EFT for Double Heavy Baryons

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Jaume Tarrús Castellà, JS, Phys. Rev. D **102**, 014012 (2020) [arXiv:2005.00552]; D **102**, 014013 (2020) [arXiv:2005.00551]

Hadrons with two heavy quarks

$$Q = b,\, c \quad , \quad q = u,\, d,\, s$$

- $QQ+$ light quarks and gluons
  - Double Heavy Baryons: QQq
  - Tetraquarks: $QQ\bar{q}\bar{q}$
  - Pentaquarks: $QQqq\bar{q}$
  - Hybrids: $QQqg$
  - ...
- $Q\bar{Q}+$ light quarks and gluons
  - Heavy Quarkonium $Q\bar{Q}$
  - Hybrids: $Q\bar{Q}g$
  - Tetraquarks: $Q\bar{Q}q\bar{q}$
  - Pentaquarks: $Q\bar{Q}qqq$
  - ...

Heavy Quarkonium

$Q\bar{Q}$ bound state , $m_Q >> \Lambda_{QCD}$, $\alpha_s(m_Q) << 1$

- Heavy quarks move slowly $v << 1$
- Non-relativistic system $\rightarrow$ multiscale problem
  - $m_Q >> m_Q v$ (Relative momentum)
  - $m_Q v >> m_Q v^2$ (Binding energy)
  - $m_Q >> \Lambda_{QCD}$
- EFTs are useful (N. Brambilla, A. Pineda, JS and A. Vairo, Rev. Mod. Phys. 77, 1423 (2005))
  - NRQCD: $m_Q \gg m_Q v, m_Q v^2, \Lambda_{QCD}$ (W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986))
  - pNRQCD (weak coupling): $m_Q v \gg m_Q v^2, \Lambda_{QCD}$ (A. Pineda, JS, Nucl.Phys.Proc.Suppl.64:428-432,1998)
  - pNRQCD (strong coupling): $m_Q v, \Lambda_{QCD} \gg m_Q v^2$ (N. Brambilla, A. Pineda, JS, A. Vairo, Nucl.Phys.B566:275,2000)

NRQCD

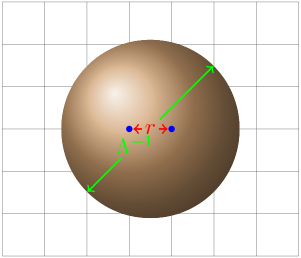
W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986)
G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125

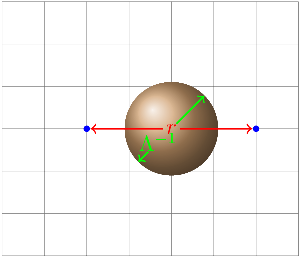
$$m_Q \quad >> \quad m_Q v \quad , \quad m_Q v^2 \quad , \quad \Lambda_{QCD}$$

$$\begin{aligned}\mathcal{L}_\psi \;=\; \psi^\dagger \Bigg\{ & iD_0 + \frac{1}{2m_Q}\mathbf{D}^2 + \frac{1}{8m_Q^3}\mathbf{D}^4 + \frac{c_F}{2m_Q}\boldsymbol{\sigma}.g\mathbf{B} + \\ & + \frac{c_D}{8m_Q^2}\left(\mathbf{D}.g\mathbf{E} - g\mathbf{E}.\mathbf{D}\right) + i\,\frac{c_S}{8m_Q^2}\boldsymbol{\sigma}.\left(\mathbf{D}\times g\mathbf{E} - g\mathbf{E}\times\mathbf{D}\right)\Bigg\}\psi\end{aligned}$$

c_F, c_D and c_S are short distance matching coefficients calculable from QCD in powers of $\alpha_{\rm s}$. They depend on m_Q and μ (factorization scale) but not on the lower energy scales.

How does the hadron look like ?





$m_Q v \sim 1/r \gg m_Q v^2 \gtrsim \Lambda_{QCD}$ $\quad$ $m_Q v \sim 1/r \gtrsim \Lambda_{QCD} \gg m_Q v^2$

weak coupling pNRQCD $\quad$ strong coupling pNRQCD

Figures: Najjar, Bali, 2009

pNRQCD weak coupling regime $\Lambda_{QCD} \lesssim m_Q v^2 \ll m_Q v$

$$\mathcal{L}_{\rm pNRQCD} = \int d^3\mathbf{r}\; {\rm Tr}\Big\{ {\rm S}^\dagger \left(i\partial_0 - h_s(\mathbf{r},\mathbf{p},\mathbf{P_R},\mathbf{S}_1,\mathbf{S}_2,\mu)\right){\rm S} +$$

$$+{\rm O}^\dagger \left(iD_0 - h_o(\mathbf{r},\mathbf{p},\mathbf{P_R},\mathbf{S}_1,\mathbf{S}_2,\mu)\right){\rm O} \Big\}$$

$$+V_A(r,\mu){\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm S} + {\rm S}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm O}\right\} +$$

$$+\frac{V_B(r,\mu)}{2}{\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r}\cdot g\mathbf{E}\,{\rm O} + {\rm O}^\dagger {\rm O}\mathbf{r}\cdot g\mathbf{E}\right\} + \mathcal{O}(\mathbf{r}^2, \frac{1}{m_Q})$$

