AARHUS UNIVERSITY
Stellar Astrophysics Centre

Time-dependent convection models: mode physics

with applications to surface effects, damping rates, amplitude ratios

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Overview of selected time-dependent convection models

Aim: estimate turbulent flux perturbations: $\delta F_{\rm c}(\omega, r)$; $\delta p_{\rm t}(\omega, r)$

applications by:

Unno (1967, 89) ➞ Gabriel (1998) ➞ Grigahcéne et al. (2004) : Dupret; Théado, …

Nonlinear mixing-length equations for a **Boussinesq fluid**
Infinite lifetime of fluid elements

Gough (1965, 77) ➞ Balmforth (1992) : Baker; Balmforth; Cunha; Gough; GH

Linearized mixing-length equations for a **Boussinesq fluid**
Finite lifetime of fluid elements (linear growth rates)

Xiong (1977, 1989) : Cheng; Deng; Xiong

Reynold's transport equations for a **Boussinesq / compressible fluid**
Third-order moments approximated with diffusion-like expressions
using parametrized length scales (closure coefficients)

For more details see, e.g., Houdek & Dupret (2015)

Nonlocal, time-dependent convection model (Gough 1977)

$$\mathcal{F}_\mathrm{c}(r) = \frac{2}{\ell}\int_{r_0-\ell/2}^{r_0+\ell/2} F_\mathrm{c}(r_0)\cos^2\left[\pi(r_0 - r)/\ell\right]\,\mathrm{d}r_0\,. \qquad F_\mathrm{c} \simeq \overline{\rho}\,\overline{c_\mathrm{p}}\,\overline{w\theta}$$

Nonlocal, time-dependent convection model (Gough 1977)

$$\mathcal{F}(r) \simeq \frac{a}{\ell} \int_{-\infty}^{\infty} F(r_0) \exp(-a|r_0 - r|/\ell)/2 \, \mathrm{d}r_0 \qquad \hat{=} \qquad \frac{\mathrm{d}^2 \mathcal{F}_\mathrm{c}}{\mathrm{d} \ln p^2} = \frac{a^2}{\alpha^2} (\mathcal{F}_\mathrm{c} - F_\mathrm{c})$$

5 convection parameters (need calibration):

(1) Mixing-length parameter: calibrated to convection-zone depth of (ASTEC) stellar models

(2) Nonlocal parameter for turbulent pressure p_t: calibrated to max(p_t) of 3D simulations

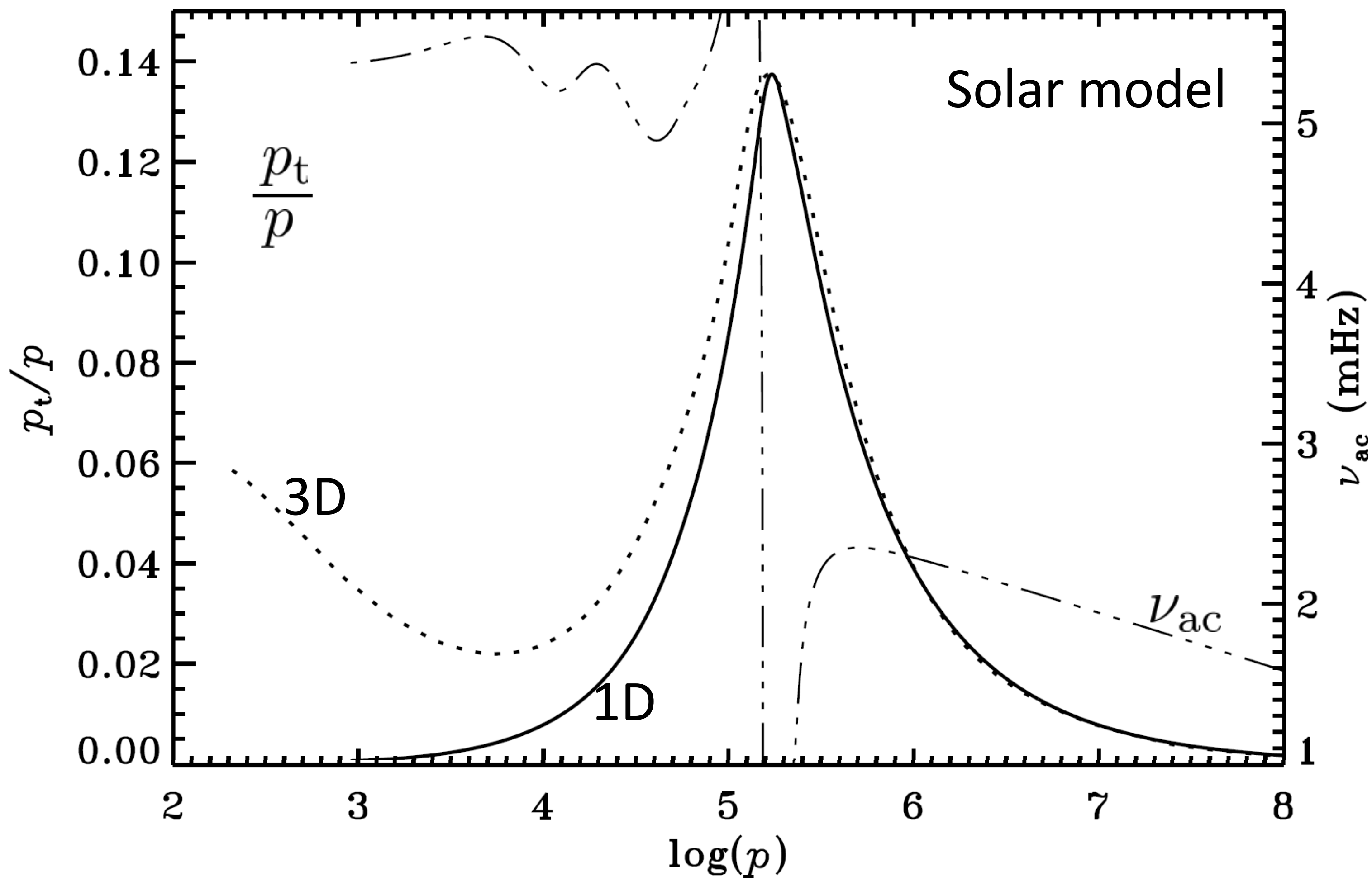
(3) Nonlocal parameter for convective heat flux: calibrated to fit Legacy linewidths

(4,5) Anisotropy Φ of convective velocity field: calibrated to fit Legacy linewidths with guidance from 3D simulations

1D models including turbulent pressure p_t

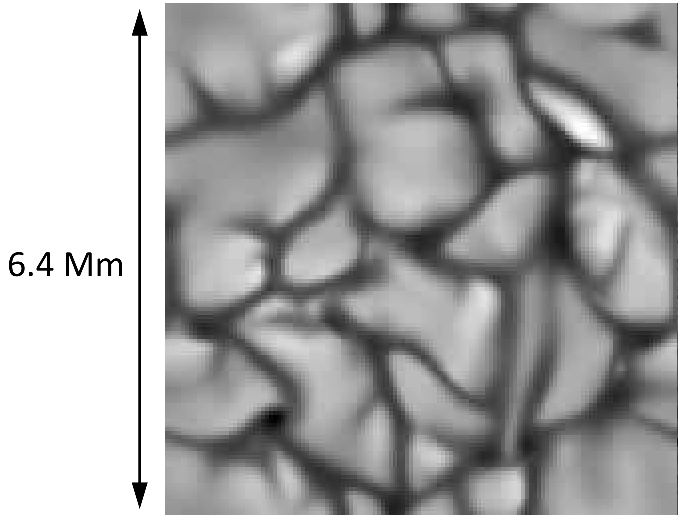
$$\frac{\partial}{\partial m}(p_\mathrm{g} + p_\mathrm{t}) = -\frac{1}{4\pi r^2}\left(\frac{Gm}{r^2}\right) \qquad p_\mathrm{t} := \overline{\rho\, ww}$$

$$\boldsymbol{u} = (u, v, w)$$



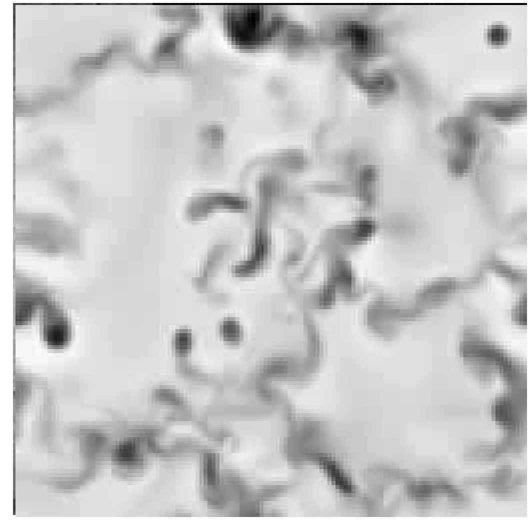
Depth-dependent **velocity anisotropy** Φ

Surface

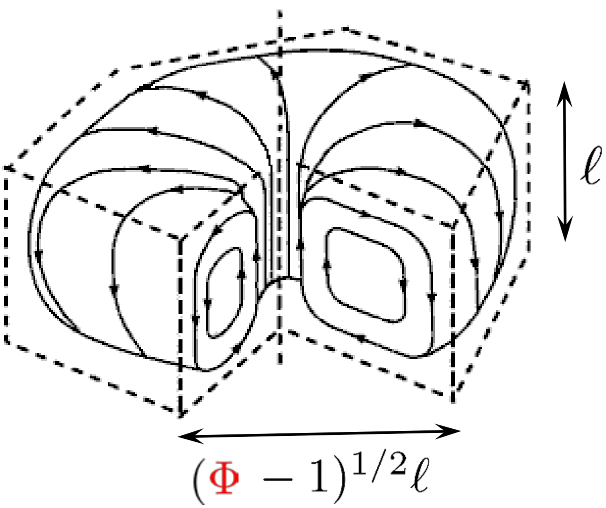


Solar 3D
simulation
128x128x127

Depth: 996 km



Definition in a generalized 1D convection model (Gough 1977)



