

(So far)

Contribution of UAB to HaSP

Rafel Escribano

Universitat Autònoma de Barcelona

HaSP meeting

Theoretical Aspects of Hadron Spectroscopy and Phenomenology

December 15, 2020

Valencia (Spain)

Research Team



Antonio Pineda

Bound states, non-relativistic QM and QFTs

pNRQCD at weak and strong coupling

pNRQED and chiral theories

OPE and renormalons

Hadronic Physics and EFTs

B-decays

Tau-decays

Muon $g-2$



Pere Masjuan

Research Team



Jaume Tarrús

EFTs for heavy hadrons (baryons and exotic)

Heavy-quark hybrids

Quarkonium hadronic transitions

Muon $g-2$

light meson decays
(CP violation, radiative corrections)

D-decays



Pablo Sánchez-Puertas

Hyperasymptotic approximation to the OPE: pole mass, plaquette and static potential

Based on

Ayala, Lobregat, Pineda: 1902.07736; 1909.01370; 1910.04090, 2005.12301 and 2009.01285

Antonio Pineda

Universitat Autònoma de Barcelona & IFAE

Theoretical aspects of Hadron Spectroscopy and Phenomenology, Valencia, Spain; 15-17 December 2020

Assumption: NonPerturbative OPE

$$\text{Observable}\left(\frac{Q}{\Lambda_{\text{QCD}}}\right) \sim \sum_{n=0}^{\infty} p_n^{(X)} \alpha_X^{n+1}(Q) \quad \text{divergent} \rightarrow \text{Principal Value regularization}$$

Borel transform

$$\sum_{n=0}^{\infty} p_n^{(X)} \left(\frac{\mu}{Q}\right) \alpha_X^{n+1}(\mu) \rightarrow B[O](t) = \sum_{n=0}^{\infty} \frac{p_n^{(X)}\left(\frac{\mu}{Q}\right)}{n!} t^n$$

Inverse Borel transform

$$\int_0^{\infty} dt e^{-t/\alpha_X(Q)} B[O](t)$$

PV regularization (scale and Scheme independent)

$$S_{\text{PV}}(\alpha(Q)) \equiv \int_{0, \text{PV}}^{\infty} dt e^{-t/\alpha_X(Q)} B[O](t)$$

$$\text{Observable}\left(\frac{Q}{\Lambda_{\text{QCD}}}\right) = S_{\text{PV}}(\alpha(Q)) + K_X^{(\text{PV})} \alpha_X^\gamma(Q) \frac{\Lambda_X^d}{Q^d} (1 + \mathcal{O}(\alpha_X(Q))) + \mathcal{O}\left(\frac{\Lambda_X^{d'}}{Q^{d'}}\right)$$

S_{PV} can only be computed in an approximated way. S_{PV} will be computed truncating the hyperasymptotic expansion in a systematic way.

Organizing the computation using superasymptotic and hyperasymptotic approximations allows for a parametric control of the error.

$$\text{Observable}\left(\frac{Q}{\Lambda_{\text{QCD}}}\right) - \sum_{n=0}^N p_n^{(X)}\left(\frac{\mu}{Q}\right) \alpha_X^{n+1}(\mu) \sim \mathcal{O}(\alpha_X^{N+2})$$

but with large coefficient!!

Truncate the sum at the minimal term \rightarrow superasymptotic approximation:

$$N = N_P \equiv |d| \frac{2\pi}{\beta_0 \alpha_X(\mu)} (1 - c \alpha_X(\mu)),$$

$$\text{Observable}\left(\frac{Q}{\Lambda_{\text{QCD}}}\right) - \sum_{n=0}^{N_P} p_n^{(X)}\left(\frac{\mu}{Q}\right) \alpha_X^{n+1}(\mu) \sim \mathcal{O}\left(e^{-|d| \frac{2\pi}{\beta_0 \alpha_X(Q)}}\right)$$

Applications

- ▶ Bottom “pole” mass

$$m_{b,\text{PV}}(4.186\text{GeV}) = 4836(\mu)_{-17}^{+8}(Z_m)_{+12}^{-11}(\alpha)_{-9}^{+8} \text{ MeV} .$$

$$\bar{\Lambda}_{\text{PV}} = 477(\mu)_{+17}^{-8}(Z_m)_{-12}^{+11}(\alpha)_{+9}^{-8} \text{ MeV} .$$

- ▶ Top “pole” mass

$$m_{t,\text{PV}}(163\text{GeV}) = 173033(\text{th})_{-28}^{+25}(\alpha)_{-123}^{+119} \text{ MeV} ,$$

$$\left[\frac{m_{t,\text{PV}}}{\bar{m}_t} - 1 \right] \times 10^5 = 6155 (\text{th})_{-17}^{+15} (\alpha)_{-75}^{+73} .$$

