Molecular nature of the P_c states

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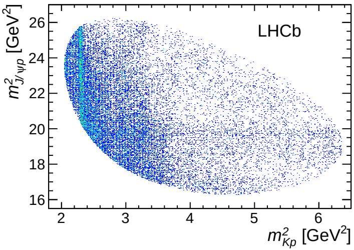
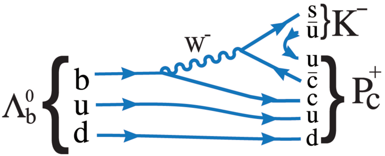
Valenica, Dec. 15-17 2020

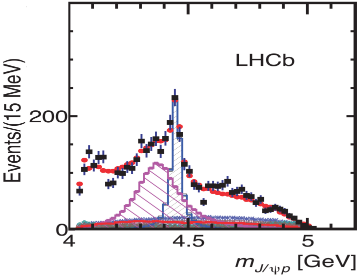
Charmonium-pentaquark states (I)

Observation of exotic structures (P_c) in $\Lambda_b^0 \to J/\psi p K^-$

LHCb, PRL 115, 072001 (2015)







$P_c(4380)^+$: $M = 4380 \pm 8 \pm 29$ MeV

$\Gamma = 205 \pm 18 \pm 86$ MeV

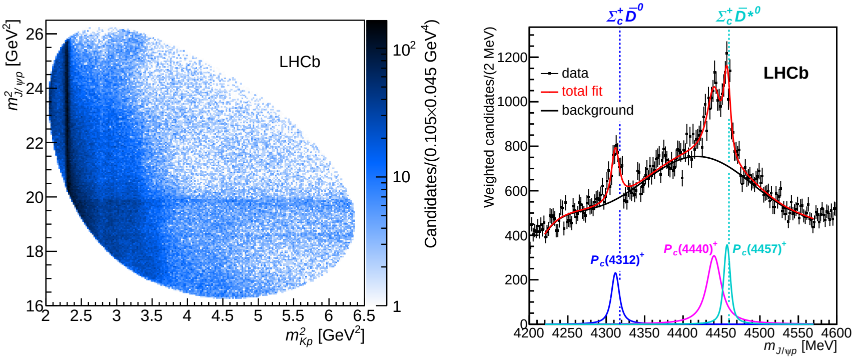
$P_c(4450)^+$: $M = 4449.8 \pm 1.7 \pm 2.5$ MeV

$\Gamma = 39 \pm 5 \pm 19$ MeV

Preferred Parity: Opposite

Charmonium-pentaquark states (II)

LHCb, PRL 122, 222001 (2019)



State	M [MeV]	Γ [MeV]	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$	$0.53 \pm 0.16^{+0.15}_{-0.13}$

Charmonium-pentaquark (theoretical)

- Compact pentaquark — Cheng et al., PRD100(2019)054002

  $P_c(4312)$, $P_c(4440)$, $P_c(4457)$: $J^P = 3/2^-, 1/2^-, 3/2^-$

- Compact diquark model — Ali et al., JHEP1910(2019)256

$3/2^-$	4240 ± 29
$3/2^+$	4440 ± 35
$5/2^+$	4457 ± 35

- $P_c(4312)$: virtual state — Fernández-Ramírez et al., PRL123(2019)092001
- K-matrix: $J/\psi p - \Sigma_c \bar{D} - \Sigma_c \bar{D}^*$ — Kuang et al., EPJC80(2020)433

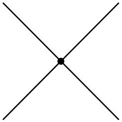
  $\hookrightarrow P_c(4312)$: $\Sigma_c \bar{D}$, $P_c(4457)$: ? cusp effect

- Molecule (HQSS) — Liu et al., PRL122,242001 (2019)

	Molecule	J^P	M (MeV)		Molecule	J^P	M (MeV)
A	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	4311.8 – 4313.0	B	$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	4306.3 – 4307.7
A	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	4376.1 – 4377.0	B	$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	4370.5 – 4371.7
A	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4440.3*	B	$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	4457.3*
A	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4457.3*	B	$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	4440.3*
A	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	4500.2 – 4501.0	B	$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	4523.2 – 4523.6
A	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	4510.6 – 4510.8	B	$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	4516.5 – 4516.6
A	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4523.3 – 4523.6	B	$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	4500.2 – 4501.0

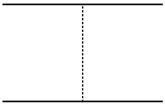
- quantum numbers? line shape? the existence of $P_c(4380)$?

