Scalar spectrum in a Graviton Soft Wall Model

by

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-The Graviton Soft Wall (GSW) model
-The spectrum of scalar glueballs
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Introduction

-The basis of our development is the so called AdS/CFT correspondence also called Maldacena's duality or gauge /gravity duality.

-Why has it become so popular (Maldacena's original paper as of 2020 almost 20000 citations)?

-It provides a powerful toolkit for studying strongly interacting quantum field theories. The usefulness of the duality results from the fact that it is a strong-weak duality: when the fields of the quantum field theory are strongly interacting, the ones in the gravitational theory are weakly interacting and thus more mathematically tractable.

-The AdS/CFT correspondence has revived the hope that QCD can be solved.

-The correspondence establishes a relation between Anti-de-Sitter spaces formulated in string theories and M-theory and Supersymmetric Yang-Mills theories which are conformal.

-Motivated by this development two different approaches have been followed for QCD thus far

a) Top-down: One starts from an N=4 Super Yang Mills theory, which is not asymptocially free, but realizes correctly chiral symmetry and confinement constructs the corresponding AdS gravity and studies the resulting dynamics. In N=4 Super Yang-Mills theory fundamental quarks have been introduced using D7 AdS branes. The mesons that appear in these theories behave in many ways similarly to the mesons in QCD.

b) Bottom-up : Proceeds the opposite way. One starts from QCD and constructs an AdS five-dimensional holographic dual which incorporates the properties of QCD.

We shall proceed in the bottom-up approach introducing it with a specific model.

The Graviton Soft Wall (GSW) model

The gravitational background of the model is and AdS metric:

$$ds^2 = \frac{R^2}{z^2} e^{\alpha\varphi(z)} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

The particles are described by fields with a correspondence

4D	5D	p	Δ	$M_5^2 R^2$
$\overline{q}(x)q(x)$	$\psi(x,z)$	0	3	-3
$Tr(F(x)^2)$	G(x,z)	0	4	0
$\overline{q}\gamma^\mu q(x)$	$V^\mu(x,z)$	1	3	0
.....	.....	.....	.....	.....

The particles are described by fields moving in the metric background with an action given by

$$\int d^5x \sqrt{g} e^{-\Phi(z)} \mathcal{L}(\psi(x,z), V_\mu(x,z), G(x,z), ...)$$

where $\Phi(z)$ is a dilaton field which describes confinement. The hadronic identity, i.e. spin and parity, of the particles is defined by each of the particles AdS bulk mass M_5^2 which is given by

$$M_5^2 R^2 = (\Delta + p)(\Delta + p - 4)$$

The equations of motion for the fields will provide the spectrum, and the mode functions (wave fuctions) which will determine decay constants and form factors.

The GSW model is defined by

$$g_{MN}(x,z) = \frac{R^2}{z^2} e^{\alpha\varphi(z)}(-1,1,1,1,1)$$

$$\Phi(z) = e^{\beta\varphi(z)}$$

To verify confinement, which manifests itself as Regge behavior, for mesons

$$\beta_S = -1 - \frac{3\alpha}{2} \qquad \beta_V = -1 - \frac{\alpha}{2}$$

$$\varphi(z) = k^2 z^2$$

Thus the whole strong dynamics for light particles is governed by two parameters α and k

The scalar glueball spectrum

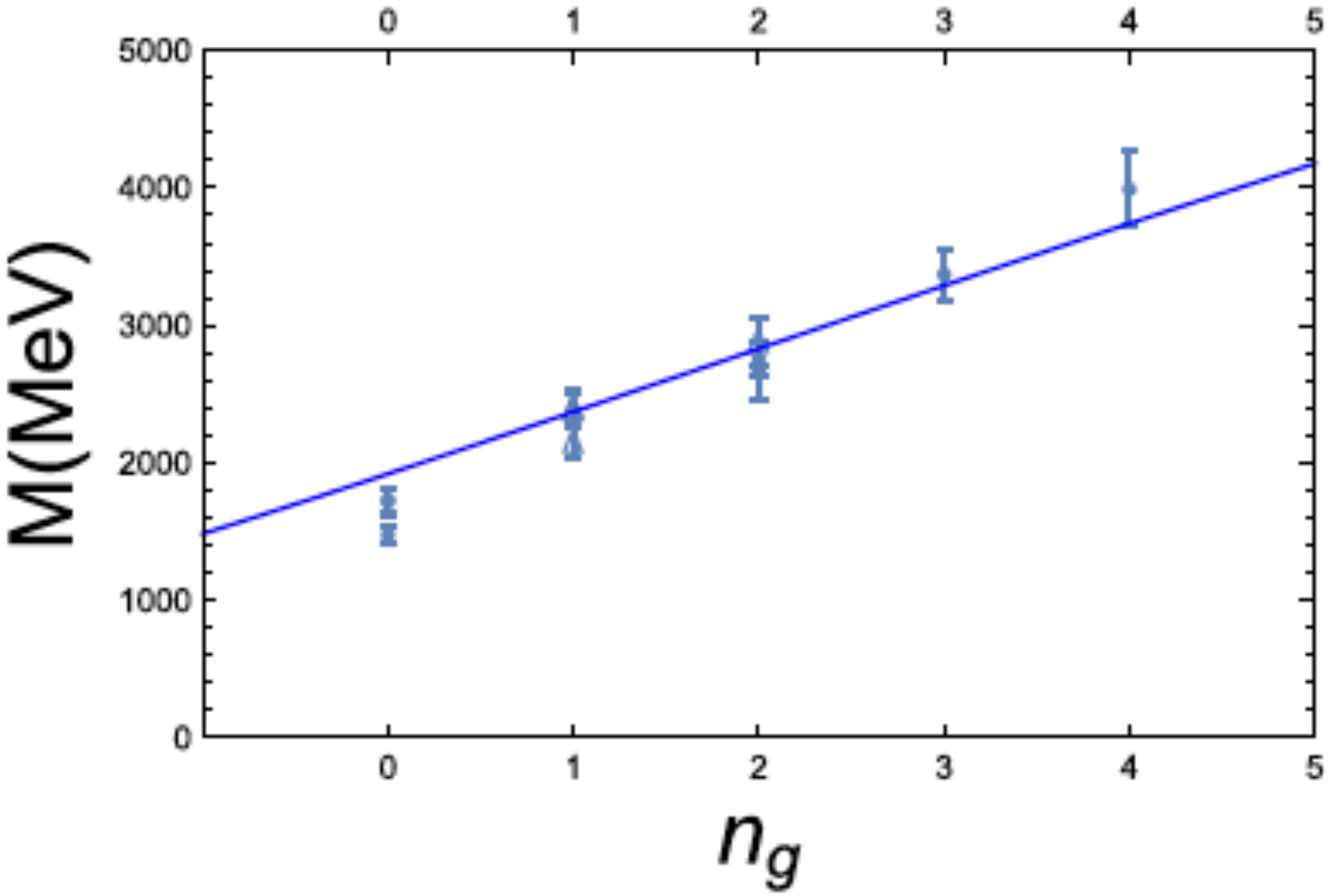
A peculiarity of our approach in the GSW model, which is the reason for the name, is that the scalar glueball arises from the scalar component of the 5D graviton and is not introduced as an independen field. Thus the Einstein equations associated with the metric provide the equation for the spectrum.

