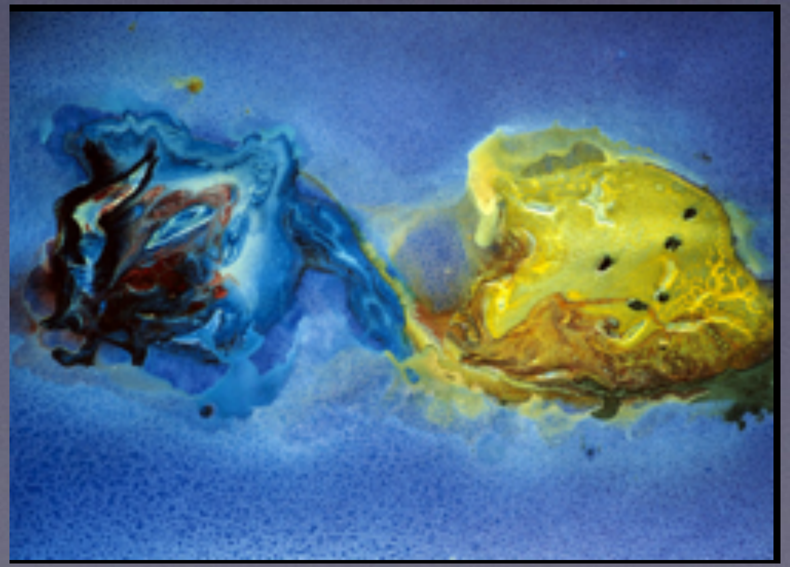
Precision determination of Standard model parameters with heavy systems and Nonrelativistic Effective Field Theories



Physik Department TUM
T30f

Nora Brambilla

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Everything with an alliance of EFTs and lattice

Material for discussion/references: NREFTs, pNRQCD

- **Effective field theories for heavy quarkonium**
  Nora Brambilla, Antonio Pineda, Joan Soto, Antonio Vairo
  Rev.Mod.Phys. 77 (2005) 1423
  e-Print: **hep-ph/0410047**

SM parameters extractions : alphas and the quark masses

Determination of the QCD coupling from the static energy and the free energy
#TUMQCD Collaboration•Alexei Bazavov(Michigan State U. and Michigan State U., East Lansing (main)) N. B. et al. (Jul 26, 2019)
Published in: Phys.Rev.D 100 (2019) 11, 114511 • e-Print: 1907.11747

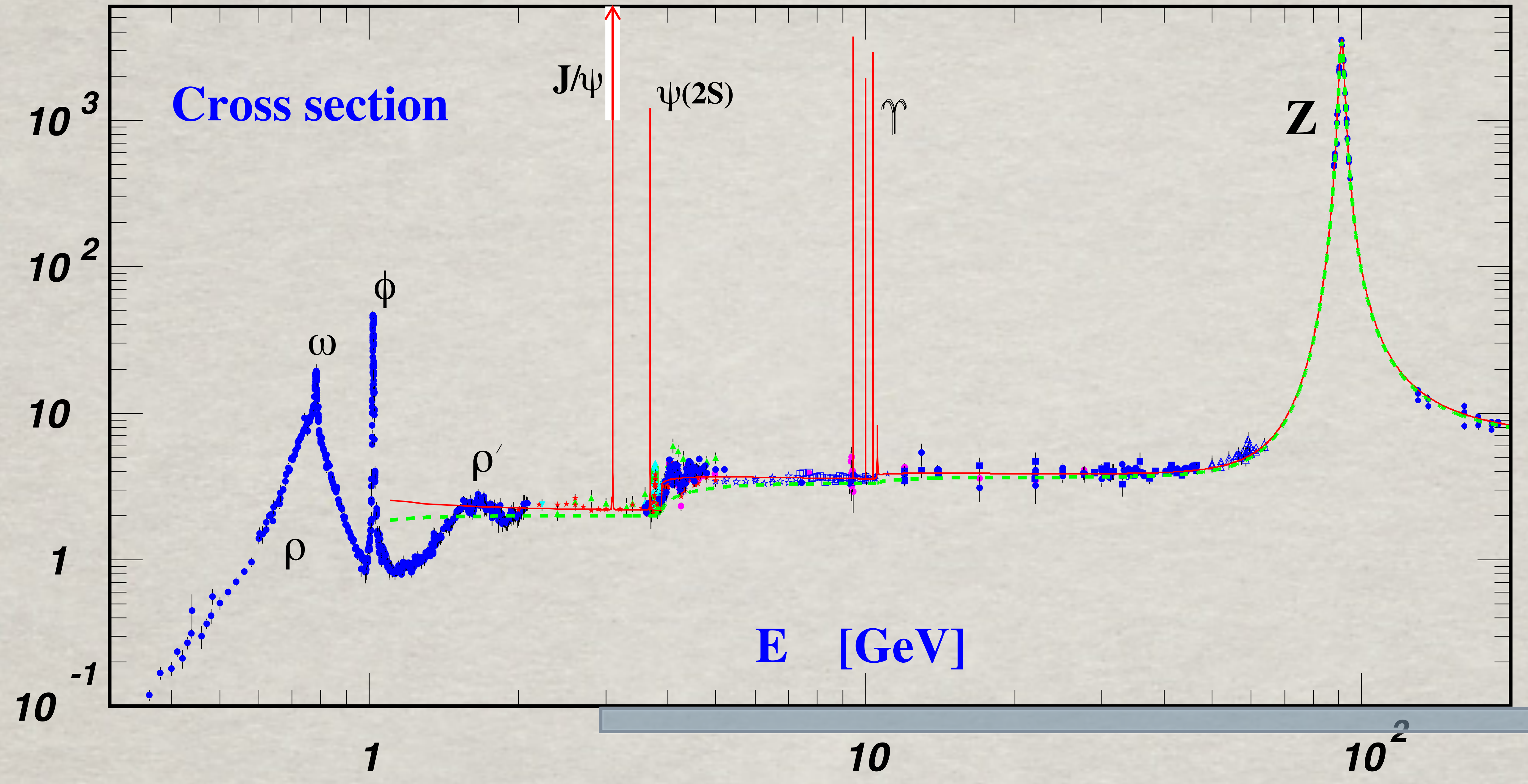
Relations between Heavy-light Meson and Quark Masses

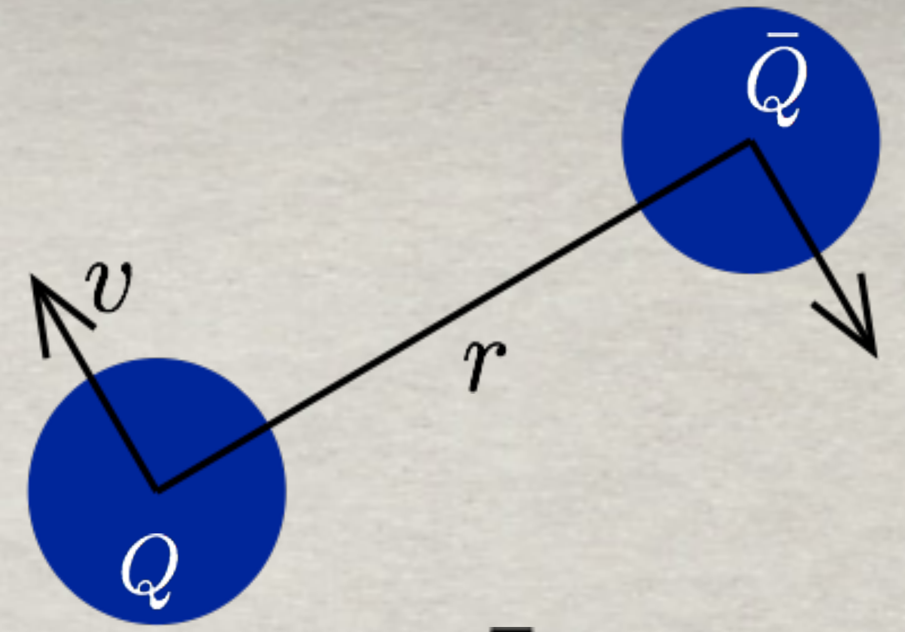
TUMQCD Collaboration•N. Brambilla(Munich, Tech. U. and TUM-IAS, Munich) et al. (Dec 13, 2017)
Published in: *Phys.Rev.D* 97 (2018) 3, 034503 • e-Print: 1712.04983

Up-, down-, strange-, charm-, and bottom-quark masses from four-flavor lattice QCD
Fermilab Lattice and MILC and TUMQCD Collaborations•A. Bazavov(Michigan State U.) N. B., et al. (Feb 12, 2018)
Published in: Phys.Rev.D 98 (2018) 5, 054517 • e-Print: 1802.04248

Lattice gauge theory computation of the static force
N. Brambilla, V. Leino, O. Philipsen, C. Reisinger, A. Vairo, M. Wagner **in preparation**

Heavy quarks offer a privileged access





with $Q, \bar{Q} = c, b, t$

$m_c \sim 1.5\text{GeV}$

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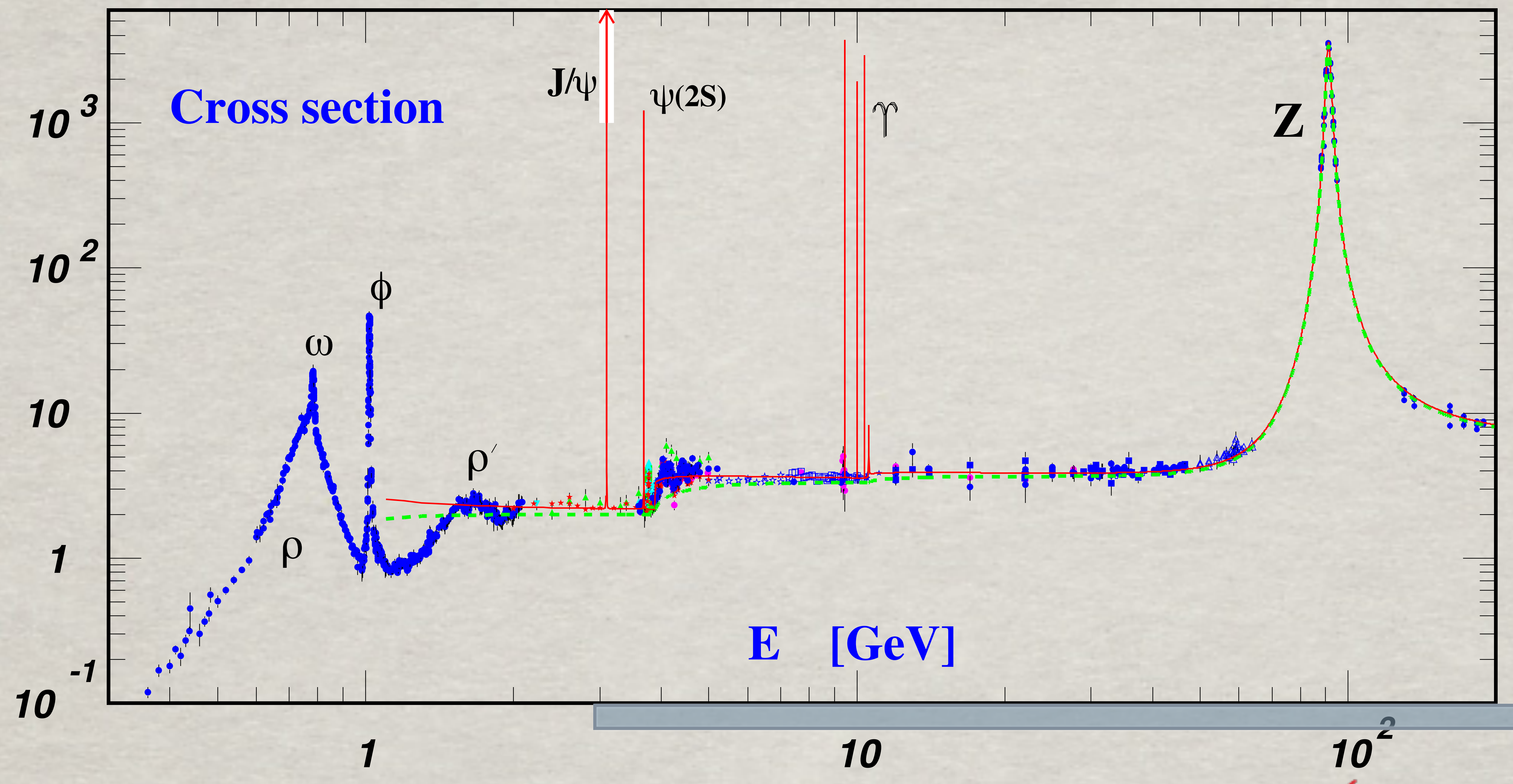

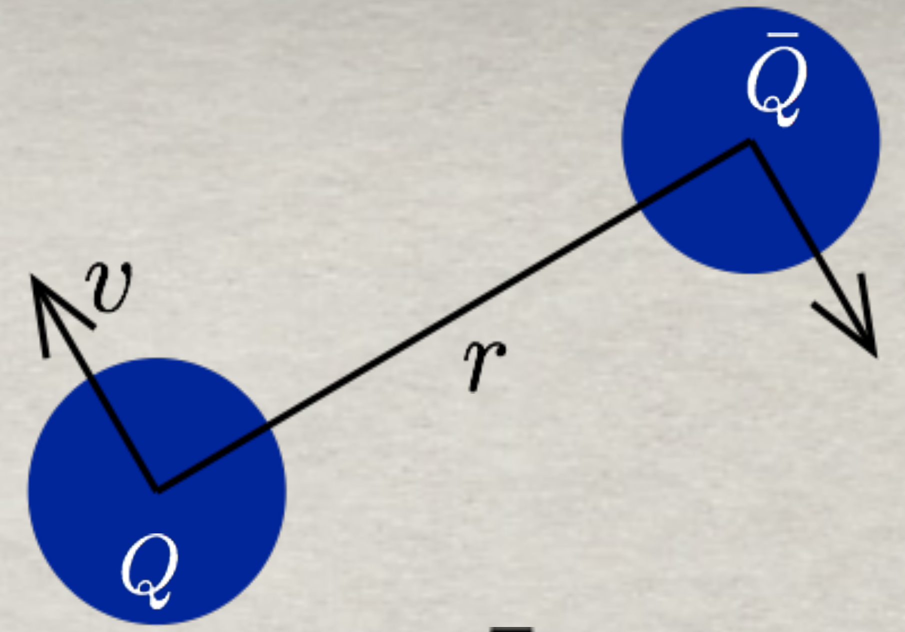

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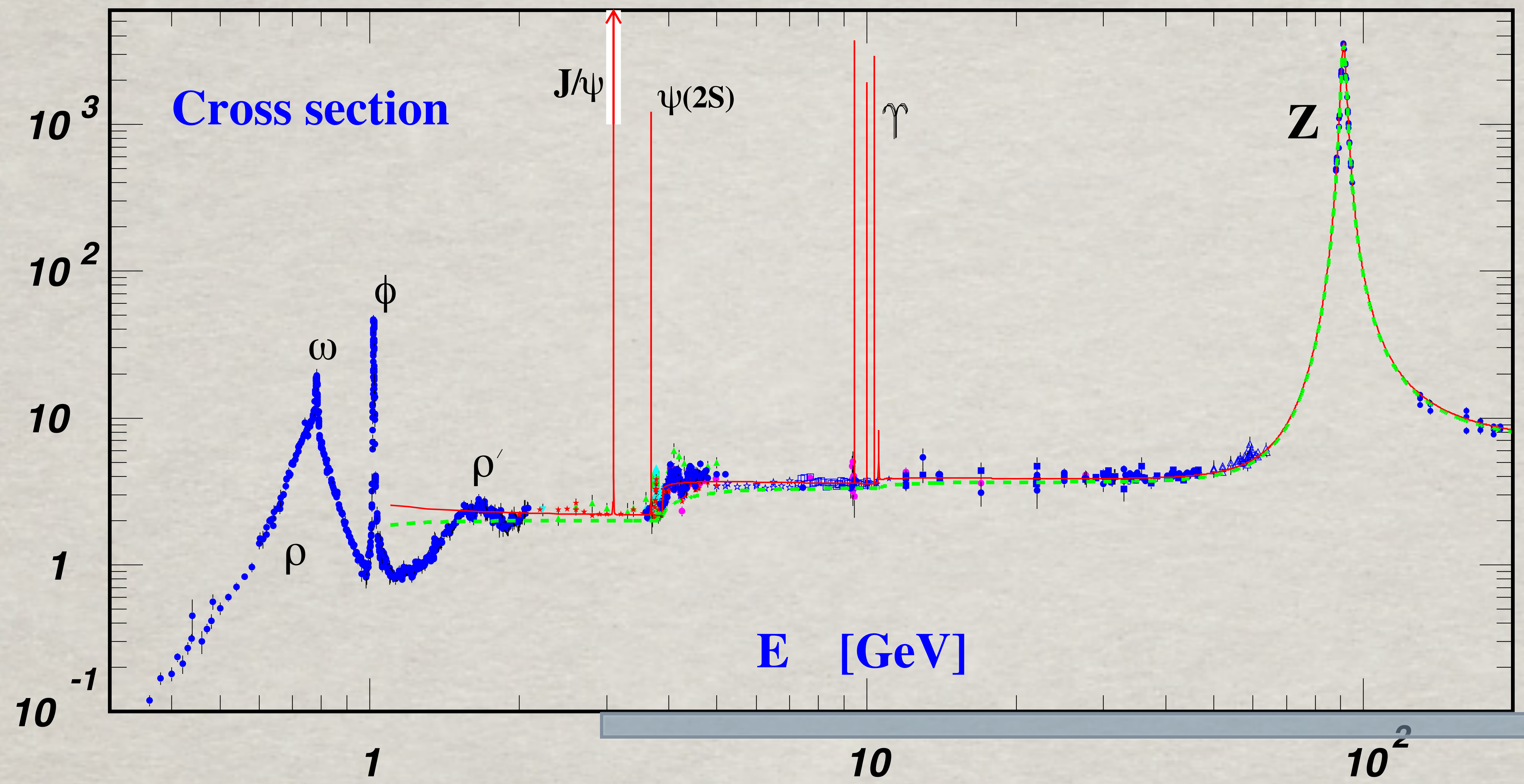

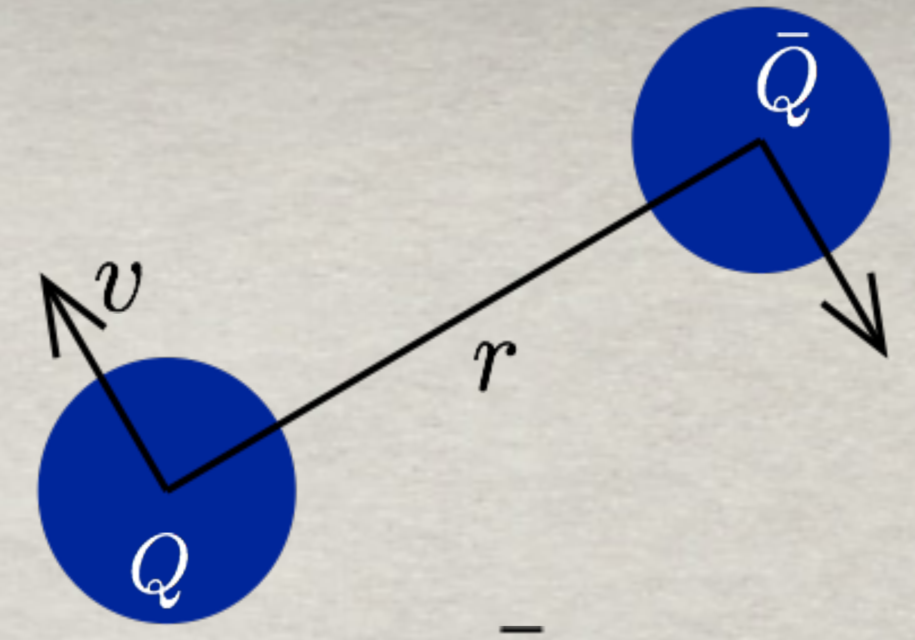

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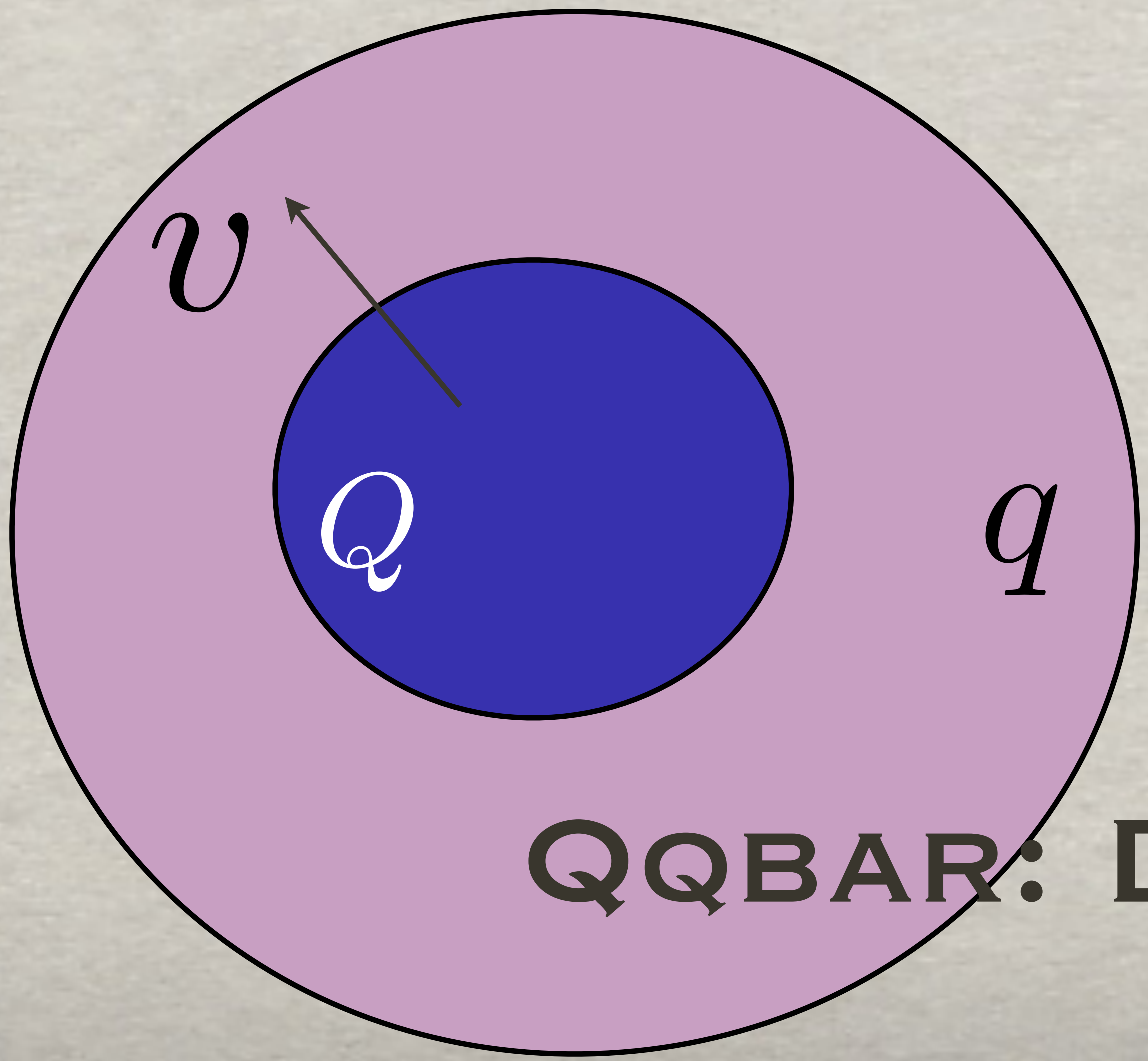
Heavy quarkonium is very different from heavy-light hadrons



different physics from the heavy light meson where only two scales exist m and Λ_{QCD}

Qqbar: D, B mesons

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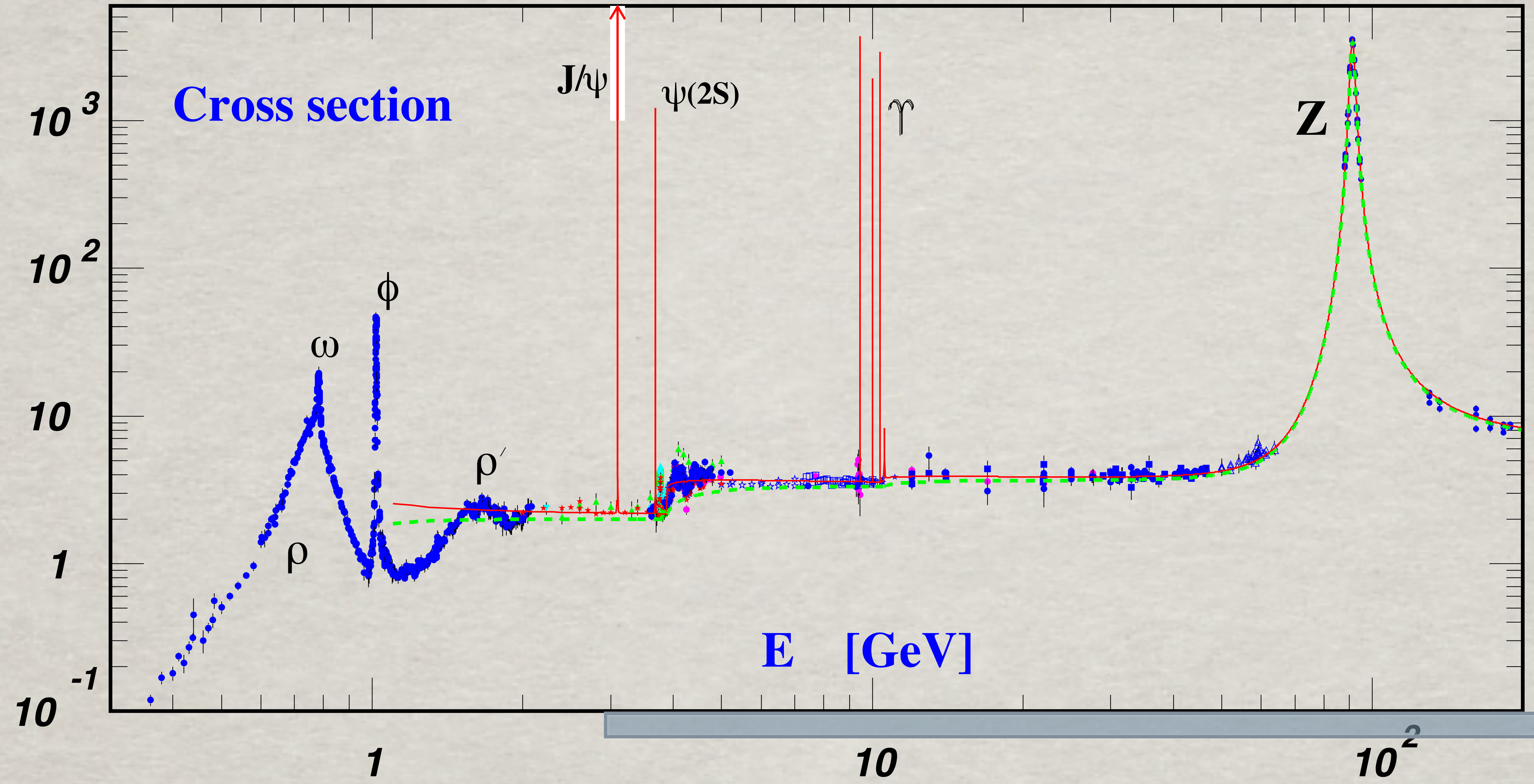

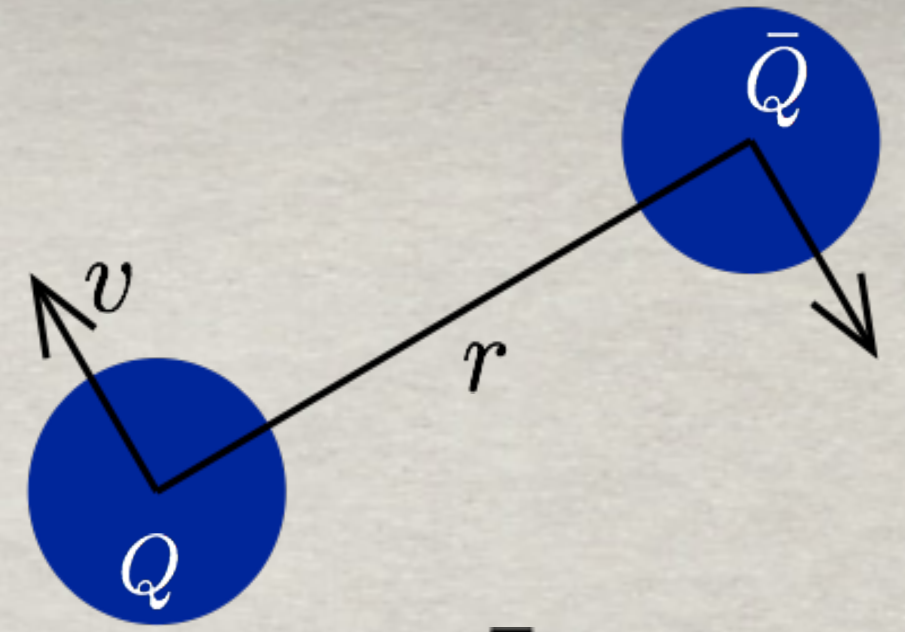

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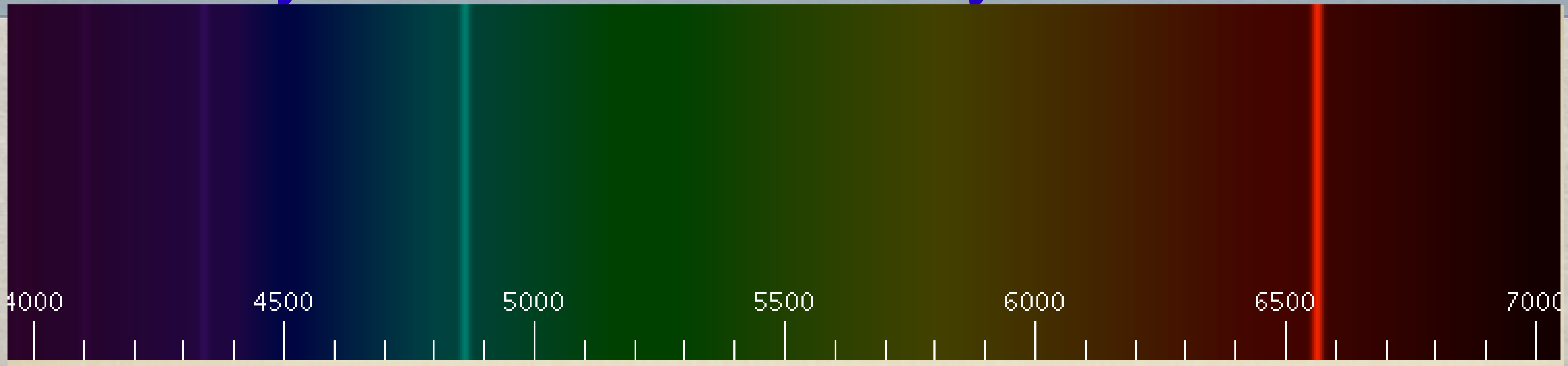
Heavy quarkonia are nonrelativistic bound systems: multiscale systems



Electromagnetic bound states: atoms, molecules,

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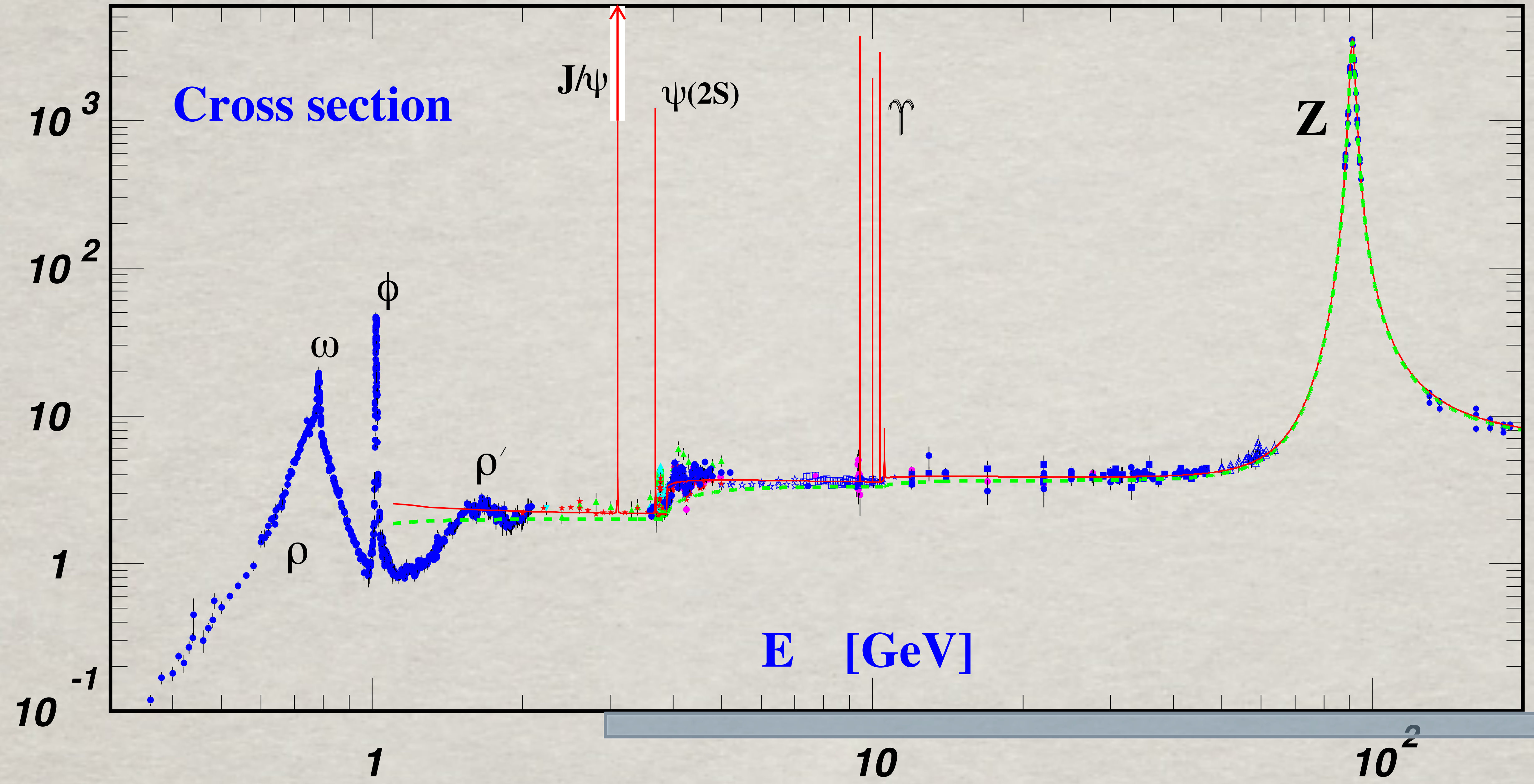