- ▶ Plaquette → confirmation of the OPE and determination of the gluon condensate:

$$\langle G^2 \rangle_{\text{PV}}(n_f = 0) = 3.15(18) r_0^{-4} .$$

- ▶ Static potential → determination of the strong coupling constant:

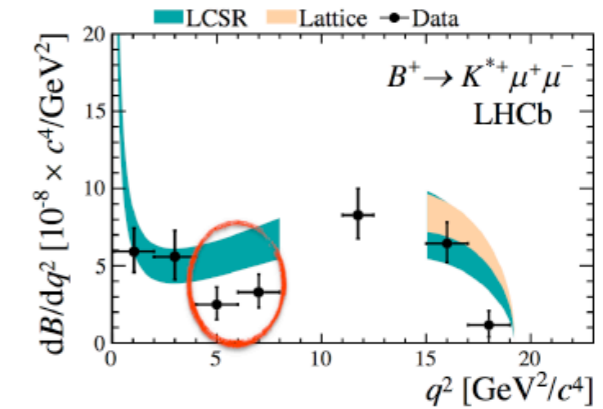
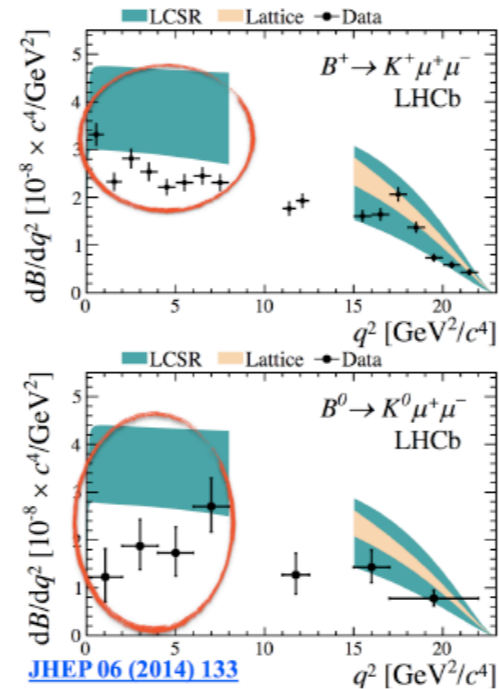
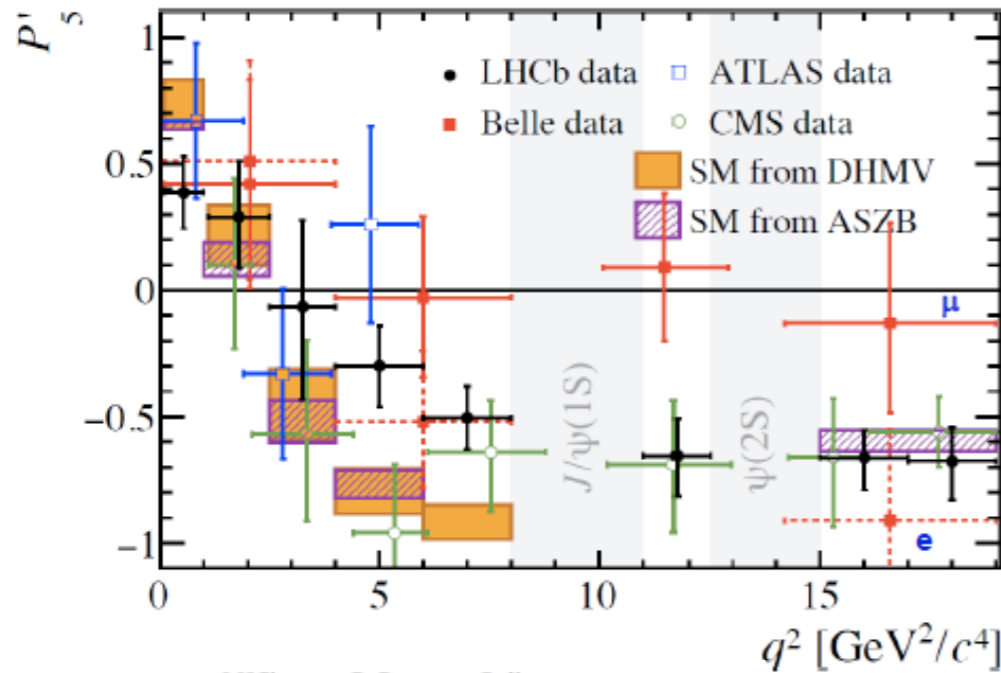
$$\Lambda_{\overline{\text{MS}}}^{(n_f=3)} = 338(12) \text{ MeV} \quad \alpha(M_Z) = 0.1181(9) .$$

