Study of the X(6900) and Compositeness

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1. Introduction

Fully-heavy-flavor tetraquark meson candidate X(6900) has been recently observed by LHCb Coll. Science Bulletin 2020 65(23) 1983

Di - J/ψ spectrum

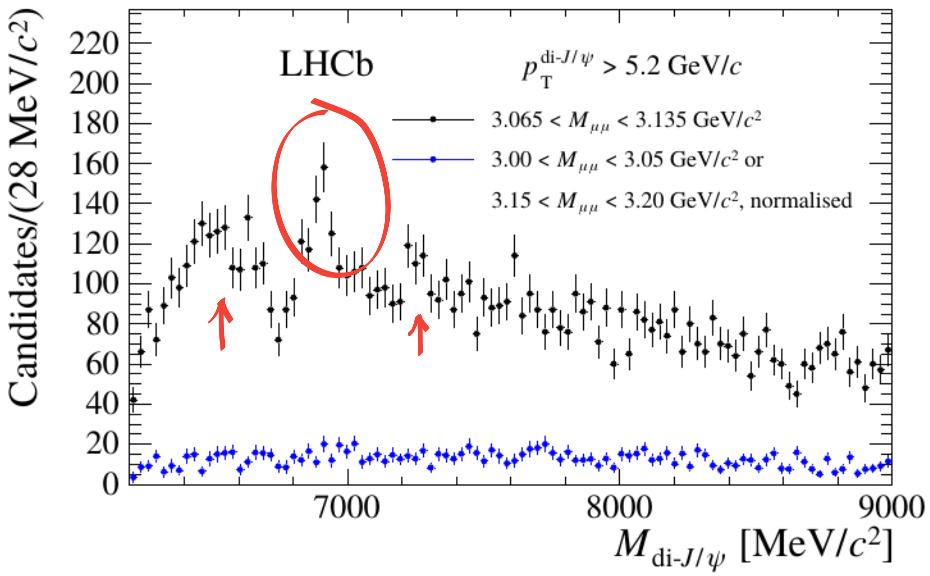


Figure 2: Invariant mass spectrum of J/ψ-pair candidates passing the $p_T^{\text{di-}J/\psi} > 5.2\,\text{GeV}/c$ requirement with reconstructed J/ψ masses in the (black) signal and (blue) background regions, respectively.

* Quark models
  $\bar{c}\bar{c}cc$
* QCD sum rules

* Dynamical features
  J/ψ, $\psi(2S)$, $\psi(3770)$
  η_c, χ_{c1}, χ_{c0}, ...

Cusp effects

Pomeron exchange

We have applied and developed related approaches to study the nature of several XYZ states in recent years.

No specific dynamical model is assumed.

Key words:

S-matrix Theory

Unitarity

CDD poles

Coupled channels

ERE

Compositeness X

$\Gamma = \sum_i \Gamma_i$ Width Saturation

$P_c(4450)$: Meißner, JAO, PLB751, 51 (2015) — X, Γ

$\Lambda_c(2595)^+$: Z.-H. Guo, JAO, PRD93, 054014 (2016) — CDD, ERE; X, Γ

$Z_b(10610)$, $Z_b(10650)$: X.-W. Kang, Z-H. Guo, J.A.O., PRD 94, 014012 (2016) — ERE; X, Γ

$X(3872)$: X.-W. Kang, J.A.O., EPJC 77, 399 (2017) — CDD, X, ERE; Event Distribut.

$Z_c(3900)$, $X(4020)$, $X_{c_1}(4140)$, $\psi(4260)$, $\psi(4660)$: R. Gao, Z.-H. Guo, X.-W. Kang, J.A.O, Adv. H.E.Phys. (2019) ERE, X

$P_c(4312)$, $P_c(4440)$, $P_c(4457)$: Z.-H. Guo, J.A.O., PLB 793, 144 (2019) ERE; X, Γ

One could also learn a good deal of phenomenology by studying the consistency among the different ideas.

Compositeness. Non-relativistic Analysis

I am going to follow J.A.O., Ann. Phys. 396, 429 (2018)

Number Operator for the free particle spicies A, $\hat{N}_A = \sum_{k} A_k^\dagger A_k = \int d^3k\, \psi_A^\dagger(x)\, \psi_A(x)$

Interaction picture

$$X_A = \frac{\langle B|\hat{N}_A|B\rangle}{n_A} \qquad n_A \equiv \text{Valence Number}$$

Examples: Deuteron $n_A = 2$

$$X_N = \frac{1}{2}\langle D|\hat{N}_N|D\rangle \qquad \hat{N}_N = \int d^3x\, \psi_N^\dagger \psi_N \,, \quad \psi_N = \begin{pmatrix} p \\ n \end{pmatrix}$$

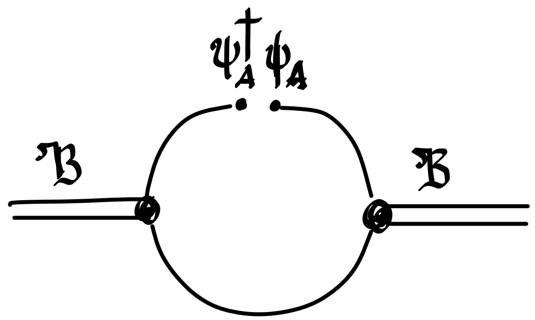
$X(6900)$ $n_V = 2$

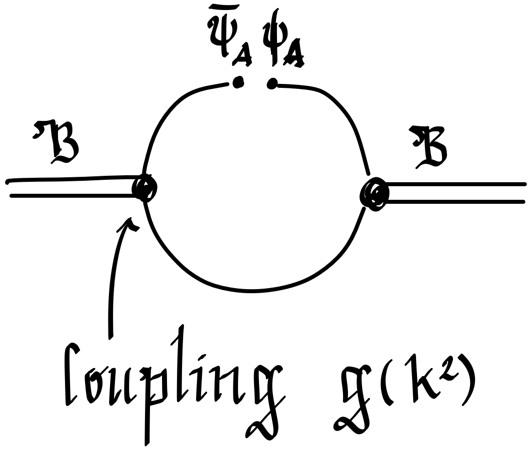
$\hat{N}_{\chi_{c0}}\,,\ \hat{N}_{\chi_{c1}}$

QFT:

$$X_A = \frac{1}{n_A}\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} d^4x\, \langle B|\, P\left[e^{-i\int d^4x' \mathcal{L}_{int}}\ \psi_A^\dagger(x)\,\psi_A(x)\right] |B\rangle$$

↑ Time-ordered Product.



