

$Z_b(10610)$ and $Z_b(10650)$ from experimental line shapes

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Theoretical Aspects of Hadron Spectroscopy and Phenomenology

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in collaboration with

E. Epelbaum, A.A. Filin, C. Hanhart, R.V. Mizuk, A.V. Nefediev, S. Ropertz, Q. Wang and J.-L. Wynn

PRD 98, 074023 (2018), PRD 99, 094013 (2019) and arXiv:2012.05034 [hep-ph]

Plan

- $Z_b(10610)$ and $Z_b(10650)$ from decays: $\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi B^{(*)} \bar{B}^*$
 $\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi \pi h_b(mP)$

Q.Wang, VB, A.A. Filin, C. Hanhart, A.V. Nefediev, and J.-L. Wynen

PRD 98, 074023 (2018)

- Predictions for their spin partner states and line shapes

VB, E. Epelbaum, A.A. Filin, C. Hanhart, A.V. Nefediev, and Q.Wang

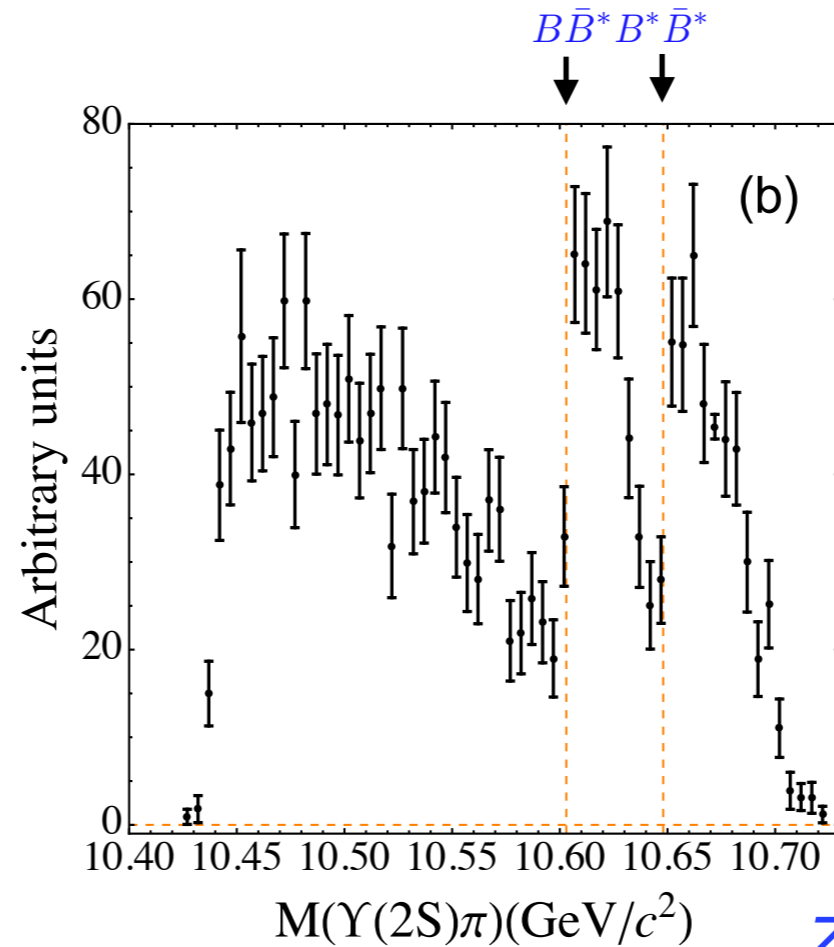
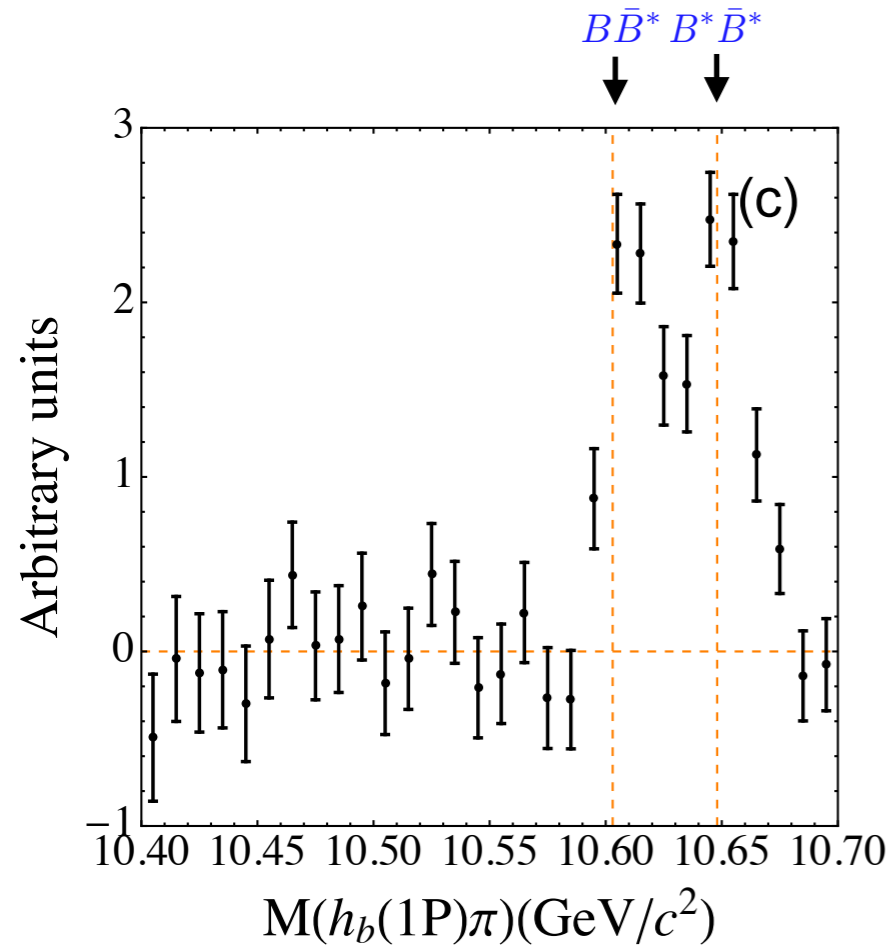
PRD 99, 094013 (2019)

- Insights into the nature of the $Z_b(10610)$ and $Z_b(10650)$
from $\Upsilon(10860) \rightarrow \Upsilon(nS) \pi^+ \pi^-$ ($n=1,2,3$)

VB, E. Epelbaum, A.A. Filin, C. Hanhart, R.V. Mizuk, A.V. Nefediev and S. Ropertz

arXiv: 2012.05034 [hep-ph]

$Z_b(10610)$ and $Z_b(10650)$ from $\Upsilon(10860)$ decays at Belle



Peaks near $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds in various channels:

$$B\bar{B}^*, B^*\bar{B}^*$$

$$\pi^\pm h_b(mP), \pi^\pm \Upsilon(nS)$$

Charged modes \implies

$Z_b^{(\prime)}$ must be made of ≥ 4 quarks

- **PDG:** $M_{Z_b} = 10607.2 \pm 2.0$ MeV, $\Gamma_{Z_b} = 18.4 \pm 2.4$ MeV
 $M_{Z'_b} = 10652.2 \pm 1.5$ MeV, $\Gamma_{Z'_b} = 11.5 \pm 2.2$ MeV

Bondar et al. PRL108, 122001(2012)
 Garmash et al. PRL116, 212001(2016)
 PRD91, 072003 (2015)

- dominant decays to open flavour channels

- $\text{Br}[\Upsilon(10860) \rightarrow \pi\pi h_b(mP)] \simeq \text{Br}[\Upsilon(10860) \rightarrow \pi\pi \Upsilon(nS)]$

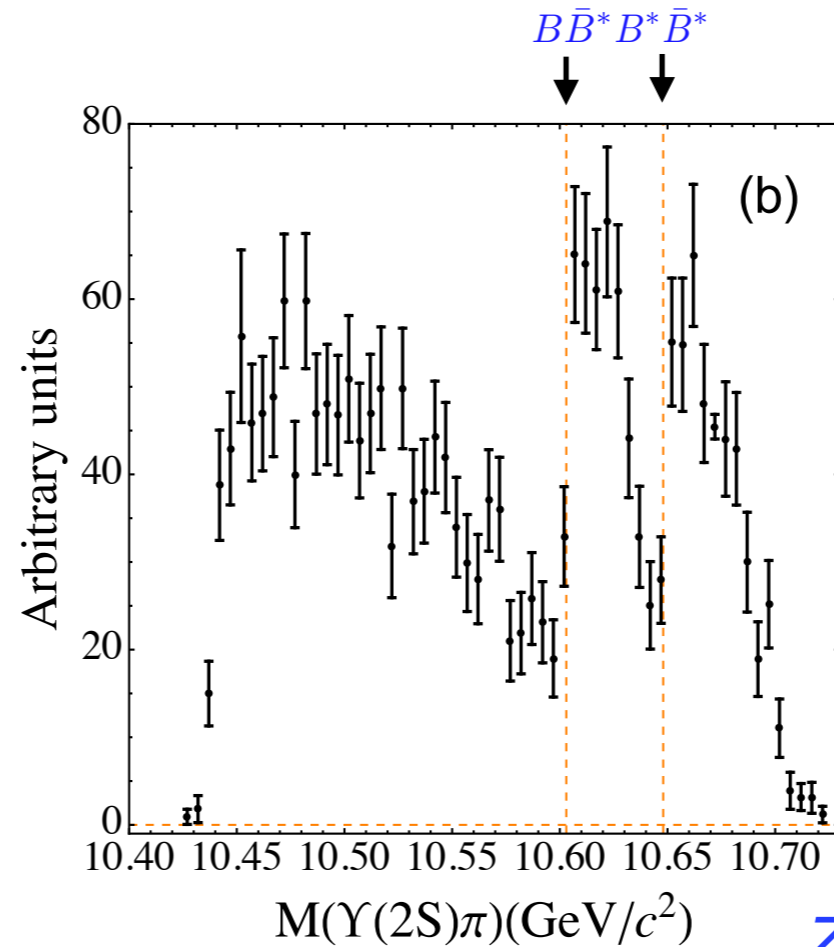
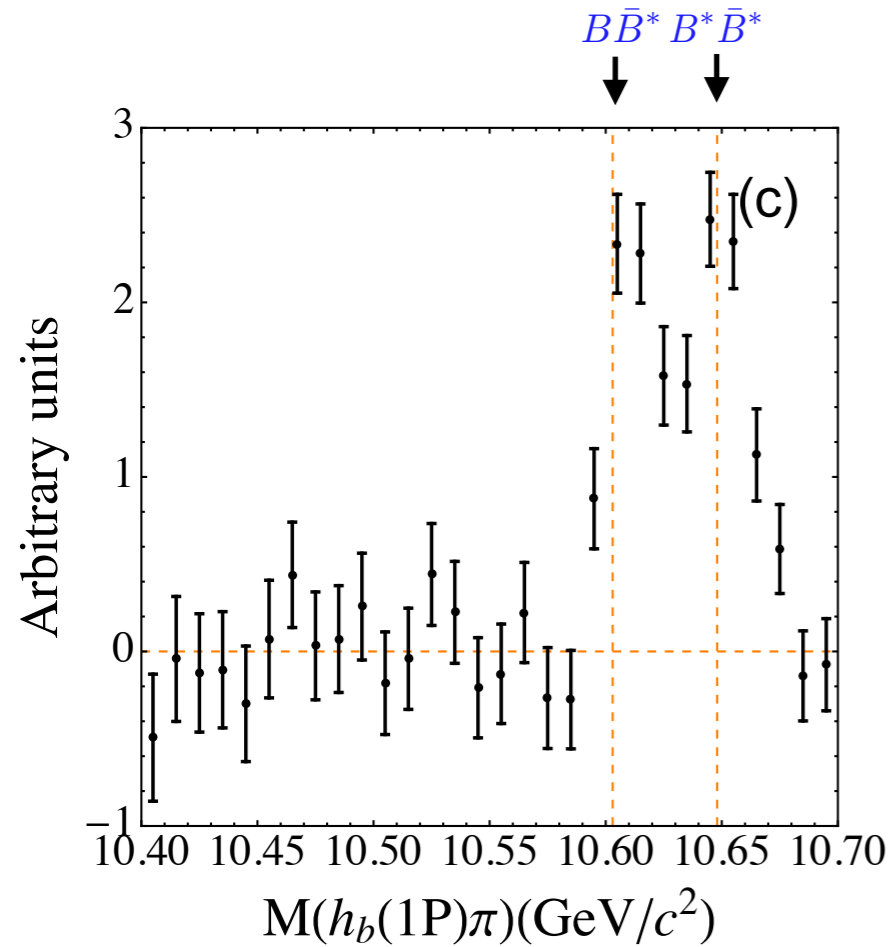
Heavy quark spin flip

No spin flip

\implies a strong hint for a large molecular component in $Z_b(10610)/Z_b(10650)$

Bondar et al. PRD 84, 054010 (2011)

$Z_b(10610)$ and $Z_b(10650)$ from $\Upsilon(10860)$ decays by Belle



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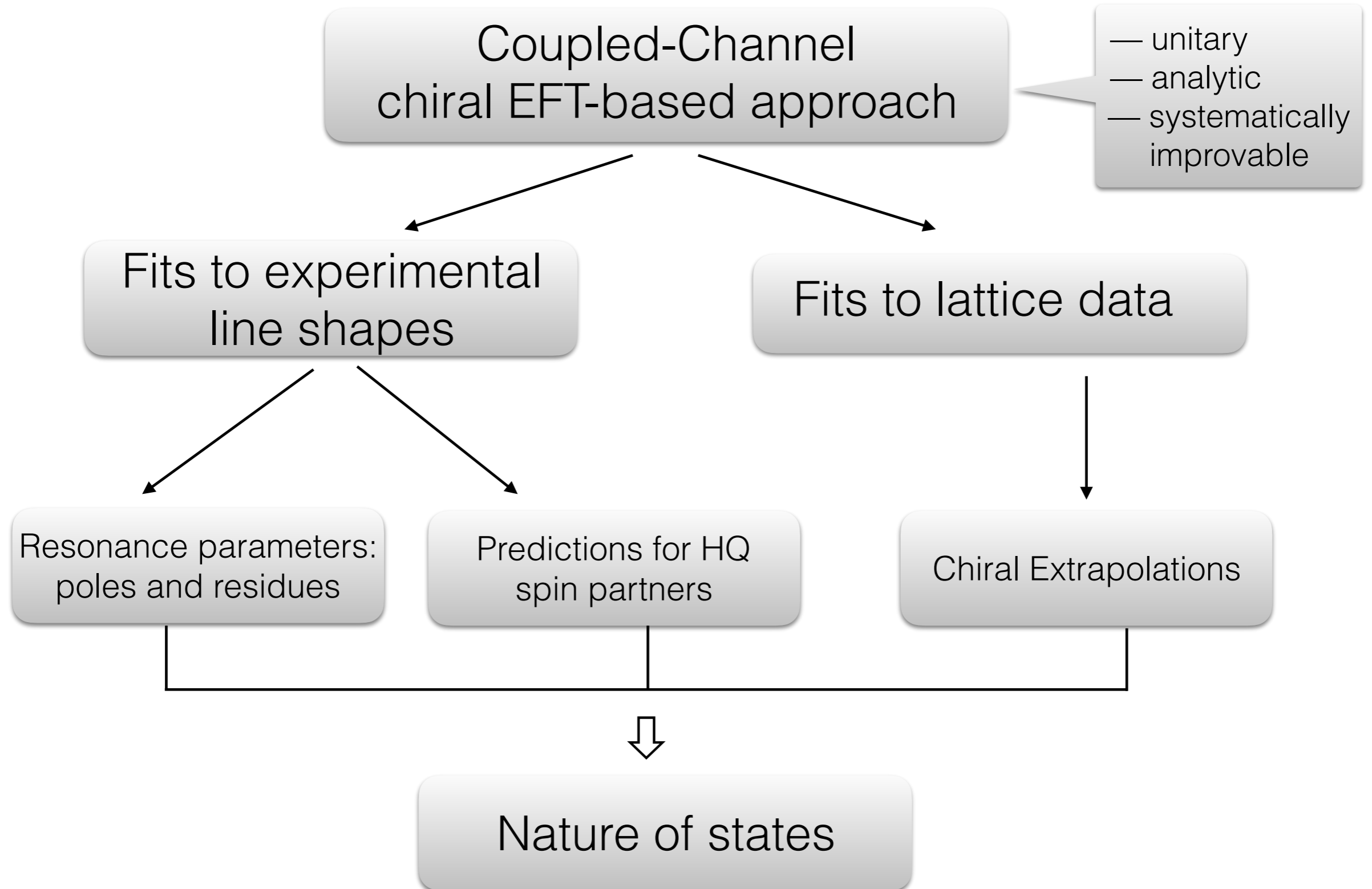
Bondar et al. PRL108, 122001(2012)
 Garmash et al. PRL116, 212001(2016)
 PRD91, 072003 (2015)

- **Exp. analysis is made using a sum of Breit-Wigner amplitudes:**

- does not account for threshold behavior
- naive coherent sum violates unitarity
- reaction dependent, no fits of all data simultaneously

How to improve?