$$\frac{d^2G(z)}{dz^2} - \left(\alpha k^2 z + \frac{3}{z}\right)\frac{dG(z)}{dz} + \left(\frac{8}{z^2} - 6\alpha k^2 - 4\alpha^2 k^4 z^2 + M^2\right)G(z) - \frac{8}{z^2}e^{\alpha k^2 z^2}G(z) = 0$$

This equation can be transformed into a Schrödinger type equation

$$-\frac{d^2g(t)}{dt^2} + \left(\frac{8}{t^2}e^{2t^2} - 15t^2 + 14 - \frac{17}{4t^2} + \right)g = \Lambda^2\phi(t),$$

where $\Lambda^2 = \dfrac{M^2}{\alpha k^2}$. This equation must be solved numerically and the experimental result depends only of one energy scale $\sqrt{\alpha}k$. Expanding the exponential to third order which leds to an exact Kummer equation is not a good approximation in this case. For $\alpha k^2 = 0.37$ GeV^2 we obtain the spectrum of the figure.



The experimental spectrum for the glueballs is the lattice spectrum of Gluodynamics.

The scalar meson spectrum

The GSW action in the scalar sector is then defined by

$$I = \int d^4x\, dz \sqrt{-\bar{g}}\, e^{\beta\varphi(z)} \left[\bar{g}^{MN} \partial_M S(x,z) \partial_N S(x,z) + M_{5m}^2 S^2(x,z)\right],$$

In the light sector we get the following Equation of Motion (EoM):

$$\partial_M(\sqrt{-g} e^{-\varphi(z)} g^{MN} \partial_N S(x,z)) = \sqrt{-g} e^{-\varphi(z)(1-\alpha)} M_{5m}^2 S(x,z).$$

for $M_5^2 R^2 = -3$ we get

$$-\frac{d^2\sigma(u)}{du^2} + \left(4u^2 + 4 + \frac{15}{4u^2} - \frac{3}{u^2} e^{2\alpha u^2}\right)\sigma(u) = \Omega^2 \sigma(u),$$

where $\Omega^2 = (2/k^2) M^2$.