Overlooking Lepton Flavor Universal New Physics in $b \rightarrow s\ell\ell$

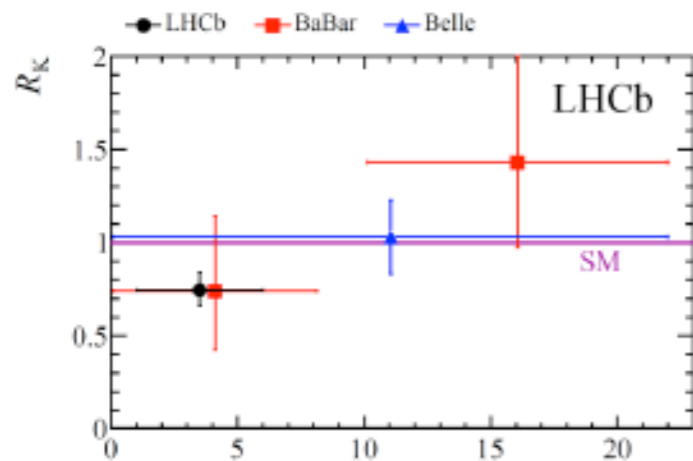
Pere Masjuan (masjuan@ifae.es), [arXiv: 1809:08447] + [arXiv: 1902:04900] + [arXiv: 1903:09578]

Consider $B^0 \rightarrow K^{*0} \mu^+ \mu^- \rightarrow K^+ \pi^- \mu^+ \mu^-$

$$\frac{d^4\Gamma(\bar{B}_d \rightarrow K^*(\rightarrow K\pi)\mu^+\mu^-)}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi} = \frac{9}{32\pi} \sum_i J_i(q^2) f_i(\theta_\ell, \theta_K, \phi)$$



$b \rightarrow s\mu^+\mu^-$ ($\times 10^7$)	bin	SM	EXP	Pull
$\text{BR}(B^0 \rightarrow K^0 \mu^+ \mu^-)$	[15,19]	0.91 ± 0.12	0.67 ± 0.12	+1.4
$\text{BR}(B^0 \rightarrow K^{*0} \mu^+ \mu^-)$	[16,19]	1.66 ± 0.15	1.23 ± 0.20	+1.7
$\text{BR}(B^+ \rightarrow K^{*+} \mu^+ \mu^-)$	[15,19]	2.59 ± 0.25	1.60 ± 0.32	+2.5
$\text{BR}(B_s \rightarrow \phi \mu^+ \mu^-)$	[15,18.8]	2.20 ± 0.17	1.62 ± 0.20	+2.2



Coherent deviation in Lepton Flavour
Dependent and Lepton Flavour Universal

Are we overlooking LFU?

LFUV observables explained by C_{il}^V
 LFD observables explained by $C_{il}^V + C_i^U$

[Algueró et al, 1809.08447]

Constrained 2D Fit (inspired from 4D+3D)

