Event Shapes for Massive Particles

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VNiVERSiDAD D SALAMANCA

In collaboration with C. Lepenik: JHEP 01 (2018) 122

A. Bris & M. Preisser: JHEP 09 (2020)

N. González Gracia (w.i.p)

+ others

HASP online metting 16-12-2020

Hadronic jets

So far we have heard (or will be hearing) about

- Hadron spectroscopy
- Dispersive approaches
- Low-energy (few) hadron production
- EFTs with hadronic d.o.f.'s
- Models

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What this talk is about

- Many hadrons produced at high energies
- First-principle computations
- Lagrangians with partonic degrees of freedom
- Quarks which do not form hadrons (top)
- EFTs for jets (!) and boosted heavy quarks

Motivation

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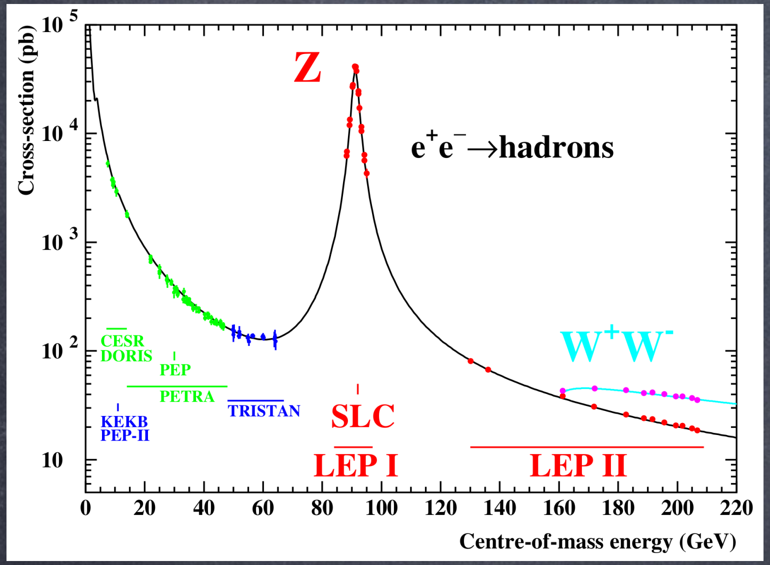
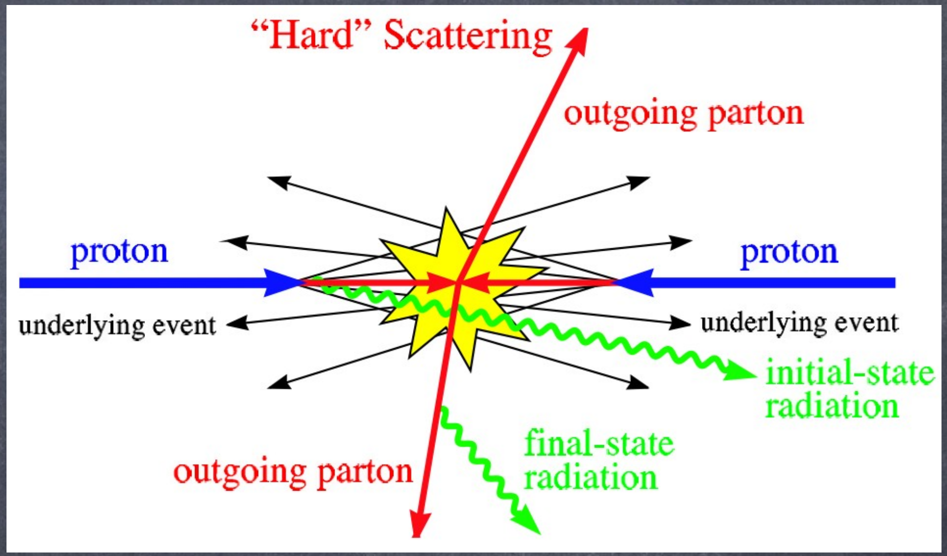
- Higher energies are cleaner because perturbative computations work
- But for inclusive observables one looses sensitivity to quark masses and the strong coupling
- In hadronic machines one gets contamination from secondary collisions (underlying event), ISR, boosted sub-jets...





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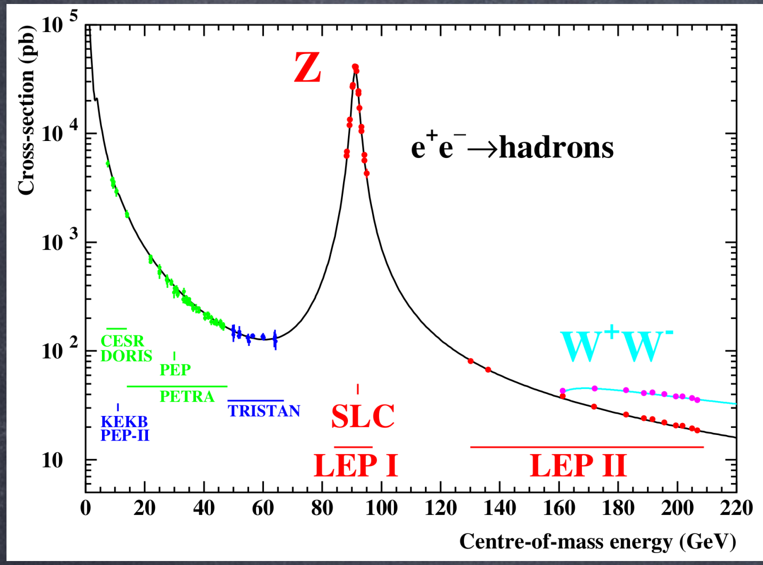

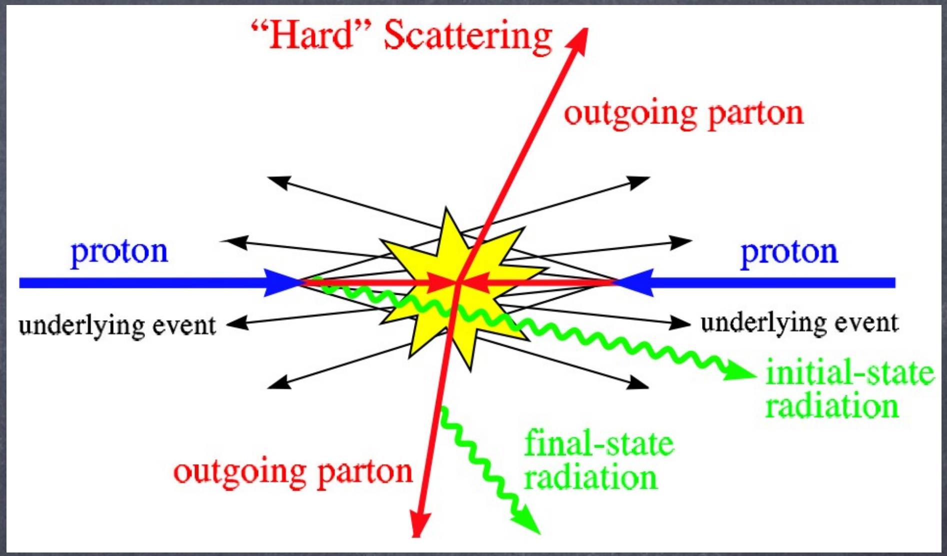

To get back some sensitivity to interesting parameters, or loose sensitivity to nasty physics, one needs to "sharpen the pencil"

The idea is keeping track of the final-state global kinematics

Jets and event shapes are an excellent tool for that

What is exactly a jet?

Need to use a specific recombination scheme or "jet algorithm"

On top of that one might use different axes (standard vs WTA)

Finally, one can use trimming or grooming techniques to remove soft radiation (e.g. soft drop) ... or ...

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We can use a continuous variable to tell us how "jetty" the event is

- Thrust $$\tau = \min_{\vec{n}}\left(1 - \frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{\sum_j |\vec{p}_j|}\right)$$

  plane normal to thrust axis defines two hemispheres

- Heavy-jet mass $$\rho = \frac{1}{Q^2}\left(\sum_{i\in\text{heavy}} p_i\right)^2$$

- C-parameter $$C = \frac{3}{2}\,\frac{\sum_{i,j}|\vec{p}_i||\vec{p}_j|\sin^2\theta_{ij}}{\left(\sum_i|\vec{p}_i|\right)^2}$$

- Jet broadening $$B_T = \frac{\sum_i |\vec{n}\times\vec{p}_i|}{2\sum_i|\vec{p}_i|}$$