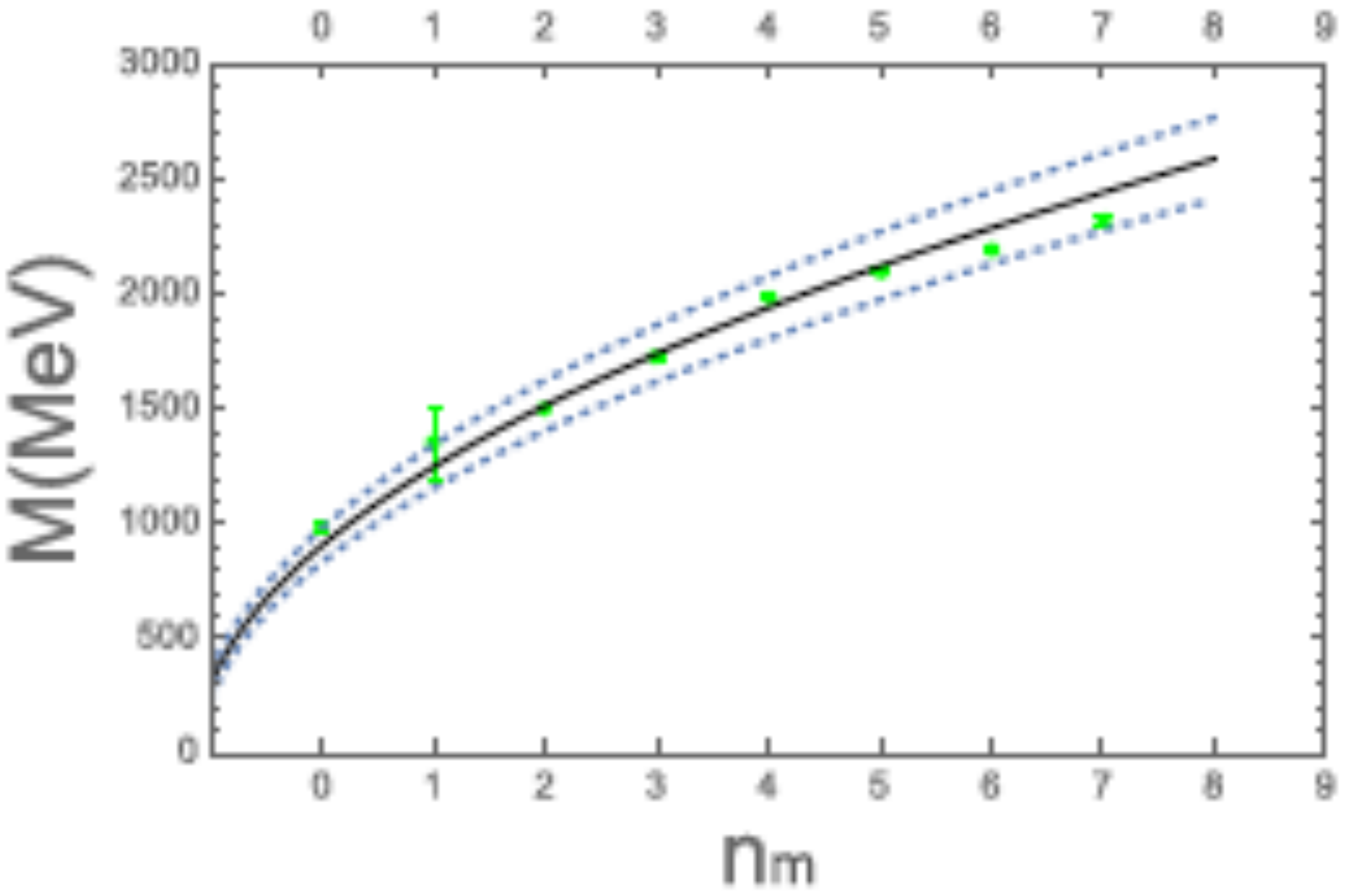
This equation has to be solved numerically but it is quite unstable specially for high values of the mode numbers. Expanding the exponential up to third order we get a Kummer equation, which can be solved exactly leading to a good approximation. Its spectrum

$$\Omega_n^2 = 4(n+1)\sqrt{4-6\alpha^2} + 4 - 6\alpha,\ n = 0, 1, 2, \ldots.$$

We have fitted the lower modes with one value of α and the upper modes with another providing in that way a theoretical error

$$\alpha = 0.55 \pm 0.04$$

The advantage of this procedure whenever possible is that we can work with analytic mode functions.



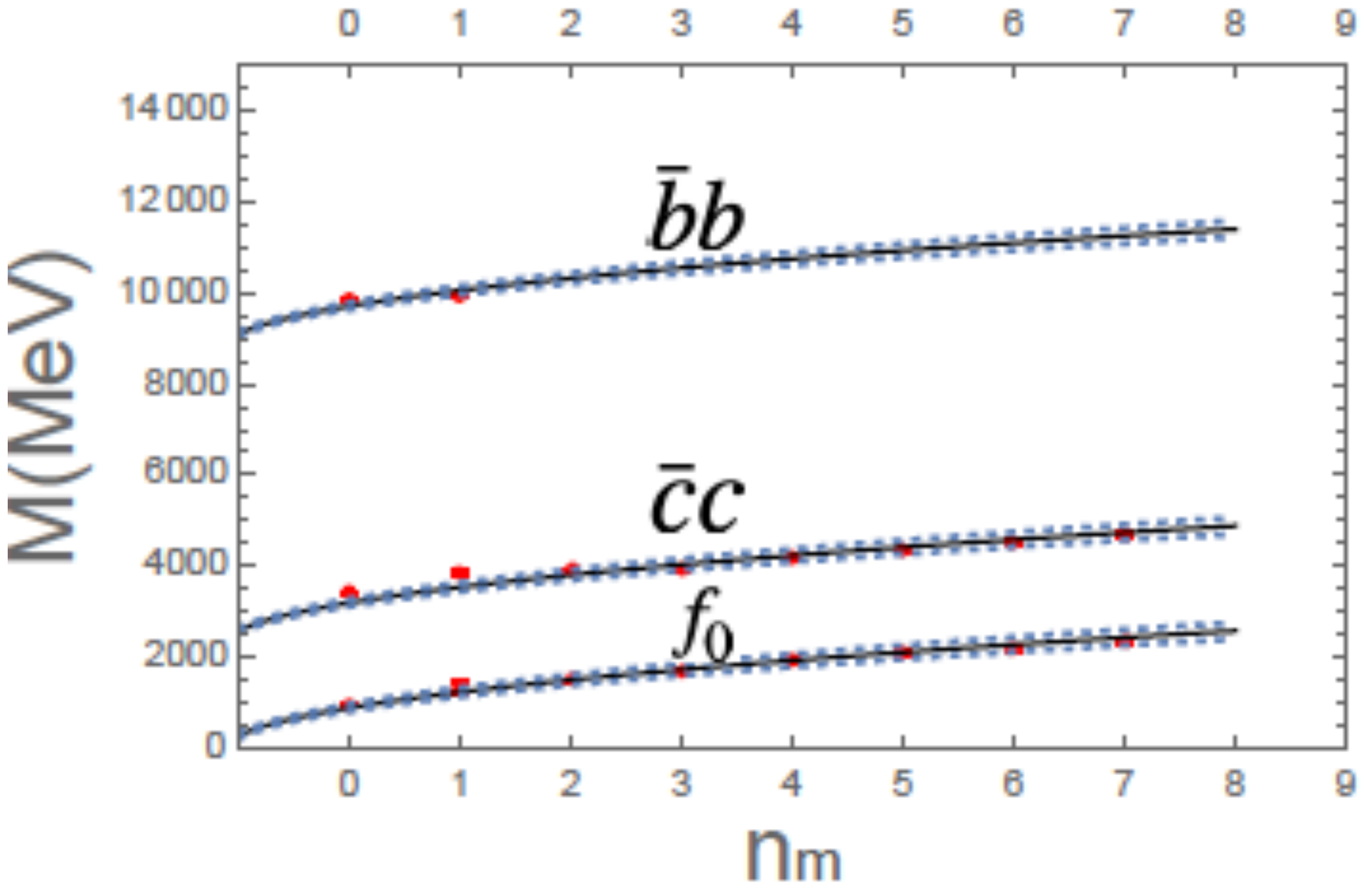
The experimental meson spectrum is the f_0 PDG spectrum excluded the $f_0(500)$.