- $h_{s,o} = \frac{\mathbf{p}^2}{m_Q} + V_{s,o}(r,\mu) + \mathcal{O}(\frac{1}{m_Q})$, quantum mechanical Hamiltonians with scale dependent potentials calculable in pertubation theory in $\alpha_{\rm s}(m_Q v)$ and $1/m_Q$ ($V_s \simeq -4\alpha_{\rm s}/3r$, $V_o \simeq \alpha_{\rm s}/6r$)
- Spin symmetry holds in $h_{s,o}$ up to $\mathcal{O}(\frac{1}{m_Q^2})$
- S=S($\mathbf{r}$, $\mathbf{R}$, t), O=O($\mathbf{r}$, $\mathbf{R}$, t) are the color singlet/octet wave function fields
- $\mathbf{E}$=$\mathbf{E}$($\mathbf{R}$, t) is the chromoelectric field

pNRQCD strong coupling regime $m_Q v^2 \ll \quad \Lambda_{QCD} \lesssim mv$

$$L_{\rm pNRQCD} = \int d^3\mathbf{x}_1 \int d^3\mathbf{x}_2 \; S^\dagger \big(i\partial_0 - h_s(\mathbf{x}_1 - \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2)\big) S,$$

$$h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2) = \frac{\mathbf{p}_1^2}{2m_Q} + \frac{\mathbf{p}_2^2}{2m_Q} + V_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2, \mathbf{S}_1, \mathbf{S}_2),$$

$$V_s = V_s^{(0)} + \frac{V_s^{(1)}}{m_Q} + \frac{V_s^{(2)}}{m_Q^2} + \cdots \, ,$$

All V_ss can be, and most of them have been, calculated on the lattice

- $V_s^{(0)}$ and $V_s^{(1)}$ are central Spin Symmetry holds
- $V_s^{(2)}$ contains spin and velocity dependent terms

pNRQCD strong coupling regime at LO

- Matching to NRQCD in the static limit $\Rightarrow$ $V_s^{(0)}$ is the ground state energy of two static color sources separated at a distance r
- Can be extracted from lattice calculations of the Wilson loop

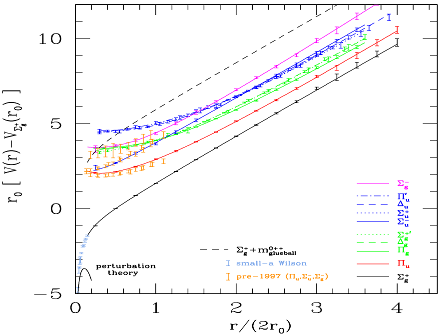


Figure: Meyer, Swanson, 2015

- Well fitted by the Cornell potential

$$V_s^{(0)} = V_{\Sigma_g^+}(r) \approx -\frac{k_g}{r} + \kappa r + E_g^{Q\bar{Q}} \quad , \; k_g = 0.489 \quad , \kappa = 0.187\, GeV^2$$

$QQ+$ light quarks and gluons

- NRQCD holds for the heavy quarks
  - If $m_Q v \gg m_Q v^2$, Λ_{QCD} $\Longrightarrow$ $\sim$ pNRQCD at weak coupling (Brambilla, Vairo, Rösch (05))
    - ★ Since $3 \otimes 3 = 3^* \oplus 6$, one has an antitriplet field and a sextet field (rather than a singlet field and an octet field from $3 \otimes 3^* = 1 \oplus 8$)

$$V_{3^*} \simeq -\frac{2\alpha_s}{3r} \quad , \quad V_6 \simeq \frac{\alpha_s}{3r} \qquad \left(V_s \simeq -\frac{4\alpha_s}{3r} \quad , \quad V_o \simeq \frac{\alpha_s}{6r} \right)$$