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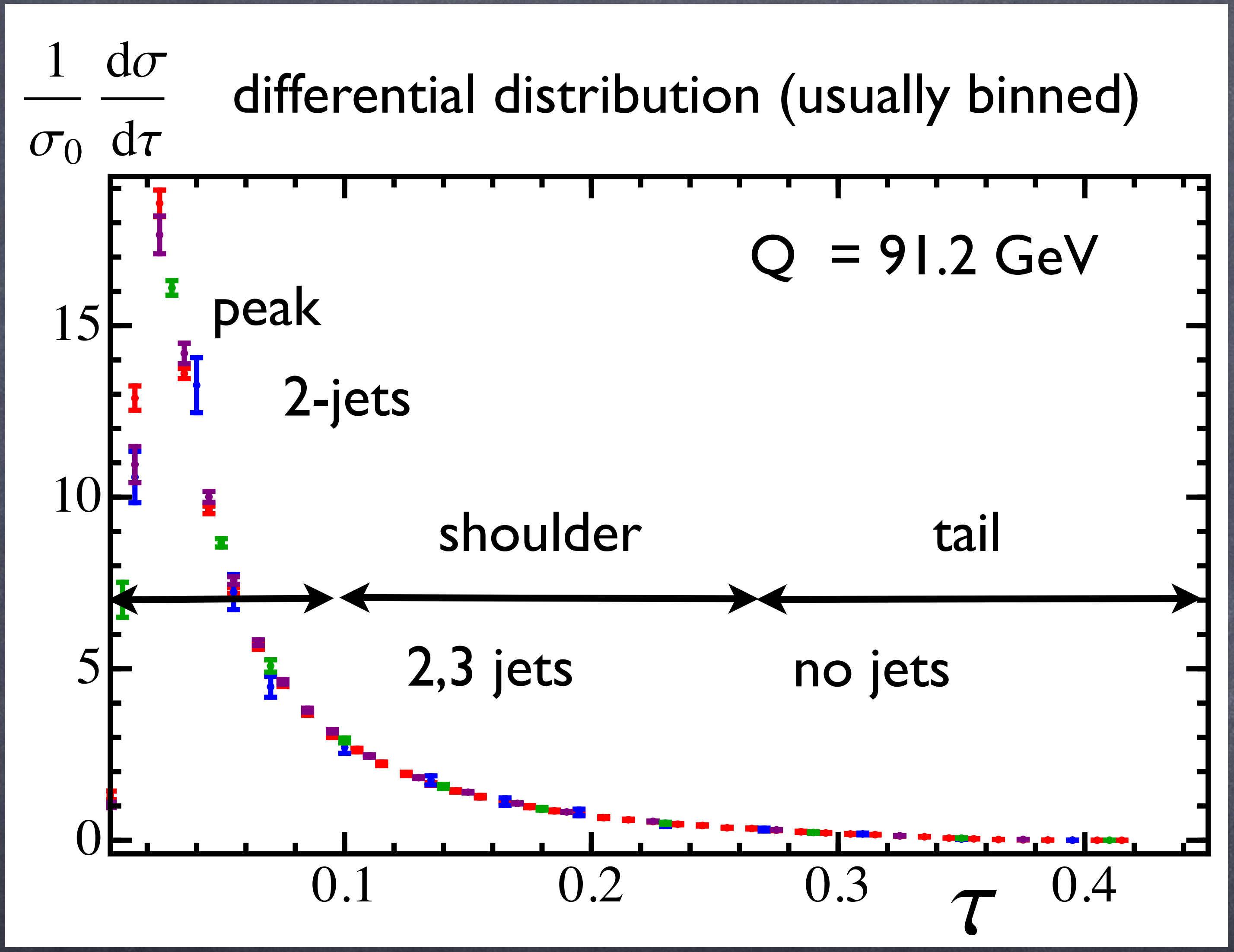
- Jet broadening $B_T = \frac{\sum_i |\vec{n}\times\vec{p}_i|}{2\sum_i|\vec{p}_i|}$

These are called event shapes and make computations simpler

In this talk I will consider these in different "massive schemes"

How do they look like?

Q = center of mass energy



Dijet configurations have small values of the event shape

EFT's for jets

Soft-collinear effective Theory

Fixed-order computations have large logs in peak and shoulder

EFT treatment allows to sum them up and treat hadronization from first principles

EFTs allow to deal with one scale at a time in mustiscale problems

For massless quarks and not-so-boosted heavy quarks: SCET

$$\frac{\mathrm{d}\hat{\sigma}^C}{\mathrm{d}\tau} = \frac{\mathrm{d}\hat{\sigma}^C_{\mathrm{SCET}}}{\mathrm{d}\tau} + \frac{\mathrm{d}\hat{\sigma}^C_{\mathrm{NS}}}{\mathrm{d}\tau} \qquad \text{(massive case)}$$

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singular terms, dominant in peak and shoulder

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hard matching coefficient

jet function (collinear radiation)

soft function (wide angle soft radiation)

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singular terms, dominant in peak and shoulder

computation to all orders in large-β_0

w.i.p. with N. González Gracia

asses ambiguities (asymptotic behavior) of various pieces

quantify hadronization corrections

closed form for anomalous dimension and matrix elements

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Sums up large logarithms through renormalisation group evolution

Dominant hadronization corrections encapsulated in soft function

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computed in at $\mathcal{O}(\alpha_s)$ in any massive scheme

[Bris, VM, Preisser: JHEP 09 (2020)]

together with analytic resummation based on expansions

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modified SCET to account for mass power corrections computed in [Lepenik & VM: JHEP 01 (2018) 122]

Absorb mass power corrections into hard and jet function

In this talk we will have time only to present [JHEP 01 (2018) 122]

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kinematic power corrections

computed in [Lepenik & VM: JHEP 01 (2018) 122]

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Massive schemes

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Originally introduced by Salam and Wicke to study hadron mass effects on hadronization [JHEP 05 (2001) 061]

Useful when considering heavy quarks to gain/loose sensitivity

Massless-particle momenta: $p^2 = 0 \longrightarrow E_0 = |\vec{p}|$ can use either

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makes a difference

Massive scheme: use both $E_p, |\vec{p}|$ maximal mass sensitivity

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P-scheme: use $\vec{p} \to \dfrac{E_p}{|\vec{p}|}\,\vec{p}$

E-scheme: use $E_p \to |\vec{p}|$

behave as massless $p_E^2 = p_P^2 = 0$

minimal mass sensitivity

same collinear limit

[A. Bris, VM, M. Praisser JHEP 09 (2020)]

+ in E-scheme leading hadronization correction is universal

F0 computations

$\mathcal{O}(\alpha_s)$ massive event-shapes at FO

[C. Lepenik, VM JHEP 01 (2018) 122]

General structure of partonic distribution at lowest two orders

$$\frac{1}{\sigma_0^C}\frac{\mathrm{d}\sigma_C}{\mathrm{d}e} = R_C^0(\hat{m})\,\delta(e-e_{\min}) + C_F\frac{\alpha_s}{\pi}A_e^C(\hat{m})\delta(e-e_{\min})$$

$$+\,C_F\frac{\alpha_s}{\pi}B_{\rm plus}^C(\hat{m})\left[\frac{1}{e-e_{\min}}\right]_+ + C_F\frac{\alpha_s}{\pi}F_e^{\rm NS}(e,\hat{m}) + \mathcal{O}(\alpha_s^2)$$

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C = V, A vector or axial-vector current

for massive primary quarks the difference matters

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$\hat{m} = \dfrac{m}{Q}$

reduced mass

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if $e_{\min}\neq 0$ then $e_{\min}(\hat{m})$ $\longrightarrow$ mass-sensitive event shape

$\mathcal{O}(\alpha_s)$ massive event-shapes at FO

[C. Lepenik, VM JHEP 01 (2018) 122]

General structure of partonic distribution at lowest two orders

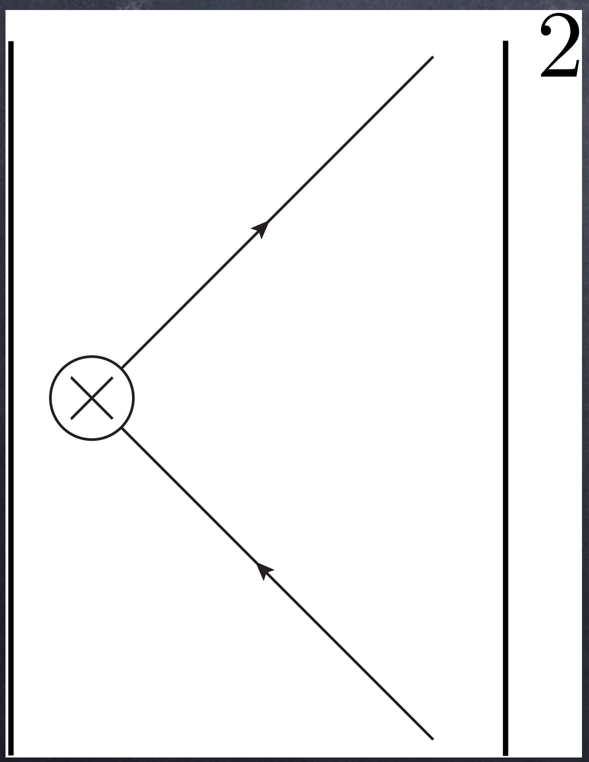
known analytically

$$\frac{1}{\sigma_0^C}\frac{\mathrm{d}\sigma_C}{\mathrm{d}e} = \boxed{R_C^0(\hat{m})}\,\delta(e-e_{\min}) + C_F\frac{\alpha_s}{\pi}A_e^C(\hat{m})\delta(e-e_{\min}) + C_F\frac{\alpha_s}{\pi}B_{\text{plus}}^C(\hat{m})\left[\frac{1}{e-e_{\min}}\right]_+ + C_F\frac{\alpha_s}{\pi}F_e^{\text{NS}}(e,\hat{m}) + \mathcal{O}(\alpha_s^2)$$