The solid line corresponds in the figure corresponds to the central value and the dotted to the error.

What about the heavy scalar mesons?
We generalize the above equation to incorporate a heavy quark scale

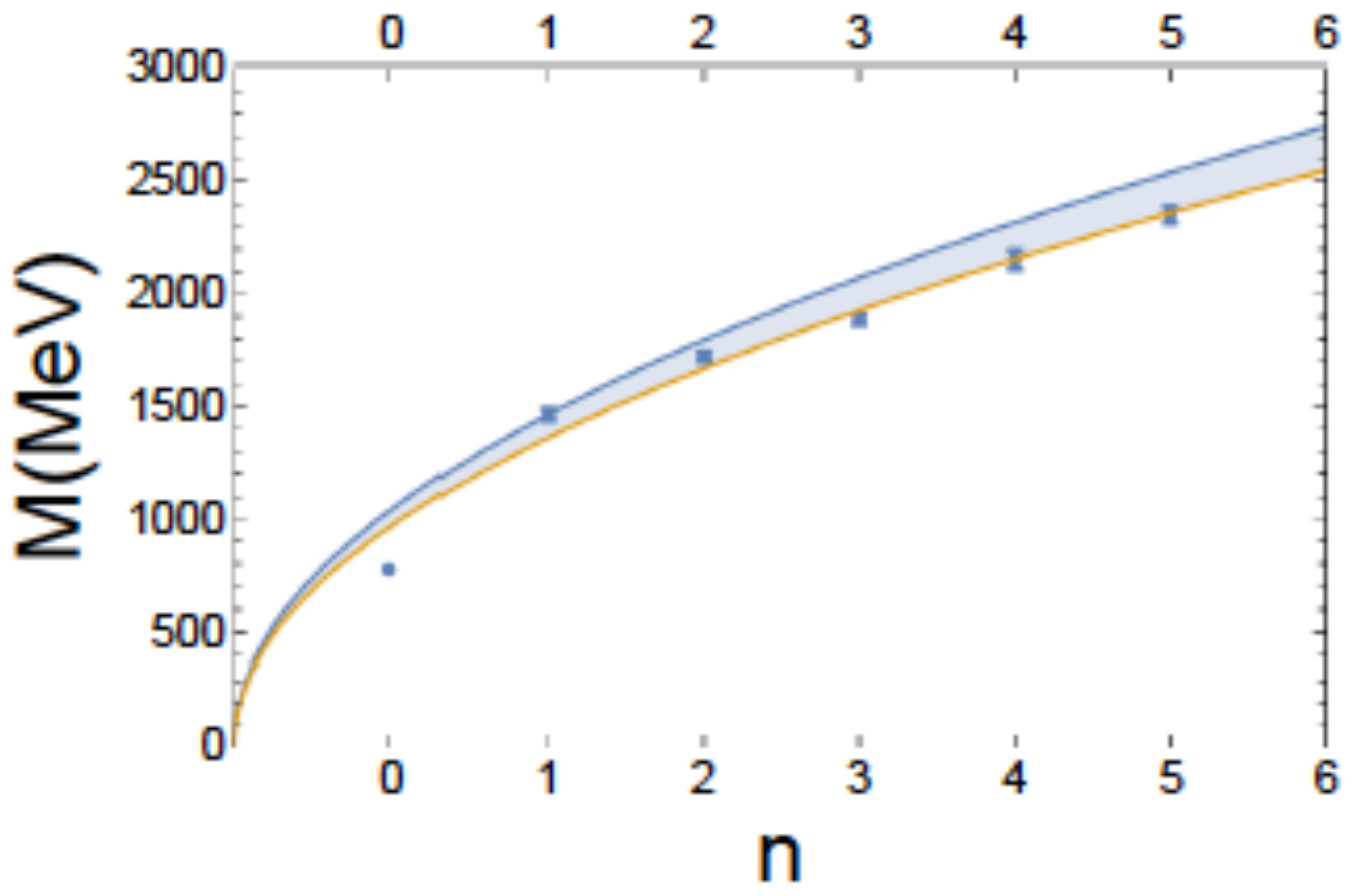
$$M_n(\alpha) = \sqrt{2}k\Omega_n(\alpha) + C$$