    - ★ The interaction of the antitriplet field with the light degrees of freedom is the same as the one of an antiquark $\Longrightarrow$ heavy quark-diquark symmetry (Savage, Wise (90); Manohar, Wise (92))
    - ★ It has been recently used to argue that stable $QQ\bar{q}\bar{q}$ tetraquarks exist in nature (Karliner, Rosner (17); Eichten, Quigg (17); Braaten, He, Mohapatra (20))
  - If $m_Q v$, $\Lambda_{QCD} \gg m_Q v^2$ $\Longrightarrow$ $\sim$ pNRQCD at strong coupling

QQ + light quarks and gluons ($m_Q v\,,\, \Lambda_{QCD} \gg m_Q v^2$)

$$\mathcal{L}_{\rm HEH} = \sum_{\kappa^p} \Psi^\dagger_{\kappa^p} \left[i\partial_t - h_{\kappa^p} \right] \Psi_{\kappa^p}$$

$$h_{\kappa^p} = \frac{\boldsymbol{p}^2}{m_Q} + \frac{\boldsymbol{P}^2}{4m_Q} + V^{(0)}_{\kappa^p}(\boldsymbol{r}) + \frac{1}{m_Q} V^{(1)}_{\kappa^p}(\boldsymbol{r}, \boldsymbol{p}) + \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

- LDF$\equiv$ light quarks + gluons, characterized by their quantum numbers (κ, p . . .)
  - $\kappa \equiv$ total angular momentum, $p \equiv$ parity
  - Quantum numbers not explicitely displayed: baryon number (B), isospin (I), strangeness (S), principal quantum number
- $V^{(0)}_{\kappa^p}$, $V^{(1)}_{\kappa^p}$, . . . must be calculated non-perturbatively
- A truncation of $\mathcal{L}_{\rm HEH}$ needed for practical calculations $\Longrightarrow$ keep a limited number of lower lying κ^p

- $V^{(0)}_{\kappa^p}$ is a $(2\kappa+1)\times(2\kappa+1)\times\mathbb{I}_{2\,Q_1}\times\mathbb{I}_{2\,Q_2}$ matrix, which can be decomposed into irreducible representations of $D_{\infty\,h}$, the symmetry group of a diatomic molecule

$$V^{(0)}_{\kappa^p}(\boldsymbol{r})=\sum_{\Lambda}V^{(0)}_{\kappa^p\Lambda}(r)\mathcal{P}_{\kappa\Lambda}$$

$\mathcal{P}_{\kappa\Lambda}$ projects onto LDF angular momenta $\pm\Lambda$ in the direction joining the two heavy quarks, $\Lambda=\kappa,\ \kappa-1,\ \dots,\ \kappa-[\kappa]$

$$\begin{aligned}
\mathcal{P}_{\frac{1}{2}\frac{1}{2}} &= \mathbb{I}^{\rm lq}_2\\
\mathcal{P}_{\frac{3}{2}\frac{1}{2}} &= \frac{9}{8}\mathbb{I}^{\rm lq}_4-\frac{1}{2}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_{3/2}\right)^2\\
\mathcal{P}_{\frac{3}{2}\frac{3}{2}} &= -\frac{1}{8}\mathbb{I}^{\rm lq}_4+\frac{1}{2}\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_{3/2}\right)^2\\
\mathcal{P}_{10} &= \mathbb{I}^{\rm lq}_3-\left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_1\right)^2\\
\mathcal{P}_{11} &= \left(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_1\right)^2\\
&\dots
\end{aligned}$$

- $V^{(1)}_{\kappa^p} = V^{(1)}_{\kappa^p \mathrm{SI}} + V^{(1)}_{\kappa^p \mathrm{SD}}$
- $V^{(1)}_{\kappa^p \mathrm{SI}}$ does not depend on the spin or orbital angular momentum of the heavy quarks $\Longrightarrow$ admits the same decomposition as $V^{(0)}_{\kappa^p}$
- $V^{(1)}_{\kappa^p \mathrm{SD}}$ depends on the spin and orbital angular momentum of the heavy quarks

$$V^{(1)}_{\kappa^p \mathrm{SD}}(\boldsymbol{r}) = \sum_{\Lambda\Lambda'} \mathcal{P}_{\kappa\Lambda} \left[V^{sa}_{\kappa^p \Lambda\Lambda'}(\boldsymbol{r}) \boldsymbol{S}_{QQ} \cdot (\mathcal{P}_{10} \cdot \boldsymbol{S}_{\kappa}) + V^{sb}_{\kappa^p \Lambda\Lambda'}(\boldsymbol{r}) \boldsymbol{S}_{QQ} \cdot (\mathcal{P}_{11} \cdot \boldsymbol{S}_{\kappa}) \right.$$
$$\left. + V^{l}_{\kappa^p \Lambda\Lambda'}(\boldsymbol{r}) \left(\boldsymbol{L}_{QQ} \cdot \boldsymbol{S}_{\kappa} \right) \right] \mathcal{P}_{\kappa\Lambda'}$$