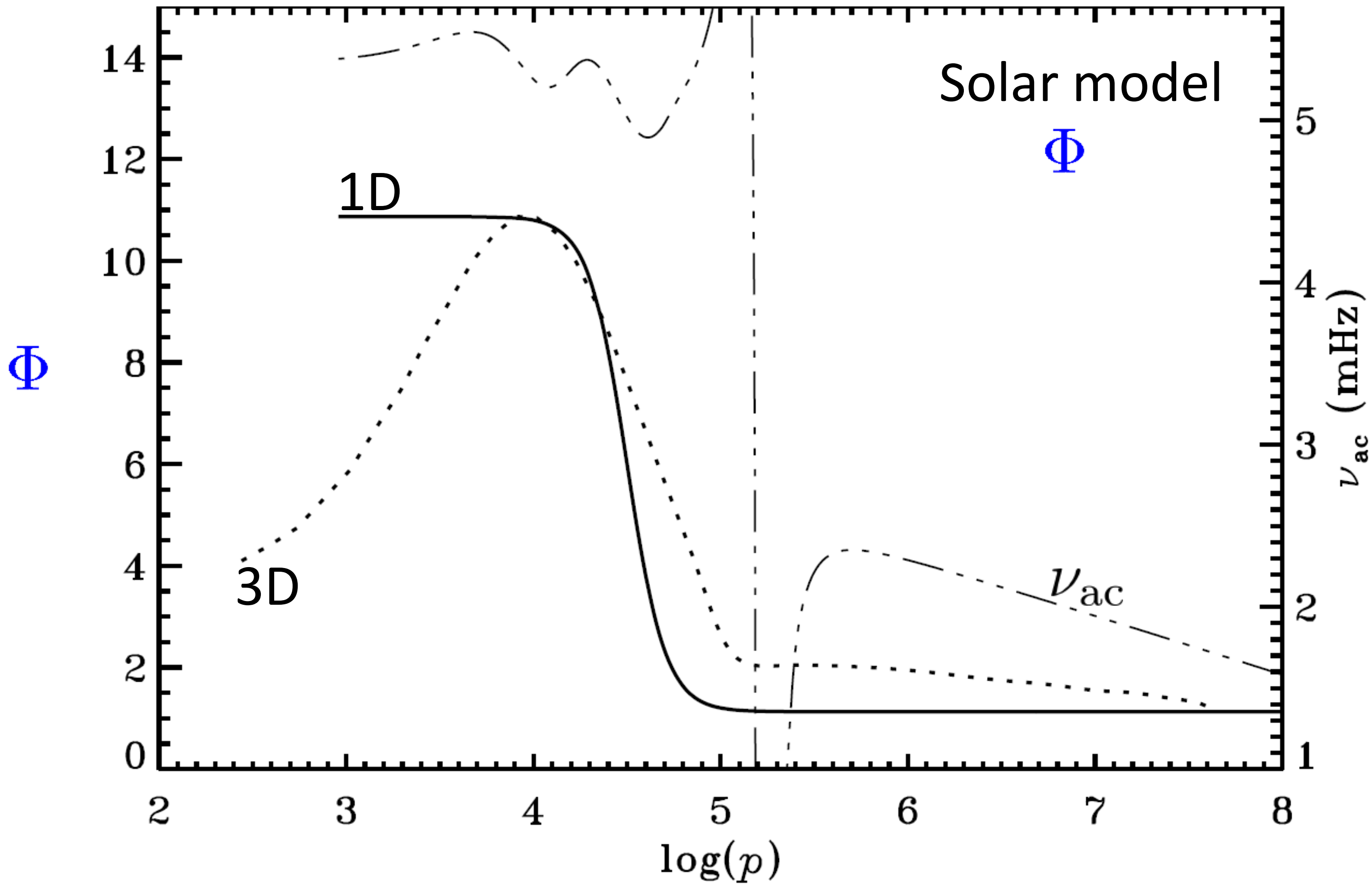
Anisotropy factor (flow geometry):

$$\Phi := \frac{\overline{uu} + \overline{vv} + \overline{ww}}{\overline{ww}}$$

$\boldsymbol{u} = (u, v, w)$ ...turbulent velocity field

Anisotropy of the turbulent velocity field

$$\Phi = \frac{\langle u^2 \rangle + \langle v^2 \rangle + \langle w^2 \rangle}{\langle w^2 \rangle}$$

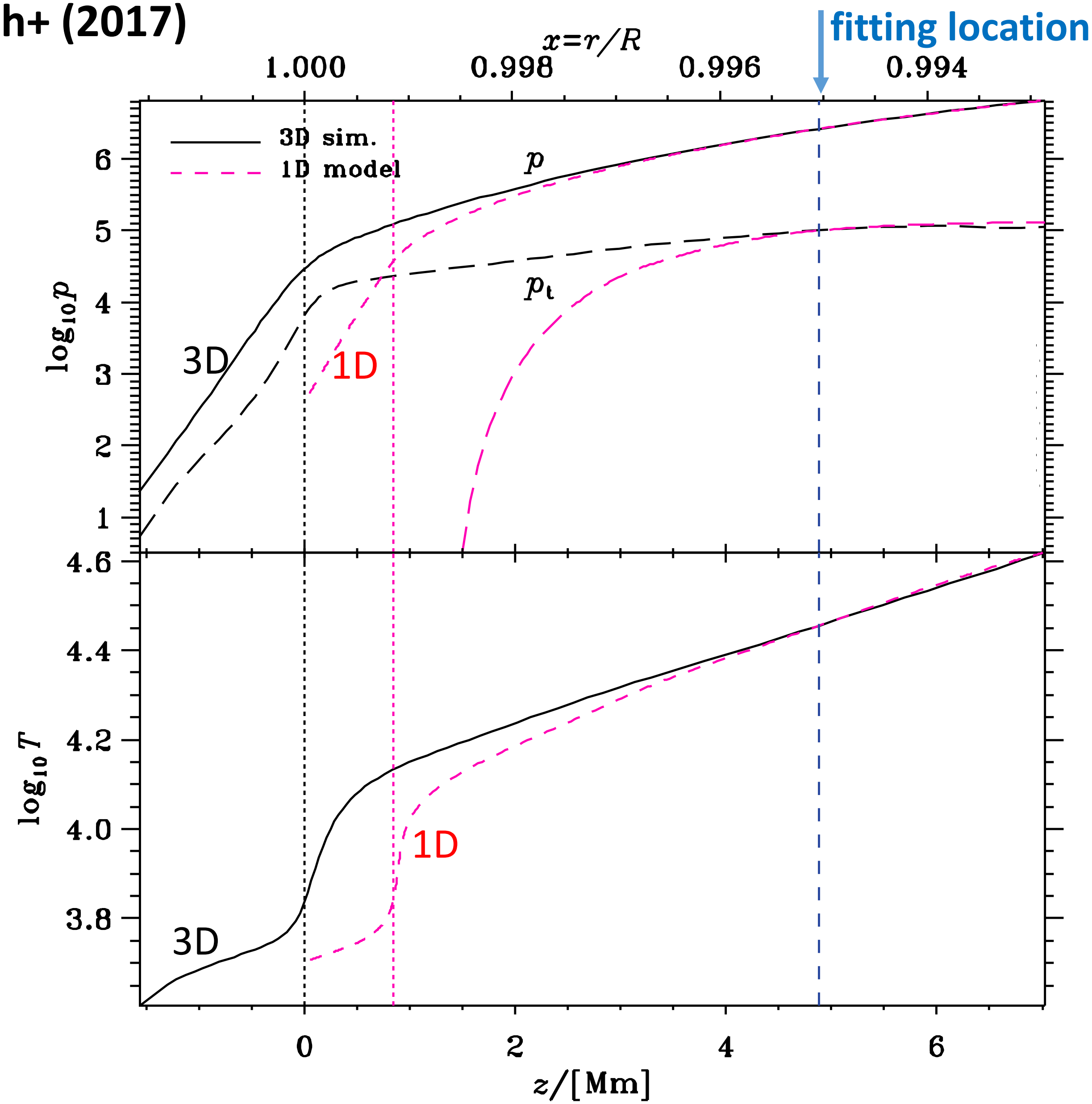


Surface effects

Surface effects

Using 3D simulations for **“Structural effects” [struct]**

Trampedach+ (2017)

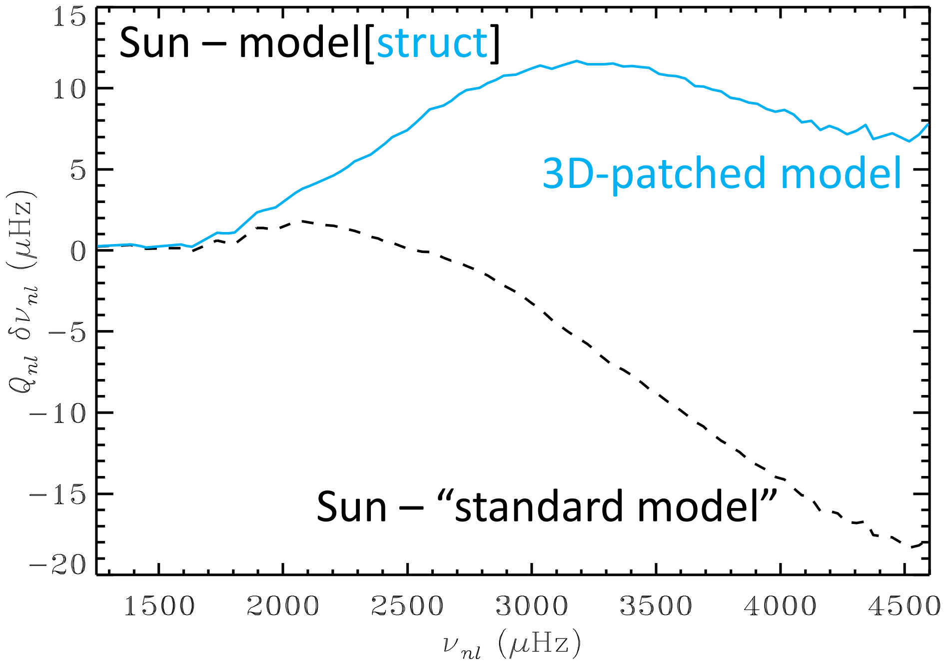


3D-patched
1D mean model

$T_{\rm eff}$ = 6901 K
log g = 4.29

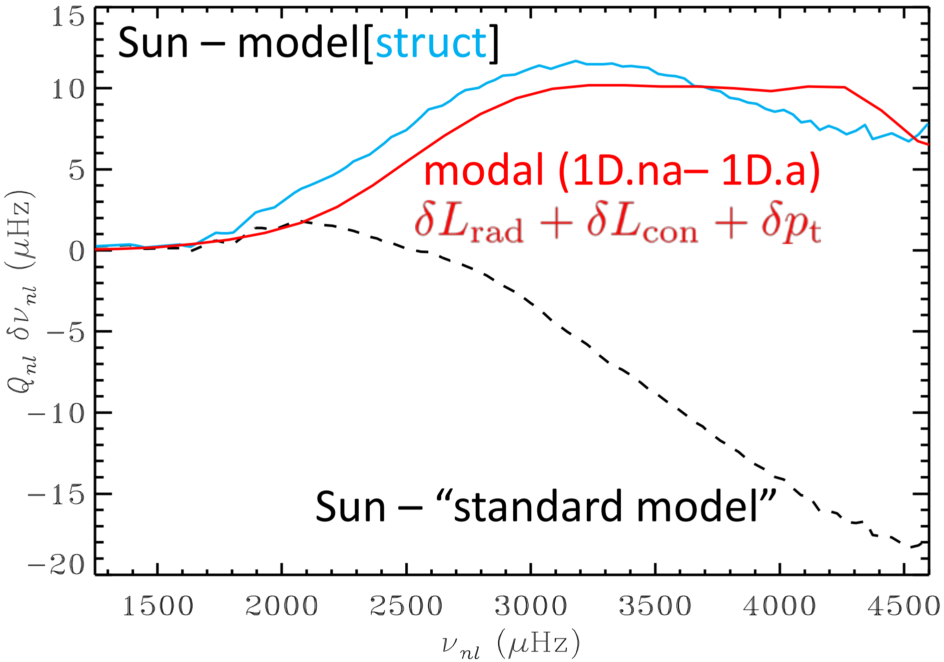
Solar model: "surface effects"