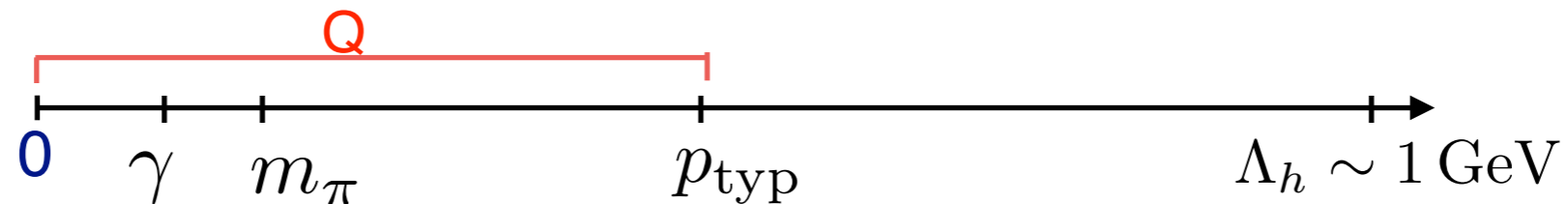
Roadmap for analysing near-threshold states



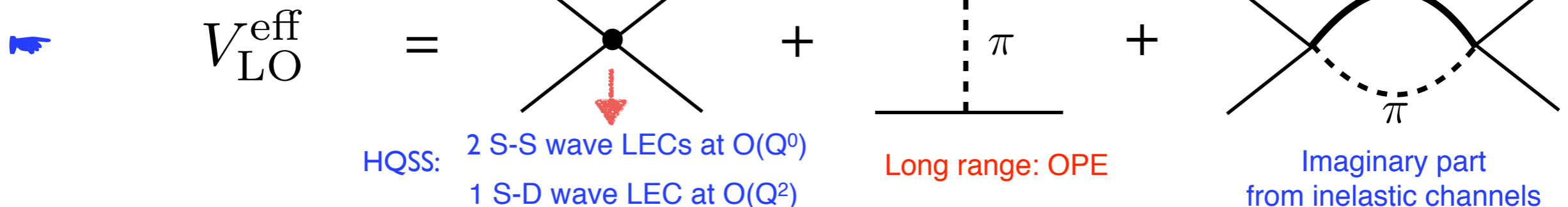
Chiral EFT approach at low energies

- Similar to nuclear EFT \Rightarrow deuteron as proton-neutron bound state, ... review: Epelbaum, Hammer, Meißner
- Elastic coupled-channel $B^{(*)}B^* \rightarrow B^{(*)}B^*$ potential to a given order in Q/Λ_h Weinberg power counting: Weinberg (1991)

▶ typical soft scale Q is quite large because of coupled-channels



$$p_{\text{typ}} = \sqrt{m \delta} \simeq 500 \text{ MeV}, \quad \delta = E_{B^*B^*}^{\text{thr}} - E_{BB^*}^{\text{thr}} = m_* - m \approx 45 \text{ MeV} \sim \text{range of validity}$$



- Amplitudes: non-perturbative solutions of coupled-channel integral equations

Formalism for line shapes $\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi\alpha$

- Input: experimental distributions for

$$\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi\alpha \quad \alpha = BB^*, B^*B^*, h_b(1P)\pi, h_b(2P)\pi$$

and branching fractions for $\alpha = B\bar{B}^*, B^*\bar{B}^*, h_b(1P)\pi, h_b(2P)\pi, \Upsilon(1S)\pi, \Upsilon(2S)\pi, \Upsilon(3S)\pi$
 Belle: Bondar et al. (2012), Garmash et al. (2016)

- $\Upsilon(mS)\pi\pi$ distributions not yet included: involve strong $\pi\pi$ FSI (come to this later!)

- Recent calculations for $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi\pi, \Upsilon(4S) \rightarrow \Upsilon(1S, 2S)\pi\pi$
 Chen et al. (2016-2017)

Production amplitudes for the events dominated by the Z_b 's poles:

$$U_{\text{el}} = \begin{array}{c} \Upsilon(5S) \\ \text{---} \end{array} \begin{array}{c} \text{---} \pi \\ \text{---} B^{(*)} \\ \text{---} \bar{B}^* \end{array} + \begin{array}{c} \text{---} \pi \\ \text{---} B \\ \text{---} \bar{B}^* \end{array} \begin{array}{c} \text{---} \pi \\ \text{---} B^{(*)} \\ \text{---} \bar{B}^* \end{array} + \begin{array}{c} \text{---} \pi \\ \text{---} \bar{B}^* \\ \text{---} \bar{B}^* \end{array} \begin{array}{c} \text{---} \pi \\ \text{---} B^{(*)} \\ \text{---} \bar{B}^* \end{array}$$

$$U_{\text{inel}} = \begin{array}{c} \Upsilon(5S) \\ \text{---} \end{array} \begin{array}{c} \text{---} \pi \\ \text{---} B \\ \text{---} \bar{B}^* \end{array} \begin{array}{c} \text{---} \pi \\ \text{---} \pi \\ \text{---} h_b(mP) \end{array} + \begin{array}{c} \text{---} \pi \\ \text{---} \bar{B}^* \\ \text{---} \bar{B}^* \end{array} \begin{array}{c} \text{---} \pi \\ \text{---} \pi \\ \text{---} h_b(mP) \end{array}$$

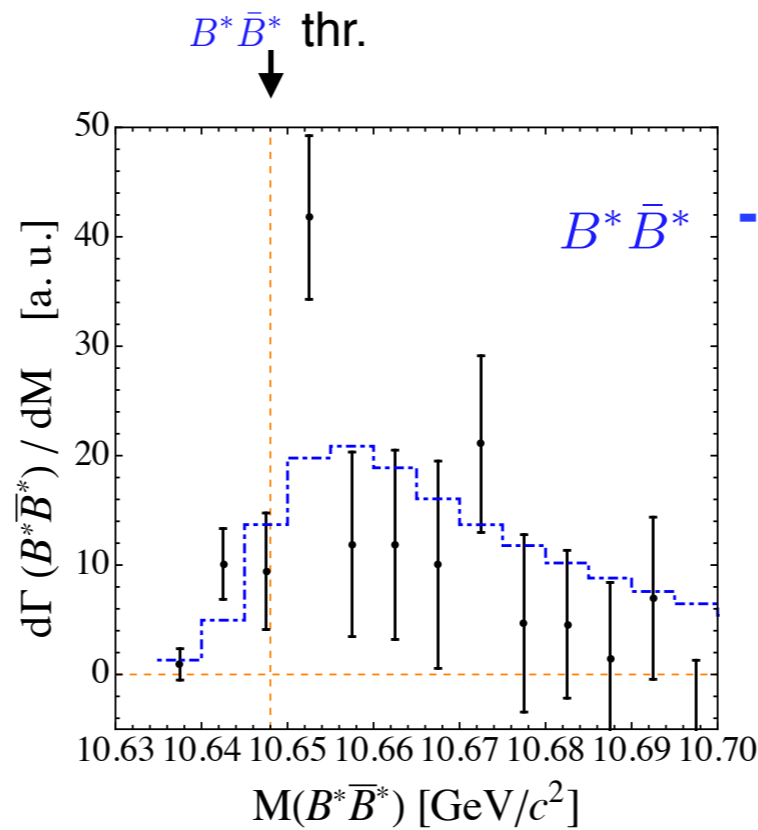
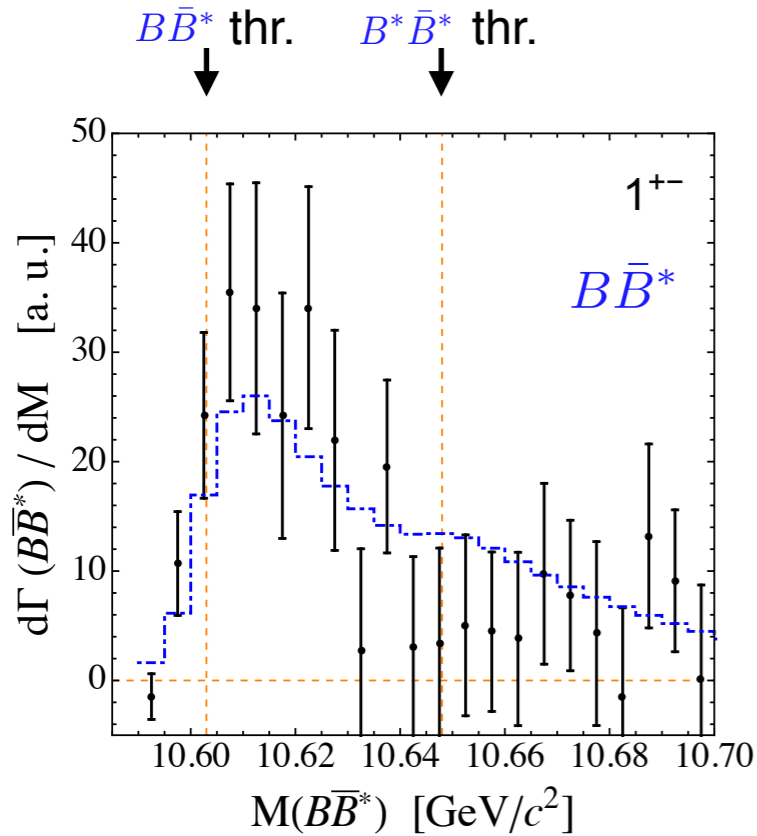
👉 Inelastic source $\Upsilon(5S) \rightarrow h_b(nP)\pi\pi$ requires flip in the HQ spin \Rightarrow suppressed by HQSS

Results: pionless theory at LO

our work:
PRD 98, 074023 (2018)

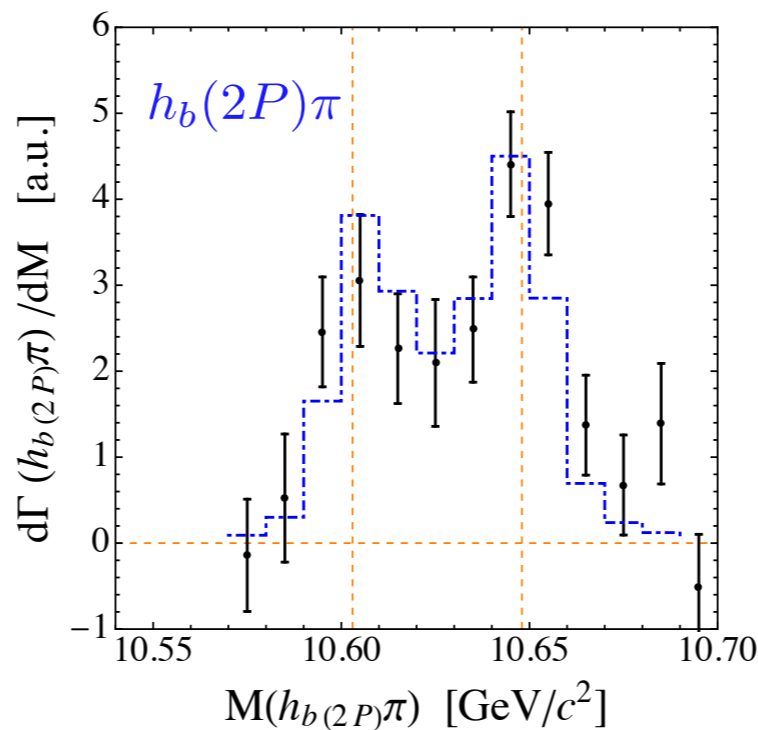
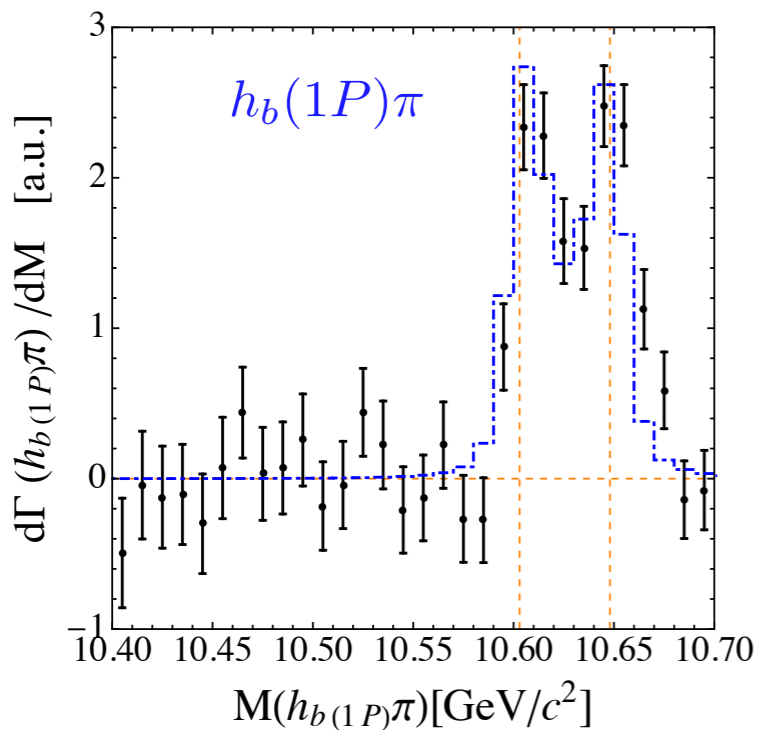
$$\chi^2 \equiv \frac{\chi^2}{\text{dof.}}$$