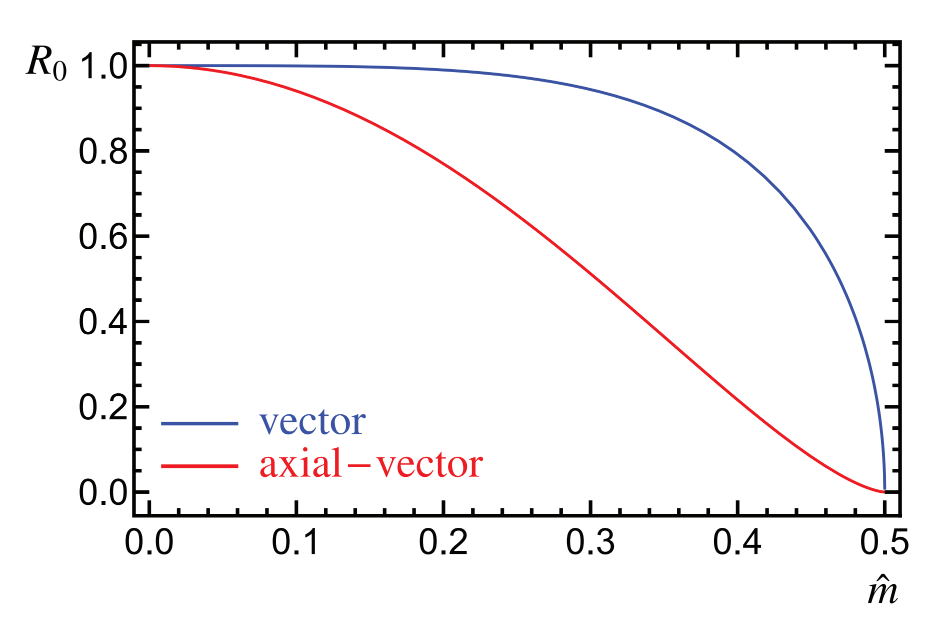
common to all event shapes

Born-normalized cross section

In terms of Feynman diagrams



x 2-particle phase space



Fixer-order massive total hadronic cross section

$\mathcal{O}(\alpha_s)$ massive event-shapes at FO

[C. Lepenik, VM JHEP 01 (2018) 122]

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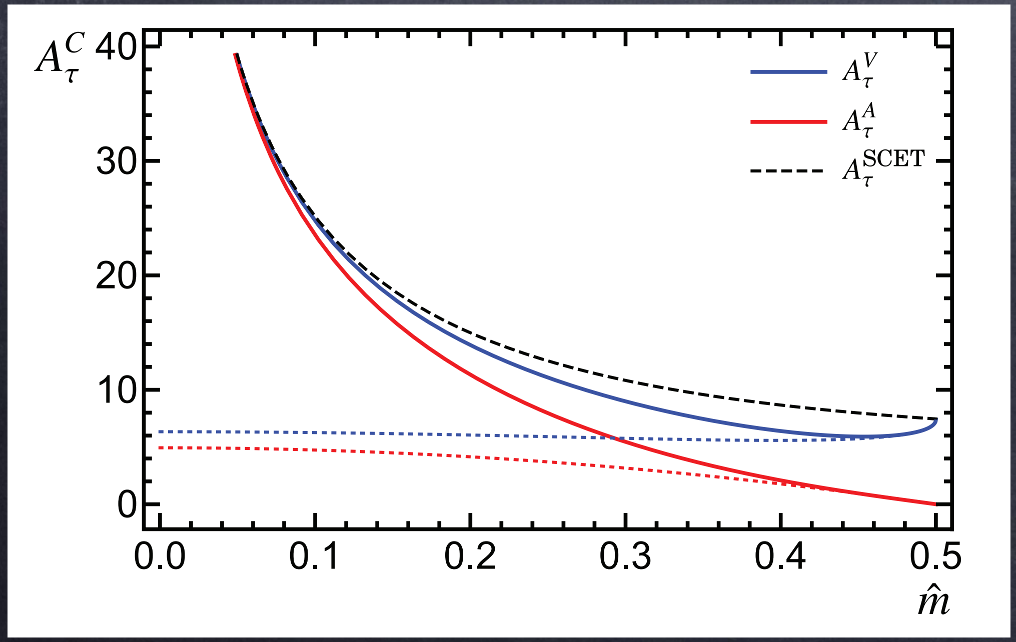
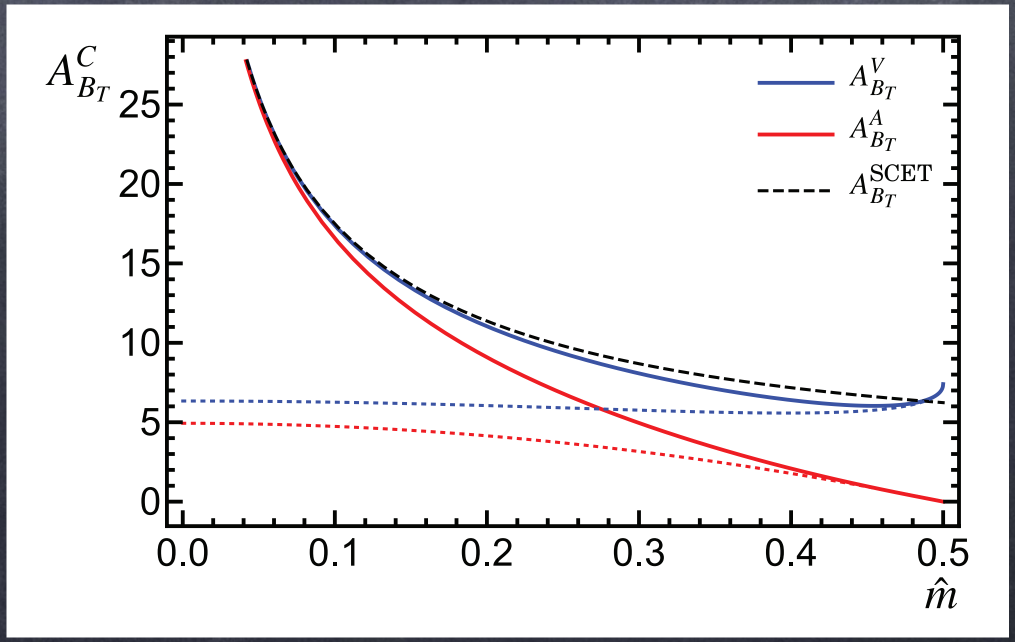
Dirac Delta

Depends on the event shape

cancellation of IR divergences takes place here

mass power correction that can be absorbed into EFT factorisation theorems





computed analytically [Lepenik, VM]

$\mathcal{O}(\alpha_s)$ massive event-shapes at FO

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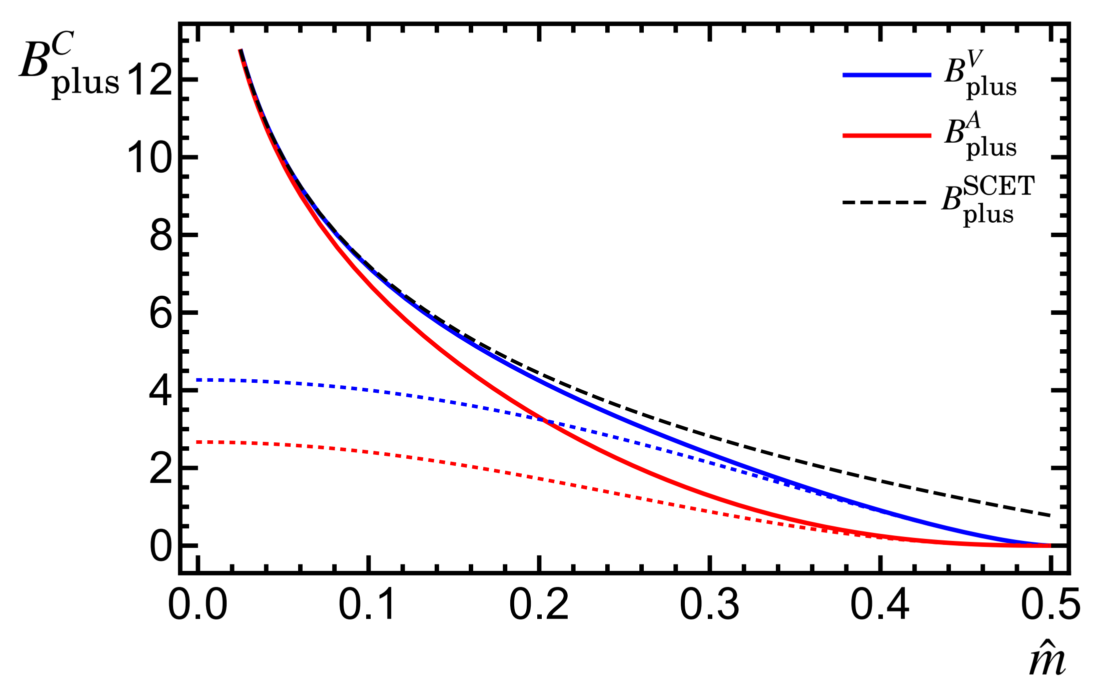
"plus" distribution

Universal (same for all event shapes)

mass power correction that can be absorbed into EFT factorisation theorems



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Depends on the event shape

non-singular terms

mass and kinematic power corrections that cannot be absorbed into EFT factorisation theorems (additive correction)

$$F_e(e,\hat{m}) \equiv \frac{B_{\rm plus}(\hat{m})}{e-e_{\min}} + F_e^{\rm NS}(e,\hat{m})$$

For a numerical determination simpler to compute non-singular + singular-non-distribution

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[C. Lepenik, VM JHEP 01 (2018) 122]