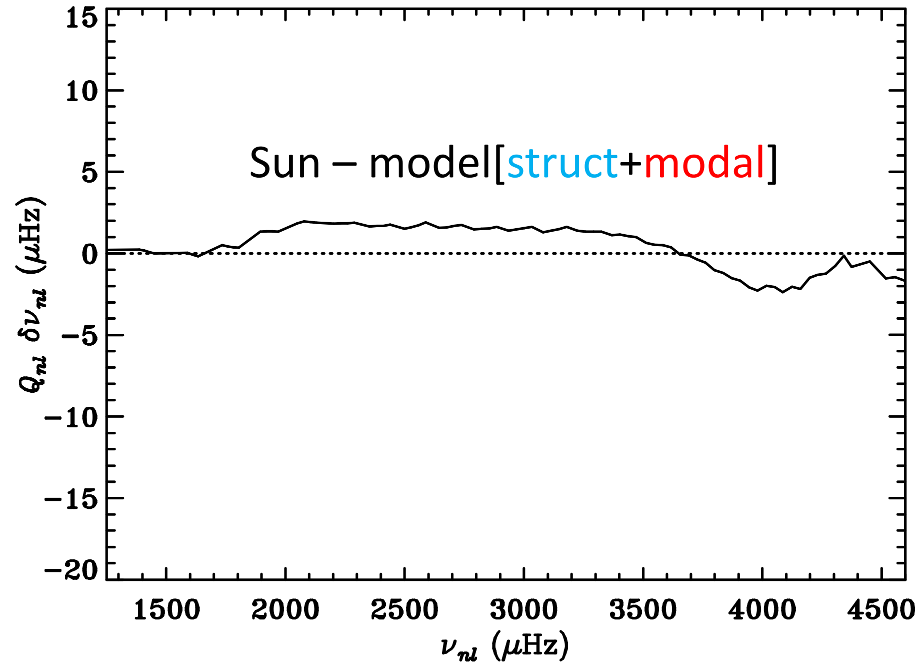
Houdek et al. (2017)



Solar model: “surface effects”

Houdek et al. (2017)

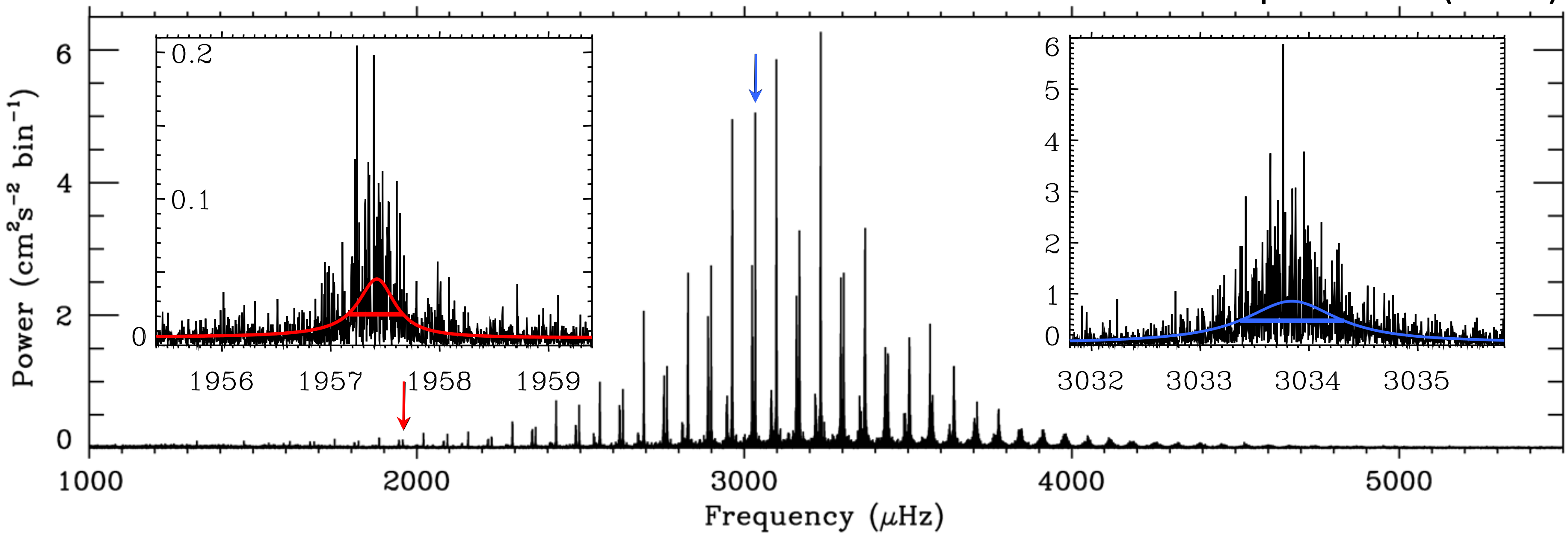




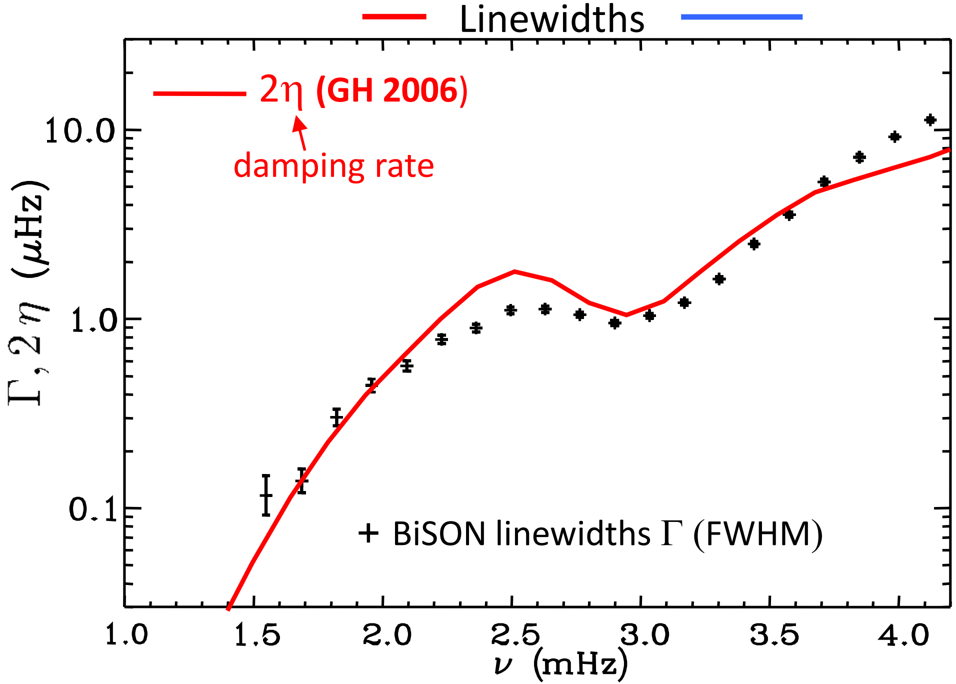
Modelled damping rates

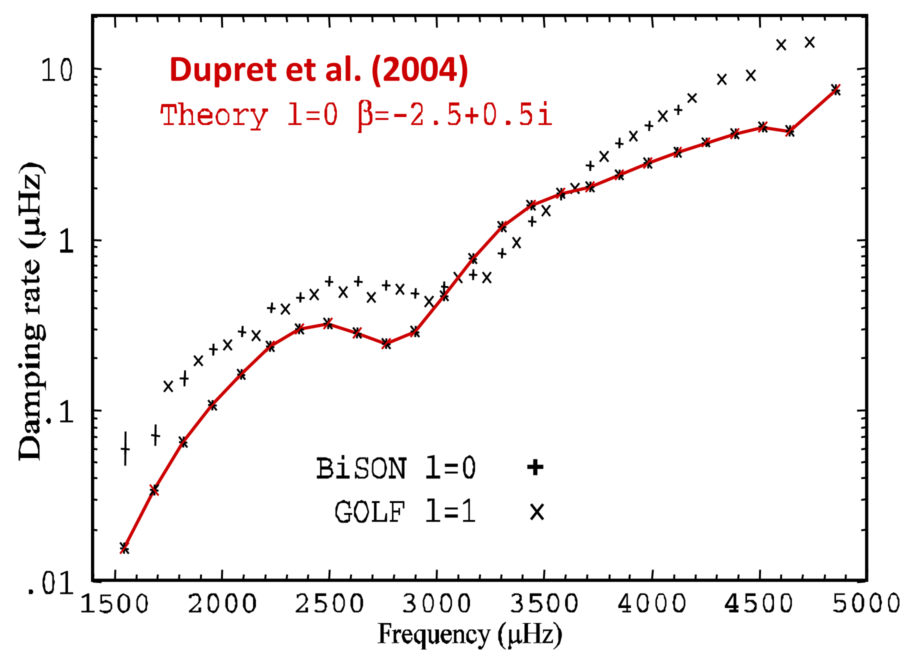
Solar linewidths / damping rates

Chaplin et al. (2004)