$$\chi^2 = 1.29$$



— π less: LO CT's

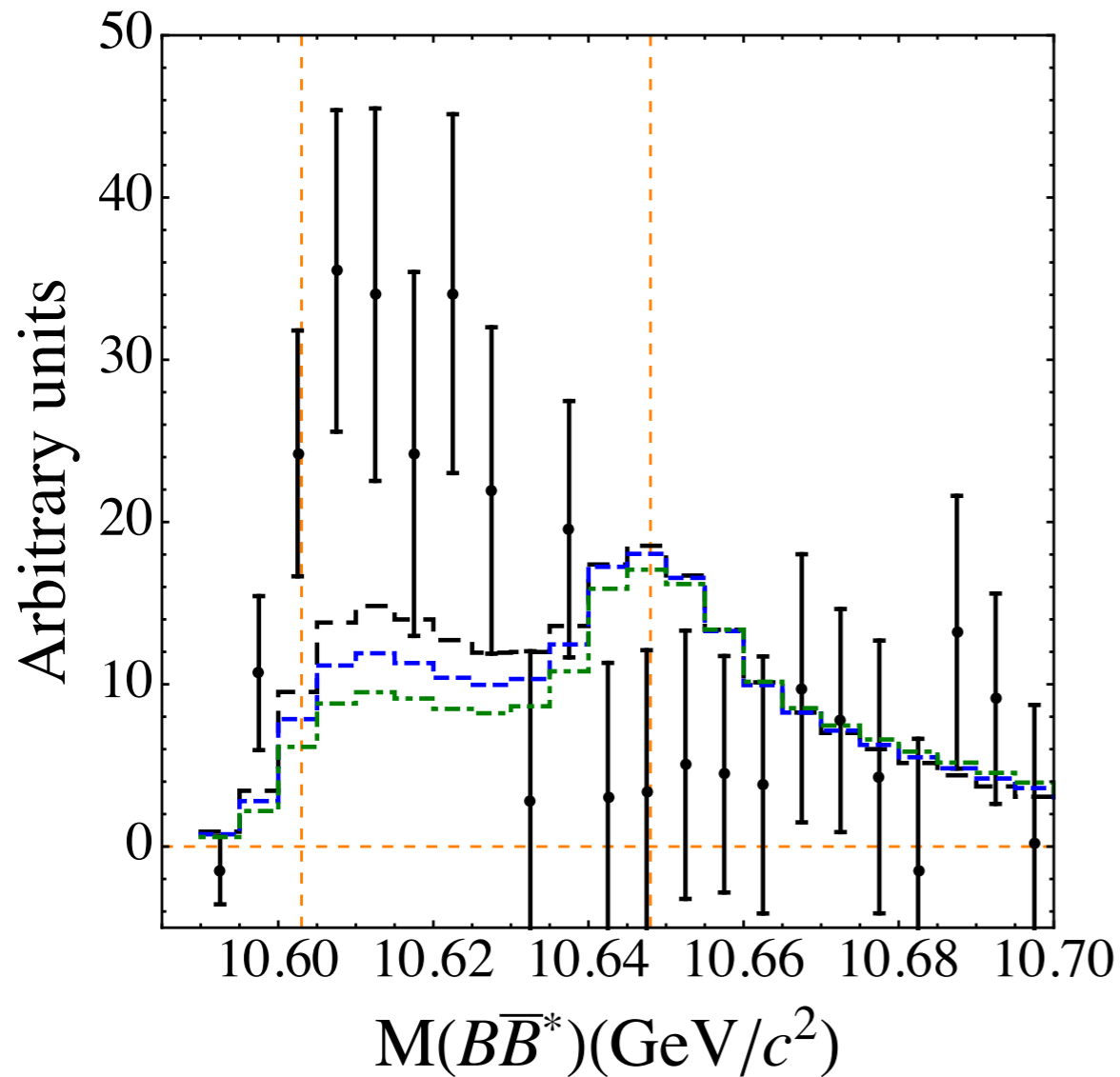
— HQSS is preserved in the potentials



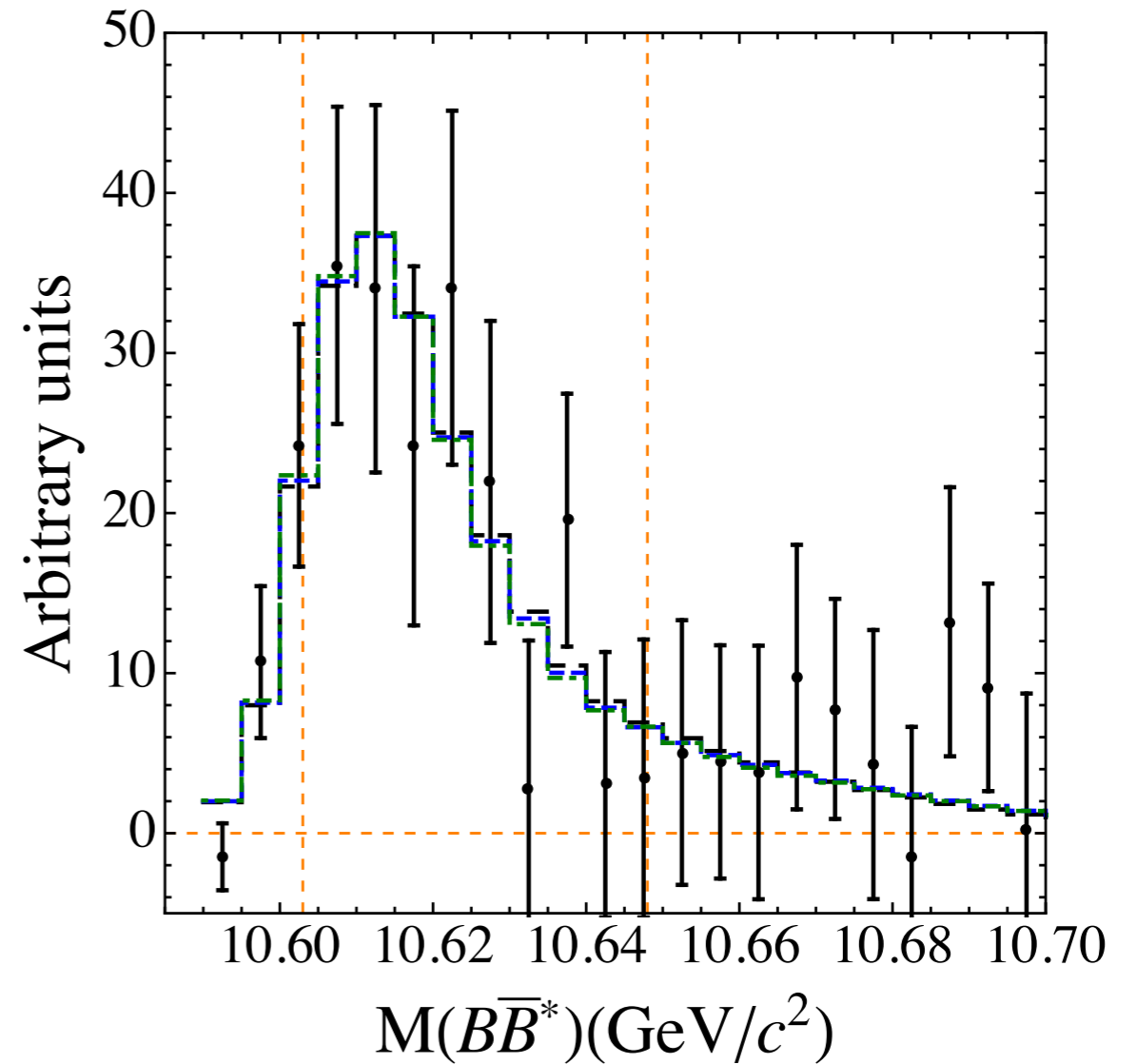
— consistent with the parameterisation by Guo et al. PRD 93, 074031 (2016)

Inclusion of OPE : regulator dependence

we use sharp cutoff $\Lambda \in [0.8 \text{ GeV}, 1.3 \text{ GeV}]$



without the SD contact term



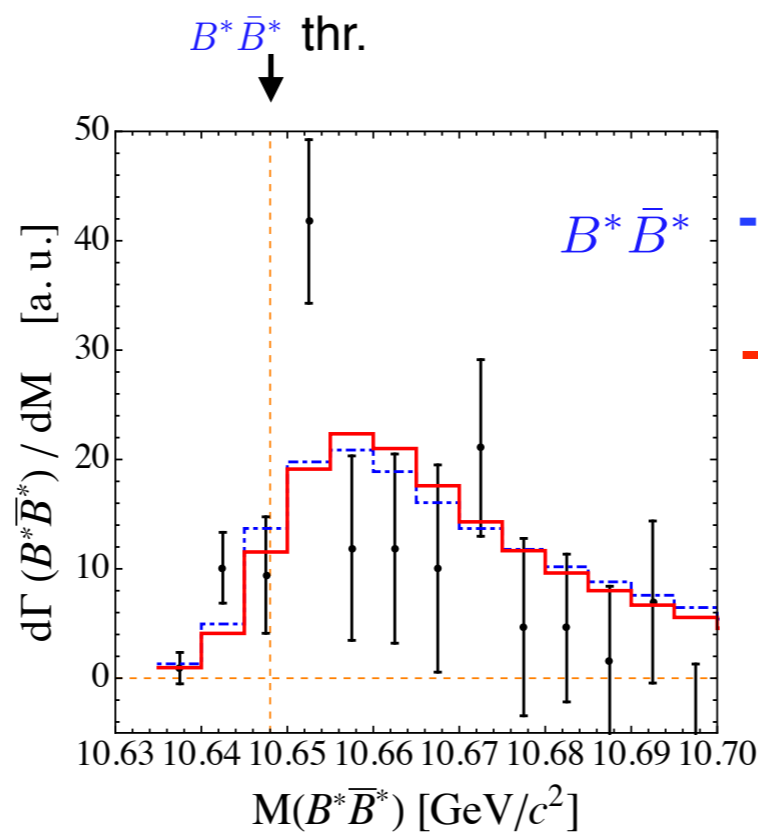
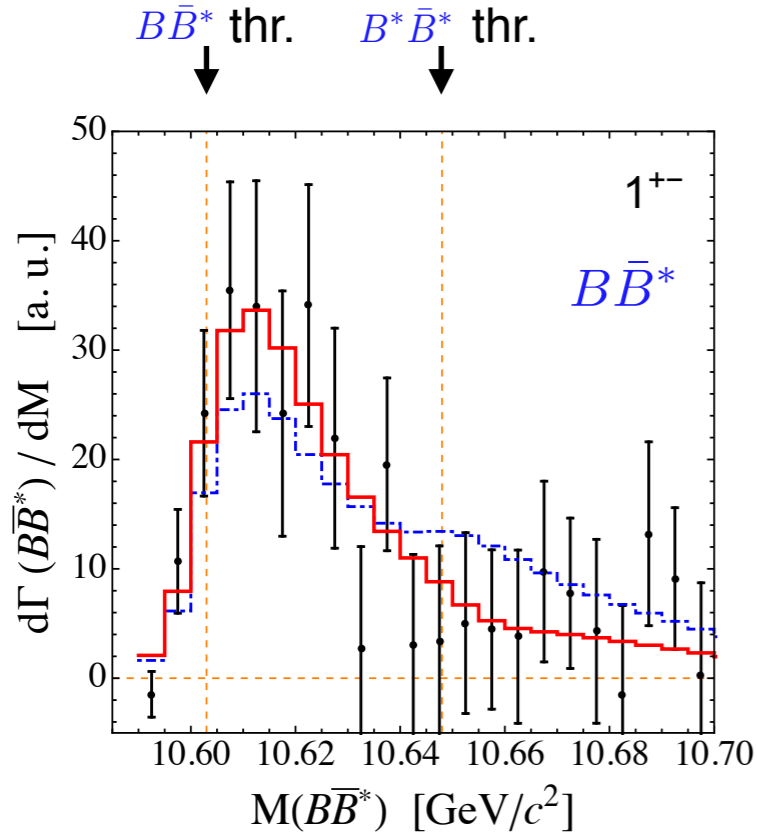
with the SD contact term

- Cutoff independence require S-wave-to-D-wave contact term to appear together with OPE

Results: LO contact terms (CT's) + OPE

our work:
PRD 98, 074023 (2018)

$$\chi^2 \equiv \frac{\chi^2}{\text{dof.}}$$



--- πless: LO CT's

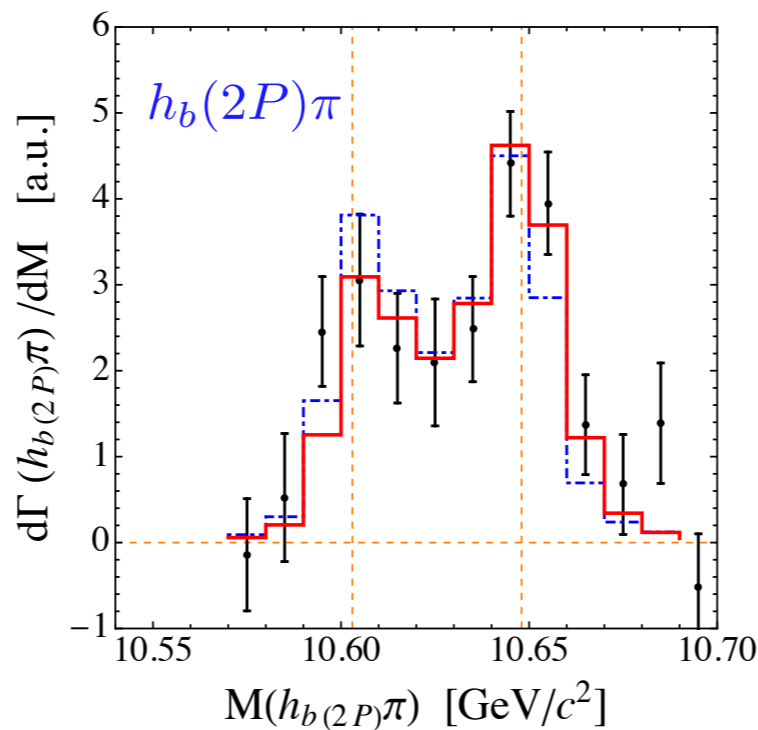
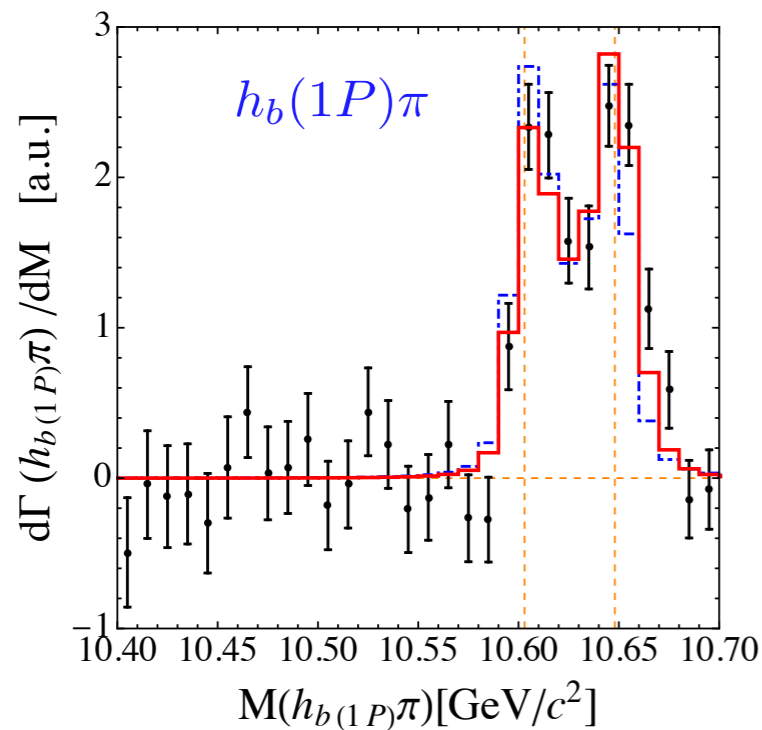
$\chi^2=1.29$