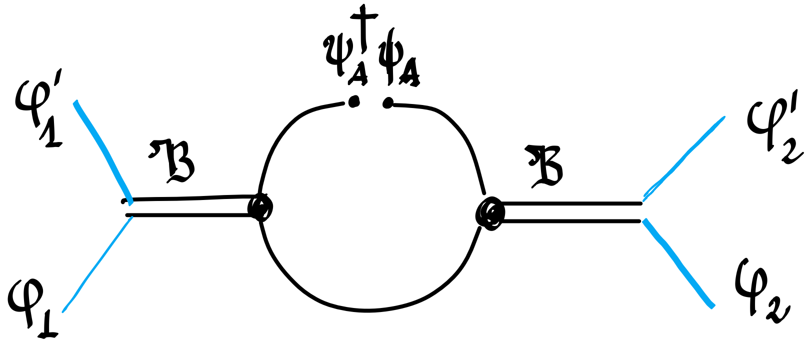


$$\chi_A = \int \frac{d^3k}{(2\pi)^3} \frac{g(k^2)^2}{\left(\frac{k^2}{2\mu} - E_B\right)^2}$$

$$g(k^2) = \frac{1}{2\pi^2} \int_0^\infty dk'\, k'^2 \, \frac{V(k,k')\, g(k'^2)}{E_B - \frac{k'^2}{2\mu}} \Bigg|_{E = E_B}$$

It has no right-hand cut

Relation with the T Matrix:



1) scattering with insertion of $\psi_A^\dagger \psi_A$.
2) Isolate double-pole contribution.

$$T(E \to E_B) \to -\frac{g_1^{on}\, g_2^{on}}{E - E_B}$$

Partial-wave amplitudes

$$A = \frac{g_1^{on}\, \chi_A\, g_2^{on}}{(E - E_B)^2}$$

Calculation in dimensional regularization:

$$g(k^2)^2 = g(\varkappa^2)^2 + c_1(k^2-\varkappa^2) + c_2(k^2-\varkappa^2)^2 + \ldots$$

$$G(E) = \int_0^\infty \frac{k^2\,dk}{2\pi^2}\,\frac{1}{k^2/2\mu - E} \qquad \varkappa^2 = 2\mu E_B .$$

$$X_A = \int \frac{d^3k}{(2\pi)^3}\,\frac{g(k^2)^2}{\left(\frac{k^2}{2\mu} - E_B\right)^2} = g(\varkappa^2)^2\,\frac{\partial G(E_B)}{\partial E_B} + c_1(2\mu)^2\int_0^\infty \frac{dk}{2\pi^2}\,\frac{k^2}{k^2-\varkappa^2} + \underbrace{c_2(2\mu)^2\int_0^\infty \frac{dk}{2\pi^2}\,k^2}_{0}$$

$$+ \underbrace{c_3(2\mu)^2\int_0^\infty \frac{dk}{(2\pi)^2}(k^2-\varkappa^2)k^2}_{0} + \ldots$$

$$X_A = g(\varkappa^2)^2\,\frac{\partial G(E_B)}{\partial E_B} + \frac{\partial g(\varkappa^2)^2}{\partial \varkappa^2}\,\frac{\mu^2|\varkappa|}{\pi}$$

For near-threshold bound states $\quad X_A \simeq \underbrace{g(\varkappa^2)^2\,\frac{\partial G(E_B)}{\partial E_B}}_{} + \mathcal{O}\left(\frac{\varkappa^2}{\Lambda^2}\right)$

↳ Specially Good Approximation

Heavy-quark meson $\quad X(6900) \quad \bar{c}\bar{c}cc$

scattering: Suppression of light-meson exchanges

Unitarized Partial-Wave Amplitudes: I am following Guo, J.A.O, PRD 93 (2016)

$$T = \left[V^{-1} - G\right]^{-1} \qquad \frac{\partial T}{\partial E} = -T\left(\frac{\partial G}{\partial E} - V^{-1}\frac{\partial V}{\partial E}V^{-1}\right)T \Rightarrow \frac{g\, g^T}{(E-E_B)^2}$$

For $E \to E_B$

Sum Rule:

$$1 = \sum_i g_i^2 \frac{\partial G_i}{\partial E} + \sum_{i,j} g_i \frac{\partial V_{ij}}{\partial E} g_j$$

$$\equiv \sum_i X_i + Z$$

We recover the previous approximation for the compositeness.

Resonances

Resonance pole E_R in another Riemann sheet.

g_i^2, $G(E_R)$ are complex

Absolute Value $\quad X_i = \left| g_i^2 \frac{\partial G_i}{\partial E} \right|_{E=E_R}$. $\quad$ Why taking the absolute value?