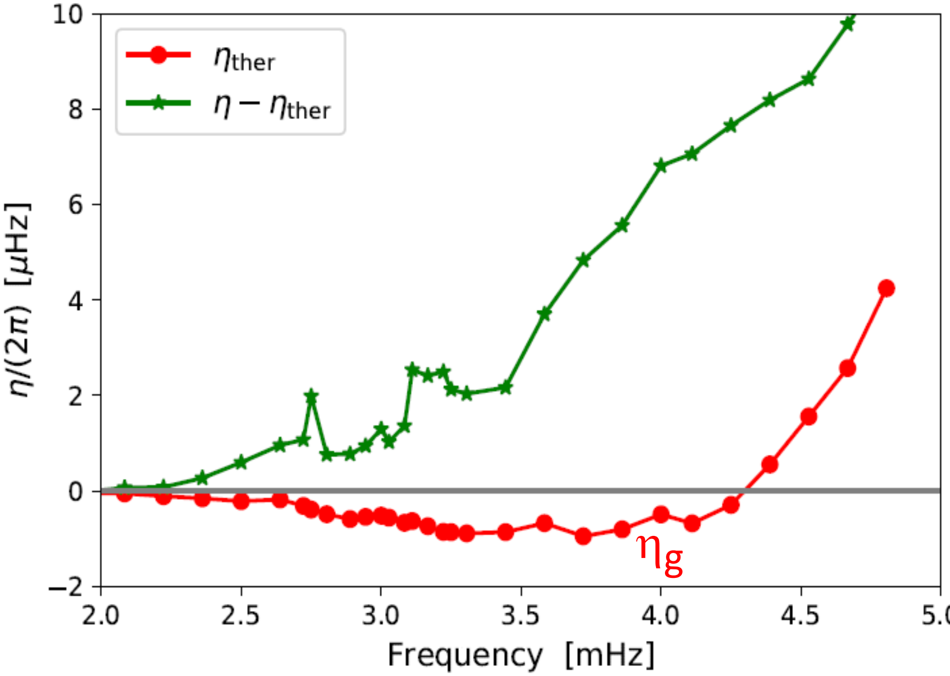
Linewidths



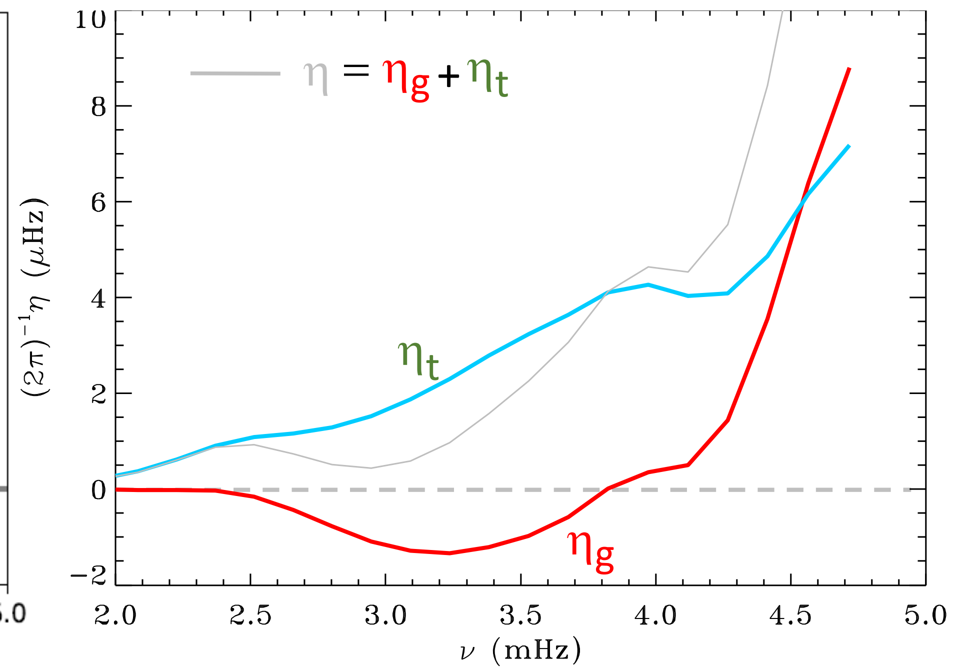


Solar linewidths

3D simulations (STAGGER): Zhou+ 2019

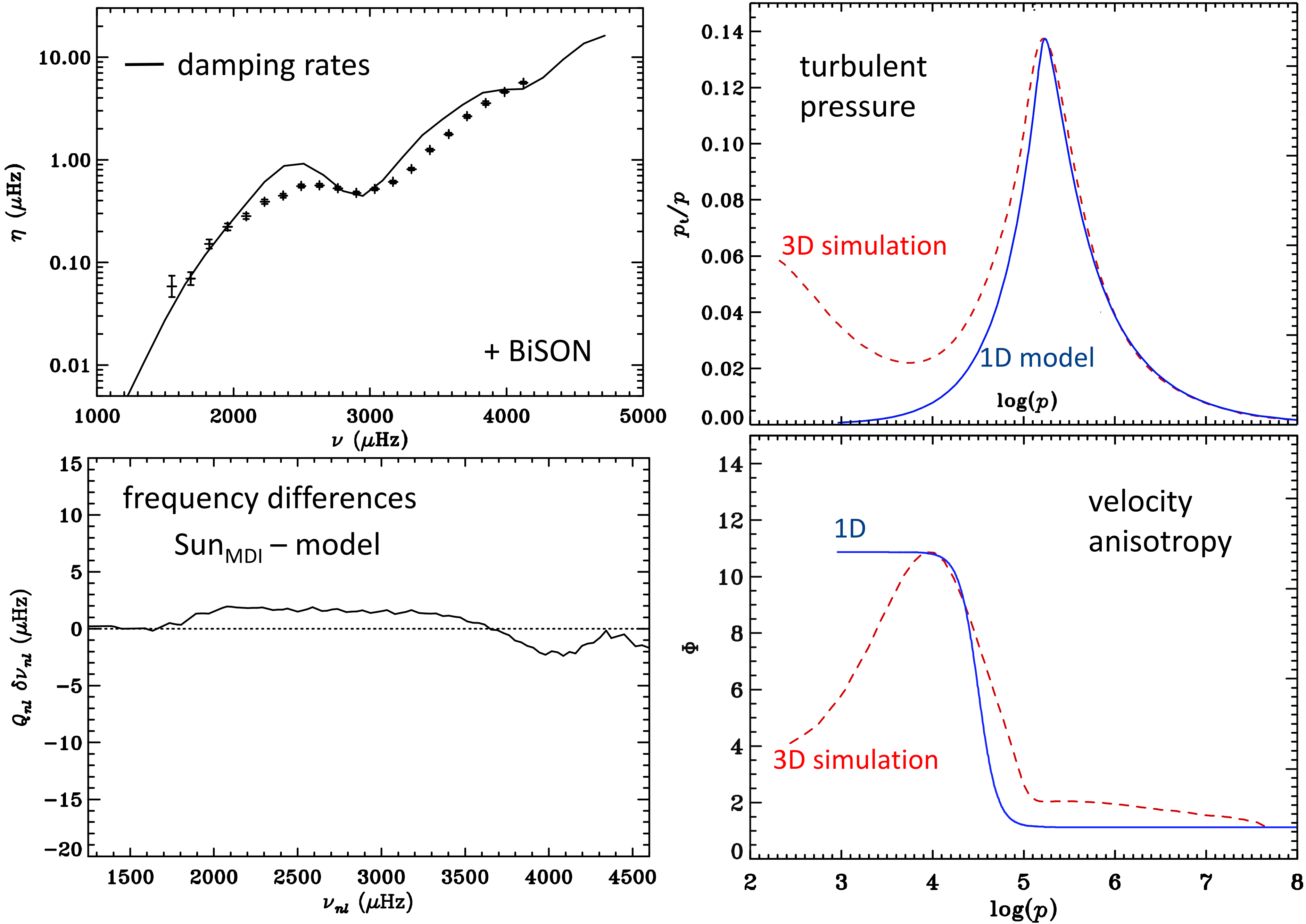


1D model (GH+ 1999, 2017)



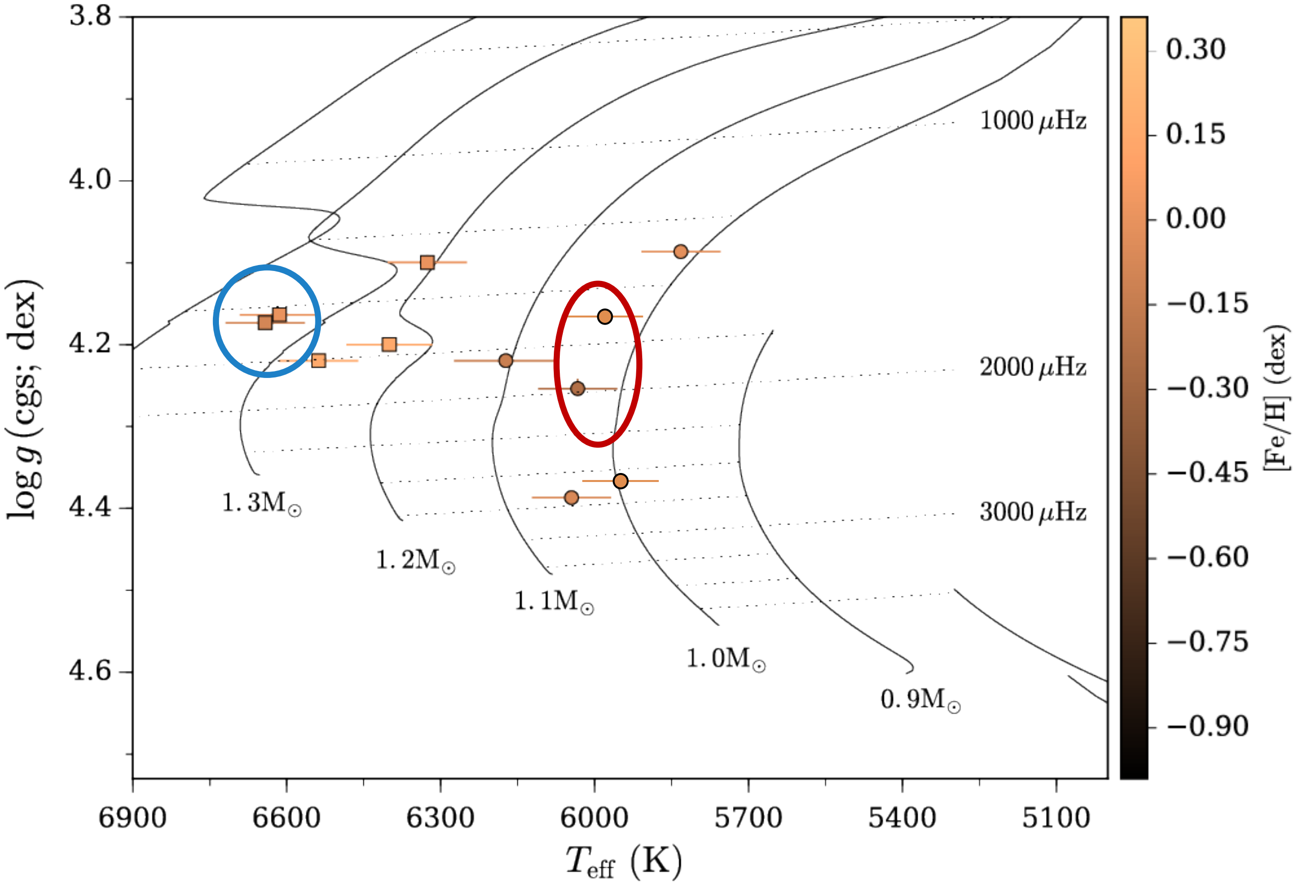
See also **Yixiao's talk,** and Belkacem+ (2019)