$$2\boldsymbol{S}_{QQ} = \boldsymbol{\sigma}_{QQ} = \boldsymbol{\sigma}_{Q_1} \mathbb{I}_{2\,Q_2} + \mathbb{I}_{2\,Q_1} \boldsymbol{\sigma}_{Q_2} \quad , \quad \mathcal{P}^{ij}_{10} = \hat{\boldsymbol{r}}^i \hat{\boldsymbol{r}}^j \quad , \quad \mathcal{P}^{ij}_{11} = \delta^{ij} - \hat{\boldsymbol{r}}^i \hat{\boldsymbol{r}}^j$$

Matching to NRQCD

- Build an NRQCD operator with the quantum numbers of Ψ_{κ^p}

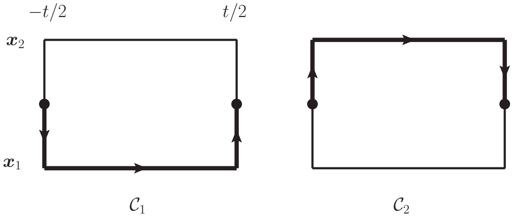
$$\mathcal{O}^{QQ}_{\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R}) = \psi^\top(t,\, \boldsymbol{x}_2)\phi^\top(t,\, \boldsymbol{R},\, \boldsymbol{x}_2)\mathcal{Q}_{QQ\kappa^p}(t,\, \boldsymbol{R})\phi(t,\, \boldsymbol{R},\, \boldsymbol{x}_1)\psi(t,\, \boldsymbol{x}_1)$$

- Impose $\quad \mathcal{O}^{h}_{\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R}) = \sqrt{Z_{h\kappa^p}}\Psi_{h\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R})\,, \quad h = QQ\,.$

$$\langle 0|T\{\mathcal{O}^{h}_{\kappa^p}(t/2)\mathcal{O}^{h\dagger}_{\kappa^p}(-t/2)\}|0\rangle = \sqrt{Z_{h\kappa^p}}\langle 0|T\{\Psi_{h\kappa^p}(t/2)\Psi^\dagger_{h\kappa^p}(-t/2)\}|0\rangle\sqrt{Z^\dagger_{h\kappa^p}}$$

  - Then at $\mathcal{O}(1)$

$$V^{(0)}_{h\kappa^p\Lambda}(\boldsymbol{r}) = \lim_{t\to\infty}\frac{i}{t}\log\left(\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\langle 1\rangle^{h\kappa^p}_{\Box}\right]\right)$$



- At $\mathcal{O}\left(\frac{1}{m_Q}\right)$, for instance,

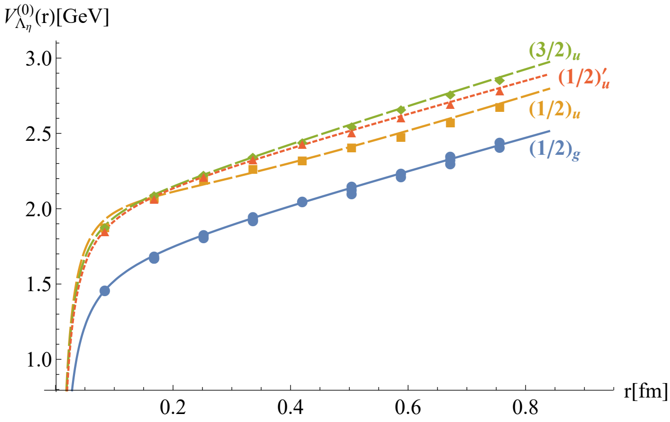
$$V^{sb}_{\kappa^p\Lambda\Lambda'} = -\, c_F \lim_{t\to\infty} \sqrt{\frac{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\right]\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda'}\right]}{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\langle 1\rangle_{\Box}^{h\kappa^p}\right]\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda'}\langle 1\rangle_{\Box}^{h\kappa^p}\right]}}$$

$$\times \frac{\ln\left(\frac{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\langle 1\rangle_{\Box}^{h\kappa^p}\right]\mathrm{Tr}[\mathcal{P}_{\kappa\Lambda'}]}{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda'}\langle 1\rangle_{\Box}^{h\kappa^p}\right]\mathrm{Tr}[\mathcal{P}_{\kappa\Lambda}]}\right)}{2t\sinh\left(\ln\sqrt{\frac{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\langle 1\rangle_{\Box}^{h\kappa^p}\right]\mathrm{Tr}[\mathcal{P}_{\kappa\Lambda'}]}{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda'}\langle 1\rangle_{\Box}^{h\kappa^p}\right]\mathrm{Tr}[\mathcal{P}_{\kappa\Lambda}]}}\right)}$$