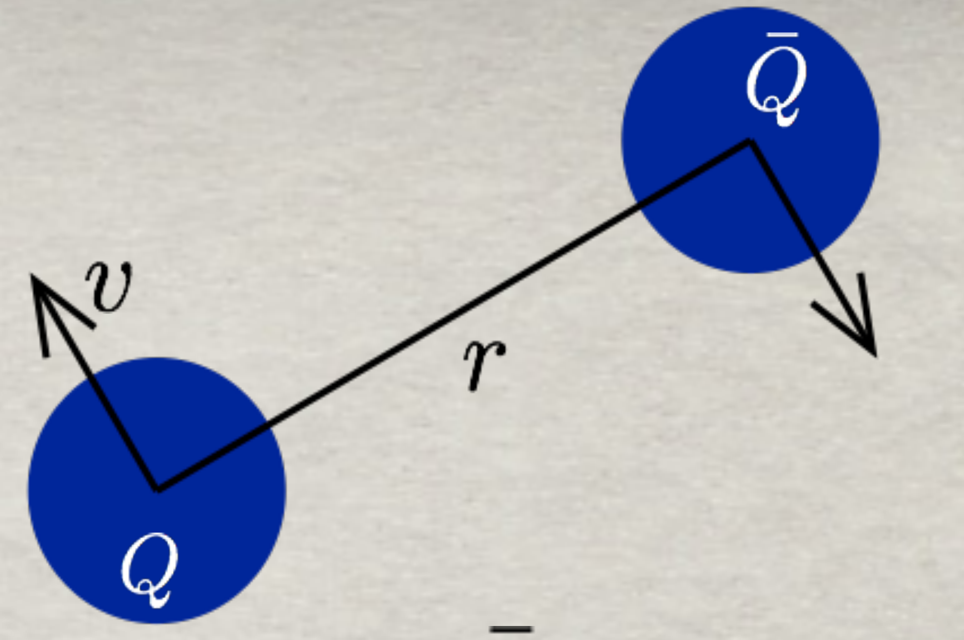

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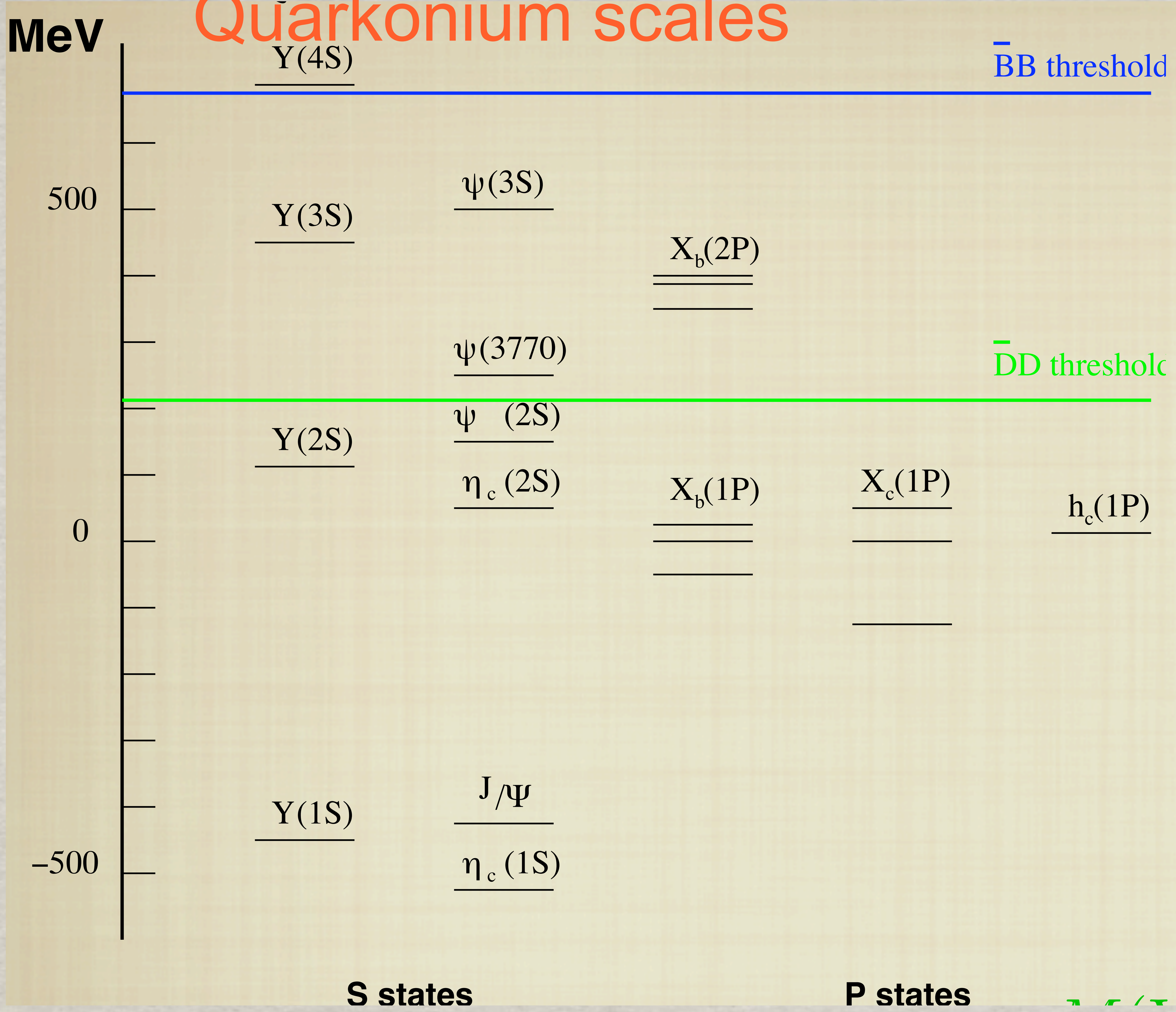
many scales: a challenge and an opportunity

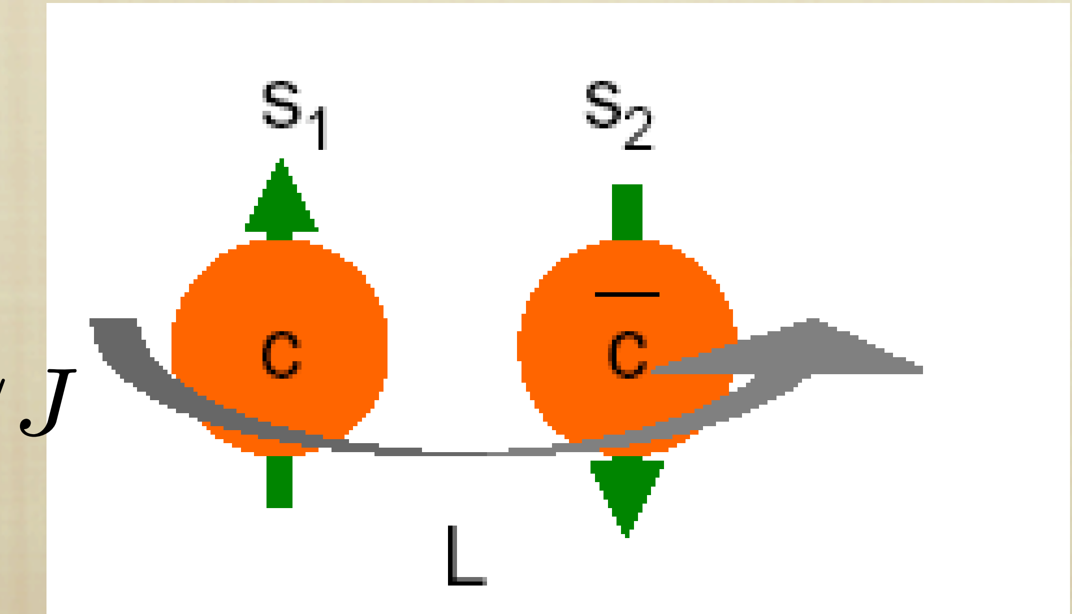
Electromagnetic bound states: atoms, molecules,

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$

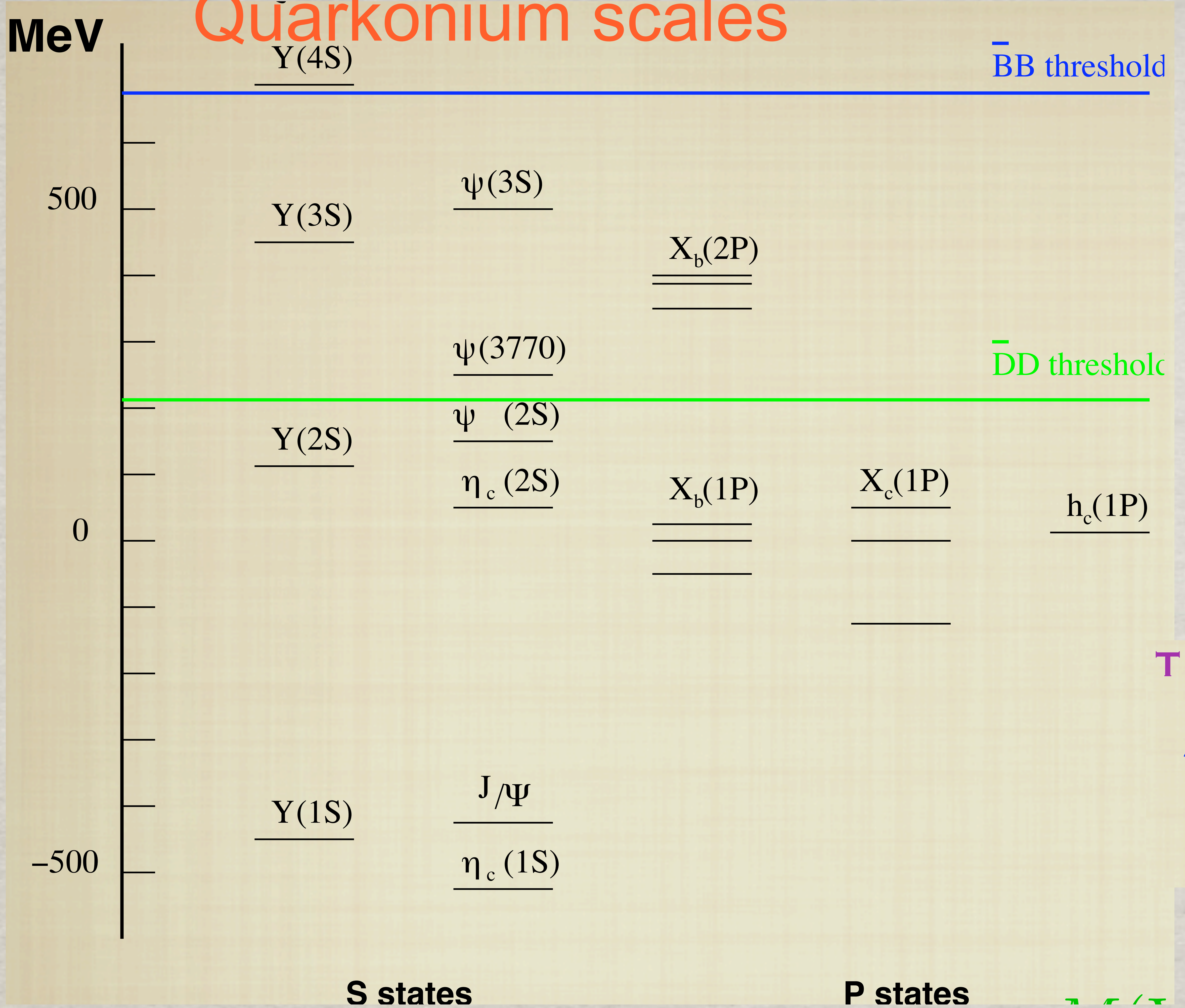


THE MASS SCALE IS PERTURBATIVE

$$m_Q \gg \Lambda_{\rm QCD}$$

$$m_b \simeq 5\,{\rm GeV}; m_c \simeq 1.5\,{\rm GeV}$$

Quarkonium scales



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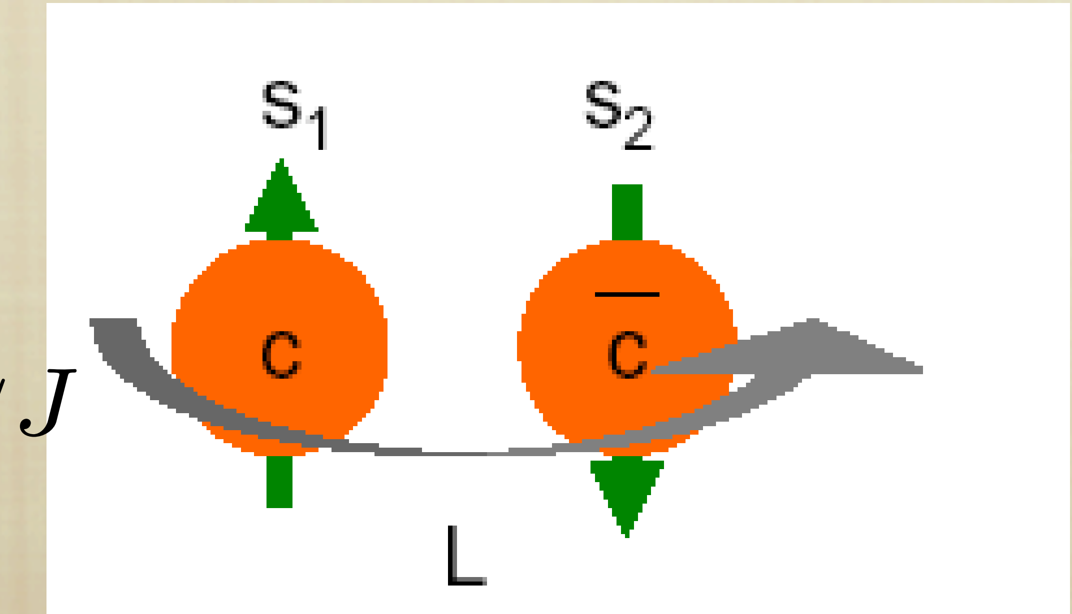

The system is nonrelativistic(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

The mass scale is perturbative

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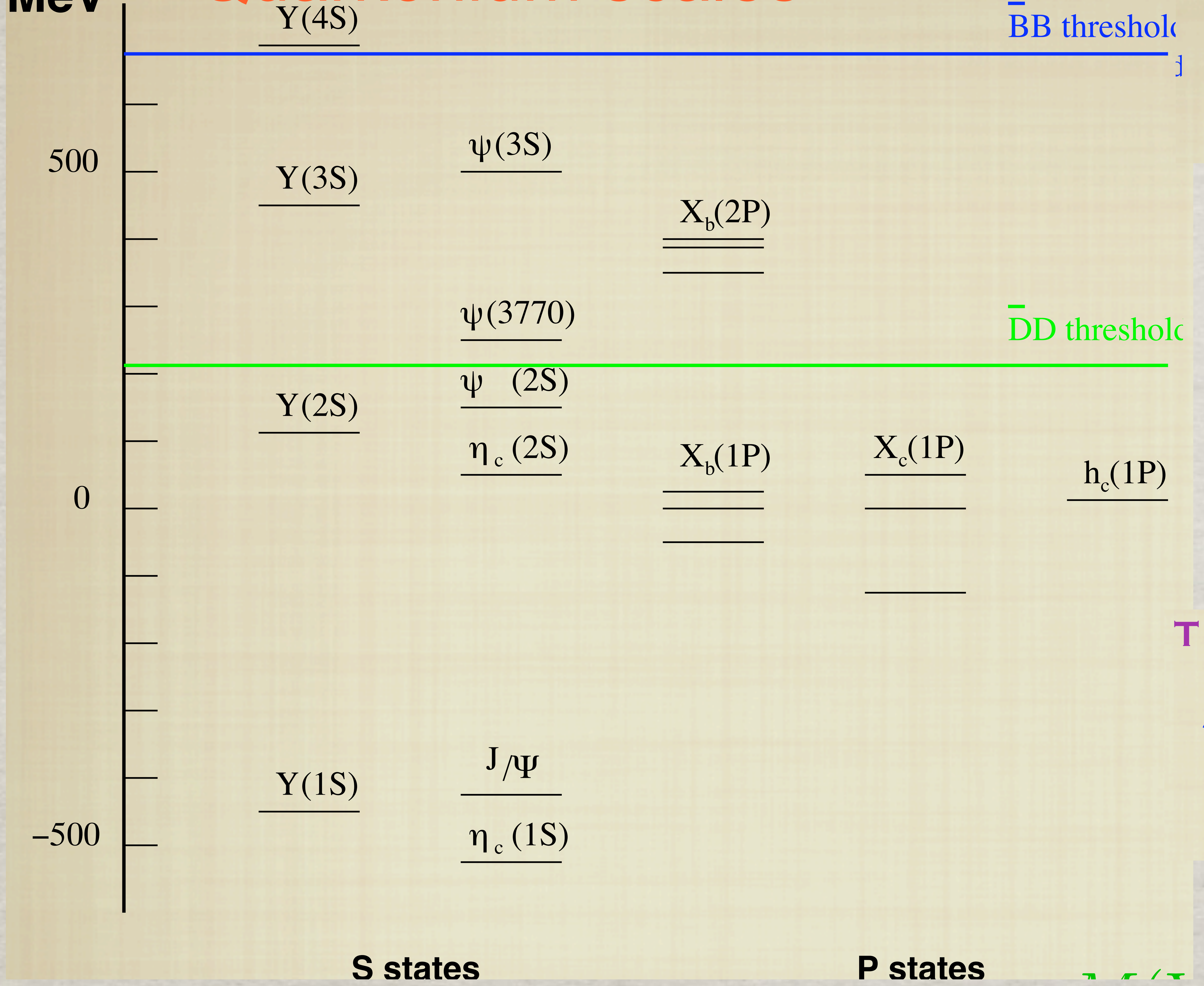

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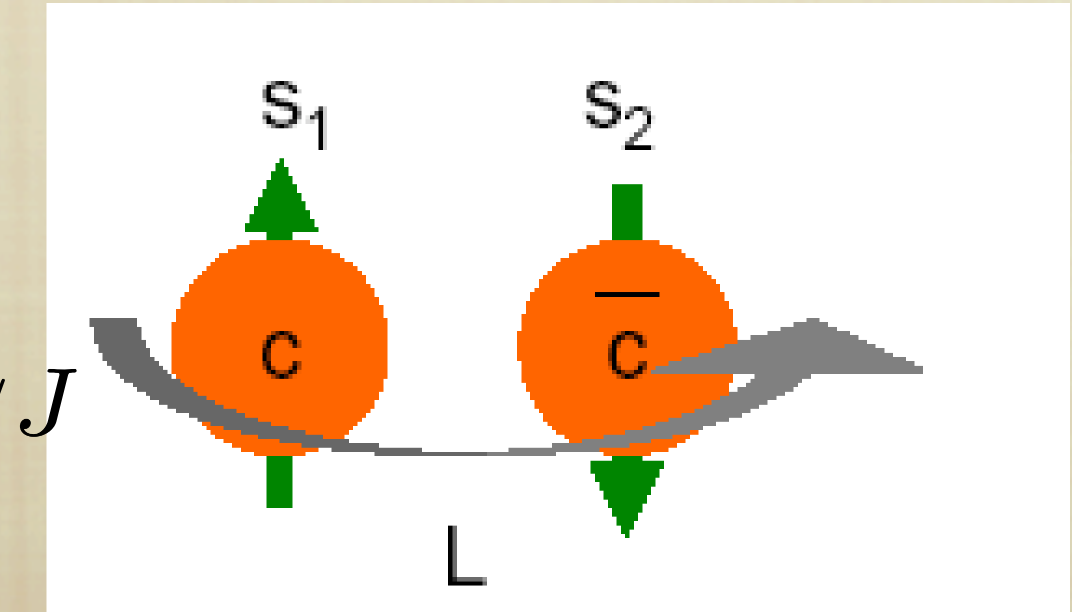

NR bound states have at least 3 scales

$$m \gg mv \gg mv^2 \quad v \ll 1$$

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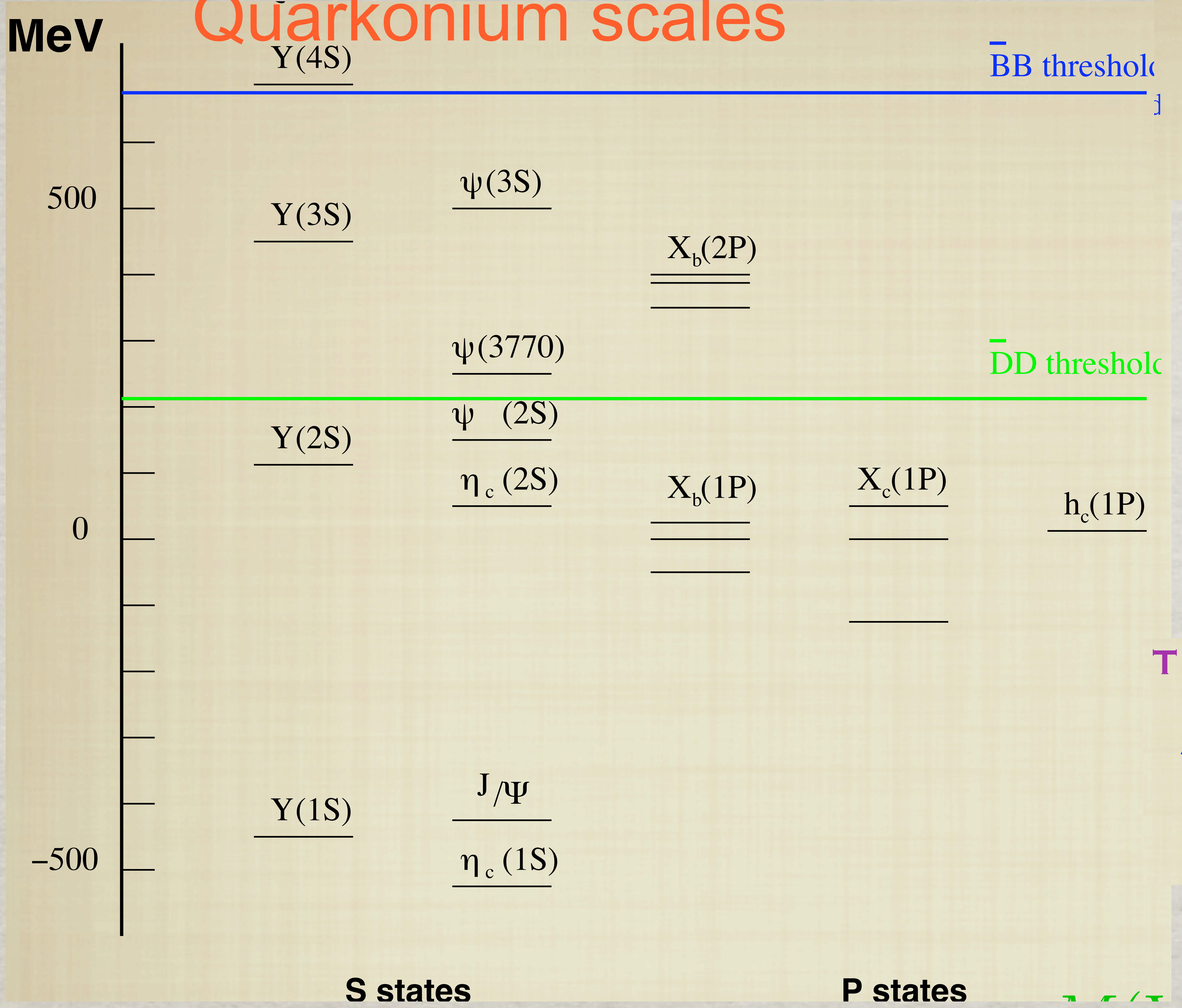

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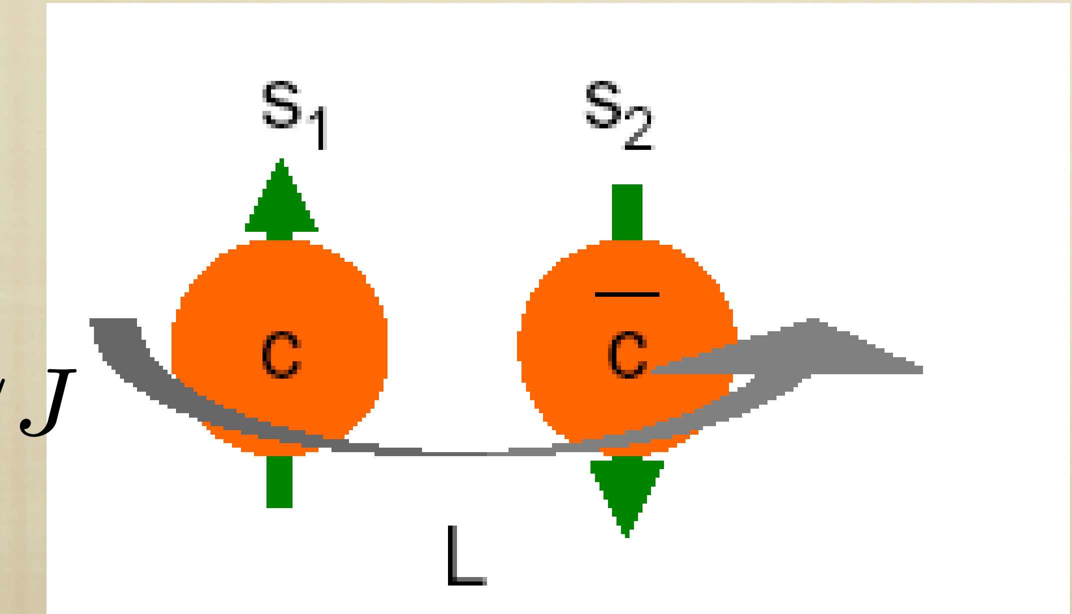

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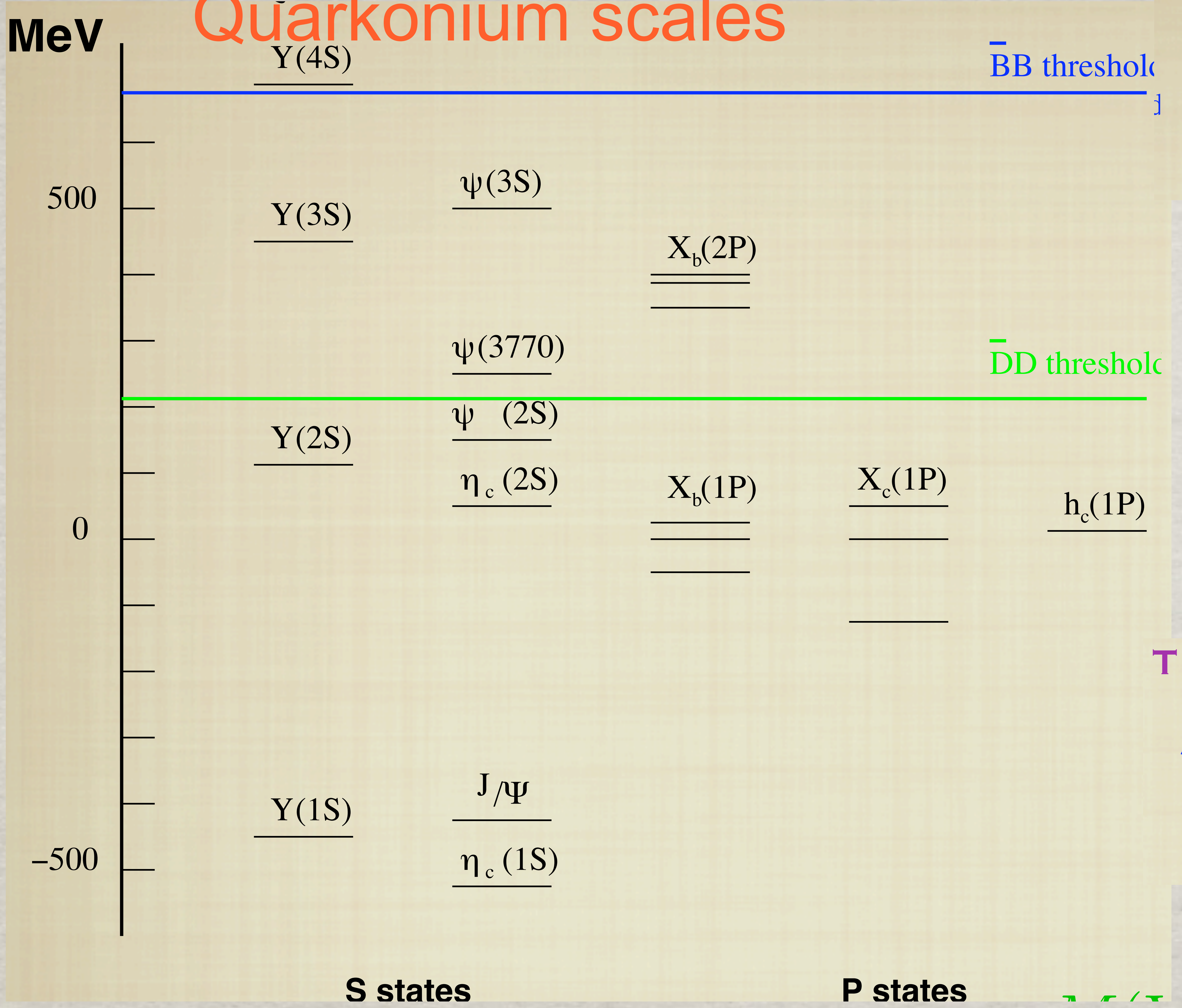

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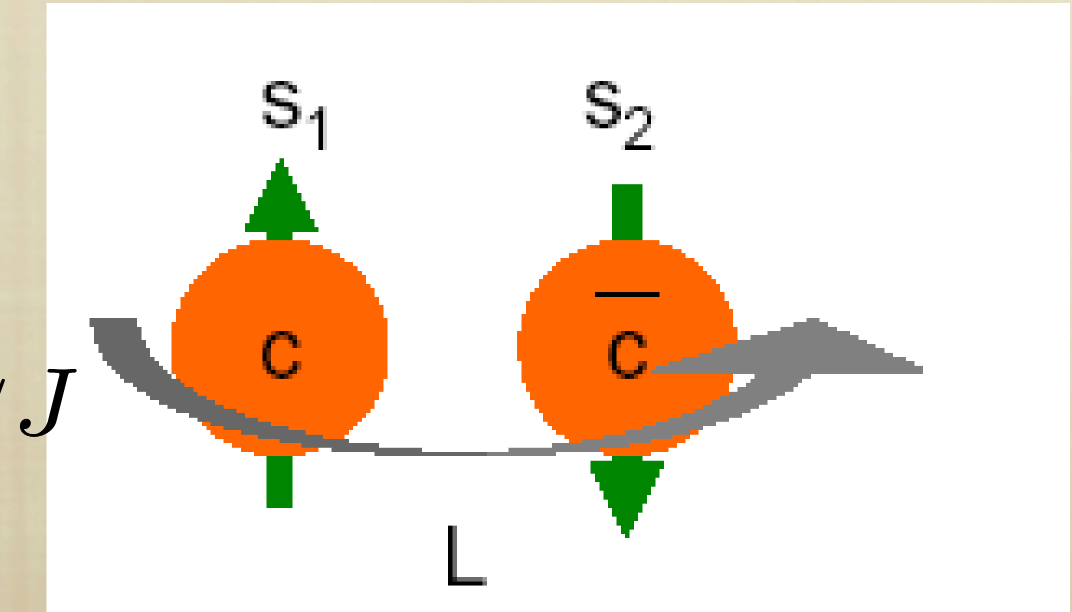

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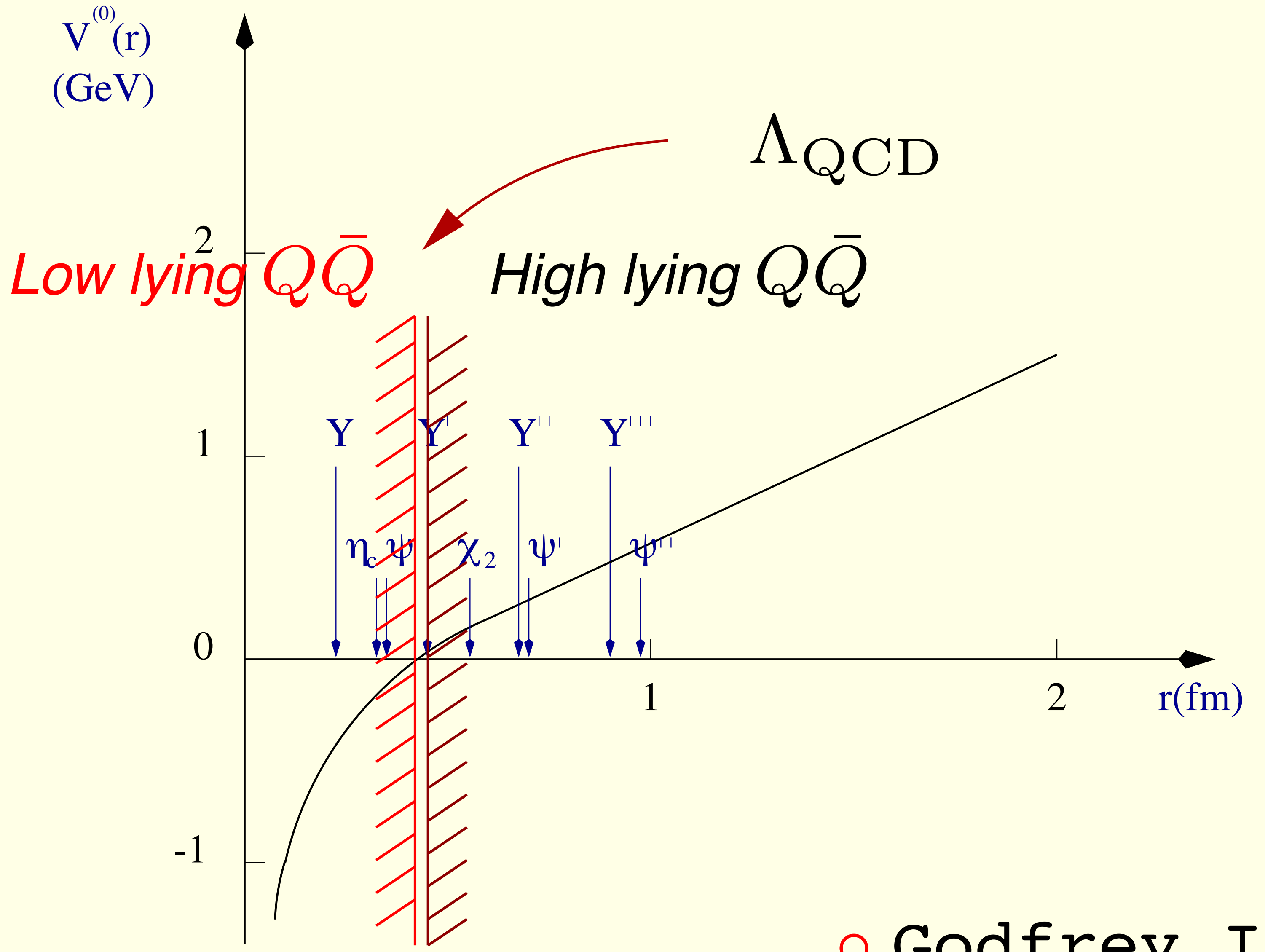
Quarkonium as a confinement and deconfinement probe

The rich structure of separated energy scales makes QQbar an ideal probe

At zero temperature

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.