Calibrated solar model



Results for selected *Kepler* stars

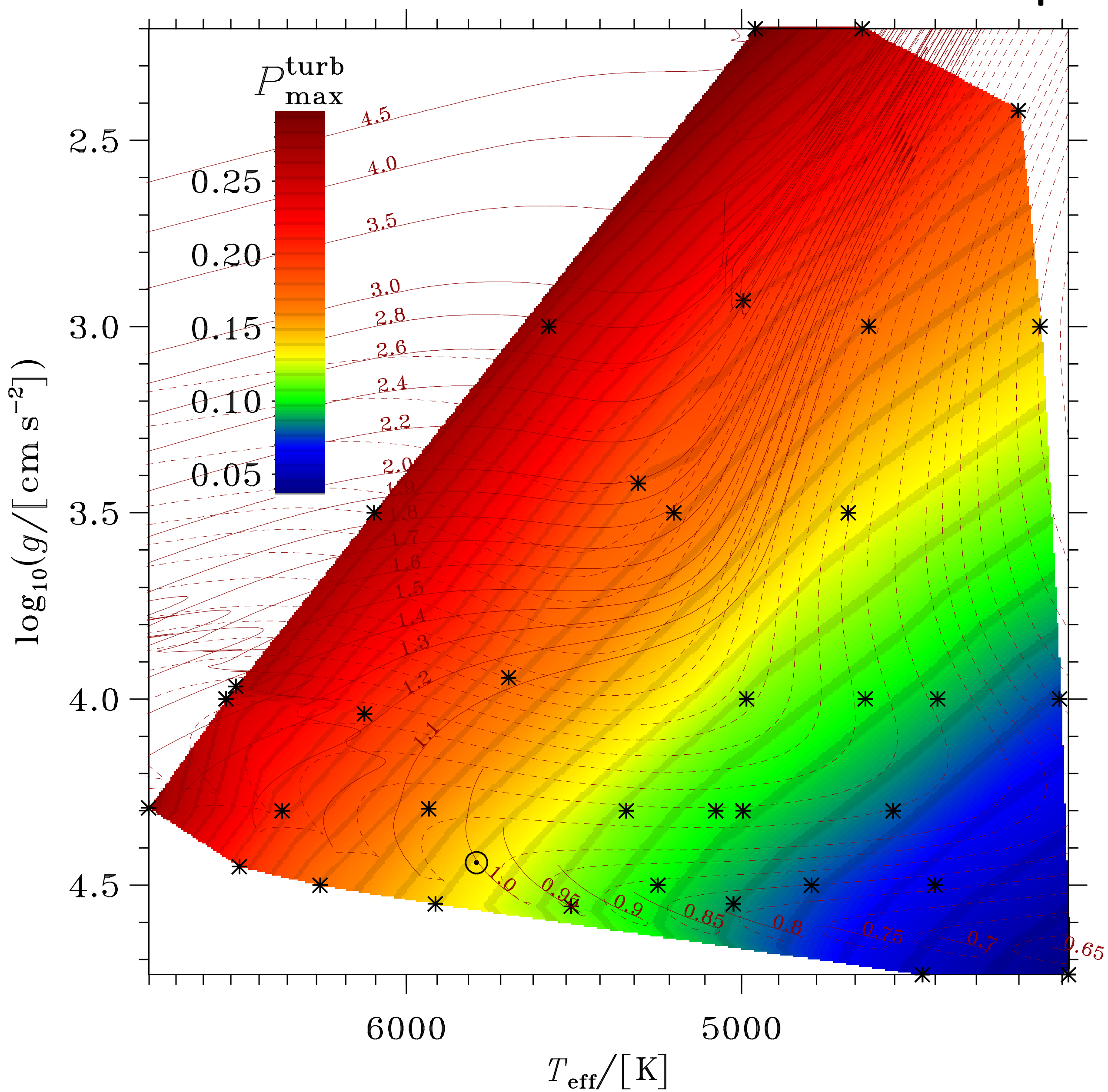
Kepler solar-type stars (LEGACY sample: Lund et al. 2017)
Selection of 12 stars for damping-rate calculations