$$2D: C_{9\mu}^V = -C_{10\mu}^V, C_9^U = C_{10}^U$$

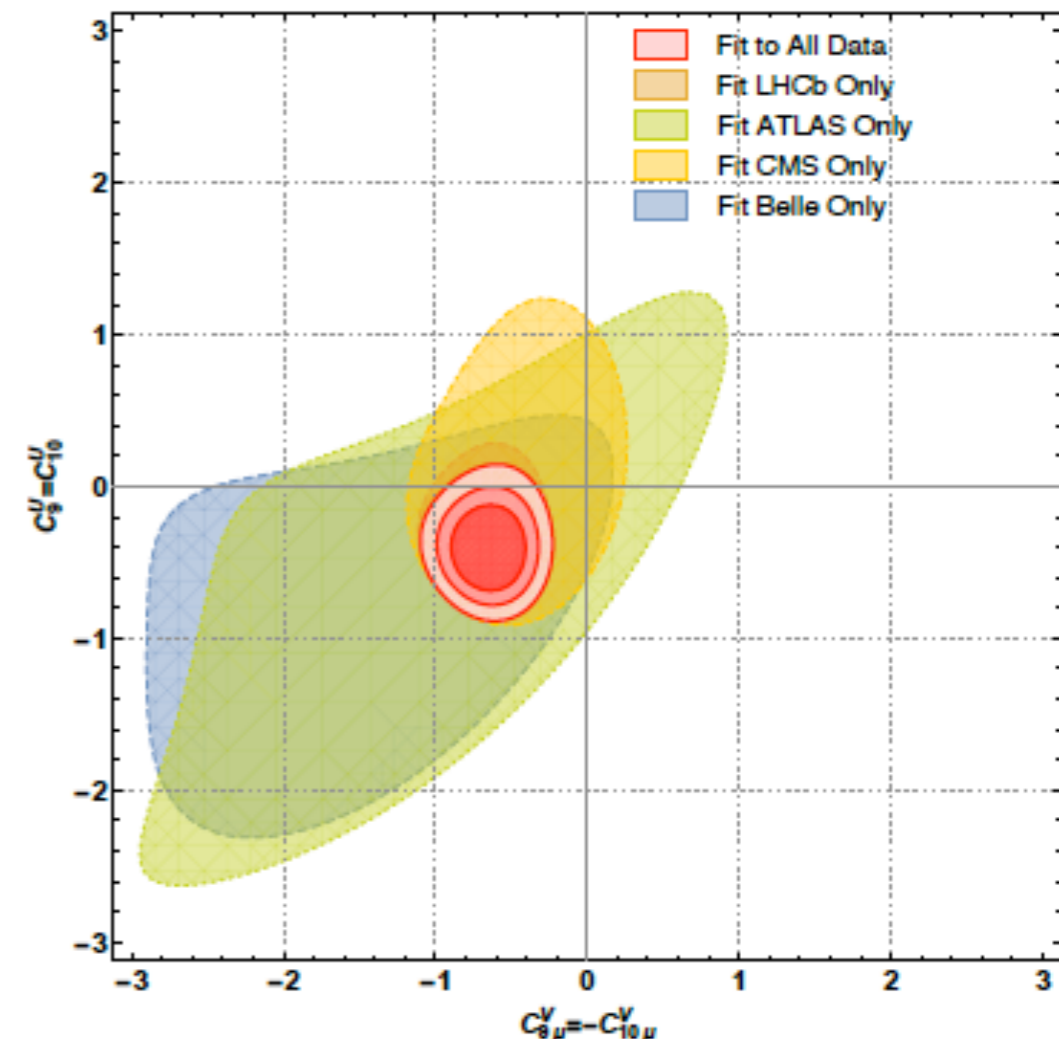
	Best-fit point	1 σ CI	2 σ CI
$C_{9\mu}^V = -C_{10\mu}^V$	-0.64	[-0.77, -0.51]	[-0.90, -0.39]
$C_9^U = C_{10}^U$	-0.44	[-0.58, -0.29]	[-0.71, -0.14]

SM excluded at **6.0 σ**

The two coefficients uncorrelated

LFUV prefers **V-A** structure while **LFU** a **V+A**

$$\text{Corr}_6 = \begin{pmatrix} 1.00 & -0.01 \\ -0.01 & 1.00 \end{pmatrix}$$



Updated results

Impact of Morion's results from March 2019

2018		Best-fit point	1σ	Pull _{SM}	p-value
Sc. 5	$C_{9\mu}^V$	-0.16	[-0.94, +0.46]	5.8	78%
	$C_{10\mu}^V$	+1.00	[+0.18, +1.59]		
	$C_9^U = C_{10}^U$	-0.87	[-1.43, -0.14]		
Sc. 6	$C_{9\mu}^V = -C_{10\mu}^V$	-0.64	[-0.77, -0.51]	6.0	79%
	$C_9^U = C_{10}^U$	-0.44	[-0.58, -0.29]		
Sc. 7	$C_{9\mu}^V$	-1.57	[-2.14, -1.06]	5.7	72%
	C_9^U	+0.56	[+0.01, +1.15]		
Sc. 8	$C_{9\mu}^V = -C_{10\mu}^V$	-0.42	[-0.57, -0.27]	5.8	74%
	C_9^U	-0.67	[-0.90, -0.42]		

2019		Best-fit point	1σ	Pull _{SM}	p-value
Sc. 5	$C_{9\mu}^V$	-0.34	[-0.93, +0.19]	5.5	72%
	$C_{10\mu}^V$	+0.69	[+0.21, +1.12]		
	$C_9^U = C_{10}^U$	-0.50	[-0.92, +0.02]		
Sc. 6	$C_{9\mu}^V = -C_{10\mu}^V$	-0.52	[-0.64, -0.41]	5.8	71%
	$C_9^U = C_{10}^U$	-0.37	[-0.52, -0.22]		
Sc. 7	$C_{9\mu}^V$	-0.91	[-1.25, -0.58]	5.5	65%
	C_9^U	-0.08	[-0.46, +0.31]		
Sc. 8	$C_{9\mu}^V = -C_{10\mu}^V$	-0.33	[-0.45, -0.22]	5.9	74%
	C_9^U	-0.72	[-0.93, -0.47]		