Source Theory : Henly, Thirring, "Elementary QFT", McGraw-Hill.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + g\rho(\vec{r},t)\phi \quad , \quad \rho(\vec{r},t) \text{ External source.}$$

Relation between Virtual Particles (Cloud) and Radiated (Real) ones

$\phi(\vec{r},t=0)$ $\qquad\qquad$ $\phi(\vec{r},t\to+\infty)$

$$\phi(\vec{r},t) = \phi^{in}(\vec{r},t) + \int d^3r'dt' \, \Delta^{ret}(\vec{r}-\vec{r}',t-t')\,\rho(\vec{r}',t')$$

$$= \phi^{out}(\vec{r},t) + \int d^3r'dt' \, \Delta^{adv}(\vec{r}-\vec{r}',t-t')\,\rho(\vec{r}',t')$$

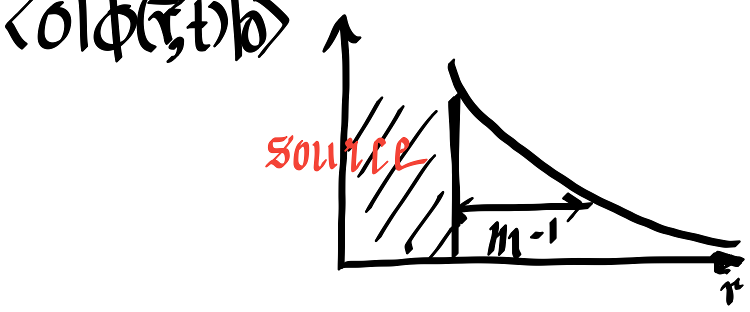
Feynman Propagator

$$\phi^{out}(\vec{r},t) = \phi^{in}(\vec{r},t) + \int d^3r'dt' \, \Delta(\vec{r}-\vec{r}',t-t')\,\rho(\vec{r}',t') \quad ; \quad \Delta = \Delta^{ret} - \Delta^{adv}$$

① Static source. Number and distribution of virtual particles:

$$a_{\vec{k}}(t=0) = A^{in}_{\vec{k}} + \frac{g\rho_{\vec{k}}}{w_k} \quad ; \quad w_k = \sqrt{m^2+\vec{k}^2} \qquad \rho_{\vec{k}} = \int d^3k\, e^{-i\vec{k}\vec{r}}\rho(\vec{r})$$

$$a_{\vec{k}}|0\rangle_{in} = \frac{g\rho_{\vec{k}}}{w_k}|0\rangle_{in} \longrightarrow \bar{n} = \int \frac{d^3k}{(2\pi)^3 2w_k^3}|g\rho_{\vec{k}}|^2 \qquad \phi(\vec{r},t) = \phi^{out}_{in}(\vec{r},t) + g\int d^3r' \frac{e^{-m|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}\rho(\vec{r}') .$$

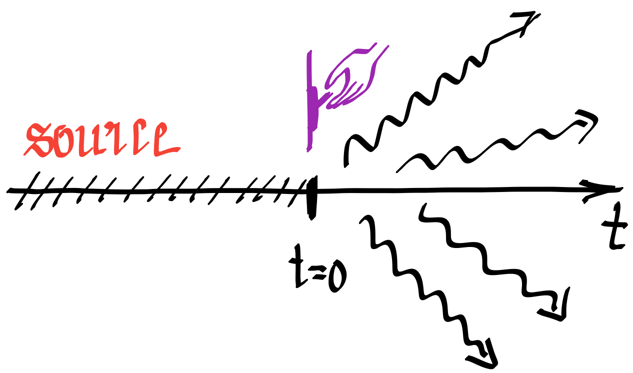


$$\bar{n} = \int \frac{d^3k}{(2\pi)^3 2\omega_k^3} |g\rho_{\vec{k}}|^2$$

Number of virtual particles in the cloud

② Sudden Event

$$\rho(\vec{r},t) = \begin{cases} 0 & , t>0 \\ g\rho(\vec{r}) & , t<0 \end{cases}$$



$$\rho_{\vec{k}}(k^0) = \int d^3r'\, dt'\, e^{ik^0t'} e^{-i\vec{k}\cdot\vec{r}} \rho(\vec{r},t) = \frac{g\rho_{\vec{k}}}{ik^0}.$$

$$A^{out}_{\vec{k}} = A^{in}_{\vec{k}} + i\rho_{\vec{k}}(\omega_k)$$

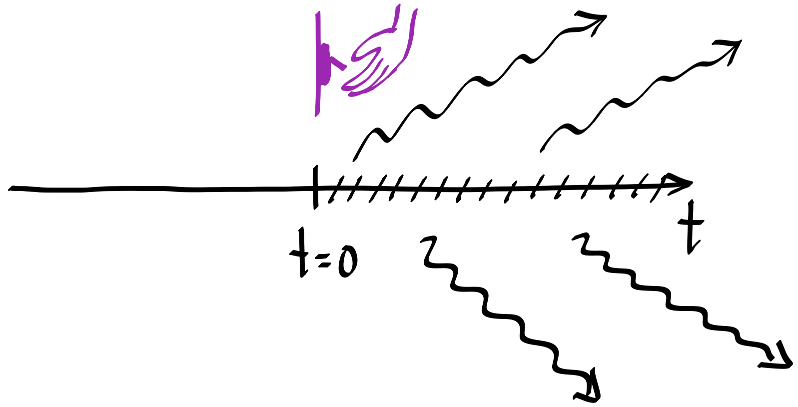
Number of radiated particles $\boxed{\bar{N} = \bar{n}}$

$$\bar{N} = \int \frac{d^3k}{(2\pi)^3 2\omega_k} |\rho_{\vec{k}}(\omega_k)|^2 = \int \frac{d^3k}{(2\pi)^3 2\omega_k^3} |g\rho_{\vec{k}}|^2 .$$

All the virtual particles become real particles and are radiated

Mimicking a decaying particle

③ $$\rho(\vec{r},t) = \begin{cases} g\rho(\vec{r})\, e^{-iM_R t}\, e^{-\frac{\Gamma}{2}t} & , t>0 \\ 0 & , t<0 \end{cases}$$



$$\rho_{\vec{k}}(k^0) = i\,\frac{g\rho_{\vec{k}}}{\left(k^0 - M + i\frac{\Gamma}{2}\right)\left(k^0 + M + i\frac{\Gamma}{2}\right)}$$

$$\bar{N} = \bar{n} = \int \frac{d^3k\, |g\rho_{\vec{k}}|^2}{(2\pi)^3 2\omega_k}\, \frac{\omega_k^2 + \Gamma^2/4}{\left[(M+\omega_k)^2 + \frac{\Gamma^2}{4}\right]\left[(M-\omega_k)^2 + \frac{\Gamma^2}{4}\right]}$$

All the radiated particles stem from the virtual particles at $t=0$.