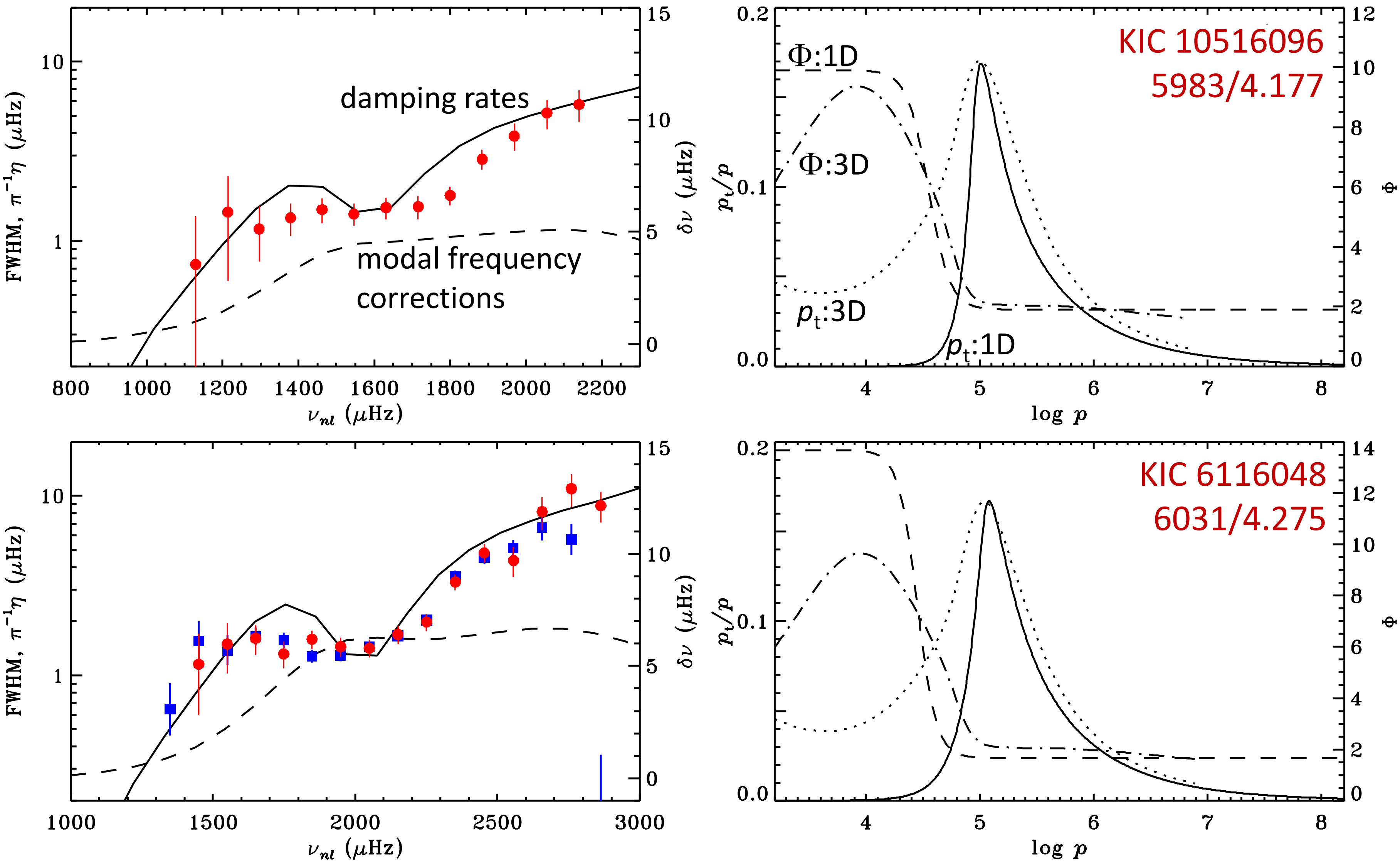


Lund+ 2017

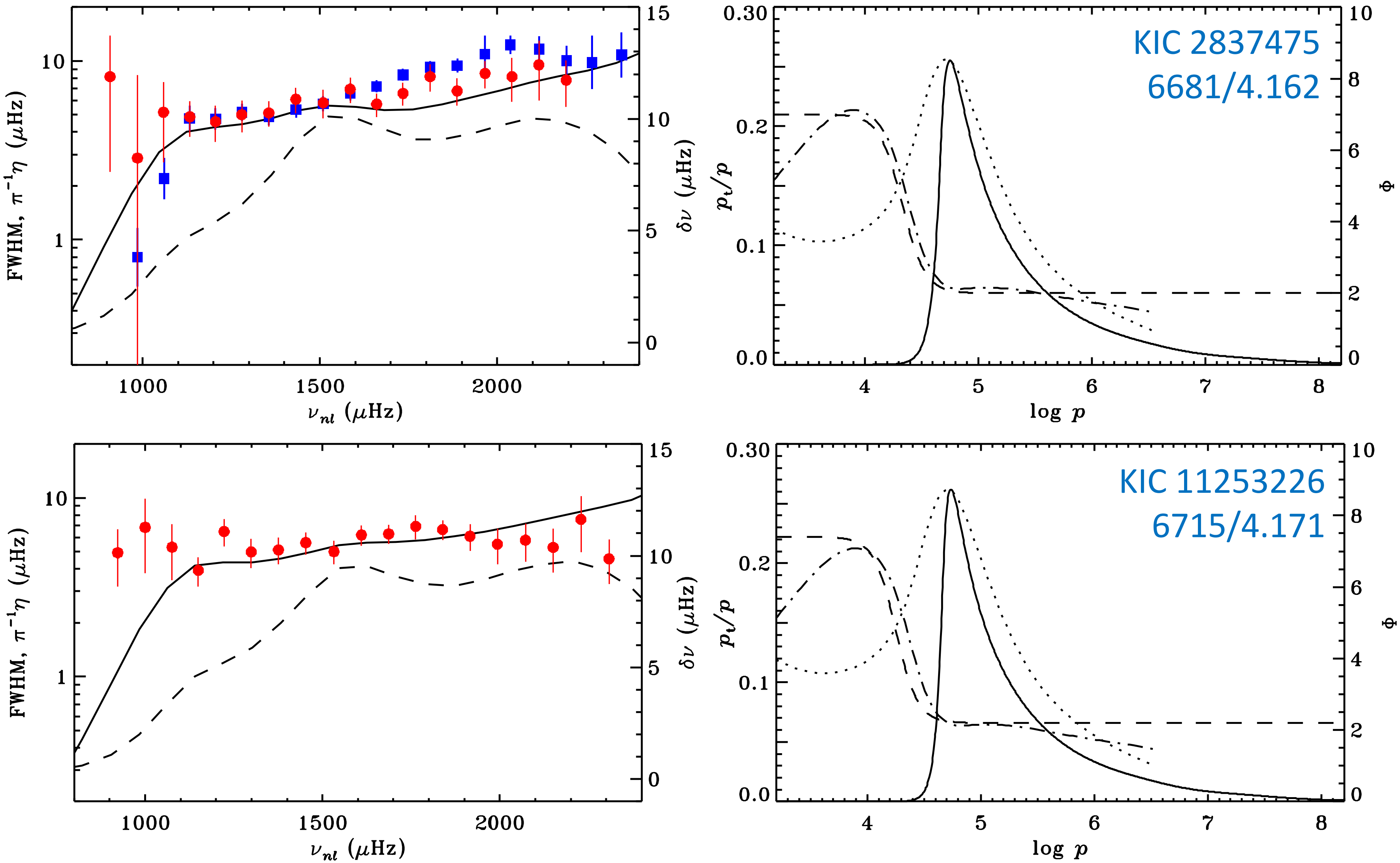
Averaged max(p_t/p) from 3D-simulations

Trampedach et al (2014)




damping rates
modal frequency corrections
FWHM, $\pi^{-1}\eta$ (μHz)
$\delta\nu$ (μHz)
ν_{nl} (μHz)
KIC 10516096
5983/4.177
Φ:1D
Φ:3D
p_t:3D
p_t:1D
p_t/p
Φ
log p
KIC 6116048
6031/4.275

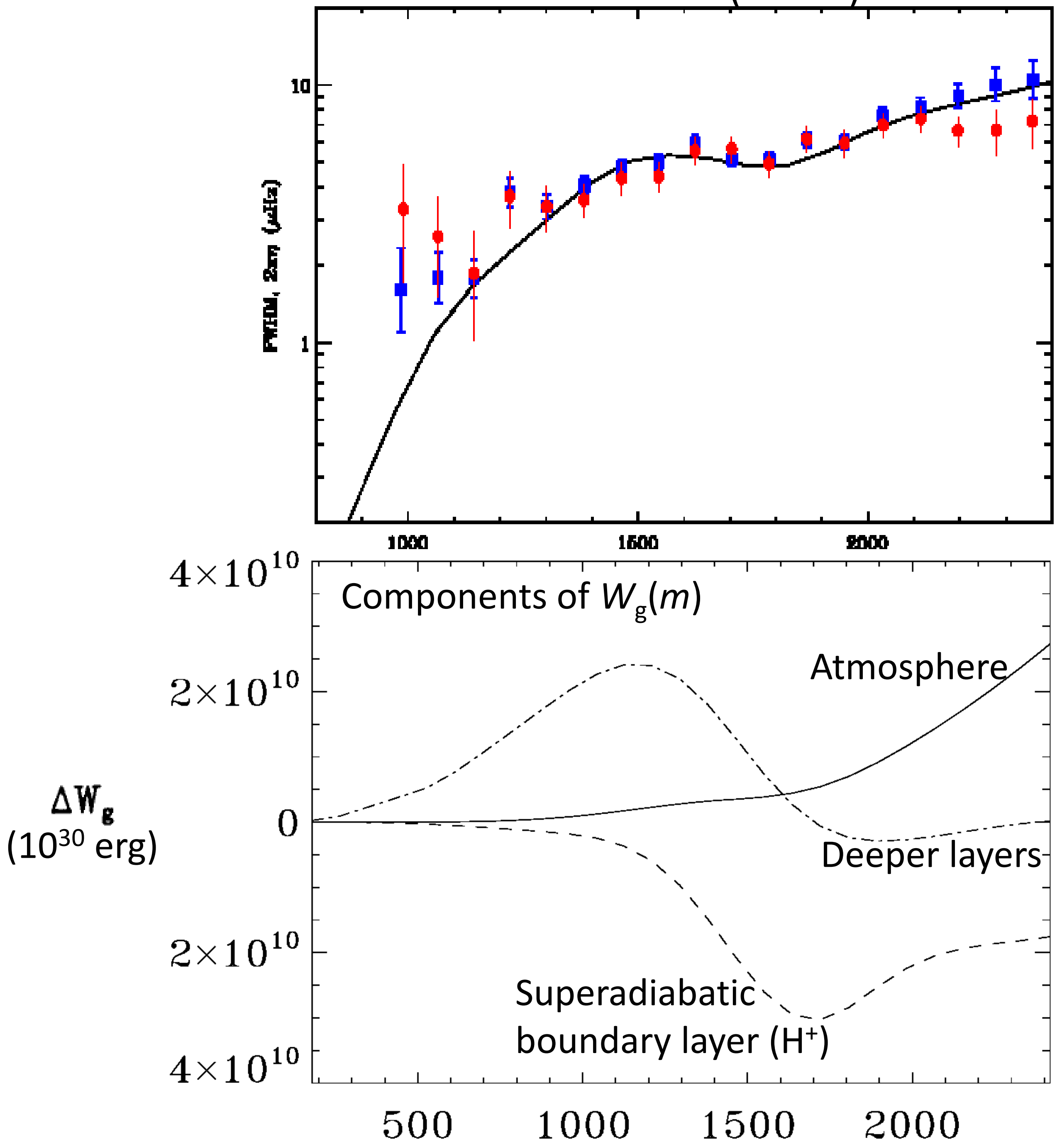
GH+ 2019



FWHM, $\pi^{-1}\eta$ (μHz)
$\delta\nu$ (μHz)
ν_{nl} (μHz)
p_t/p
Φ
log p
KIC 2837475
6681/4.162
KIC 11253226
6715/4.171

GH+ 2019

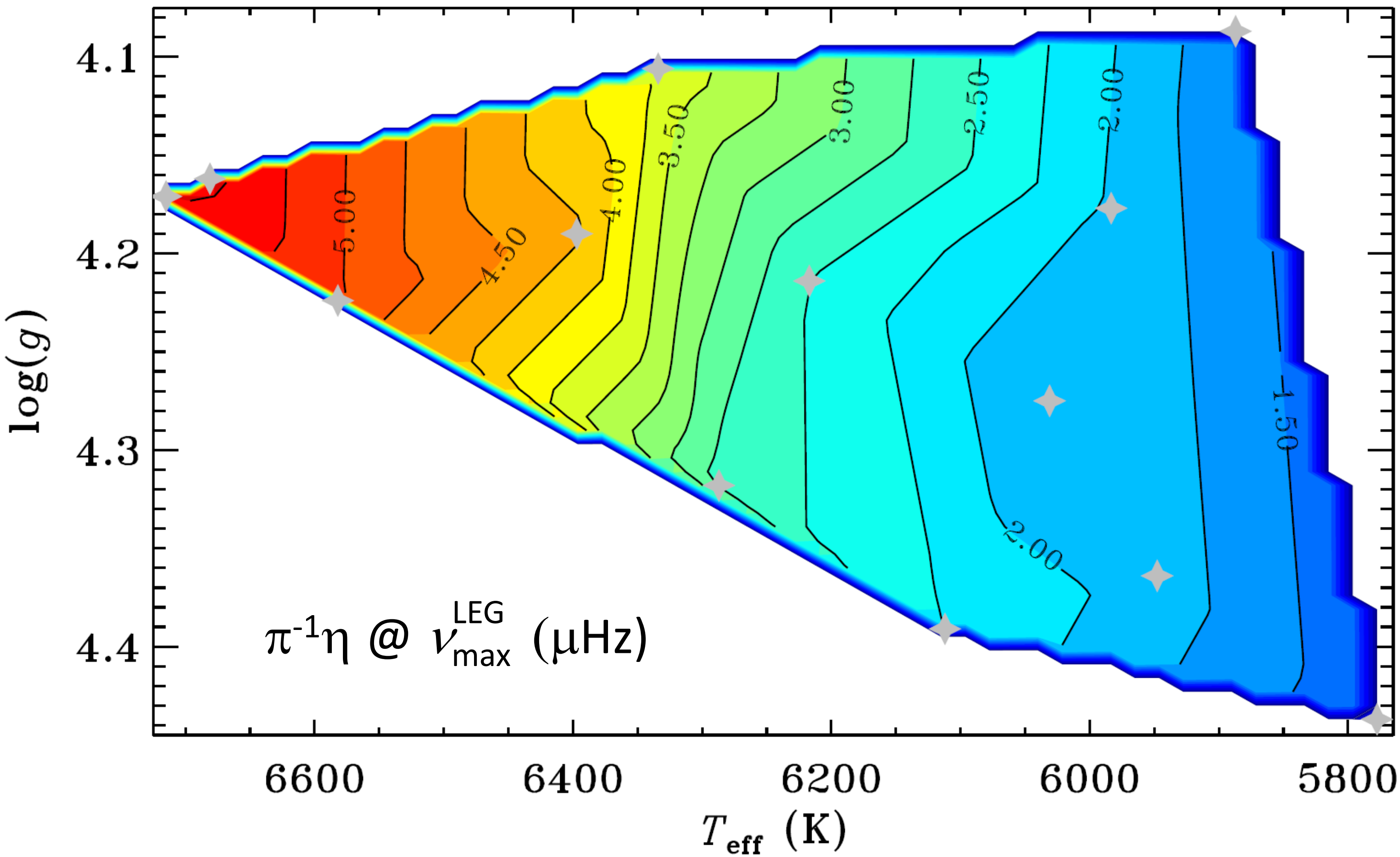
9139163 (Punto)