Changed

Sc. 7: If only V-NP preference for LFUV-NP

$$C_{9\mu}^V + C_9^U = -0.98$$

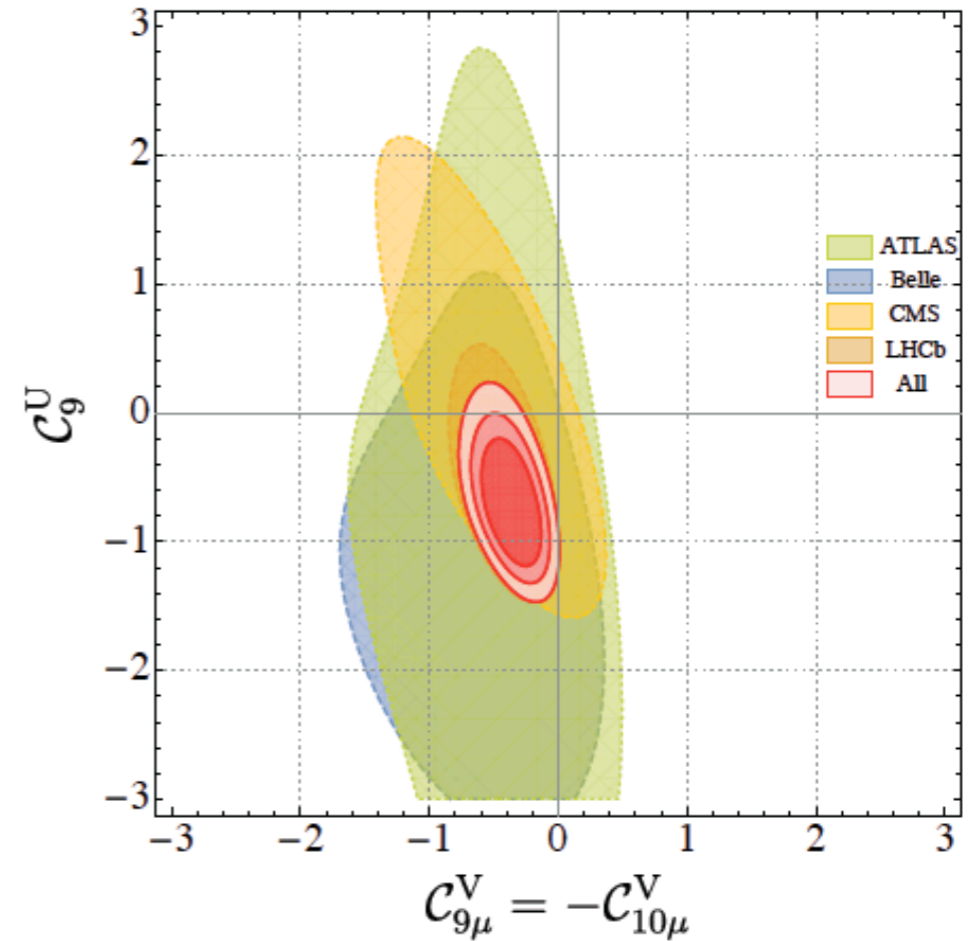
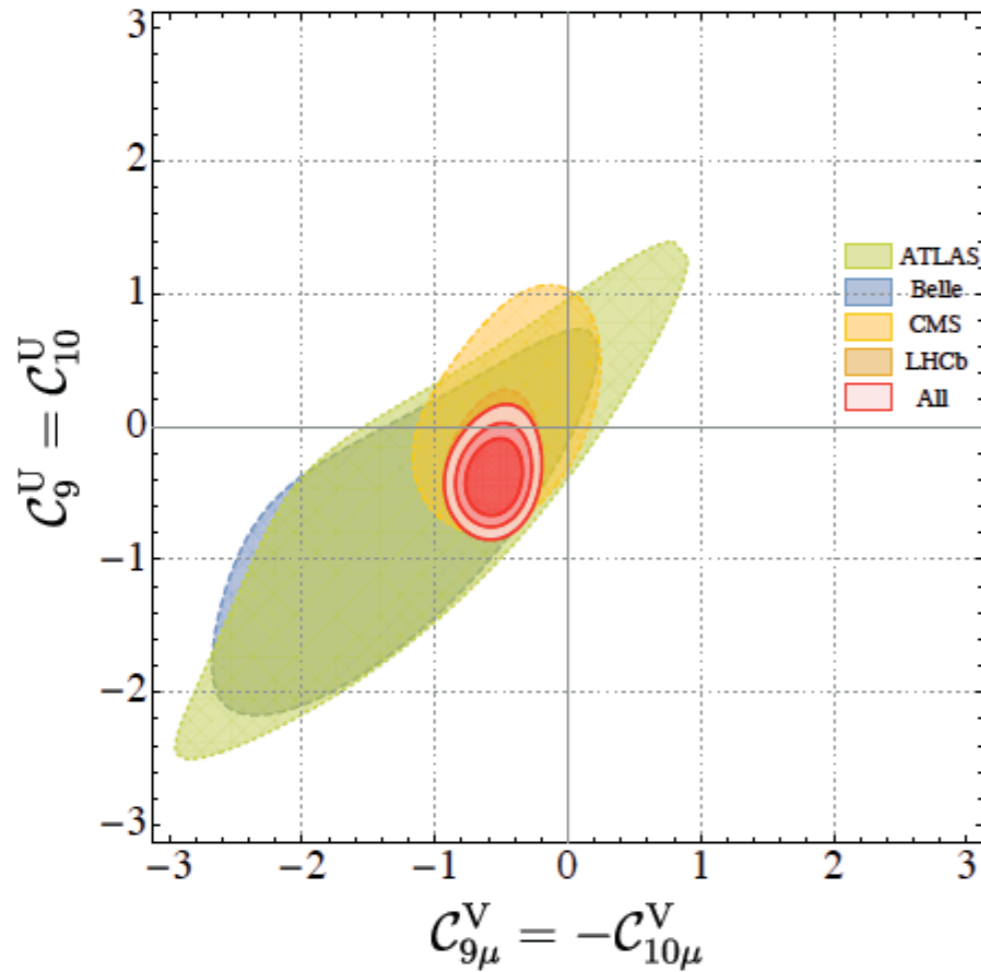
Unchanged