Effective Lagraingian $\Sigma_c^{(*)}\bar{D}^{(*)}$, $J/\psi p$, $\eta_c p$, $\Lambda_c \bar{D}^{(*)}$



- Contact Lagrangian

$$\begin{aligned}
\mathcal{L} &= -C_a \vec{S}_c^{\dagger} \cdot \vec{S}_c \mathrm{Tr}[\bar{H}_c^{\dagger} \bar{H}_c] \\
&\quad -C_b i\epsilon_{ijk} (S_c^{\dagger})_j (S_c)_k \mathrm{Tr}[\bar{H}_c^{\dagger} \sigma_i \bar{H}_c]. \\
&\quad +C_c \Big(S_{ab}^{i\dagger} T_{ca} \langle \bar{H}_c^{\dagger} \sigma^i \bar{H}_b \rangle - T_{ca}^{\dagger} S_{ab}^{i} \langle \bar{H}_b^{\dagger} \sigma^i \bar{H}_c \rangle \Big) \\
&\quad +C_d T_{ab}^{\dagger} T_{ba} \langle \bar{H}_c^{\dagger} \bar{H}_c \rangle
\end{aligned}$$



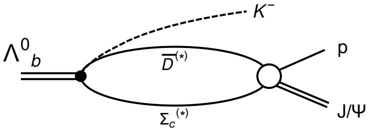
- One-pion exchange (OPE)

$$\begin{aligned}
\mathcal{L}_{DD\pi} &= \frac{g}{4} \langle \sigma \cdot u_{ab} \bar{H}_b \bar{H}_a^{\dagger} \rangle, \\
\mathcal{L}_{\Sigma_c \Sigma_c \pi} &= i \frac{3}{2} g_1 \epsilon_{ijk} \langle \bar{S}_i u_j S_k \rangle. \\
\mathcal{L}_{\Sigma_c \Lambda_c \pi} &= -\frac{1}{\sqrt{2}} g_3 (S_{ab}^{i\dagger} u_{bc}^{i} T_{ca} + T_{ab}^{\dagger} u_{bc}^{i} S_{ca}^{i}),
\end{aligned}$$

☞ Nonrelativistic superfield for the heavy-quark spin doublets

$$\begin{aligned}
\vec{S}_c &= \frac{1}{\sqrt{3}} \vec{\sigma} \Sigma_c + \vec{\Sigma}_c^{*}, \\
\bar{H}_c &= \frac{1}{2} \Big(-\bar{D} + \vec{\sigma} \cdot \vec{\bar{D}}^{*} \Big).
\end{aligned}$$

$\Lambda_b^0 \to K^- J/\psi p$



- ☞ $m_{J/\psi p} \sim 4440$ MeV
  $\hookrightarrow |\mathbf{p}| \sim 810$ MeV
  $\hookrightarrow J/\psi p(S),\ J/\psi p(D)$
- ☞ Effective Lagrangian for $\bar{D}^{(*)}\Sigma_c^{(*)} \to J/\psi p\ (\eta_c p)$: $J = -\eta_c + \sigma \cdot \psi$

$$\mathcal{L} = \frac{g_S}{\sqrt{3}} N^\dagger \sigma^i \bar{H} J^\dagger S^i - \sqrt{3} g_D N^\dagger \sigma^i \bar{H} (\partial^i \partial^j - \frac{1}{3}\delta^{ij}\partial^2) J^\dagger S^j,$$

- ☞

$$\text{channels} \quad \begin{cases} \underline{\Sigma_c^{(*)}\bar{D}^{(*)}(S/D)}, \Lambda_c \bar{D}^{(*)}(S/D) & \to \ \alpha, \beta, \gamma \\ J/\psi p(S/D), \eta_c p(S/D) & \to \ i, j, k \end{cases}$$

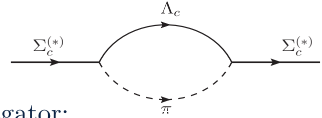
- ☞ Weak production: S-wave $\Sigma_c^{(*)}\bar{D}^{(*)}$.