In the limit $\Gamma \to 0$

$$\bar{N} = \bar{n} \to \frac{1}{4}\int \frac{d^3k}{(2\pi)^3 2\omega_k}\, \frac{|g\rho_{\vec{k}}|^2}{(M-\omega_k)^2 + \frac{\Gamma^2}{4}}$$

Effective Range Expansion:

$$T_{II}(E) = \left[a^{-1} + \frac{r}{2} k^2 - i \sqrt[II]{2\mu E} \right]$$

2nd Riemann sheet $\sqrt[II]{z} = \sqrt{|z|}\, e^{i\frac{\varphi}{2}}$ $\varphi \in [2\pi, 4\pi]$

Resonance Pole: $k_R = \sqrt{2\mu E_R} \equiv k_r + i k_i$, $k_i < 0$; $0 > \frac{a}{2} > r$

$$\lim_{k \to k_R} T \to \frac{\gamma_k^2}{k - k_R}$$

$\gamma_k^2 = -\frac{k_i}{k_r} = \text{tg}\phi \leq 1$ $\phi \in [0, \frac{\pi}{4}]$

For $M_R > 0$ (ctgϕ otherwise)

$$X = -i\gamma_k^2 \longrightarrow |X| = \gamma_k^2 \leq 1$$

$$= \left(\frac{2r}{a} - 1 \right)^{-\frac{1}{2}}$$

Probalistic interpretation.

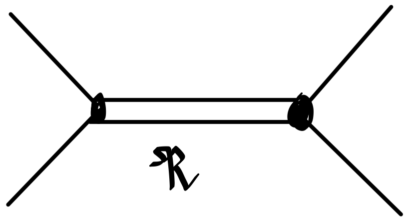
One should not take $\Re(X)$. It is always zero.

$$X + Z = 1 \to \Re X + \Re Z = 1, \quad \Im X + \Im Z = 0$$

S-matrix transformations

Guo, J.A.O, PRD 93 (2016)

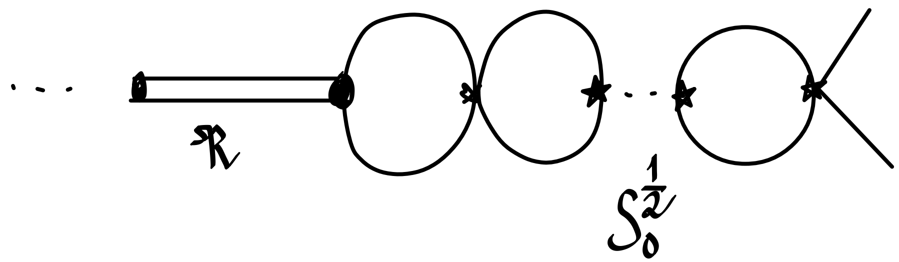
$$S = S_0^{\frac{1}{2}}\left[I - \frac{i\, w\, w^T}{s - s_R}\right] S_0^{\frac{1}{2}}$$



Resonant decay

Vector of couplings $\quad w \propto \begin{bmatrix} |g_1^{on}| \\ \vdots \\ |g_n^{on}| \end{bmatrix}$

The non-resonant terms generate rescattering in the resonant process



$$S_0^{-\frac{1}{2}}\, S\, S_0^{-\frac{1}{2}} = I - i\,\frac{w\, w^T}{s - s_R}$$

Pure resonant S-matrix

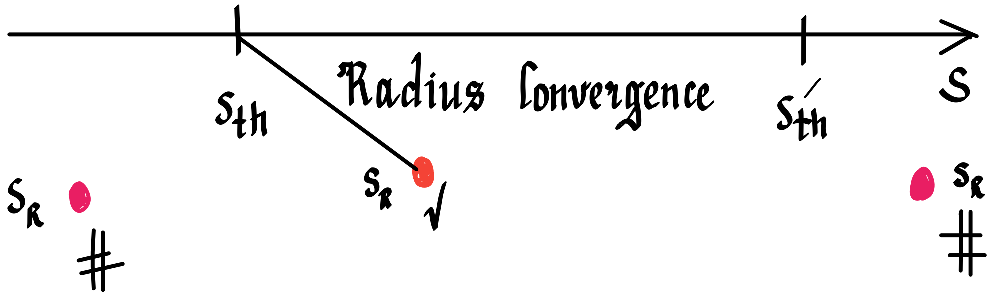
In terms of it X_i is not yet real because $\dfrac{\partial G_i(s_R)}{\partial s_R}$ is complex $\longrightarrow$ Extra diagonal unitary transformation

$$e^{i\phi_i}\, |g_i^{on}|^2\, \frac{\partial G_i(s_R)}{\partial s_R} = |g_i^{on}|^2 \left|\frac{\partial G_i(s_R)}{\partial s_R}\right| = |X_i|$$

Interpretation of Vector of Couplings

$$W \propto \begin{bmatrix} |g_1^{on}| \\ \vdots \\ |g_n^{on}| \end{bmatrix}$$

This requires the resonant effects to be continued analytically on the physical axis



Condition:

$$s_{th} < \mathrm{Re}\, s_R < s'_{th}$$

Recall condition for the CKE study $M_R > M_{th}$

$$X = -\frac{k_i}{k_r} = tg\phi \le 1 \quad , \quad tg 2\phi = \frac{\Gamma}{2|E_R - M_{th}|}$$

Dynamical Study for the X(6900)

Partial-wave amplitude without left-hand cut

$$T = \left[\sum_i \frac{\gamma_i}{s - m^2_{i,CDD}} - G(s) \right]^{-1}$$

J.A.O, Oset, PRD 60 (1999)

$\uparrow$ Source for extra structure near the threshold

$\uparrow$ Right-hand cut. Branch point singularity.

☆ Since we have only one resonance → only one CDD pole is included

☆ This is more general than the ERE

$$\frac{\gamma}{k^2 - k_0^2} = -\frac{\gamma}{k_0^2} - \frac{\gamma}{k_0^4} k^2 + \ldots$$

It blows up for $k_0^2 \to 0$.

