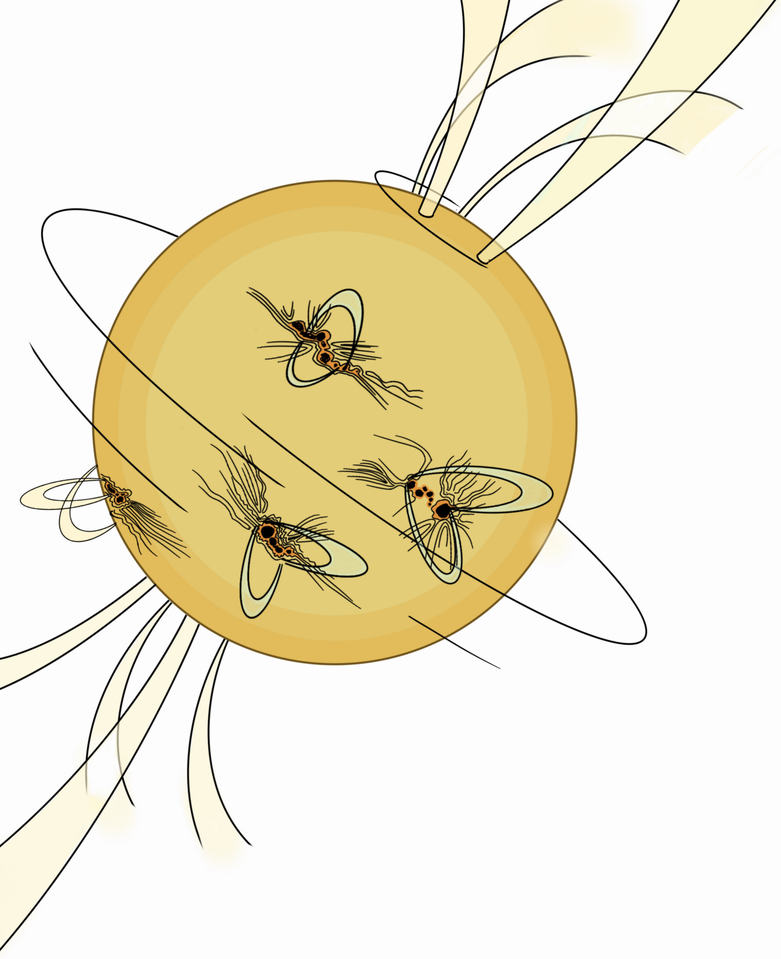


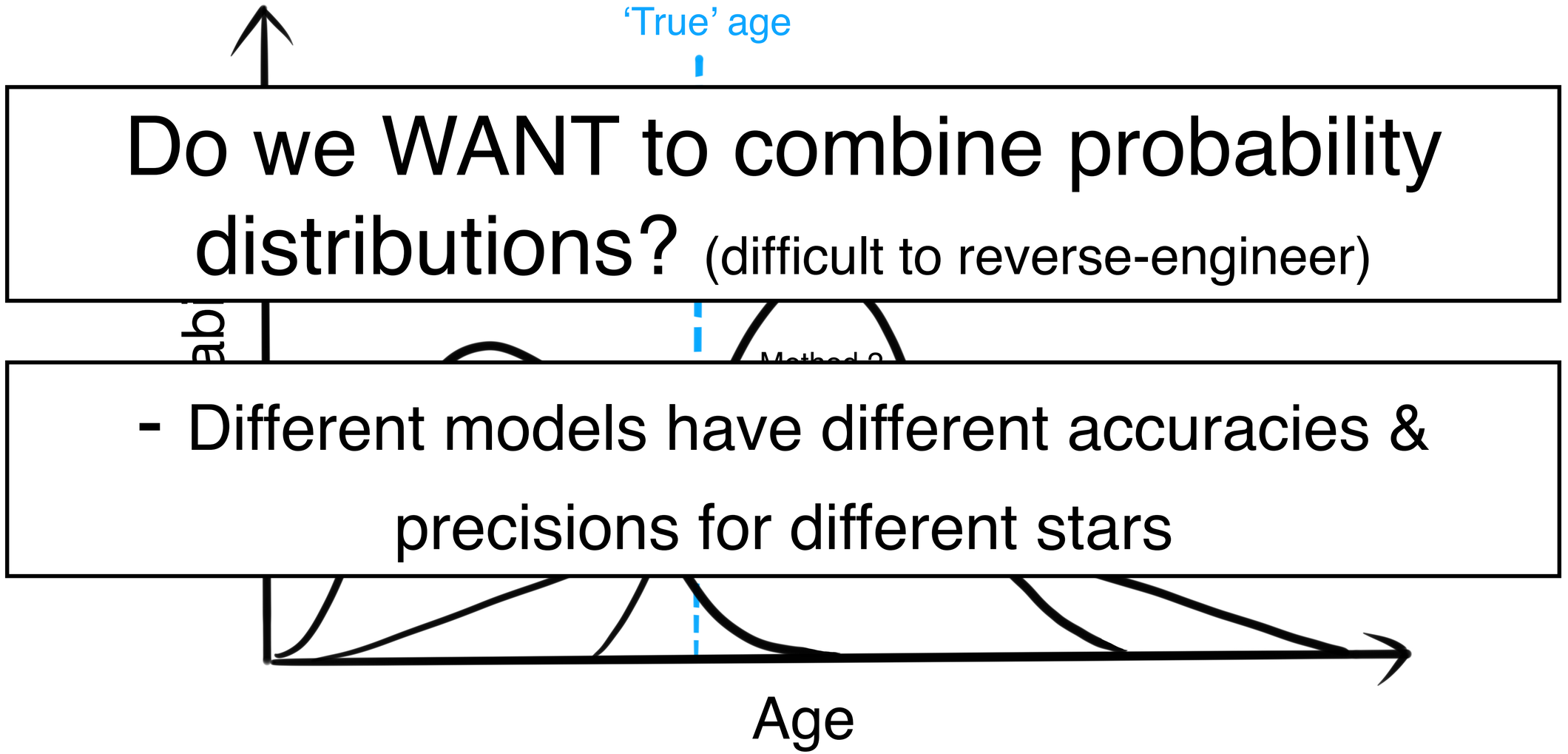
Final stellar parameters:
Combining methods

Ruth Angus

Assistant Curator, Department of Astrophysics, American Museum of Natural History
Associate Research Scientist, Center for Computational Astrophysics, Flatiron Institute
Assistant Adjunct Professor, Department of Astronomy, Columbia University

What is the final age?

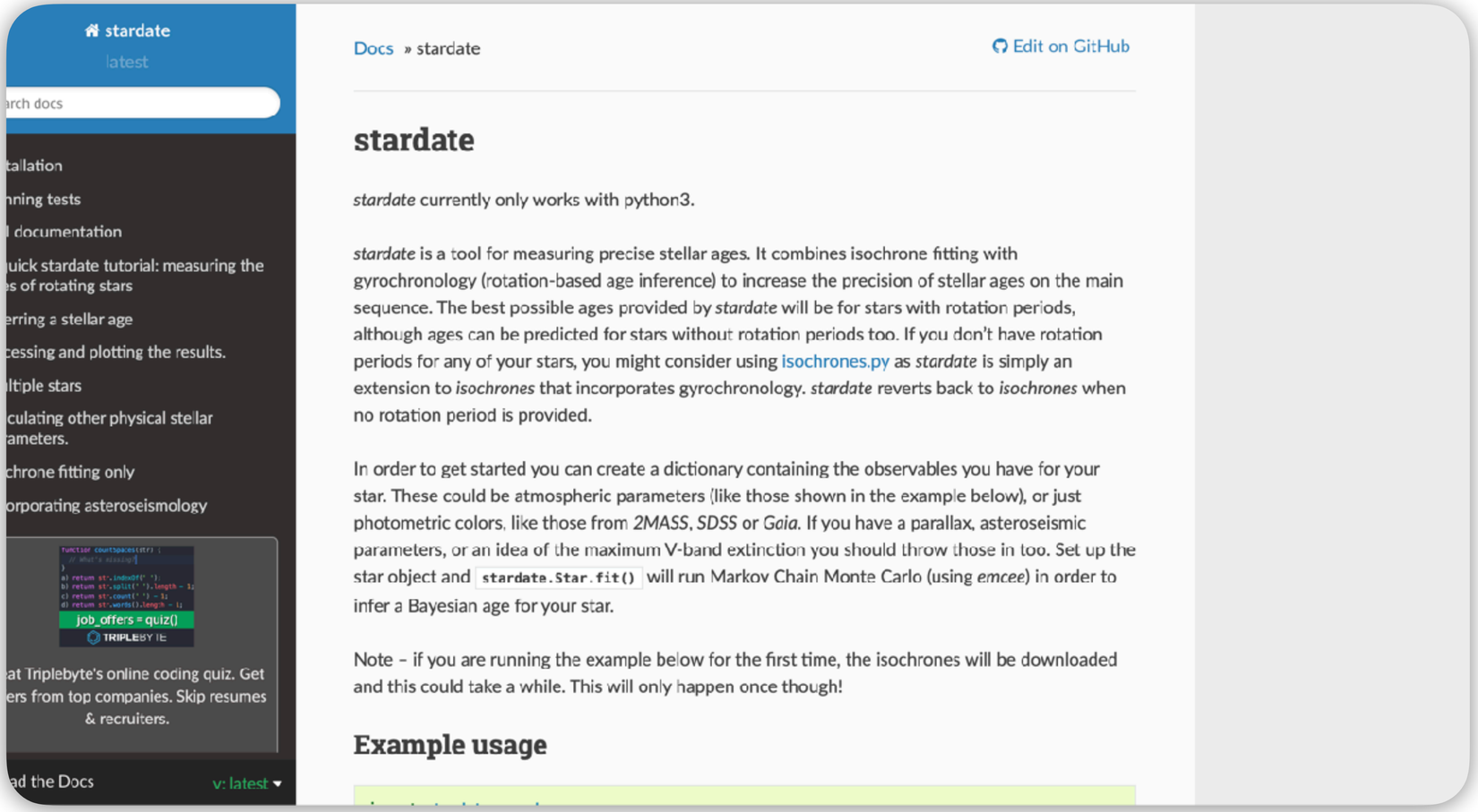
How do you **combine** probability distributions?



stardate (Cross-published in AJ and JOSS)

