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Revisiting the a_0 resonances in photon-photon scattering

Bachir Moussallam

in collaboration with:
Junxu Lu (Beihang U., Beijing)

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Introduction:

- a_0 resonances seen in recent photon-photon experiment [Belle (2009)] different from previous results ?

- $a_0(980)$ seen long ago [Astier(1967)] (in $K_0 K^-$) ,[Ammar(1969)] (in $\pi\eta$)
  - Shape strongly distorted by $K\overline{K}$ threshold
  - Ambiguity in width determination [Flatté (1976)]
  - Lattice QCD [J.Dudek et al.,PR D93,094506 (2016)]: $a_0(980)$ pole on 4th Riemann sheet ?

- $a_0(1450)$ indications from multimeson final states [$\bar{p}p \to \eta\pi\pi, K\overline{K}\pi$], broad resonance ($\Gamma \simeq 265$ MeV)

- Belle claims:
  - $a_0(980)$ has a Breit-Wigner shape
  - $a_0(1450)$ narrow resonance: $\Gamma = 65^{+2.1}_{-5.4}$ MeV

- Our approach: photon-photon amplitudes are "simple" FSI problem
  - Apply theoretical techniques (ChPT, dispersive/Omnès)
  - Perform combined fits $\gamma\gamma \to \pi\eta, K\overline{K}$
  - Extract information on strong T-matrix elements $[\pi\eta \to \pi\eta,\ \pi\eta \to K\overline{K},\ K\overline{K} \to K\overline{K}]$
  - Resonances : extrapolate to complex energies

ChPT approach to photon-photon:

- First calculation: $\gamma\gamma \to \pi^0\pi^0$ [J.Bijnens,F.Cornet, NP B296(1988)557]
  - No tree contribution, loop is finite

$$H_{++}(s,t) = \frac{1}{8\pi^2}\left(\frac{s-m_\pi^2}{F_\pi^2}\,G_\pi(s) + \frac{s}{4F_\pi^2}G_K(s)\right)$$

  - Simple structure, loop function:

$$G_P(s) = -1 - \frac{m_P^2}{s}\left(\log\frac{\sqrt{1-4m_P^2/s}+1}{\sqrt{1-4m_P^2/s}-1}\right)^2$$

  - When $s \to 0$: $G_P(s) \sim s/(12m_P^2)$ soft photon theorem

- $\gamma\gamma \to \pi^0\eta$ very similar [Ll.Amettler, PL B278,185 (1992)]

$$L_{++}(s,t) = \frac{1}{8\pi^2}\left(\frac{\sqrt{3}(9s - m_\pi^2 - 8m_K^2 - 3m_\eta^2)}{36F_\pi^2} G_K(s) + \frac{B_0(m_d - m_u)}{3\sqrt{3}(m_\eta^2 - m_\pi^2)F_\pi^2}(4m_\pi^2 - 3s)G_\pi(s)\right)$$

  - Soft pion theorem: Adler zero $s_A = m_\eta^2 + O(m_\pi^2)$
  - Isospin breaking part: relevant for $\eta \to \pi^0\gamma\gamma$

- One loop is not enough
  - Resummation of higher loops: replace tree-level amplitudes by unitarized amplitudes [J.Oller,E.Oset NP A629(1998)739]
  - More general: dispersion relations/Omnès method [M.Gourdin,A.Martin, Nuov.Cim.17(1960)224, D. Morgan, M. Pennington, PL B192(1987)207]

Analyticity:

- Structure of partial waves can be complicated
  [J. Kennedy, T.D. Spearman PR D126,1596 (1961)]

  But quite simple for two-photons amplitudes:
  $l_{0++}(s)$, $k_{0++}(s)$: analytic functions of s

  - One left-hand cut: $[-\infty, 0]$
  - One right-hand cut $[(m_\eta + m_\pi)^2, \infty]$