Recall that ERE is more general than an elastic Flatté parameterization.

Kang, Guo, J.A.O, PRD 94 (2016)

$3^{1/2}$ channels are included :

Threshold [MeV]

X(6900)

I : $M = 6905 \pm 11 \pm 7$ $\Gamma = 80 \pm 19 \pm 33$

II : $M = 6886 \pm 11 \pm 11$ $\Gamma = 168 \pm 33 \pm 69$

(1)	$J/\psi\, J/\psi$	6193.8	$(\eta_c \eta_c)$
(2)	$\chi_{c0}\, \chi_{c0}$	6829.4	
(3)	$\chi_{c1}\, \chi_{c1}$	7021.3	

$$V(s) = \begin{bmatrix} 0 & b_{12} & b_{12}/\sqrt{3} \\ b_{12} & \frac{b_{22}}{M^2_{J/\psi}}(s - M^2_{CDD}) & \frac{b_{22}/\sqrt{3}}{M^2_{J/\psi}}(s - M^2_{CDD}) \\ b_{12}/\sqrt{3} & \frac{b_{22}/\sqrt{3}}{M^2_{J/\psi}}(s - M^2_{CDD}) & \frac{b_{22}/3}{M^2_{J/\psi}}(s - M^2_{CDD}) \end{bmatrix}$$

S-wave scattering

Heavy-quark symmetry $1/\sqrt{3}$

Subtraction constant in $G(s)$:

$$x_\pm = \frac{s + m_1^2 - m_2^2}{2s} \pm \frac{q_j(s)}{\sqrt{s}} \; ; \quad G(s) = \frac{-1}{16\pi^2}\left\{ a(\mu^2) + \log\frac{m^2}{\mu^2} - x_+ \log\frac{x_+ - 1}{x_+} - x_- \log\frac{x_- - 1}{x_-} \right\}$$

Natural size estimate : $a(\Lambda^2) = -2\log\left(1 + \sqrt{1 + \frac{m_X^2}{\Lambda^2}}\right) \simeq -3$

Matching at threshold with $G_\Lambda(s)$, $\Lambda \simeq 1$ GeV , a three-momentum cutoff .

T-Matrix: $T(s) = [I - V(s)\cdot G(s)]^{-1}\cdot V(s)$

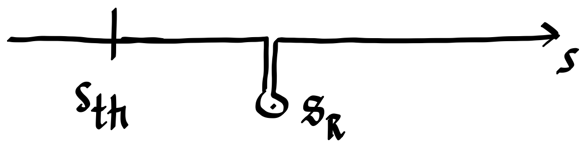
J/ψ J/ψ Event distribution: $$\frac{dN(s)}{d\sqrt{s}} = |B_1(s)|^2 \frac{q_1}{M^2_{J/\psi}}$$

Production Amplitudes: $$B(s) = \underbrace{[I - V(s)\,G(s)]^{-1}}_{\text{Final-State interactions}}\cdot P(s)$$

$$P(s) = \begin{bmatrix} 0 \\ d_2 \\ d_2/\sqrt{3} \end{bmatrix}$$

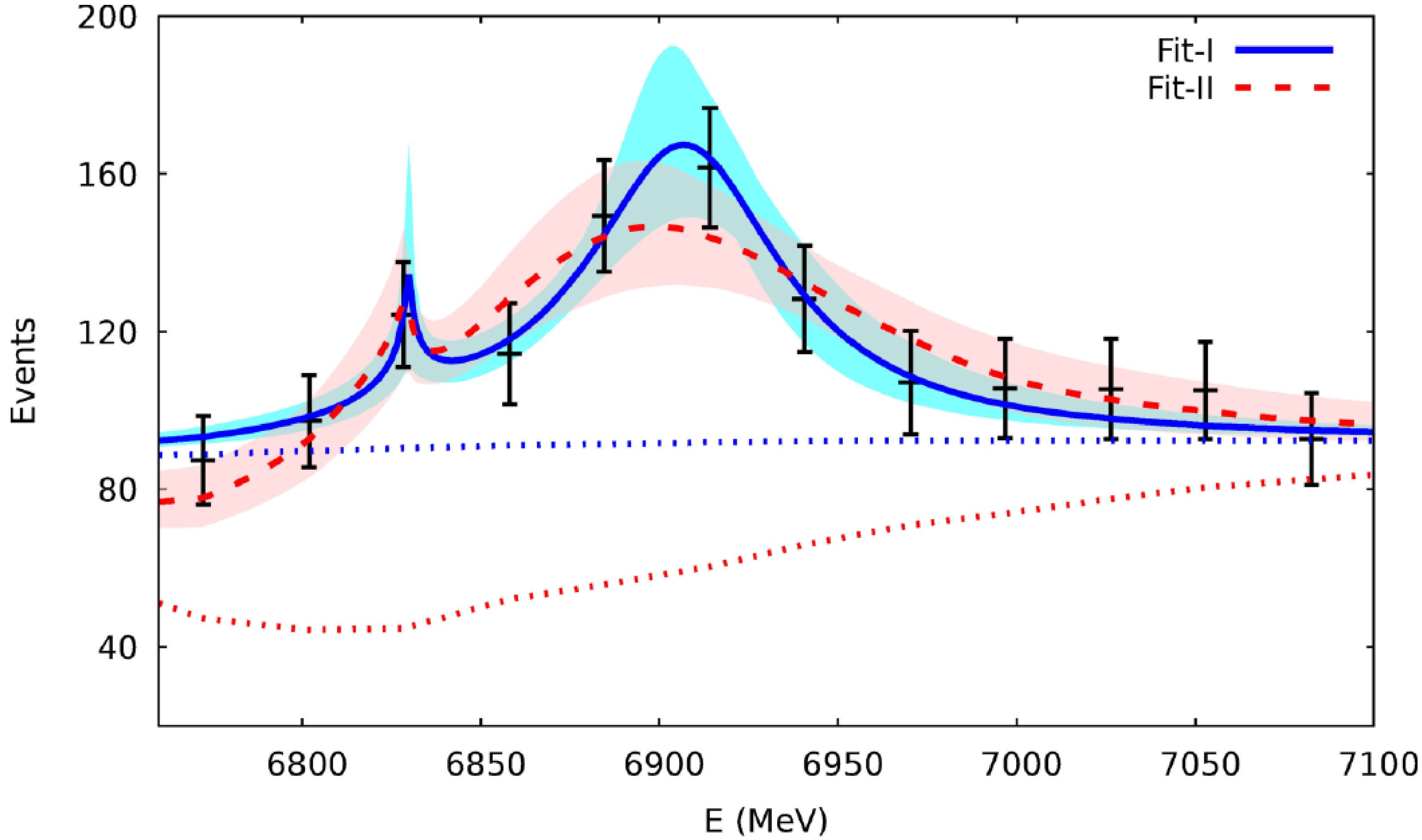
4 free parameters: b_{12}, d_{22}, M^2_{CDD}, d_2