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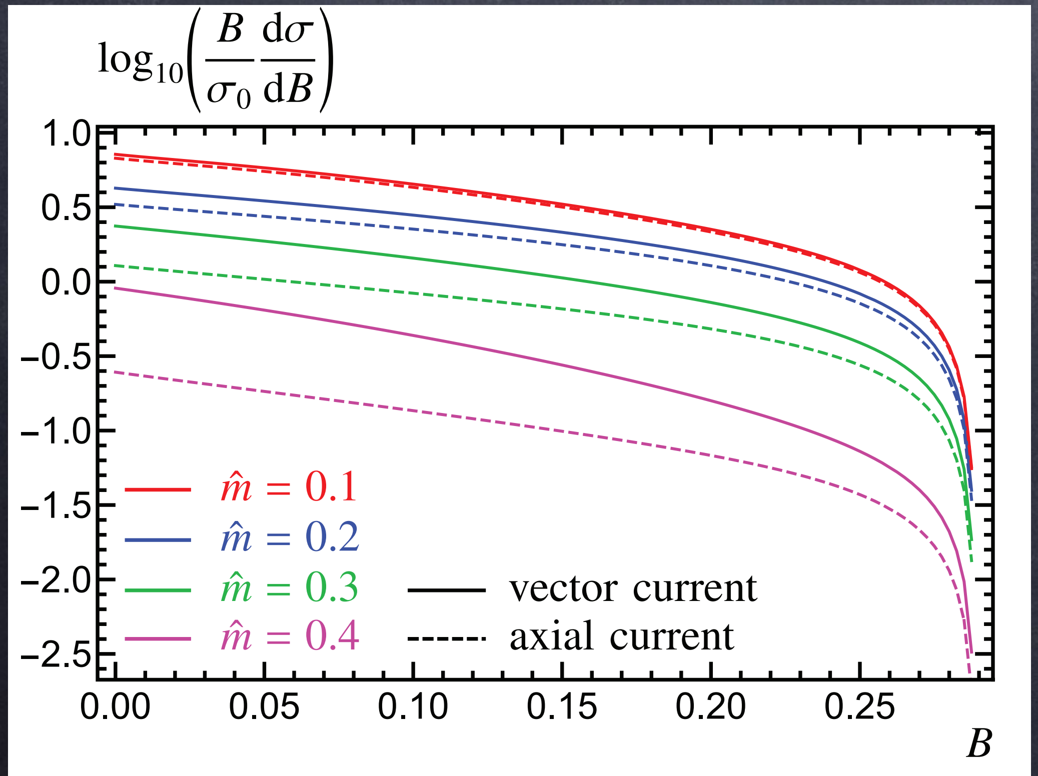

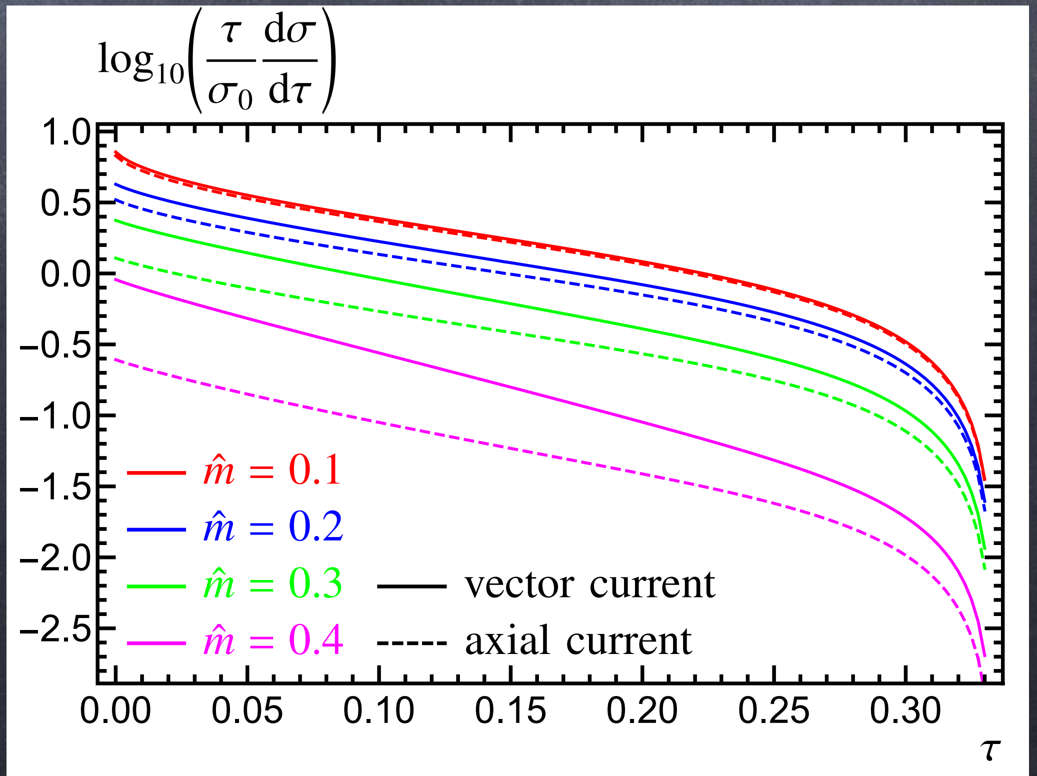

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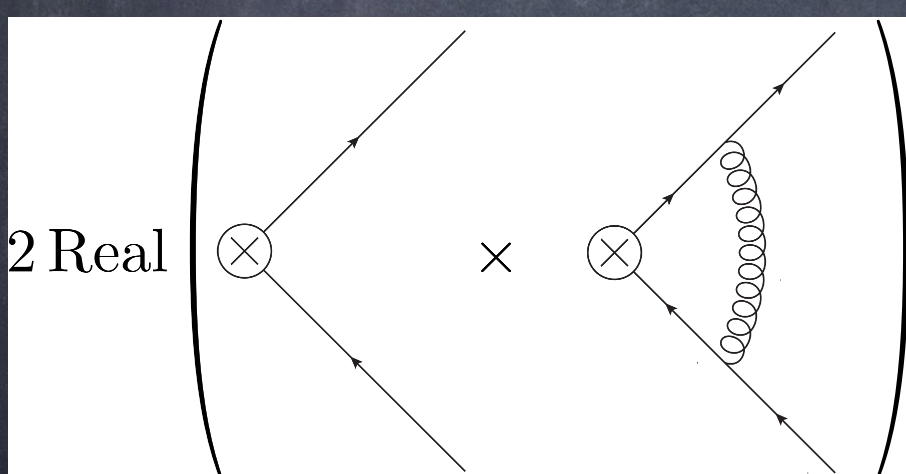
In terms of Feynman diagrams at $\mathcal{O}(\alpha_s)$ one has



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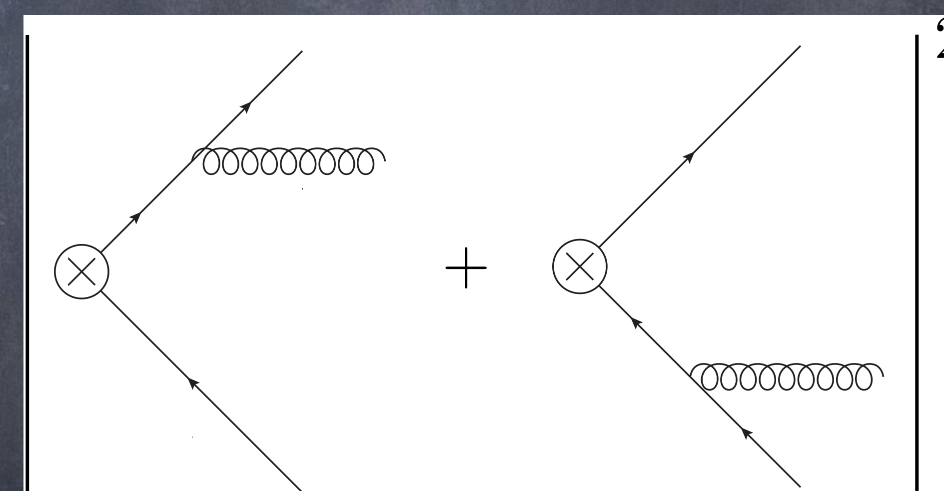


x 2-particle phase space Φ_2

x 3-particle phase space $\mathrm{d}\Phi_3$

both in d-dimensions

Virtual contributions

[C. Lepenik, VM JHEP 01 (2018) 122]

Massive quark form factors have two contributions:

$$V^\mu = \left[1 + C_F \frac{\alpha_s}{\pi} \boxed{A(\hat{m})}\right] \gamma^\mu + C_F \frac{\alpha_s}{\pi} \frac{B(\hat{m})}{2m} (p_1 - p_2)^\mu \,,$$

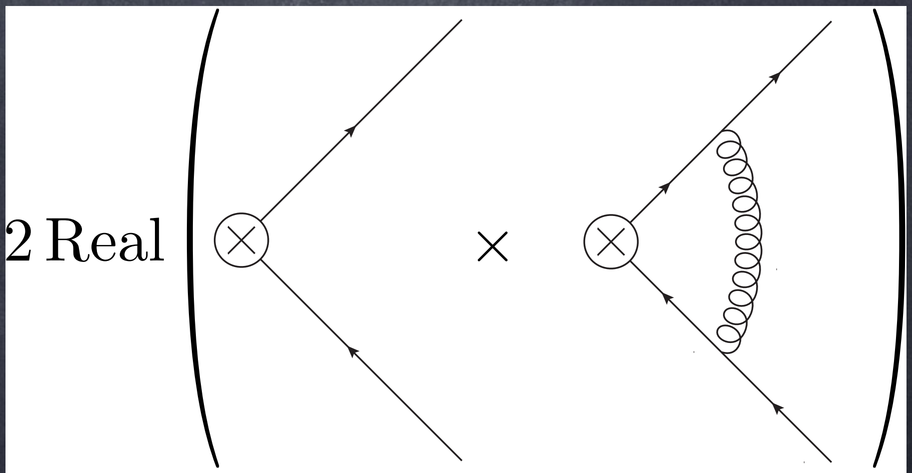
$$A^\mu = \left[1 + C_F \frac{\alpha_s}{\pi} \boxed{C(\hat{m})}\right] \gamma^\mu \gamma_5 + C_F \frac{\alpha_s}{\pi} \frac{D(\hat{m})}{2m} \gamma_5 \, q^\mu \,,$$