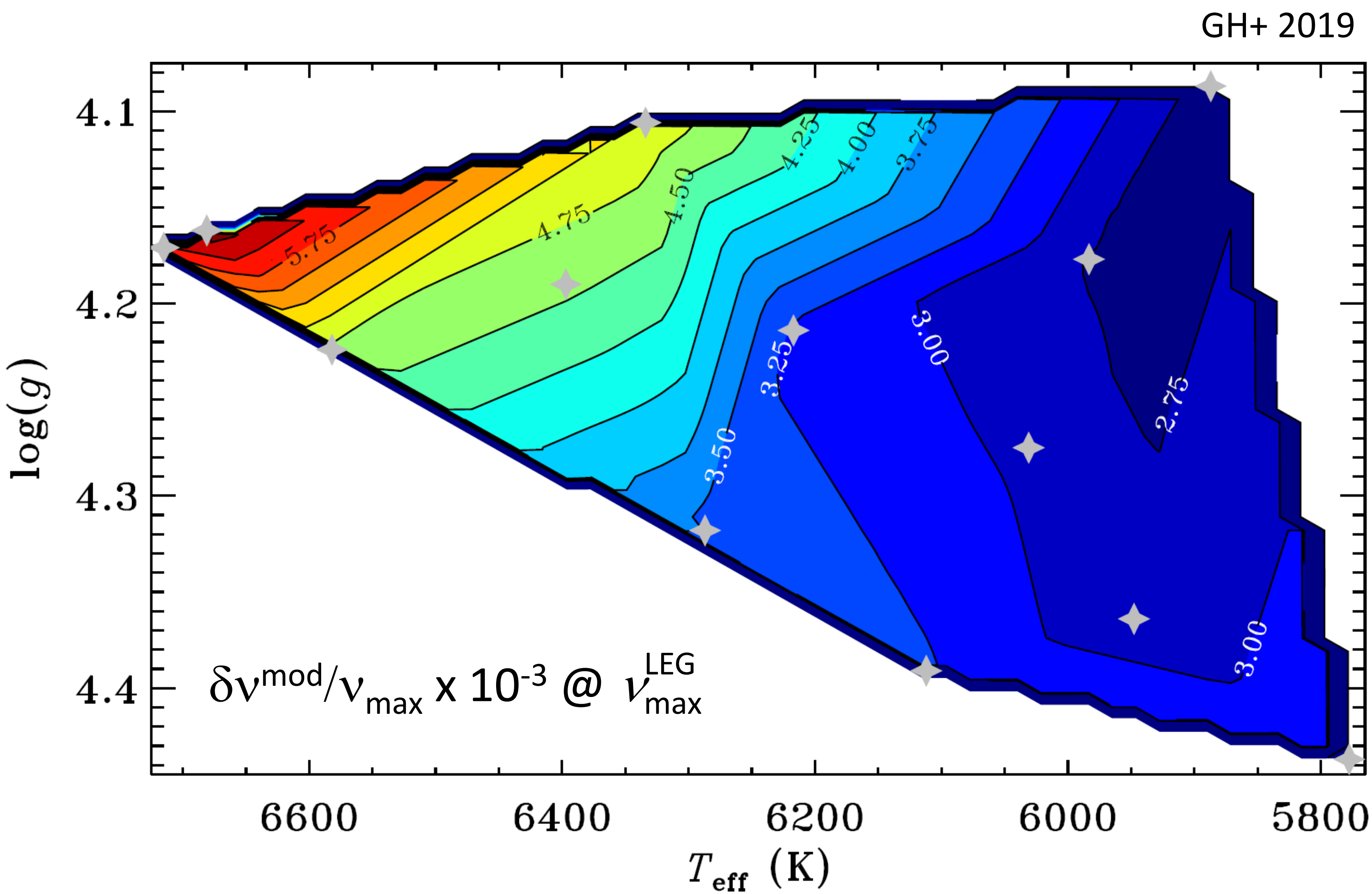
T_{eff} = 6397 K
log g = 4.190

Modelled damping rates, $\pi^{-1}\eta$ @ $\nu_{\max}^{\mathrm{LEG}}$ (μHz)

GH+ 2019



Relative modal frequency corrections $\delta\nu^{mod}/\nu_{max}$ @ ν_{max}^{LEG}



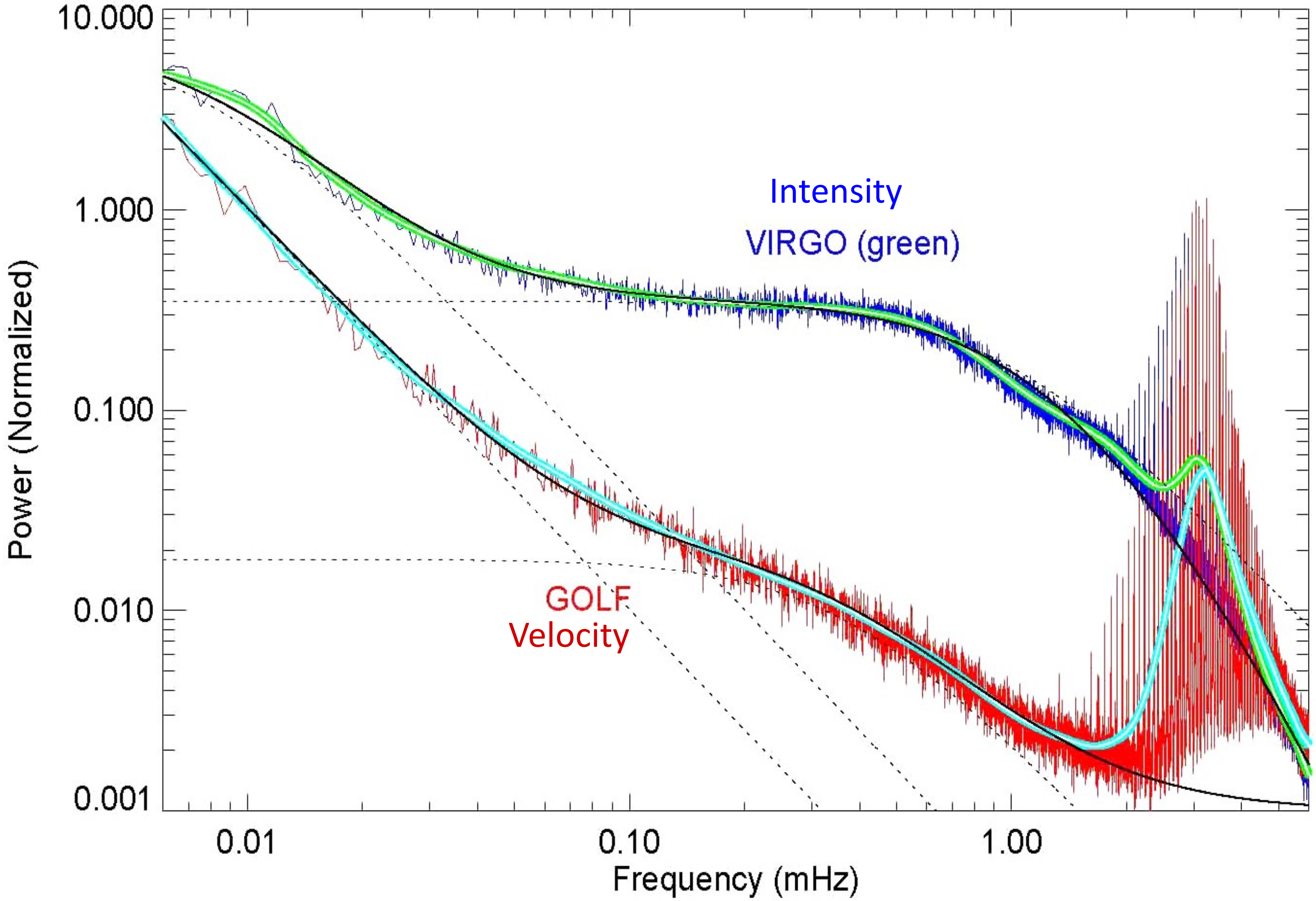
Amplitude ratios
WP126300
(GH, L. Bigot, R. Trampedach, F. Kupka, M.N. Lund)

Amplitude ratios
WP 126300

Max. amplitudes

- photometry ~ 3.6 ppm
- spectroscopy ~ 15 cm/sec

max. amplitude ratio: $\Delta L/\Delta V \simeq$ 0.24 ppm/cm s^{-1}



Courtesy of Christensen-Dalsgaard

Solar-type pulsations

Amplitude ratio: $$\underbrace{\Delta L/\Delta V}_{\text{observable}} =: \underbrace{\frac{\delta L/L}{\omega_{\rm r} r \delta r/r}}_{\text{model}}$$

Stochastic excitation model

(Goldreich & Keeley 1977; Balmforth 1992; Samadi+ 2001; Chaplin, Houdek+ 2005)

eigenfunction δr

Energy supply rate:
(Reynolds stress contribution)

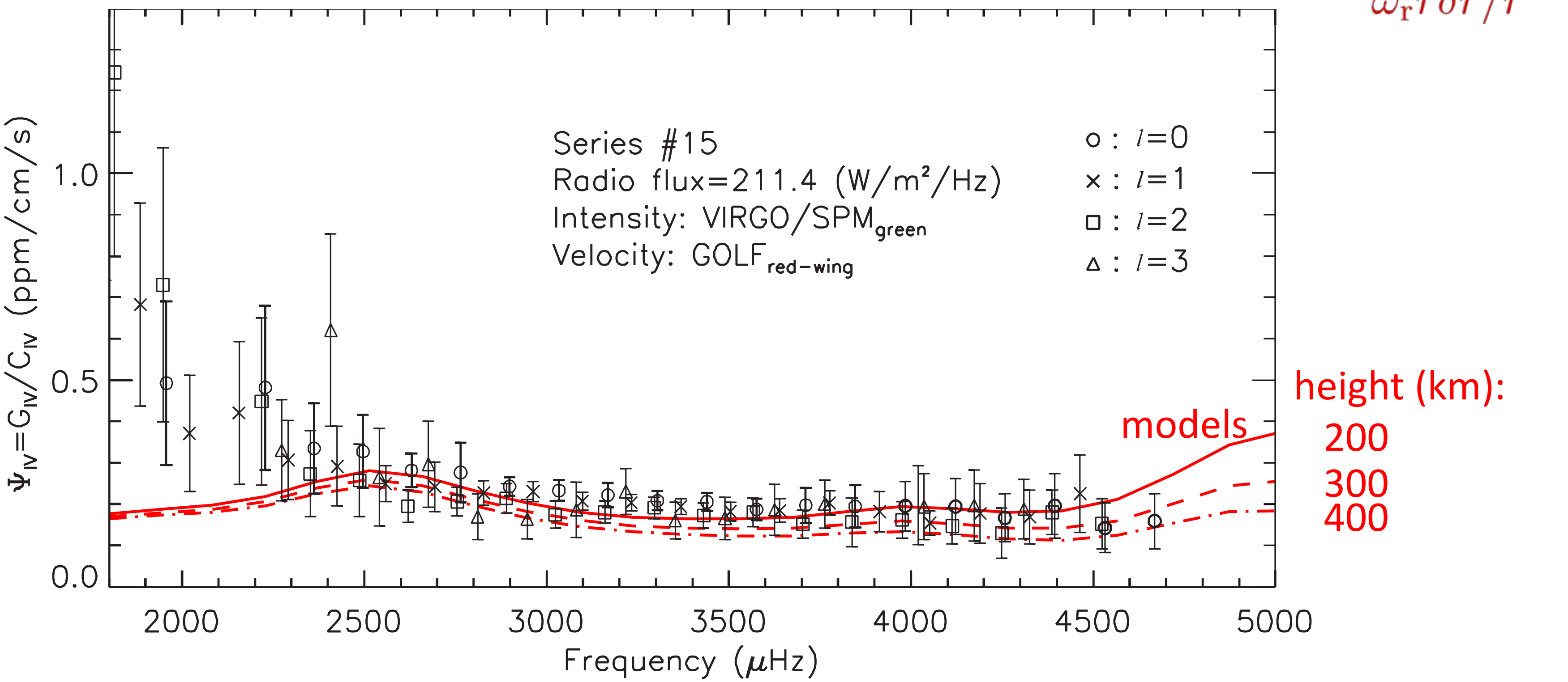
$$P \propto I^{-1} \int_0^R \ell^3 \left(r \frac{\partial\, \delta r}{\partial r} p_{\rm t} \right)^2 \mathcal{S}\, \mathrm{d}r$$