— πful 1: LO CT's + OPE $\chi^2=0.95$

- S-wave central OPE is weak

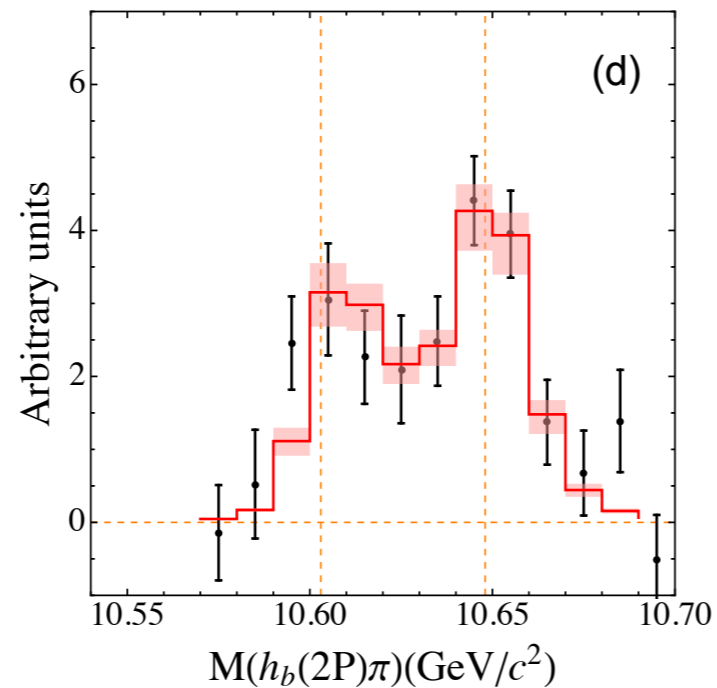
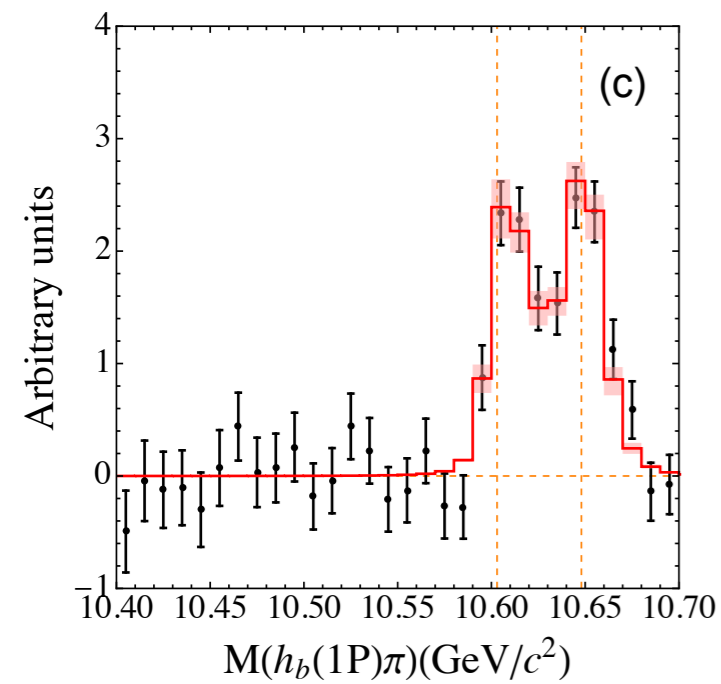
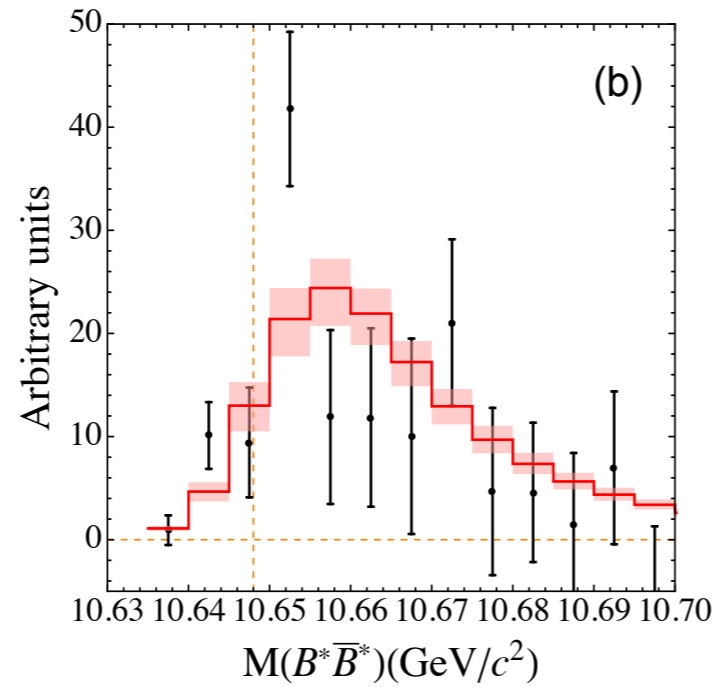
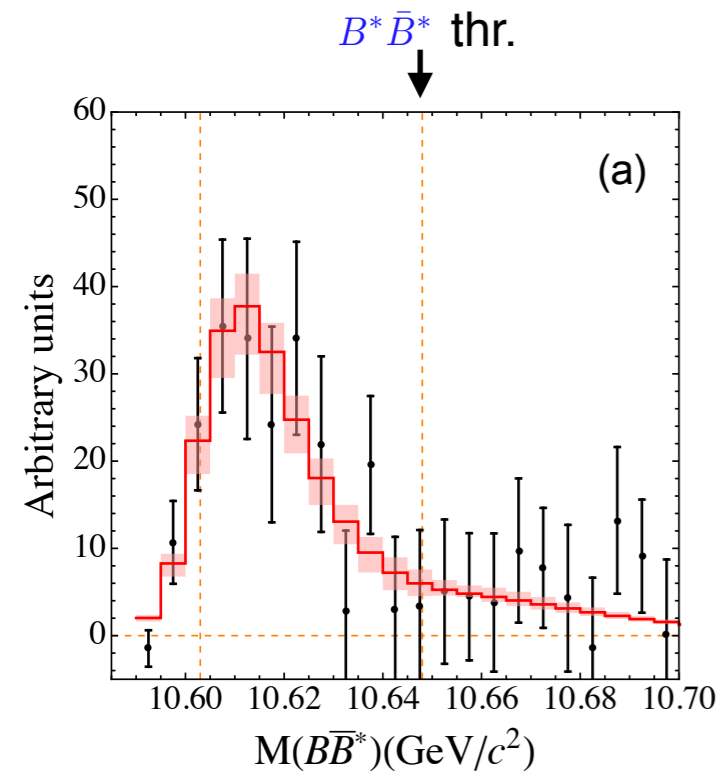
- S-wave-to-D-wave tensor forces from OPE are important

our work: JHEP 1706, 158 (2017)



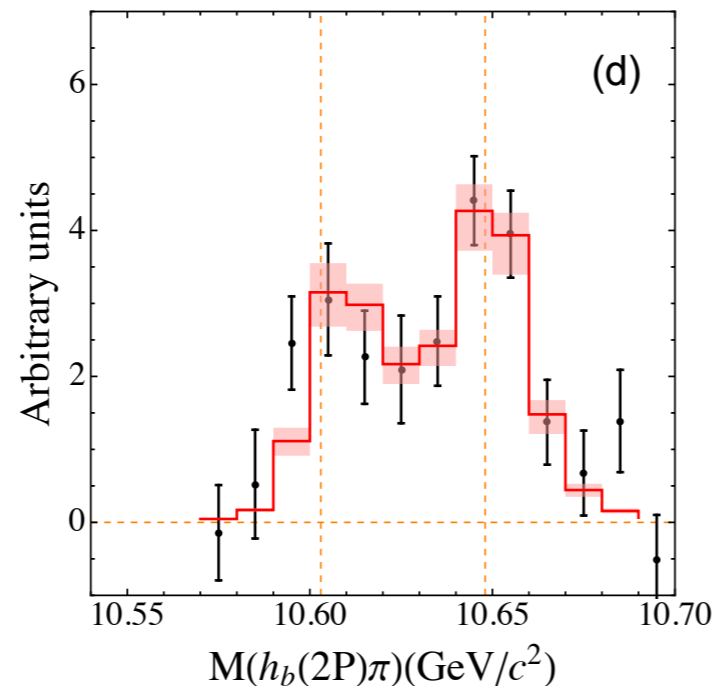
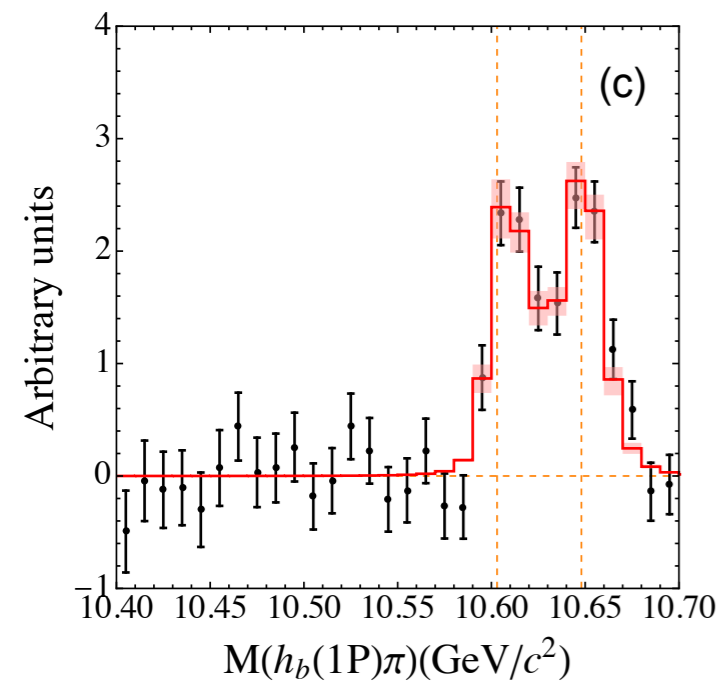
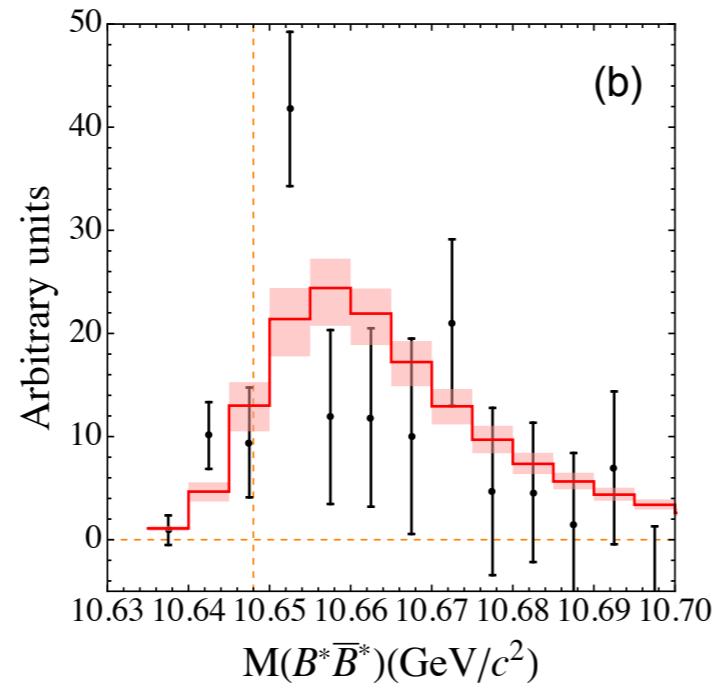
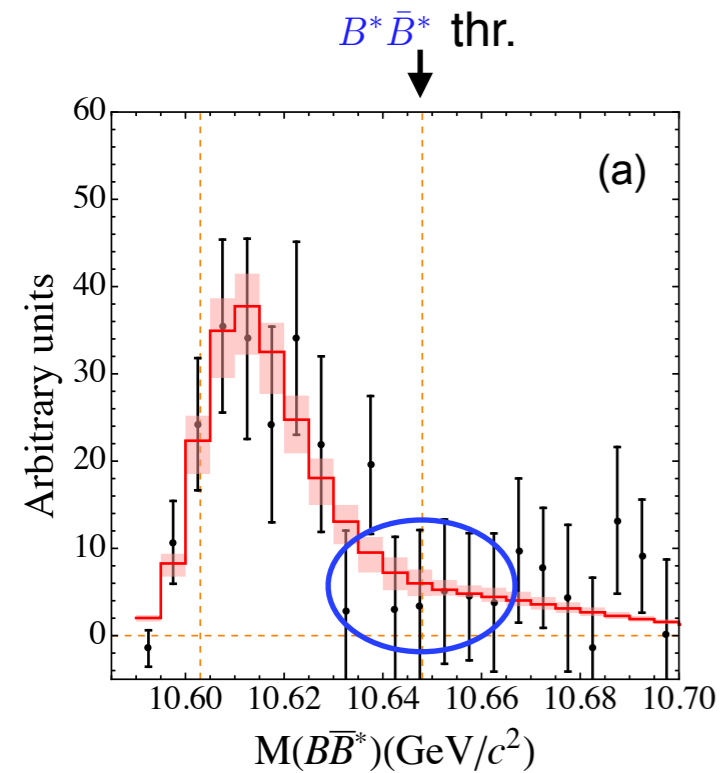
Residual effect from OPE results in a quantitative improvement of the fit

Final remarks



- All LECs are extracted from the best fit including 1σ errors
- Visible effect from OPE
- Natural suppression of higher-order terms
- Data are consistent with HQSS respecting interactions

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- All LECs are extracted from the best fit including 1σ errors
- Visible effect from OPE
- Natural suppression of higher-order terms
- Data are consistent with HQSS respecting interactions
- Data: no pronounced coupled-channel structure around B^*B^* threshold.