Results look simpler in terms of the quark velocity $v = \sqrt{1 - 4\hat{m}^2}$

We also define $L_v \equiv \log\left(\frac{1+v}{2\hat{m}}\right)$

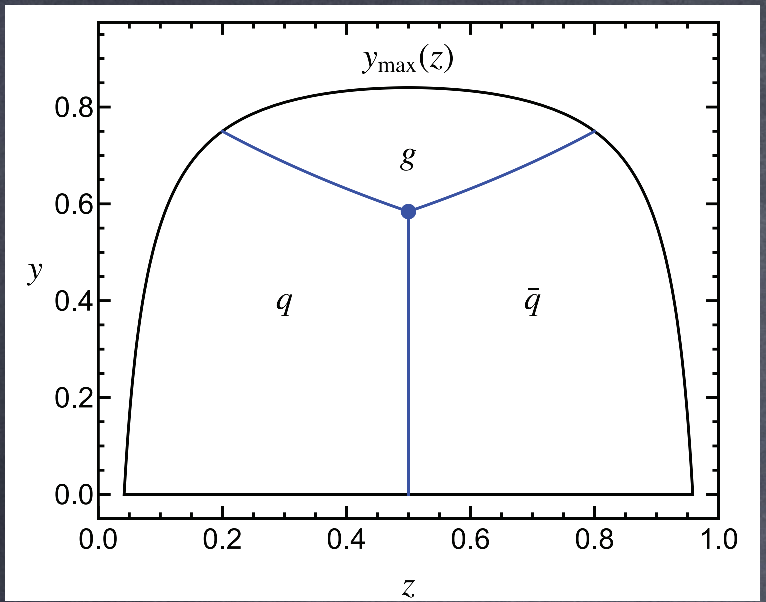
A and C are IR divergent and contain $\log\left(\frac{m}{\mu}\right)$

IR divergence and explicit μ dependence cancels when adding real radiation



Real radiation

[C. Lepenik, VM]



Use symmetric variables such that soft singularities are located at y = 0

$$E_g = \frac{y\,Q}{2} \qquad E_q = \frac{Q}{2}(1 - z\,y)$$

Useful to consider the di-jet expansion of the event-shape measurement function

$$\hat{e}(z,y) = e_{\min} + y\,f_e(z) + \mathcal{O}(y^2) \equiv \bar{e}(y,z) + \mathcal{O}(y^2)$$

will appear again

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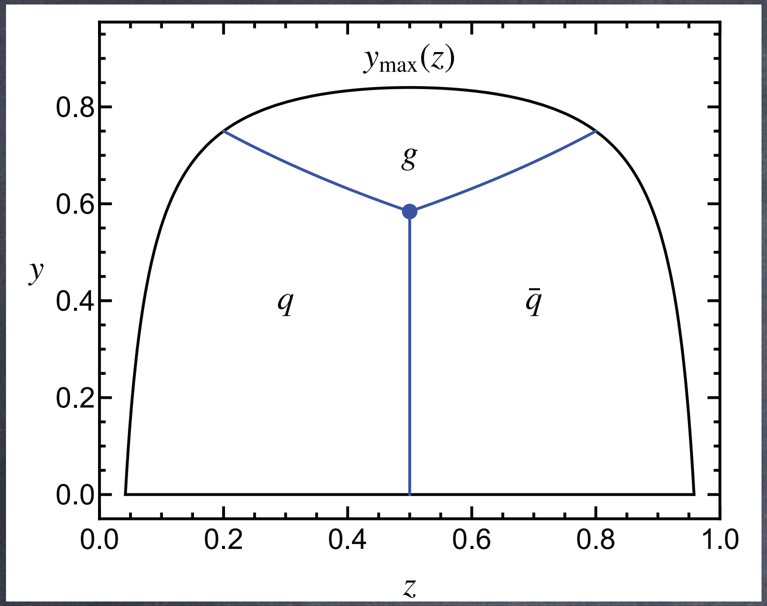

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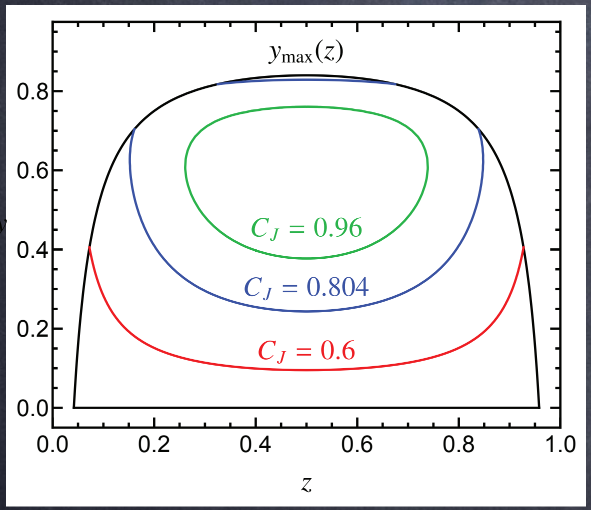

Isobaric event-shape lines have in general non-analytic form. May or may not cut the phase-space boundary. They become the y = 0 boundary at their min value and shrink to a point at their max

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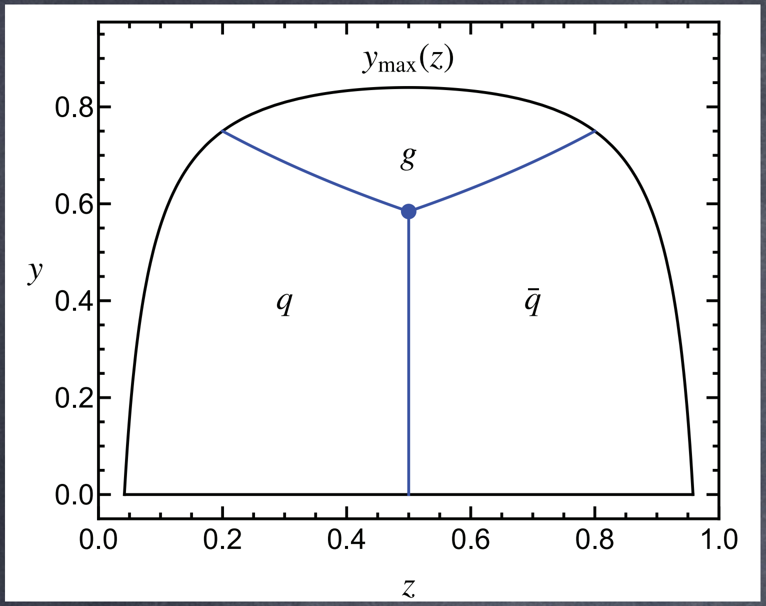

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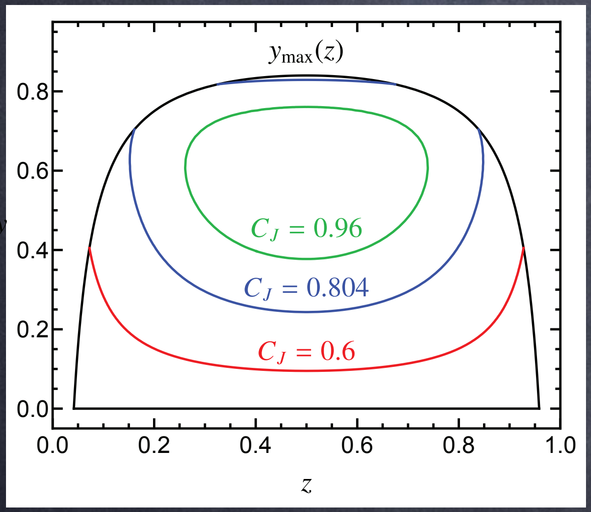

Isobaric event-shape lines have in general non-analytic form. May or may not cut the phase-space boundary. They become the $y = 0$ boundary at their min value and shrink to a point at their max

Using some distribution identities one can extract IR divergences and isolate the singular terms

$$\left[\frac{\log^n(bx)}{bx}\right]_+ = \frac{1}{b}\left\{\frac{\log^{n+1}(b)}{n+1}\,\delta(x) + \sum_{i=0}^{n}\binom{n}{i}\log^{n-i}(b)\left[\frac{\log^n(x)}{x}\right]_+\right\}$$

$$x^{-1+\varepsilon} = \frac{1}{\varepsilon}\delta(x) + \sum_{n=0}\frac{\varepsilon^n}{n!}\,\frac{\log^n(x)}{x}\bigg|_+$$

Analytic results

[C. Lepenik, VM]