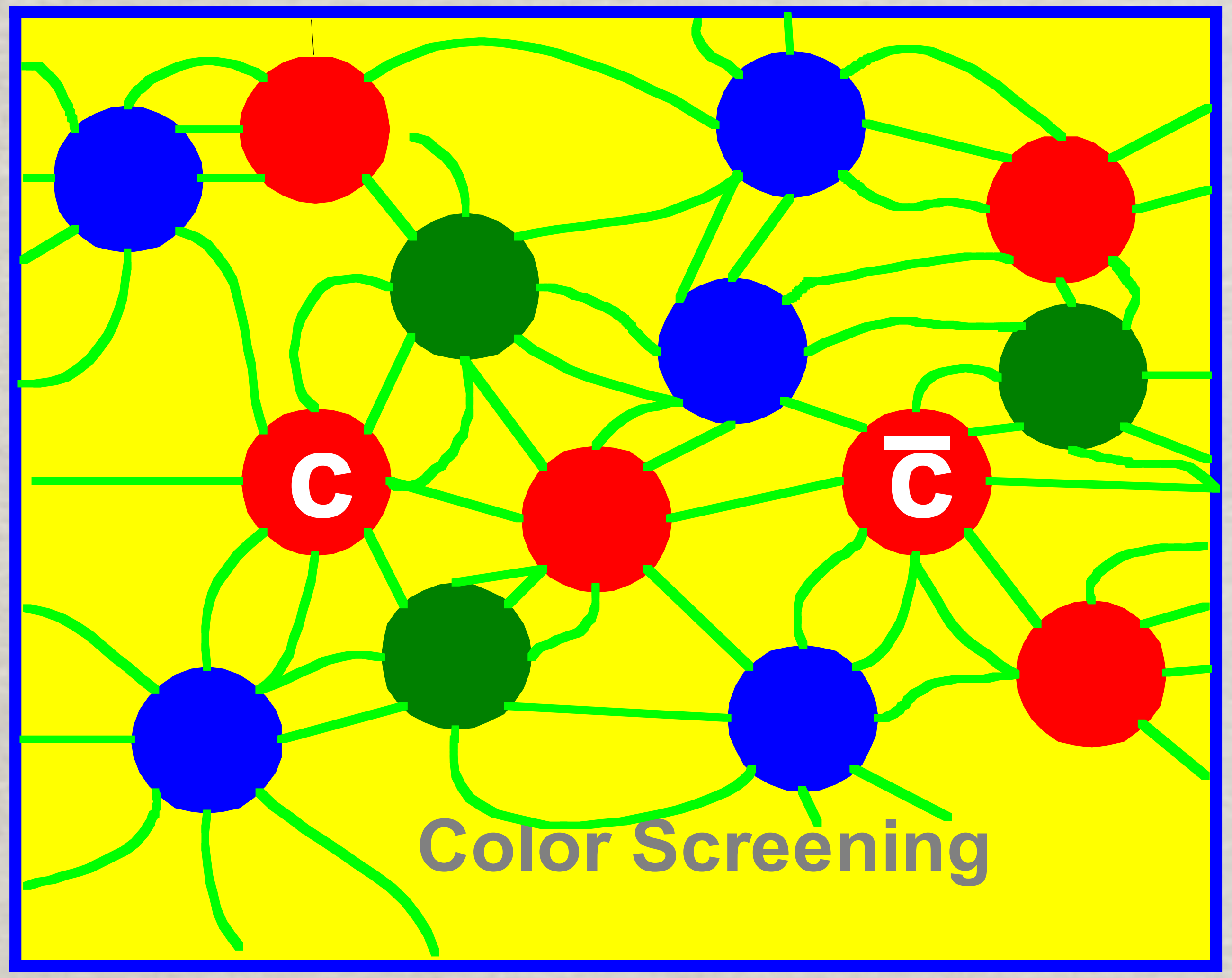


○ Godfrey Isgur PRD 32(85)189

quarkonia probe the perturbative (high energy) and non perturbative region (low energy) as well as the transition region in dependence of their radius r

Quarkonium as a confinement and deconfinement probe

At finite temperature T they are sensitive to the formation of a quark gluon plasma via color screening



Debye charge screening $m_D \sim gT$

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Matsui Satz 1986

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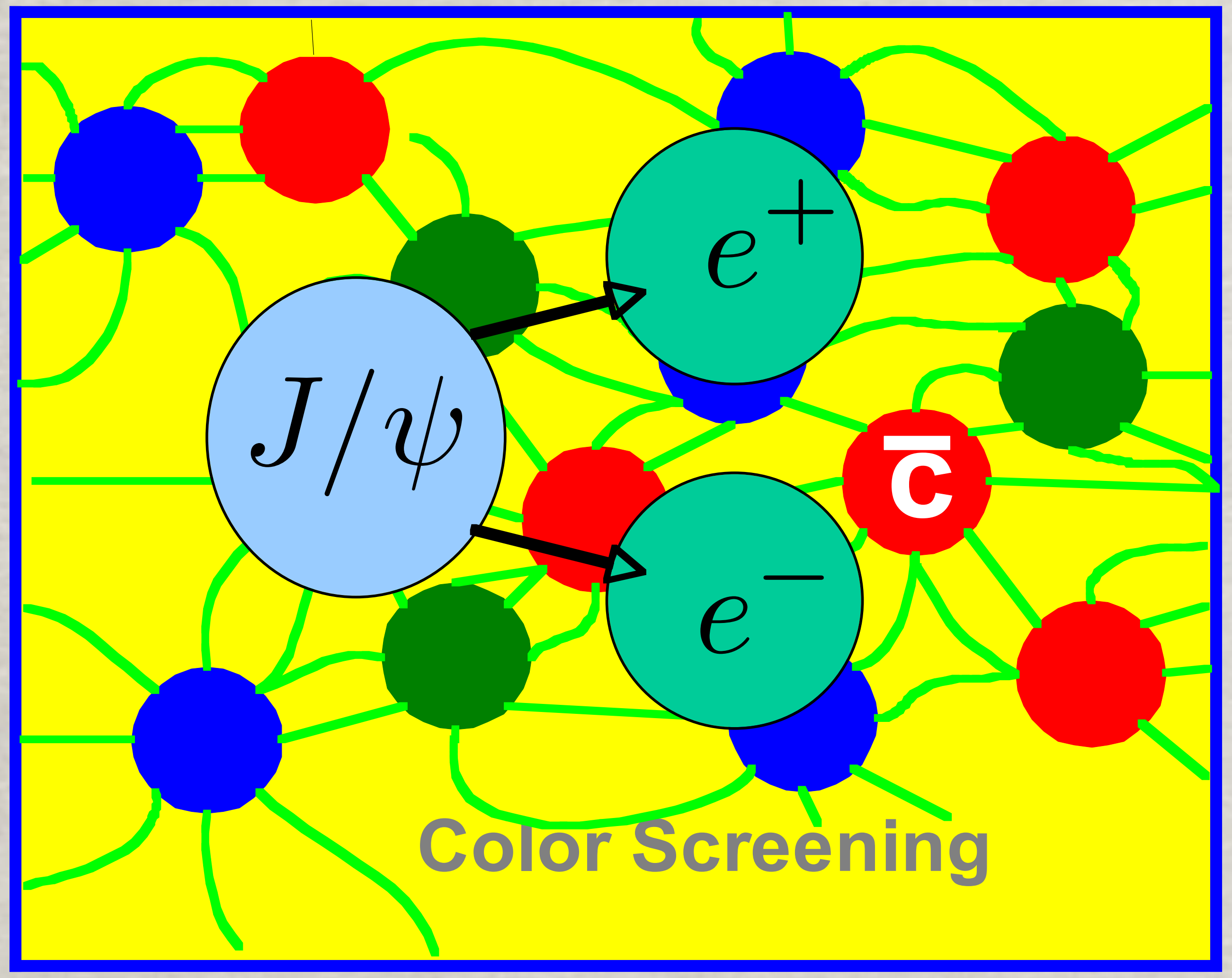

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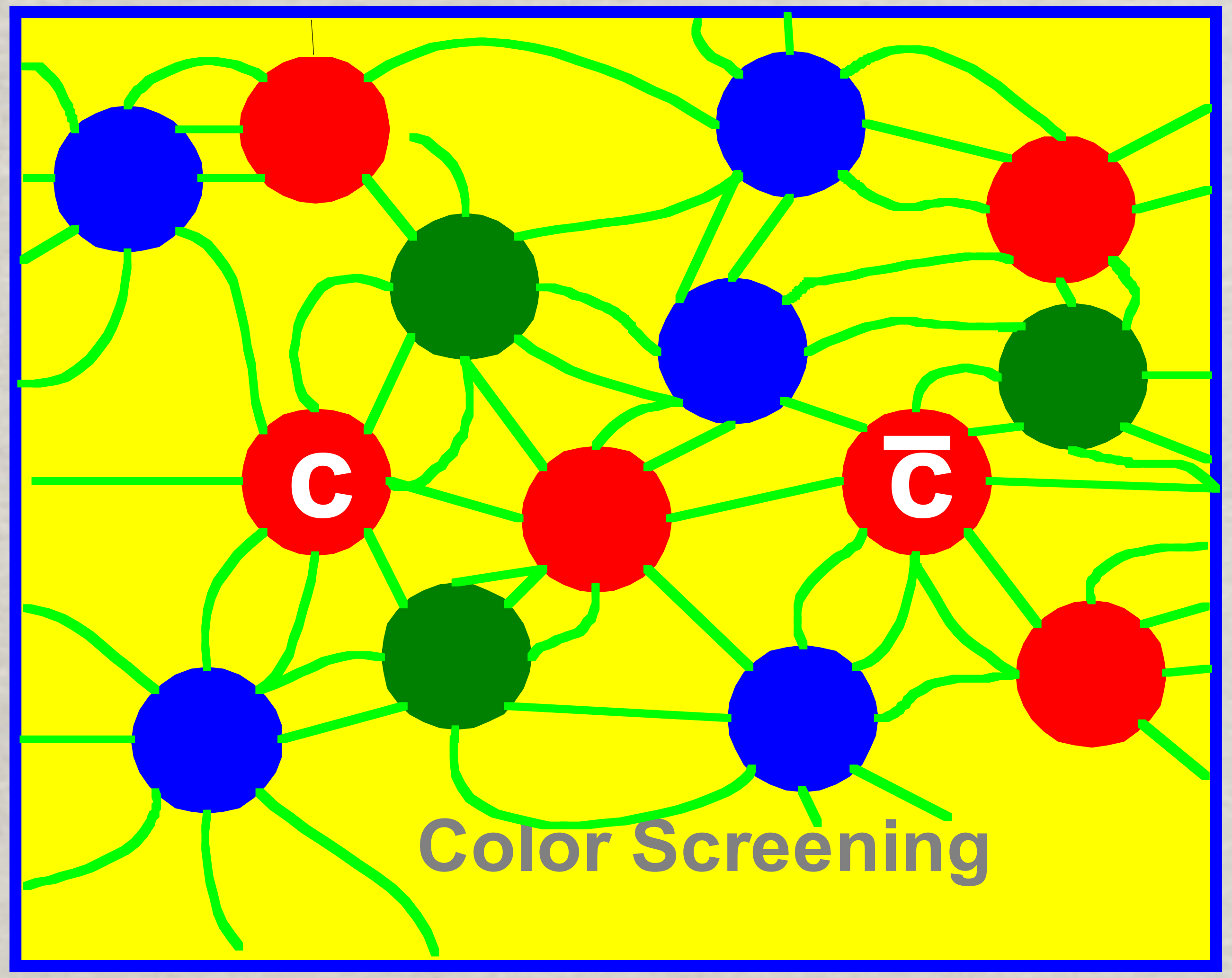

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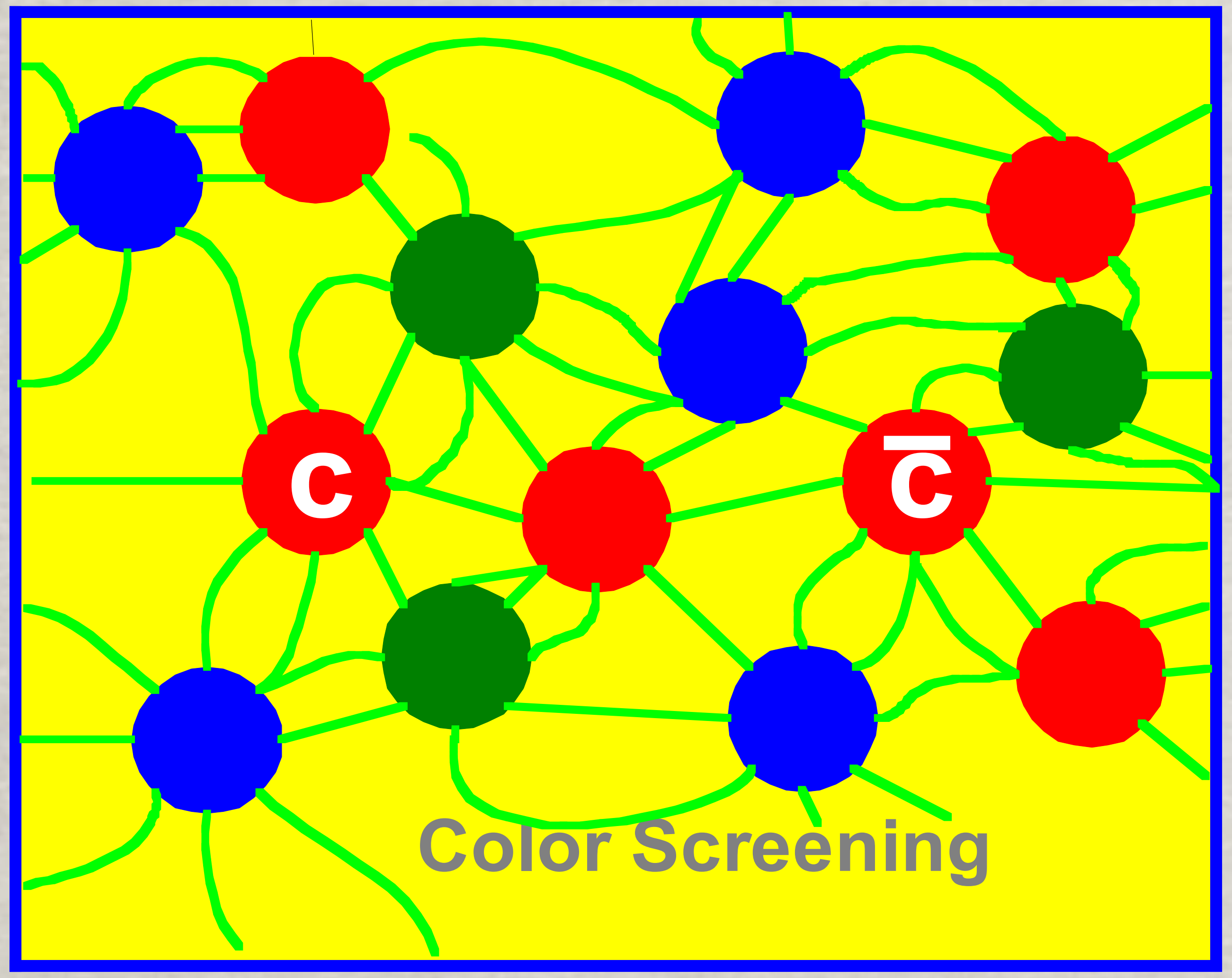

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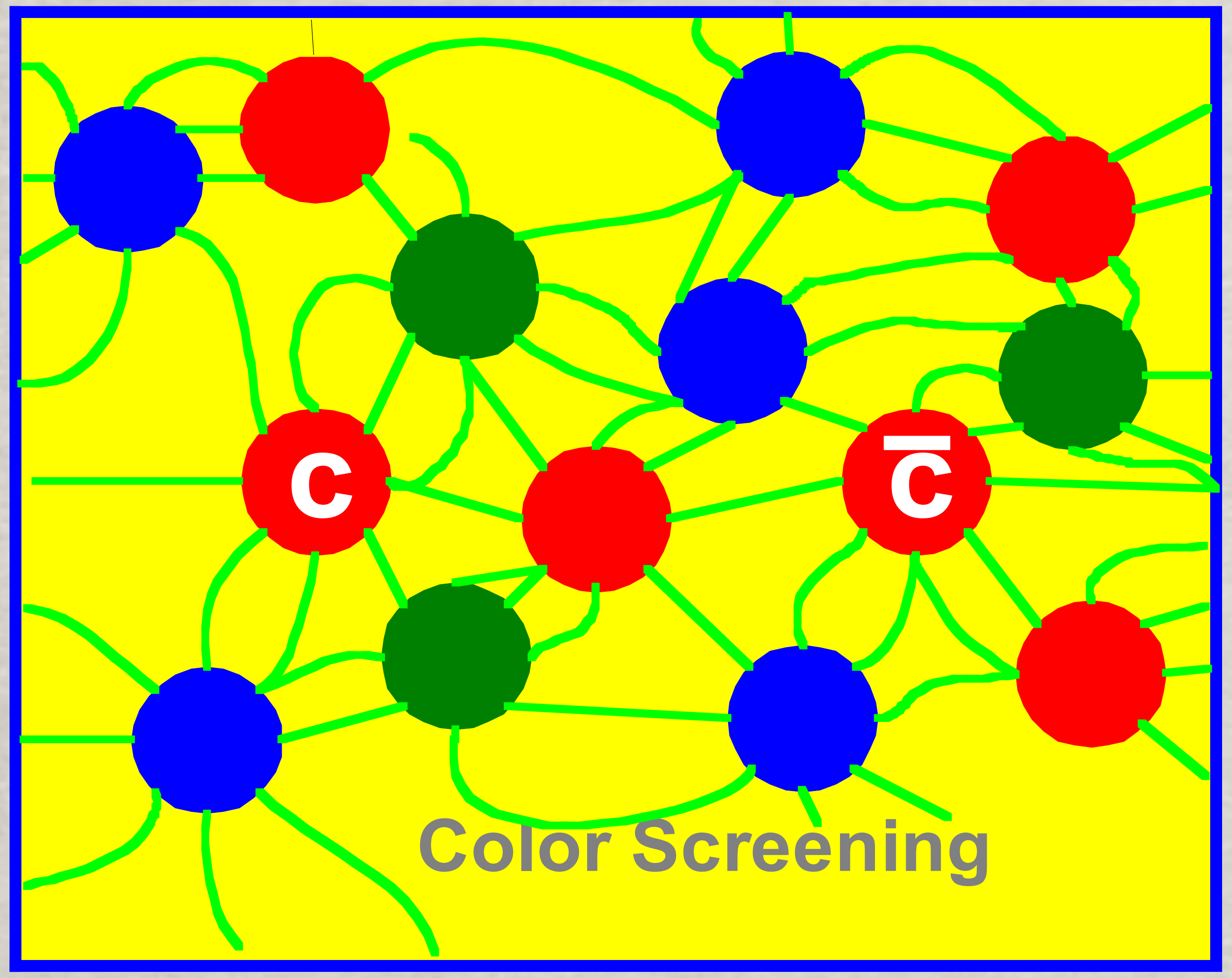

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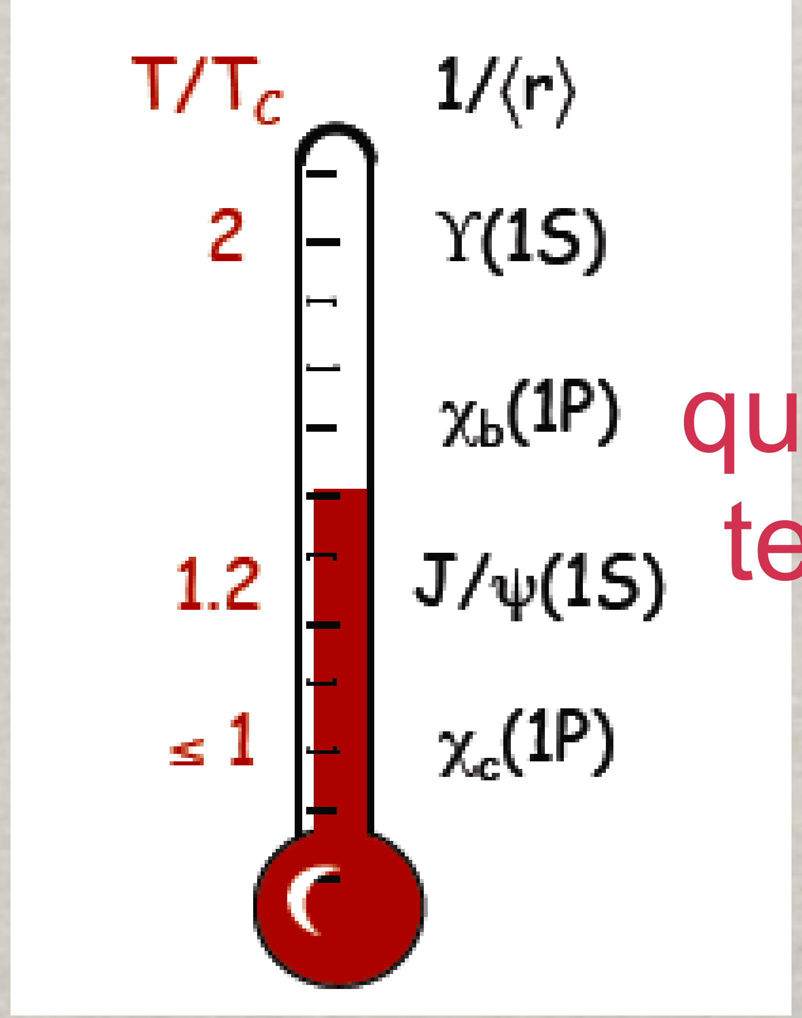

quarkonia dissociate at different temperature in dependence of their radius: they are a Quark Gluon Plasma thermometer

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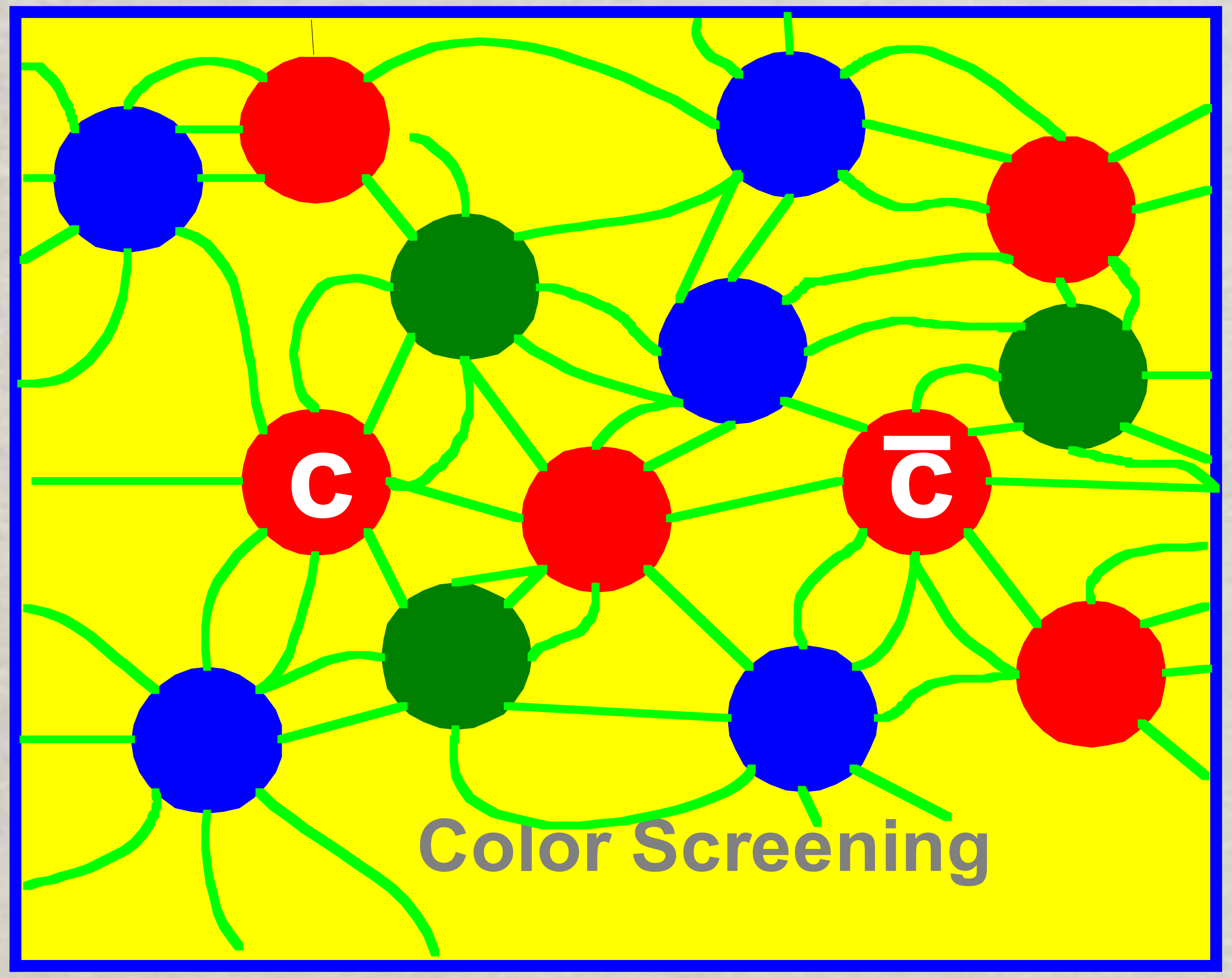

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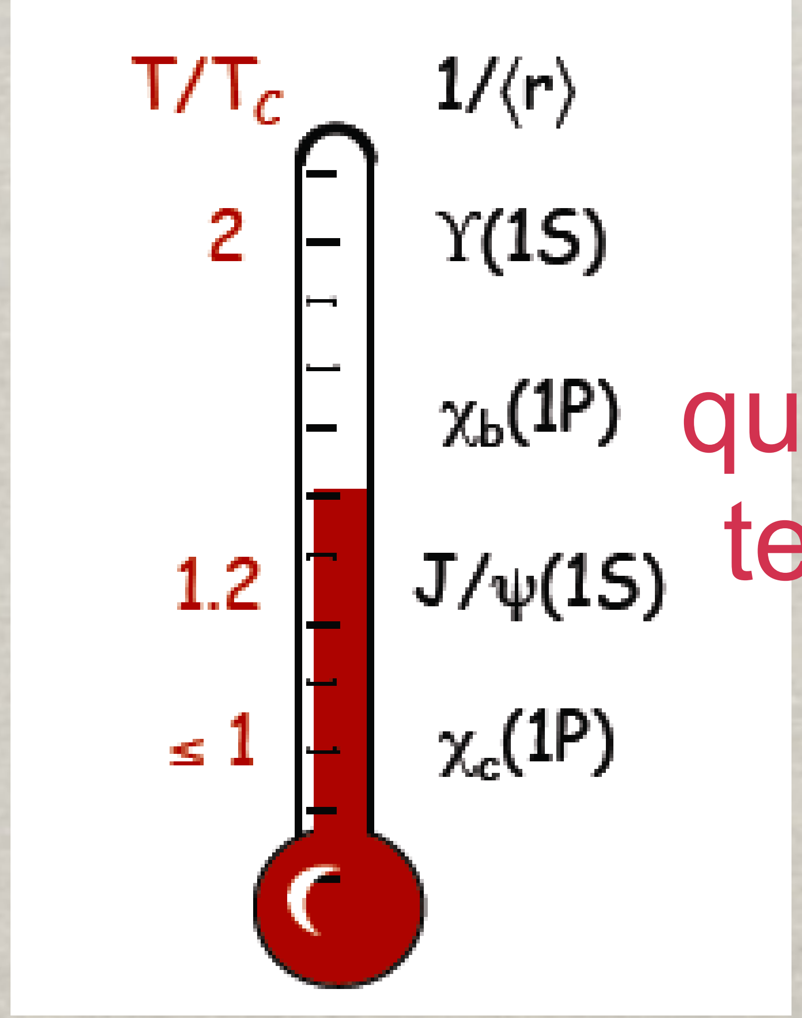

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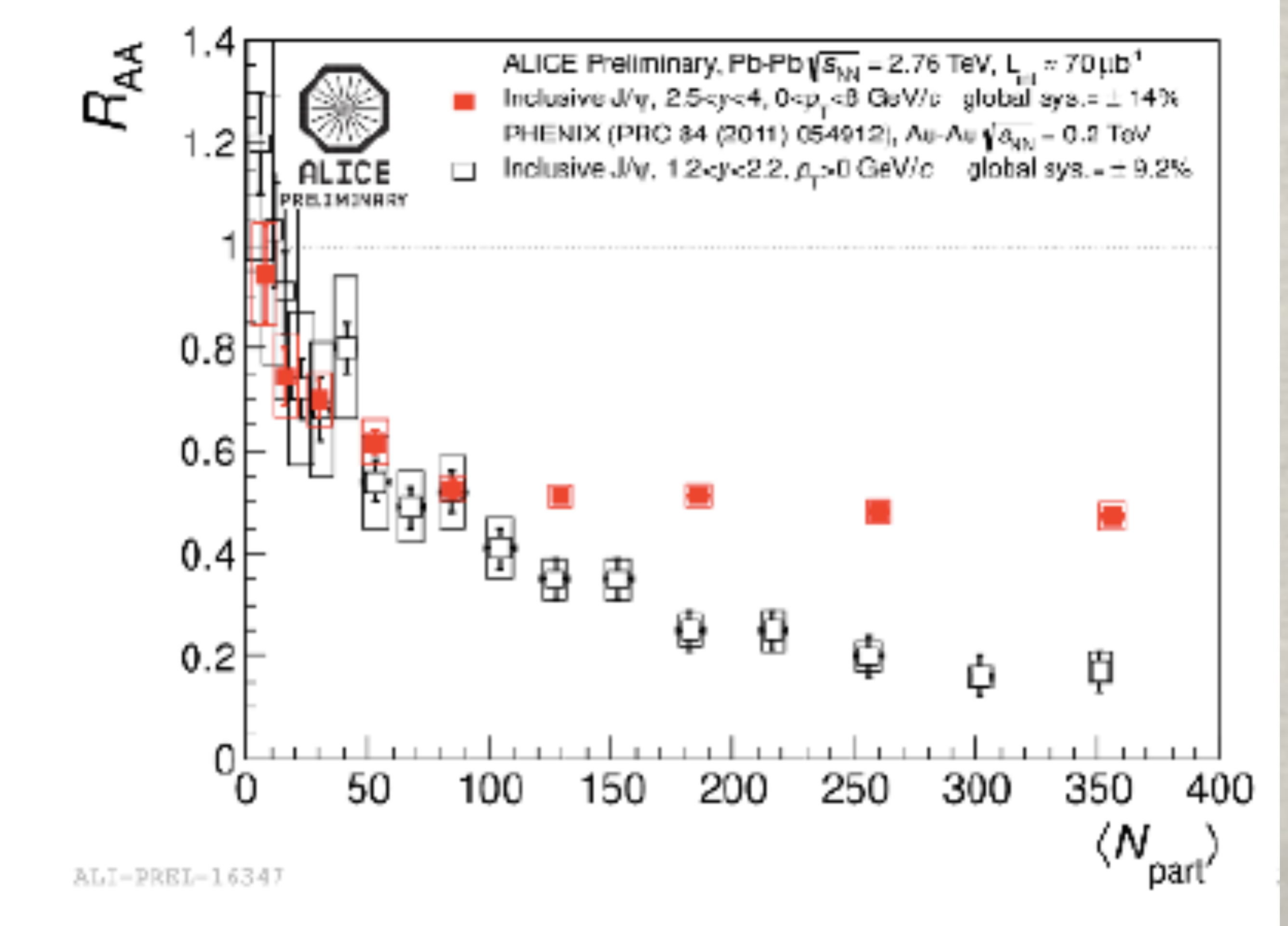
nuclear modificatio

$$R_{AA} = \frac{Yield^{q\bar{q}}_{AA}}{\langle N_{coll}\rangle \times Yield^{q\bar{q}}_{pp}}$$

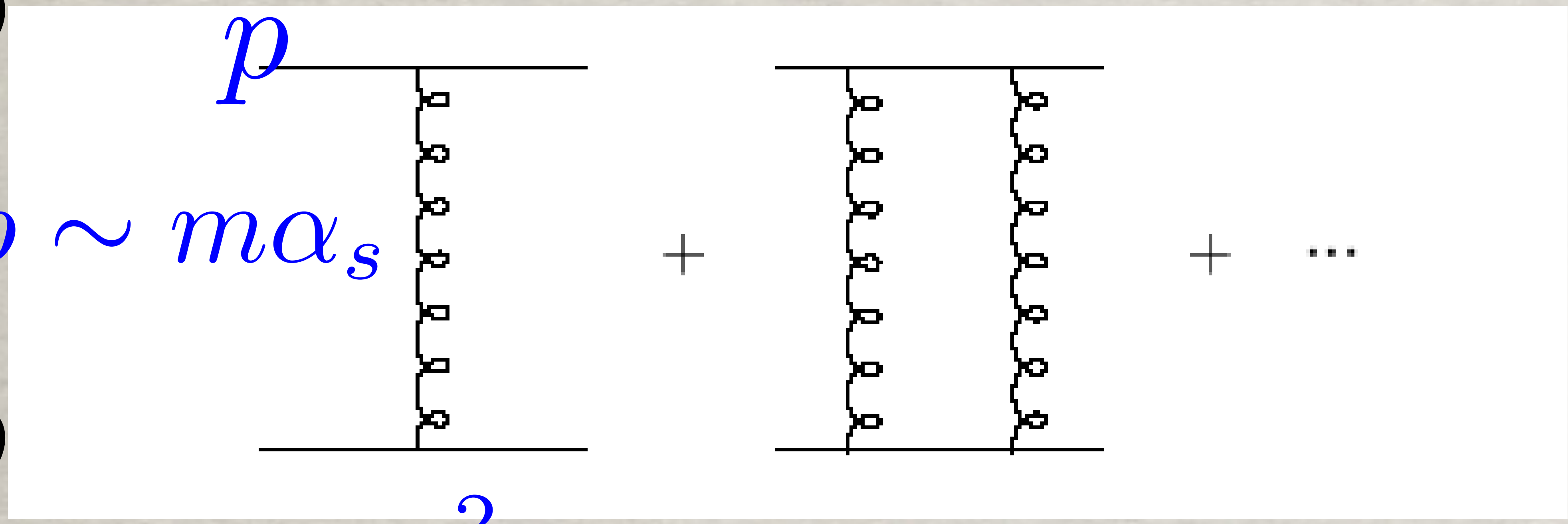


QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim v$

Q $\quad p$

$p \sim m\alpha_s$



$\bar{Q}$

$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

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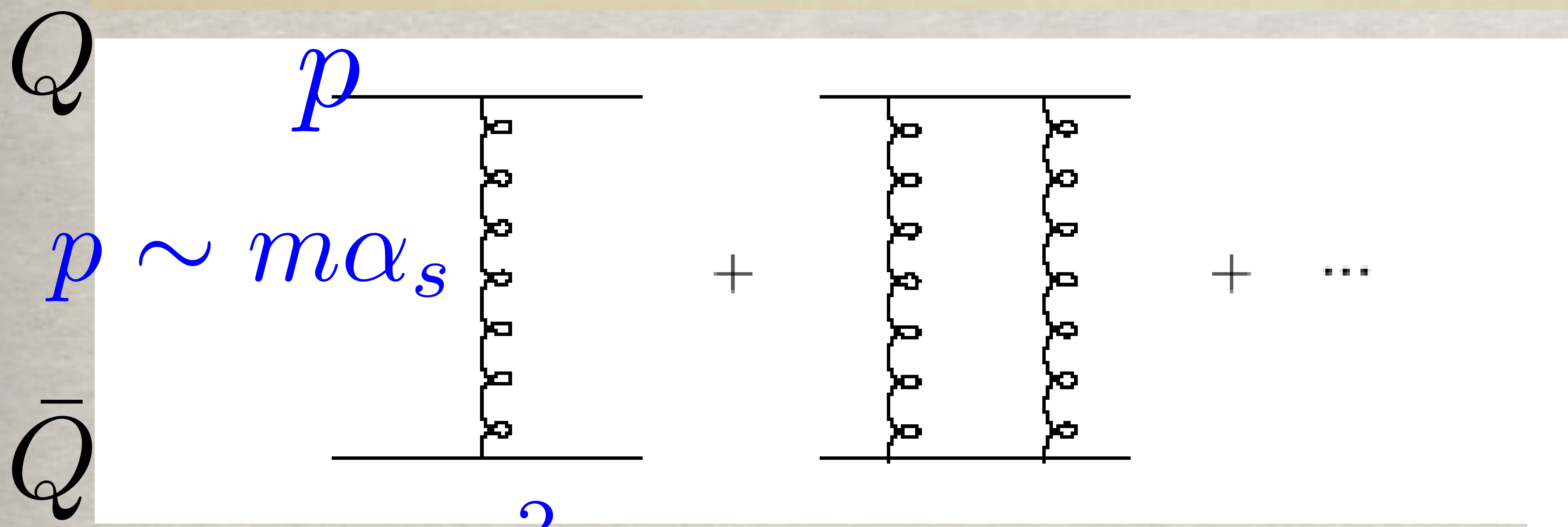

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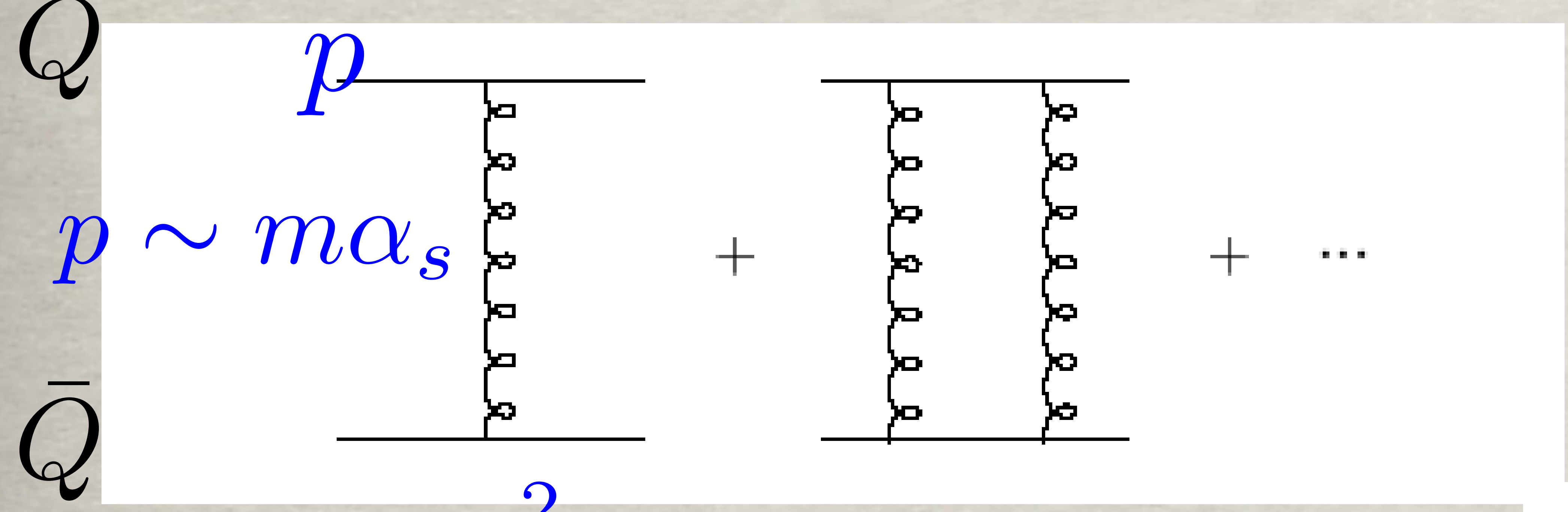