- Asymptotic behaviour ?
  Integrate over $z = \cos(\theta)$ from 0 to 1
  - $z \simeq 0$: $t \sim u \sim -s \to -\infty$ [S.Brodsky,G.Lepage, PR D24,1808 (1981), M.Diehl,P.Kroll,C.Vogt, PL B532,99 (2002)]

$$L_{++}(s,t) << L_{+-}(s,t) \sim \frac{\alpha_s(-s)}{s}$$

  - $z \simeq 1$:Regge regime

$$L_{++}(s,t) \sim \beta_V(t)(\alpha' s)^{\alpha_V + \alpha' t}$$

  - Conclusion: partial wave $l_{0++}(s) \lesssim \sqrt{s}$

- Dispersion relations with one subtraction:

$$l_{0++}(s) = s\left[\frac{1}{\pi}\int_{-\infty}^{s_V} ds' \frac{\mathrm{Im}\,[l_{0++}(s')]}{s'(s'-s)} + \frac{1}{\pi}\int_{m_+^2}^{\infty} ds' \frac{\mathrm{Im}\,[l_{0++}(s')]}{s'(s'-s)}\right]$$

$$k^1_{0++}(s) = k^{1,Born}_{0++}(s) + s\left[\frac{1}{\pi}\int_{-\infty}^{s_{K^*}} ds' \frac{\mathrm{Im}\,[k^1_{0++}(s')]}{s'(s'-s)} + \frac{1}{\pi}\int_{m_+^2}^{\infty} ds' \frac{\mathrm{Im}\,[k^1_{0++}(s')]}{s'(s'-s)}\right]$$

  - $k^{1,Born}_{0++}(s)$: $\gamma\gamma \to K^+K^-$ QED Born amplitude ($I = 1$)
  - Soft-photon theorem implemented: no free parameter
  - Left-cut: light resonance exchanges : ρ, ω, K^*,... in cross-channels $\gamma\pi \to \gamma\eta$, $\gamma K \to \gamma K$

Unitarity:

- Two-channel unitarity ($\sqrt{s} \lesssim 1.4 GeV$)

$$\mathrm{Im}\begin{pmatrix} l_{0++}(s) \\ k_{0++}(s) \end{pmatrix} = \begin{pmatrix} T^*_{11}(s) & T^*_{12}(s) \\ T^*_{21}(s) & T^*_{22}(s) \end{pmatrix} \begin{pmatrix} \sigma_{\pi\eta}(s) & 0 \\ 0 & \sigma_{KK}(s) \end{pmatrix} \begin{pmatrix} l_{0++}(s) \\ k_{0++}(s) \end{pmatrix}$$

  - Generate Muskhelishvili-type integral equations [R.Omnès, Nuov.Cim.8, 316 (1958)]
  - Amplitudes expressed in terms of Omnès-Muskhelishvili matrix $\boldsymbol{\Omega}(s)$
  - Ω-matrix must be determined numerically from T-matrix

- Omnès with two subtraction parameters:
(parameters absorb high-energy contributions)

$$\begin{pmatrix} l_{0++}(s) \\ k^1_{0++}(s) \end{pmatrix} = \begin{pmatrix} 0 \\ k^{1,Born}_{0++}(s) \end{pmatrix} + s\,\Omega(s) \begin{pmatrix} b_l + L_1(s) + R_1(s) \\ b_k + L_2(s) + R_2(s) \end{pmatrix}$$

– $L_1(s)$, $L_2(s)$: left-cut integrals

$$L_i(s) = \frac{s - s_A}{\pi} \int_{-\infty}^{s_V} \frac{ds'}{s'(s'-s_A)(s'-s)} D_{i1}(s')\,\mathrm{Im}\,[l^V_{0++}(s')] + \frac{s - s_A}{\pi} \int_{-\infty}^{s_{K^*}} \frac{ds'}{s'(s'-s_A)(s'-s)} D_{i2}(s')\,\mathrm{Im}\,[k^V_{0++}(s')]$$

with $D_{ij}(s) = [\Omega^{-1}]_{ij}$

– $R_1(s)$, $R_2(s)$: integrals of Born amplitude

$$R_i(s) = -\frac{s - s_A}{\pi} \int_{4m_K^2}^{\infty} \frac{ds'}{s'(s' - s_A)(s' - s)} \, \mathrm{Im}\left[D_{i2}(s')\right] k_{0++}^{1,Born}(s')$$

– Adler zero at $s = s_A$ in l_{0++} :

$$\Omega_{11}(s_A) b_l + \Omega_{12}(s_A) b_k = 0$$

Two-channel T-matrix model ($J = 0, I = 1$)

[M. Albaladejo, B.M., EPJ C75,488 (2015)]

- K-matrix representation: $\mathbf{T}(s) = (1 - \mathbf{K}(s)\boldsymbol{\Phi}(s))^{-1}\mathbf{K}(s)$
  - Unitarity: $\mathbf{K}(s)$ real in $s > (m_\eta + m_\pi)^2$

$$\mathrm{Im}\,\boldsymbol{\Phi}(s) = \begin{pmatrix} \sigma_{\pi\eta}(s) & 0 \\ 0 & \sigma_{KK}(s) \end{pmatrix}$$