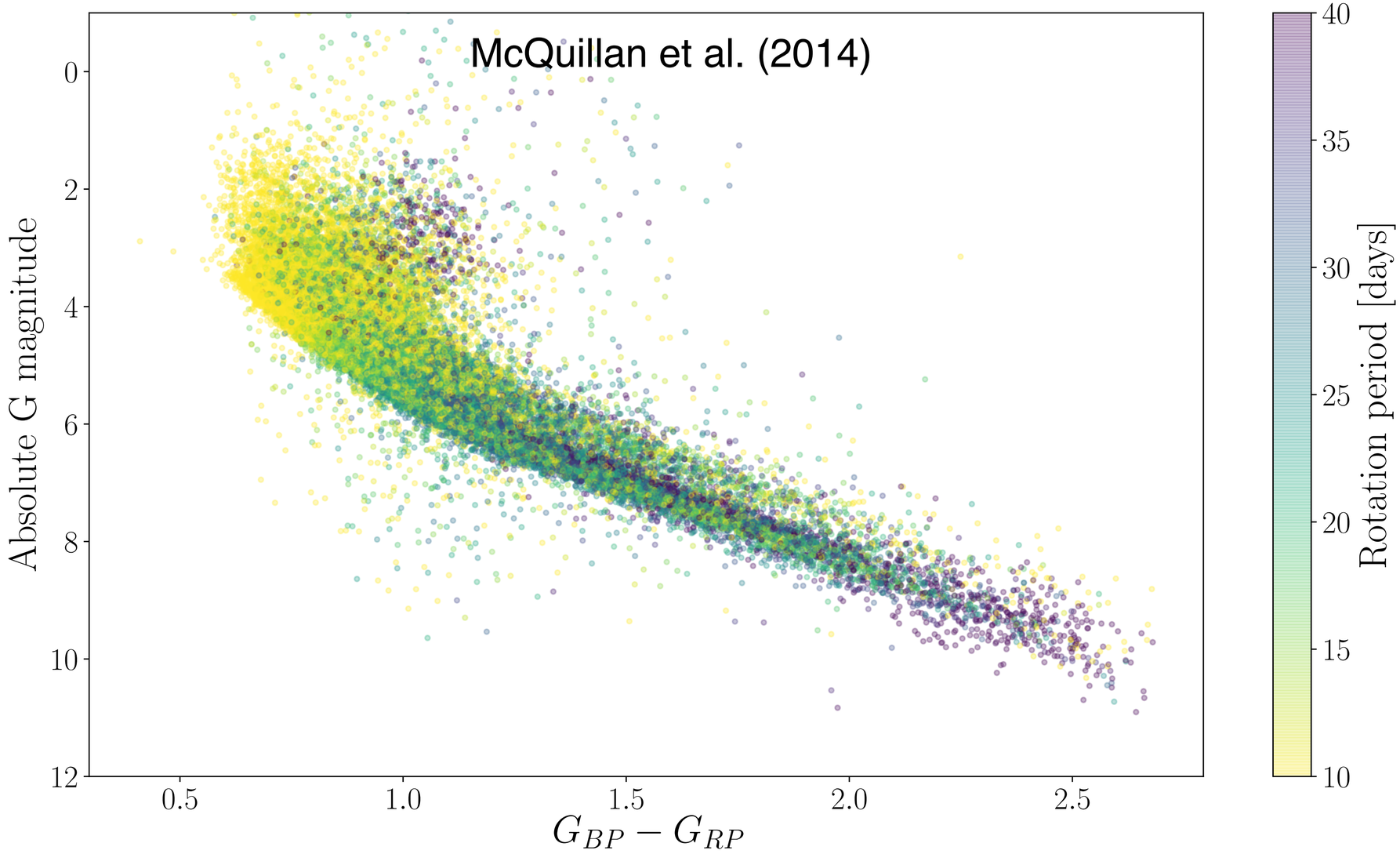
Angus et al (2019a, b)

stardate.readthedocs.io
github/ruthangus/stardate



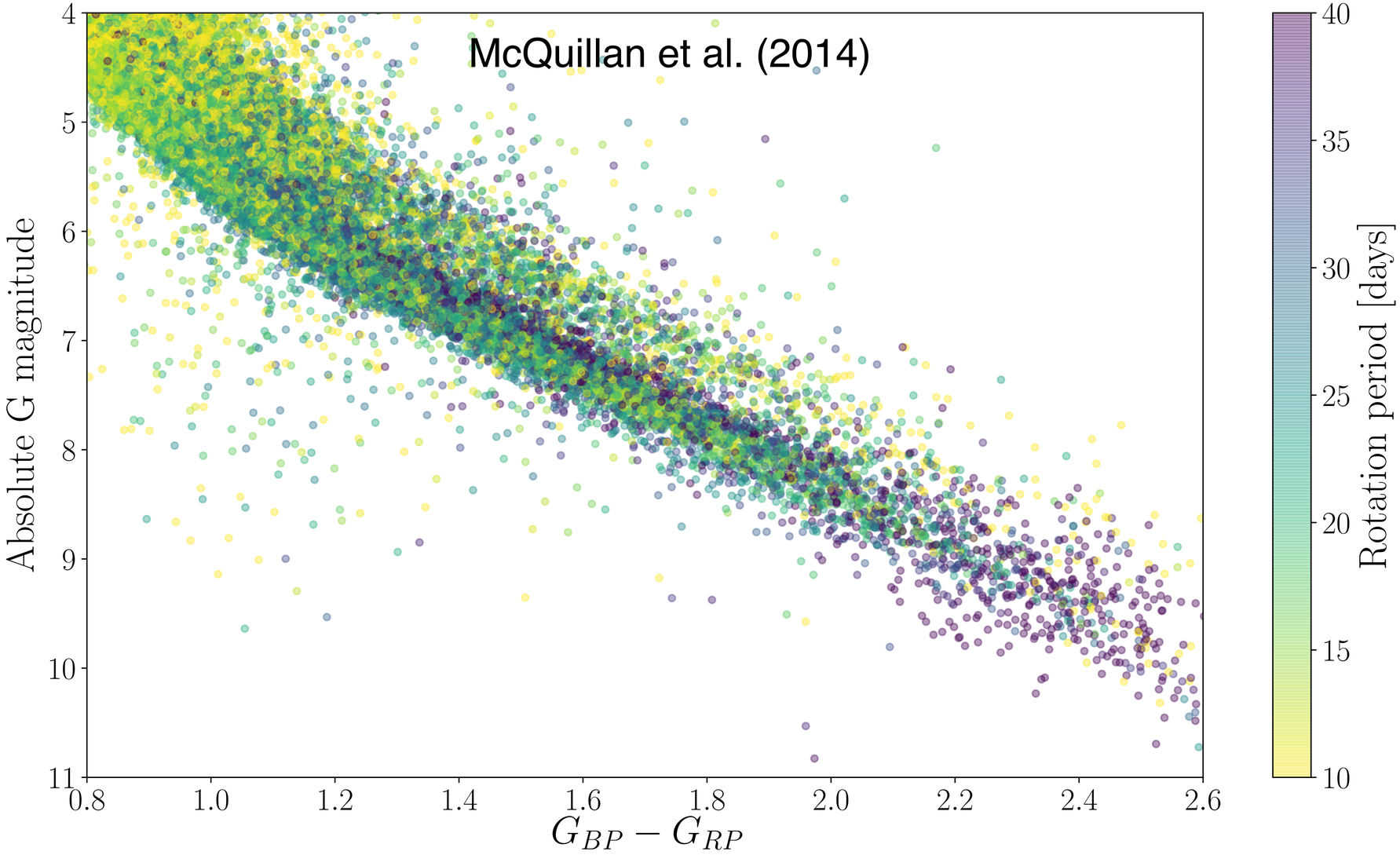
Built on isochrones.py (Morton, 2018)