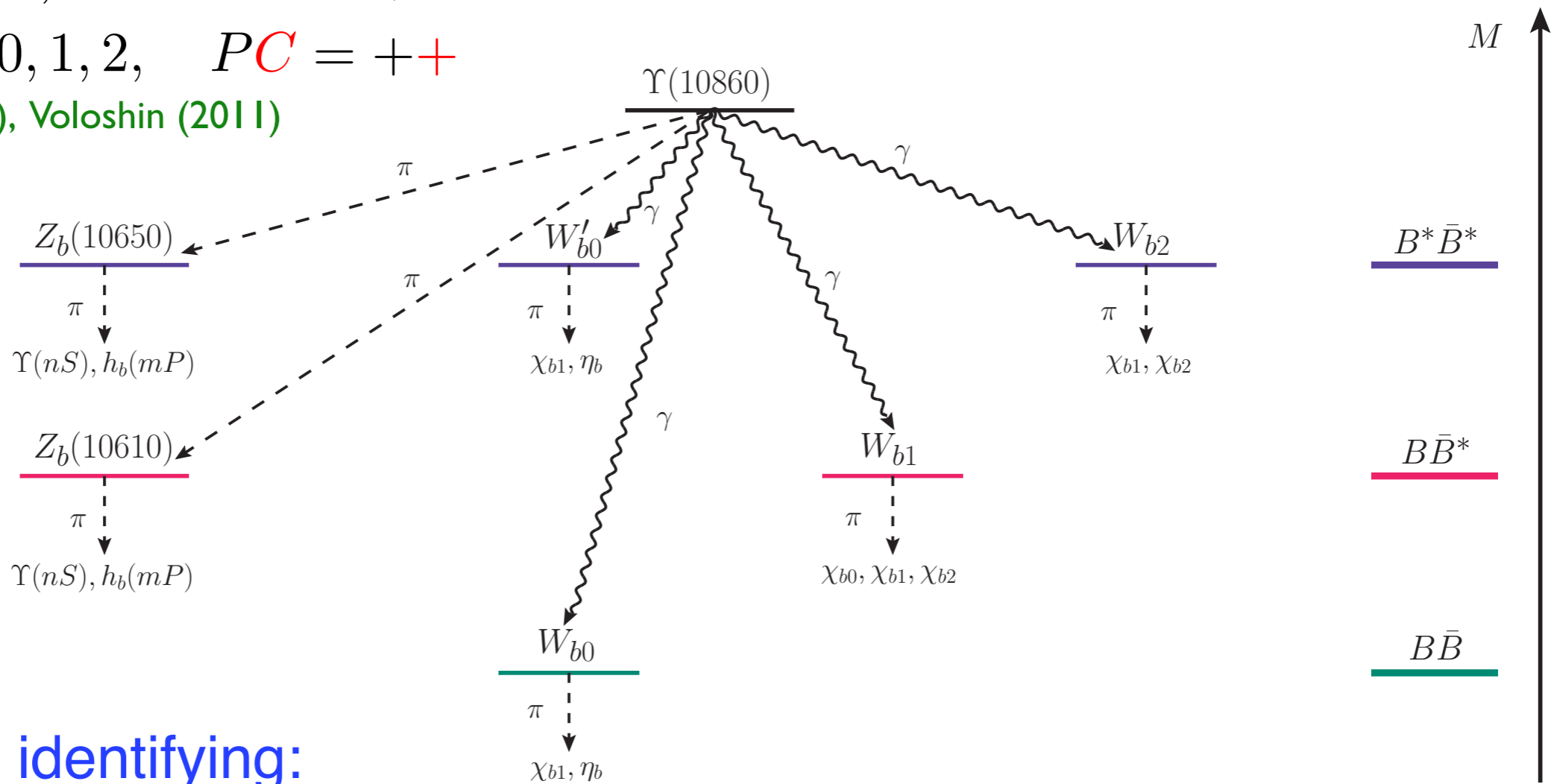
$Z_b(10650) \rightarrow B\bar{B}^*$ is suppressed

Applications: spin partners of $Z_b(10610)/Z_b(10650)$

$$Z_b^{(\prime)} : J = 1, \quad PC = +- -$$

$$W_{bJ} : J = 0, 1, 2, \quad PC = +++$$

Bondar et al. (2011), Voloshin (2011)



Difficulties in identifying:

$\Upsilon(10860) \rightarrow \gamma W_{bJ} \rightarrow$ final state

$\alpha=1/137$ penalty

$\Upsilon(10860) \rightarrow \pi\pi W_{b0} \rightarrow$ final state

very limited phase space

$\Upsilon(10860) \not\rightarrow \pi\pi W_{b1}, \pi\pi W'_{b0}, \pi\pi W_{b2}$

not possible

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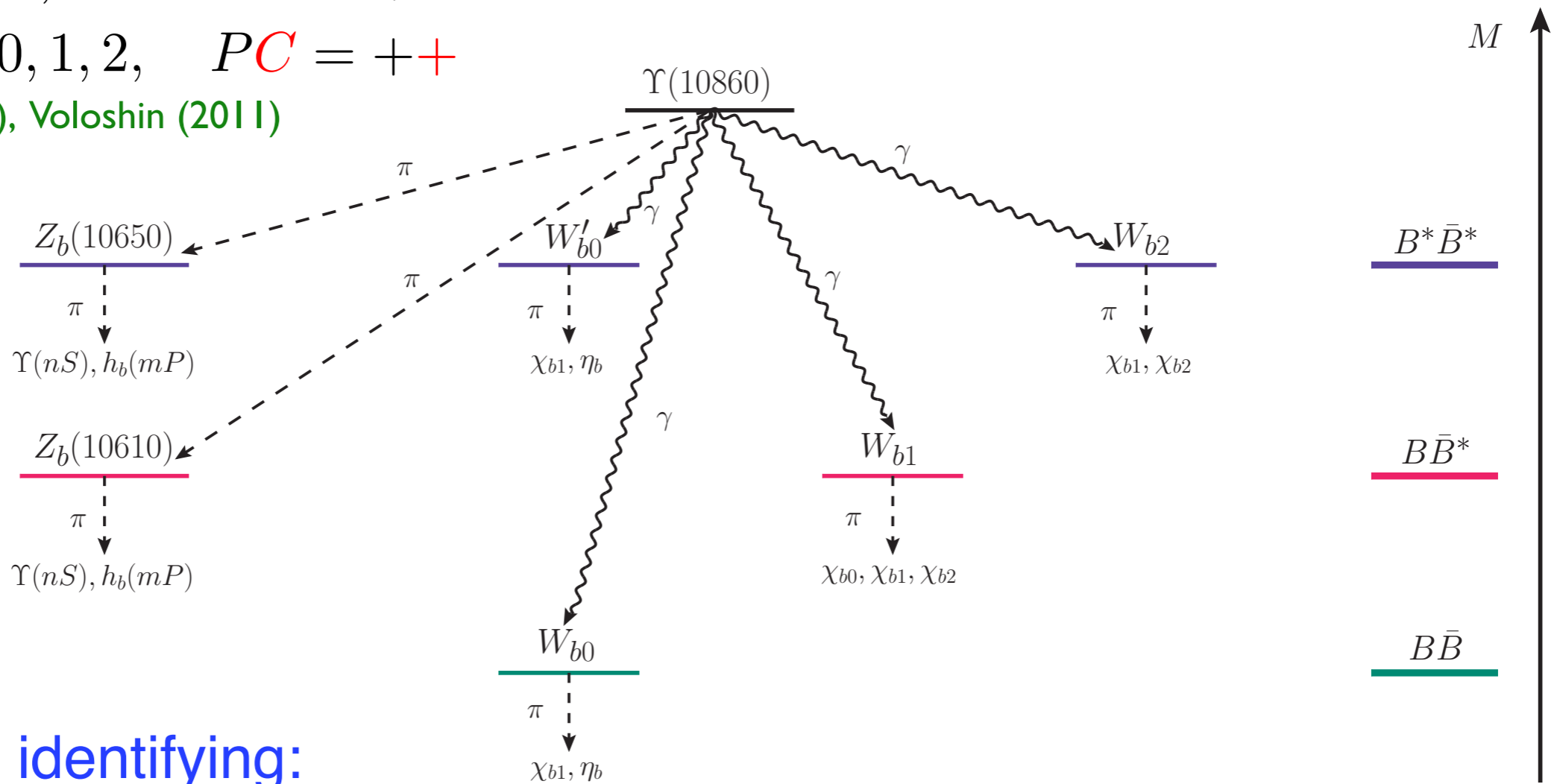
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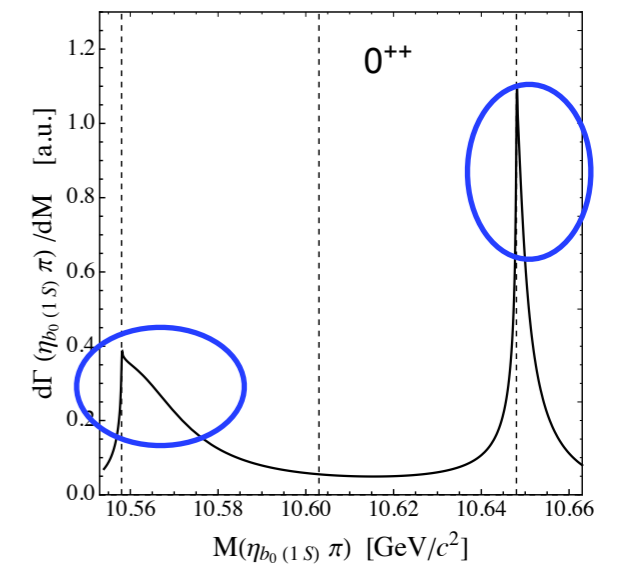
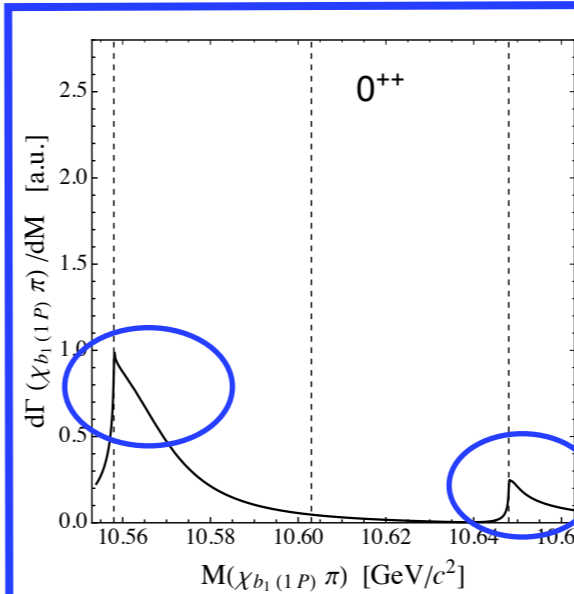
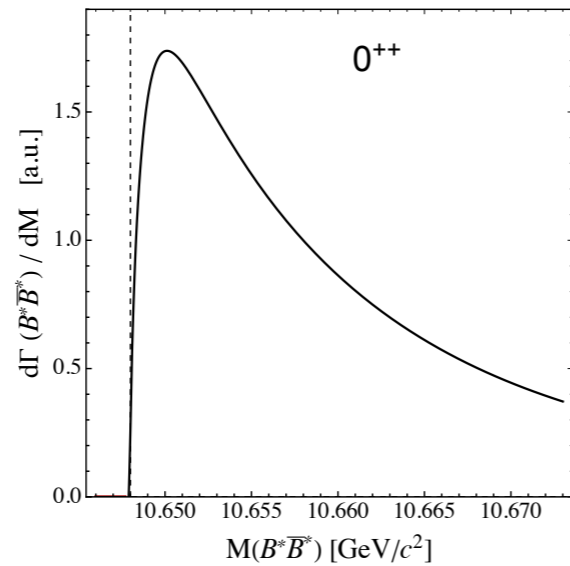
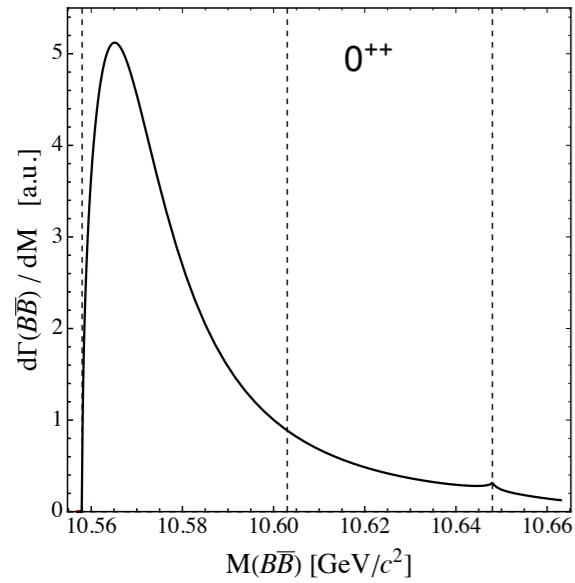
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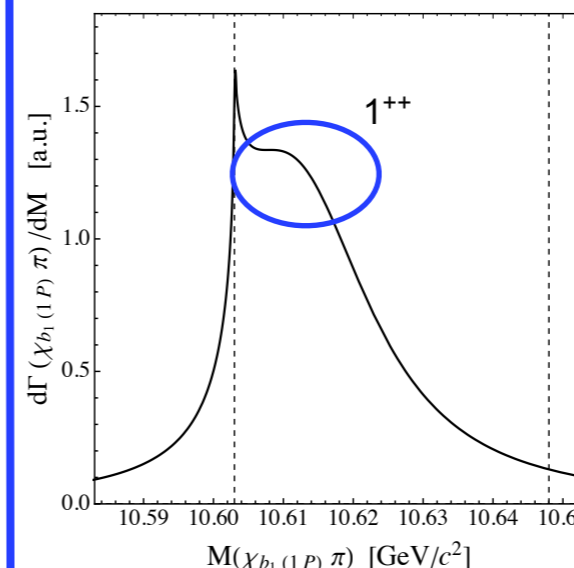
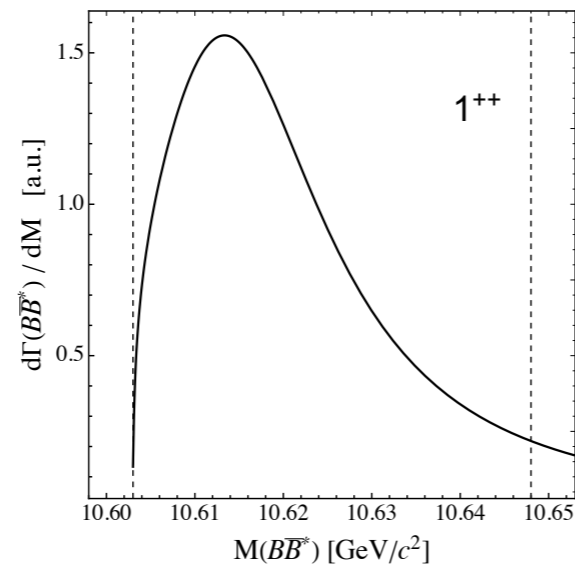
Good news: large statistics by BELLE II!