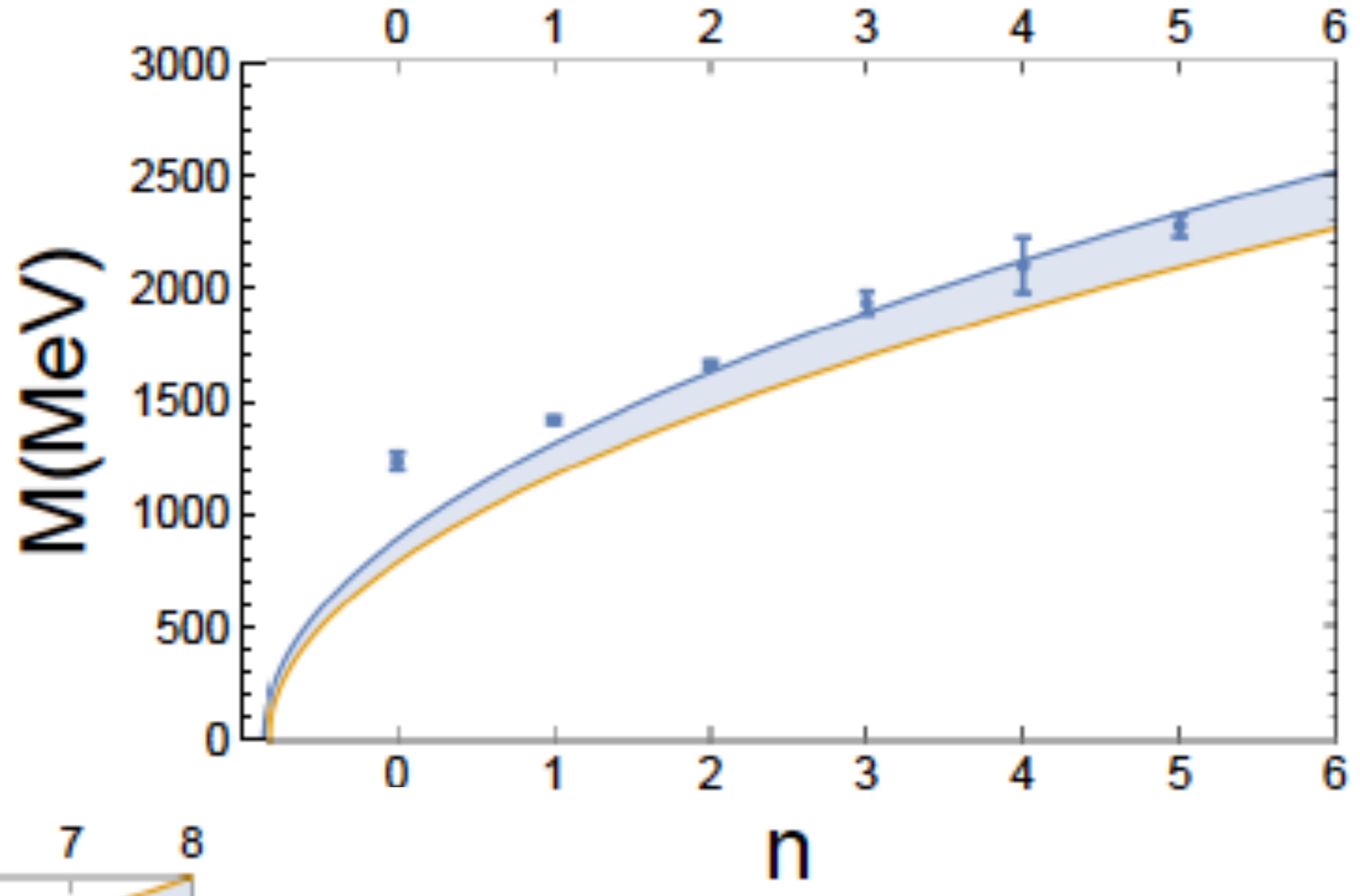
where C depends on flavor.
With
C_C = 2400 (~2* ($m_C = 1270 \pm 20$)) MeV
and
C_B = 8700 MeV (~2*($m_B = 4180^{+30}_{-20}$))