Sc. 8: Presence of V-LFU favours slightly $L_q \otimes L_\ell$

Updated results

Impact of Morion's results from March 2019

[arXiv: 1903:09578]



Assuming loop-level scale
of NP and no MFV

$$\Lambda_i^L \sim \frac{v}{s_\omega g} \frac{1}{\sqrt{2|V_{tb}V_{ts}^*|}} \frac{1}{|C_i^{\text{NP}}|^{1/2}}$$

Scenario 6:

$$\{C_{9\mu}^V = -C_{10\mu}^V, C_9^U = C_{10}^U\}$$

LFUV-NP $L_q \otimes L_l$

$$\Lambda_i^{\text{LFUV}} \sim 3.9 \text{ TeV}$$

LFU-NP $L_q \otimes R_l$

$$\Lambda_i^{\text{LFU}} \sim 4.6 \text{ TeV}$$

Scenario 8:

$$\{C_{9\mu}^V = -C_{10\mu}^V, C_9^U\}$$

LFUV-NP $L_q \otimes L_l$

$$\Lambda_i^{\text{LFUV}} \sim 4.6 \text{ TeV}$$

LFU-NP $L_q \otimes V_l$

$$\Lambda_i^{\text{LFU}} \sim 3.3 \text{ TeV}$$

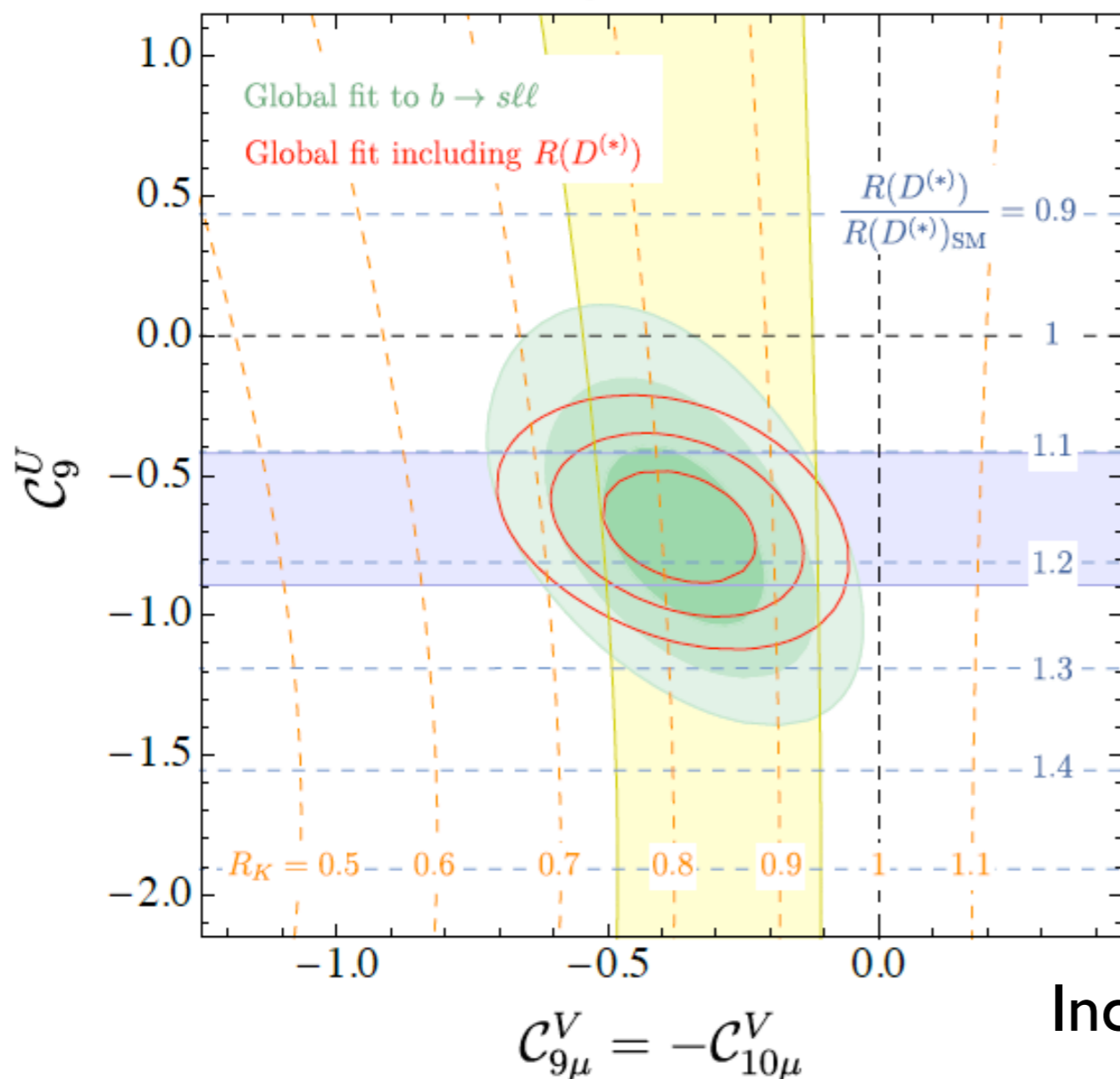
Updated results

Impact of Morion's results from March 2019

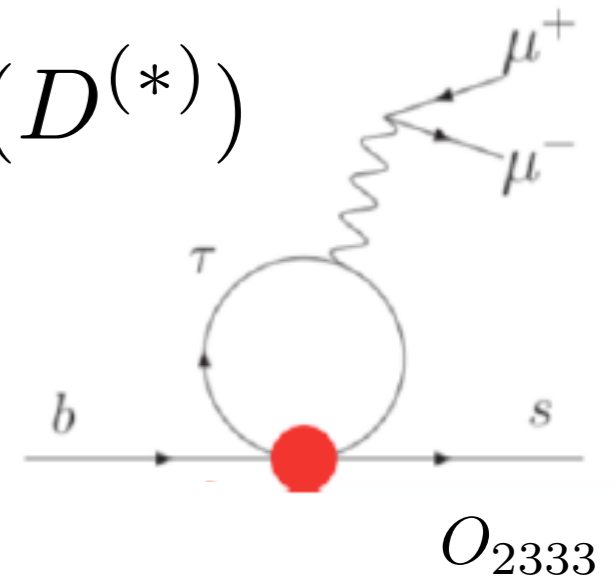
[arXiv: 1903:09578]

Scenario 8: $\{C_{9\mu}^V = -C_{10\mu}^V, C_9^U\}$ $b \rightarrow s\ell\ell \leftrightarrow b \rightarrow c\tau\nu$

SMEFT $C^{(1)} = C^{(3)}$ scenario (dim. 6 model of an SU(2) singlet vector leptoquark)



LFU C_9^U explains $R(D^{(*)})$



$$C_9^U \approx 7.5 \left(1 - \sqrt{\frac{R_{D^{(*)}}}{R_{D^{(*)};SM}}} \right) \left(1 + \frac{\log(\Lambda^2/(1\text{TeV}^2))}{10.5} \right)$$

LFUV $C_{9\mu}^V = -C_{10\mu}^V$ explains $b \rightarrow s\ell\ell$
from O_{2322}