With some analytic manipulations we get

$$A_e(\hat{m}) = A_e^{\rm real}(\hat{m}) + R_C^{\rm virt}(\hat{m})\,,$$

$$A_e^{\rm real}(\hat{m}) = -\frac{P(Q,\varepsilon)}{2}\int\!{\rm d}z\biggl\{M_C^1(z,\hat{m}) + M_C^0(z,\hat{m})\biggl[\frac{1}{\varepsilon} + 2\log\Bigl(\frac{\mu}{Q}\Bigr) - \log\biggl(\frac{z(1-z)-\hat{m}^2}{[f_e(z)]^2}\biggr)\biggr]\biggr\},$$

$$B_{\rm plus}(\hat{m}) = \int{\rm d}z\, M_C^0(z,\hat{m})\,,$$

$$\begin{aligned}F^{\rm NS} = \int{\rm d}z\,{\rm d}y\,\biggl\{ M_C^{\rm hard}(y,z)\delta[e-\hat{e}(y,z)] + \frac{M_C^0(z,\hat{m})}{y}\biggl[\delta[e-\hat{e}(y,z)] \\ - \Theta[y-y_{\rm max}(z)]\,\delta[e-\bar{e}(y,z)] - \delta[e-\bar{e}(y,z)]\biggr]\biggr\} \equiv F^{\rm NS}_{\rm hard} + F^{\rm NS}_{\rm soft}\,.\end{aligned}$$

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real-radiation squared matrix element, summed over polarisations

$$\frac{|\sum_{\rm spin}\mathcal{M}_C|^2}{4\sigma_0^C} = \frac{256\pi^2\alpha_s\tilde{\mu}^{2\varepsilon}C_F}{y^2}M_C(y,z,\hat{m},\varepsilon)$$

and split into various pieces

$$M_C(y,z,\hat{m},\varepsilon) = M_C^0(z,\hat{m}) + \varepsilon M_C^1(z,\hat{m}) + yM_C^{\rm hard}(y,z) + \mathcal{O}(\varepsilon^2)$$

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B_{\rm plus}(\hat{m}) &= \int \mathrm{d}z\, M_C^0(z,\hat{m})\,,\\
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IR divergence and explicit μ dependence cancels when adding virtual contribution

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$$\boxed{B_{\rm plus}(\hat{m}) = \left(\frac{3-v^2}{2\,v^2}\right)\left[(1+v^2)L_v - v\right]}$$

integral done analytically

Analytic results

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$$A_e^V(\hat m) = (1+2\hat m^2)\Big\{(1-2\hat m^2)\Big[{\rm Li}_2\Big(-\frac{v(1+v)}{2\hat m^2}\Big) - 3\,{\rm Li}_2\Big(\frac{v(1-v)}{2\hat m^2}\Big) + 2\log^2(\hat m) + \pi^2 - 2\log^2\Big(\frac{1+v}{2}\Big)\Big] + 2v\,[\log(\hat m)-1] - 2I_e(\hat m)\Big\} + (4+v^2-16\hat m^4)L_v\,,$$

$$A_e^A(\hat m) = v^2\Big\{(4+v^2)L_v + 2v\,[\log(\hat m)-1] - 2I_e(\hat m) + (1-2\hat m^2)\times\Big[{\rm Li}_2\Big(-\frac{v(1+v)}{2\hat m^2}\Big) - 3\,{\rm Li}_2\Big(\frac{v(1-v)}{2\hat m^2}\Big) + \pi^2 + 2\log^2(\hat m) - 2\log^2\Big(\frac{1+v}{2}\Big)\Big]\Big\}$$

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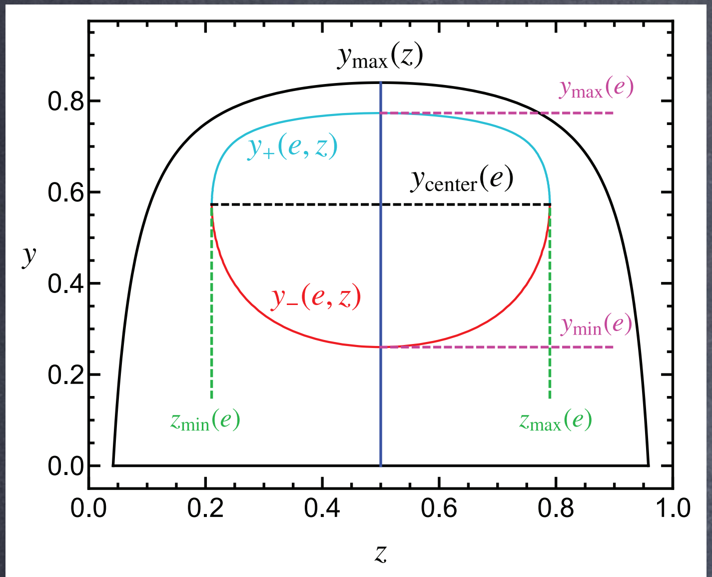
found analytic form in all cases

$$I_e(\hat m) = \frac{1}{2}\int {\rm d}z\,\frac{(1-z)z-\hat m^2}{(1-z)^2z^2}\log[f_e(z)]$$

only event-shape dependent part

Numerical strategy [C. Lepenik, VM]

It is straightforward to find analytic expressions for the event-shape measurement in terms of the two phase-space variables z, y



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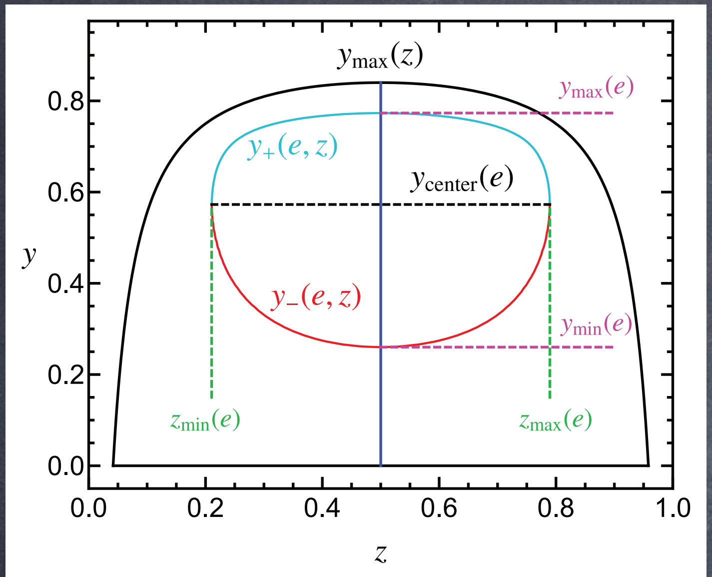

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Use python to determine all boundaries and compute all numerical integrals

$$F_e(e,\hat{m}) = 2\int_{z_{\min}(e)}^{1/2} \mathrm{d}z \sum_{y=y_\pm(e,z)} \frac{M_C(y,z)}{y\left|\frac{\mathrm{d}\hat{e}(y,z)}{\mathrm{d}y}\right|}$$

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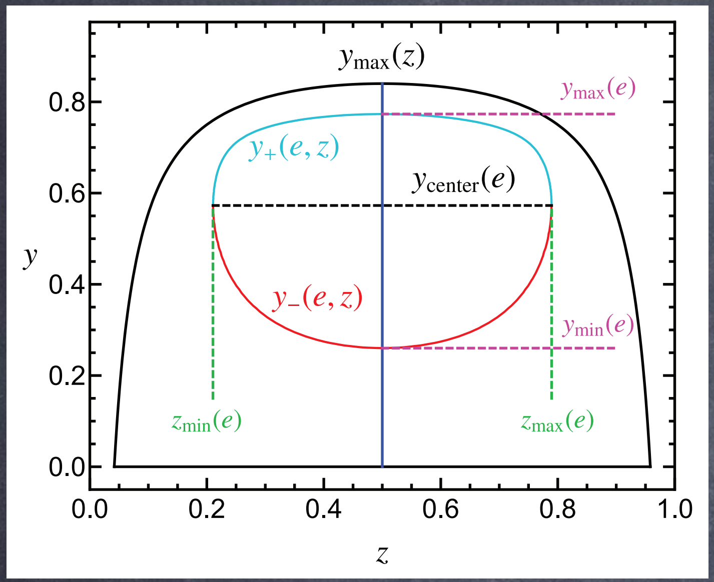

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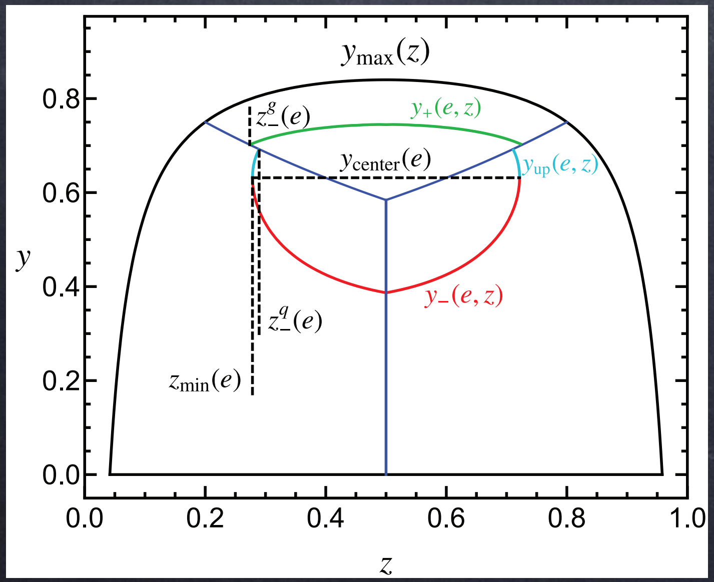
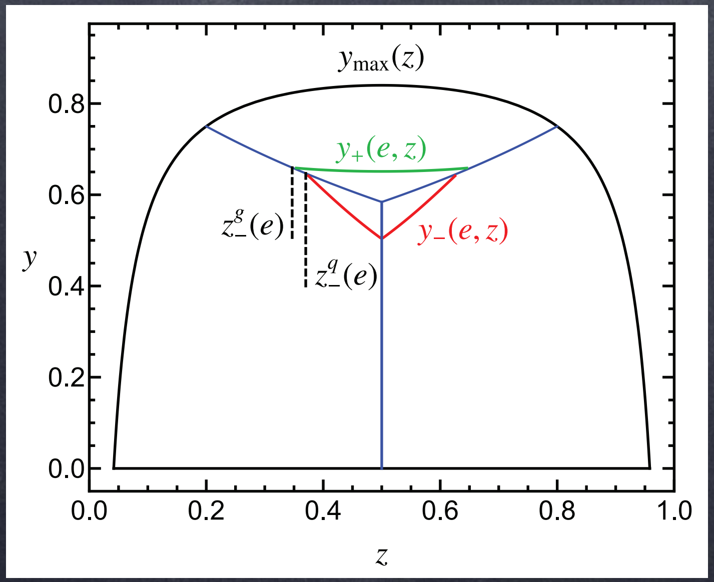
Some event shapes need special treatment