Lippman-Schwinger equaitons

$$U^J_\alpha(E,p) = P^J_\alpha(E,p) - \sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} V^J_{\alpha\beta}(E,p,q) G_\beta(E,q) U^J_\beta(q),$$

$$U^J_i(E,p) = \sum_\beta \int \frac{d\mathbf{q}^3}{(2\pi)^3} \mathcal{V}_{\beta i} G_\beta(E,q) U^J_\beta(q).$$

- $\Gamma(\Sigma_c^* \to \Lambda_c \pi) = 15.0$ MeV, $\Gamma(\Sigma_c \to \Lambda_c \pi) = 1.86$ MeV,
  $\hookrightarrow \sim \Gamma(P_c)$
- The self-energy function $\tilde{\Sigma}_R^{(*)}(s) \sim ig^2 \frac{p^3}{\sqrt{s}}$



- Two-body propagator:

$$G_\beta(E,\mathbf{q}) = \frac{m_{\Sigma_c^{(*)}} m_{D^{(*)}}}{E_{\Sigma_c^{(*)}}(\mathbf{q}) E_{D^{(*)}}(\mathbf{q})} \frac{1}{E_{\Sigma_c^{(*)}}(\mathbf{q}) + E_{D^{(*)}}(\mathbf{q}) - E - \frac{\tilde{\Sigma}_R^{(*)}(s)}{2E_{\Sigma_c^{(*)}}(\mathbf{q})}},$$

$s = \left(E - E_{D^{(*)}}(\mathbf{q})\right)^2 - \mathbf{q}^2$ is the off-shellness of $\Sigma_c^{(*)}$.

Fit Schemes

☞ The effective potentials

$$V^J_{\alpha\beta} = V^J_{\mathrm{CT},\alpha\beta} + V^J_{\mathrm{OPE},\alpha\beta} + \mathcal{G}^J_{\alpha\beta},$$

The effective contributions from the $J/\psi p$ and $\eta_c p$ bubble loop ($k \sim 0.9$ GeV)

$$\mathcal{G}^J_{\alpha\beta} = \sum_i$$

$$= R_{\mathcal{G}} - \sum_i i \frac{1}{2\pi E} m_{\psi(\eta_c)} m_p g^{(\prime)J}_{\alpha i} g^{(\prime)J}_{\beta i} k^{2l_i+1}.$$

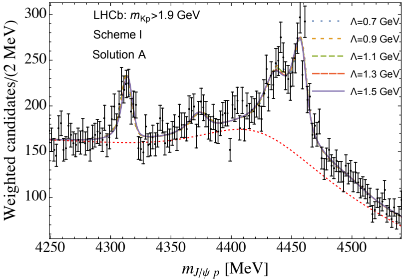
☞ Fit schemes:

- Scheme I: pure contact potential w/o $\Lambda_c \bar{D}^{(*)}$
- Scheme II: Scheme I + elastic OPE w/o $\Lambda_c \bar{D}^{(*)}$ (+ CT for $\Lambda_c \bar{D}^{(*)}$)
- Scheme III: Scheme II + S-D counter term w/o $\Lambda_c \bar{D}^{(*)}$

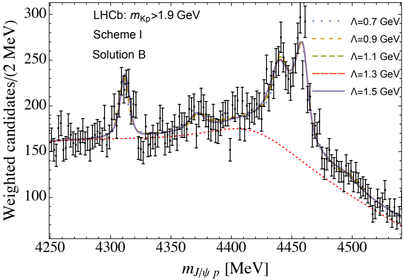
  ↪ coupled channel
- Scheme IV: contact + OPE + S-D counter terms w/ $\Lambda_c \bar{D}^{(*)}$