Stars spin more slowly over time



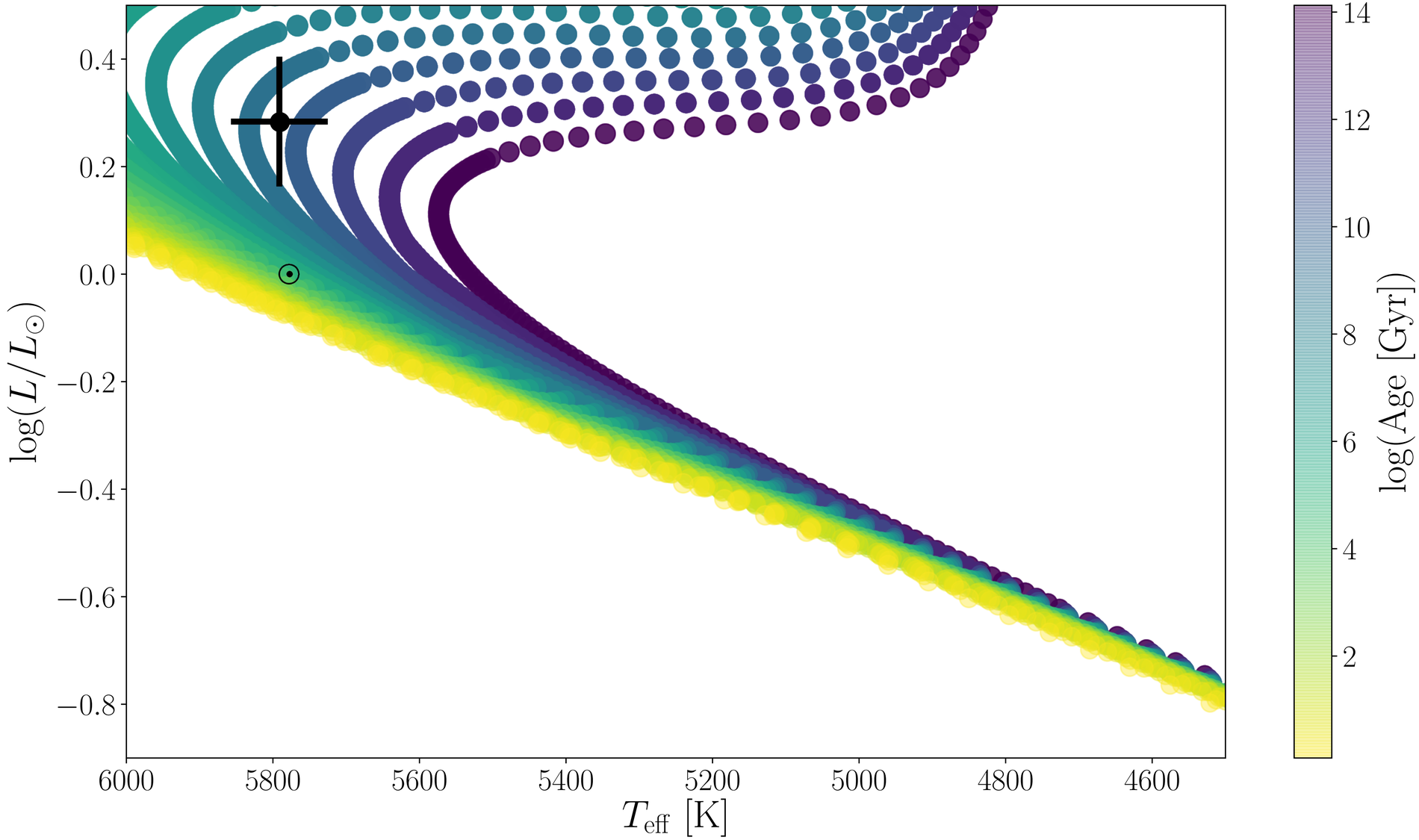
@ruthangus

Stars spin more slowly over time



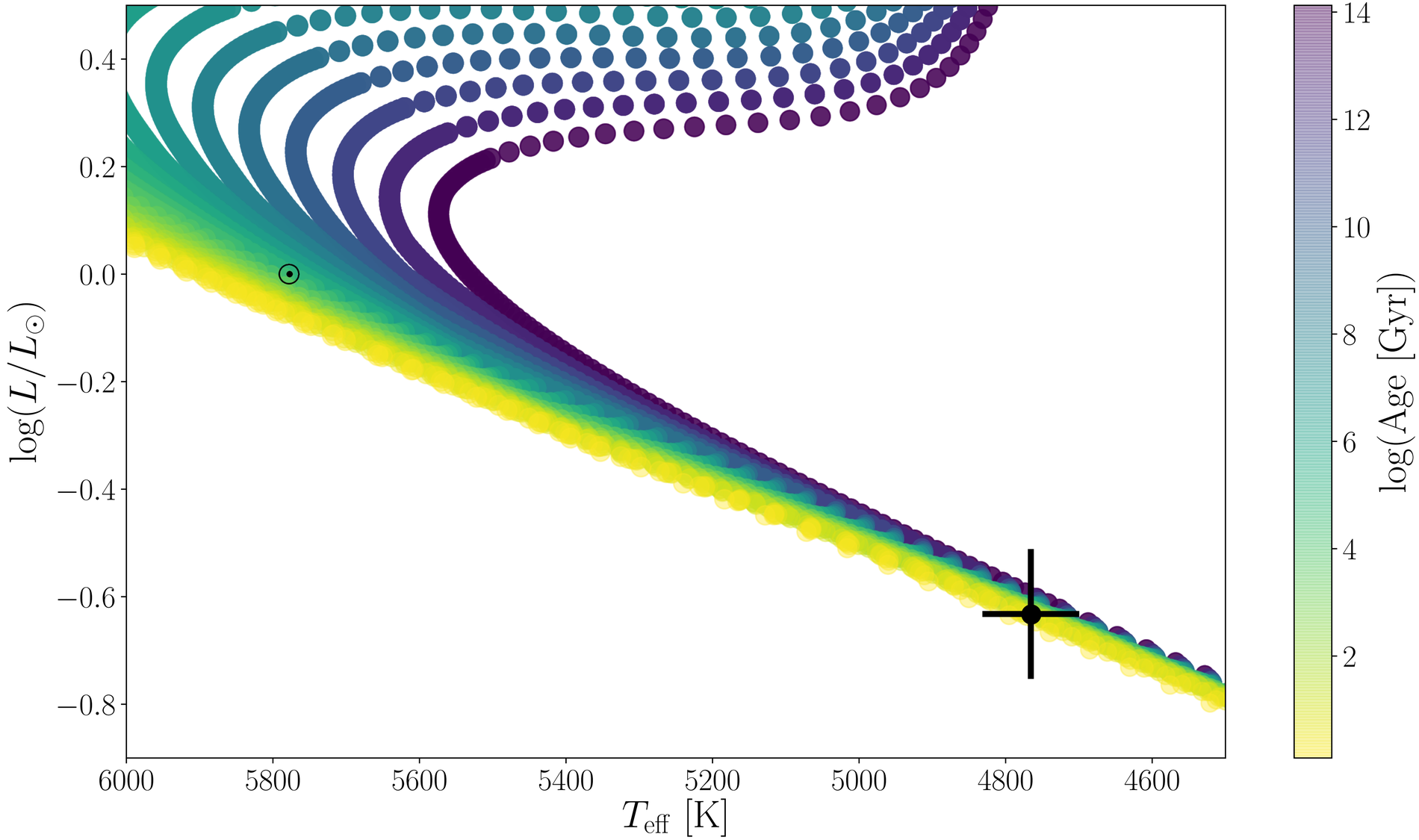
@ruthangus

We use MIST isochrones (Dotter + 2016, Choi + 2016)



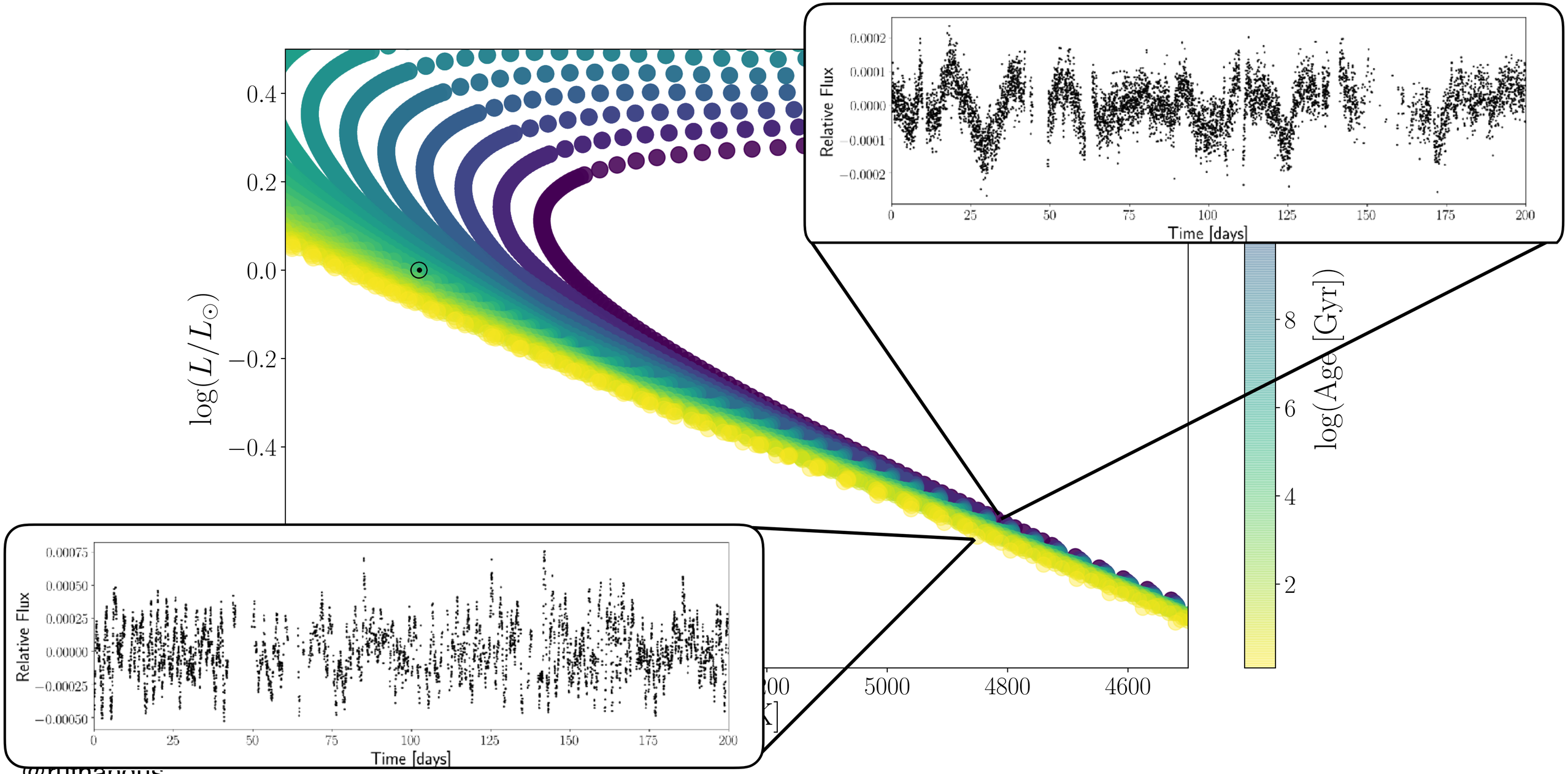
@ruthangus

We use MIST isochrones (Dotter + 2016, Choi + 2016)

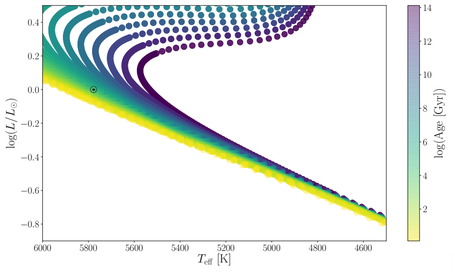


@ruthangus

We use MIST isochrones (Dotter + 2016, Choi + 2016)



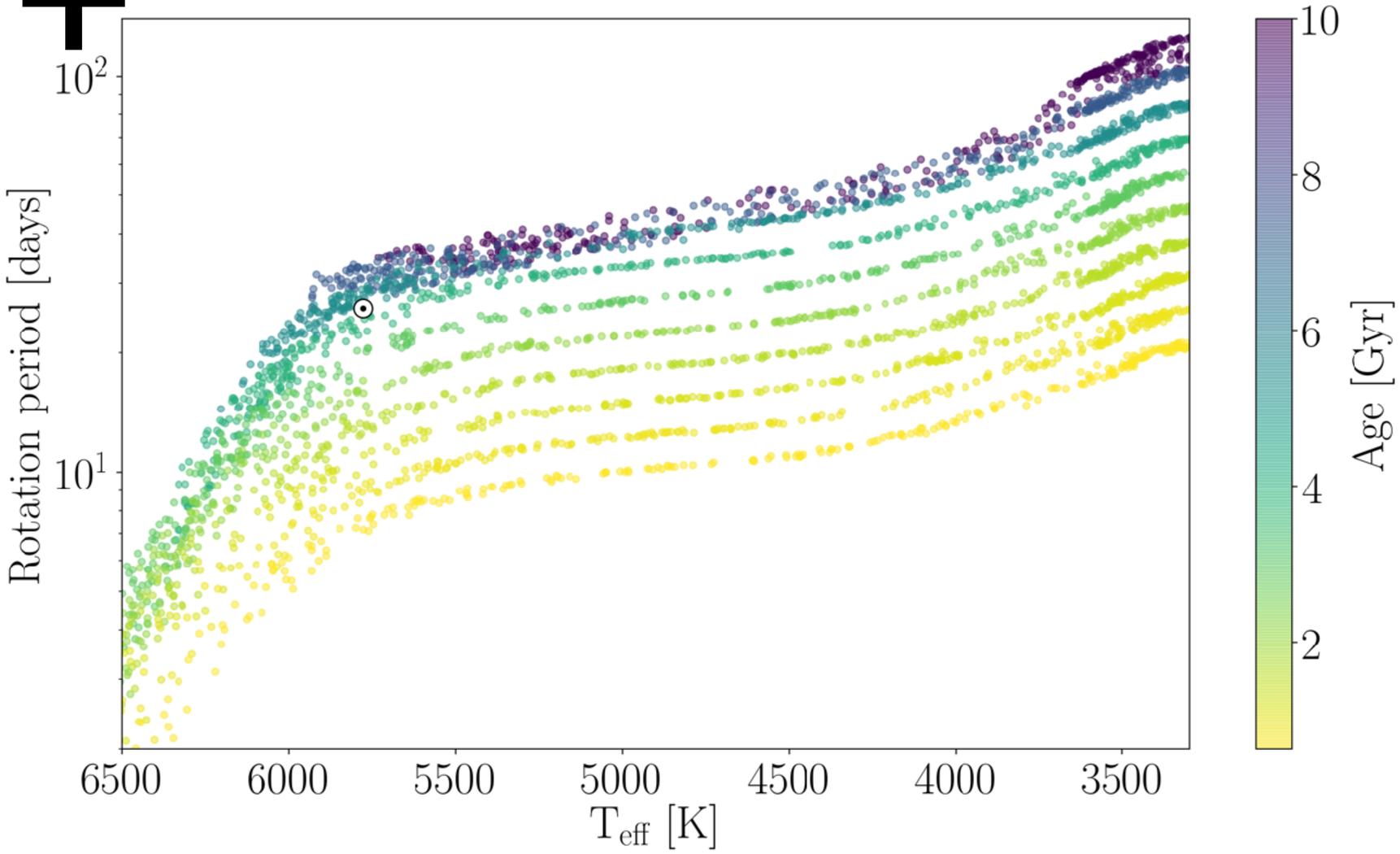
@ruthangus



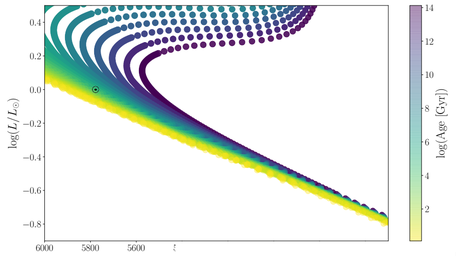
We use MIST isochrones (Dotter + 2016, Choi + 2016)