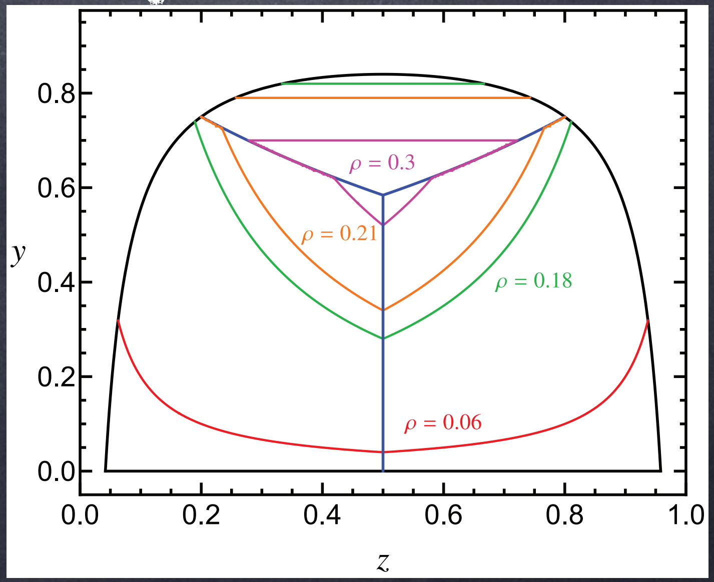
E-scheme broadening



E-scheme thrust



Heavy-jet-mass



Applications

Applications

- Strong coupling determination

  Abbate, Fickinger, Hoang, VM, Stewart [PRD 83 (2011) 074021, 86 (2012) 094002]

  Hoang, Kolodrubetz, VM, Stewart [PRD 91 (2015) 9, 094017-094018]

  Hoang, VM, Schwartz, Stewart [w.i.p.]

- Top quark mass calibration

  Buttenschoen, Dehnadi, Hoang, VM, Preisser, Stewart

  [PRL 117 (2016) 23, 232001]

- Top quark mass determination at N^3LL

  Bachu, Hoang, VM, Pathak, Stewart [w.i.p.]

- Secondary massive quark radiation

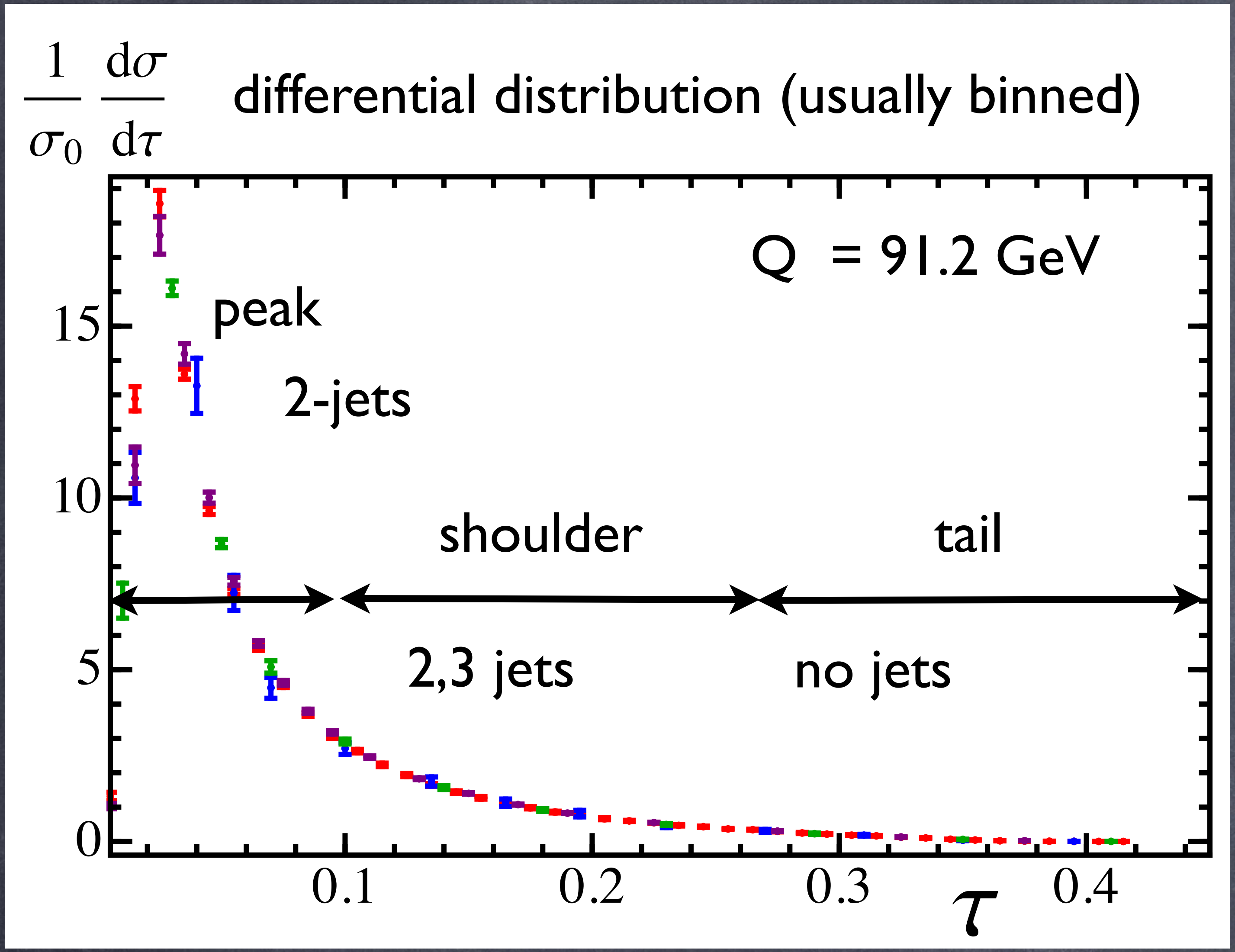
  Pietrulewicz, Gritschacher, Hoang, Jemos, VM

  [PRD 90 (2014) 11, 114001]

Backup slides

Massive schemes

How do they look like?



Q = center of mass energy

Dijet configurations have small values of the event shape

Therefore peak clearly visible at low values (hadronization important)

Moderate non-perturbative effects in shoulder: good for fits

Peak and shoulder need large-log resummation — EFT called for!

Massive schemes

Originally introduced by Salam and Wicke to study hadron mass effects on hadronization [JHEP 05 (2001) 061]

Useful when considering heavy quarks to gain/loose sensitivity

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P-scheme: use $\vec{p} \to \frac{E_p}{|\vec{p}|}\vec{p}$

E-scheme: use $E_p \to |\vec{p}|$

behave as massless $p_E^2 = p_P^2 = 0$

minimal mass sensitivity

same collinear limit

[A. Bris, VM, M. Praisser JHEP 09 (2020)]

+ in E-scheme leading hadronization correction is universal

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[C. Lepenik, VM]

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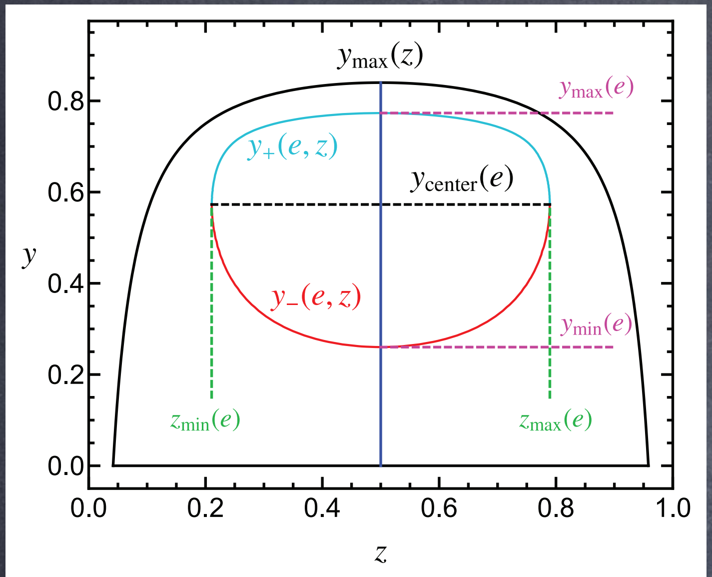

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Use python to determine all boundaries and compute all numerical integrals