Resonant Poles:



$$G_j^{II}(s) = G_j^{I}(s) - \frac{i\, q_j(s)}{4\pi\sqrt{s}}$$

1st $(+,+,+)$ 3rd $(-,-,+)$ 4th $(+,-,+)$ 5th $(-,-,-)$ Riemann sheets



	χ^2/d.o.f	$a(\mu)$	M_{CDD}	b_{22}	b_{12}	d_2
Fit-I	$1.6/(12-3)$	-3.0^*	6910^*	10817^{+8378}_{-2096}	151^{+153}_{-99}	2213^{+2106}_{-316}
Fit-II	$4.9/(12-3)$	-3.0^*	6885^*	21073^{+15141}_{-7359}	484^{+239}_{-112}	3645^{+1325}_{-714}

Residua at the resonance pole: $T_{kj} \underset{s \to s_R}{\Longrightarrow} - \frac{g_k^{on} g_j^{on}}{s - s_R}$

	Mass (MeV)	Width/2 (MeV)	$\lvert g_1^{on} \rvert$ (GeV)	$\lvert g_2^{on} \rvert$ (GeV)	$\lvert g_3^{on} \rvert$ (GeV)	X_1	X_2	X_3	$X = \sum_{i=1}^{3} X_i$
Fit-I	$M_{CDD}=6910$ 6907^{+5}_{-3}	33^{+14}_{-10}	$4.6^{+2.5}_{-2.8}$	$9.7^{+1.4}_{-2.6}$	$5.6^{+0.8}_{-1.5}$	$0.01^{+0.01}_{-0.01}$	$0.13^{+0.04}_{-0.06}$	$0.03^{+0.01}_{-0.01}$	$0.17^{+0.04}_{-0.07}$
Fit-II	6885 6892^{+2}_{-2}	80^{+24}_{-17}	$10.3^{+1.8}_{-1.4}$	$6.9^{+1.4}_{-1.9}$	$4.0^{+0.8}_{-1.1}$	$0.05^{+0.02}_{-0.01}$	$0.06^{+0.03}_{-0.03}$	$0.01^{+0.01}_{-0.01}$	$0.13^{+0.03}_{-0.03}$