And a very simple gyrochronology model (Angus + 2019b)

+



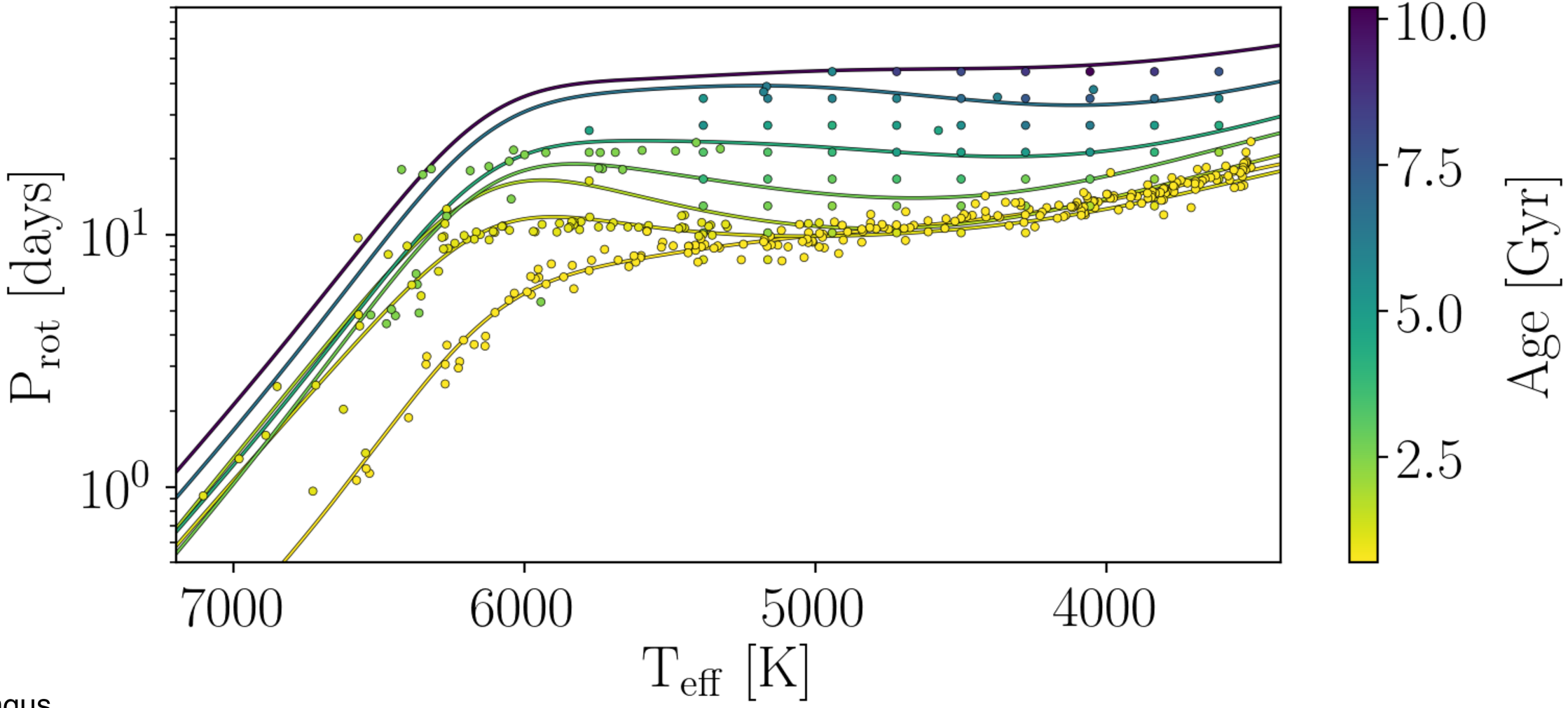
@ruthangus



We use MIST isochrones (Dotter + 2016, Choi + 2016)

To be replaced by GP-based gyrochronology model

+



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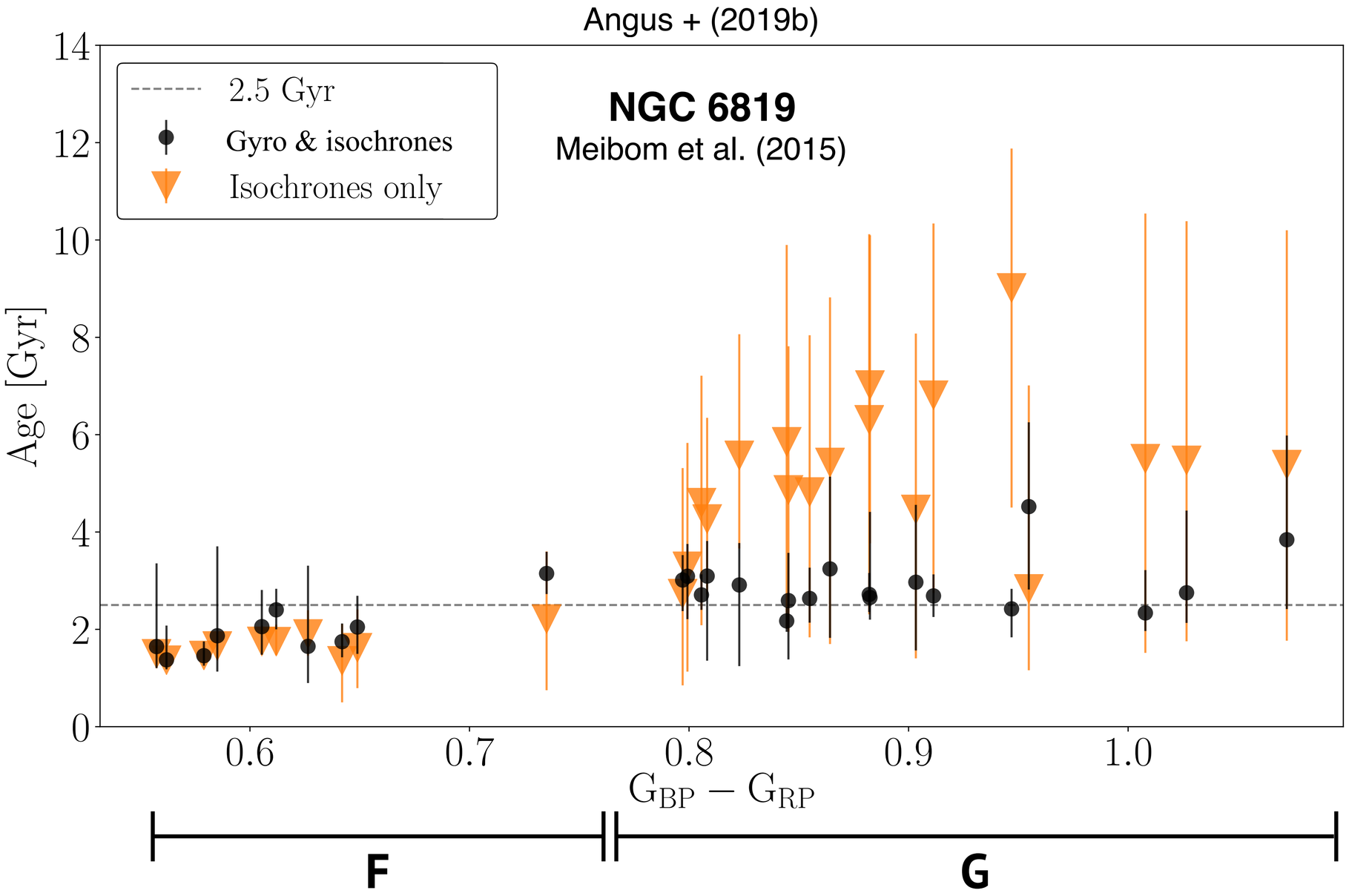
Combining methods can be useful



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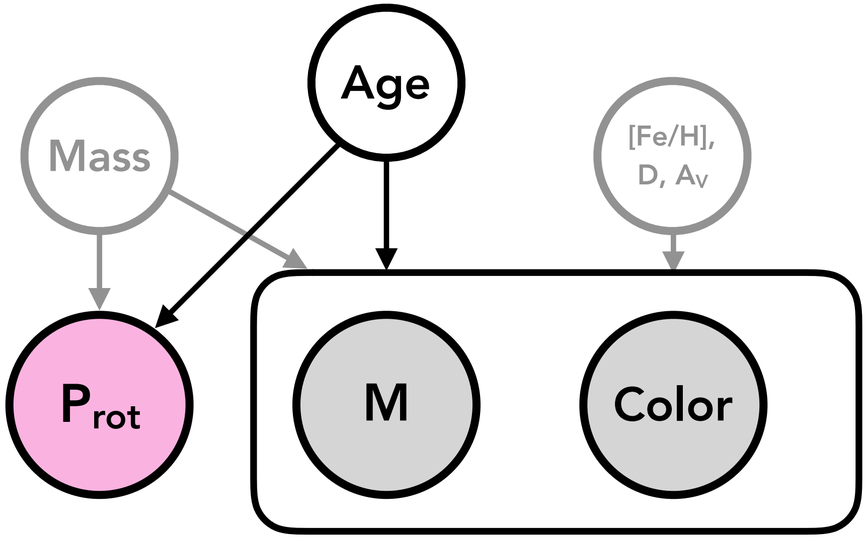
Combining methods in stardate