$$\times \int_{-t/2}^{t/2} dt' \frac{\mathrm{Tr}\left[(\boldsymbol{S}_\kappa\cdot\mathcal{P}_{11})\cdot\left(\mathcal{P}_{\kappa\Lambda}\langle g\boldsymbol{B}(t',\boldsymbol{x}_1)\rangle_{\Box}^{h\kappa^p}\mathcal{P}_{\kappa\Lambda'}\right)\right]}{\mathrm{Tr}\left[(\boldsymbol{S}_\kappa\cdot\mathcal{P}_{11}^{\mathrm{c.r.}})\cdot(\mathcal{P}_{\kappa\Lambda}\boldsymbol{S}_\kappa\mathcal{P}_{\kappa\Lambda'})\right]}$$

QQq ($m_Q v$, $\Lambda_{QCD} \gg m_Q v^2$)

- We apply the general results to the case $B = 1/3$, $I = 1/2$, $S = 0$
- There is available lattice data (Najjar, Bali (09)): $N_f = 2$, $a \simeq 0.084$ fm, $L \simeq 1.3$ fm, $m_\pi \simeq 783$ MeV



$O(3)$	$D_{\infty h}$
$(1/2)^+$	$(1/2)_g$
$(3/2)^-$	$(1/2)_u$, $(3/2)_u$
$(1/2)^-$	$(1/2)'_u$

$$
\begin{aligned}
\mathcal{L}_{QQq} =& \Psi^\dagger_{(1/2)^+}\left[i\partial_t - h_{(1/2)^+}\right]\Psi_{(1/2)^+} + \Psi^\dagger_{(3/2)^-}\left[i\partial_t - h_{(3/2)^-}\right]\Psi_{(3/2)^-} \\
&+ \Psi^\dagger_{(1/2)^-}\left[i\partial_t - h_{(1/2)^-}\right]\Psi_{(1/2)^-}
\end{aligned}
$$

$$
h_{\kappa^p} = \frac{\boldsymbol{p}^2}{m_Q} + \frac{\boldsymbol{P}^2}{4m_Q} + V^{(0)}_{\kappa^p}(\boldsymbol{r}) + \frac{1}{m_Q} V^{(1)}_{\kappa^p}(\boldsymbol{r},\, \boldsymbol{p})
$$

- At $\mathcal{O}(1)$

$$
\begin{aligned}
\mathcal{L}^{\rm LO}_{QQq} \simeq& \Psi^\dagger_{(1/2)^+}\left(i\partial_t - \frac{\boldsymbol{p}^2}{m_Q} + V^{(0)}_{(1/2)^+}(r)\right)\Psi^\dagger_{(1/2)^+} + \Psi^\dagger_{(1/2)^-}\left(i\partial_t - \frac{\boldsymbol{p}^2}{m_Q} + V^{(0)}_{(1/2)^-}(r)\right) \\
&+ \Psi^\dagger_{(3/2)^-}\left(i\partial_t - \frac{\boldsymbol{p}^2}{m_Q} + V^{(0)}_{(3/2)^-(1/2)}(r)\mathcal{P}_{\frac{3}{2}\frac{1}{2}} + V^{(0)}_{(3/2)^-(3/2)}(r)\mathcal{P}_{\frac{3}{2}\frac{3}{2}}\right)\Psi^\dagger_{(3/2)^-}
\end{aligned}
$$

$$V_{(1/2)^+} = E_{(1/2)_g} = -\frac{2}{3}\frac{\alpha_s(\nu_{\rm lat})}{r} + \frac{c_2 r + c_1}{c_3 r + 1} + \sigma r$$

$$V_{(3/2)^-(1/2)} = E_{(1/2)_u} = -\frac{2}{3}\frac{\alpha_s(\nu_{\rm lat})}{r} + \frac{b_3 r^2 + b_2 r + b_1}{b_5 r^2 + b_4 r + 1} + \sigma r$$

$$V_{(3/2)^-(3/2)} = E_{(3/2)_u} = -\frac{2}{3}\frac{\alpha_s(\nu_{\rm lat})}{r} + \frac{b_7 r^2 + b_6 r + b_1}{b_9 r^2 + b_8 r + 1} + \sigma r$$

$$V_{(1/2)^-} = E_{(1/2)'_u} - \frac{2}{3}\frac{\alpha_s(\nu_{\rm lat})}{r} + \frac{c_5 r + c_4}{c_6 r + 1} + \sigma r$$