Scheme I: pure contact potential w/o $\Lambda_c\bar{D}^{(*)}$

Solution A



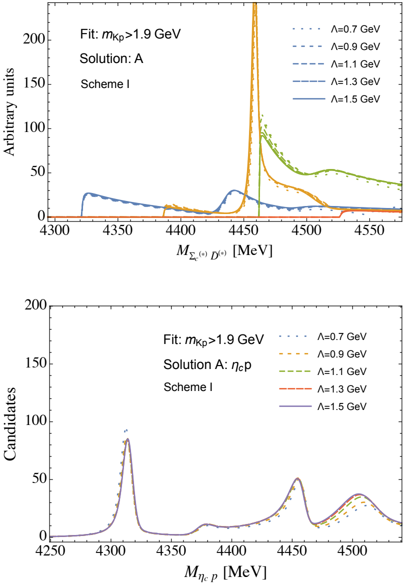
Solution B



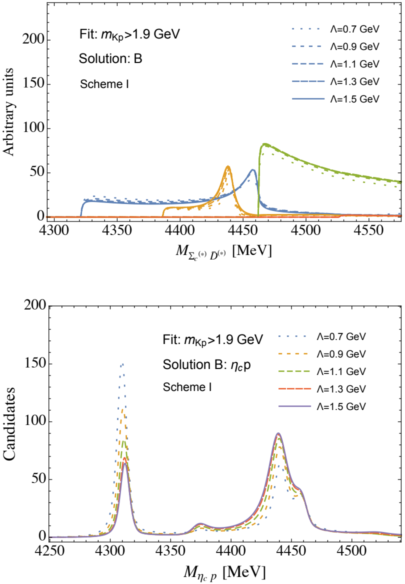
- ☞ $\Lambda_{\text{soft}} \sim 0.7$ GeV
- ☞ Cutoff-independent for both solution A and B
- ☞ No need for $\Lambda_c\bar{D}^{(*)}$

Scheme I: pure contact potential w/o $\Lambda_c\bar{D}^{(*)}$

Solution A



Solution B



Scheme I: pole positions

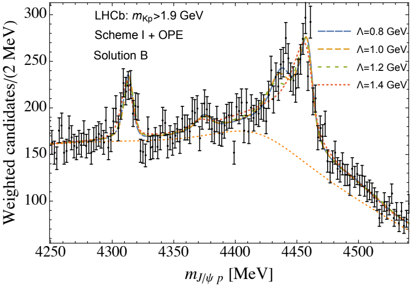
		Solution A		Solution B	
	DC ([MeV])	J^P	Pole [MeV]	J^P	Pole [MeV]
$P_c(4312)$	$\Sigma_c\bar{D}$ (4321.6)	$\frac{1}{2}^-$	$4314(1)-4(1)i$	$\frac{1}{2}^-$	$4312(2)-4(2)i$
$\mathbf{P_c(4380)}^*$	$\Sigma_c^*\bar{D}$ (4386.2)	$\frac{3}{2}^-$	$4377(1)-7(1)i$	$\frac{3}{2}^-$	$4375(2)-6(1)i$
$P_c(4440)$	$\Sigma_c\bar{D}^*$ (4462.1)	$\frac{1}{2}^-$	$4440(1)-9(2)i$	$\frac{3}{2}^-$	$4441(3)-5(2)i$
$P_c(4457)$	$\Sigma_c\bar{D}^*$ (4462.1)	$\frac{3}{2}^-$	$4458(2)-3(1)i$	$\frac{1}{2}^-$	$4462(4)-5(3)i$
P_c	$\Sigma_c^*\bar{D}^*$ (4526.7)	$\frac{1}{2}^-$	$4498(2)-9(3)i$	$\frac{1}{2}^-$	$4526(3)-9(2)i$
P_c	$\Sigma_c^*\bar{D}^*$ (4526.7)	$\frac{3}{2}^-$	$4510(2)-14(3)i$	$\frac{3}{2}^-$	$4521(2)-12(3)i$
P_c	$\Sigma_c^*\bar{D}^*$ (4526.7)	$\frac{5}{2}^-$	$4525(2)-9(3)i$	$\frac{5}{2}^-$	$4501(3)-6(4)i$

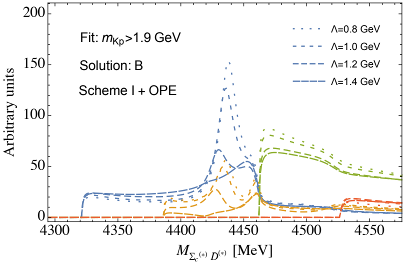
☞ * NOT the broad $P_c(4380)$ reported by LHCb in 2015