Line shapes for spin partners in $\Upsilon(10860) \rightarrow \gamma W_{bJ} \rightarrow$ final state



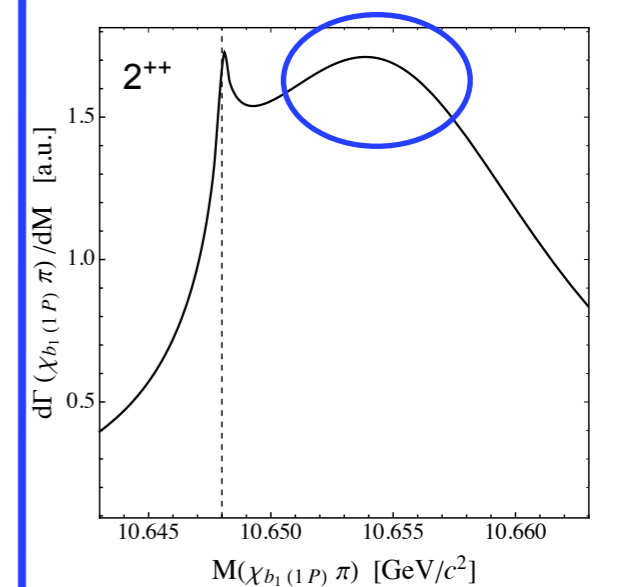
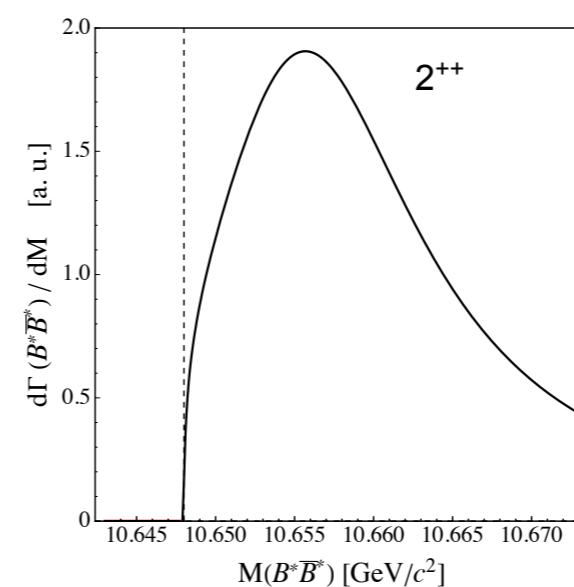
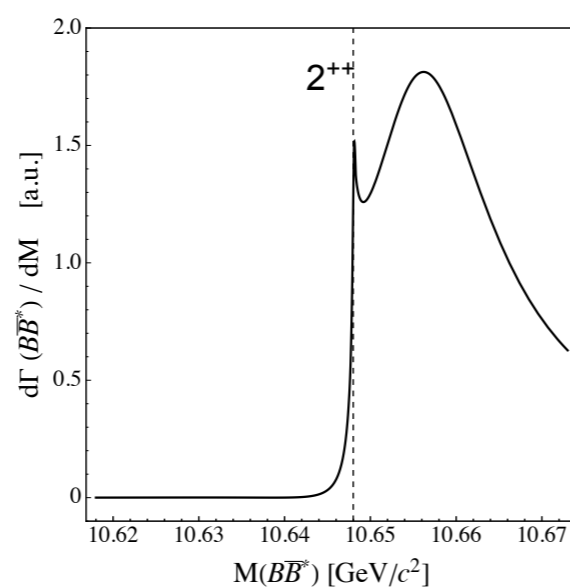
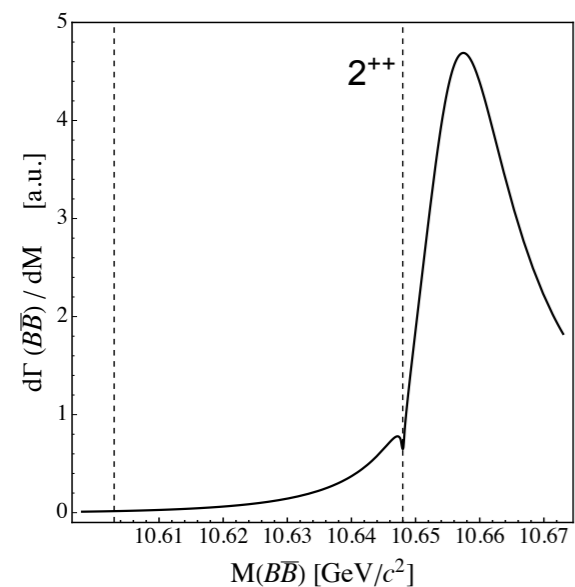
our work:

PRD 99, 094013 (2019)

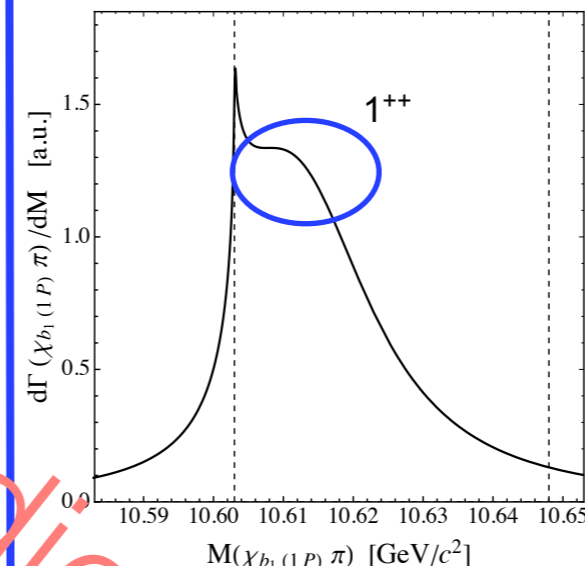
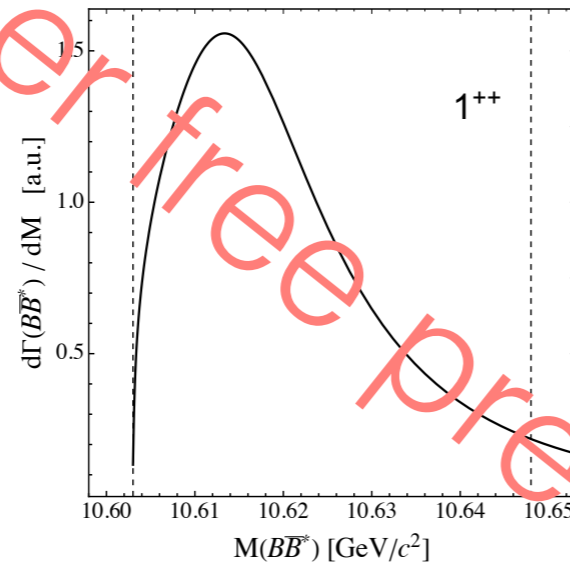
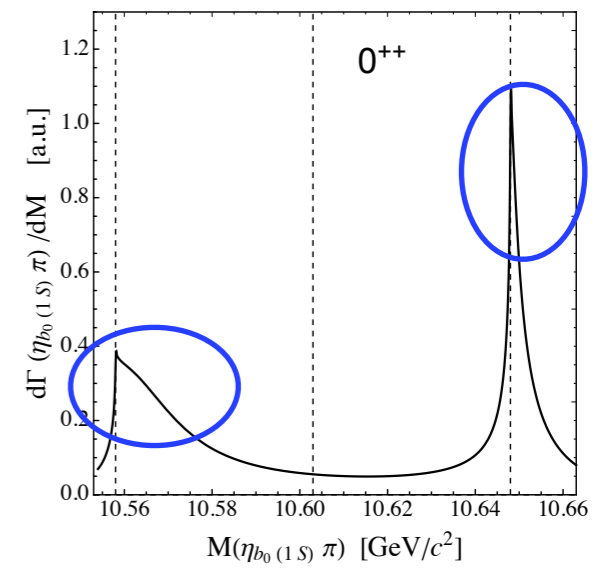
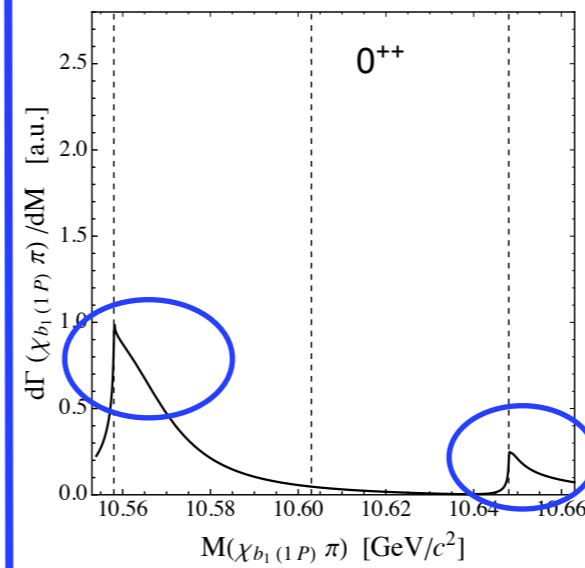
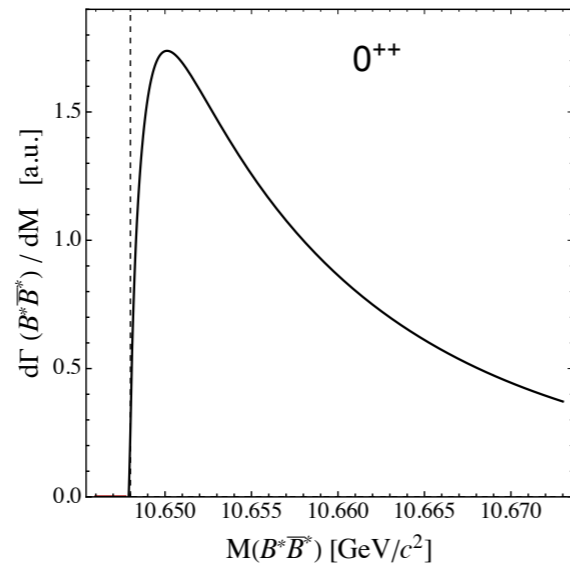
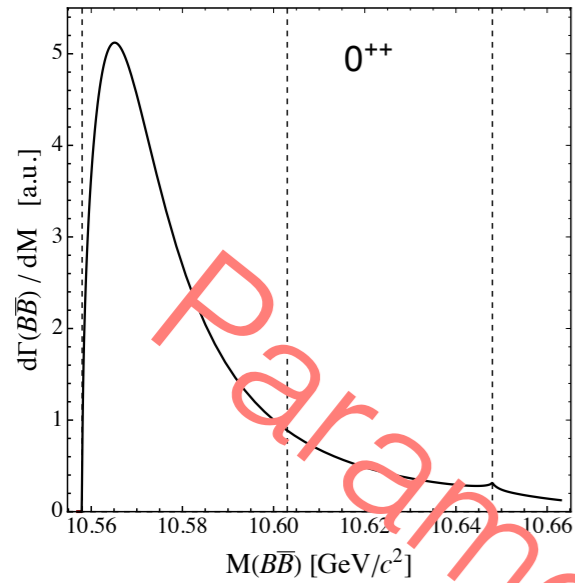


Inelastic channels:

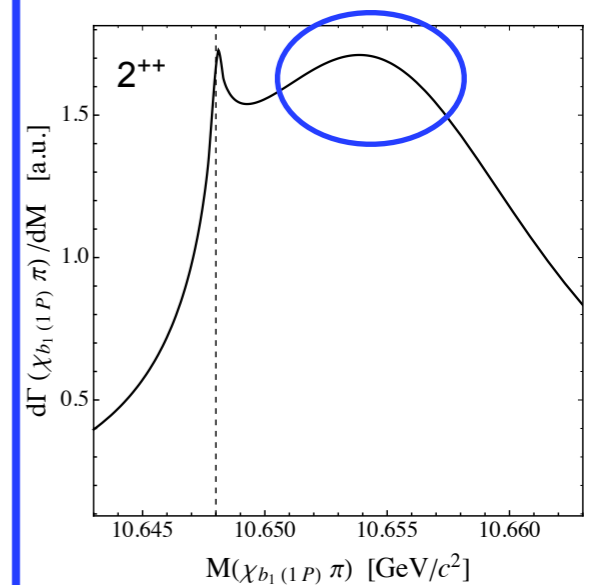
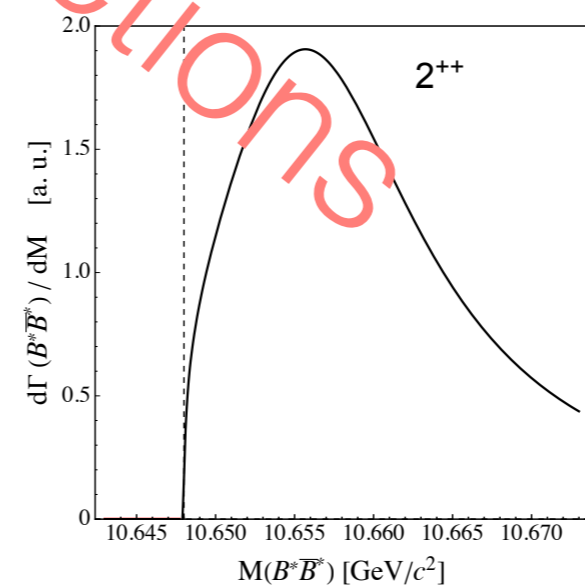
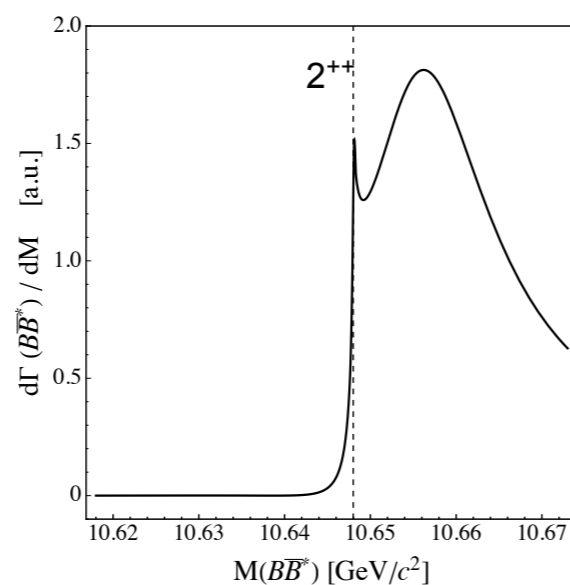
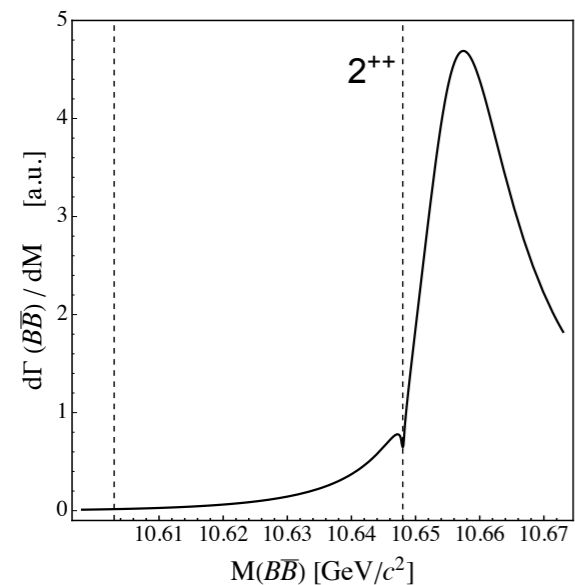
spin partners as peaks
or enhancements
above thresholds