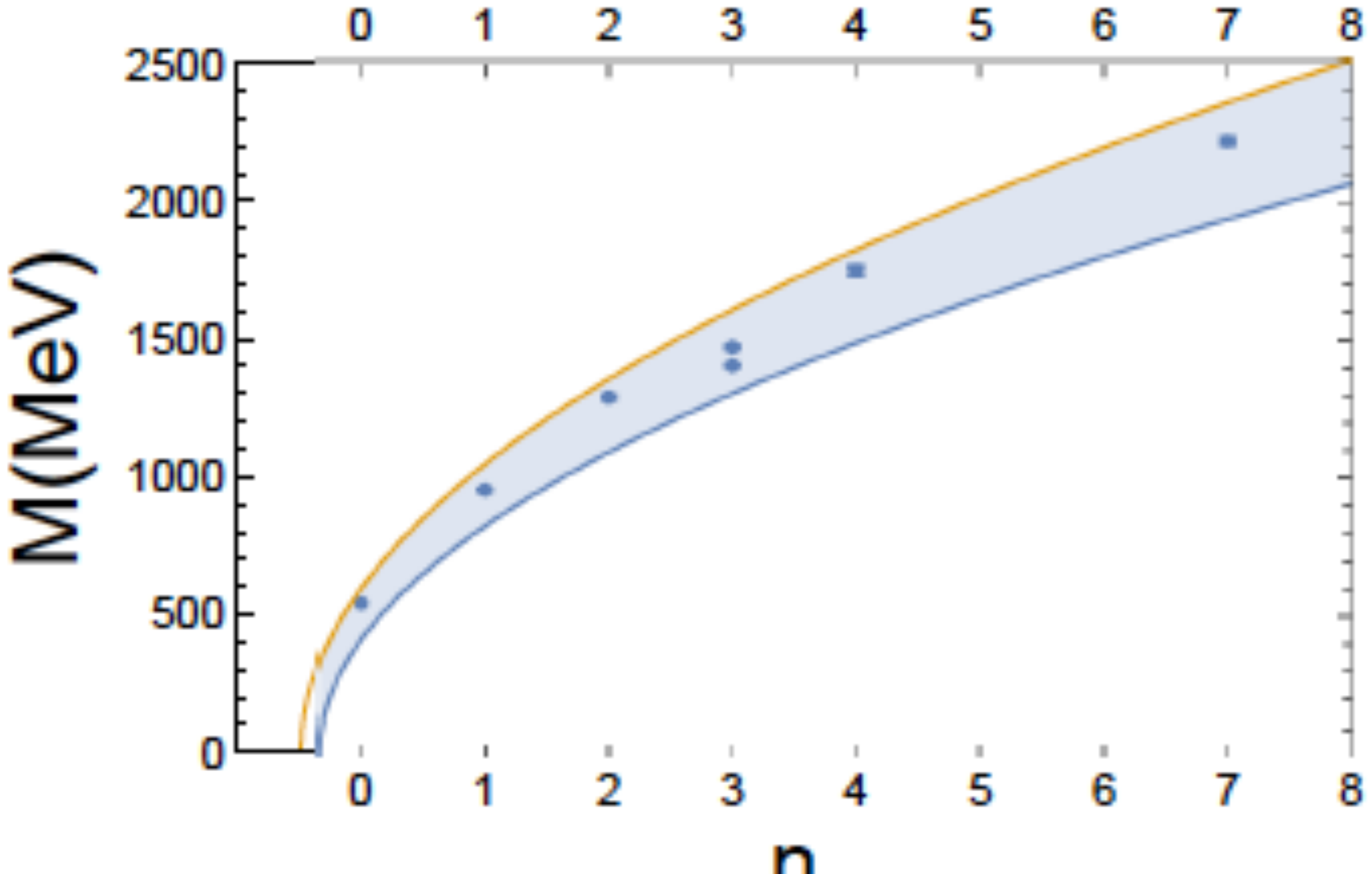


Comments on unpublished results

We have applied the model with fixed $\alpha = 0.55 \pm 0.04$ and $\alpha k^2 = 370$ MeV^2 to ρ, a_1, η and high spin glueballs.







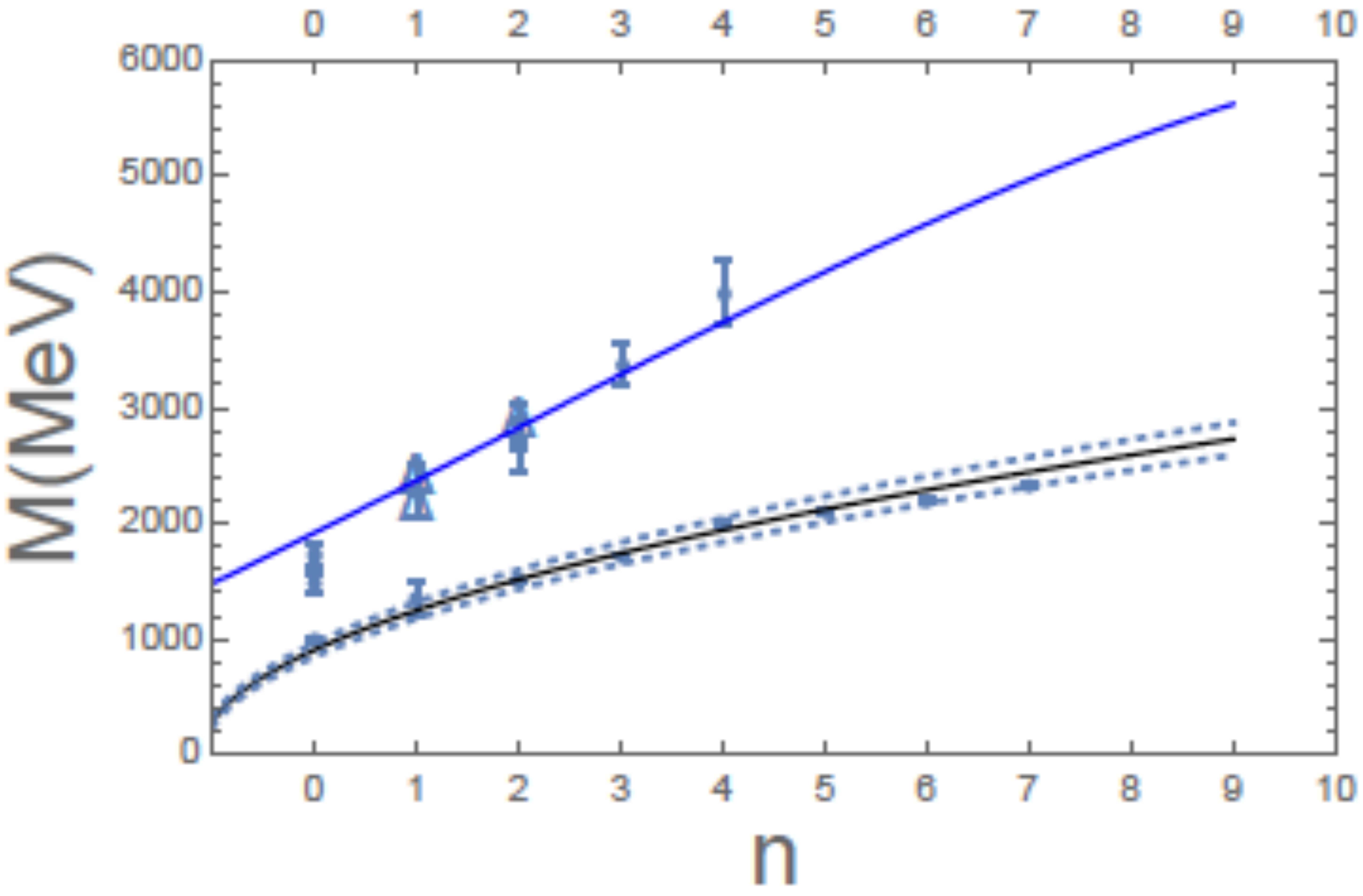
J^{PC}	M&P	Ky	My	Ll	Mta	Sz	This work
1^{--}	3850 ± 140	3830 ± 130	3240 ± 480	3950	3990	3001	3081 ± 23
3^{--}	4130 ± 290	4200 ± 245	4330 ± 460	4150	4160	4416	4219 ± 17
5^{--}				5050	5260	5498	5362 ± 13
7^{--}				5900			6508 ± 11