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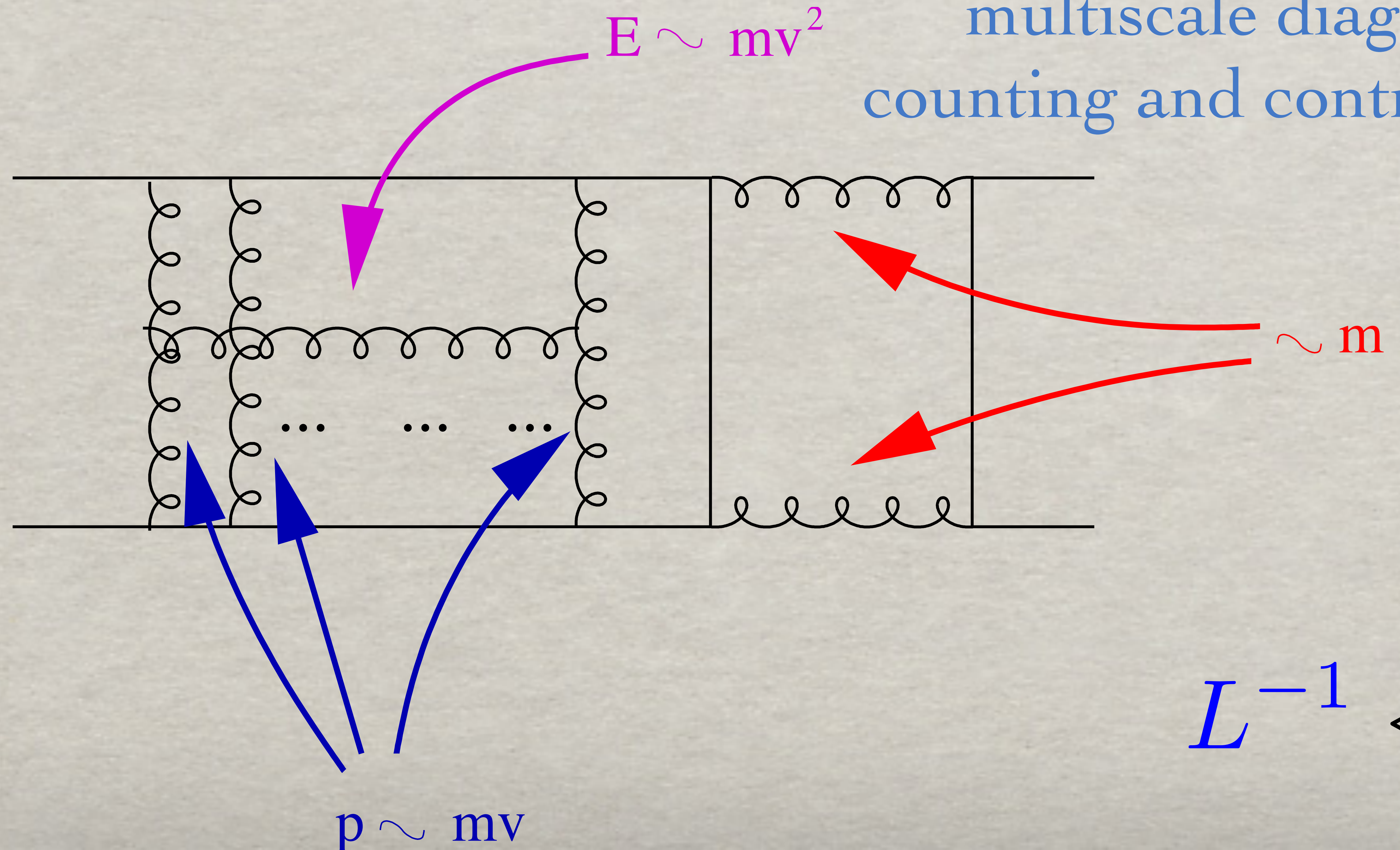
multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

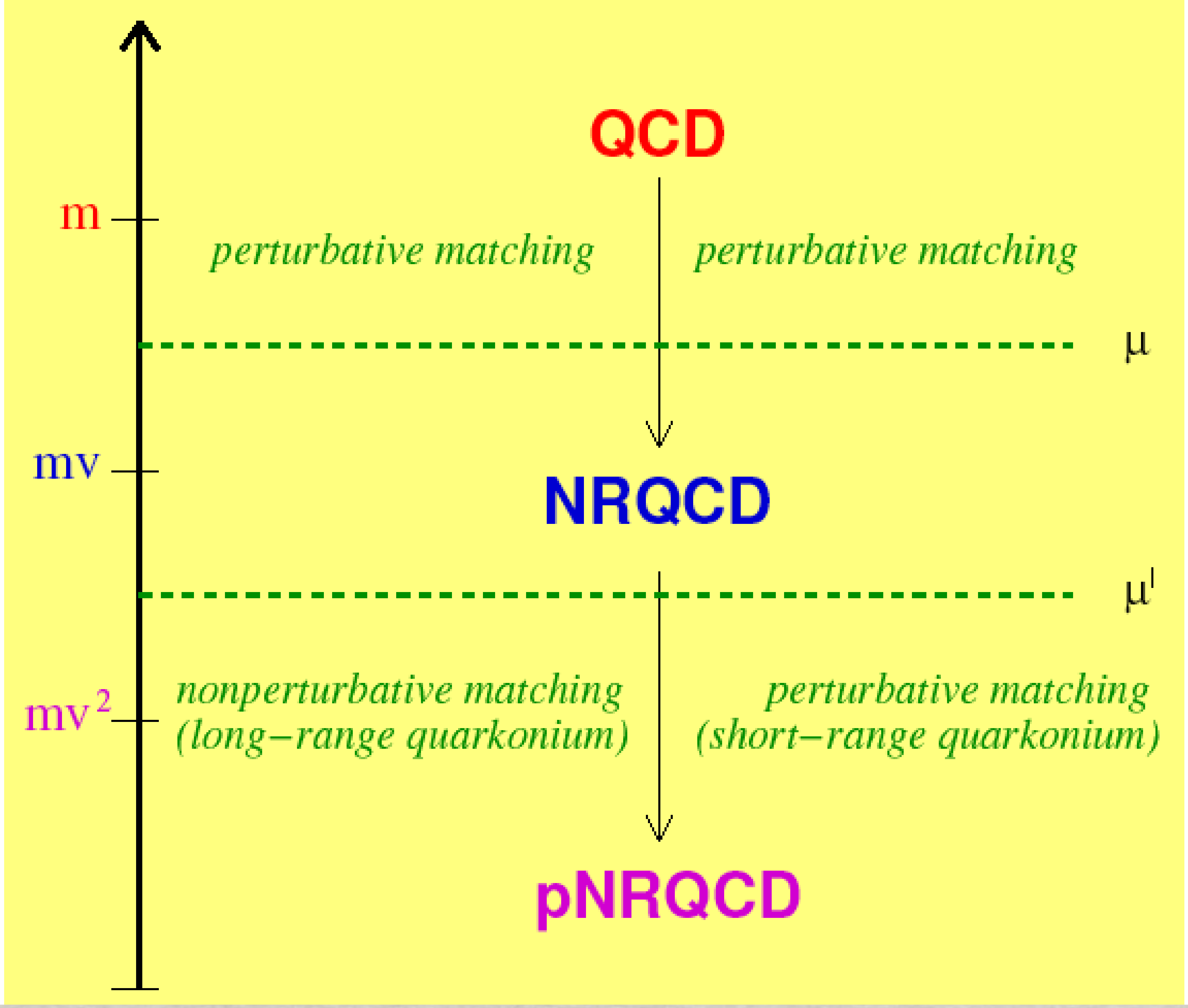


Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

Quarkonium with Non relativistic Effective Field Theories

Color degrees of freedom
3X3bar=1+8
singlet and octet QQbar



Hard

Soft
(relative
momentum)

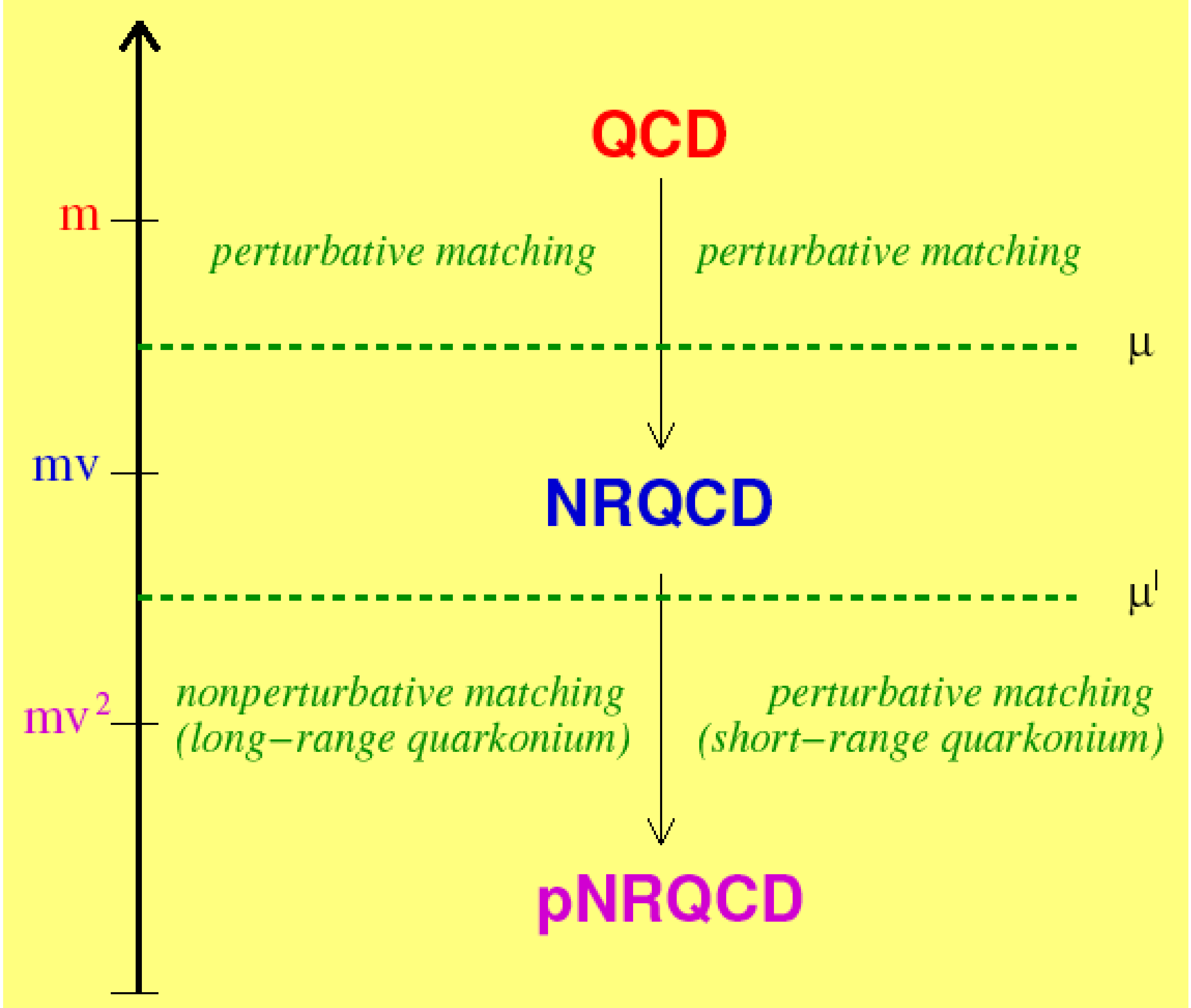
Ultrasoft
(binding energy)

$$\mathcal{L}_{\rm EFT} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

Quarkonium with Non relativistic Effective Field Theories

Color degrees of freedom
3X3bar=1+8
singlet and octet QQbar



Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft
(relative
momentum)

Ultrasoft
(binding energy)

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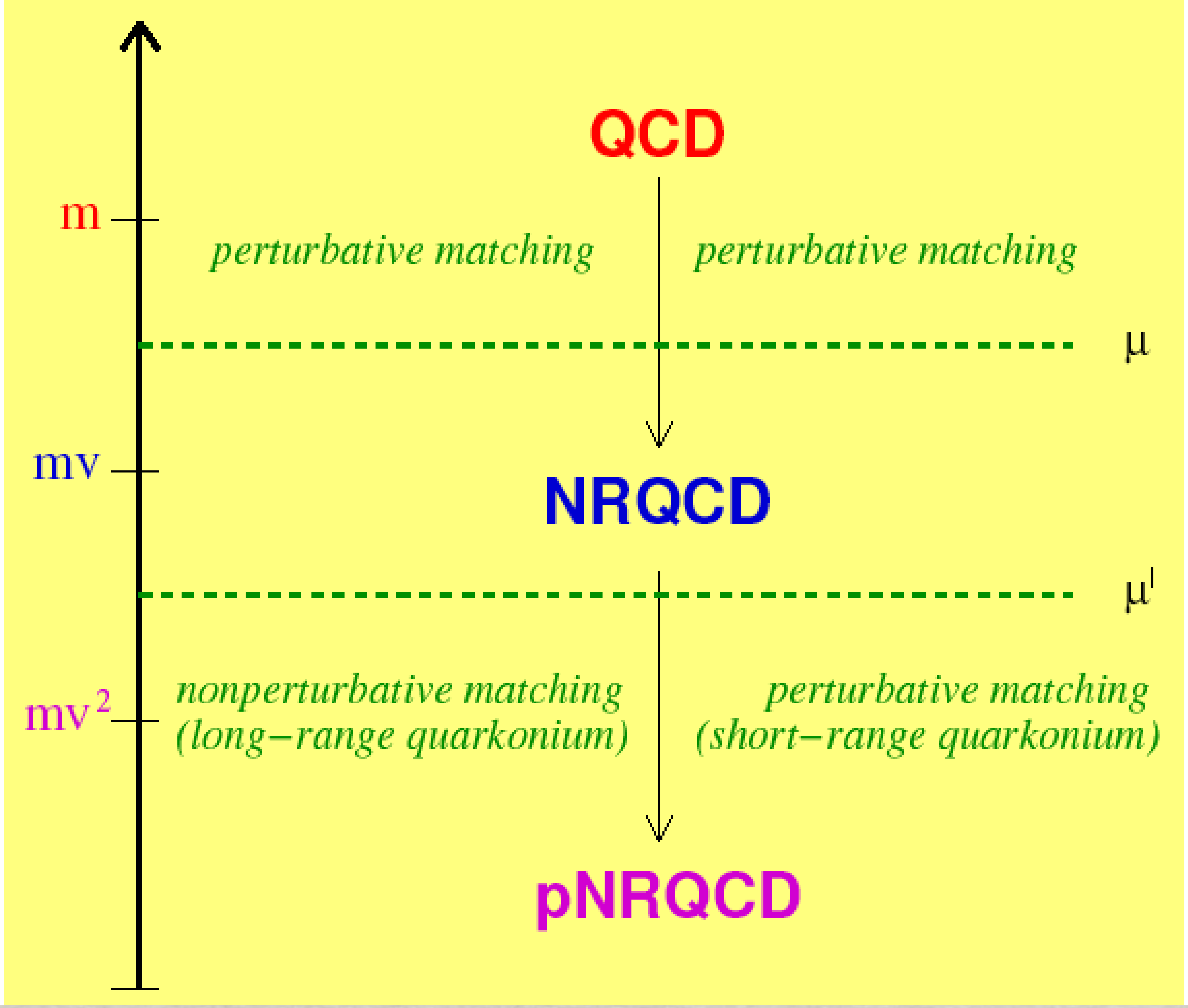

Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft
(relative
momentum)

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

Ultrasoft
(binding energy)

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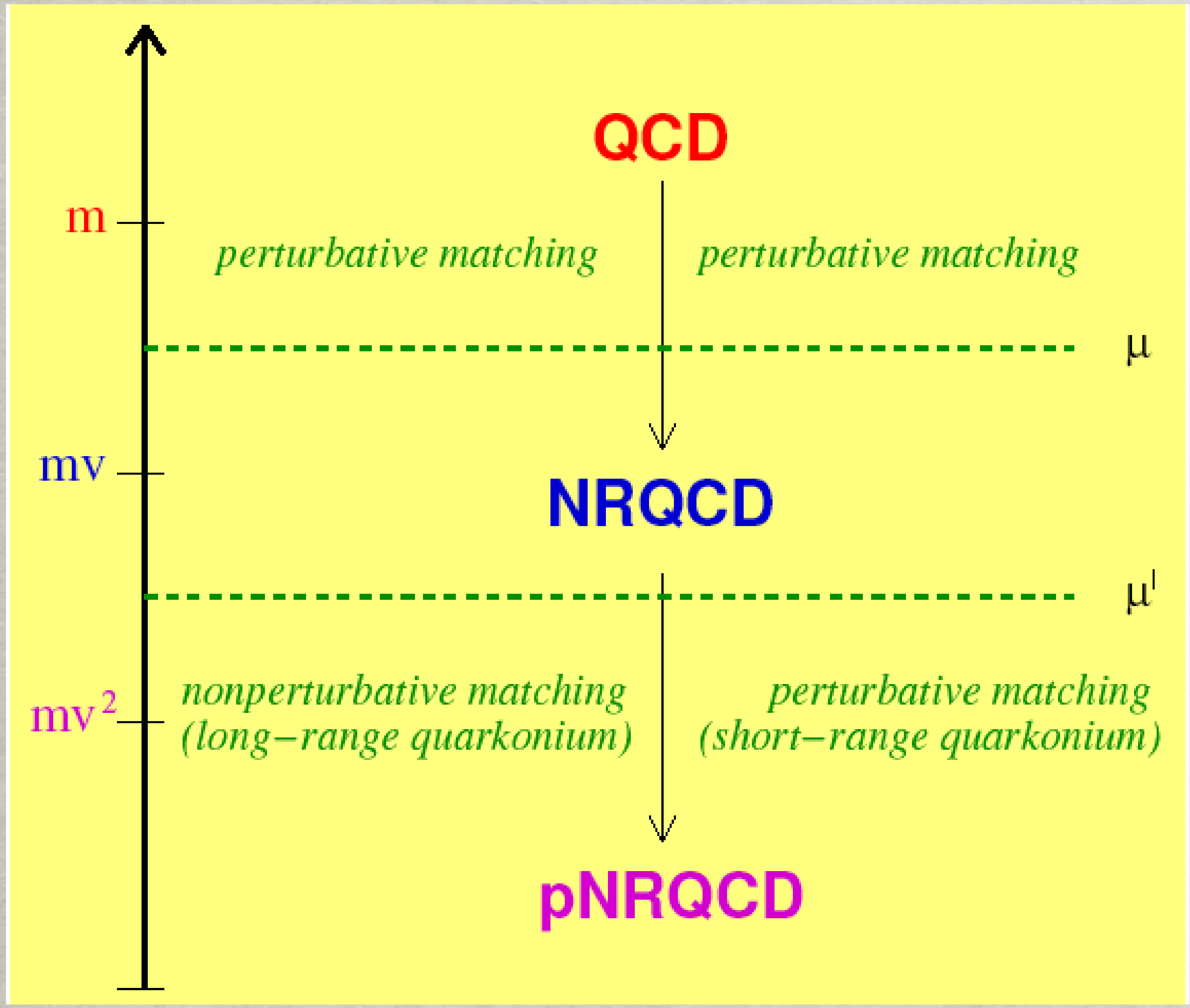
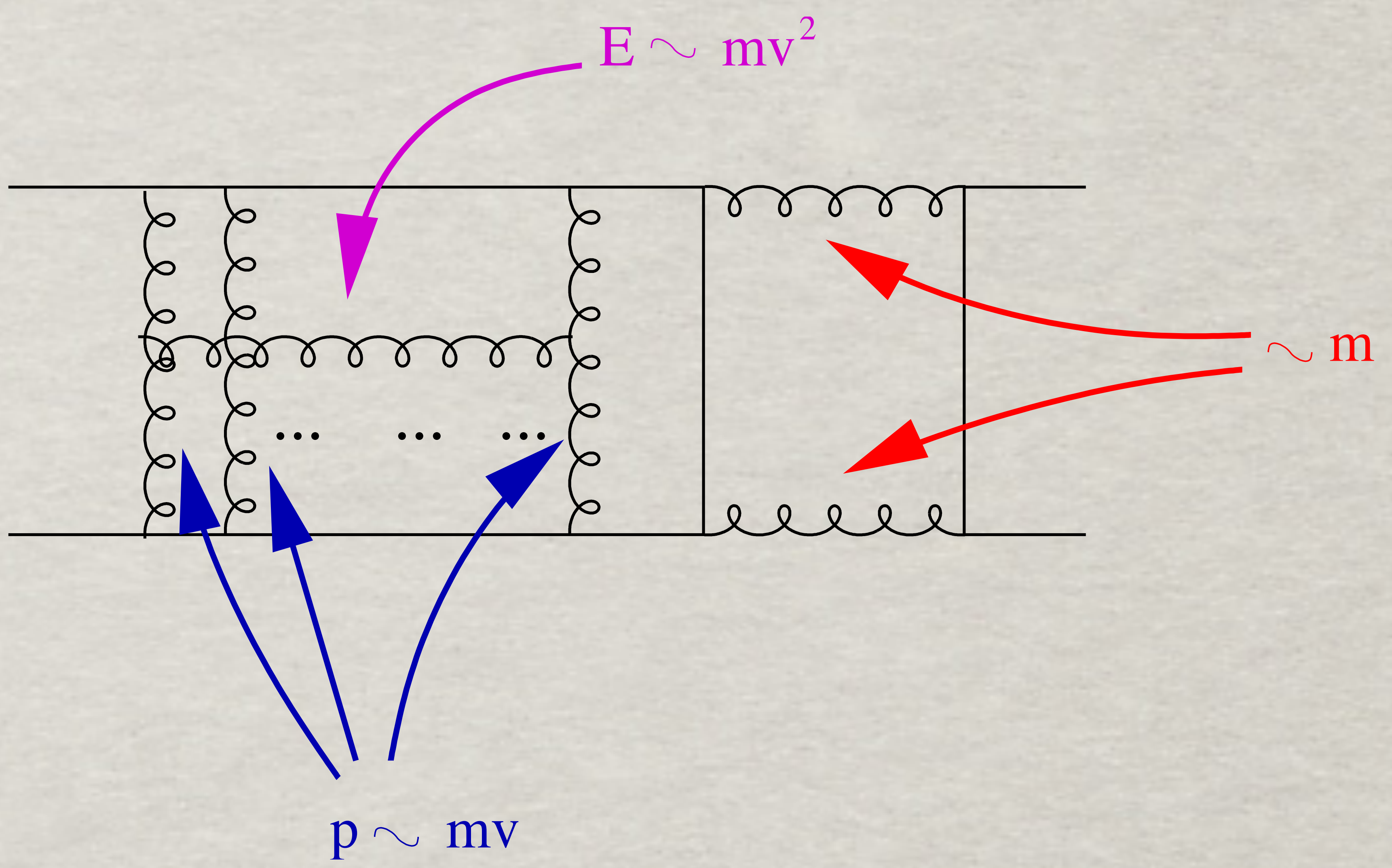
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)





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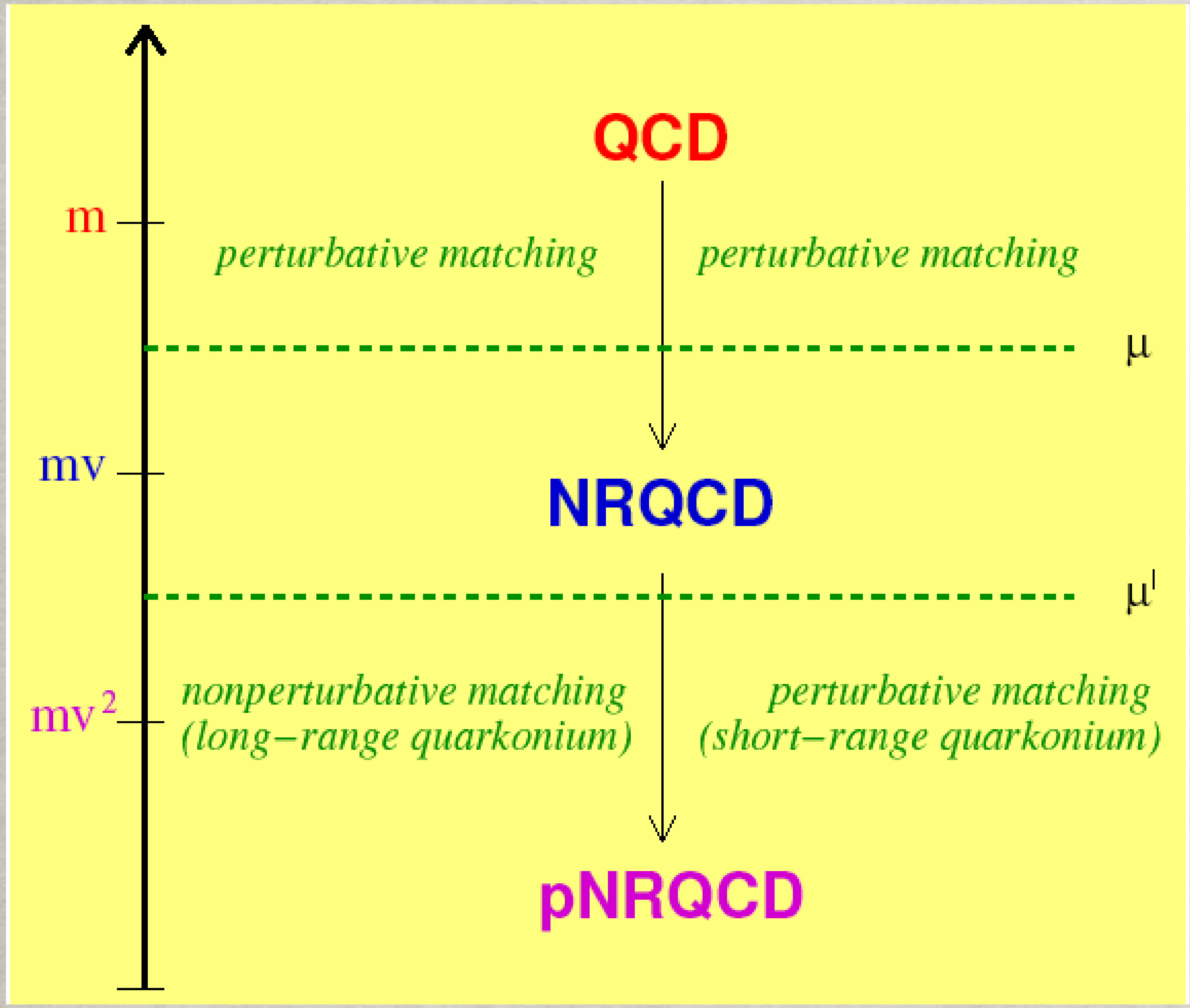