Total compositeness $X < 0.2$ Overwhelming bare component

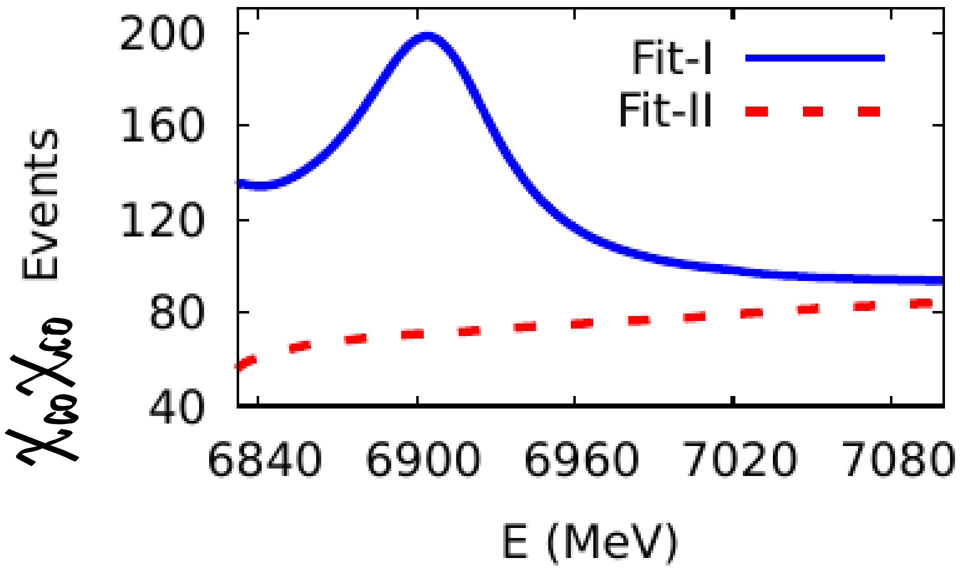
$X(6900)$ LHCb

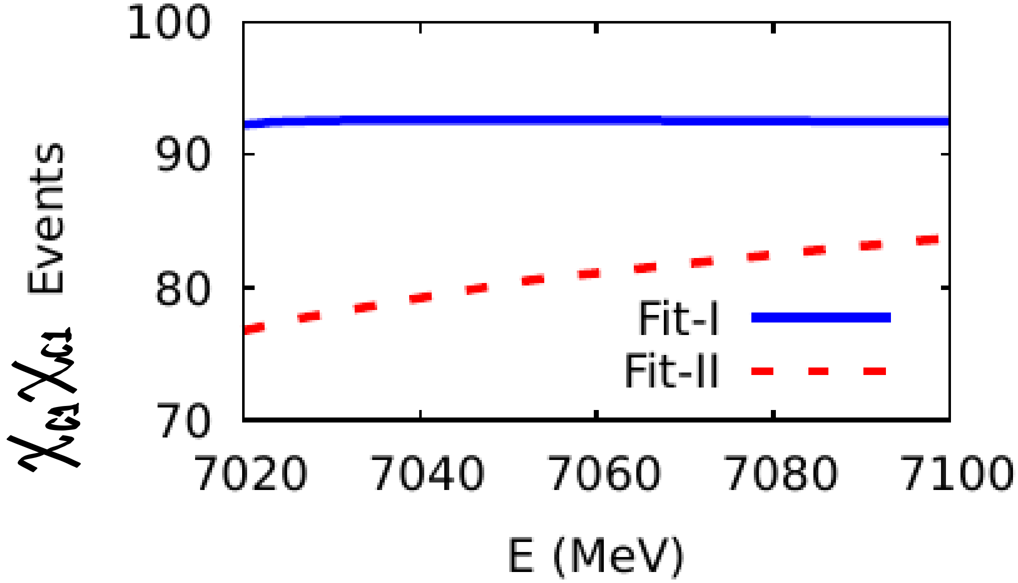
I: $M = 6905 \pm 11 \pm 7$ $\Gamma = 80 \pm 19 \pm 33$

II: $M = 6886 \pm 11 \pm 11$ $\Gamma = 168 \pm 33 \pm 69$

In agreement $M_{CDD} \simeq M_R$
which drives the Morgan's pole-counting criterion

— Similar pole positions in 5th Riemann sheet. $G^{II}_{\chi_{c1}\chi_{c1}}$



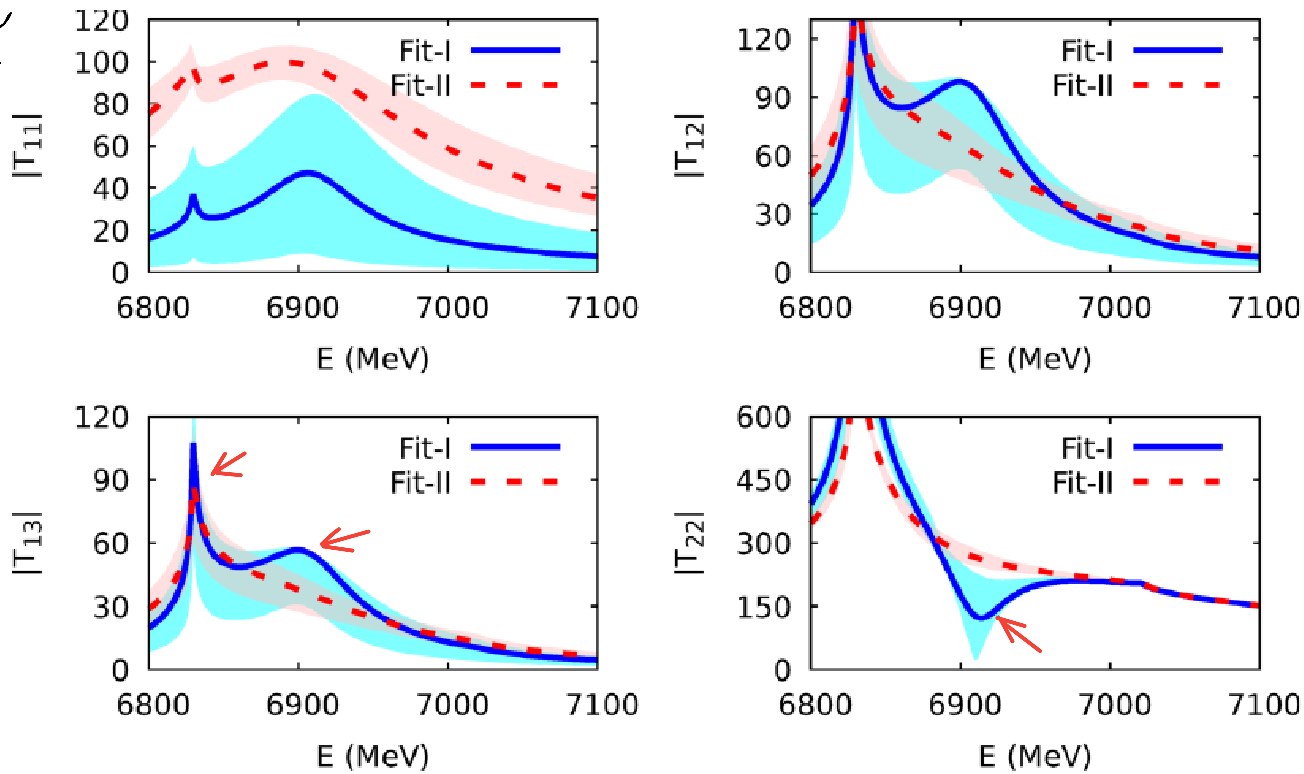


Its measurement could discriminate between Fits I and II

$\chi_{c0}\chi_{c0}$ cusp effect

The resonance is much more manifest for Fit I

Dip for $\chi_{c0,1}\chi_{c0,1} \to \chi_{c0,1}\chi_{c0,1}$



Saturation of width and compositeness:

$$X = X_1 + X_2 + X_3 = |g_1^{on}|^2 \left|\frac{\partial G_1^{II}}{\partial s}\right| + |g_2^{on}|^2 \left|\frac{\partial G_2^{II}}{\partial s}\right| + \left|\frac{g_2^{on}}{\sqrt{3}}\right|^2 \left|\frac{\partial G_3}{\partial s}\right| \quad \text{at } s = s_R$$

$$\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 = |g_1^{on}|^2 \frac{q_1}{8\pi M_R^2} + |g_2^{on}|^2 \int_{M_{th,2}}^{M_R + n\Gamma} dw \frac{q_2(w)}{16\pi^2 w^2} \frac{\Gamma}{(M_R - w)^2 + \Gamma^2/4}$$

$$+ \frac{|g_2^{on}|^2}{3} \int_{M_{th,3}}^{M_R + n\Gamma} dw \frac{q_3(w)}{16\pi^2 w^2} \frac{\Gamma}{(M_R - w)^2 + \Gamma^2/4}$$

From the pole position: M_R, Γ
but X is not known a priori.