  - $\mathbf{K} = \mathbf{K}_{(2)} + \mathbf{K}_{(4)} + \mathbf{K}_{(6)}$
    $\mathbf{K}_{(2)}, \mathbf{K}_{(4)}$ chiral exp. from matching to $\mathbf{T}_{(2)} + \mathbf{T}_{(4)}$
  - matching of $T_{(4)}^{K\overline{K}\to K\overline{K}}(s)$ approximate

- 6 Parameters:

$$[K_{(6)}]_{ij}(s) = \lambda \frac{g_i g_j}{16\pi}\left(\frac{1}{m_8^2 - s} - \frac{1}{m_8^2}\right)$$

and

$$\boldsymbol{\Phi}(s) = \begin{pmatrix} \alpha_1 + \beta_1 s + 16\pi \bar{J}_{\pi\eta}(s) & 0 \\ 0 & \alpha_2 + \beta_2 s + 16\pi \bar{J}_{KK}(s) \end{pmatrix}$$

model allows for two resonance poles below 1.5 GeV

- 7 parameters in $\gamma\gamma$ amplitudes:
  - 1 param. in Omnès S-waves (assuming Adler zero $s_A \simeq m_\eta^2$)
  - 6 D-wave parameters (couplings of a_2, f_2, m_{a_2}, Γ_{a_2})

Experimental data

- $d\sigma_{\gamma\gamma\to\pi^0\eta}/dz$: 448 data points in range $[0.85 - 1.39]$ GeV [S. Uehara et al. (Belle), PR D80,032001 (2009)]

- $d\sigma_{\gamma\gamma\to K_S K_S}/dz$: 240 data points in range $[1.105 - 1.395]$ GeV [S. Uehara et al.(Belle), PTEP 12,123C01 (2013)]

- Also considered $\sigma_{\gamma\gamma\to K^+K^-}$: 7 data points [H. Albrecht et al. (ARGUS), Z.Phys. C48,183 (1990)]

  NOTE: must combine $\gamma\gamma \to (K\overline{K})_{I=1}$ with $\gamma\gamma \to (K\overline{K})_{I=0}$, taken from [R. García-Martín, B.M., EPJ C70,155 (2010)]

Fits to the experimental data

- Good fits are obtained provided all parameters (6+7) are varied

- Two different minimums found by MINUIT
  Fit I: $\chi^2 = 428$

  Fit II: $\chi^2 = 439$

	α_1	$\beta_1(\mathrm{GeV}^{-2})$	α_2	$\beta_2(\mathrm{GeV}^{-2})$	$m_8(\mathrm{GeV})$	λ
fit I	4.00(8)	−2.23(4)	−0.545(5)	0.167(6)	1.304(4)	0.47(4)
fit II	0.98(3)	−4.07(1)	−0.495(1)	−0.18(1)	0.900(2)	1.064(1)

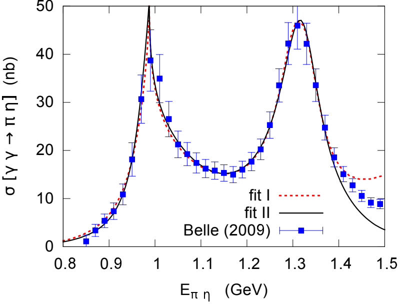
→ Pole mass m_8 quite different

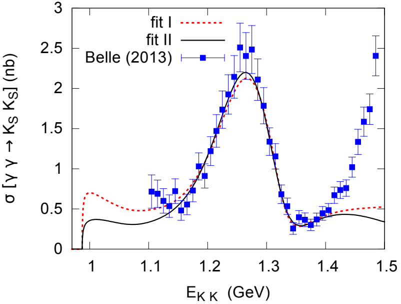
Resonance poles

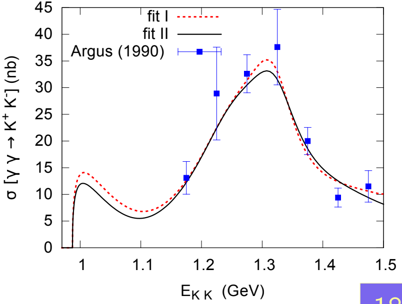
$M - i\Gamma/2$ (MeV)	Fit I	Fit II
$a_0(980)$	1020.3-i 49.3	1000.7-i 36.6
$a_0(1450)$	1314.4-i 24.5	1420.9-i 174.4