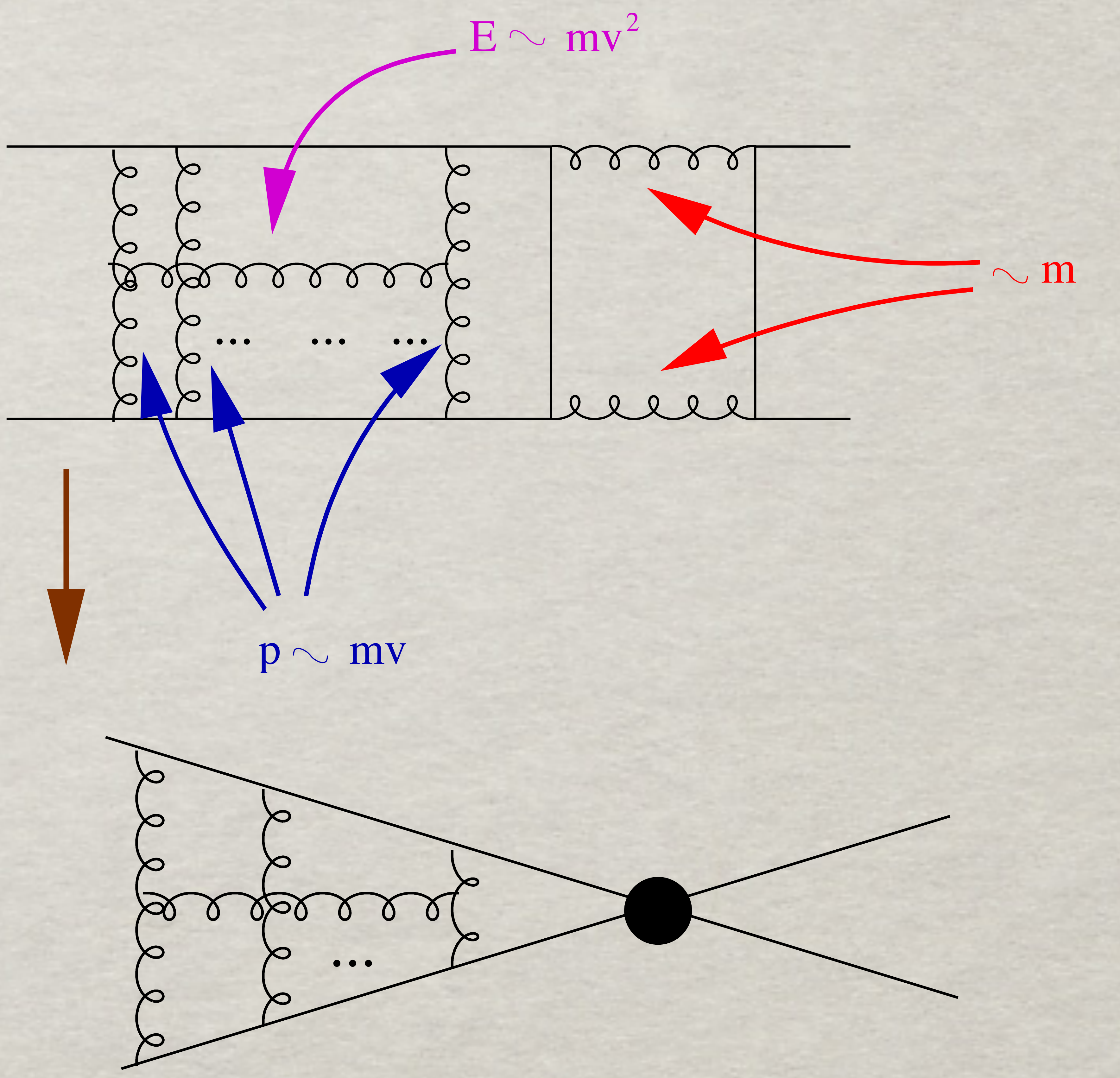

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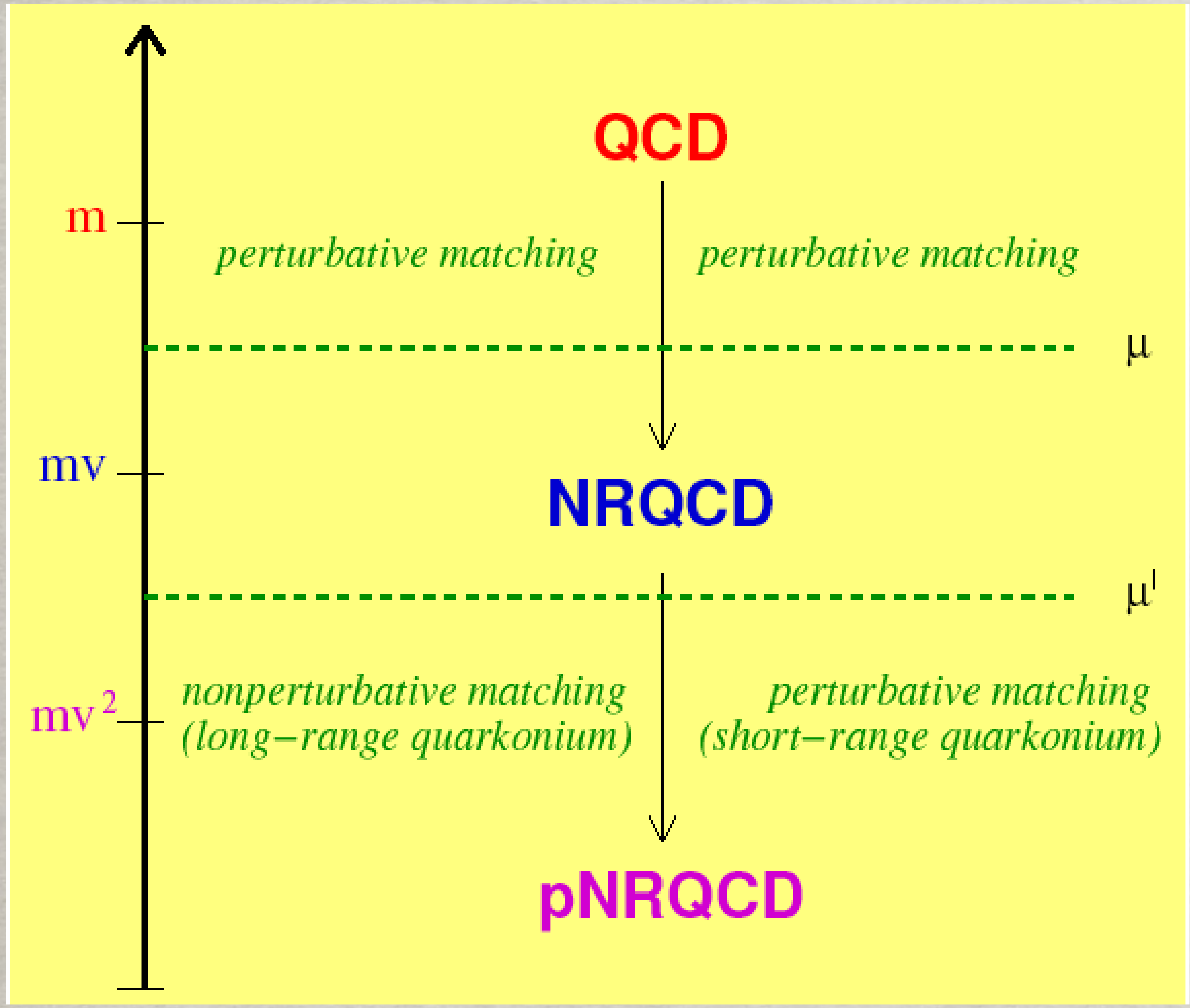
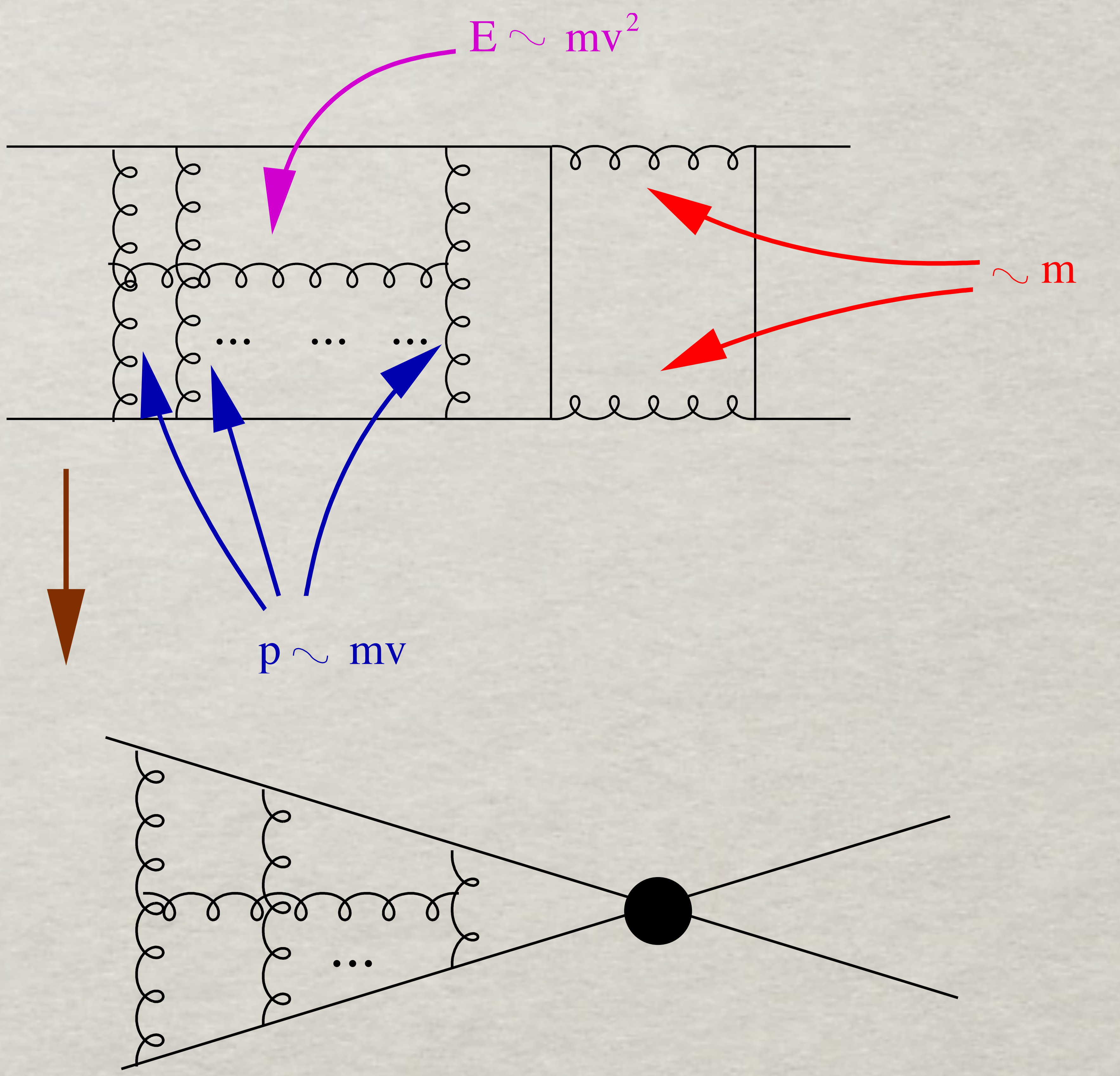
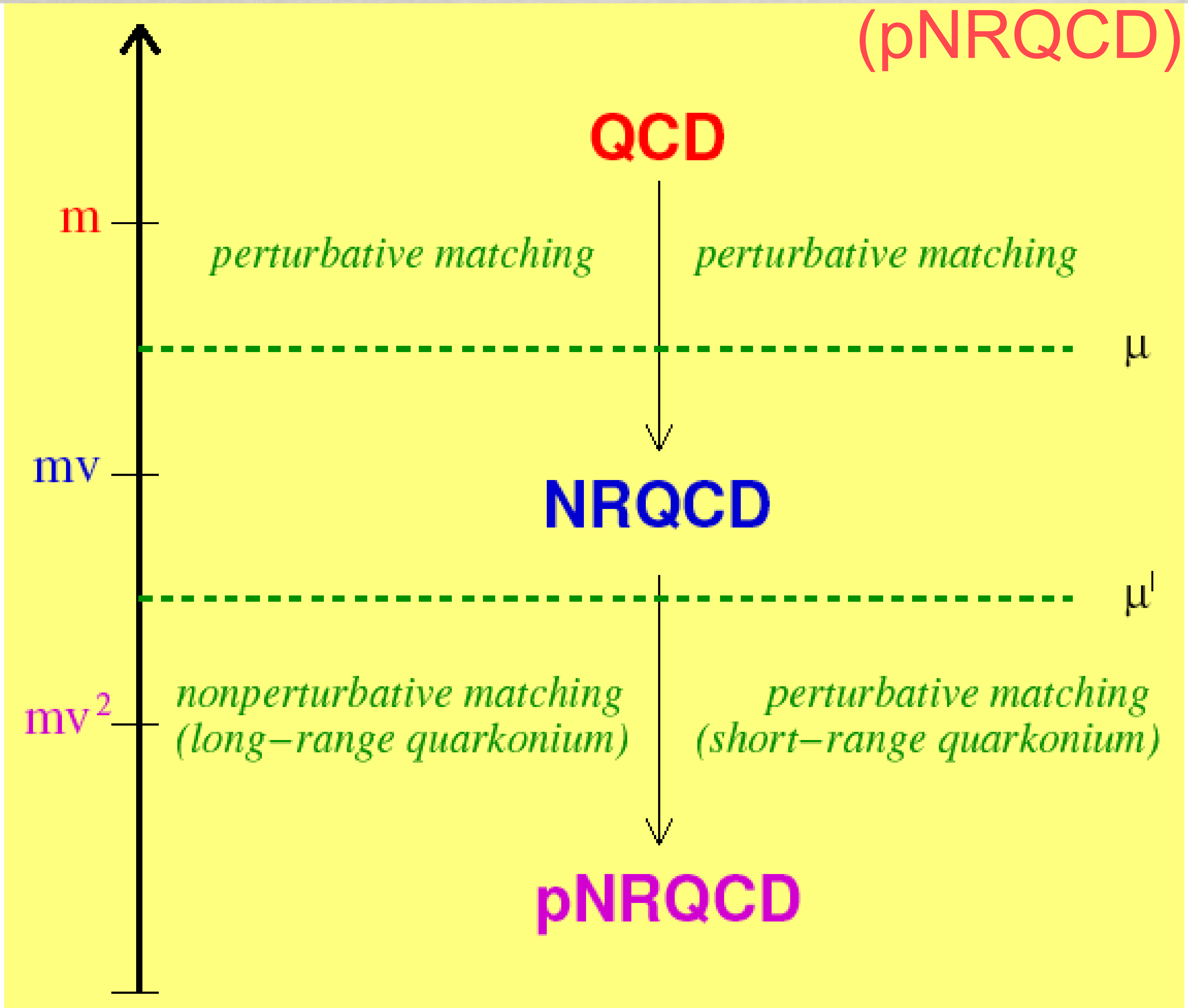
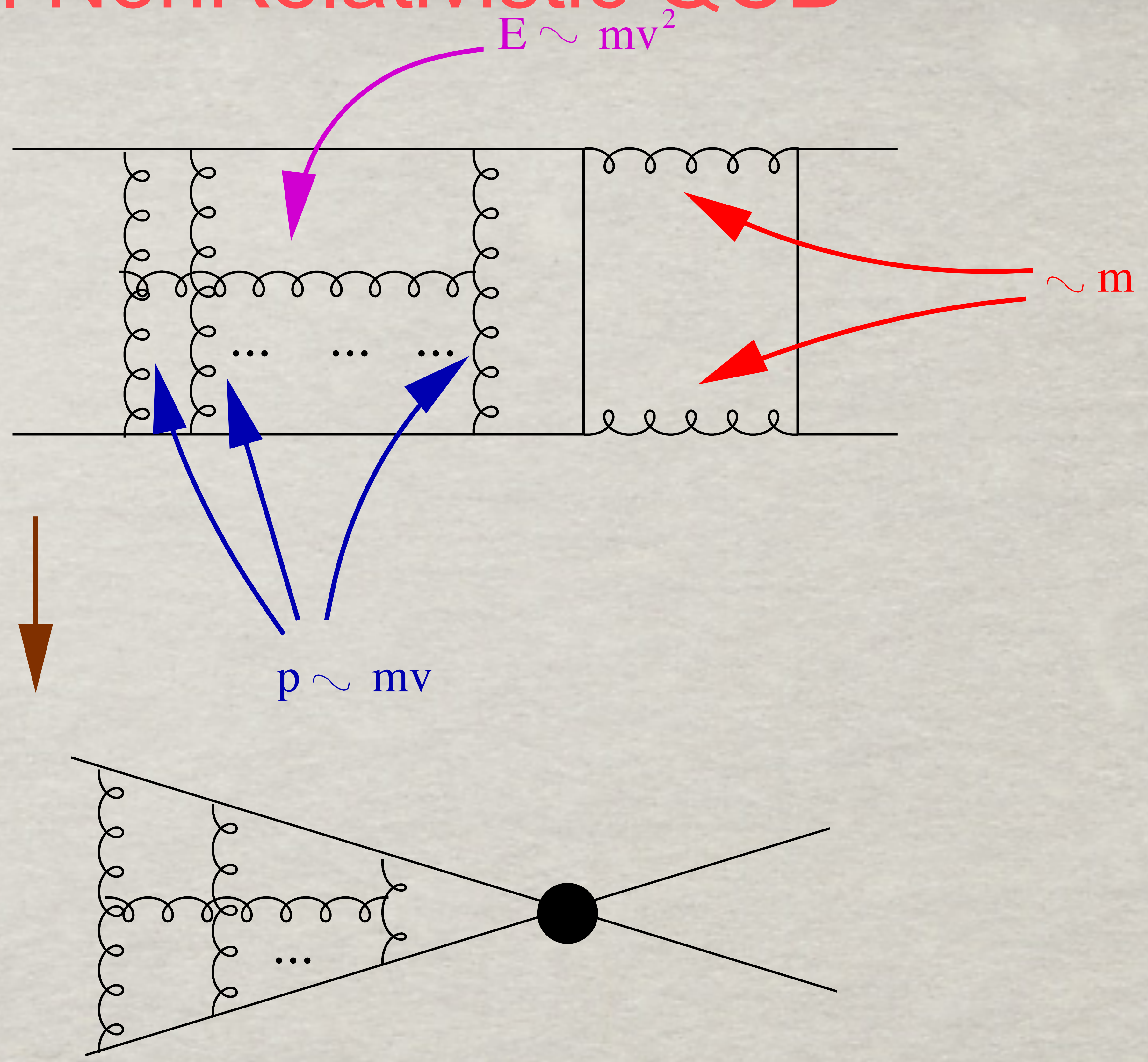




$$\mathcal{L}_{\mathrm{NRQCD}} = \sum_n c(\alpha_{\mathrm{s}}(m/\mu)) \times \frac{O_n(\mu, \lambda)}{m^n}$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





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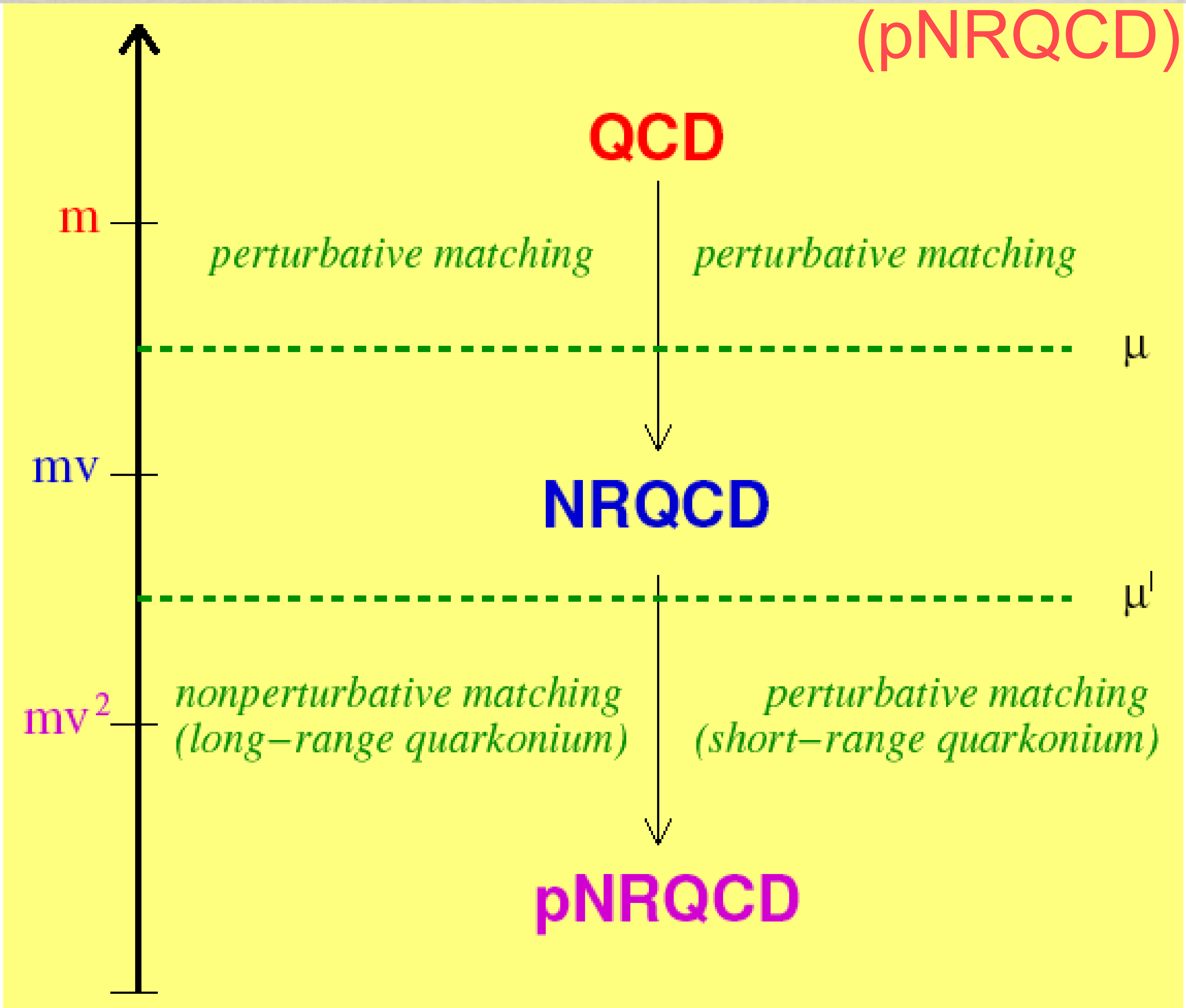

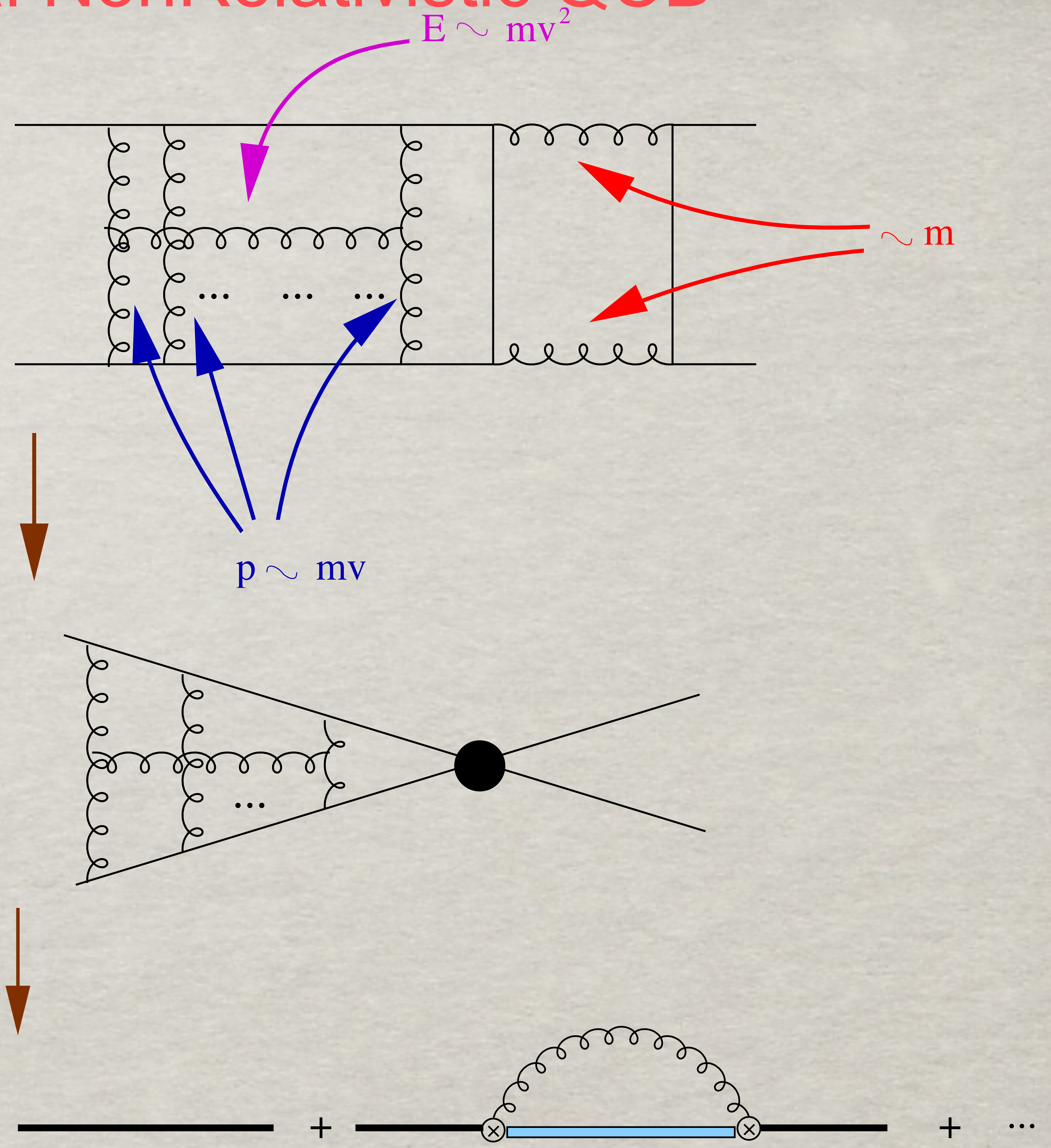

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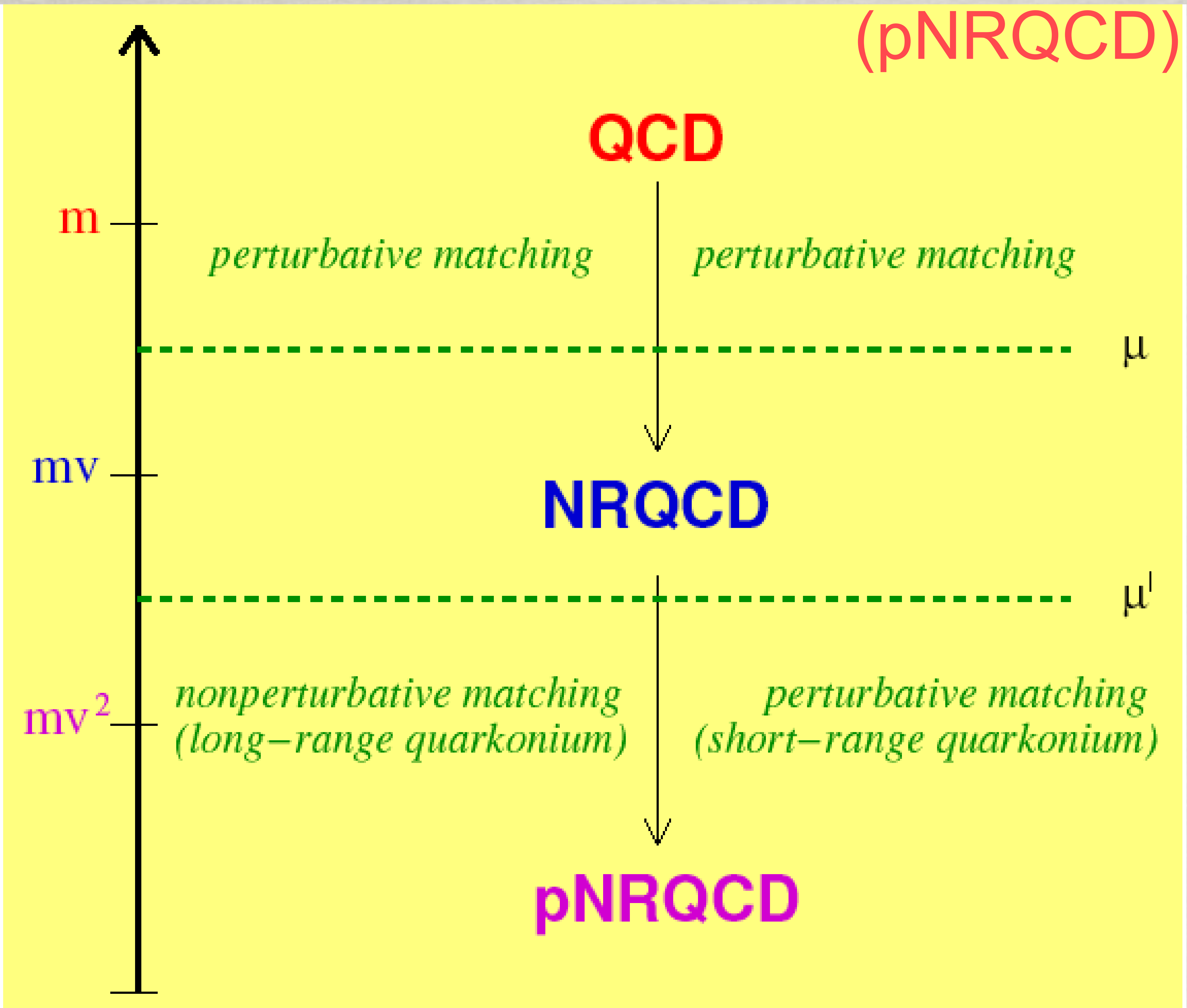
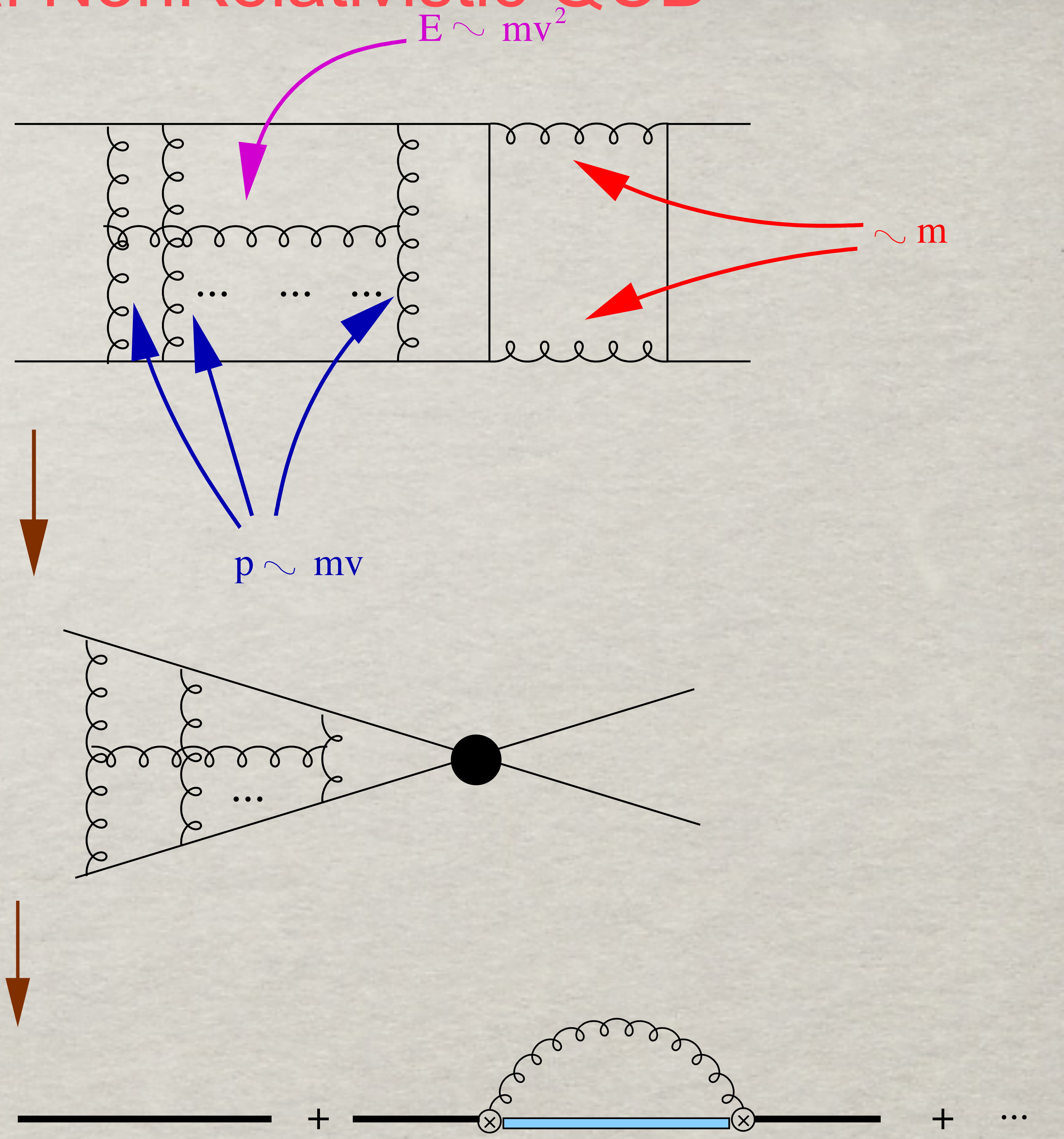
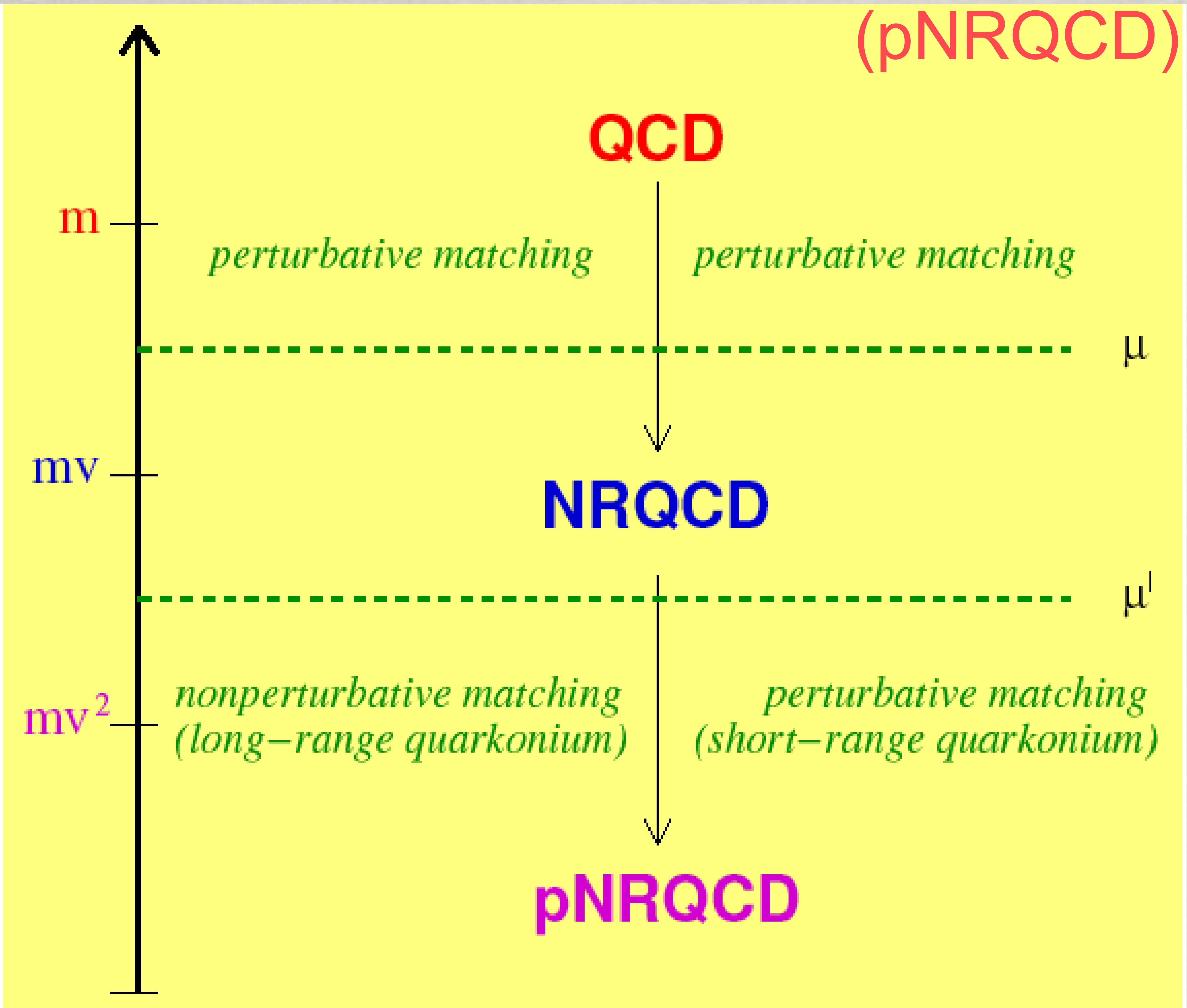
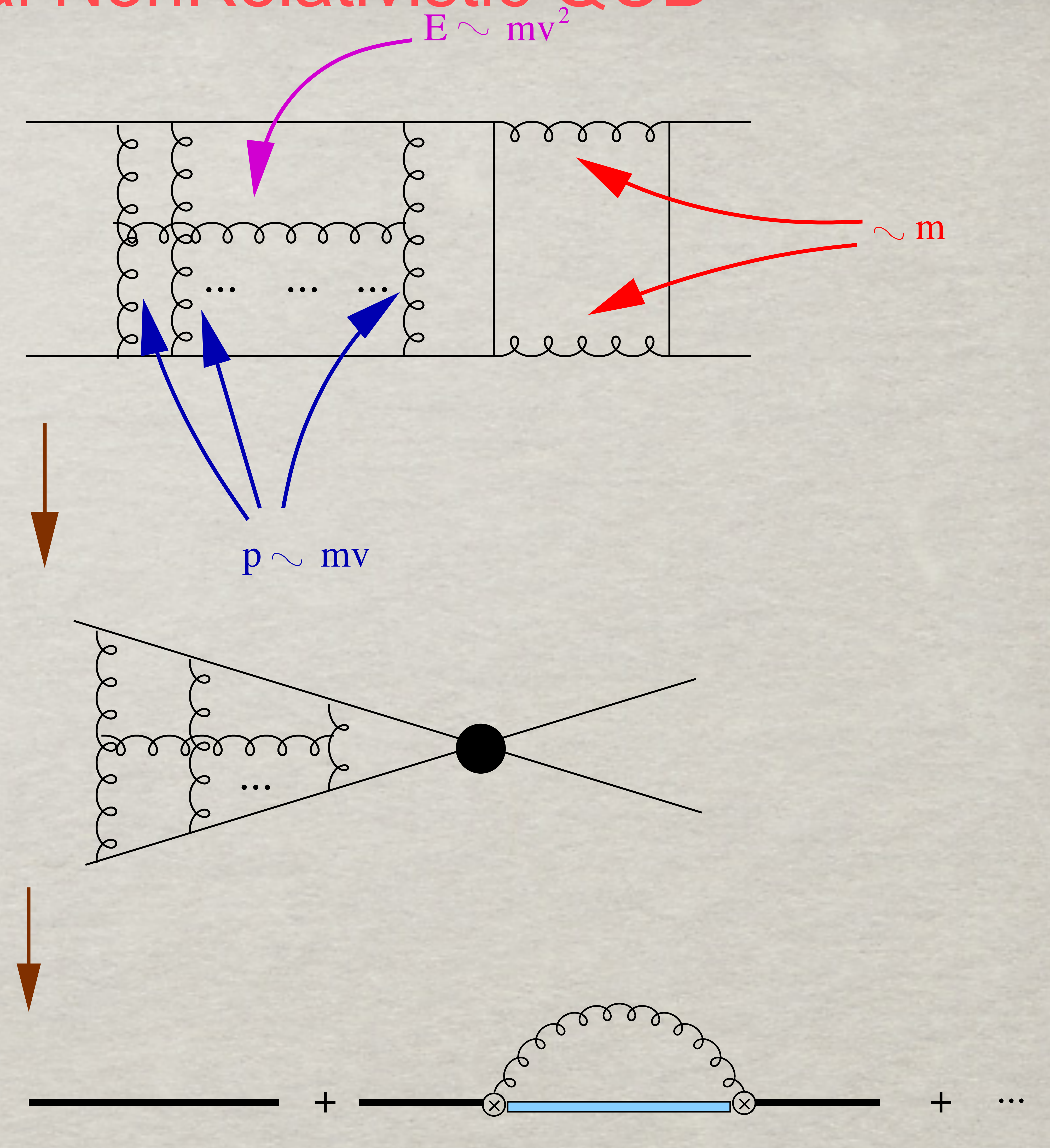