One can compute the cumulative

$$\Sigma(e_c) = \frac{1}{\sigma_0}\int_0^{e_c} \mathrm{d}e\, \frac{\mathrm{d}\sigma}{\mathrm{d}e} = R_0(\hat m)\,\Theta(e_c - e_{\rm min}) + C_F\frac{\alpha_s}{\pi}\,\Sigma^1(e_c)$$

with a trick $\qquad \theta(x) + \theta(-x) = 1$

$$\Sigma^1_C(\bar e_c) = R_1^C(\hat m) - 2\int_{z_{\rm min}(e)}^{1/2} \mathrm{d}z\, \mathcal{M}_C[\,y_+(e,z), y_-(e,z), z, \hat m]$$

with

$$\int_{y_1}^{y_2} \mathrm{d}y\, \frac{M_C(y,z)}{y} \equiv \mathcal{M}_C(y_1,y_2,z,\hat m) = M_C^0(z,\hat m)\log\!\left(\frac{y_1}{y_2}\right) + M_C^2(z,\hat m)\,(y_1 - y_2) + \frac{1}{2}M_C^3(z,\hat m)\,(y_1^2 - y_2^2)$$

Virtual contributions

[C. Lepenik, VM JHEP 01 (2018) 122]

Massive quark form factors have two contributions:

$$V^\mu = \left[1 + C_F \frac{\alpha_s}{\pi} A(\hat{m})\right]\gamma^\mu + C_F \frac{\alpha_s}{\pi}\frac{B(\hat{m})}{2m}(p_1 - p_2)^\mu\,,$$

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A and C are IR divergent

Results look simpler in terms of the quark velocity $v = \sqrt{1 - 4\hat{m}^2}$

We also define $L_v \equiv \log\left(\frac{1+v}{2\hat{m}}\right)$

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$$\mathrm{Re}[B(\hat{m})] = \frac{2\,\hat{m}^2}{v}L_v\,,$$

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$$A_{\mathrm{reg}}(\hat{m}) = \frac{3}{2}\,v\,L_v - 1 + \frac{1+v^2}{4v}\left[\pi^2 - 2L_v^2 - 2\,\mathrm{Li}_2\left(\frac{2v}{1+v}\right)\right]$$

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Massive quark form factors have two contributions:

$$V^\mu = \left[1 + C_F \frac{\alpha_s}{\pi} A(\hat{m})\right]\gamma^\mu + C_F \frac{\alpha_s}{\pi}\frac{B(\hat{m})}{2m}(p_1 - p_2)^\mu\,,$$
$$A^\mu = \left[1 + C_F \frac{\alpha_s}{\pi} C(\hat{m})\right]\gamma^\mu\gamma_5 + C_F \frac{\alpha_s}{\pi}\frac{D(\hat{m})}{2m}\gamma_5\, q^\mu\,,$$

A and C are IR divergent

Results look simpler in terms of the quark velocity $v = \sqrt{1-4\hat{m}^2}$

We also define $L_v \equiv \log\left(\dfrac{1+v}{2\hat{m}}\right)$ $\qquad P(Q,\varepsilon) = \dfrac{\Phi_2^{m=0}|_{\varepsilon\to 0}}{\Phi_2^{m=0}}$

In terms of those we get

$$\frac{1}{\sigma_0^C}\frac{\mathrm{d}\sigma^V_{2,\mathrm{virt}}}{\mathrm{d}e} = \frac{\alpha_s}{\pi} C_F R_C^{\mathrm{virt}}(\hat{m})\,\delta(e - e_{\min})\,,$$
$$R_V^{\mathrm{virt}}(\hat{m}) = P(Q,\varepsilon)\, v^{1-2\varepsilon}\Big\{2\,\mathrm{Re}[A(\hat{m})](1+2\hat{m}^2-\varepsilon) - v^2\mathrm{Re}[B(\hat{m})]\Big\}$$
$$= P(Q,\varepsilon)v\left\{(3-v^2)\mathrm{Re}[A] - [1+2\log(v)]\left(\frac{1+v^2}{v}L_v - 1\right) - v^2\,\mathrm{Re}[B] + \mathcal{O}(\varepsilon)\right\}$$
$$R_A^{\mathrm{virt}}(\hat{m}) = 2\,P(Q,\varepsilon)v^{3-2\varepsilon}(1-\varepsilon)\,\mathrm{Re}[C(\hat{m})]$$
$$= 2\,P(Q,\varepsilon)v^3\left\{\mathrm{Re}[C] - [1+2\log(v)]\left(\frac{1+v^2}{v}L_v - \frac{1}{2}\right) + \mathcal{O}(\varepsilon)\right\}.$$

$\mathcal{O}(\alpha_s)$ massive event-shapes at FO

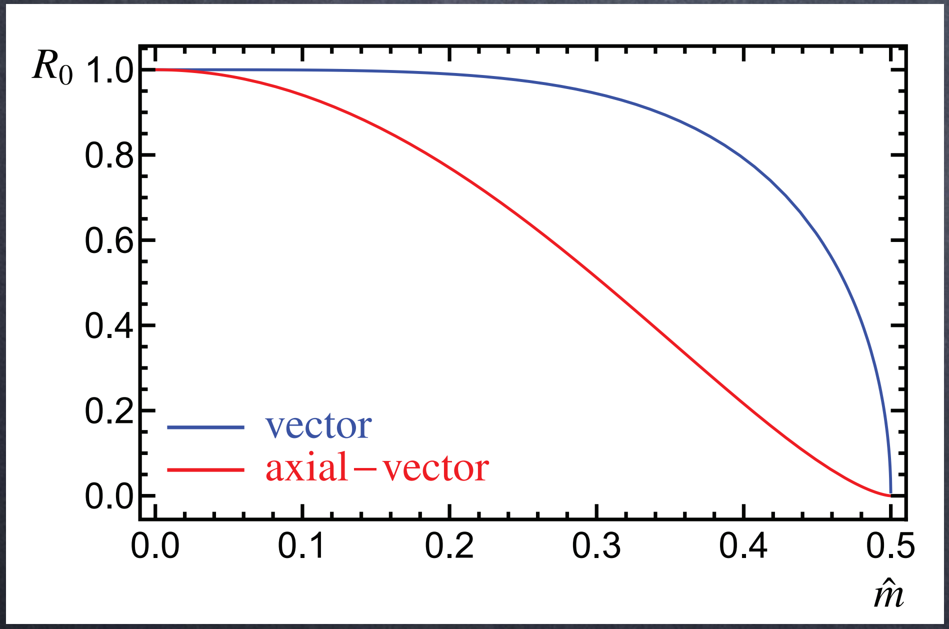
[C. Lepenik, VM JHEP 01 (2018) 122]

General structure of partonic distribution at lowest two orders

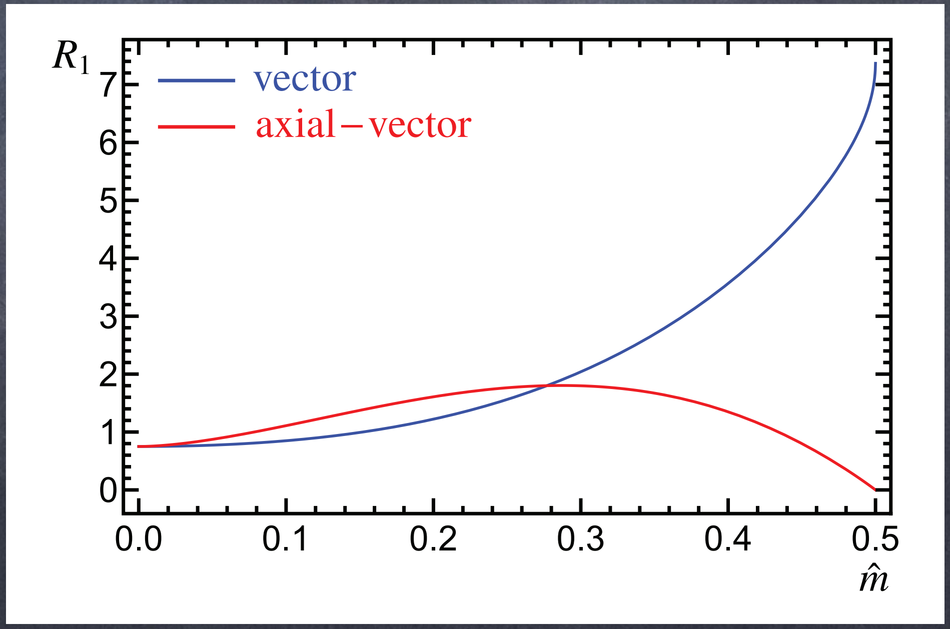
$$\frac{1}{\sigma_0^C}\frac{\mathrm{d}\sigma_C}{\mathrm{d}e} = \boxed{R_C^0(\hat{m})}\,\delta(e-e_{\min}) + C_F\frac{\alpha_s}{\pi}A_e^C(\hat{m})\delta(e-e_{\min}) + C_F\frac{\alpha_s}{\pi}B^C_{\rm plus}(\hat{m})\left[\frac{1}{e-e_{\min}}\right]_+ + C_F\frac{\alpha_s}{\pi}F_e^{\rm NS}(e,\hat{m}) + \mathcal{O}(\alpha_s^2)$$

known analytically

Born-normalized cross section



One-loop correction to total cross section



Fixer-order massive total hadronic cross section

$$R(\hat{m}) = R_0(\hat{m}) + C_F\frac{\alpha_s}{4\pi}R_1(\hat{m}) + \mathcal{O}(\alpha_s^2)$$