Line shapes for spin partners in $\Upsilon(10860) \rightarrow \gamma W_{bJ} \rightarrow$ final state



Inelastic channels:
spin partners as peaks
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above thresholds



our work:
PRD 99, 094013 (2019)

Parameter free predictions

Pole positions and residues

our work: PRD 99, 094013 (2019)

J^{PC}	State	Threshold	E_B w.r.t. threshold, [MeV]	Residue at pole
1^{+-}	Z_b	$B\bar{B}^*$	$(-2.3 \pm 0.5) - i(1.1 \pm 0.1)$	$(-1.2 \pm 0.2) + i(0.3 \pm 0.2)$
1^{+-}	Z'_b	$B^*\bar{B}^*$	$(1.8 \pm 2.0) - i(13.6 \pm 3.1)$	$(1.5 \pm 0.2) - i(0.6 \pm 0.3)$
0^{++}	W_{b0}	BB	$(2.3 \pm 4.2) - i(16.0 \pm 2.6)$	$(1.7 \pm 0.6) - i(1.7 \pm 0.5)$
0^{++}	W'_{b0}	$B^*\bar{B}^*$	$(-1.3 \pm 0.4) - i(1.7 \pm 0.5)$	$(-0.9 \pm 0.3) - i(0.3 \pm 0.2)$
1^{++}	W_{b1}	$B\bar{B}^*$	$(10.2 \pm 2.5) - i(15.3 \pm 3.2)$	$(1.3 \pm 0.2) - i(0.4 \pm 0.2)$
2^{++}	W_{b2}	$B^*\bar{B}^*$	$(7.4 \pm 2.8) - i(9.9 \pm 2.2)$	$(0.7 \pm 0.1) - i(0.3 \pm 0.1)$

All Z_b 's and W_{bJ} 's are:

- **virtual states** in a scheme with just $O(Q^0)$ contact interactions
- **resonances** in a scheme when OPE is included

Conclusion: Z_b 's and W_{bJ} 's are consistent with molecular scenario

Insights into the nature of the Z_b and Z_b'
from $\Upsilon(10860) \rightarrow \Upsilon(nS) \pi^+ \pi^-$ ($n=1,2,3$)

arXiv: hep-ph 2012.05034

10.12.2020

VB, E.Epelbaum, A.A.Filin, C.Hanhart, R.V. Mizuk, A.Nefediev, and S. Ropertz

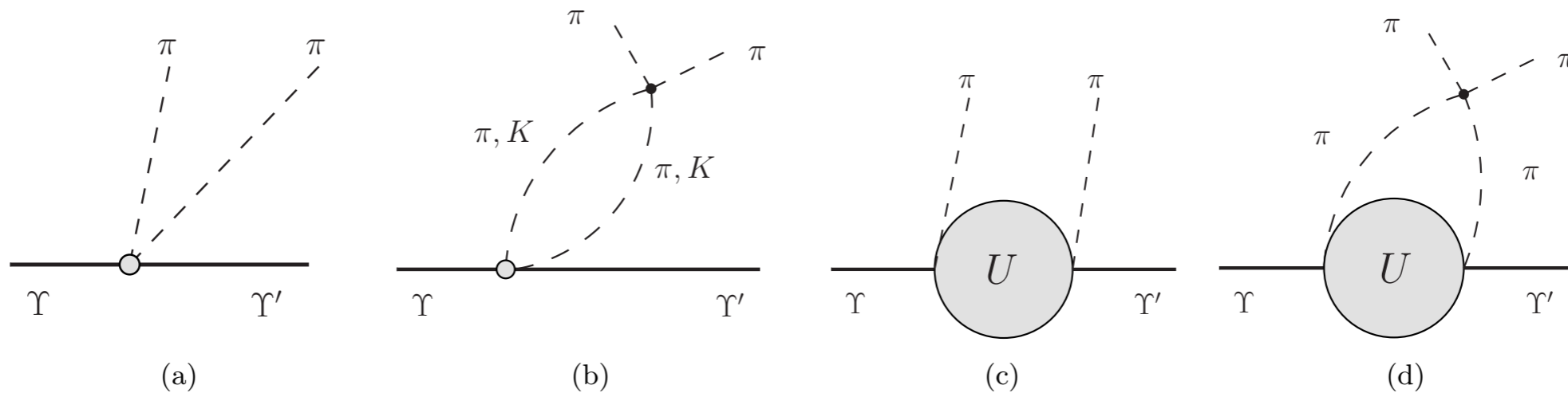
$\Upsilon(10860) \rightarrow \Upsilon(nS) \pi^+ \pi^-$: Goals

VB, E. Epelbaum, A.A. Filin, C. Hanhart, R.V. Mizuk, A. Nefediev, and S. Ropertz

hep-ph 2012.05034

- High-statistic data by Belle
- A significant nonresonant contribution from the $\pi\pi$ system \Rightarrow Dalitz plot analysis

- Production: contact (a-b) and coupled-channel via B-meson loops (c-d) formulated above



Employ U from a simple but realistic contact scheme

- Dispersive approach to account for the $\pi\pi$ - KK final-state interaction (FSI),
 KK component is especially important for $\Upsilon(1S)$
- Dalitz plot analysis of Belle data
- Check consistency with previous results

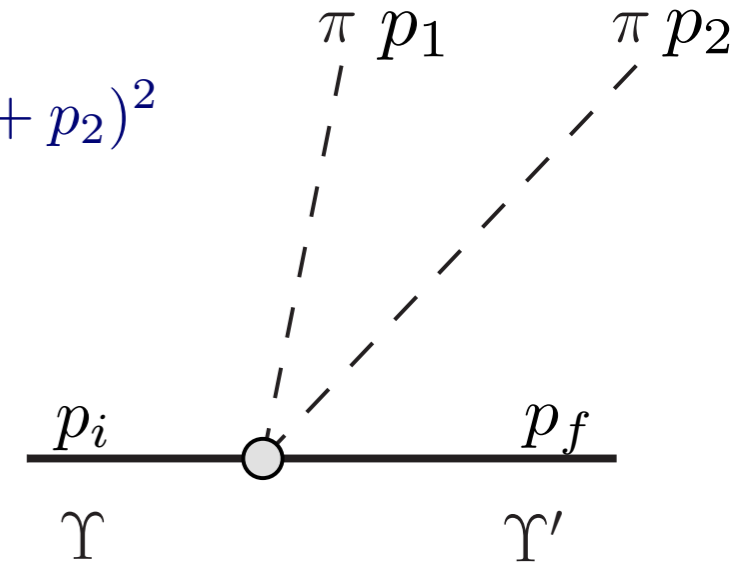
Kinematics for $\Upsilon(10860) \rightarrow \pi^+ \pi^- \Upsilon(nS)$

Mandelstam variables:

$$s = (p_1 + p_2)^2, \quad t = (p_f + p_1)^2, \quad u = (p_f + p_2)^2$$

$$s + t + u = m_i^2 + m_f^2 + 2m_\pi^2$$

$$p_i^2 = m_i^2, \quad p_f^2 = m_f^2, \quad p_1^2 = p_2^2 = m_\pi^2$$



Kinematical relations:

$$t(s, z) = \frac{1}{2}(m_i^2 + m_f^2 + 2m_\pi^2 - s) + \frac{1}{2}k(s)z$$

$$u(s, z) = \frac{1}{2}(m_i^2 + m_f^2 + 2m_\pi^2 - s) - \frac{1}{2}k(s)z$$

$$z \equiv \cos \theta = \hat{p}_1 \cdot \hat{p}_f$$

helicity angle

$$k(s) = \frac{1}{s} \sqrt{\lambda(s, m_i^2, m_f^2) \lambda(s, m_\pi^2, m_\pi^2)}$$

λ - Källén triangle function

Production amplitude:

$$M^{\text{full}} = M(s, t, u) \epsilon_{\Upsilon(10860)} \cdot \epsilon_{\Upsilon(nS)}^*$$

Double differential production rate:

$$\frac{d^2 \text{Br}}{ds dt} = \mathcal{N} |M(s, t, u)|^2$$

\mathcal{N} - overall normalization

Dispersion relations for $\pi\pi$ -KK FSI

S-wave projection: $M_0(s) = \frac{1}{2} \int_{-1}^{+1} dz M(s, t, u) \equiv M_0^L + M_0^R$

Left-hand cut piece \leftarrow \rightarrow Right-hand cut piece with FSI

Dispersive reconstruction
of $\hat{M}_0^R(s)$ via $\hat{M}_0^L(s)$

$$\hat{M}_0(s) = \hat{M}_0^L(s) + \frac{\hat{\Omega}_0(s)}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\hat{\Omega}_0^{-1}(s') \hat{T}(s') \hat{\sigma}(s') \hat{M}_0^L(s')}{s' - s - i0}$$

$\pi\pi$ -KK scattering amplitude: $\hat{T}(s) = \begin{pmatrix} T_{\pi\pi \rightarrow \pi\pi} & T_{\pi\pi \rightarrow K\bar{K}} \\ T_{K\bar{K} \rightarrow \pi\pi} & T_{K\bar{K} \rightarrow K\bar{K}} \end{pmatrix} = \begin{pmatrix} \frac{\eta e^{2i\delta} - 1}{2i\sigma_\pi} & g e^{i\psi} \\ g e^{i\psi} & \frac{\eta e^{2i(\psi-\delta)} - 1}{2i\sigma_K} \end{pmatrix}$

R. Garcia-Martin et al., PRD83, 074004 (2011), I. Caprini et al., EPJC72, 1860 (2012), P. Buettiker et al., EPJC33, 409 (2004), L.Y.Dai et al., PRD90, 036004 (2014).

multichannel Omnès function:

$$\hat{\Omega}_0(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\hat{T}^*(s') \hat{\sigma}(s) \hat{\Omega}_0(s')}{s' - s - i0}$$

$$\hat{\sigma}(s) = \text{diag}\{\sigma_\pi, \sigma_K\}$$

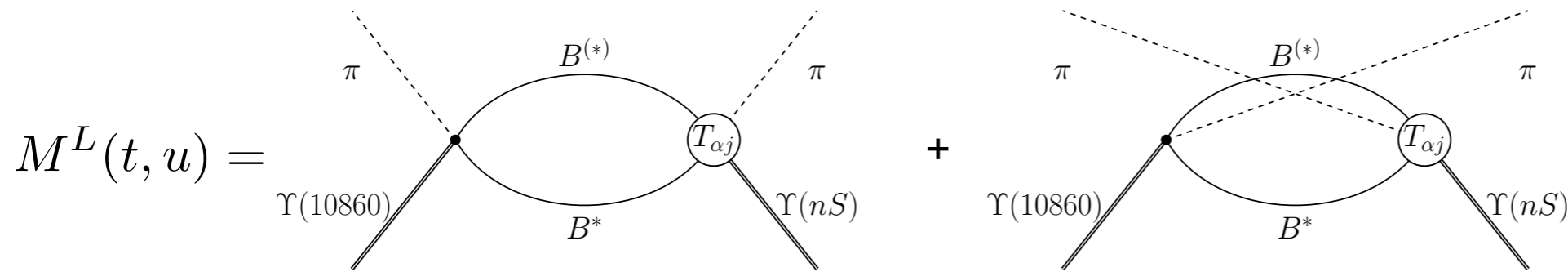
$$\sigma_P(s) = \sqrt{1 - s_P^{\text{th}}/s}$$

$$P = \pi, K$$

Production via $\pi\pi$ mode:

$$\hat{M}_0^L = ([M_0^L]_{\pi\pi}, 0)^T$$

Left-hand cut production amplitude



$Z_b(10610)/Z_b(10650)$ are poles in the coupled-channel amplitudes

$$M^L(t, u) = U(t) + U(u) = -\frac{1}{\pi} \int_{(m_\pi + m_{\Upsilon(1S)})^2}^{\infty} d\mu^2 \text{Im} U(\mu^2) \left(\frac{1}{t - \mu^2} + \frac{1}{u - \mu^2} \right)$$

S-wave
 $M_0^L(s)$

$$M_{0,\text{stable}}^L(s; \mu) = \frac{1}{2} \int dz \left(\frac{1}{t - \mu^2} + \frac{1}{u - \mu^2} \right) \equiv \left(-\frac{2}{\sigma_\pi(s)} \right) \frac{\text{Disc } C_0(s)}{2\pi i} \quad C_0(s, \mu) \equiv C_0(m_i^2, s, m_f^2, \mu^2, m_\pi^2, m_\pi^2)$$

—standard scalar loop function

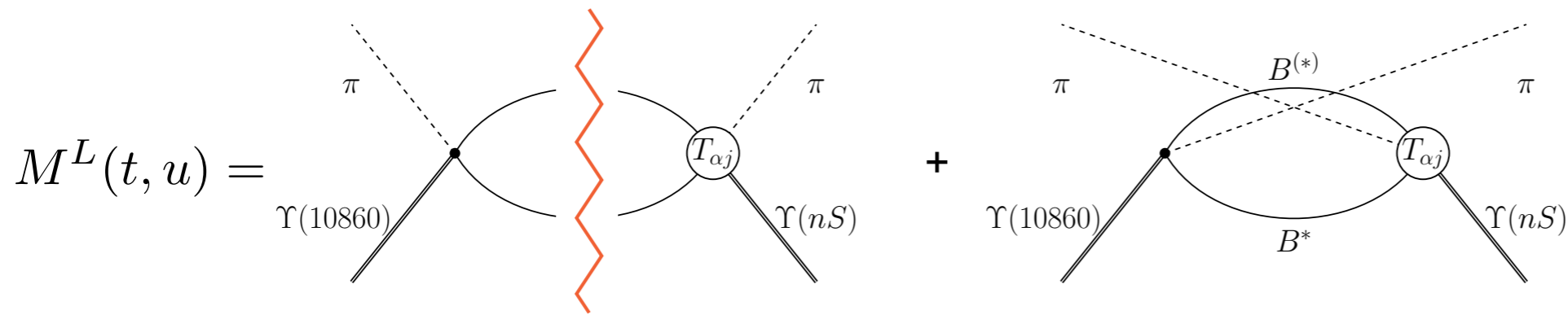
— Take care about anomalous thresholds

- $C_0(s, \mu)$ is an analytic function of μ only if anomalous contributions (AC) are included from the branch point of the Logarithm

AC emerges if $\mu^2 < \mu_{\text{crit}}^2 \equiv \frac{1}{2}(m_f^2 + m_i^2) - m_\pi^2$