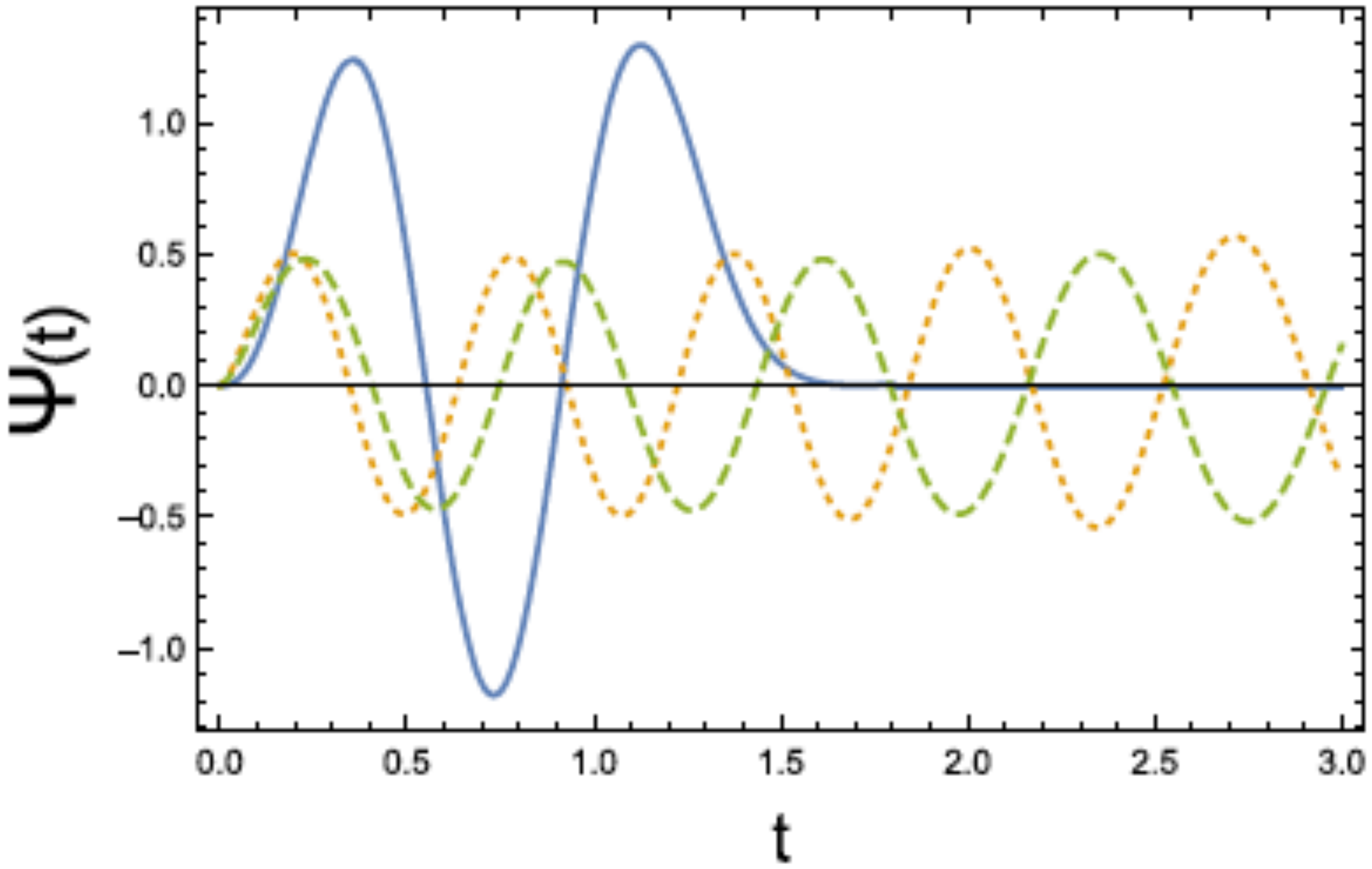
Odd spin glueballs

J^{PC}	M&P	Ky	My	Gy	Sk	Mtb	This work
2^{++}	2400 ± 145	2390 ± 150	2150 ± 130	2620 ± 50	2420	2590	2515 ± 29
4^{++}			3640 ± 150		3990	3770	3649 ± 19
6^{++}			4360 ± 460			4600	4790 ± 15

Even spin glueballs

A prediction about meson-glueball mixing




1.0
0.5
0.0
−0.5
−1.0
Ψ(t)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
t

The holographic LF representation of the EoM, in AdS space can be recast in the form of a LF Hamiltonian [8].