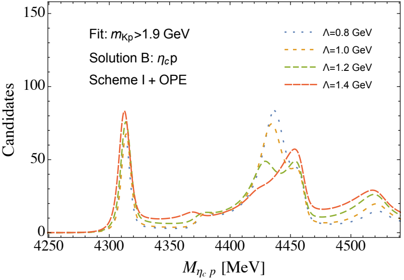
☞ Bound states with respect to the dominant channel (DC)

Scheme II: scheme I + OPE w/o $\Lambda_c \bar{D}^{(*)}$

- ☞ No solution A
- ☞ Solution B: Cut-off dependent
- ☞ $\Lambda_{\rm soft} \sim 700$ MeV

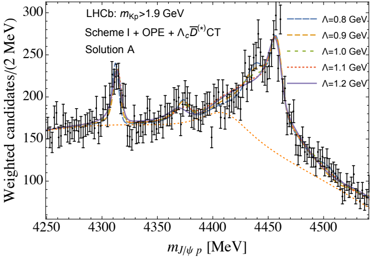


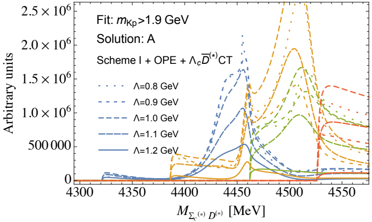




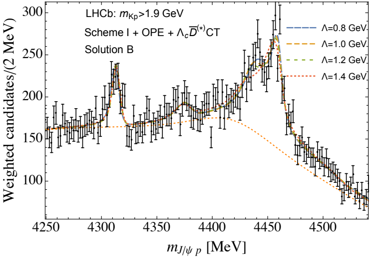
Scheme I + OPE + CT for $\Lambda_c \bar{D}^{(*)}$

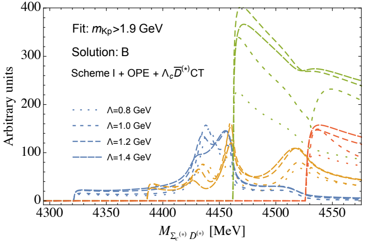
Solution A





Solution B

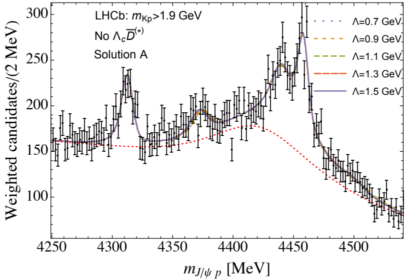




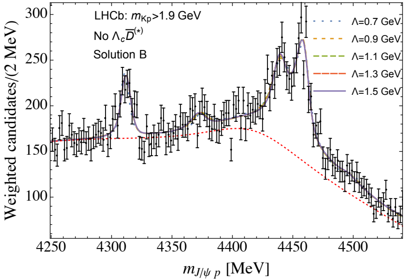
- Cut-off dependent
- $\Lambda_{\rm soft} \sim 900$ MeV $\Lambda_c \bar{D}^{(*)}$

Scheme III: contact + OPE + S-D w/o $\Lambda_c\bar{D}^{(*)}$

Solution A



Solution B

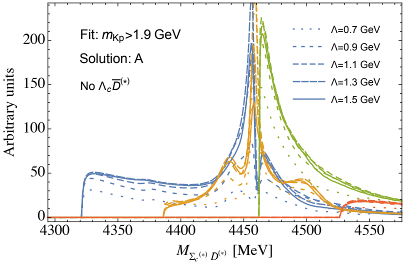


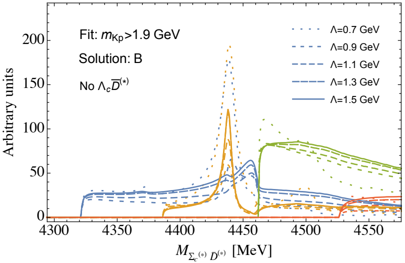
☞ $\Lambda_{\rm soft} \sim 0.7$ GeV

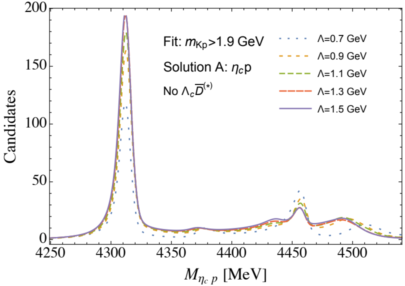
☞ Cutoff-independent for both solution A and B