$$\mathcal{L}_{\mathrm{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





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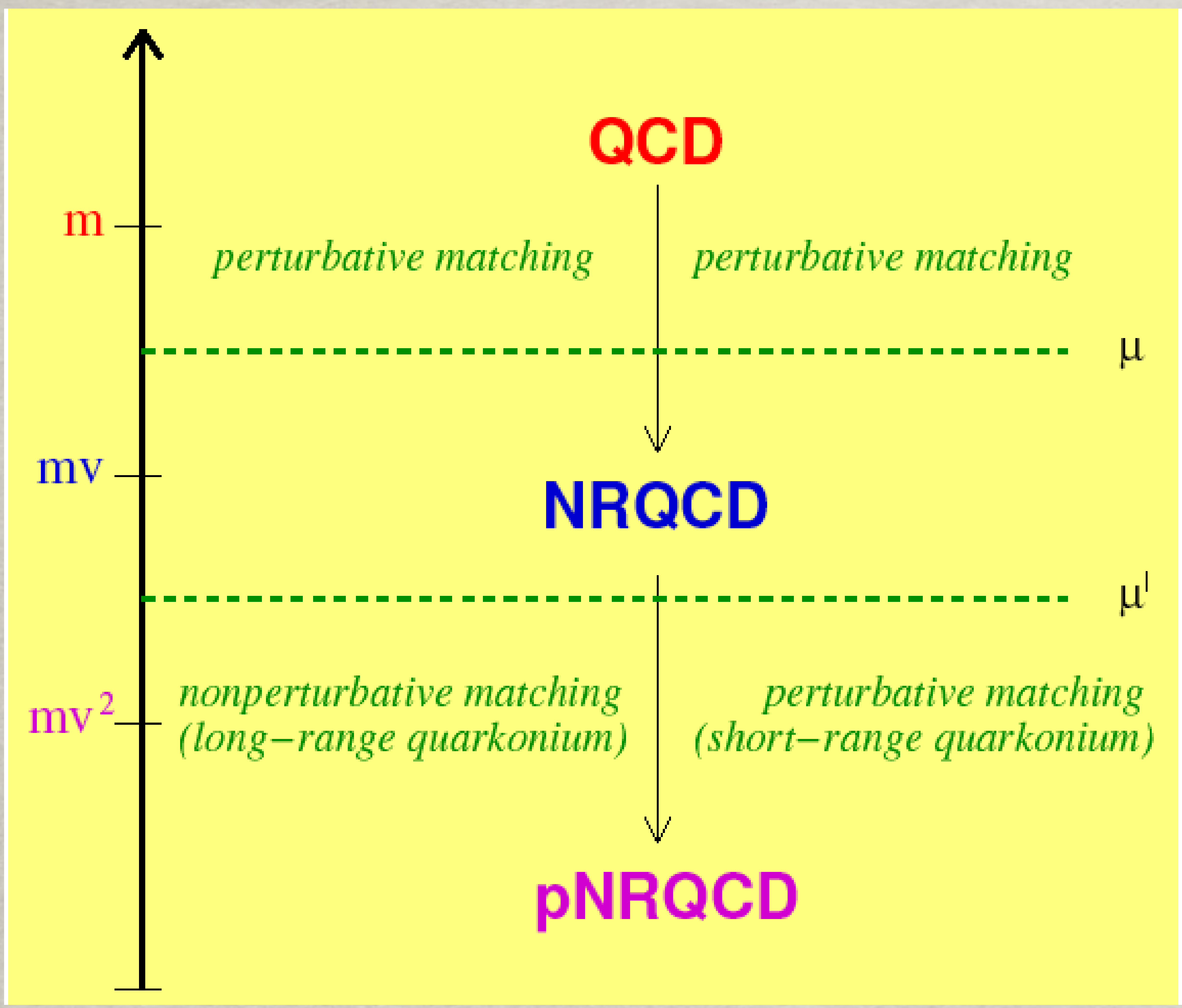
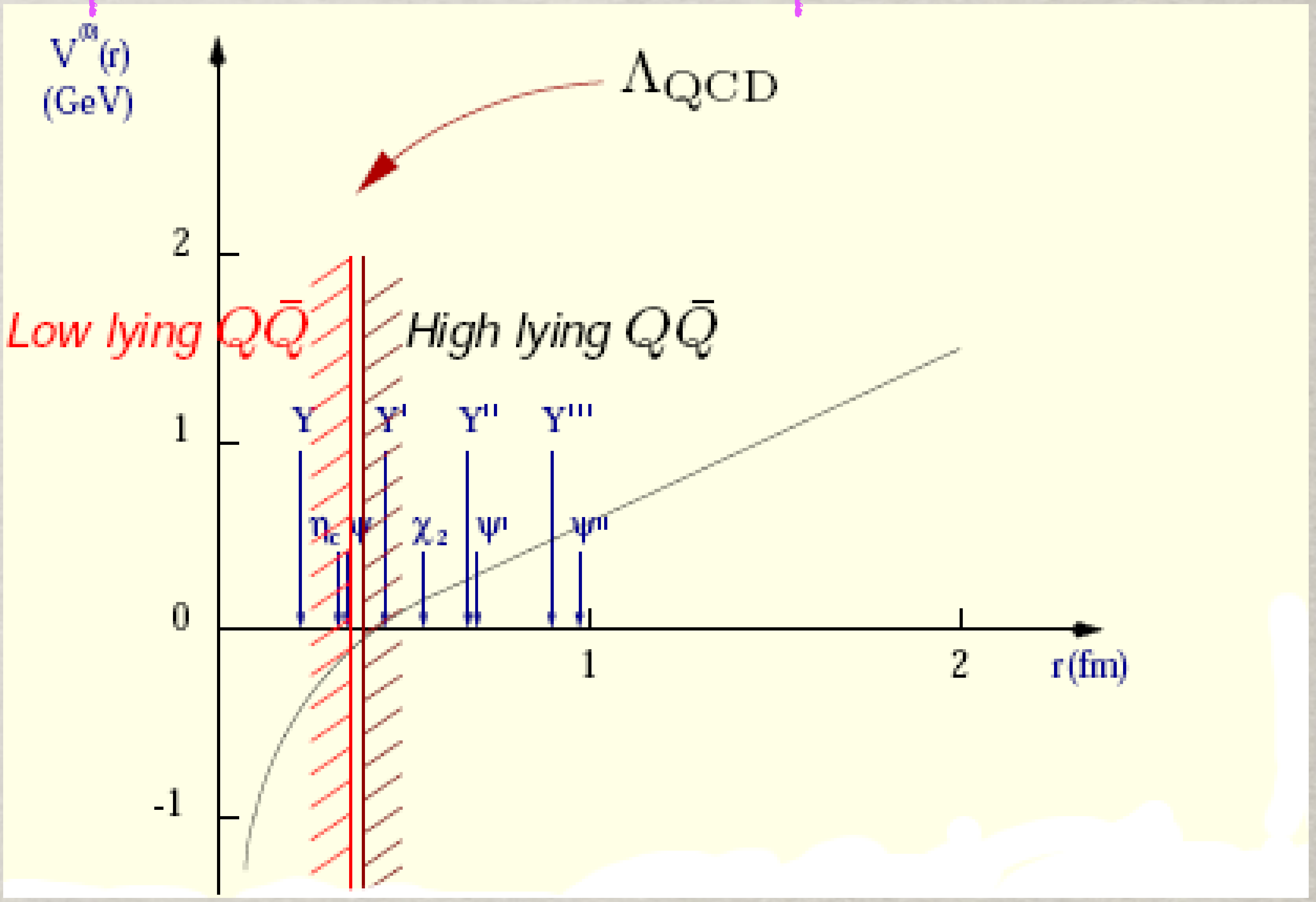
Quarkonium with NR EFT: pNRQCD



weakly coupled pNRQCD

strongly coupled pNRQCD



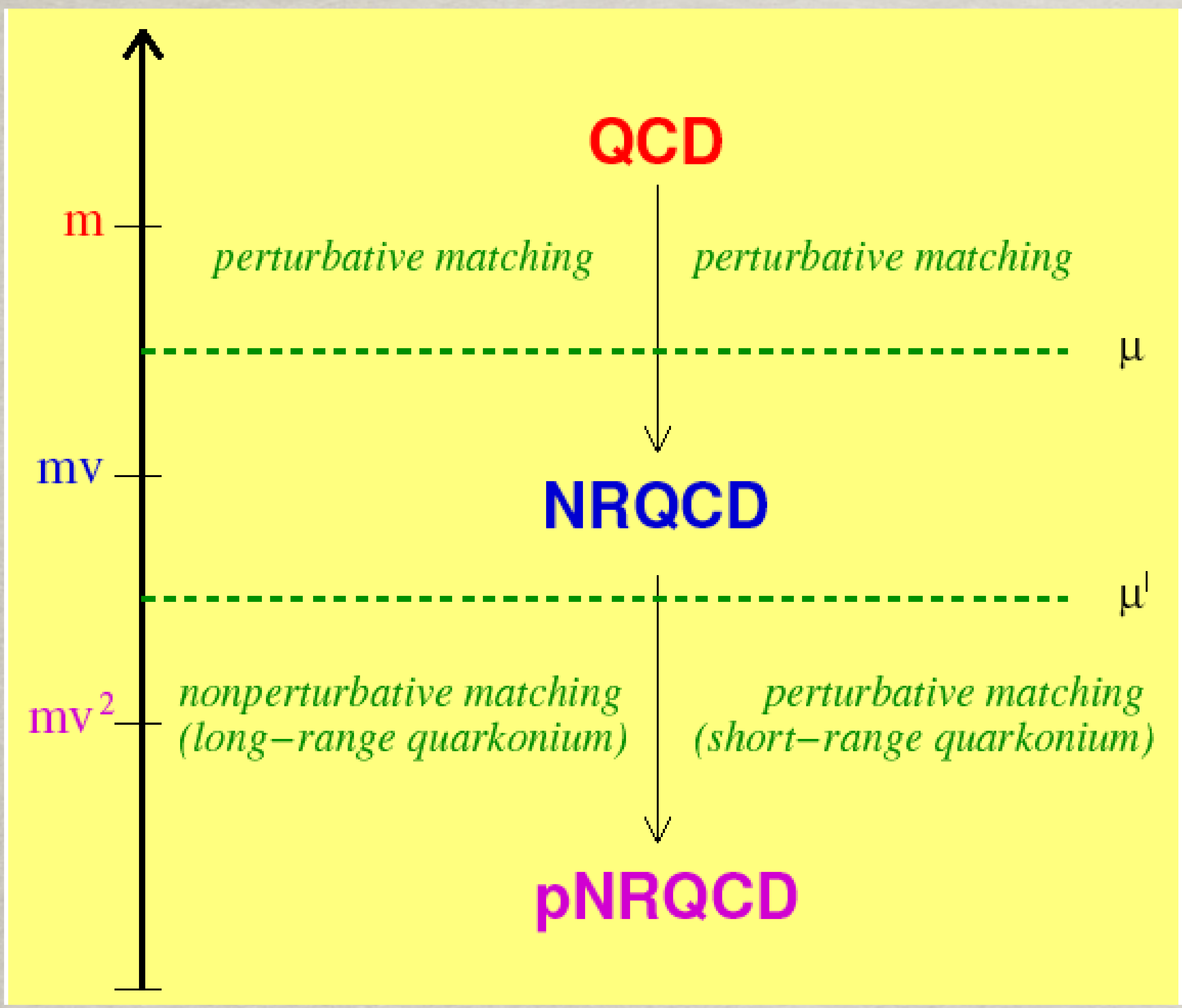
A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{QCD}$

In QCD another scale is relevant Λ_{QCD}

Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, Pineda, Soto 00--014
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

Quarkonium with NR EFT: pNRQCD



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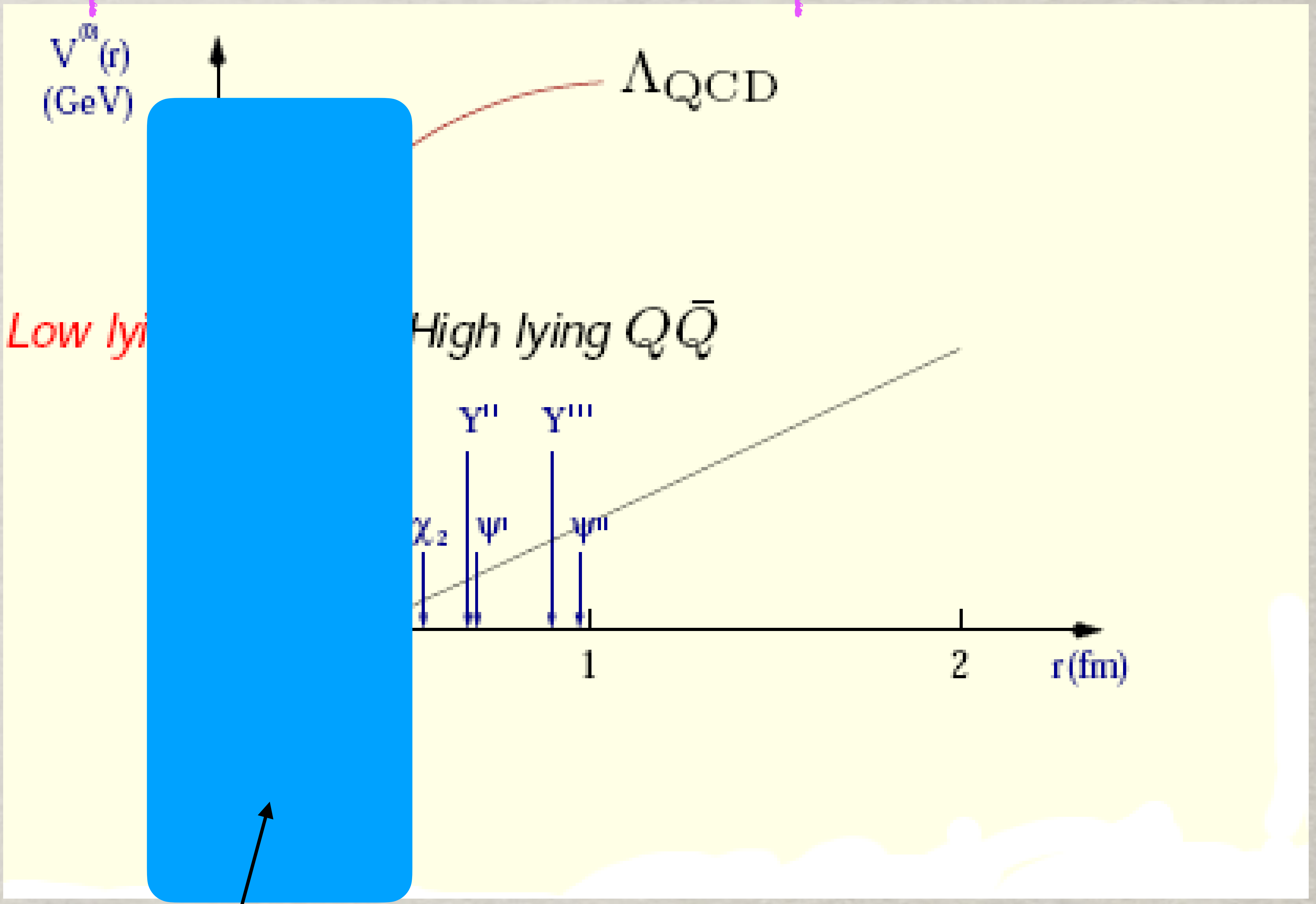
weakly coupled pNRQCD

strongly coupled pNRQCD



A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{QCD}$

this the region where to extract precise determinations of alphas and the quark masses

Notice: additional scales smaller than m can be integrated out combining with other EFTs

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Example: quarkonium in thermal medium, T<m, use Hard Thermal EFT (HTL) loop to resum the scale T

N. B., Ghiglieri,Petreczky, Vairo 08

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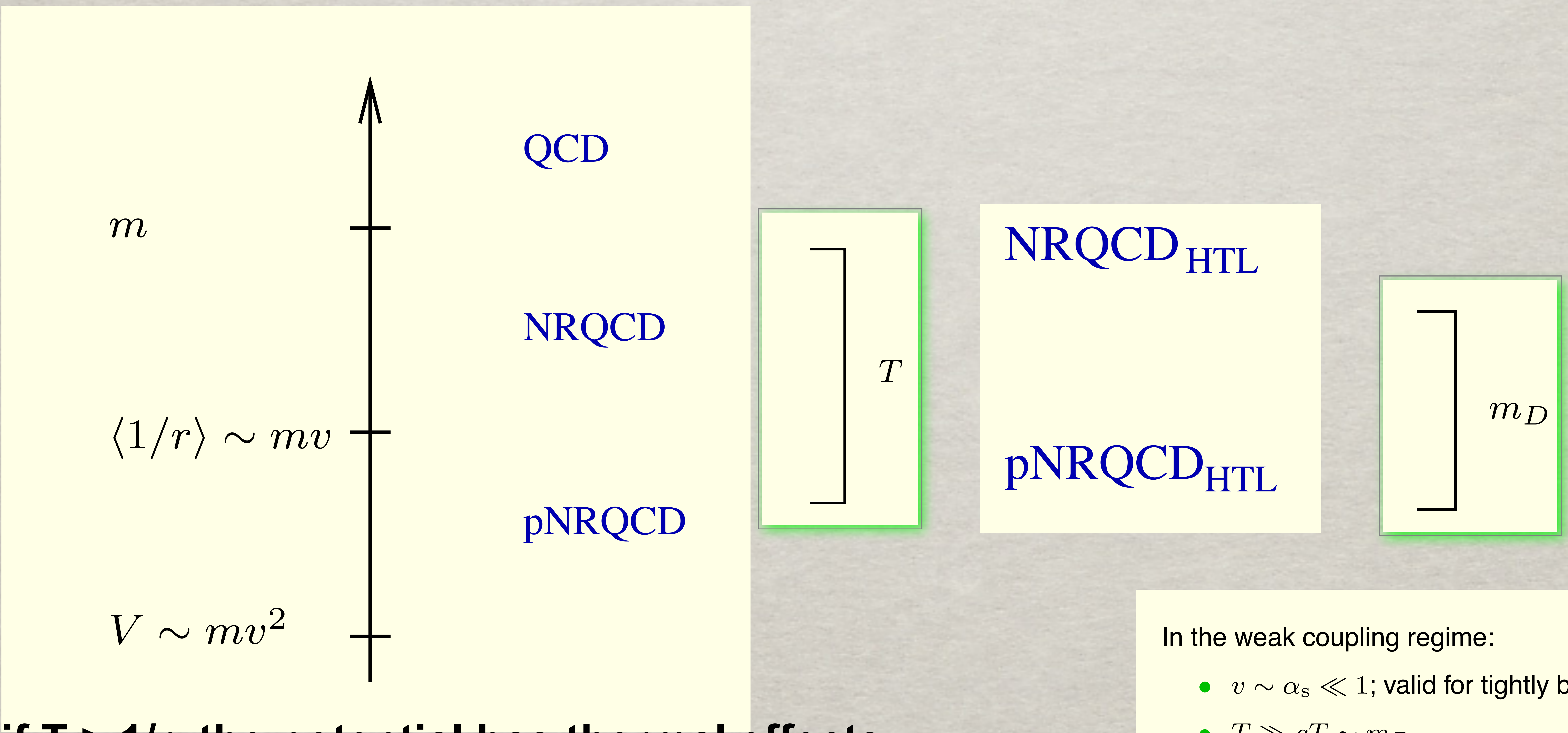


if T > 1/r the potential has thermal effects that can be systematically calculated with pNRQCD

In the weak coupling regime:

- $v \sim \alpha_s \ll 1$; valid for tightly bound states: $\Upsilon(1S)$, J/ψ, ...
- $T \gg gT \sim m_D$.

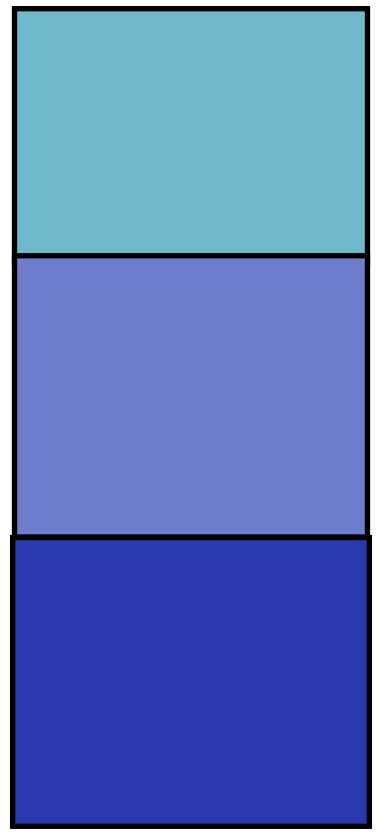
Effects due to the scale Λ_{QCD} will not be considered.

Low energy (nonperturbative) factorized effects depend on the size of the physical system

$$r \ll \frac{1}{\Lambda_{QCD}}$$

lowest quarkonia states

The EFT factorizes the low energy nonperturbative part.
Depending on the physical system:



$\langle F^2(0)\rangle$ $\Upsilon(1S)$, ... local condensates

$\Lambda_{\rm QCD} \sim t$ $\langle F(t)F(0)\rangle$ annihilations, short range $c\bar{c}$, $b\bar{b}$, gluelumps, ... non local condensates

$\Lambda_{\rm QCD} \sim loop$ $\langle e^{ig\oint dx A(x)}\rangle$ long range $c\bar{c}$, $b\bar{b}$, hybrids, glueballs, ... Wilson loops

The more extended the physical object, the more we probe the non-perturbative vacuum.

excited quarkonia states

$$r \sim \frac{1}{\Lambda_{QCD}}$$

$$r \ll \frac{1}{\Lambda_{QCD}} \qquad T$$

quarkonia in a hot medium

$$r \sim \frac{1}{\Lambda_{QCD}}$$

quarkonia and exotics close and above threshold

Low energy (nonperturbative) facto
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lowest
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Depending on the phy

condensates

Λ ...ions, short range $c\bar{c}$, $b\bar{b}$, gluelumps, ...
non local condensates

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Wilson loops

...ed the physical object, the more we probe
th... ...turbative vacuum.

THESE OBJECTS SHOULD BE
CALCULATED ON THE LATTICE
which is why we founded the TUMQCD
lattice collaboration

excited
quarkonia states

$r \sim \frac{1}{\Lambda_{QCD}}$

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THESE OBJECTS SHOULD BE CALCULATED ON THE LATTICE

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THESE EFFECTS ARE SUPPRESSED FOR SMALL SYSTEMS

$r \ll \frac{1}{\Lambda_{QCD}}$

lowest quarkonia states

excited quarkonia states

$r \sim \frac{1}{\Lambda_{QCD}}$

$r \ll \frac{1}{\Lambda_{QCD}}$ T

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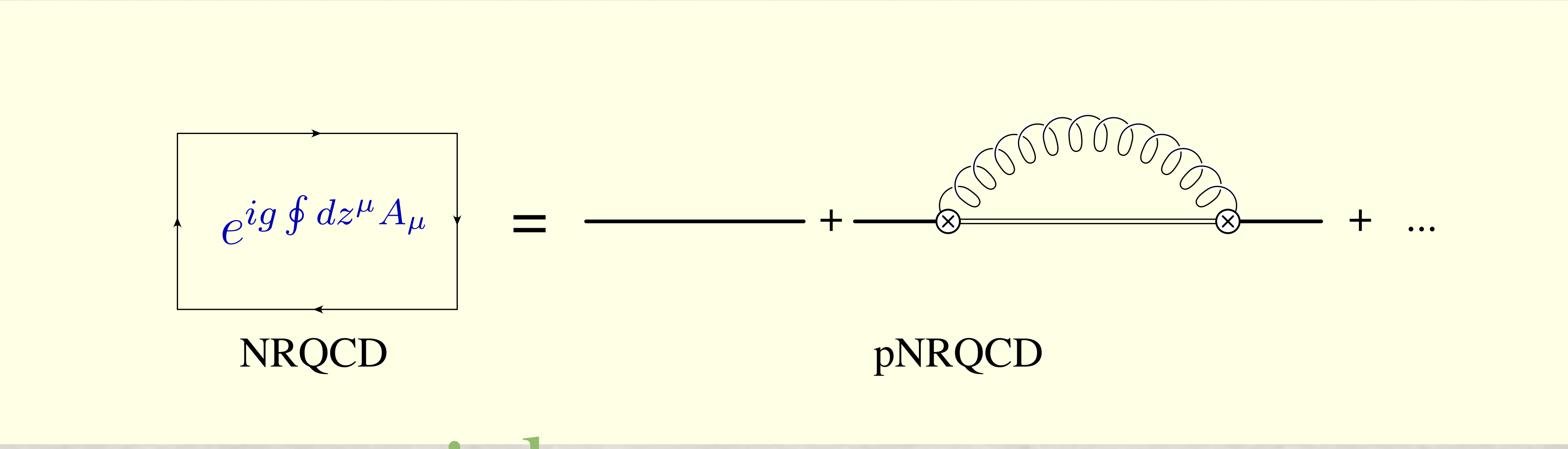
quarkonia and exotics close and above threshold

QCD singlet static potential and singlet static energy

QCD singlet static potential and singlet static energy



potential

$$\lim_{T\to\infty} \frac{i}{T} \ln\langle \square \rangle = V_s(r,\mu) - i\frac{g^2}{N_c} V_A^2 \int_0^\infty dt\, e^{-it(V_o - V_s)} \langle \mathrm{Tr}(r\cdot E(t)\, r\cdot E(0))\rangle(\mu) + \dots$$

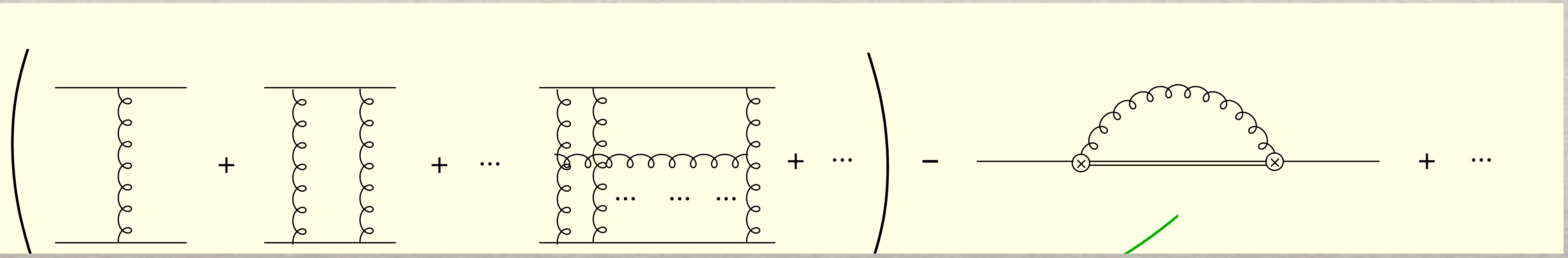
static energy

ultrasoft contribution

contributes from 3 loops

The potential is a Wilson coefficient of the EFT.
In general, it undergoes renormalization, develops scale dependence and satisfies renormalization group equations, which allow to resum large logarithms.

$V =$



Quarkonium singlet static potential at N^4LO

$$V_s(r,\mu) = -C_F \frac{\alpha_s(1/r)}{r} \left[1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 \right.$$

$$+ \left(\frac{16\,\pi^2}{3} C_A^3 \ln r\mu + a_3 \right) \left(\frac{\alpha_s(1/r)}{4\pi}\right)^3$$

$$\left. + \left(a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2 C_A^3 \beta_0 (-5 + 6\ln 2) \right) \ln r\mu + a_4 \right) \left(\frac{\alpha_s(1/r)}{4\pi}\right)^4 \right]$$

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a_1 Billoire 80

a_2 Schroeder 99, Peter 97

coeff $ln r\mu$ N.B. Pineda, Soto, Vairo 99

a_4^{L2}, a_4^L N.B., Garcia, Soto, Vairo 06

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

Quarkonium singlet static potential at N^4LO

$$
\begin{aligned}
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& \left. + \left(a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\, C_A^3 \beta_0 (-5 + 6\ln 2)\right)\ln r\mu + a_4\right)\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\right]
\end{aligned}
$$

a_1 Billoire 80

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coeff $ln r\mu$ N.B. Pineda, Soto, V...

a_4^{L2}, a_4^L N.B., Garcia, Sot...

3LOOPS REDUCES TO 1 LOOP IN THE EFT

4LOOPS REDUCES TO 2LOOPS IN THE EFT

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

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\end{aligned}
$$

Two problems:

1)Bad convergence of the series due to large beta_0 terms

2) Large logs

Quarkonium singlet static potential at N^4LO

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$$

Two problems:

for long it was believed that such series was not convergent problem for any phenomenological application

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Quarkonium singlet static potential at N^4LO

$$V_s(r,\mu) = -C_F \frac{\alpha_s(1/r)}{r}\Bigg[1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2\left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 + \left(\frac{16\,\pi^2}{3} C_A^3 \ln r\mu + a_3\right)\left(\frac{\alpha_s(1/r)}{4\pi}\right)^3 + \left(a_4^{L2}\ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2 C_A^3 \beta_0(-5 + 6\ln 2)\right)\ln r\mu + a_4\right)\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\Bigg]$$

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1)Bad convergence of the series due to large beta_0 terms

2) Large logs

The eft cures both:

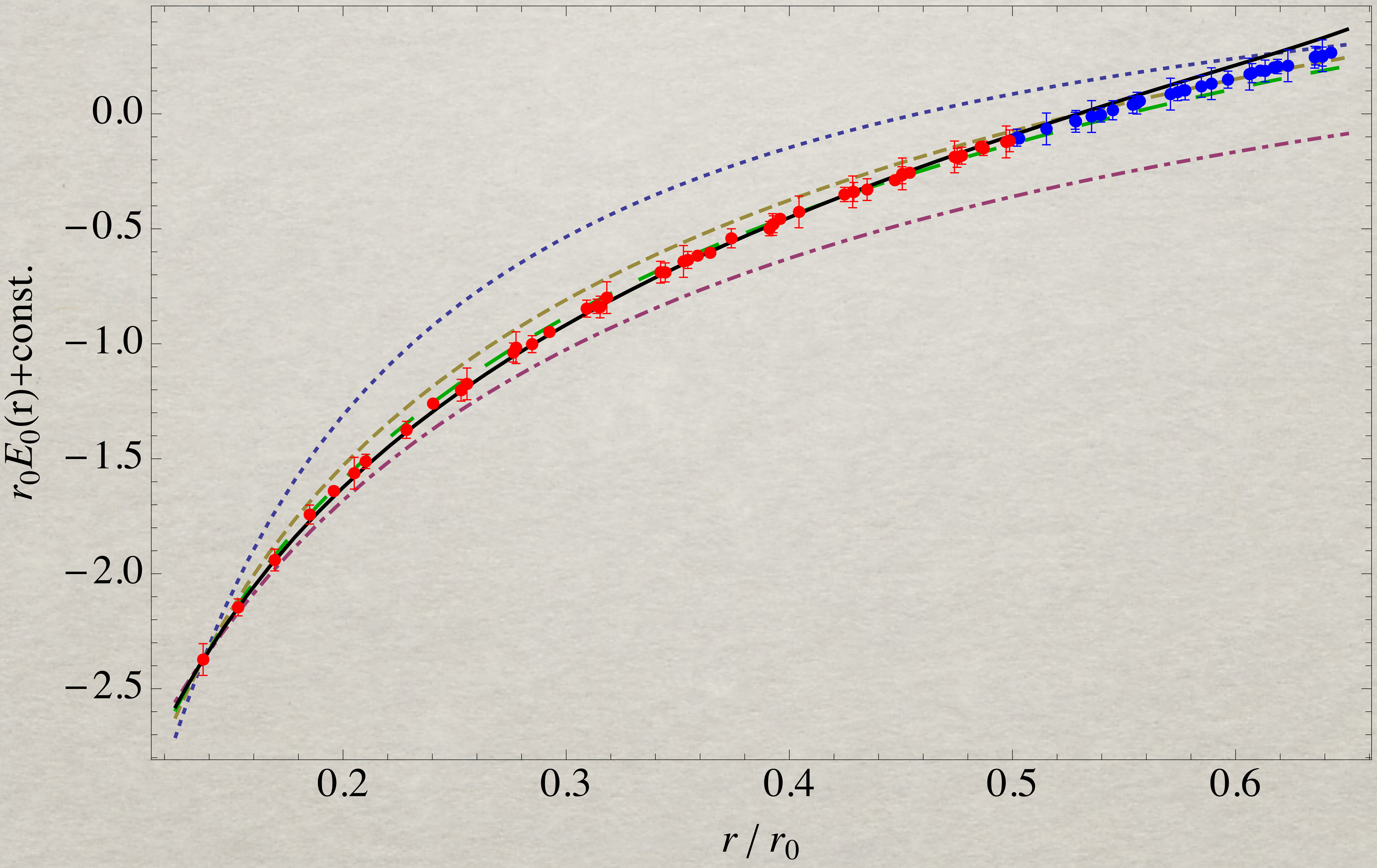
1) Renormalon subtracted scheme

Beneke 98, Hoang, Lee 99, Pineda 01, N.B. Pineda Soto, Vairo 09

2) Renormalization group summation of the logs up to N^3LL $(\alpha_s^{4+n} \ln^n \alpha_s)$ N. B Garcia, Soto Vairo 2007, 2009, Pineda, Soto

QQbar singlet static energy at N^3LI in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014, with Weber 2019



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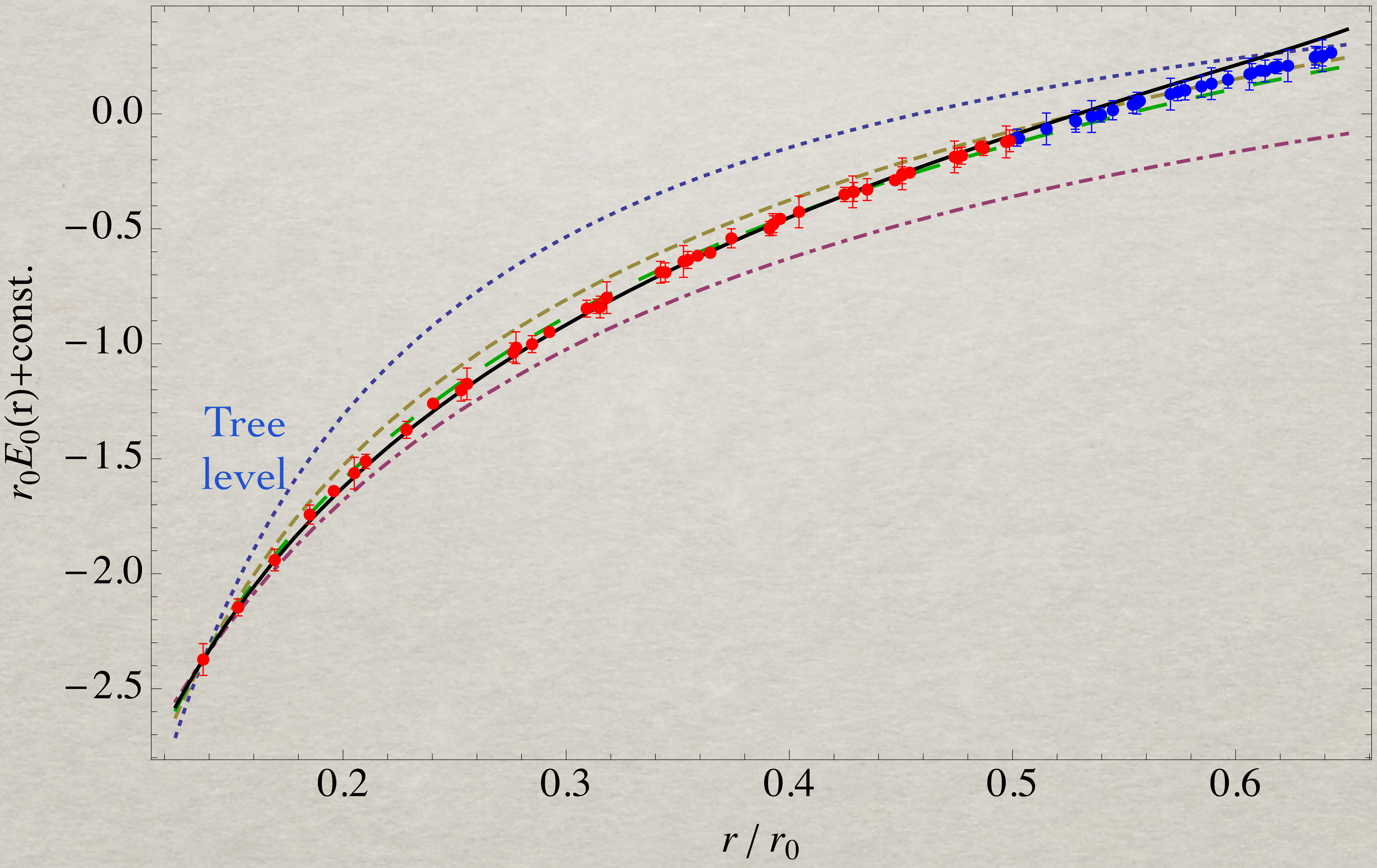

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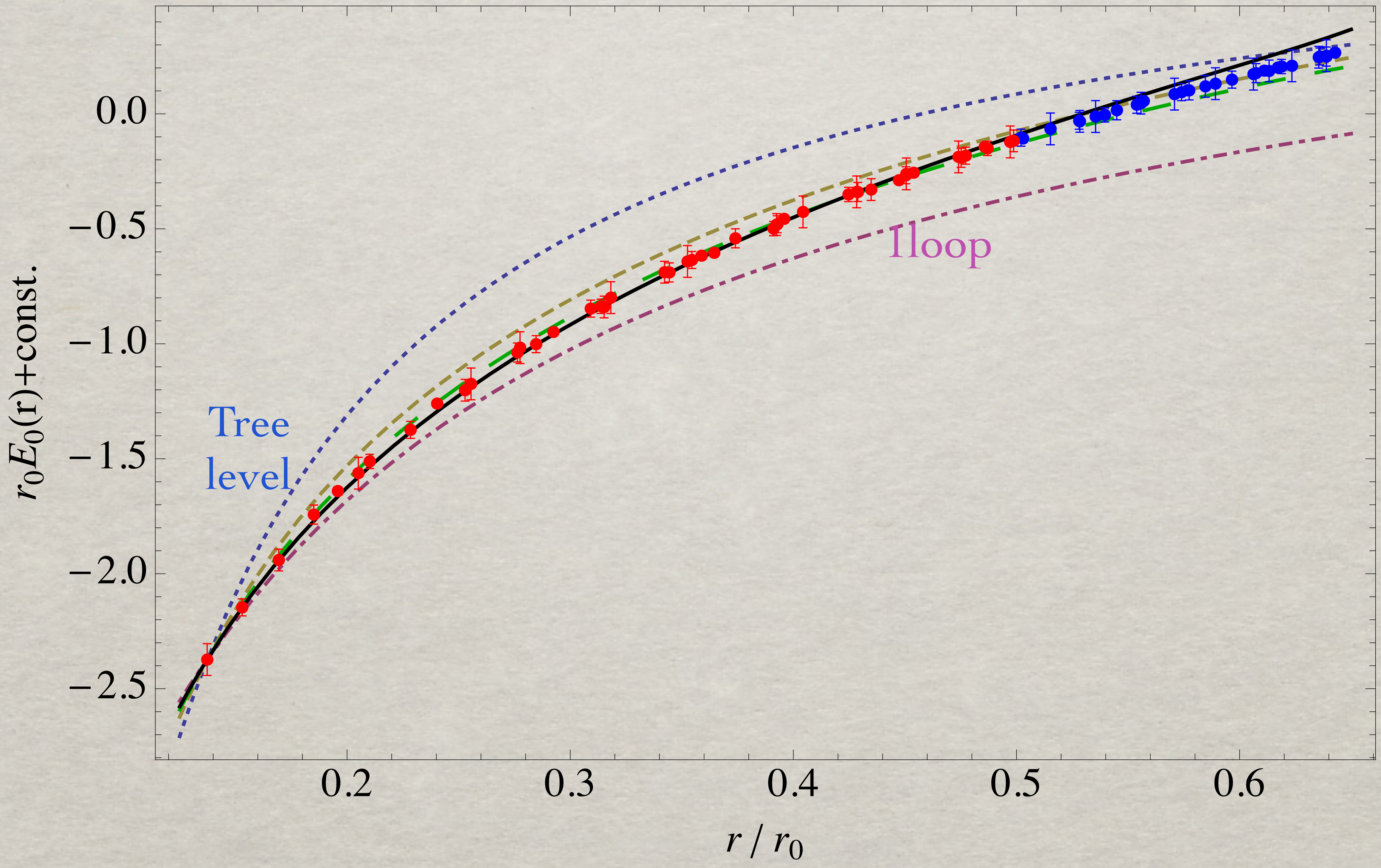