- $b_1 = c_1 + E^{\rm latt}(1a)_{(1/2)_u} - E^{\rm latt}(1a)_{(1/2)_g}$, $c_4 = c_1 + E^{\rm latt}(1a)_{(1/2)'_u} - E^{\rm latt}(1a)_{(1/2)_g}$
- $c_1 \simeq 1.95$ GeV from the fit to the one-loop potential of the short distance points
- $\sigma = 0.21$ GeV^2 fixed (standard value for $Q\bar{Q}$ systems, assumed equal for QQ ones)
- Remaining parameters obtained from fits to all data
- c_1, b_1, c_4 subtracted to get the binding energies (E_b)

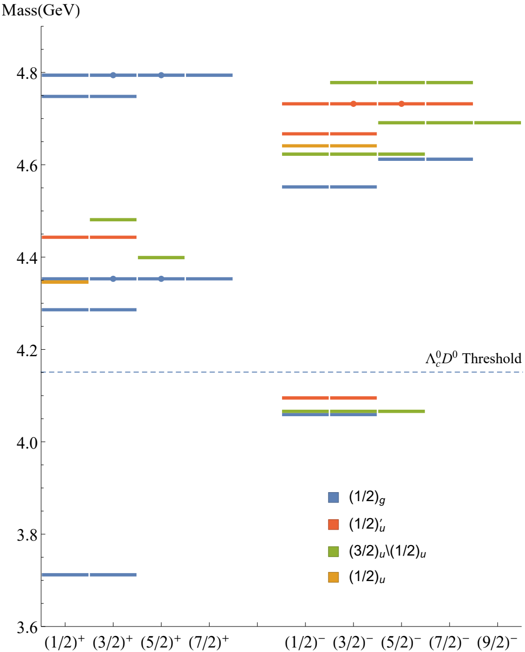
$$M^{(0)}_{(1/2)_g} = 2m_Q + E_b + \overline{\Lambda}_{(1/2)^+}$$
$$M^{(0)}_{(3/2)_u \backslash (1/2)_u} = 2m_Q + E_b + \overline{\Lambda}_{(1/2)^+} + E^{\rm latt}(1a)_{(1/2)_u} - E^{\rm latt}(1a)_{(1/2)_g}$$
$$M^{(0)}_{(1/2)'_u} = 2m_Q + E_b + \overline{\Lambda}_{(1/2)^+} + E^{\rm latt}(1a)_{(1/2)'_u} - E^{\rm latt}(1a)_{(1/2)_g}$$

- We take (Bazavov et al. (18))

$$m_c = 1.392(11)\ \mathrm{GeV}\,,\ m_b = 4.749(18)\ \mathrm{GeV}\,,\ \overline{\Lambda}_{(1/2)^+} = 0.555(31)\ \mathrm{GeV}$$

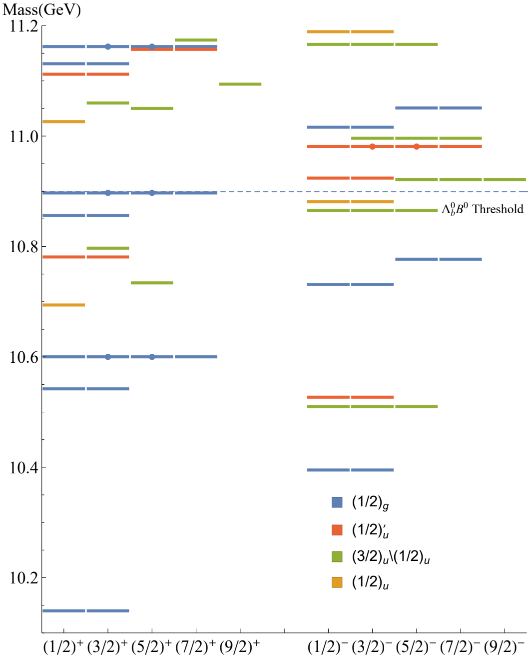
- If we tune c_1 to reproduce the Ξ_{cc}^{++} mass $\Longrightarrow$ our spectra move down by 91 MeV
- Uncertainties:
  - ⋆ Input parameters: 53 MeV for *ccq*, 67 MeV for *bbq* (global shift)
  - ⋆ Lattice data + modeling of the potentials $\lesssim$ 10 MeV
  - ⋆ $1/m_Q$ corrections: $\sim$ 100 MeV for *ccq*, $\sim$ 30 MeV for *bbq*