$$H_{LC}|\Psi_n >= M^2|\Psi_n > .$$

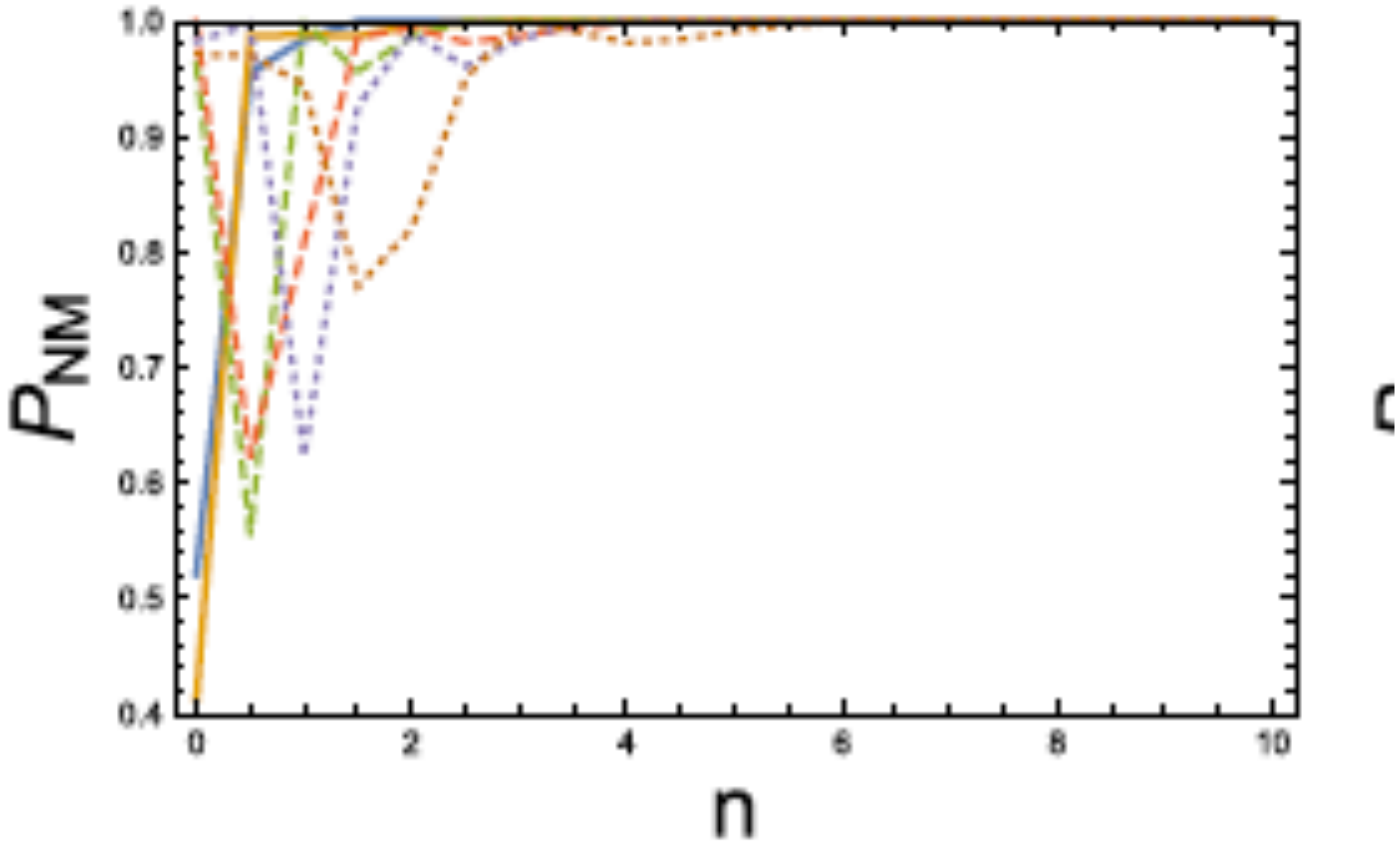
In the AdS/QCD LF framework the above relation becomes a Schrödinger type equation

$$\left(-\frac{d^2}{dt^2} + V(t)\right)\Psi(t) = \Lambda^2\Psi(t)$$

$$\Psi_n(t) = \langle t|\Psi_n\rangle$$

$$\langle\Psi_n|\Psi_n\rangle = \int dt\ |\Psi_n(t)|^2 = 1$$

Mixing ~ Overlap probability ~ $|\langle\Psi_m|\Phi_g\rangle|^2$



1.0
0.9
0.8
0.7
0.6
0.5
0.4
P_{NM}
0
2
4
6
8
10
n

Conclusions

- The AdS/CFT correspondence offers a procedure to treat a strong coupling field theory as a weak AdS gravitaty theory, and viceversa.
- The Bottom-Up approach aims at defining a gravity theory in 5D which manifests the strong coupling properties of QCD.
- The effective theory depends on a background metric, a dilaton and a free field lagrangian with an AdS mass which characterises the QCD bound states: spin and parity.
- The models described up to now relate large N QCD with AdS, thus our results are large N dominant.
- We have shown that one can have a 10% fit to all the light (heavy) experimental meson spectrum and with the lattice QCD results for the glueballs with only two (+2) parameters.
- We have seen some peculiarities worth mentioning

  +the $f_0(500)$ does not fit into the scheme—> it is not a $\bar{q}q$ state

  +the $\rho(1570)$ does not fit into the scheme—> It has been eliminated from the PDG and it is assumed to be an OZI decay of the $\rho(1700)$

  +the a,(1260) seems to be too high mass (huge width 200-600)

  +the $\eta(1405)$ and $\eta(1475)$ seem to be the same particle (PDG comments)

  +prediction: heavy glueballs will be pure states.

The π in this model like the η. It requires a deeper study !

Thank you for
your attention