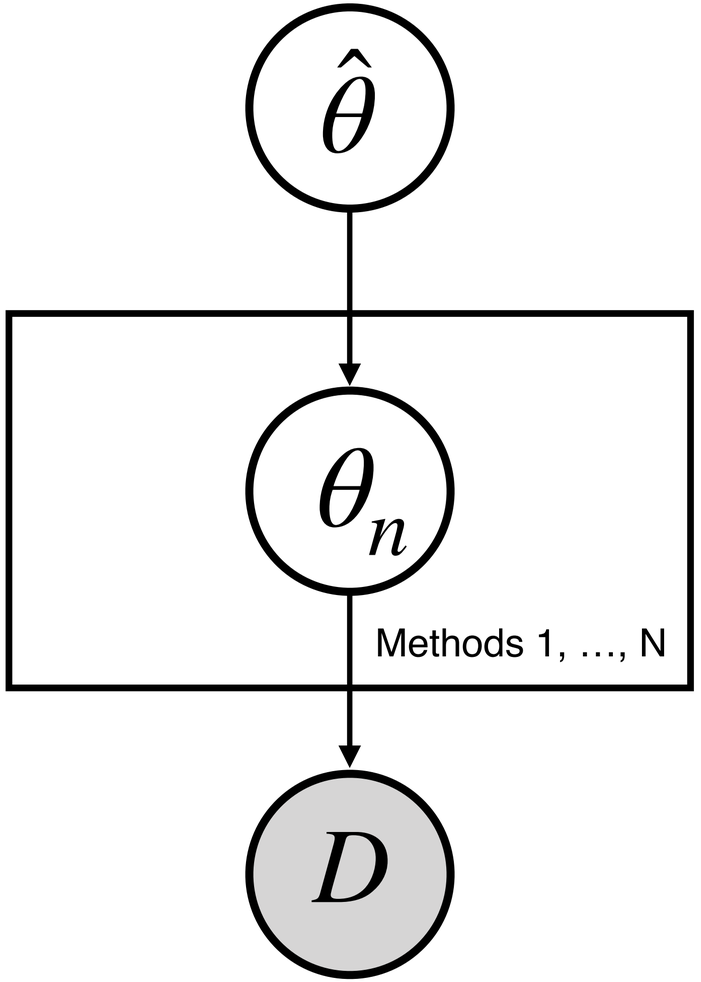
$$p(\theta \mid D) \propto p(\theta)\ p(D_{\rm iso} \mid \theta)\ p(P_{\rm rot} \mid \theta)$$

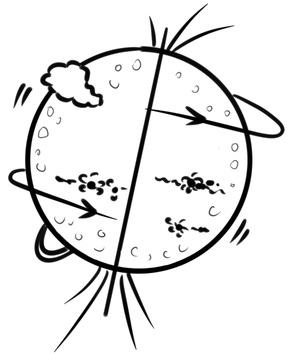
This is like augmenting MIST models with a rotation dimension



$\theta =$ Mass, Age, [Fe/H], etc

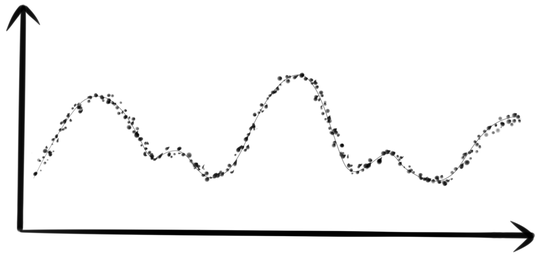
Combining methods in *general*: a hierarchical problem





True star

Observed star
(noisy)



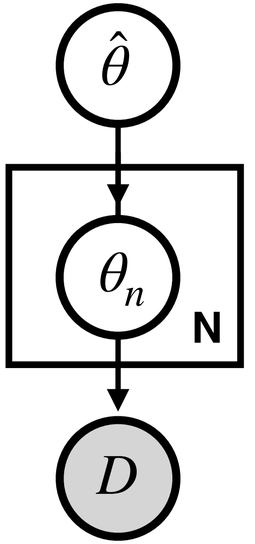
Data

$\theta =$ Mass, Age, [Fe/H], etc

Combining methods

We want:

$$p(\hat{\theta} \mid D)$$



$\theta =$ Mass, Age, [Fe/H], etc

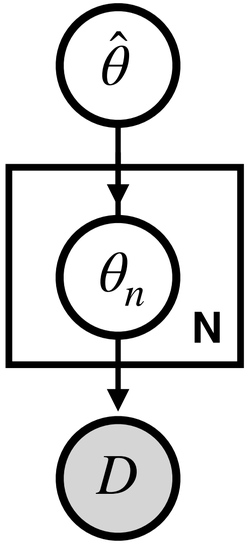
Combining methods

We want:

$$p(\hat{\theta} \mid D)$$

We have:

$$p(\hat{\theta})\ p(\{\theta_n\} \mid \hat{\theta})\ p(D \mid \{\theta_n\})$$



$\theta =$ Mass, Age, [Fe/H], etc

Combining methods

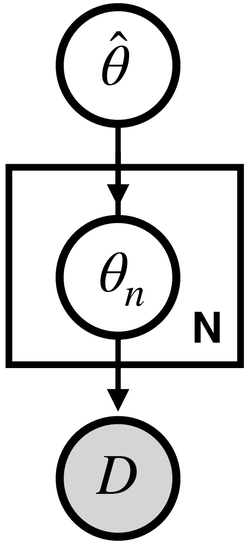
We want:

$$p(\hat{\theta} \mid D)$$

We have:

$$p(\hat{\theta})\ p(\{\theta_n\} \mid \hat{\theta})\ p(D \mid \{\theta_n\})$$

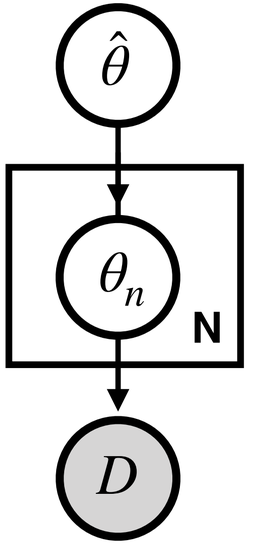
Marginalise over individual parameter measurements



$\theta =$ Mass, Age, [Fe/H], etc

Combining methods

$$p(\hat{\theta} \mid D) \propto \int p(\hat{\theta})\ p(\{\theta_n\} \mid \hat{\theta})\ p(D \mid \{\theta_n\})\ d\{\theta_n\}$$

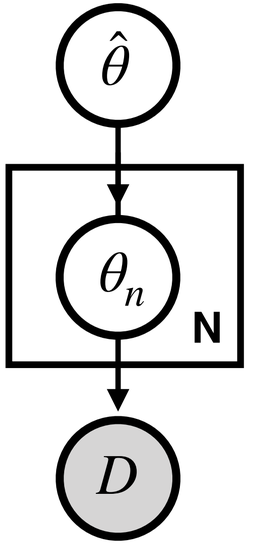


θ = Mass, Age, [Fe/H], etc

Combining methods

$$p(\hat{\theta} \mid D) \propto p(\hat{\theta}) \int p(\{\theta_n\} \mid \hat{\theta})\ p(D \mid \{\theta_n\})\ d\{\theta_n\}$$

Prior comes out of the integral

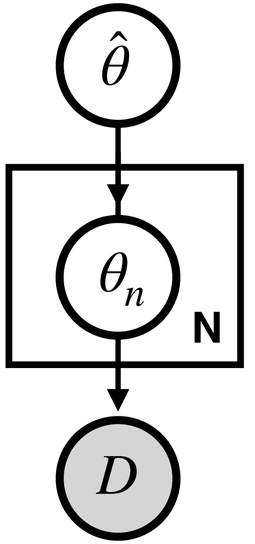


θ = Mass, Age, [Fe/H], etc

Combining methods

$$p(\hat{\theta} \mid D) \propto p(\hat{\theta}) \prod_{n=1}^{N} \int p(\theta_n \mid \hat{\theta})\ p(D \mid \theta_n)\ d\theta_n$$

Assumption of independence

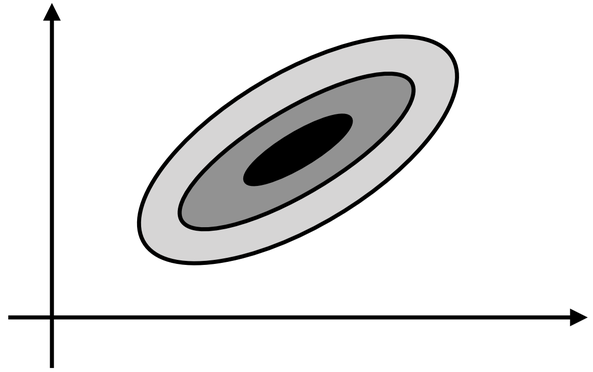


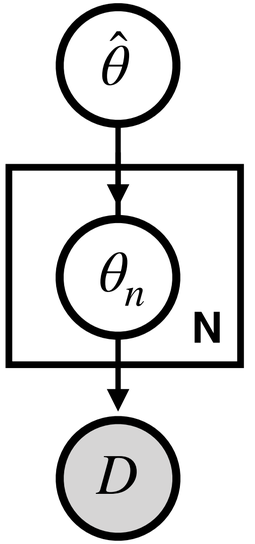
θ = Mass, Age, [Fe/H], etc

Combining methods

$$p(\hat{\theta}\,|\,D) \propto p(\hat{\theta}) \prod_{n=1}^{N} \int p(\theta_n\,|\,\hat{\theta})\; p(D\,|\,\theta_n)\; d\theta_n$$

The likelihood of the data,
given the measured parameters



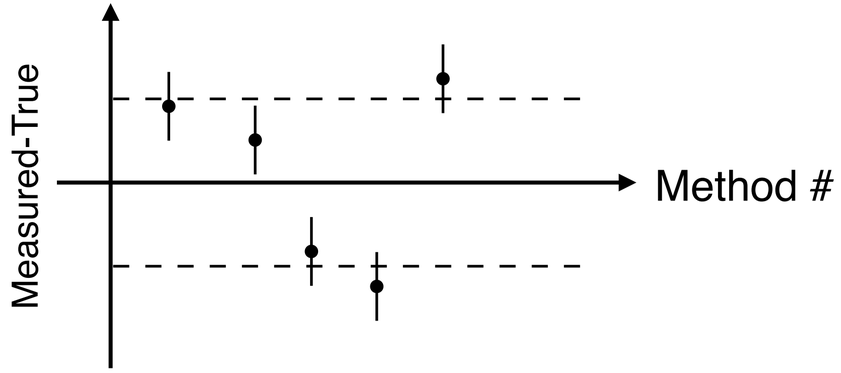


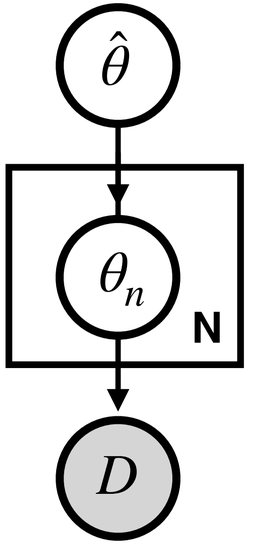
θ = Mass, Age, [Fe/H], etc

Combining methods

$$p(\hat{\theta}\,|\,D) \propto p(\hat{\theta}) \prod_{n=1}^{N} \int p(\theta_n\,|\,\hat{\theta})\; p(D\,|\,\theta_n)\; d\theta_n$$