Ξ_{cc}



Mass(GeV)
4.8
4.6
4.4
4.2
4.0
3.8
3.6
$\Lambda_c^0 D^0$ Threshold
$(1/2)_g$
$(1/2)'_u$
$(3/2)_u\backslash(1/2)_u$
$(1/2)_u$
$(1/2)^+$ $(3/2)^+$ $(5/2)^+$ $(7/2)^+$
$(1/2)^-$ $(3/2)^-$ $(5/2)^-$ $(7/2)^-$ $(9/2)^-$

Ξ_{bb}



Mass(GeV)
11.2
11.0
10.8
10.6
10.4
10.2
$\Lambda_b^0 B^0$ Threshold
$(1/2)_g$
$(1/2)'_u$
$(3/2)_u \backslash (1/2)_u$
$(1/2)_u$
$(1/2)^+$ $(3/2)^+$ $(5/2)^+$ $(7/2)^+$ $(9/2)^+$
$(1/2)^-$ $(3/2)^-$ $(5/2)^-$ $(7/2)^-$ $(9/2)^-$

- At $\mathcal{O}(1/m_Q)$
  - There is no lattice data available
  - We can derive general properties of the splittings for $\kappa = 1/2$ (only three unknown potentials)

$$V^{(1)}_{(1/2)^{\pm}\mathrm{SD}}(\boldsymbol{r}) = V^{s1}_{(1/2)^{\pm}}(r)\boldsymbol{S}_{QQ}\cdot\boldsymbol{S}_{1/2} + V^{s2}_{(1/2)^{\pm}}(r)\boldsymbol{S}_{QQ}\cdot\left(\boldsymbol{\mathcal{T}}_2\cdot\boldsymbol{S}_{1/2}\right) + V^{l}_{(1/2)^{\pm}}(r)\left(\boldsymbol{L}_{QQ}\cdot\boldsymbol{S}_{1/2}\right)$$

$$(\boldsymbol{\mathcal{T}}_2)^{ij} = \hat{\boldsymbol{r}}^i\hat{\boldsymbol{r}}^j - \frac{1}{3}\delta^{ij}$$

  - Let us write,

$$M_{njl\ell} = M^{(0)}_{nl} + M^{(1)}_{njl\ell} + \ldots$$

    - ★ $n =$ principal quantum number
    - ★ $j =$ total angular momentum
    - ★ $l =$ orbital angular momentum
    - ★ $\ell =$ orbital angular momentum $+$ LDF angular momentum
    - ★ Pauli principle $\Longrightarrow$ $s_{QQ} = 1$ for l even, $s_{QQ} = 0$ for l odd

- For $l=0$,

$$M^{(1)}_{nj0\frac{1}{2}}=\frac{1}{2}\left(j(j+1)-\frac{11}{4}\right)\frac{\langle V^{s1}_{(1/2)\pm}\rangle_{n0}}{m_Q}\quad\Longrightarrow\quad 2M_{n\frac{3}{2}0\frac{1}{2}}+M_{n\frac{1}{2}0\frac{1}{2}}=3M^{(0)}_{n0}$$

- For $l=1$,

$$M^{(1)}_{nj1j}=\frac{1}{2}\left(j(j+1)-\frac{11}{4}\right)\frac{\langle V^{l}_{(1/2)\pm}\rangle_{n1}}{m_Q}\quad\Longrightarrow\quad 2M_{n\frac{3}{2}1\frac{3}{2}}+M_{n\frac{1}{2}1\frac{1}{2}}=3M^{(0)}_{n1}$$

- For $l=2$,

$$M^{(1)}_{n\frac{1}{2}2\frac{3}{2}}=\frac{1}{2}\frac{\langle V^{s1}_{(1/2)\pm}\rangle_{n2}}{m_Q}-\frac{1}{3}\frac{\langle V^{s2}_{(1/2)\pm}\rangle_{n2}}{m_Q}-\frac{3}{2}\frac{\langle V^{l}_{(1/2)\pm}\rangle_{n2}}{m_Q}$$

$$M^{(1)}_{n\frac{3}{2}2\frac{3}{2}}=\frac{1}{5}\frac{\langle V^{s1}_{(1/2)\pm}\rangle_{n2}}{m_Q}-\frac{2}{15}\frac{\langle V^{s2}_{(1/2)\pm}\rangle_{n2}}{m_Q}-\frac{3}{2}\frac{\langle V^{l}_{(1/2)\pm}\rangle_{n2}}{m_Q}$$