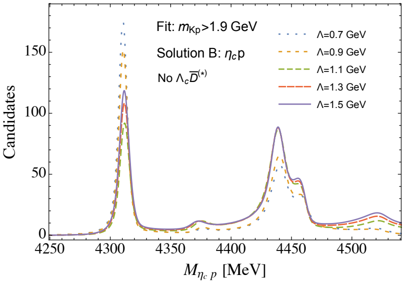
Scheme III: contact + OPE + S-D w/o $\Lambda_c\bar{D}^{(*)}$

Solution A $\Lambda_{\rm soft} \sim 0.7$ GeV Solution B





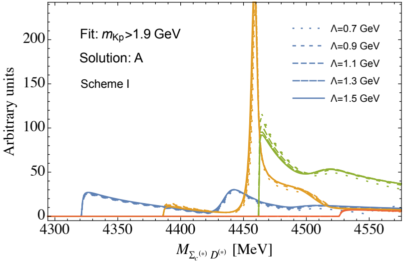


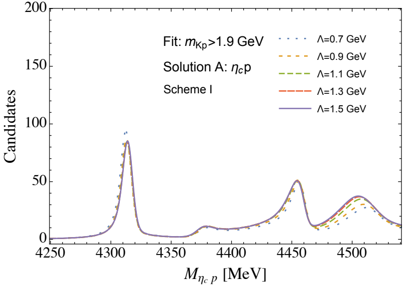


Scheme I vs Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ $\Lambda_{\rm soft}\sim 0.7$ GeV

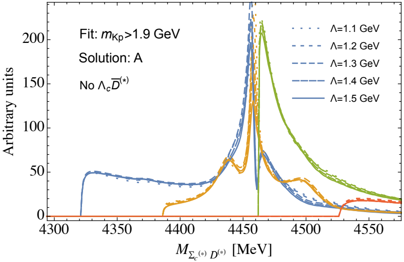
Solution A

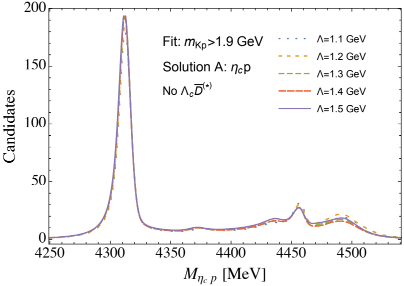
Scheme I: pure contact





Scheme III: contact + OPE + SD

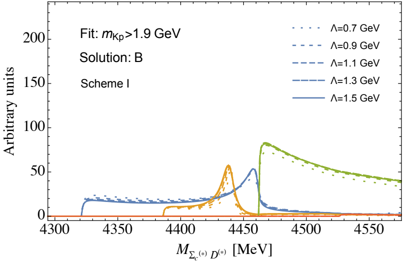




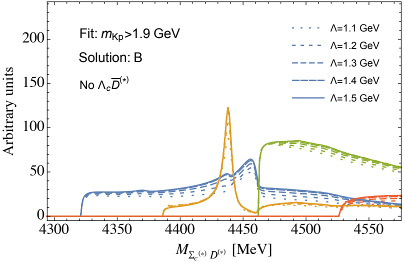
Scheme I vs Scheme III w/o $\Lambda_c \bar{D}^{(*)}$ $\Lambda_{\rm soft} \sim 0.7$ GeV