For $\Upsilon(3S)$: $\mu_{\text{crit}} = 10.6097 \text{ GeV}$
 very close to **$Z_b(10610)$ pole**

Left-hand cut production amplitude



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$$M^L(t, u) = U(t) + U(u) = -\frac{1}{\pi} \int_{(m_\pi + m_{\Upsilon(1S)})^2}^{\infty} d\mu^2 \text{Im} U(\mu^2) \left(\frac{1}{t - \mu^2} + \frac{1}{u - \mu^2} \right)$$

S-wave



$$M_0^L(s)$$

$$M_{0,\text{stable}}^L(s; \mu) = \frac{1}{2} \int dz \left(\frac{1}{t - \mu^2} + \frac{1}{u - \mu^2} \right) \equiv \left(-\frac{2}{\sigma_\pi(s)} \right) \frac{\text{Disc } C_0(s)}{2\pi i} \quad C_0(s, \mu) \equiv C_0(m_i^2, s, m_f^2, \mu^2, m_\pi^2, m_\pi^2)$$

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$$\text{For } \Upsilon(3S): \mu_{\text{crit}} = 10.6097 \text{ GeV}$$

very close to $Z_b(10610)$ pole

- $\text{Im } M_0^L(s)$: Leading contribution is from the $B^{(*)}\bar{B}^*$ cuts, these states can be on shell subleading one— from inelastic channels

Subtractions and matching to chiral contact amplitudes

$$\hat{M}_0(s) = \hat{M}_0^L(s) + \frac{\hat{\Omega}_0(s)}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\hat{\Omega}_0^{-1}(s') \hat{T}(s') \hat{\sigma}(s') \hat{M}_0^L(s')}{s' - s - i0}$$

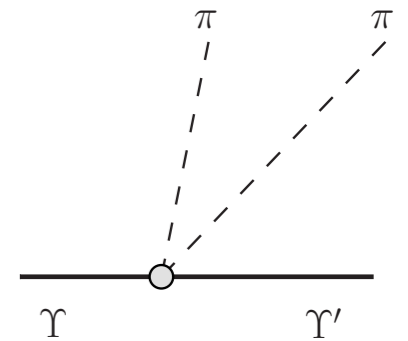
— Dispersive Integral is convergent but details of $\pi\pi$ at large s are known badly

⇒ 2 subtractions with real coefficients

— matching to chiral expansion

$$\mathcal{L}_{\Upsilon\Upsilon'\Phi\Phi} = \frac{c_1}{2} \langle J^\dagger J' \rangle \langle u_\mu u^\mu \rangle + \frac{c_2}{2} \langle J^\dagger J' \rangle \langle u_\mu u_\nu \rangle v^\mu v^\nu + \text{h.c.}$$

$$J = \Upsilon \cdot \sigma + \eta_b$$



$$u_\mu = i (u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) \quad u = \exp\left(\frac{i\Phi}{\sqrt{2}f}\right)$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$$\Rightarrow \hat{M}_0^\chi(s) = \left(M_0^{\chi,\pi\pi}(s), \frac{2}{\sqrt{3}} M_0^{\chi,KK}(s) \right)^T$$

$$M_0^{\chi,PP}(s) = -\frac{2}{f_P^2} \sqrt{m_\Upsilon m_{\Upsilon'}} \left\{ c_1 (s - 2m_P^2) + \frac{c_2}{2} \left[s + q^2 \left(1 - \frac{\sigma_P^2(s)}{3} \right) \right] \right\}$$

Chen et al. PRD93, 034030 (2016),
PRD95, 034022 (2017)

Final results for $M(s,t,u)$

$$M(s, t, u) = \underbrace{M^L(t, u)}_{\substack{\text{coupled-channel} \\ \text{production amplitude,} \\ \text{contains all partial waves}}} + \underbrace{\hat{\Omega}_0(s)}_{\substack{\text{chiral contact} \\ \text{term}}} \left(\underbrace{\hat{M}_0^{\chi, \pi\pi}(s)}_{\substack{\text{chiral contact} \\ \text{term}}} + \underbrace{\hat{I}_0^{(2)}(s)}_{\substack{\text{dispersive} \\ \text{integral}}} \right) + \underbrace{\Omega_2(s)}_{\substack{\text{D-wave contribution}}} \underbrace{M_2^{\chi, \pi\pi}(s)}_{\substack{\text{D-wave contribution}}} P_2(z)$$

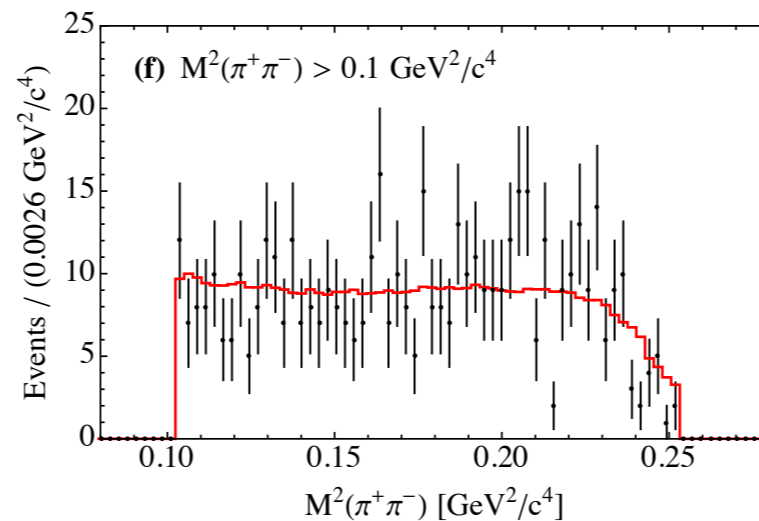
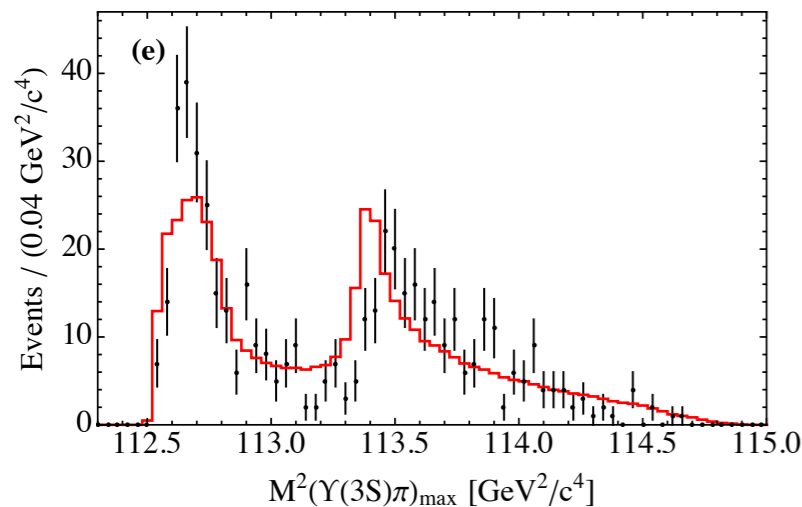
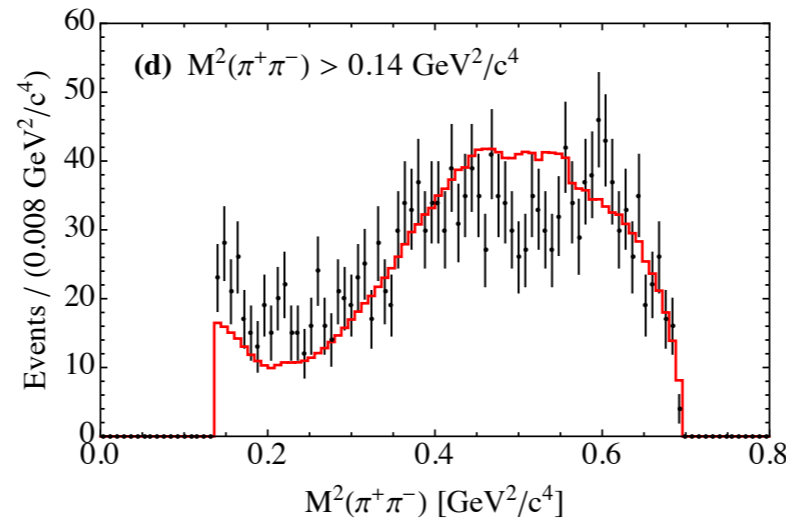
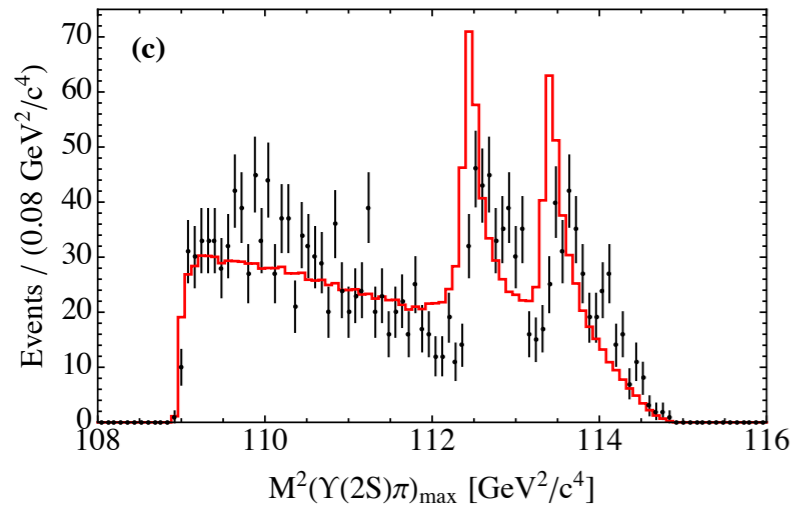
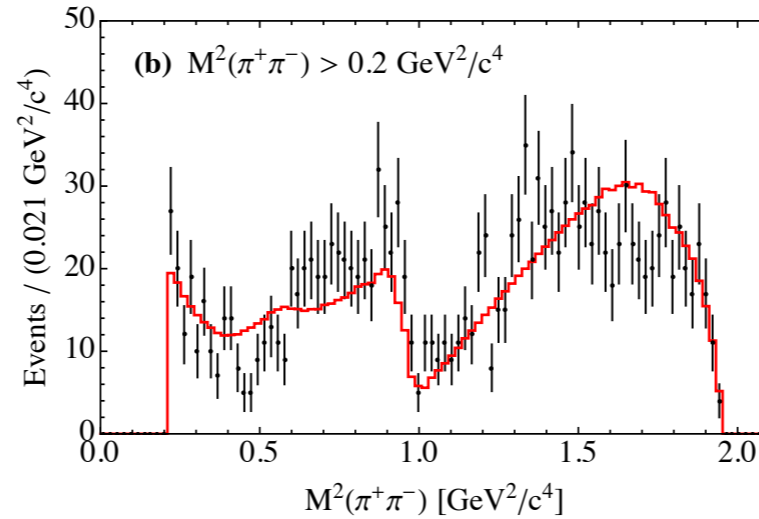
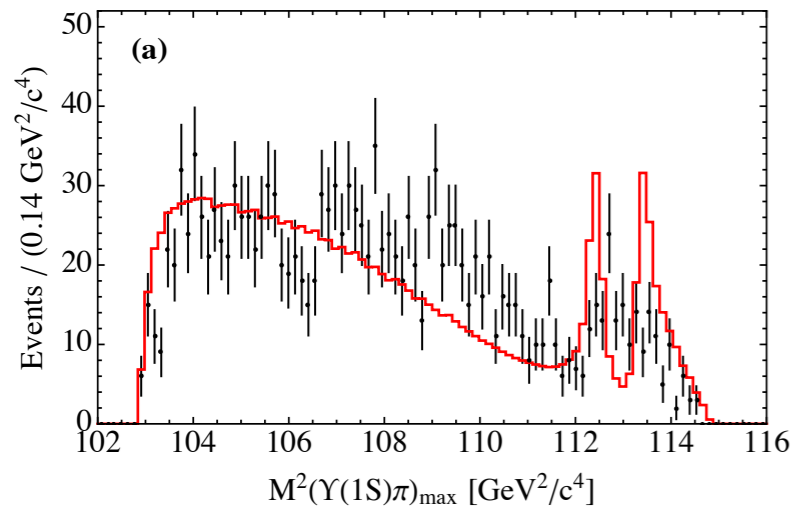
$$\hat{I}_0^{(2)}(s) = \frac{s^2}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\hat{\Omega}_0^{-1}(s') \hat{T}(s') \hat{\sigma}(s') \text{Re} M_s^L(s')}{s' - s - i0} + \frac{i}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\hat{\Omega}_0^{-1}(s') \hat{T}(s') \hat{\sigma}(s') \text{Im} M_s^L(s')}{s' - s - i0} + \hat{I}_0^{\text{anom}}(s)$$

- All the parameters from a coupled-channel approach in $M^L(t,u)$ fixed from data to the decays

$$\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi B^{(*)} \bar{B}^* \quad \text{and} \quad \Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi\pi h_b(mP)$$

- Correct for efficiency and resolution in $M(\pi Y(nS))^2$, add coherent experimental background and make maximum likelihood fits to Dalitz plot
- Parameters in the fits: overall normalization \mathcal{N} and chiral LECs $c1$ and $c2$

Results for $M(\pi\Upsilon(nS))^2$ and $M(\pi\pi)^2$ projections



$\Upsilon(1S)$ and $\Upsilon(2S)$

➡ $\pi\pi$ -KK FSI very important

Key contribution to:

— right shoulder in $M(\pi\pi)^2$

— left shoulder in $M(\pi\Upsilon(nS))^2$ $n=1,2$

— dip region ~ 1 GeV in $M(\pi\pi)^2$

$\Upsilon(3S)$

➡ Completely dominated by
 $M^L(t,u) = U(t)+U(u)$

➡ $\pi\pi$ FSI is not important

➡ Very reasonable overall description

➡ Peaks of the Z_b 's, consistent with $B^{(*)}B^*$ and $\pi\pi h_b(mP)$, are not exactly in accord with $\pi\Upsilon(nS)$

Summary

- Line shapes in c and b -sectors can be systematically analysed within an EFT approach consistent with chiral and heavy quark symmetries, analyticity and unitarity
- We analyse the line shapes $\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi\alpha \implies$ poles and residues of the $Z_b^{(')}$
- Line shapes for spin partners of $Z_b^{(')}$ states and their poles are predicted parameter free $\implies W_{bJ}$ can be searched for at Belle II
 - A similar analysis of the LHCb P_c states from $\Lambda_b \rightarrow K P_c \rightarrow K J/\Psi p$
Talk by Meng-Lin Du on Thursday
- The Dalitz plot analysis of $\Upsilon(10860) \rightarrow \pi Z_b^{(')} \rightarrow \pi\pi\Upsilon(nS)$ including $\pi\pi$ -KK FSI yields very reasonable results with all parameters from $Z_b^{(')}$ fixed

\implies Strong evidence that the line shapes relevant for $Z_b^{(')}$ states can be understood within a molecular scenario!

Next step: A combined analysis of all channels within the same framework