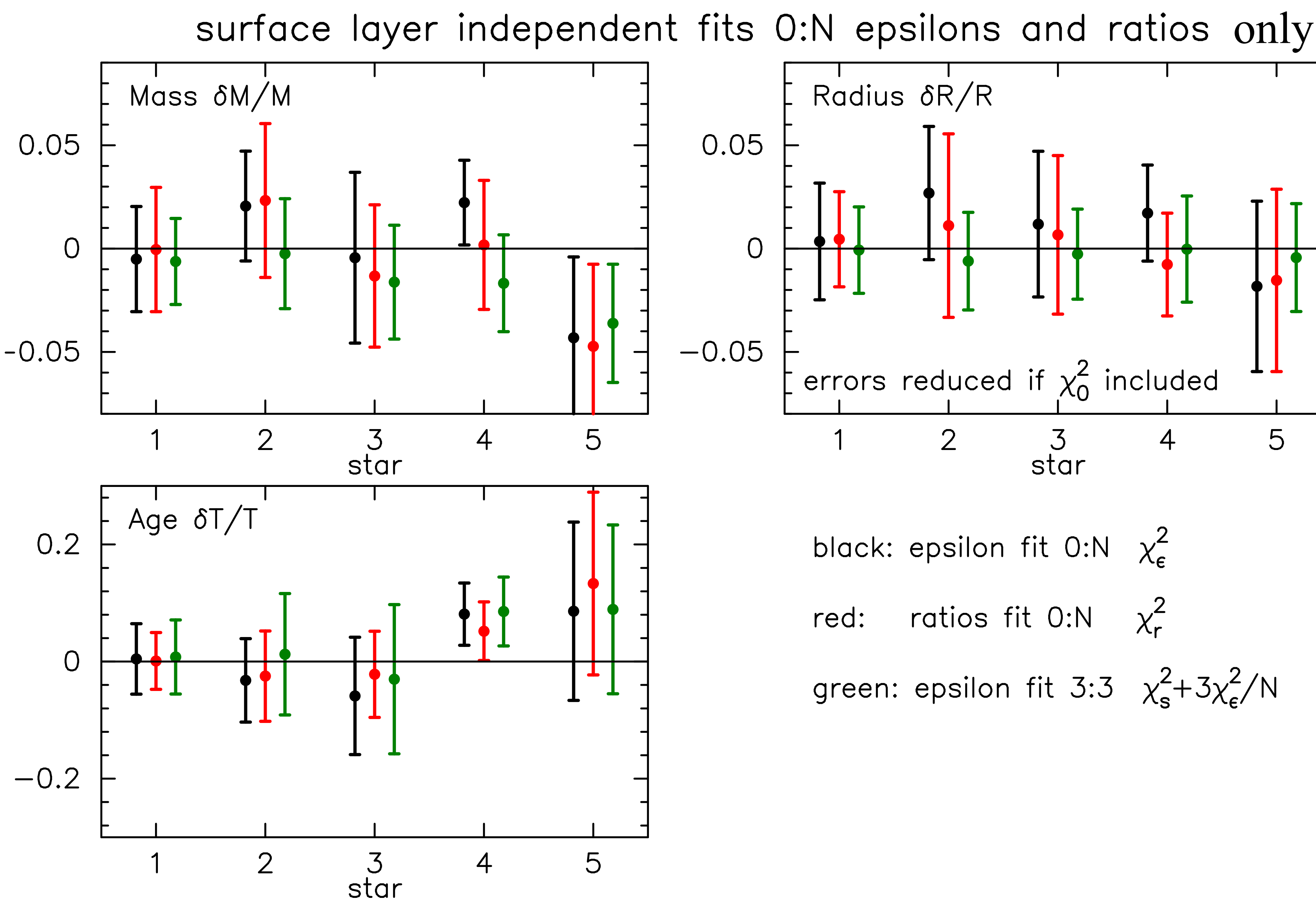




1 r10_r02_ratios_ν_3:1
2 r10_r02_ratios_ν_3:3
3 r10_r02_ratios_ν_3:N
4 r10_r02_ratios_ν_0:N
5 r10_r02_ratios_n_3:1
6 r10_r02_ratios_n_3:3
7 r10_r02_ratios_n_3:N
8 r10_r02_ratios_n_0:N

weights:
3:1={LTF}:{ratios/N}, 3:3={LTF}:{3xratios/N}
3:N={LTF}:{ratios} 0:N=ratios alone

But not for other stars??


surface layer independent fits 0:N epsilons and ratios only
Mass δM/M
Radius δR/R
Age δT/T
errors reduced if χ_0^2 included
star
black: epsilon fit 0:N χ_ϵ^2
red: ratios fit 0:N χ_r^2
green: epsilon fit 3:3 $\chi_s^2+3\chi_\epsilon^2/N$