- $a_0(980)$ pole on 2nd Riemann sheet
- $a_0(980)$ mass slightly higher than PDG
  Differs from [Danilkin et al., PR D96,114018 (2017)]
  4th sheet: $\sqrt{s_{pol}} = 1120^{+70}_{-20} - i140^{+40}_{-70}$ (MeV).
- Belle's narrow $a_0(1450)$ recovered in fit I

- Integrated cross-sections:





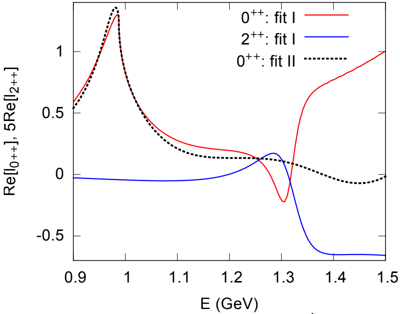


→ $a_0(980)$: cusp peak at $\sqrt{s} = 2m_{K^+} = 987.354$ (MeV)

→ Cross-section $\gamma\gamma \to K_S K_S$ small [Oller,Oset (1998)]

→ Not much difference between the two fits

- Why no sharp $a_0(1450)$ peak in Fit I ?



→ Coherent interference between $J^{\lambda\lambda'} = 0^{++}$ and 2^{++} amplitudes. Angular functions:

$$d^0_{00}(\theta) - d^2_{00}(\theta) = \sqrt{2} d^2_{20}(\theta)$$

No S-wave effect left !

→ Coincidence of fast energy variations seem unphysical

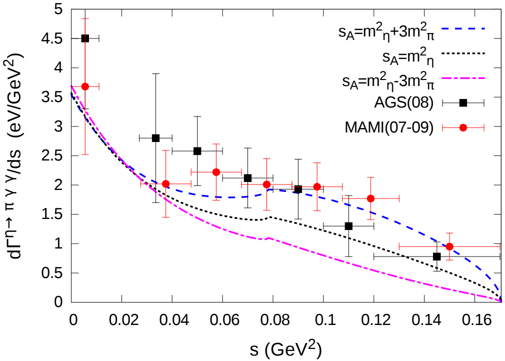
Decay mode: $\eta \to \pi^0\gamma\gamma$

- Interest in the 90's [J.Ng,D. Peters,PR D46,5034(1992), L. Amettler et al.,PL B276,185 (1992), E.Oset et al.,PR D67,073013(2003)]
  Tensions with experimental data (factor 2)

- Improved in recent measurements [S.Prakhov et al.(AGS),PR C78,015206 (2008), B. Nefkens (MAMI) PR C90,025206 (2014)]

- Deduced from dispersive $\gamma\gamma$ amplitudes
  $0 \leqslant s \leqslant (m_\eta - m_\pi)^2$ but:
  - → Isospin violating contribution: $\gamma\gamma \to \pi^+\pi^-$, $\pi^+\pi^- \to \pi^0\eta$ should be included in unitarity
  - → Sensitivity to position of Adler zero

Results for $\eta \to \pi^0\gamma\gamma$



- Our calc. includes $\pi^+\pi^- \to \pi^0\eta$ from Khuri-Treiman solutions [M.Albaladejo, BM, EPJ C77,508(2017)] $\pi^+\pi^-$ produces visible cusp

- Adler zero $s_A = m_\eta^2 + 3m_\pi^2$ favoured

Conclusions

- We constructed dispersive/Omnès S-wave amplitudes $\gamma\gamma \to \pi^0\eta, (K\overline{K})_{I=1}$ (+model T-matrix with some flexibility)
- Compared with data $\gamma\gamma \to \pi\eta$, $K_S K_S$ ($K^+ K^-$)
- Possibility of narrow $a_0(1450)$ confirmed (but requires curious cancellation between 0^{++} and 2^{++})
- $a_0(980)$ ordinary second sheet pole:

$$M - i\Gamma/2 = 1000.7^{+12.9}_{-0.7} - i\,36.6^{+12.7}_{-2.3}\ (\mathrm{MeV})$$

  - Unlike lattice QCD ? (but $m_\pi = 391$ MeV)
  - Specific shape of $a_0(980)$ peak requires more statistics