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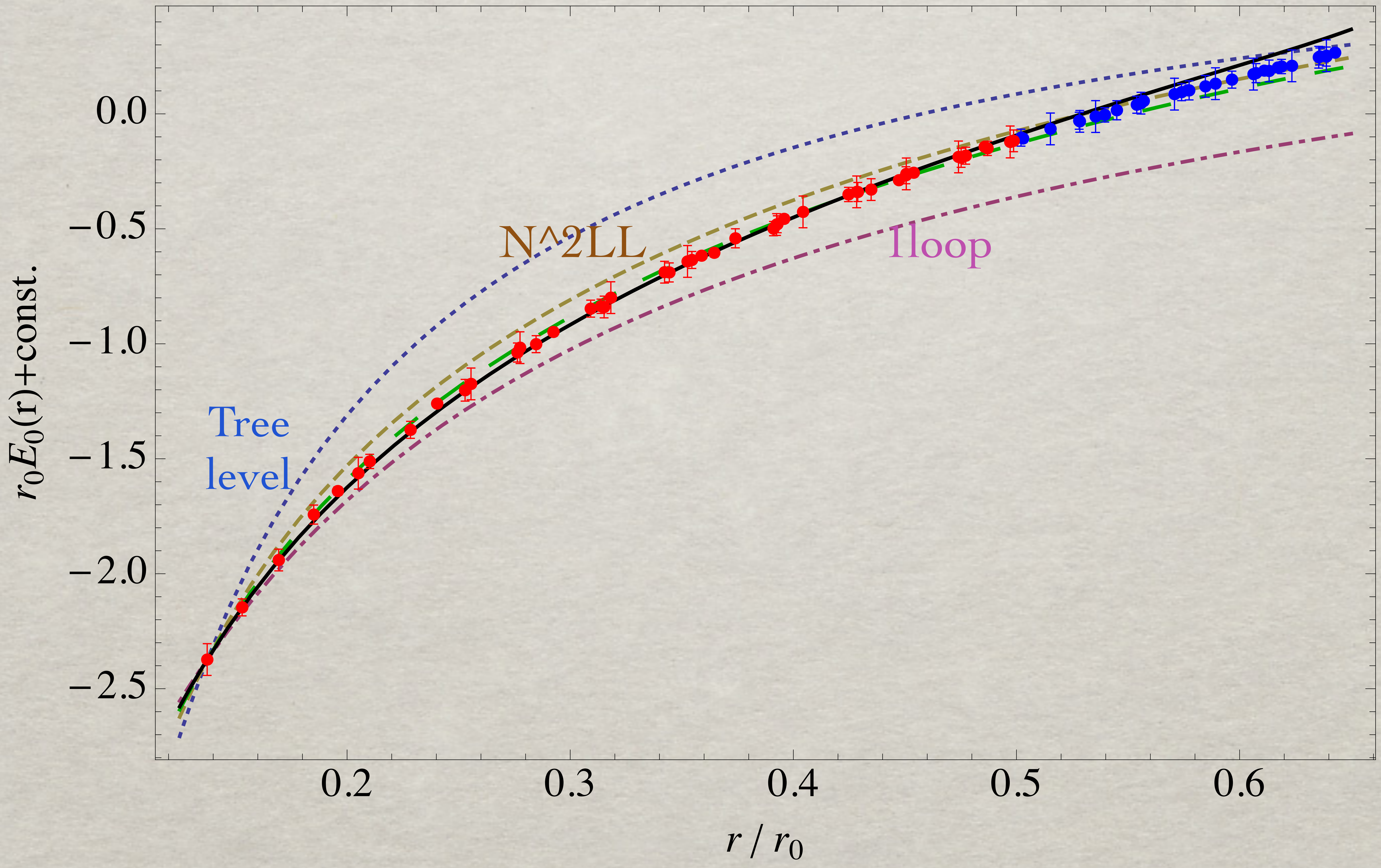

QQbar singlet static energy at N^3Ll in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

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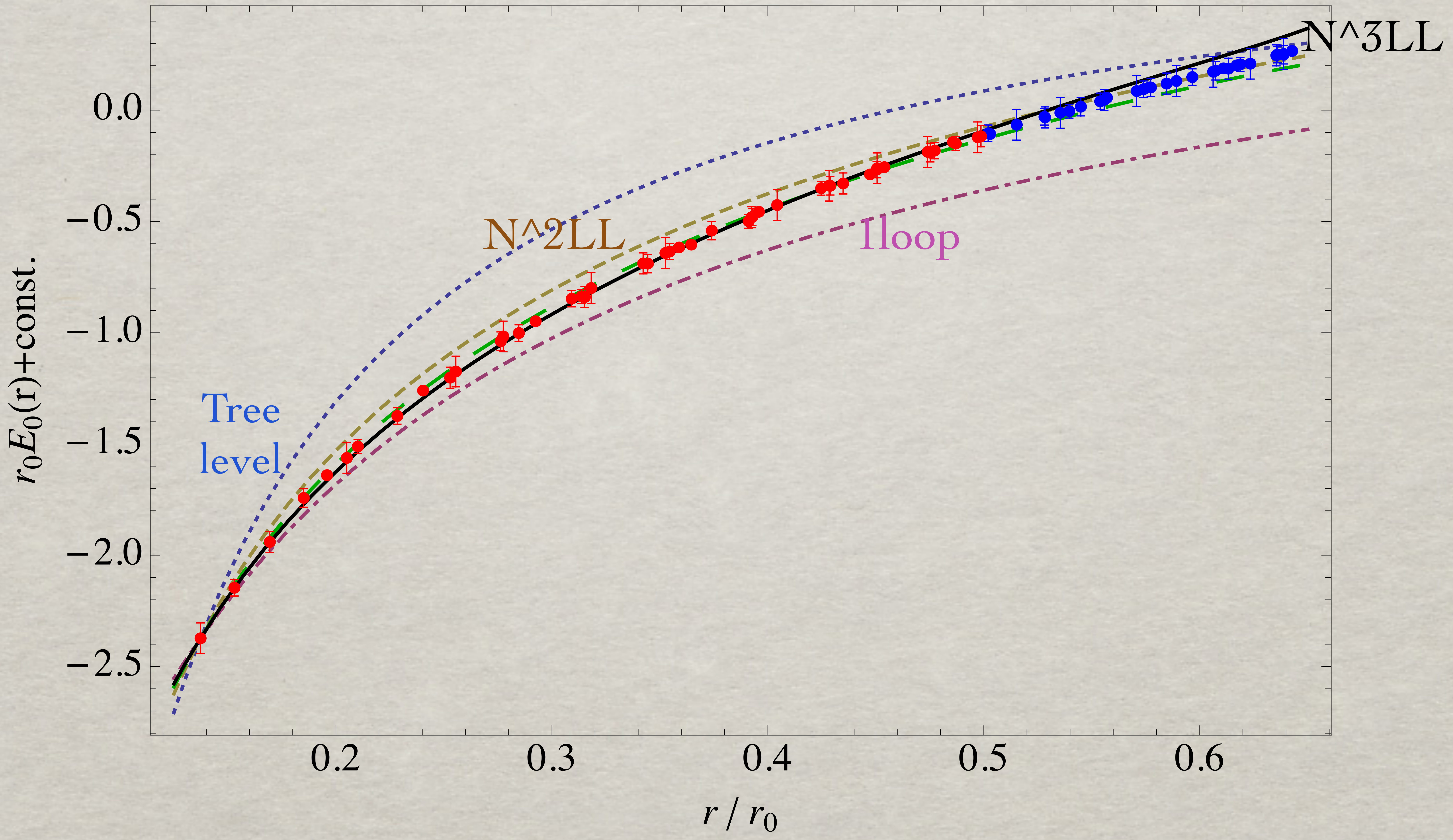

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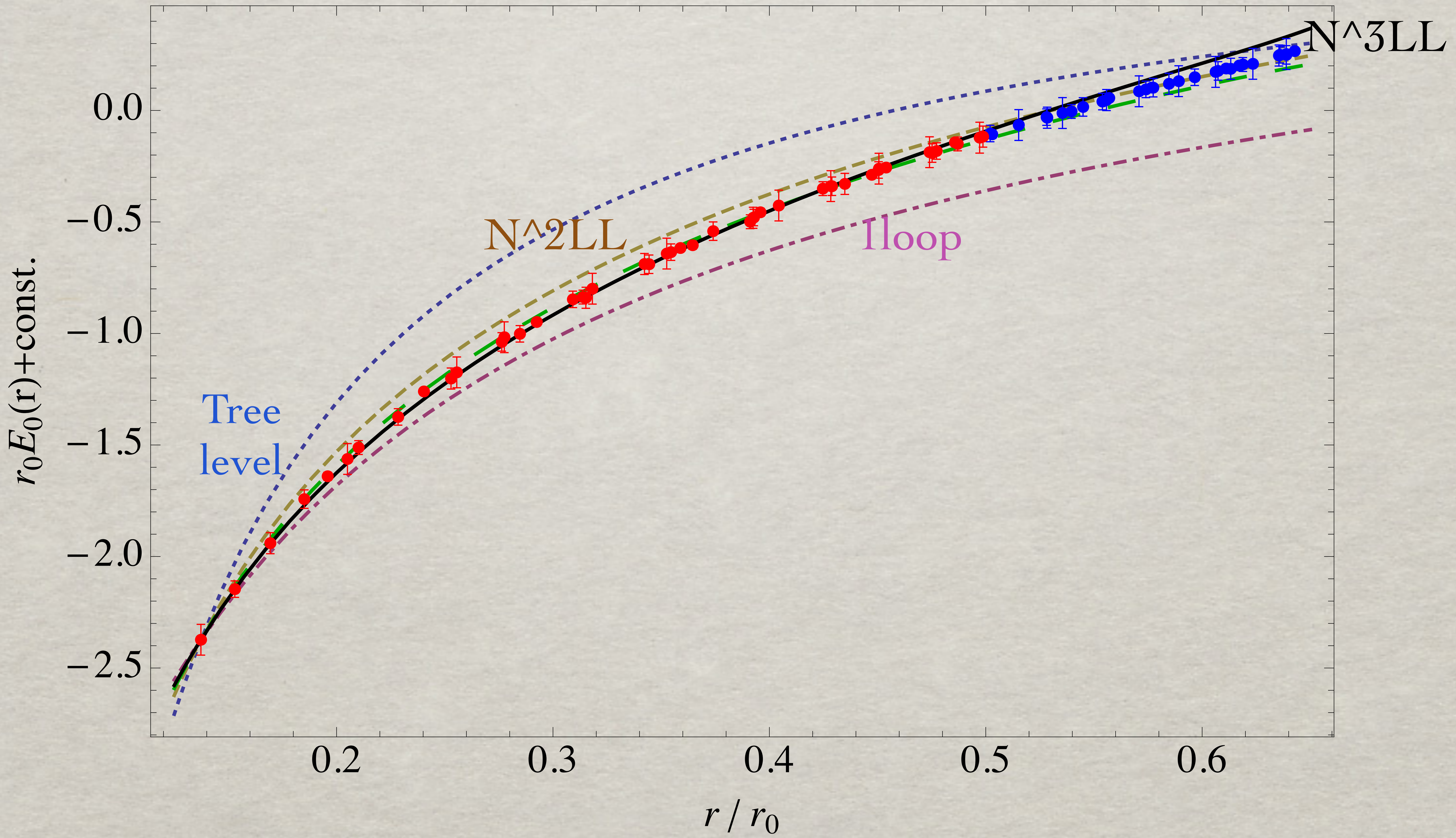

Good convergence to the lattice data

Lattice data less accurate in the unquenched case

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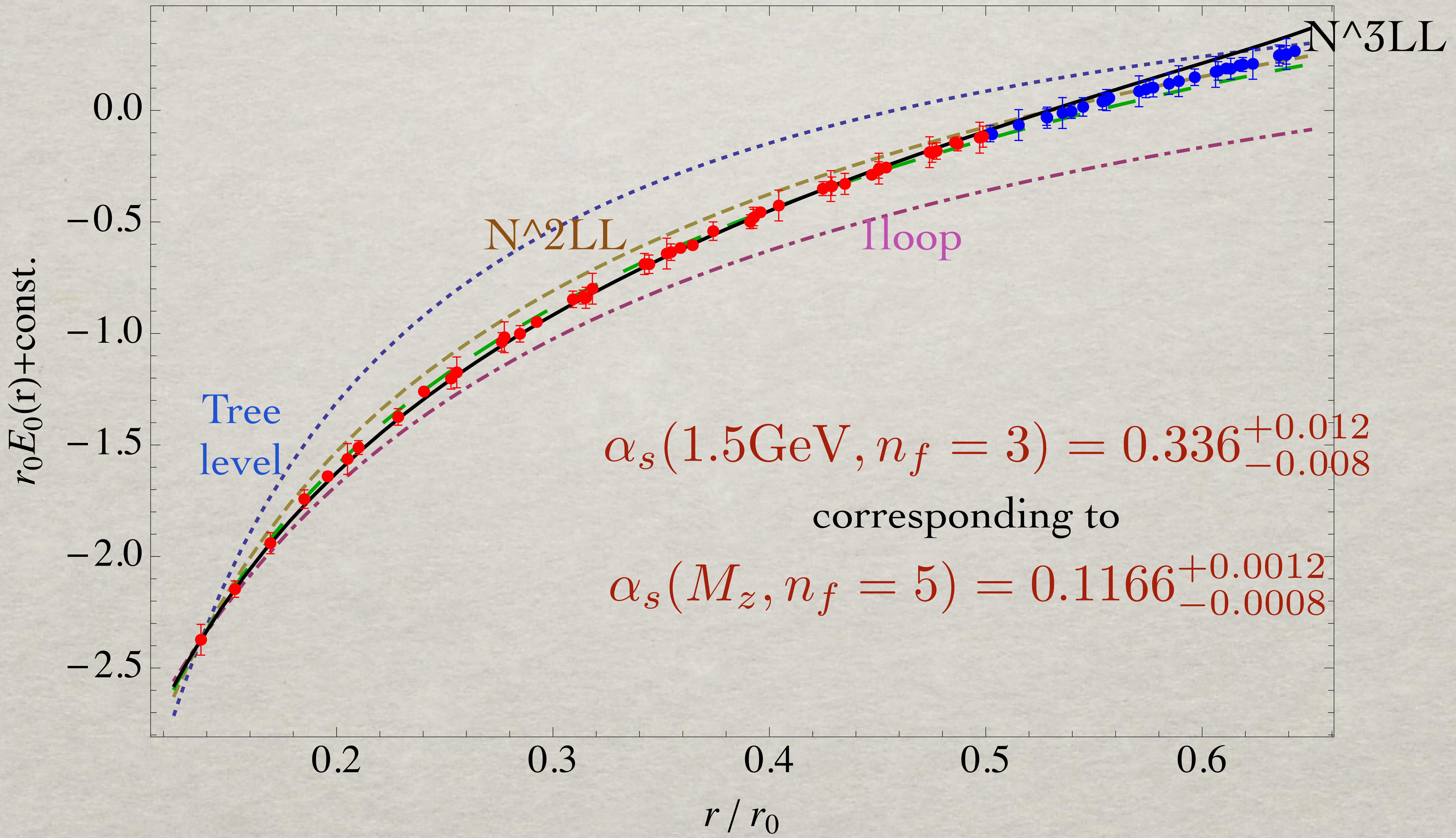

Good convergence to the lattice data

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- e-Print: 1907.11747

Extraction of alphas from the static energy

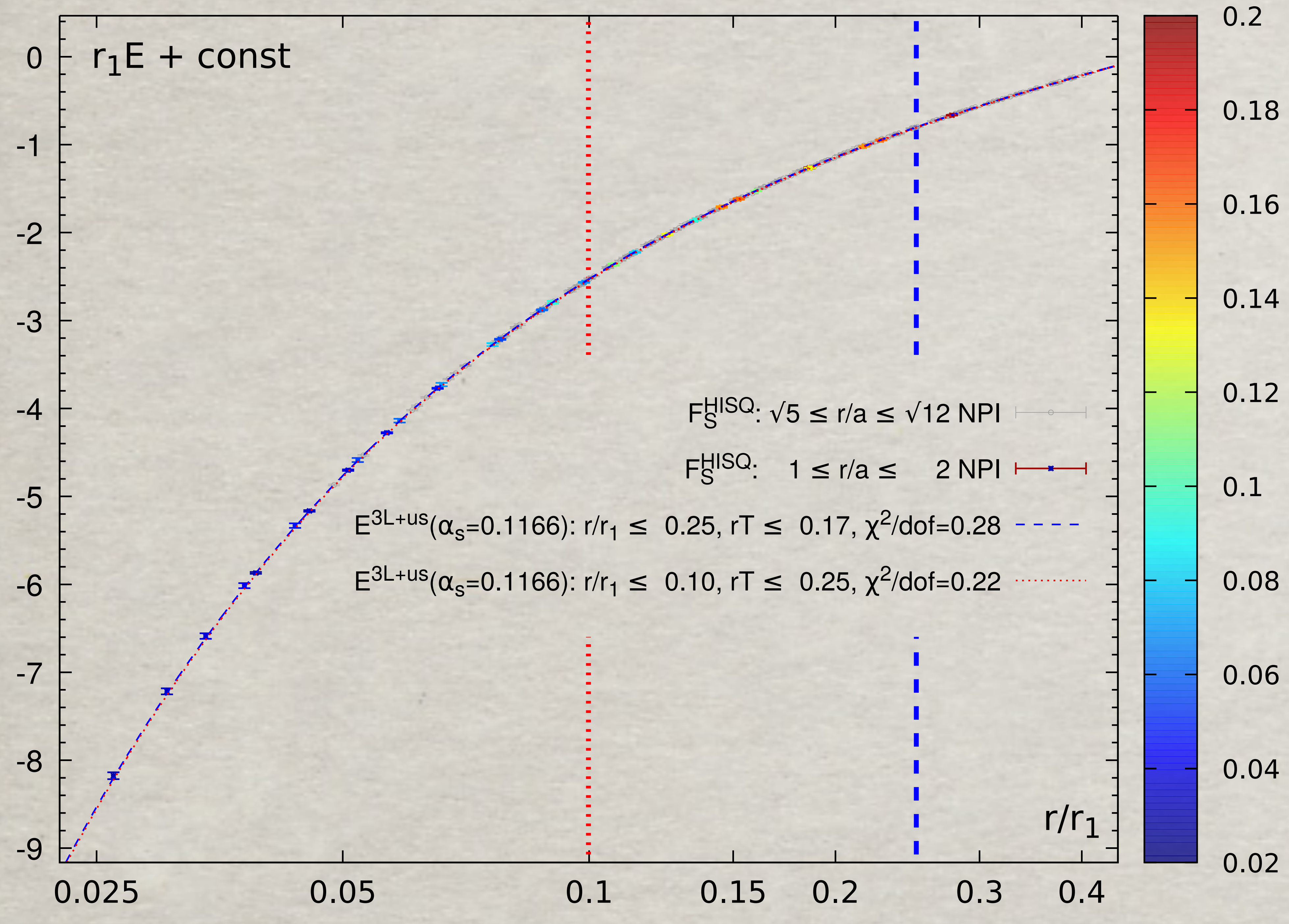

FIG. 10. Normalized lattice data and weak-coupling result for the static energy in units of r_1. We use a logarithmic scale for the coordinate axis. The colored or gray bullets show the nonperturbatively improved (NPI) lattice data for $r/a \leq 2$ or $2 < r/a \leq \sqrt{12}$. The *three-loop with leading ultra-soft resummation* with standard scales is shown for the $\alpha_s(M_Z)$ grid values corresponding to the best fits for the r and rT intervals as indicated. The vertical lines of the same color indicate max(r) of the fits.

$$\alpha_s(M_Z, N_f = 5) = 0.11660^{+0.00110}_{-0.00056}, \tag{4}$$

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or in terms of $\Lambda_{\overline{\rm MS}}^{N_f=3}$ as

$$\Lambda_{\overline{\rm MS}}^{N_f=3} = 314.0^{+15.5}_{-8.0}\ {\rm MeV}, \tag{6}$$

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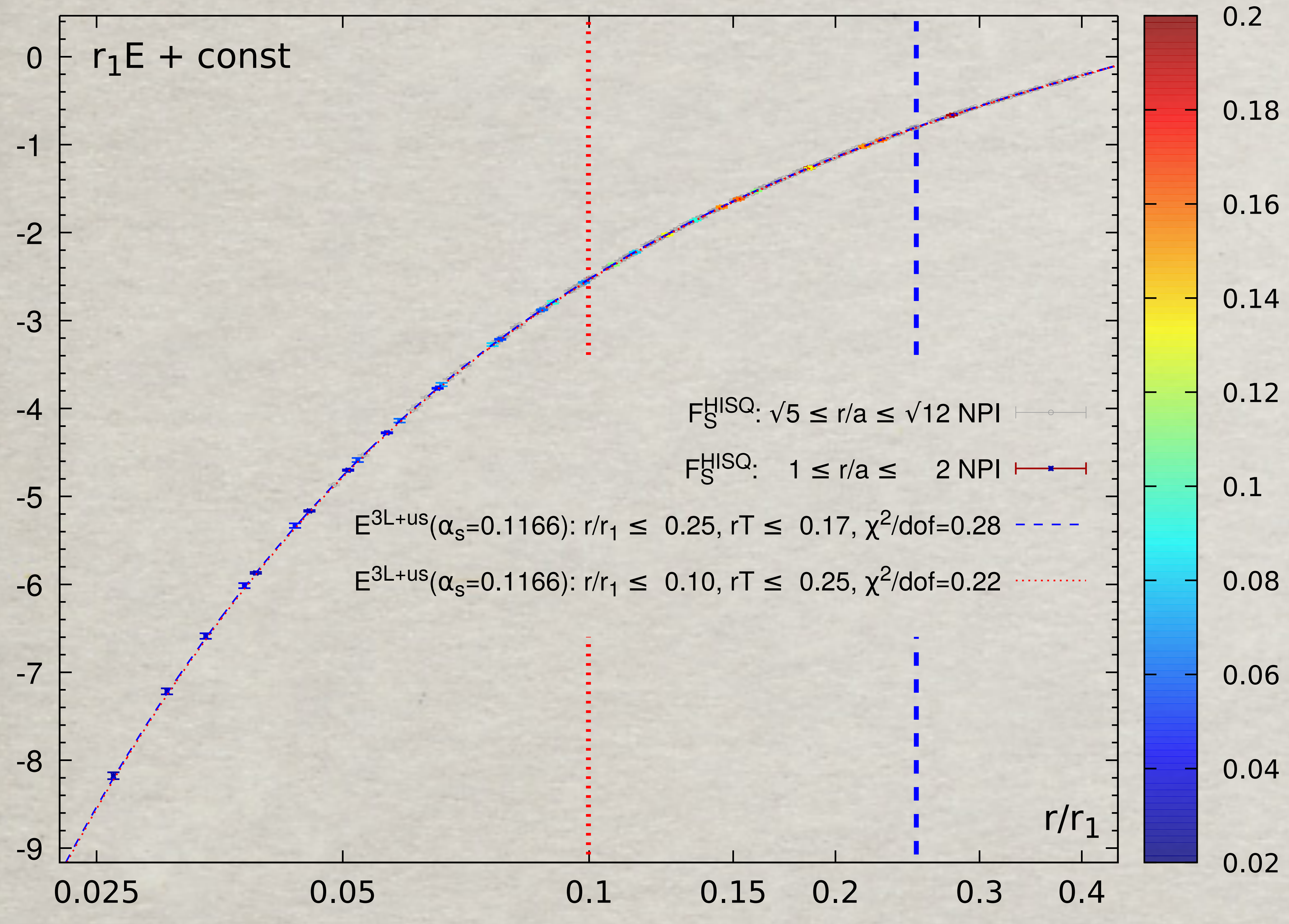

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$$F_S(r,T) = -T\ln\left(\frac{1}{N_c}\langle {\rm Tr}\left[W(r)W^\dagger(0)\right]\rangle\right).$$

At distances much smaller than the inverse temperature $rT \ll 1$, we can write using pNRQCD [4]

$$F_S(r,T) = V_s(r,\mu_{us}) + \delta F_S(r,T,\mu_{us}), \tag{11}$$

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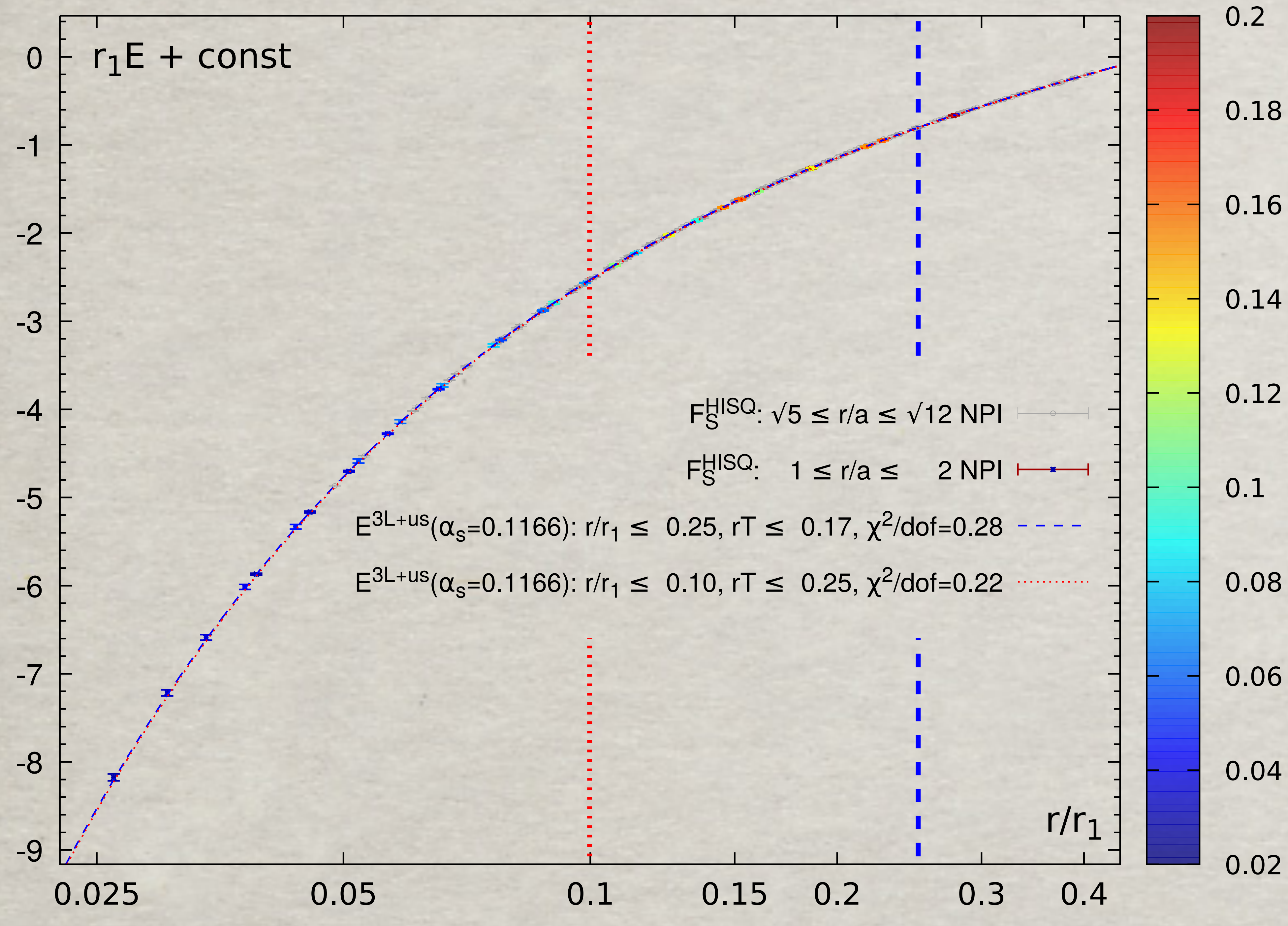

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Determination of alphas from an hyperasymptotic approximation to the energy of a static quark-antiquark pai

Cesar Ayala(Santa Maria U., Valparaiso), Xabier Lobregat(Barcelona, IFAE), Antonio Pineda(Barcelona, IFAE)

- : 2005.12301

$$\Lambda_{\overline{\rm MS}}^{(n_f=3)} = 338(12)\ \text{MeV and } \alpha(M_z) = 0.1181(9).$$

different power counting on the log resumed contribution, that given that the constant at 4 loop is not known may have an impact

Not all of the presently available perturbative information has been used:

N^3LL, finite mass effects, ...

- because the data are not sensitive to it. More precise lattice data on finer lattices and with more data points at short distances could take advantage of it.

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- A possible systematic effect is due to the finite lattice spacing.
  -> continuum extrapolation

To further improve the extraction of alphas we (A. Kronfeld, N. Brambilla, R. Delgado, W. Leino, P. Petreczky, S. Steinbeisser, A. Vairo with G. von Hippel) 2020 :

—calculate on the lattice (in 2+1+1) the static energy minus twice the energy of a static quark and a strange antiquark —> well defined continuum limit

—use $a \sim 0.025$fm

—calculate the one loop static energy in lattice perturbation theory

—make a global fit using the information of the leading cutoff effects

α_s from the force

well defined in the continuum limit

N. Brambilla, V. Leino, O. Philipsen, C. Reisinger, A. Vairo, M. Wagner 2020

It is possible to compute the force directly from the lattice:

$$F(r) = -\lim_{T\to\infty} \frac{\left\langle \mathrm{Tr}\,\mathrm{P}\,\hat{\mathbf{r}}\cdot g\mathbf{E}(t,\mathbf{r})\exp\left\{ig\oint_{r\times T} dz^\mu\, A_\mu\right\}\right\rangle}{\left\langle \mathrm{Tr}\,\mathrm{P}\exp\left\{ig\oint_{r\times T} dz^\mu\, A_\mu\right\}\right\rangle}$$

- Vairo MPLA 31 (2016) 1630039

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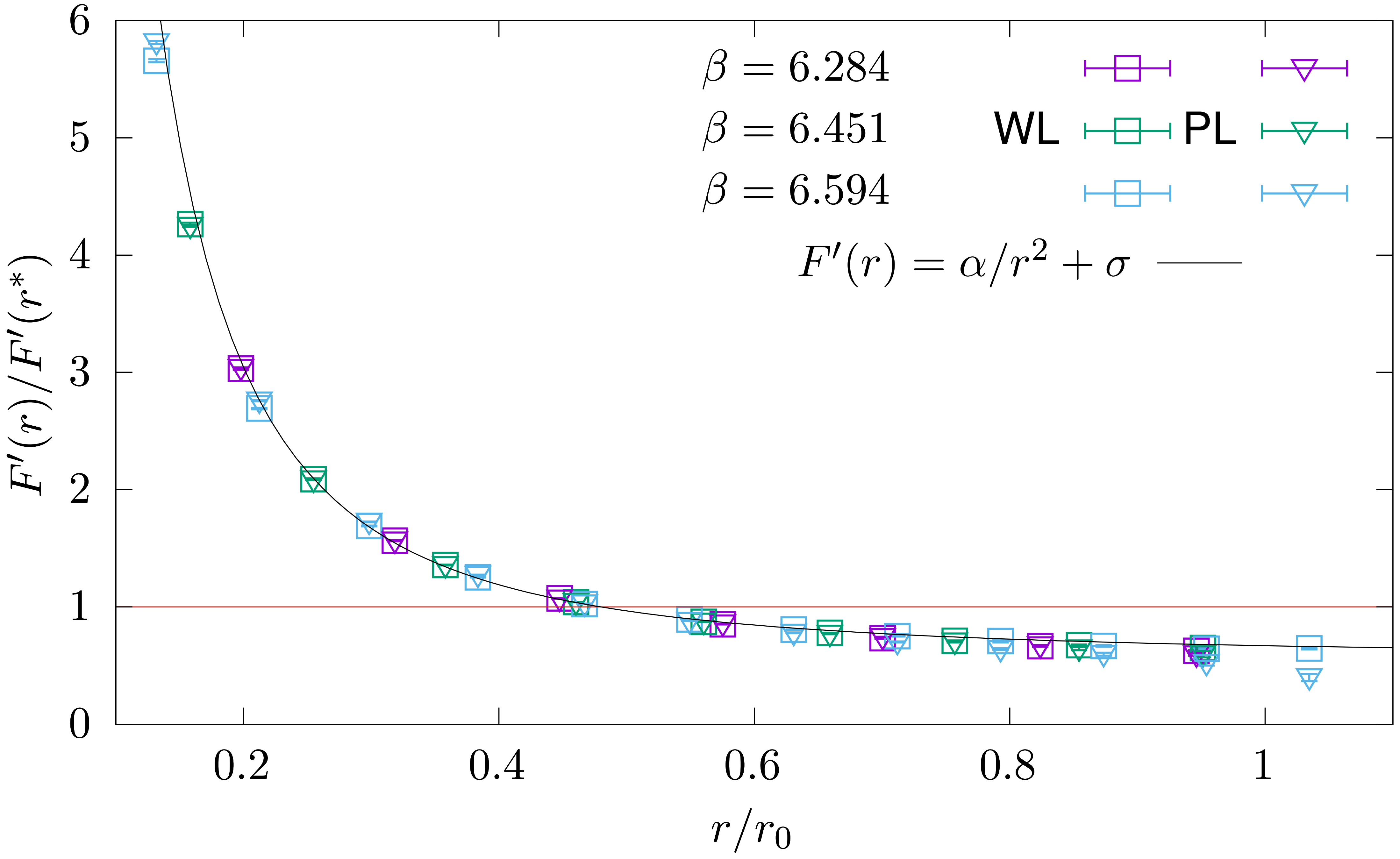
$F'(r)/F'(r^*) = F(r)/F(r^*)$ as a function of r for $r^* = 0.48\, r_0 \approx 0.24\,\mathrm{fm}$ obtained with generalized Wilson loops (open symbols) and generalized Polyakov loops (closed symb



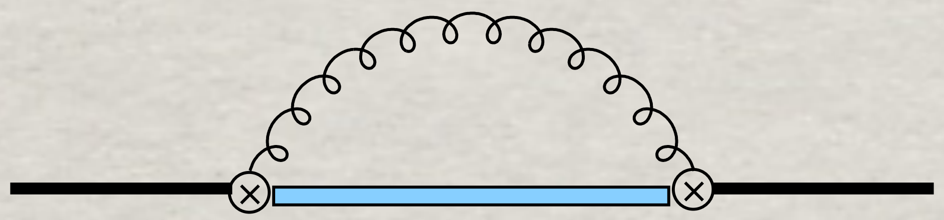
For comparison we also show $\partial_r V_{\mathrm{para}}(r)/\partial_r V_{\mathrm{para}}(r^*)$.