Amplitude ratios are independent of stochastic excitation model (*P*)!

Sun: Virgo and GOLF (data: Jimenez+ 2002)

Amplitude ratios $\Delta L/\Delta V$ (ppm cm^{-1} s)

$$\Delta L/\Delta V =: \frac{\delta L/L}{\omega_{\rm r} r\delta r/r}$$

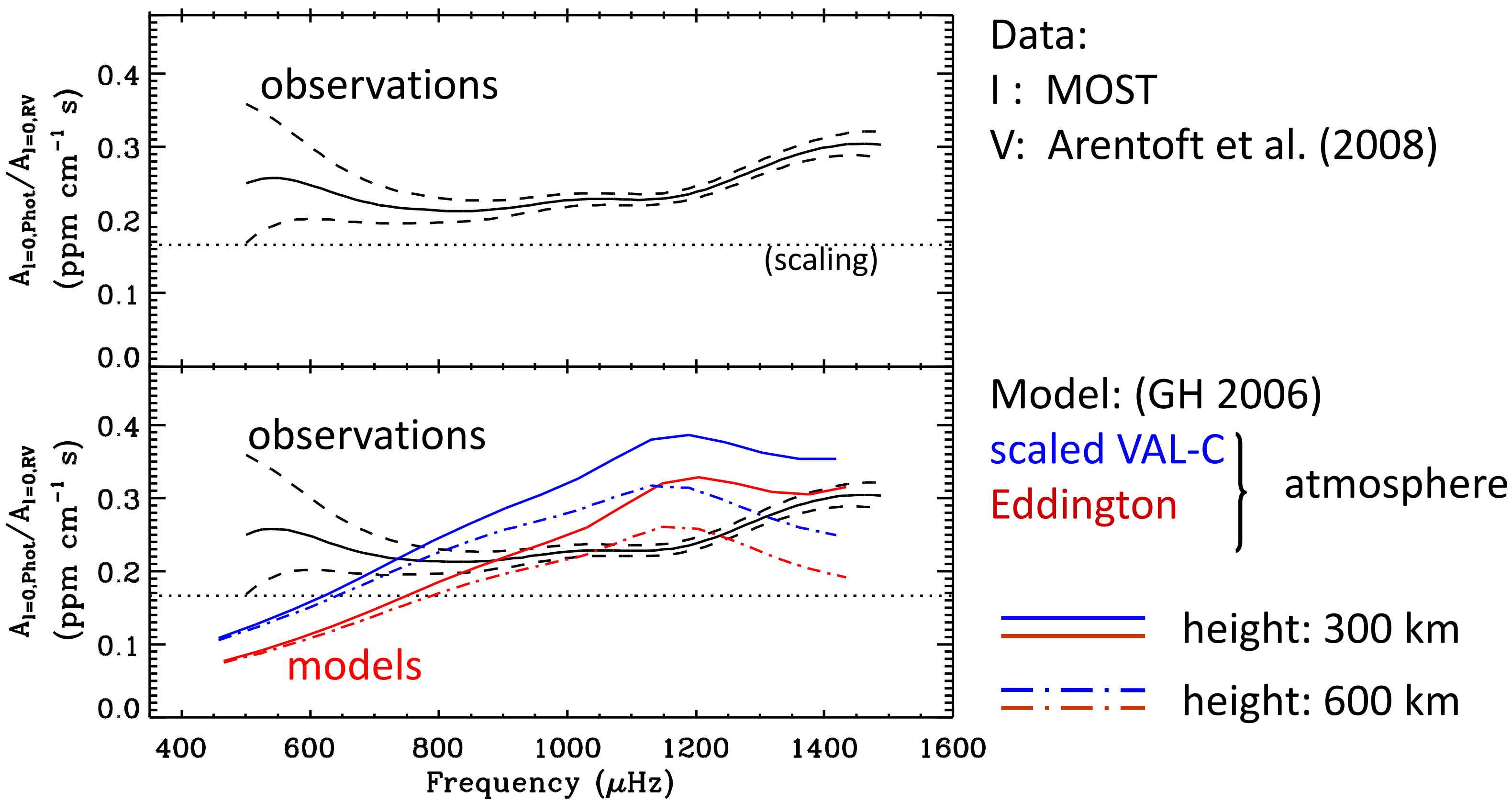


Amplitude ratios are independent of stochastic excitation model (P)!

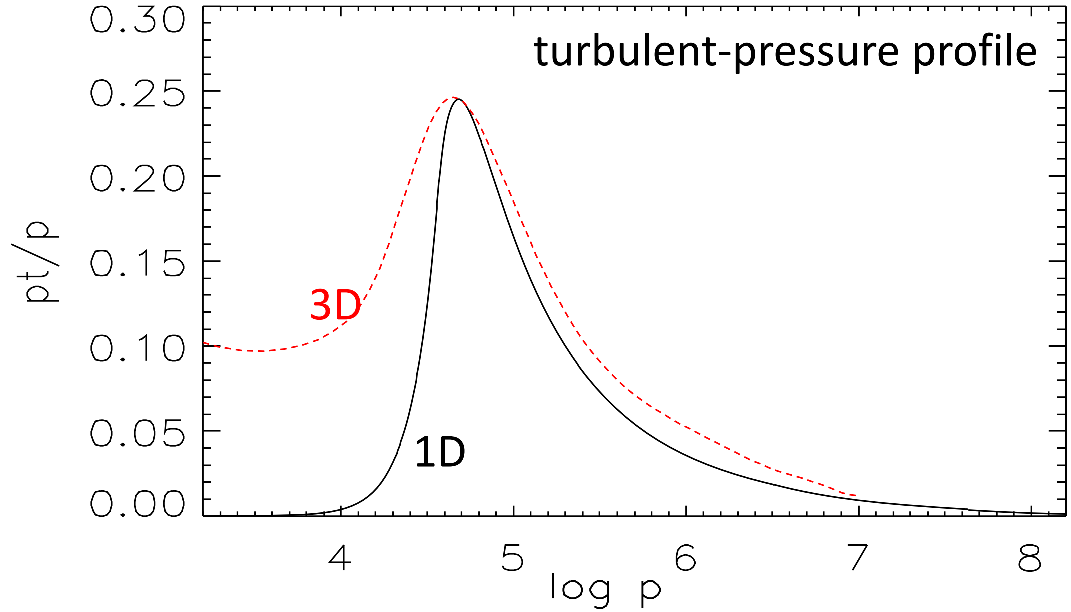
See also Houdek (2011, PLATO conference Berlin; 2012, Hakone conference)

Amplitude ratios in a model for Procyon A
(MOST satellite + ground-based observing campaign)

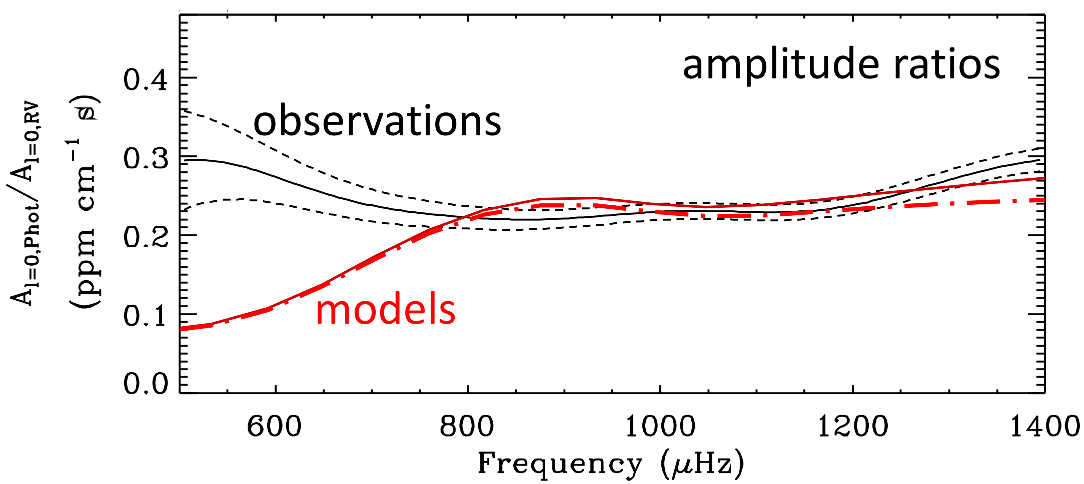
Huber+ (2011)



Amplitude ratios in a model for **Procyon A**
(**MOST** satellite + **ground-based** observing campaign)



1D Model: (GH+ in prep.)
calibrated to 3D simulations
(Trampedach+ 2014)



Model: (GH+ in prep.)
scaled VAL-C atmosphere

height: 400 km

height: 600 km

Summary / conclusions

- Frequency-dependent linewidths and 3D simulation results can be used to (further) calibrate stellar models.

- Such calibrated stellar models also provide modal frequency corrections.

- Need for updated 3D simulation grids with various values for metallicity *Z*.

- Total frequency corrections in others stars require 3D-calibrated evol. models.

- There is still need to implement the physics of kinetic energy flux and convective back-warming into (time-dependent) 1D convection models.