The probability of the measured parameters, given the true parameters



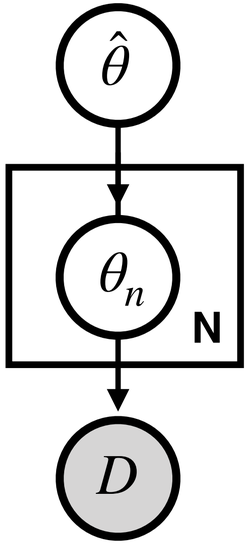


θ = Mass, Age, [Fe/H], etc

Combining methods: a hierarchical problem

$$p(\hat{\theta} \mid D) \propto p(\hat{\theta}) \prod_{n=1}^{N} \int p(\theta_n \mid \hat{\theta})\ p(D \mid \theta_n)\ d\theta_n$$

Marginalise over measured parameters

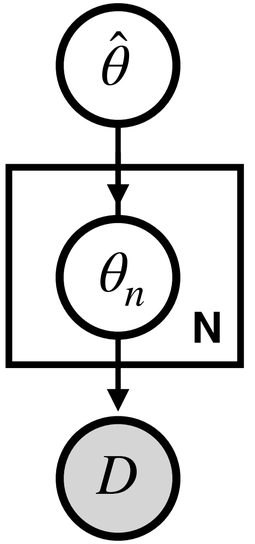


θ = Mass, Age, [Fe/H], etc

Combining methods: a hierarchical problem

$$p(\hat{\theta} \mid D) \propto p(\hat{\theta}) \prod_{n=1}^{N} \int p(\theta_n \mid \hat{\theta})\ p(D \mid \theta_n)\ d\theta_n$$

Take product over methods

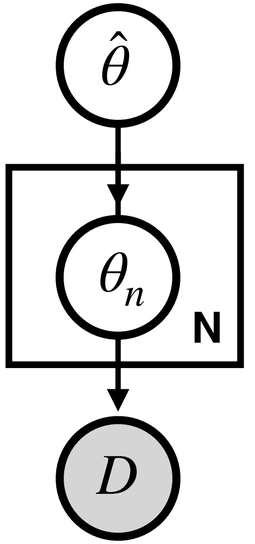


$\theta =$ Mass, Age, [Fe/H], etc

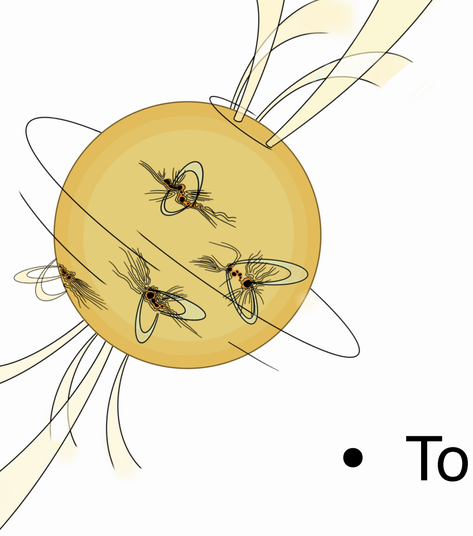
Combining methods: a hierarchical problem

$$p(\hat{\theta} \mid D) \propto p(\hat{\theta}) \prod_{n=1}^{N} \int p(\theta_n \mid \hat{\theta})\ p(D \mid \theta_n)\ d\theta_n$$

If you combine models **during inference**,
this is relatively straightforward



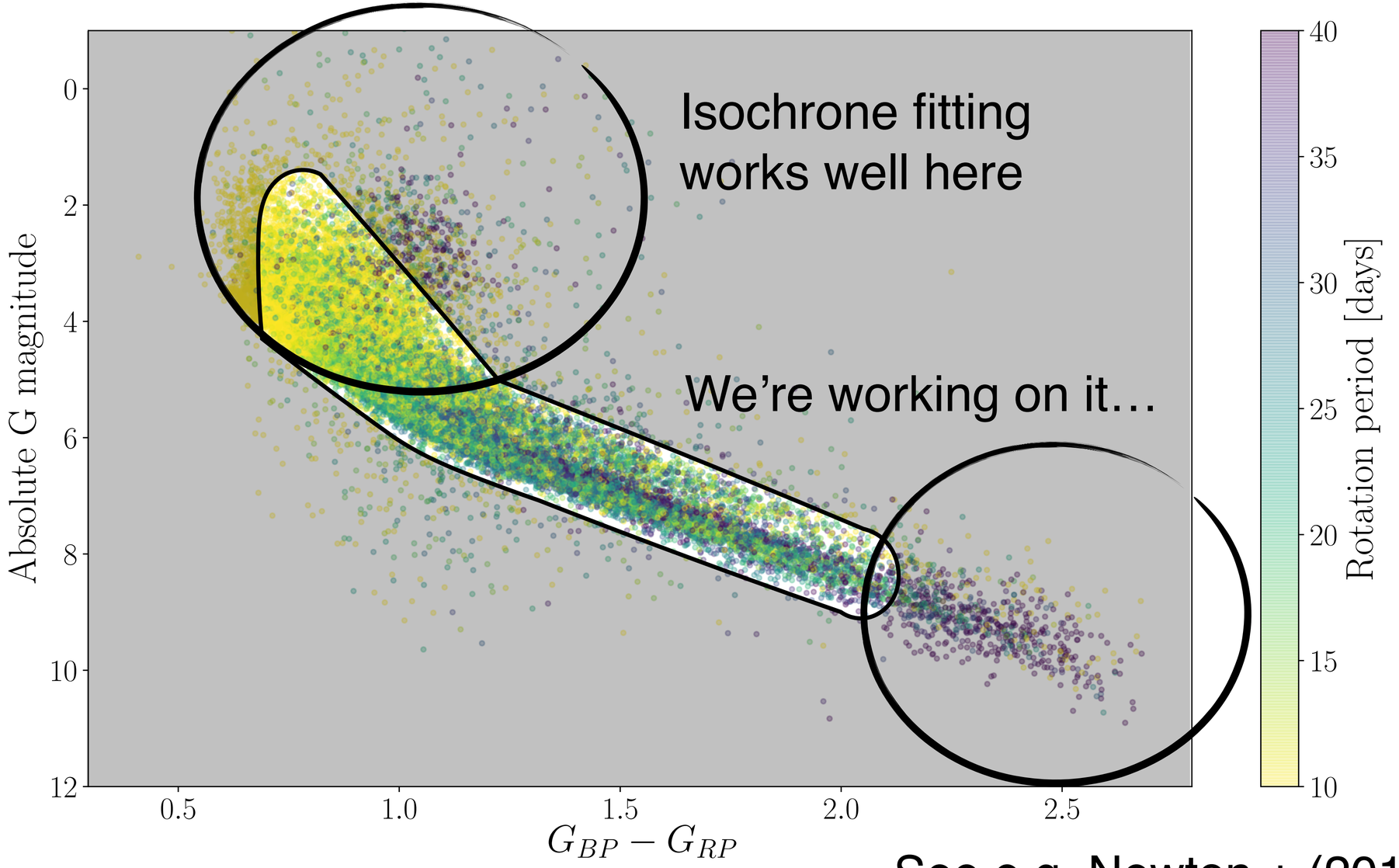
$\theta =$ Mass, Age, [Fe/H], etc



Summary:

- To combine stellar parameters, marginalise over methods.
- Easiest if likelihood functions are combined directly.
- If not possible, combine approximations to likelihood functions.
- Decide if this is the best approach.
- Publish prior values along with posterior samples.

stardate **only** applies gyrochronology where calibrated

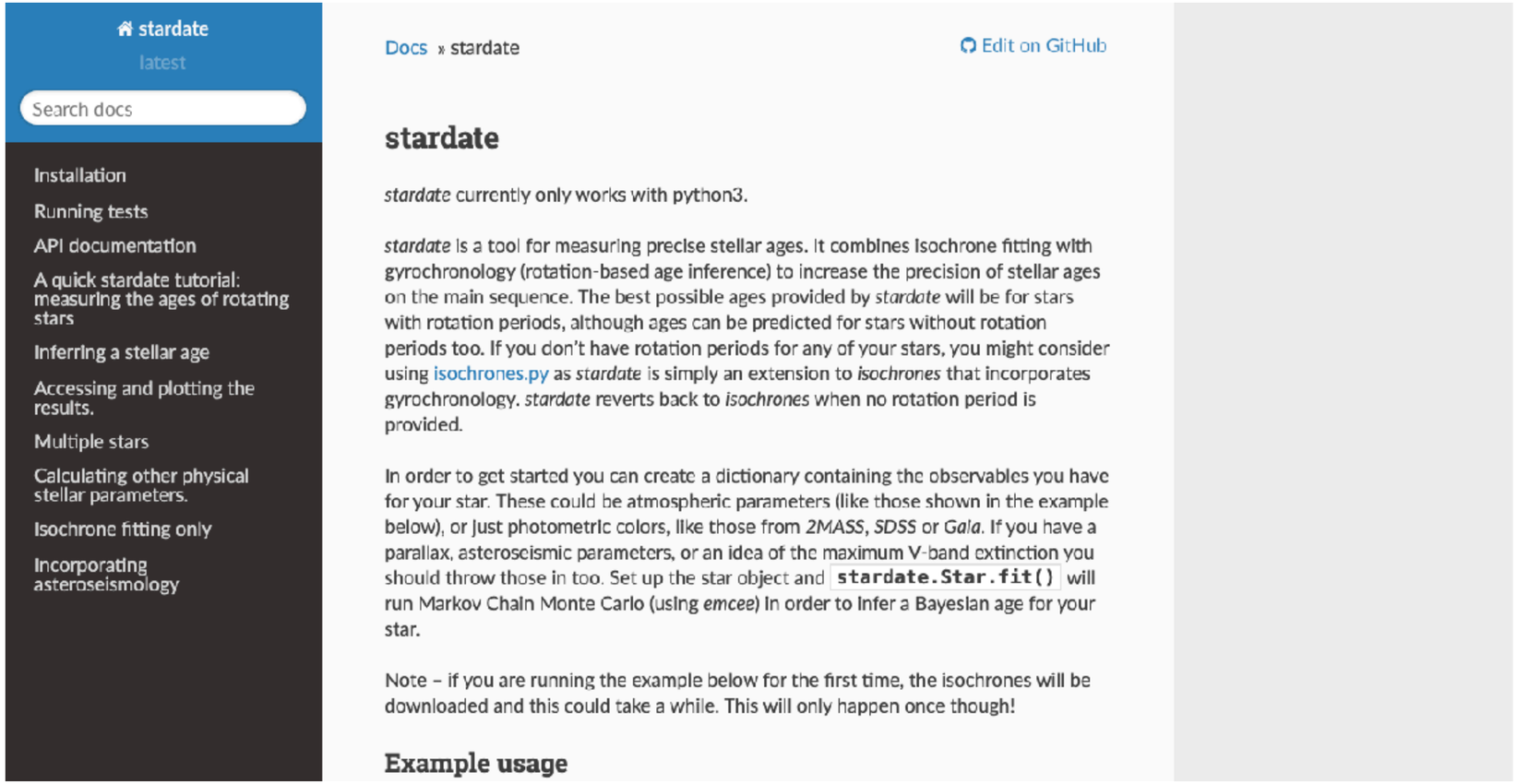


See e.g. Newton + (2016, 17, 18),
Kiman + (2019)

stardate (Cross-published in AJ and JOSS)

www.theoj.org/joss-papers/joss.01469/10.21105.joss.01469.pdf
ArXiv: 1908.07528

stardate.readthedocs.io
github/ruthangus/stardate



stardate (Cross-published in AJ and JOSS)

www.theoj.org/joss-papers/joss.01469/10.21105.joss.01469.pdf
ArXiv: 1908.07528

stardate.readthedocs.io
github/ruthangus/stardate

In order to get started you can create a dictionary containing the observables you have for your star. These could be atmospheric parameters (like those shown in the example below), or just photometric colors, like those from *2MASS*, *SDSS* or *Gaia*. If you have a parallax, asteroseismic parameters, or an idea of the maximum V-band extinction you should throw those in too. Set up the star object and `stardate.Star.fit()` will run Markov Chain Monte Carlo (using *emcee*) in order to infer a Bayesian age for your star.