Channel	$\lvert g_1^{on}\rvert$ (GeV)	$\lvert g_2^{on}\rvert$ (GeV)	Γ_1 (MeV)	Γ_2 (MeV)	Γ_3 (MeV)	X_1	X_2 Largest	X_3
$X(6900)$-I								
$X = 0.1$	$7.1^{+2.2}_{-2.1}$	$6.8^{+0.6}_{-0.8}$	$64.7^{+42.5}_{-33.4}$	$15.2^{+5.3}_{-4.6}$	$0.1^{+0.3}_{-0.1}$	$0.02^{+0.02}_{-0.01}$	$0.06^{+0.01}_{-0.01}$	$0.01^{+0.00}_{-0.00}$
$X = 0.4$	$0.6^{+5.7}_{-0.5}$	$15.4^{+0.5}_{-0.5}$	$0.4^{+49.7}_{-0.4}$	$79.2^{+11.1}_{-12.5}$	$0.4^{+1.8}_{-0.4}$	$0.00^{+0.02}_{-0.01}$	$0.33^{+0.01}_{-0.01}$	$0.07^{+0.01}_{-0.01}$
$X(6900)$-II								
$X = 0.1$	$11.3^{+2.7}_{-3.5}$	$5.0^{+1.6}_{-3.1}$	$160.7^{+83.3}_{-83.6}$	$7.0^{+6.8}_{-6.1}$	$0.3^{+0.1}_{-0.3}$	$0.06^{+0.03}_{-0.03}$	$0.03^{+0.03}_{-0.03}$	$0.01^{+0.01}_{-0.01}$
$X = 0.4$	$8.9^{+3.0}_{-5.5}$	$15.0^{+0.5}_{-1.0}$	$100.9^{+79.0}_{-86.2}$	$64.3^{+13.7}_{-16.9}$	$2.8^{+3.7}_{-2.6}$	$0.04^{+0.03}_{-0.03}$	$0.31^{+0.03}_{-0.03}$	$0.06^{+0.01}_{-0.01}$
$X = 0.9$	$1.0^{+6.6}_{-1.0}$	$23.7^{+1.5}_{-1.4}$	$1.3^{+71.9}_{-1.3}$	$159.9^{+44.4}_{-38.9}$	$6.8^{+10.1}_{-4.8}$	$0.00^{+0.03}_{-0.00}$	$0.76^{+0.02}_{-0.02}$	$0.14^{+0.01}_{-0.02}$

For increasing X the Γ_2, X_2 increase; Γ_1, X_1 decrease

Taking X from the dynamical study plus fit to data:

Fit I: $|g_1^{on}| = 6.2$ GeV, $|g_2^{on}| = 9.5$ GeV, $\Gamma_1 = 49.7$ MeV $\quad X_1 = 0.02$

$\Gamma_2 = 30.1$ MeV $\quad X_2 = 0.13$

$\Gamma_3 = 0.2$ MeV $\quad X_3 = 0.03$

Fit II: $|g_1^{on}| = 11.1$ GeV, $|g_2^{on}| = 6.7$ GeV, $\Gamma_1 = 154.7$ MeV $\quad X_1 = 0.06$

$\Gamma_2 = 12.8$ MeV $\quad X_2 = 0.06$

$\Gamma_3 = 0.5$ MeV $\quad X_3 = 0.01$

The consistency indicates that the

Branching ratios are calculated properly

ERE study

$\chi_{c0}\chi_{c0}$ uncoupled S-wave scattering

$$T = \left(-\frac{1}{a} + \frac{1}{2} r k^2 - ik\right)^{-1}$$

Consistency requirement $X = \left(\frac{2r}{a} - 1\right)^{-1/2} \simeq 1$

From the total compositeness + width saturation study:
If X increases then Γ_1 and X_1 decrease.

A near threshold CDD implies

$$r = -\frac{\gamma}{\mu(M_{CDD} - M_{th})^2} \longrightarrow \infty$$

$$\delta a = -\frac{M_{th} - M_{CDD}}{\gamma} \longrightarrow 0$$

$\Big\}\ X \underset{\#}{\longrightarrow} 0$

Resonance	Mass (MeV)	Width (MeV)	Threshold (MeV)	a (fm)	r (fm)	X
$X(6900)$-I	6905 ± 13	80 ± 38	$\chi_{c0}\chi_{c0}$ (6829.4)	-0.18 ± 0.07	-1.52 ± 0.69	0.25 ± 0.11
$X(6900)$-II	6886 ± 16	168 ± 77	$\chi_{c0}\chi_{c0}$ (6829.4)	-0.32 ± 0.06	-0.72 ± 0.26	0.53 ± 0.16

We take into account next the decay branching ratio into $\chi_{c0}\chi_{c0}$

Adjust the pole position $M_R - i\frac{\Gamma_2}{2}$ removing the $J/\psi J/\psi$ decay width

The real part of the self-energy is driven by the nearest threshold
$\Rightarrow M_R$ is kept with the same value

Fit I: $\Gamma_2 = 40^{+11}_{-20}$ MeV, $a = -0.1$ fm, $r = -3.0$ fm, $X = 0.13$

Fit II: $\Gamma_2 = 26^{+11}_{-14}$ MeV, $a = -0.09$ fm, $r = -4.1$ fm, $X = 0.10$

From the dynamical study and fit to the data $X_2 = 0.13^{+0.04}_{-0.06}$ (Fit I)

$X_2 = 0.06 \pm 0.03$ (Fit II)

Great Compatibility
between different approaches

Conclusions

We have discussed compositeness for non-relativistic systems for bound states and resonances

Number operator, Source theory, ERE, S-matrix transformations

Dynamical study for the $X(6900)$:

Near-threshold S-matrix parameterization with one CDD pole

No specific dynamical model is assumed

This is more general than an ERE or a Flatté parameterization

Compositeness $X < 0.2$

Overwhelming bare nature of the X(6900)

Connection with the Morgan's counting-pole rule

the ERE study

the compositeness and width saturation approach

All these approaches are sensibly and sensitively compatible