$$M^{(1)}_{n\frac{5}{2}2\frac{3}{2}}=-\frac{3}{10}\frac{\langle V^{s1}_{(1/2)\pm}\rangle_{n2}}{m_Q}+\frac{1}{5}\frac{\langle V^{s2}_{(1/2)\pm}\rangle_{n2}}{m_Q}-\frac{3}{2}\frac{\langle V^{l}_{(1/2)\pm}\rangle_{n2}}{m_Q}$$

$$\Longrightarrow\quad M_{n\frac{3}{2}2\frac{3}{2}}=\frac{1}{8}\left(5M_{n\frac{1}{2}2\frac{3}{2}}+3M_{n\frac{5}{2}2\frac{3}{2}}\right)$$

$$M^{(1)}_{n\frac{3}{2}2\frac{5}{2}} = -\frac{7}{10}\frac{\langle V^{s1}_{(1/2)\pm}\rangle_{n2}}{m_Q} + \frac{2}{15}\frac{\langle V^{s2}_{(1/2)\pm}\rangle_{n2}}{m_Q} + \frac{\langle V^{l}_{(1/2)\pm}\rangle_{n2}}{m_Q}$$

$$M^{(1)}_{n\frac{5}{2}2\frac{5}{2}} = -\frac{1}{5}\frac{\langle V^{s1}_{(1/2)\pm}\rangle_{n2}}{m_Q} + \frac{4}{105}\frac{\langle V^{s2}_{(1/2)\pm}\rangle_{n2}}{m_Q} + \frac{\langle V^{l}_{(1/2)\pm}\rangle_{n2}}{m_Q}$$

$$M^{(1)}_{n\frac{7}{2}2\frac{5}{2}} = \frac{1}{2}\frac{\langle V^{s1}_{(1/2)\pm}\rangle_{n2}}{m_Q} - \frac{2}{21}\frac{\langle V^{s2}_{(1/2)\pm}\rangle_{n2}}{m_Q} + \frac{\langle V^{l}_{(1/2)\pm}\rangle_{n2}}{m_Q}$$

$$\Longrightarrow \quad M_{n\frac{5}{2}2\frac{5}{2}} = \frac{1}{12}\left(7M_{n\frac{3}{2}2\frac{5}{2}} + 5M_{n\frac{7}{2}2\frac{5}{2}}\right)$$

- If we take $M_{n\frac{1}{2}1\frac{1}{2}} = M_{\Xi^{++}_{cc}} = 3621.2 \pm 0.7$ MeV,

$$\Longrightarrow \quad M_{n\frac{3}{2}1\frac{1}{2}} = M_{\Xi^{*++}_{cc}} = 3757(68)\ \text{MeV}$$

Comparison with lattice QCD

- Ξ_{cc}
  - Ground state doublet

Ref.	δ_{hf} [MeV]	spin avg.
Briceno *et al.* (2012)	53(94)	3630(50)
Namekawa *et al.* (2013)	101(36)	3672(20)
Brown *et al.* (2014)	82.8(9.2)	3665(36)
Alexandrou *et al.* (2014)	84(58)	3624(33)
Bali *et al.* (2015)	85(9)	3666(13)
Padmanath *et al.* (2015)	94(12)	3700(6)
Alexandrou *et al.* (2017)	76(41)	3657(25)
Our values	136(44)	3712(63)

  - First excitation ($(1/2, 3/2)^-$ doublet, P-wave excitation): agreement with spin average of Padmanath *et al.* (2015) and Bali *et al.* (2015)
  - Higher excitations below threshold ($(1/2, 3/2, 5/2)^-$ triplet, the ground state of $(3/2)_u - (1/2)_u$ and $(1/2, 3/2)^-$ doublet, the ground state of $(1/2)'_u$): qualitative agreement with Padmanath *et al.* (2015)

- Spin splitting formula for $l = 2$: agreement with Padmanath *et al.* (2015) within 2 MeV

- Ξ_{bb}
  - Only lattice NRQCD results for the ground state doublet available
  - Agreement with the spin averages of Lewis *et al.* (2008) and Brown *et al.* (2014), and compatible with Mohanta *et al.* (2019)

Conclusion

- We have worked out an EFT framework for hadrons with two heavy quarks (QQ or $Q\bar{Q}$) and LDF with arbitrary quantum numbers at NLO in the $1/m_Q$ expansion when $m_Q v\,,\, \Lambda_{QCD} \gg m_Q v^2$
- Non-perturbative potentials are needed as inputs
- We have applied the formalism to QQq
  - Ξ_{cc}
    - Overall compatibility with existing lattice QCD results
    - Excellent agreement for the $l = 2$ hyperfine splitting formula
  - Ξ_{bb}
    - Compatibility with existing lattice NRQCD results for the ground state
    - Complete spin average spectrum below threshold based entirely on QCD presented for the first time