Exploratory study, with the force calculated at smaller r and unquenched we can extract alphas in an independent way with different systematics

the quarkonium singlet potential can all be calculated in perturbation theory

$$V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

Small systems: QQ energies at $m\alpha_s^5$

$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

$$E_n = \langle n|H_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\, \langle n|\mathbf{r}e^{it(E_n^{(0)} - H_o)}\mathbf{r}|n\rangle\, \langle \mathbf{E}(t)\, \mathbf{E}(0)\rangle(\mu)$$

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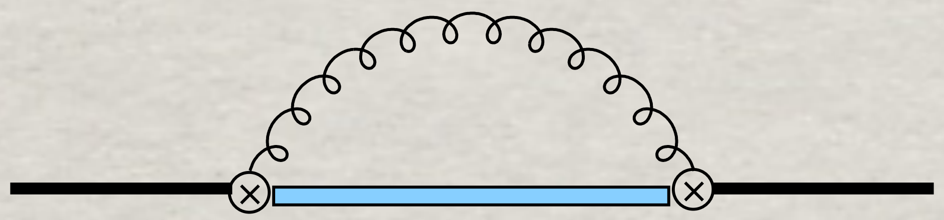
$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

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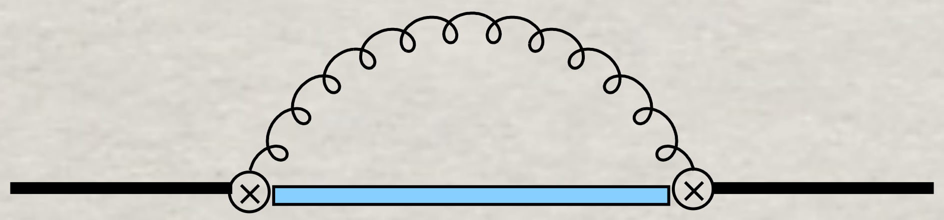
$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$  $$|n\rangle$$

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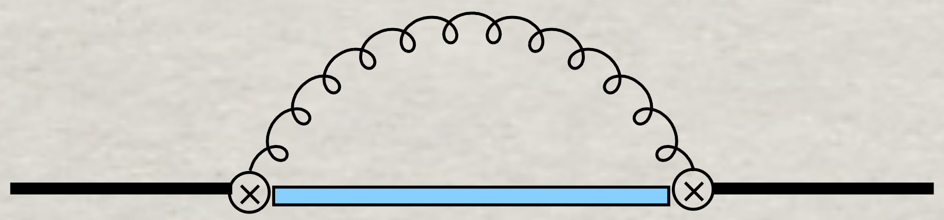

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many recent results by Mateu, Entem, Ortega , Pineda, Peset et collaborators

Extraction of quark masses from heavy-light meson masses

- HQET description of a HL meson mass in terms of its heavy quark mass

$$M_H = m_h + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2(m_h)}{2m_h} + \mathcal{O}(1/m_h^2)$$

  - $\bar{\Lambda}$: energy of light quarks and gluons inside the system
  - $\mu_\pi^2/2m_h$: kinetic energy of the heavy quark inside the system
  - $\mu_G^2(m_h)/2m_h$: hyperfine energy due to heavy quark's spin
    (can be estimated from B^*-B splitting $\Rightarrow \mu_G^2(m_b) \approx 0.35\,\mathrm{GeV}^2$)
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- For the heavy quark mass, we use the minimal renormalon subtracted (MRS) scheme [PRD97, 034503 (2018)]
  - removes the leading infrared renormalon from the pole mass
  - has an asymptotic expansion identical to the perturbative pole mass
    (does not spoil the HQET power counting)
  - is a gauge- and scale-independent scheme;
    it does not introduce any factorization scale (unlike, *e.g.*, the RS or kinetic scheme)

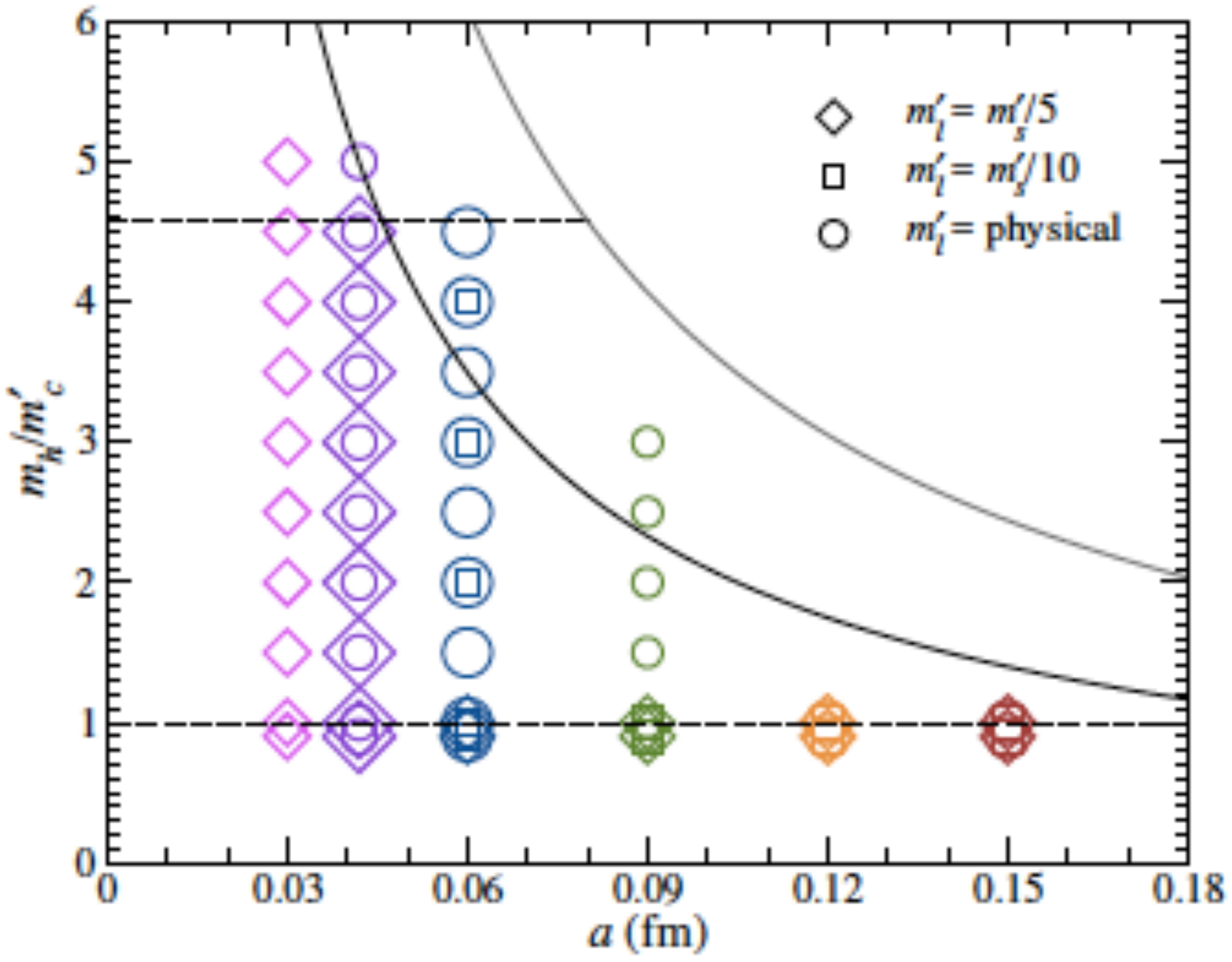
MILC ensembles with (2+1+1)-flavors of dynamical quarks

- Ensembles with physical mass for the strange quark:

$\approx a$ (fm)	m_l/m_s	size	L (fm)	$M_\pi L$	M_π (MeV)
0.15	1/5	$16^3 \times 48$	2.38	3.8	314
0.15	1/10	$24^3 \times 48$	3.67	4.0	214
0.15	1/27	$32^3 \times 48$	4.83	3.2	130
0.12	1/5	$24^3 \times 64$	3.00	4.5	299
0.12	1/10	$24^3 \times 64$	2.89	3.2	221
0.12	1/10	$32^3 \times 64$	3.93	4.3	216
0.12	1/10	$40^3 \times 64$	4.95	5.4	214
0.12	1/27	$48^3 \times 64$	5.82	3.9	133
0.09	1/5	$32^3 \times 96$	2.95	4.5	301
0.09	1/10	$48^3 \times 96$	4.33	4.7	215
0.09	1/27	$64^3 \times 96$	5.62	3.7	130
0.06	1/5	$48^3 \times 144$	2.94	4.5	304
0.06	1/10	$64^3 \times 144$	3.79	4.3	224
0.06	1/27	$96^3 \times 192$	5.44	3.7	135
0.042	1/5	$64^3 \times 192$	2.91	4.34	294
0.042	1/27	$144^3 \times 288$	6.12	4.17	134
0.03	1/5	$96^3 \times 288$	3.25	4.84	294

- The fermion action is "highly improved staggered quark" (HISQ) action
- Physical-mass ensembles at most lattice spacings

Heavy-light mesons with HISQ action



- We have 24 Ensembles:
  - 6 lattice spacings
  - several sea masses
- We calculate masses of pseudoscalar mesons for various light and heavy quarks with masses:
  - light valence: $m_{ud} \lesssim m_v \lesssim m_s$
  - heavy valence: $m_c \lesssim m_h \lesssim m_b$
- We use only $am_h < 0.9$ to avoid large discretization errors

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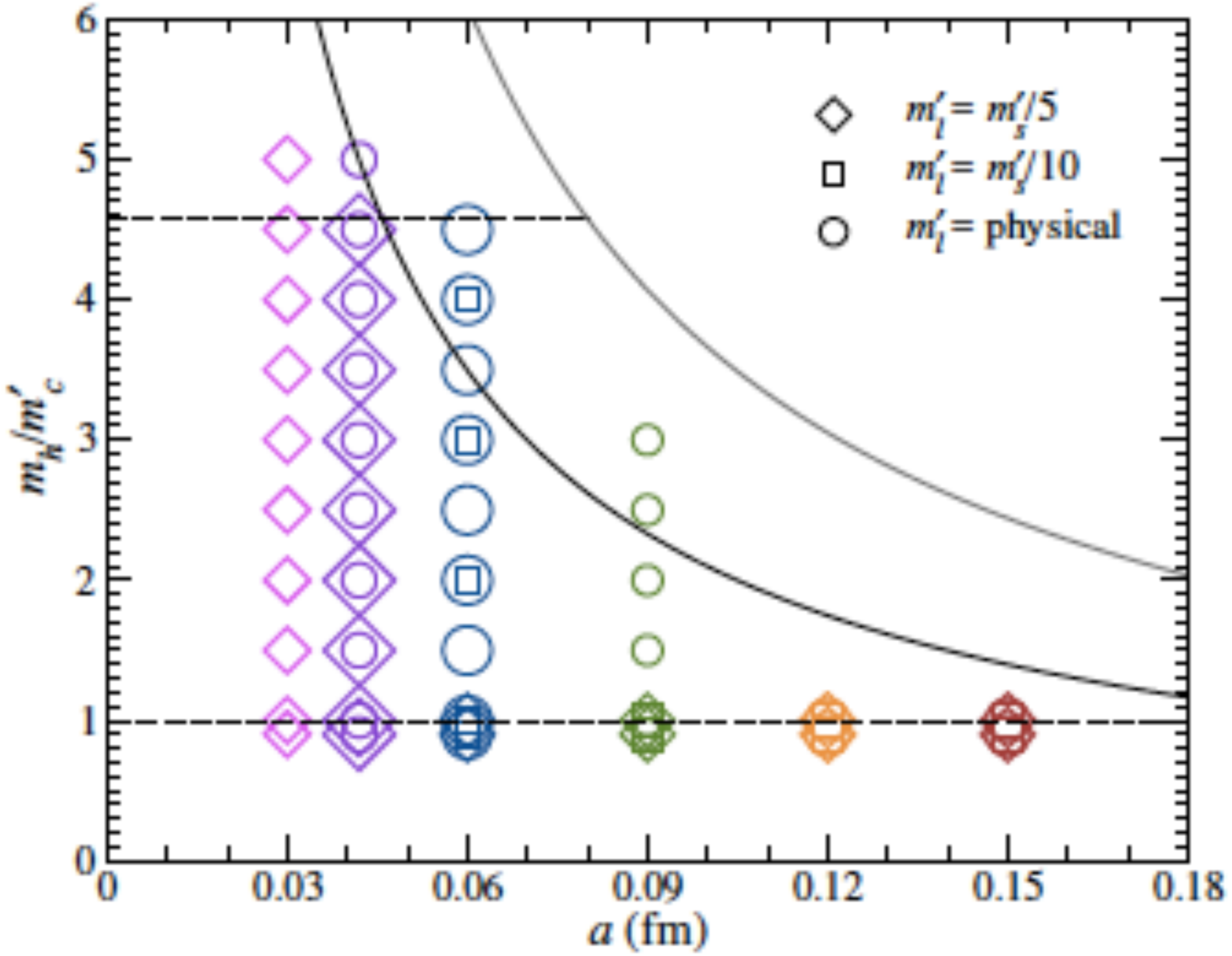

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EFT description of heavy-light meson masses

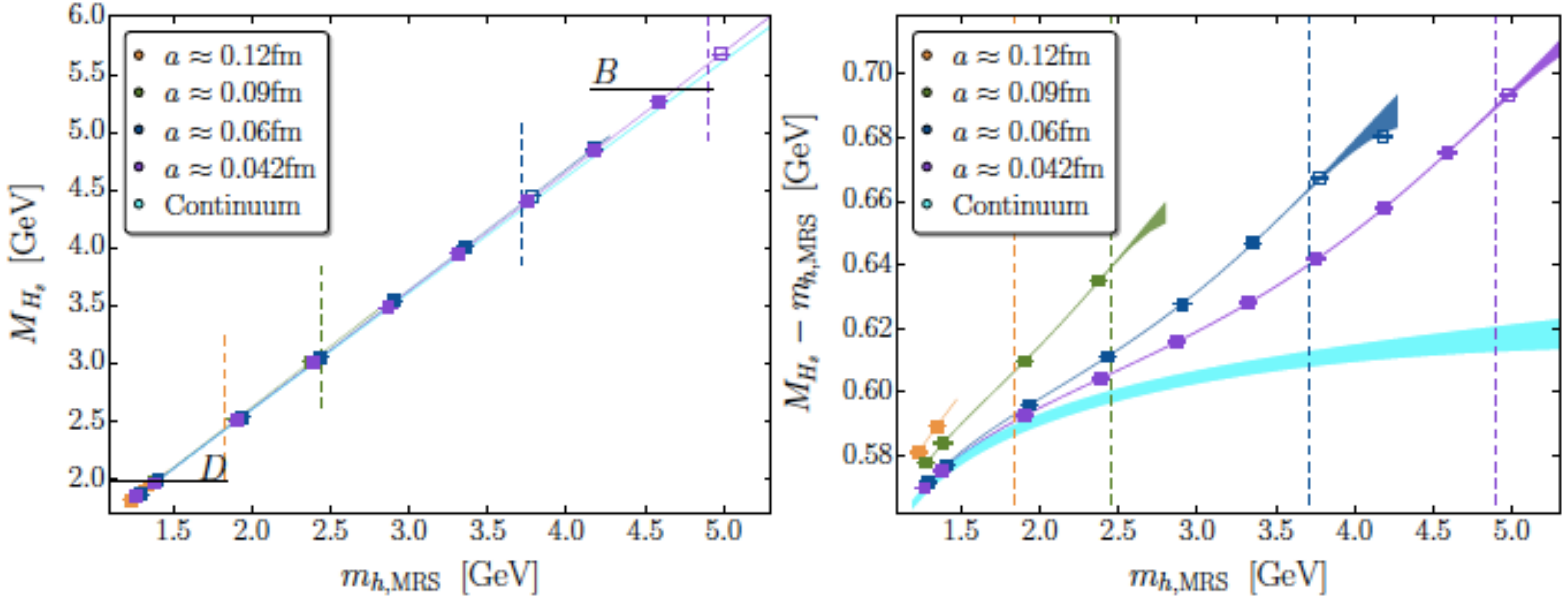
We employ HQET and heavy-meson staggered ChPT to describe the dependence of meson masses on both heavy and light quark masses and incorporate taste-breaking lattice artifacts

- Include HMrPQASχPT and higher order HQET terms

$$M_H = m_{h,\mathrm{MRS}} + \overline{\Lambda}_{\mathrm{MRS}} + \frac{\mu_\pi^2 - \mu_G^2(m_h)}{2m_{h,\mathrm{MRS}}} + \text{HMrPQAS}\chi\text{PT} + \text{higher order HQET}$$

- $m_{h,\mathrm{MRS}}$ is a function of am_h/am_{p4s} and $am_{p4s,\overline{\mathsf{MS}}}(2\ \mathrm{GeV})$
- The higher order terms are typically polynomials in dimensionless, "natural" expansion parameters:
  - Light-quark and gluon discretization: $(a\Lambda)^2$ with $\Lambda = 600\ \mathrm{MeV}$
  - Heavy-quark discretization: $(2am_h/\pi)^2$
  - Light valence and sea quark mass effects: $B_0 m_q/(4\pi^2 f_\pi^2)$
  - HQET: $\Lambda/m_{h,\mathrm{MRS}}$ with $\Lambda = 600\ \mathrm{MeV}$
- Our fit function has 77 parameters and 384 data points

A snapshot of the fit and data



Dashed lines: $am_h \approx 0.9$; open symbols: data points omitted from fit
Vertical axis: heavy-strange meson masses
Horizontal axis: the fit values for the RS mass projected to continuum (no lattice artifacts)

- The combined-correlated fit gives $\chi^2/\mathrm{d.o.f} \approx 1$, $p = 0.3$
- After extrapolating to continuum, experimental masses of D_s and B_s with EM effects subtracted are used to determine the charm- and bottom-quark masses

Results for the strange, charm and bottom quarks

- The strange quark masses in a theory with 4 active flavors:

$$m_{s,\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 92.52(40)_{\mathrm{stat}}(18)_{\mathrm{syst}}(52)_{\alpha_s}(12)_{f_\pi,\mathrm{PDG}}\ \mathrm{MeV}$$

- For quark mass ratios:

$$m_c/m_s = 11.784(11)_{\mathrm{stat}}(17)_{\mathrm{syst}}(00)_{\alpha_s}(08)_{f_\pi,\mathrm{PDG}}$$

$$m_b/m_s = 53.93(7)_{\mathrm{stat}}(8)_{\mathrm{syst}}(1)_{\alpha_s}(5)_{f_\pi,\mathrm{PDG}}$$

$$m_b/m_c = 4.577(5)_{\mathrm{stat}}(7)_{\mathrm{syst}}(0)_{\alpha_s}(1)_{f_\pi,\mathrm{PDG}}$$

- For heavy quarks:

$$\overline{m}_c = 1273(4)_{\mathrm{stat}}(1)_{\mathrm{syst}}(10)_{\alpha_s}(1)_{f_\pi,\mathrm{PDG}}\ \mathrm{MeV}$$

$$\overline{m}_b^{(n_f=5)} = 4197(12)_{\mathrm{stat}}(1)_{\mathrm{syst}}(8)_{\alpha_s}(1)_{f_\pi,\mathrm{PDG}}\ \mathrm{MeV}$$

  where $\overline{m}_h = m_{h,\overline{\mathrm{MS}}}(m_{h,\overline{\mathrm{MS}}})$.

- Uncertainties:

"stat") Statistics and EFT fit

"syst") Various systematic uncertainties in inputs: FV, EM, topological charge freezing, contamination from higher order states...

α_s) Uncertainty in the strong coupling constant $\alpha_{s,\overline{\mathrm{MS}}}(5\,\mathrm{GeV}; n_f{=}4) = 0.2128(25)$ [HPQCD, arXiv:1408.4169]

$f_{\pi,\mathrm{PDG}}$) Uncertainty in the PDG value of $f_{\pi^\pm} = 130.50(13)$ MeV, which is used for scale setting

Results for HQET parameters

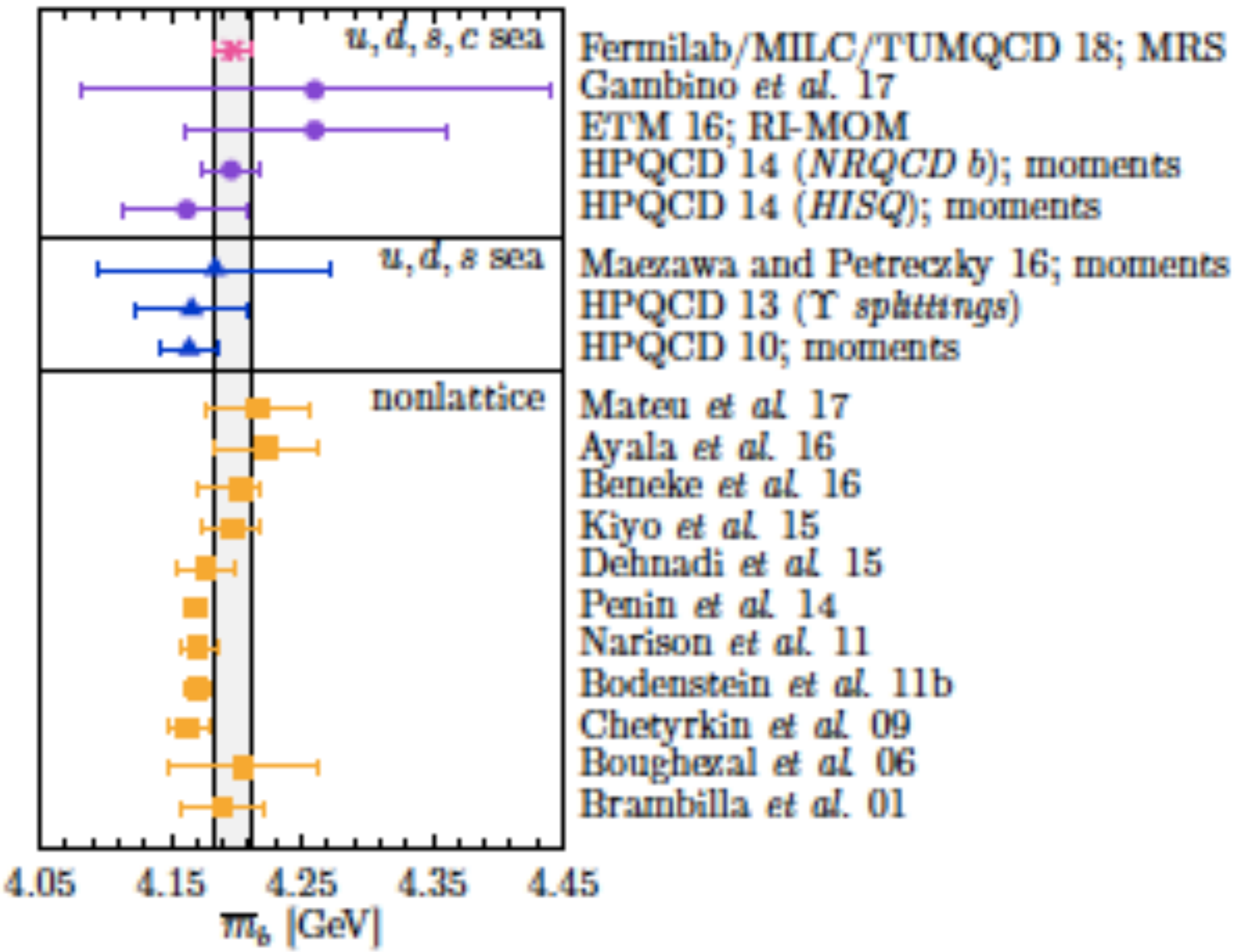
- For HQET parameters we have

$$\overline{\Lambda}_{\mathrm{MRS}} = 552(25)_{\mathsf{stat}}(6)_{\mathsf{syst}}(16)_{\alpha_s}(2)_{f_\pi,\mathsf{PDG}}\ \mathrm{MeV}$$
$$\mu_\pi^2 = 0.06(16)_{\mathsf{stat}}(14)_{\mathsf{syst}}(06)_{\alpha_s}(00)_{f_\pi,\mathsf{PDG}}\ \mathrm{GeV}^2$$
$$\mu_G^2(m_b) = 0.38(01)_{\mathsf{stat}}(01)_{\mathsf{syst}}(00)_{\alpha_s}(00)_{f_\pi,\mathsf{PDG}}\ \mathrm{GeV}^2$$

(Note that the prior value of $\mu_G^2(m_b)$ is set to $0.35(7)\,\mathrm{GeV}^2$ [Gambino and Schwanda, arXiv:1307.4551])

Comparison

Our result is shown as a magenta burst, with the gray band showing how it compares directly with the other lattice and nonlattice results;
see [arXiv:1802.04248 [hep-lat]] for details.



Comparison

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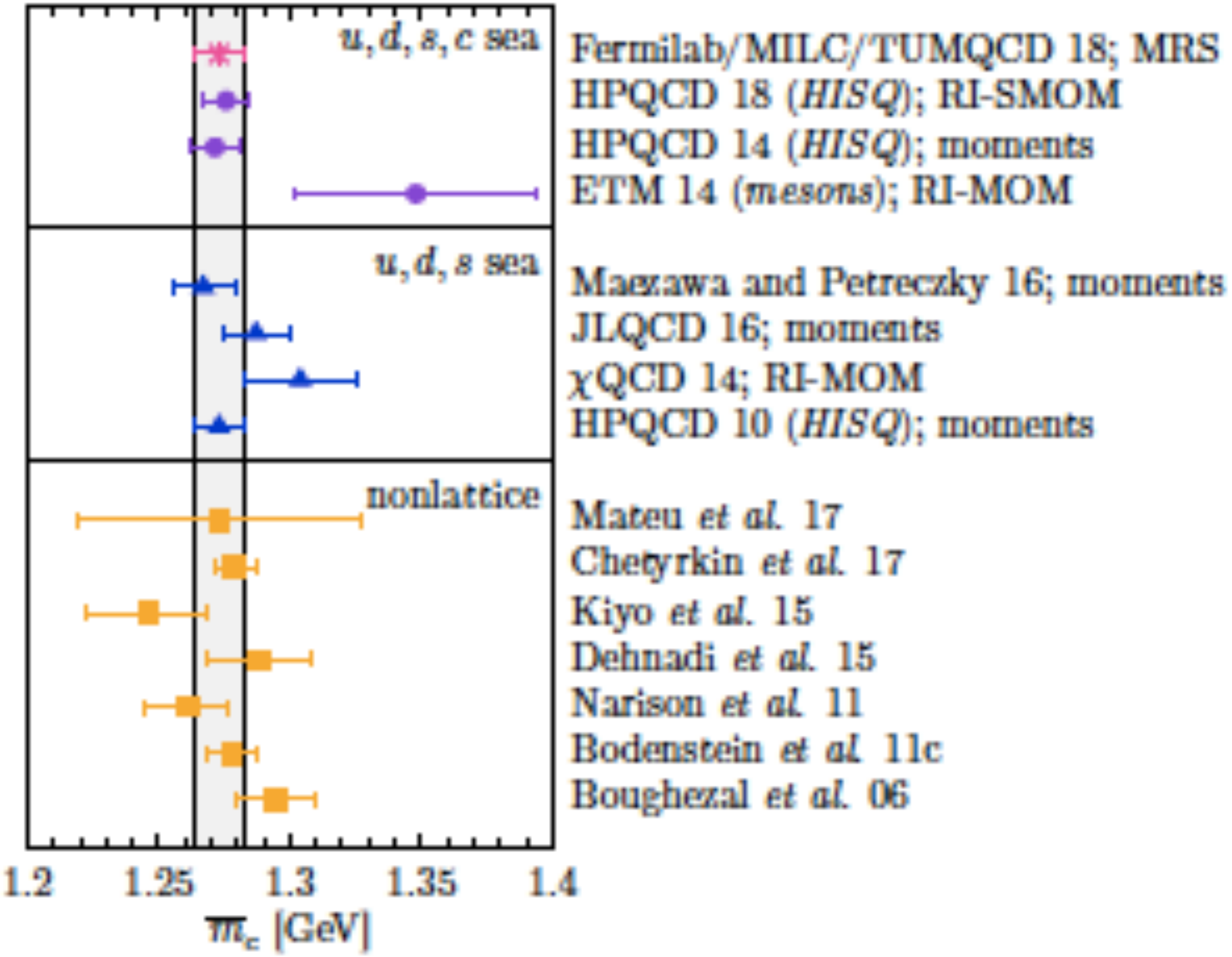


Outlook

Heavy quark system and especially quarkonium are golden systems to study strong interactions

Nonrelativistic EFTs allow us to treat these systems in the realm of QCD

In this way these systems become suitable for the precise extraction of SM parameters like alphas and the masses

The alliance of EFTs and lattice make these extractions competitive

They are also complementary and offers different systematics

These methods and these calculations offer a unique insight in the interplay of different scale in QCD and in pattern of nonperturbative effects

backup

Minimal renormalon subtracted mass

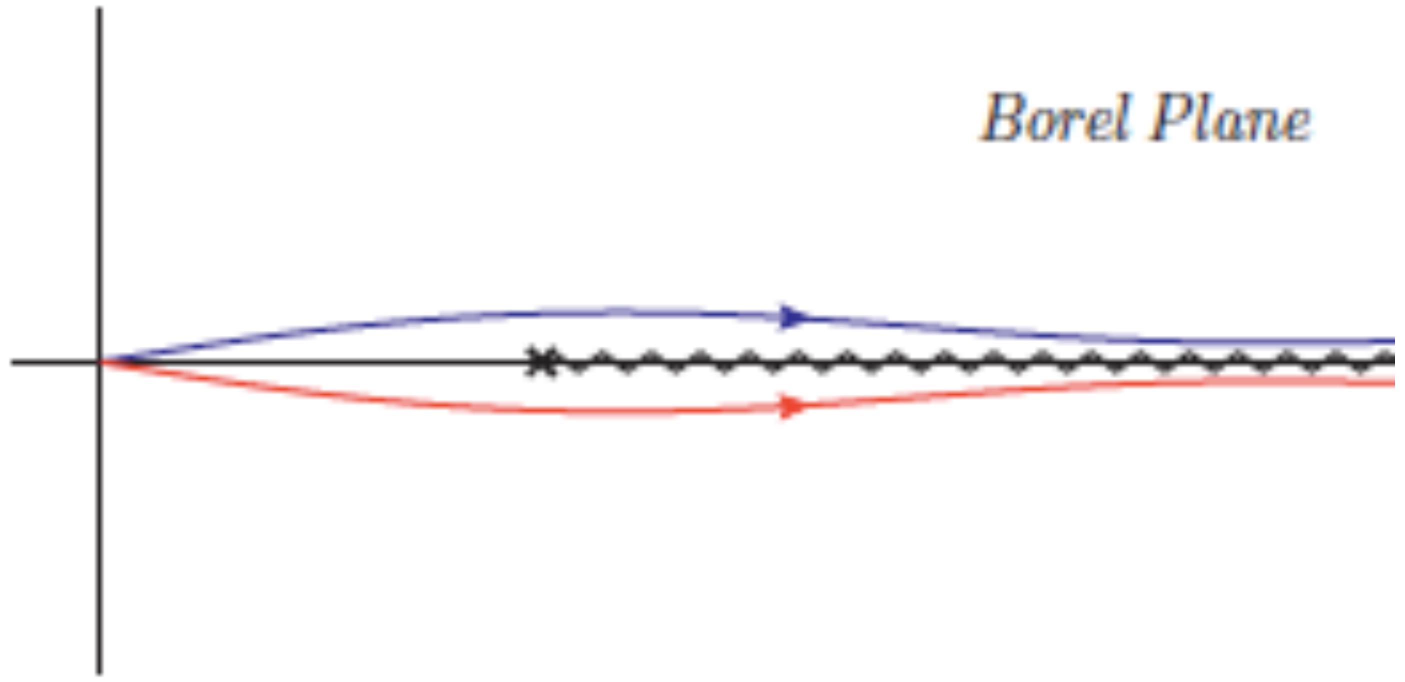
- The pole mass can be calculated at each order in perturbation theory

$$m_{\text{pole}} = \overline{m}\left(1 + \sum_{n=0}^{N} r_n \alpha_s^{n+1}(\overline{m}) + \mathcal{O}(\alpha_s^{N+2})\right)$$

  - $\overline{m}$ is the $\overline{\text{MS}}$ mass at scale $\mu = \overline{m}$
  - The series diverges because $r_n \propto (2\beta_0)^n \Gamma(n+b+1)$ as $n \to \infty$
- The divergent expression can be interpreted using the Borel transform

involves an integral of form $\int_0^\infty dz \, \frac{e^{-z/(2\beta_0\alpha_s)}}{(1-z)^{1+b}}$

with $b = \beta_1/(2\beta_0^2)$



- The idea in the MRS scheme is to divide the integral as

$$\int_0^1 dz \, \frac{e^{-z/(2\beta_0\alpha_s)}}{(1-z)^{1+b}} \quad \to \quad \mathcal{J}_{\text{MRS}}(\mu)$$

$$\int_1^\infty dz \, \frac{e^{-z/(2\beta_0\alpha_s)}}{(1-z)^{1+b}} \quad \to \quad \delta m \propto (-1)^b \Lambda_{\text{QCD}}$$

  and subtract the ambiguous term δm from the pole mass

- The MRS mass is defined as

$$\begin{aligned} m_{\mathrm{MRS}} &= m_{h,\mathrm{pole}} - \delta m \\ &= \overline{m}\left(1 + \sum_{n=0}^{\infty}\left[r_n - R_n\right]\alpha_s^{n+1}(\overline{m})\right) + \mathcal{J}_{\mathrm{MRS}}(\overline{m}) + \Delta m_{(c)} \end{aligned}$$

$\overline{m}$:	$\overline{\mathrm{MS}}$ mass at scale $\mu = \overline{m}$
r_n:	coefficients relating the $\overline{\mathrm{MS}}$ mass to the perturbative pole mass
$-R_n$:	subtracting the leading renormalon from the perturb. series
$\mathcal{J}_{\mathrm{MRS}}$:	contribution from the leading renormalon (see backup slides)
$\Delta m_{(c)}$:	for contribution from the charm quark [arXiv:1407.2128]

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- For a theory with $n_l = 3$ massless quarks, and $R_0 = 0.535$:

$$r_n - R_n = (-0.1106,\ -0.0340,\ 0.0966,\ 0.0162, \ldots)$$

The smallness of $r_n - R_n$ reduces the truncation error in our work

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With the MRS mass for heavy quarks, we proceed to map bare quark masses to the MRS mass

Mapping bare quark masses to the $\overline{\text{MS}}$ and MRS masses

- Introduce a "reference mass", and construct the identity (up to lattice artifacts)

$$m_{h,\text{MRS}} = m_{r,\overline{\text{MS}}}(\mu) \frac{\overline{m}_h}{m_{h,\overline{\text{MS}}}(\mu)} \frac{m_{h,\text{MRS}}}{\overline{m}_h} \frac{am_h}{am_r}$$

1) First factor: a fit parameter (we set $am_r = am_{p4s}$ and $\mu = 2$ GeV)
2) Second factor: running factor governed by the mass anomalous dimension (the five-loop result is known [JHEP 1410 (2014) 076])
3) Third factor:

$$m_{h,\text{MRS}} = \overline{m}_h \left(1 + \sum_{n=0}^{3} [r_n - R_n] \alpha_s^{n+1}(\overline{m}_h) + \mathcal{O}(\alpha_s^5)\right) + \mathcal{J}_{\text{MRS}}(\overline{m}_h) + \Delta m_{(c)}$$

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$$\alpha_{\overline{MS}}(5\,\mathrm{GeV}; n_f = 4) = 0.2128(25)$$ [HPQCD, arXiv:1408.4169]

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- Discretization errors should be incorporated as powers of $(am_h)^2$ and $(a\Lambda)^2$

- $\mathcal{J}_{\mathrm{MRS}}(\mu)$ is defined as

$$\mathcal{J}_{\mathrm{MRS}}(\mu) = \frac{R_0}{2\beta_0}\mu e^{-1/[2\beta_0\alpha_{\mathrm{g}}(\mu)]}\sum_{n=0}^{\infty}\frac{1}{n!(n-b)}\left(\frac{1}{2\beta_0\alpha_{\mathrm{g}}(\mu)}\right)^n$$

where $b = \beta_1/(2\beta_0^2)$, R_0 is the overall normalization of the leading renormalon in the pole mass, and $\alpha_{\mathrm{g}}(\mu)$ is the coupling constant in the scheme with

$$\beta\left(\alpha_{\mathrm{g}}(\mu)\right) = -\frac{\beta_0\alpha_{\mathrm{g}}^2(\mu)}{1-(\beta_1/\beta_0)\alpha_{\mathrm{g}}(\mu)}$$

- For the relations between the RS and MRS schemes:

$$m_{\mathsf{RS}}(\nu_f) = m_{\mathrm{MRS}} - \mathcal{J}_{\mathrm{MRS}}(\nu_f)$$
$$\overline{\Lambda}_{\mathsf{RS}}(\nu_f) = \overline{\Lambda}_{\mathrm{MRS}} + \mathcal{J}_{\mathrm{MRS}}(\nu_f)$$