Note – if you are running the example below for the first time, the isochrones will be downloaded and this could take a while. This will only happen once though!

Example usage

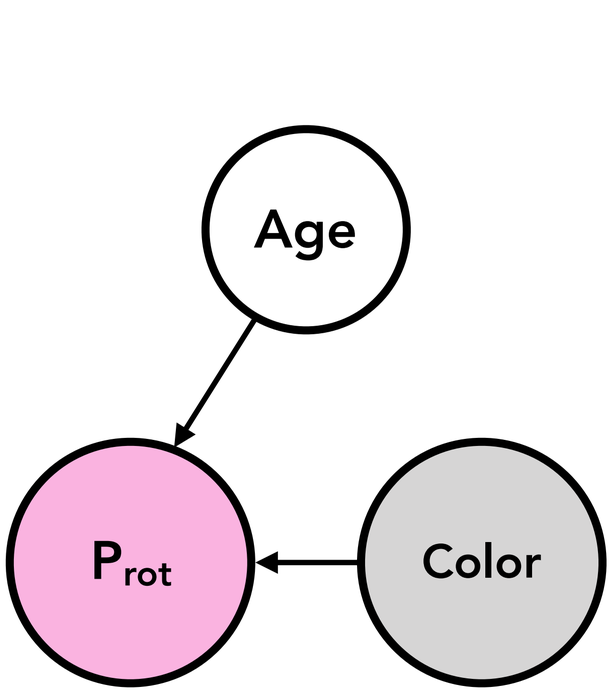
```
import stardate as sd

# Create a dictionary of observables
iso_params = {"teff": (4386, 50),      # Teff with uncertainty.
              "logg": (4.66, .05),     # logg with uncertainty.
              "feh": (0.0, .02),       # Metallicity with uncer
              "parallax": (1.48, .1),  # Parallax in milliarcs
              "maxAV": .1}             # Maximum extinction

prot, prot_err = 29, 3

# Set up the star object.
star = sd.Star(iso_params, prot=prot, prot_err=prot_err)  # He

# Run the MCMC
star.fit(max_n=1000)
```

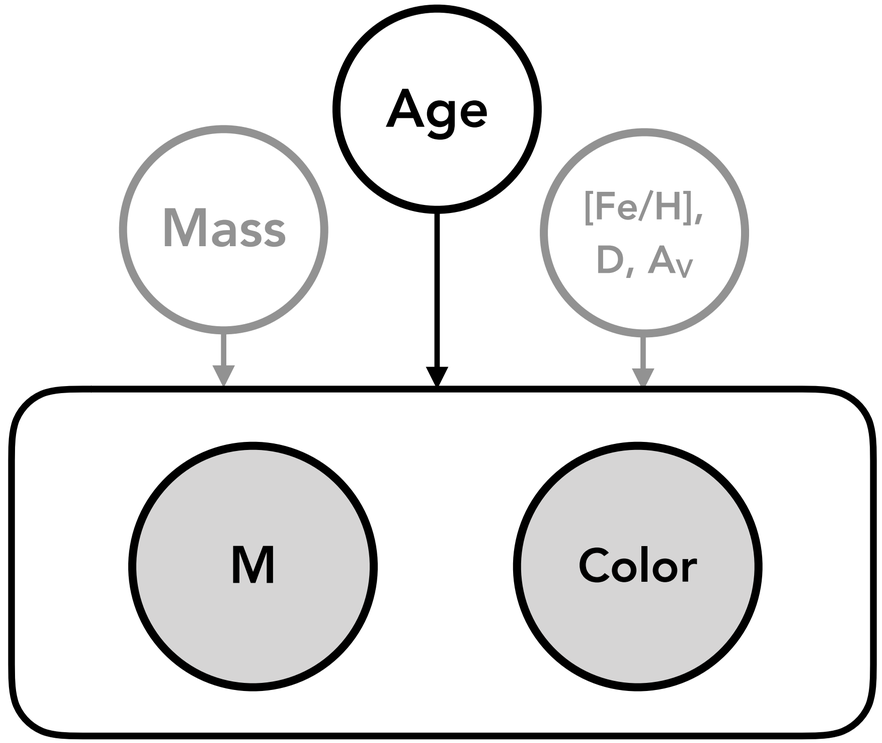
@ruthangus

How to **combine** age information

$$\mathscr{L} = p(P_{\rm rot} \,|\, {\rm Age}, {\rm color})$$

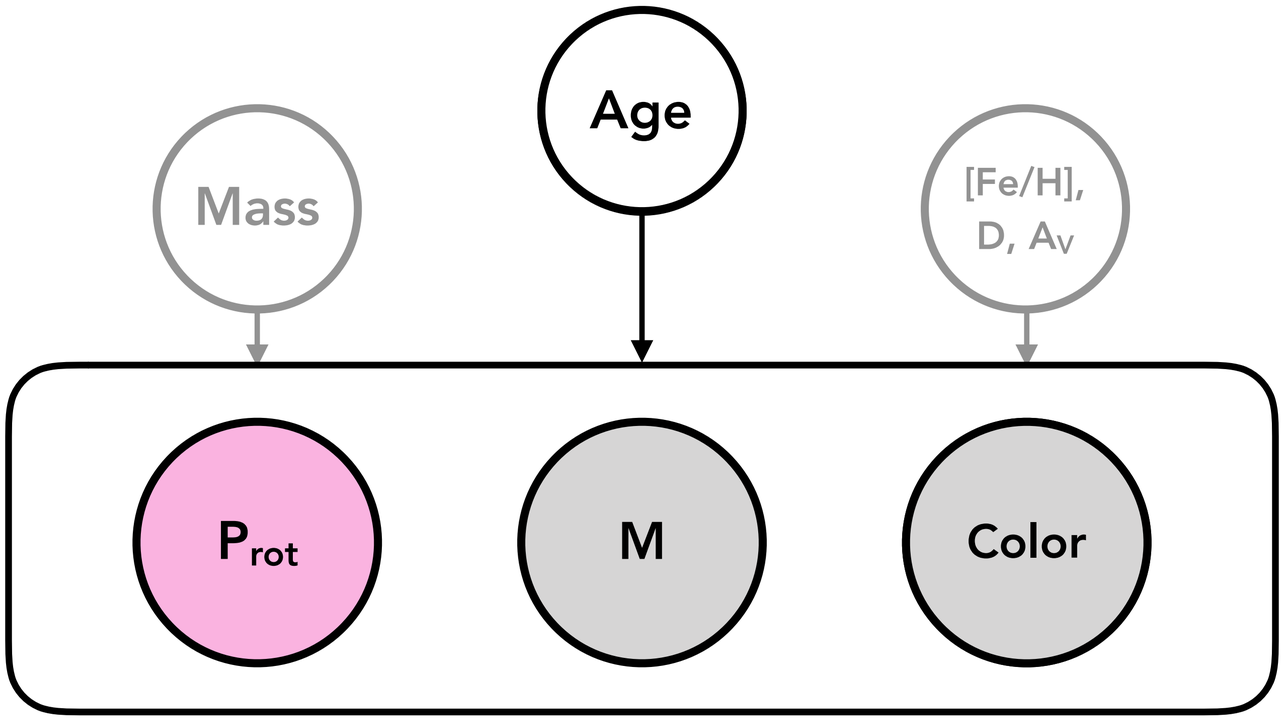
$$\mathscr{L} = p(M, {\rm color} \,|\, {\rm Age}, {\rm Mass}, [{\rm Fe/H}])$$





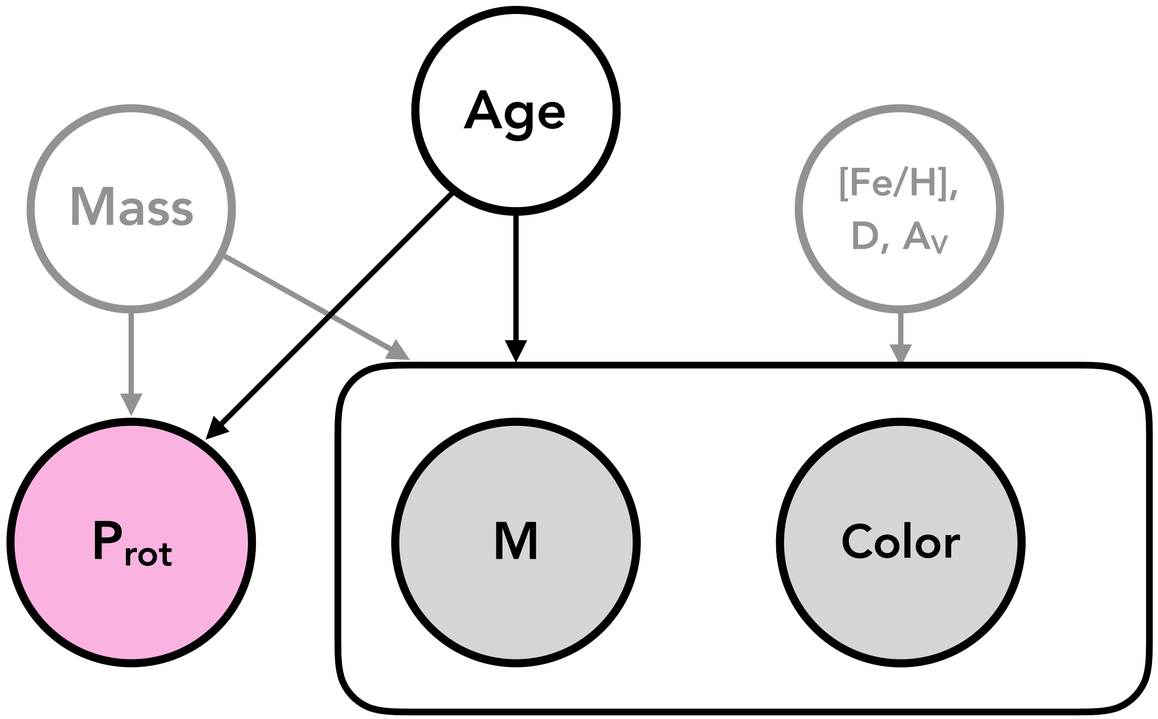
How to **combine** age information

$$\mathscr{L} = p(M, \text{color}, P_{\text{rot}} \mid \text{Age}, \text{Mass}, [\text{Fe/H}])$$