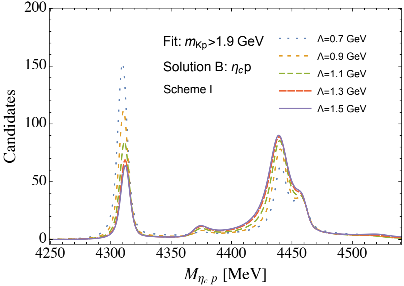
Solution B

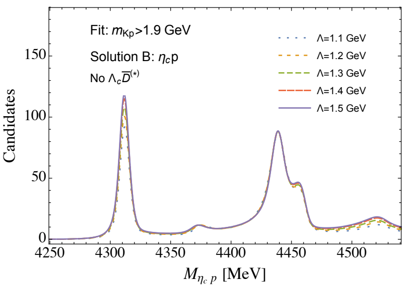
Scheme I: pure contact



Scheme III: contact + OPE + SD

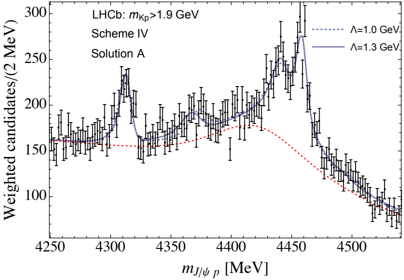




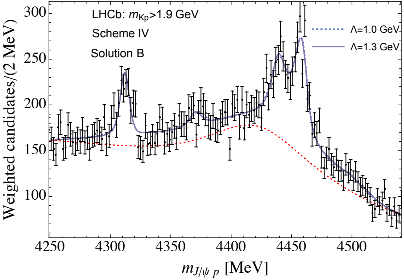


Scheme IV: CT + OPE + SD w/ $\Lambda_c \bar{D}^{(*)}$ $\Lambda_{\rm soft} \sim 0.9$ GeV

Solution A



Solution B



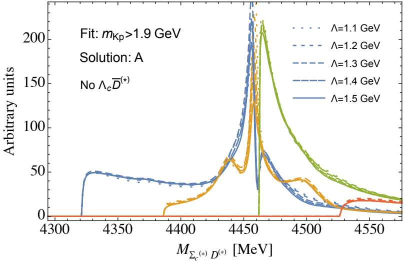
☞ Cutoff-independent for both solution A and B

☞ ? uncertainty

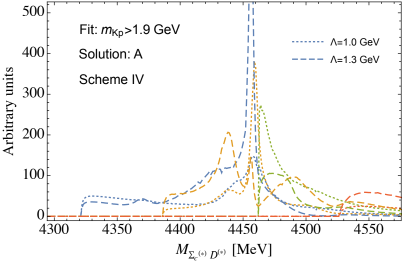
Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ vs Scheme IV w/ $\Lambda_c\bar{D}^{(*)}$

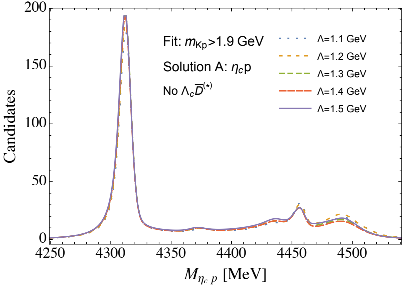
Solution A

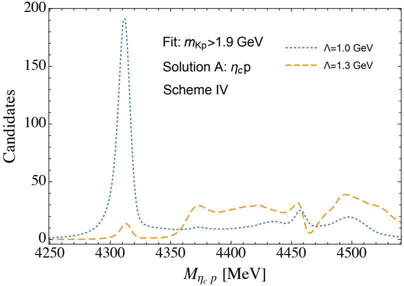
Scheme III: $\Lambda_{\rm soft} \sim 0.7$ GeV



Scheme IV: $\Lambda_{\rm soft} \sim 0.9$ GeV



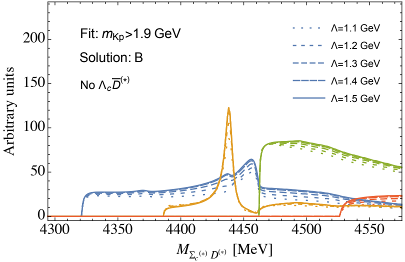




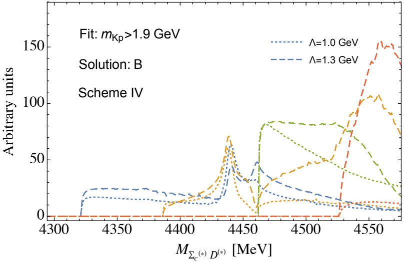
Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ vs Scheme IV w/ $\Lambda_c\bar{D}^{(*)}$