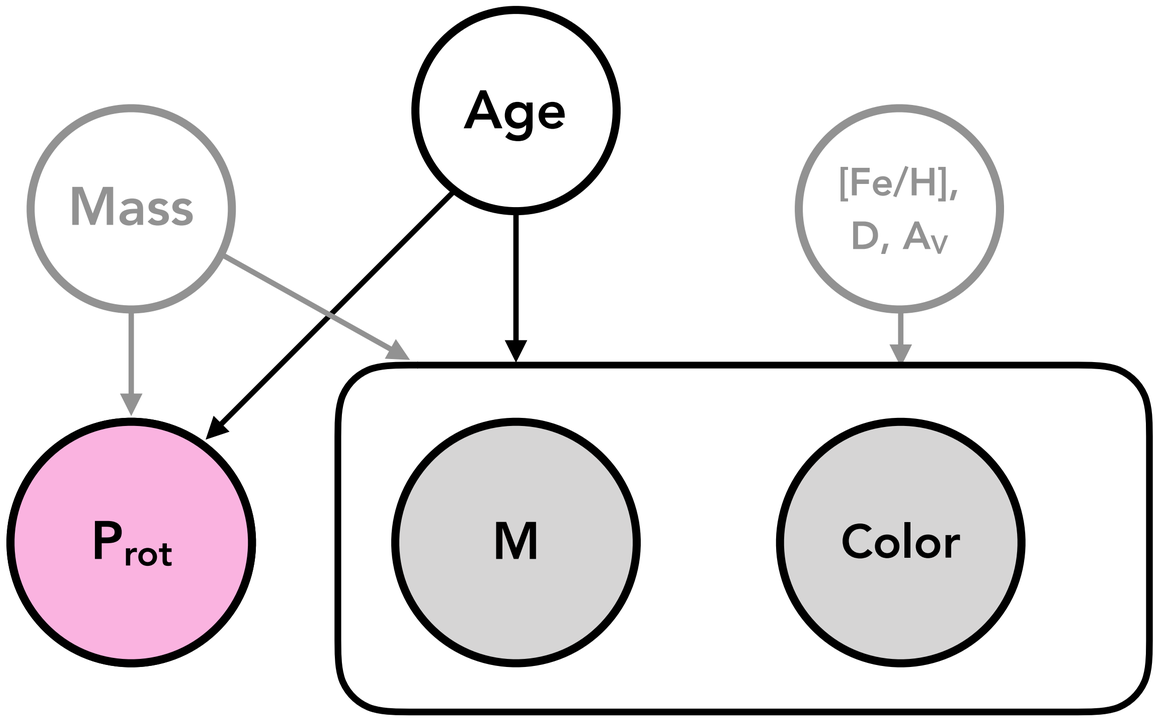
How to **combine** age information

$$\mathscr{L} = p(M, \text{color} \mid \text{Age}, \text{Mass}, [\text{Fe/H}])\; p(P_{\text{rot}} \mid \text{Age}, \text{Mass})$$



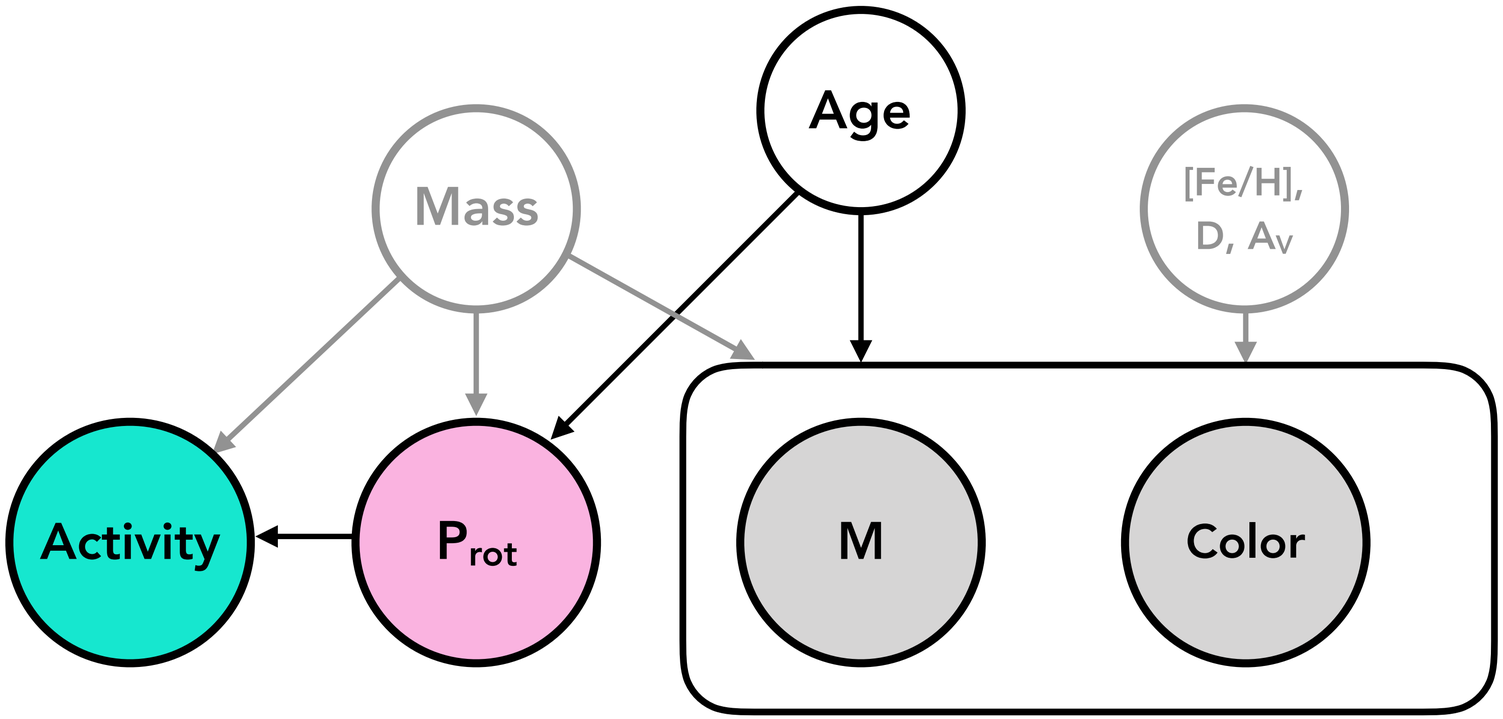
Adding activity

$$\mathscr{L} = p(M, \text{color} \mid \text{Age}, \text{Mass}, [\text{Fe/H}])\ p(P_{\text{rot}} \mid \text{Mass}, \text{Age})$$



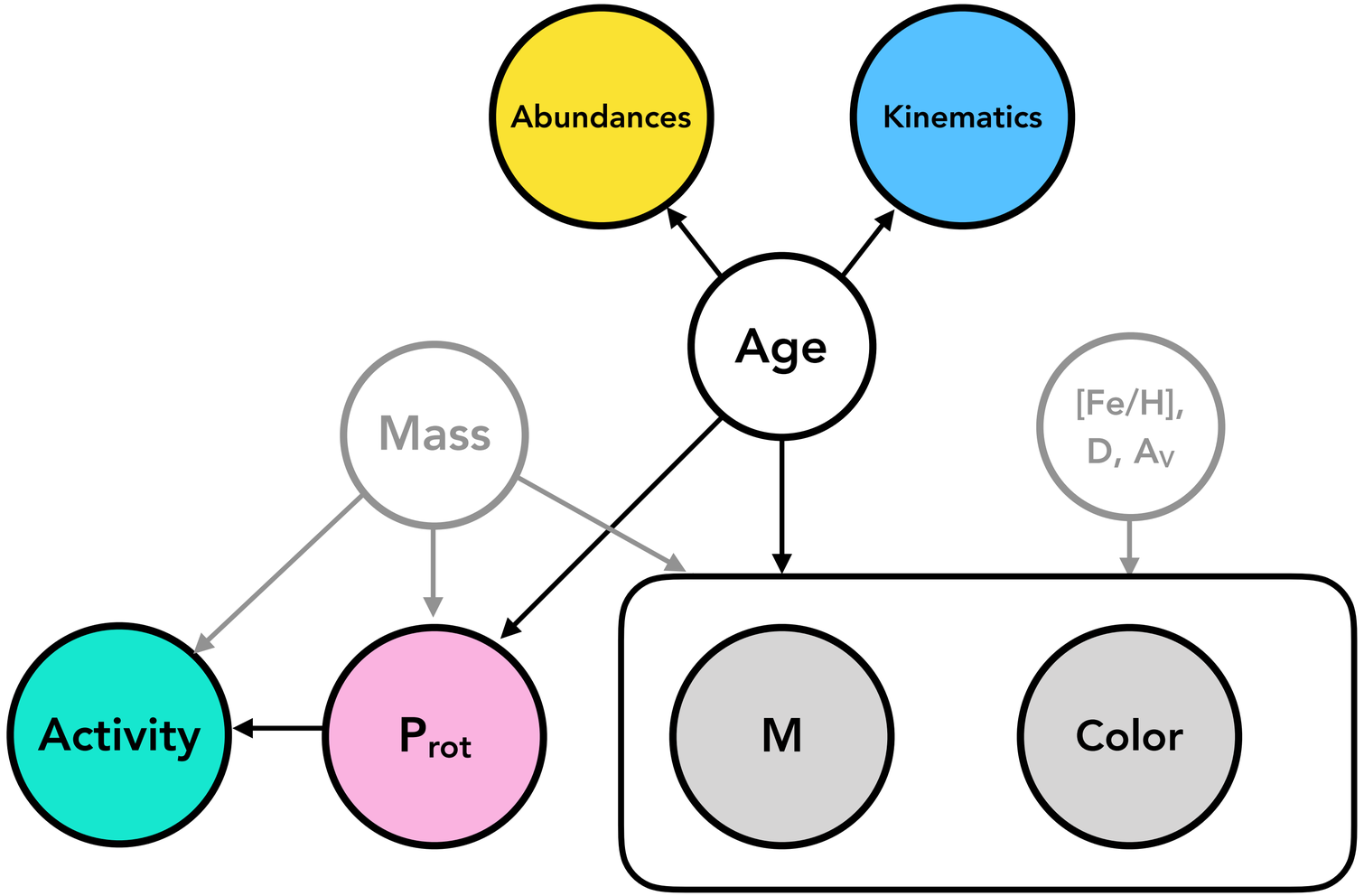
Adding activity

$$\mathscr{L} = p(M, \text{color} \mid \text{Age}, \text{Mass}, [\text{Fe/H}])\ p(P_{\text{rot}} \mid \text{Mass}, \text{Age})\ p(\text{Activity} \mid P_{\text{rot}}, \text{Mass})$$



Adding other age indicators

$$\mathscr{L} = p(M, \text{color} \mid \text{Age}, \text{Mass}, [\text{Fe/H}])\ p(P_{\text{rot}} \mid \text{Mass}, \text{Age})\ p(\text{Activity} \mid P_{\text{rot}}, \text{Mass})$$
$$\times\, p(\text{Abundances} \mid \text{Age})\ p(\text{Kinematics} \mid \text{Age})$$



…etc