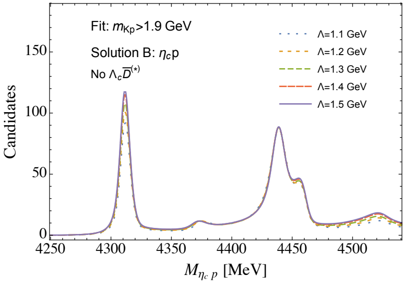
Solution B

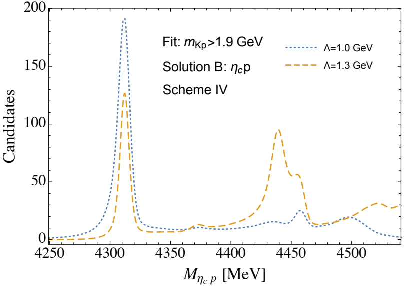
Scheme III: $\Lambda_{\rm soft} \sim 0.7$ GeV



Scheme IV: $\Lambda_{\rm soft} \sim 0.9$ GeV



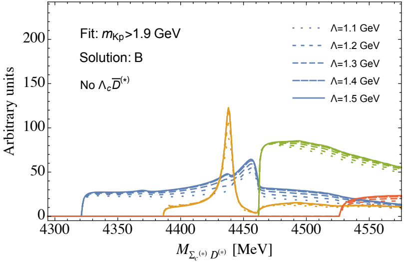




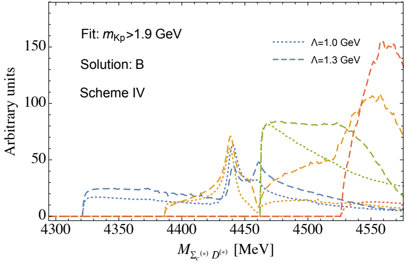
Scheme III w/o $\Lambda_c\bar{D}^{(*)}$ vs Scheme IV w/ $\Lambda_c\bar{D}^{(*)}$

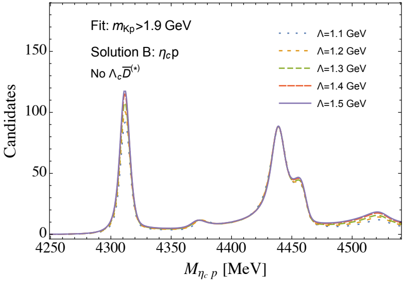
Solution B :)

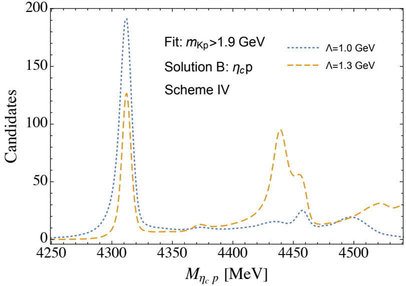
Scheme III: $\Lambda_{\rm soft} \sim 0.7$ GeV



Scheme IV: $\Lambda_{\rm soft} \sim 0.9$ GeV







Summary & Outlook

- Solving Lippmann-Schwinger equation with respect to
  - Unitarity, three-body cut
    $\hookrightarrow$ width of $\Sigma_c^{(*)}$
  - Coupled-channels
    $\hookrightarrow$ cut-off independence: OPE $\rightarrow$ SD counter term
  - Heavy quark spin symmetry
    $\hookrightarrow$ 7 $\Sigma_c^{(*)}\bar{D}^{(*)}$ molecular states
- $\Lambda_{\mathrm{cutoff}} = 1.3$ GeV
  $\hookrightarrow$ $\Lambda_{\mathrm{cutoff}} \gg \Lambda_{\mathrm{soft}}$
  - Solution A is scheme dependent
  - Solution B is consistent for all cut-off independent schemes
    $P_c(4440)$: $J^P = \frac{3}{2}^-$, $P_c(4457)$: $J^P = \frac{1}{2}^-$ preferred ?
- Formalism consistent
  $\hookrightarrow$ we can not say much about $\Lambda_c\bar{D}^{(*)}$ interaction without data in this channel.
- A narrow $P_c(4380)$, different from the broad one reported by LHCb in 2015.

Thank you very much for your attention!