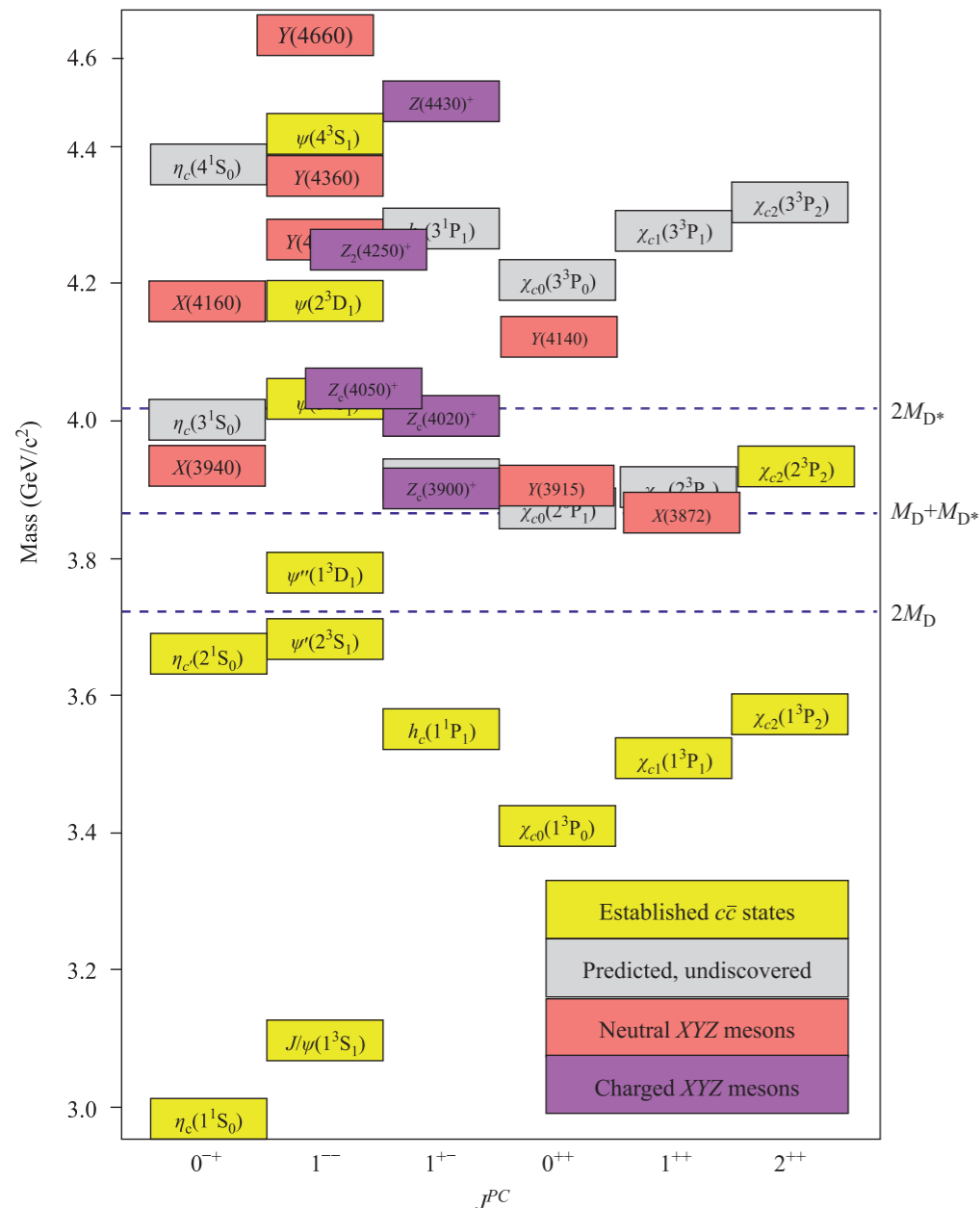
Inclusion of R(D) yields a SM pull of 7σ

Exotic Quarkonium

N. Brambilla, G. Krein, J. Tarrús, A. Vairo; Phys.Rev. D97 (2018) no.1, 016016

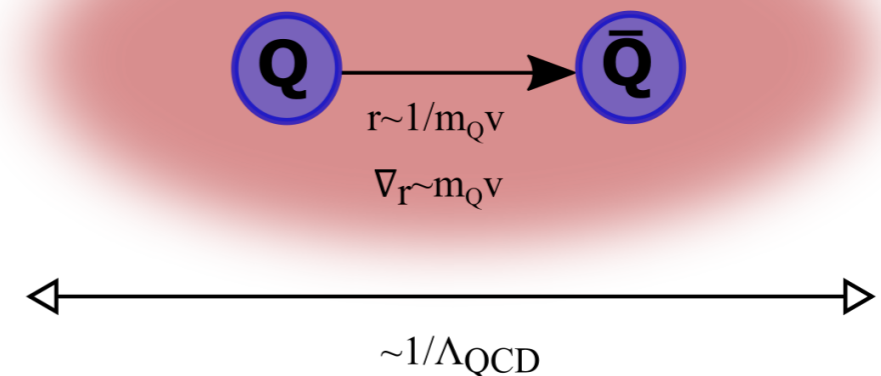
N. Brambilla, W-K. Lai, J. Segovia, J. Tarrús, A. Vairo; Phys.Rev. D99 (2019) no.1, 014017

J. Tarrús, G. Krein; Phys.Rev. D98 (2018) no.1, 014029



- ▶ Many states not fitting the traditional quark model have been observed in the $c\bar{c}$ and $b\bar{b}$ spectrum (XYZ).

$$E_{\text{heavy}} \sim m_Q v^2 \quad E_{\text{light}} \sim \Lambda_{\text{QCD}}$$



- ▶ Two main properties help us describe this exotic states

- * $m_Q \gg \Lambda_{\text{QCD}}$, non relativistic heavy quarks.
- * Adiabatic expansion between the heavy quark and gluon/light-quark dynamics.

- ▶ Analog system to diatomic molecules.

- ▶ We exploit this gaps between the scales of the system at the Lagrangian level to build EFTs to describe this states.

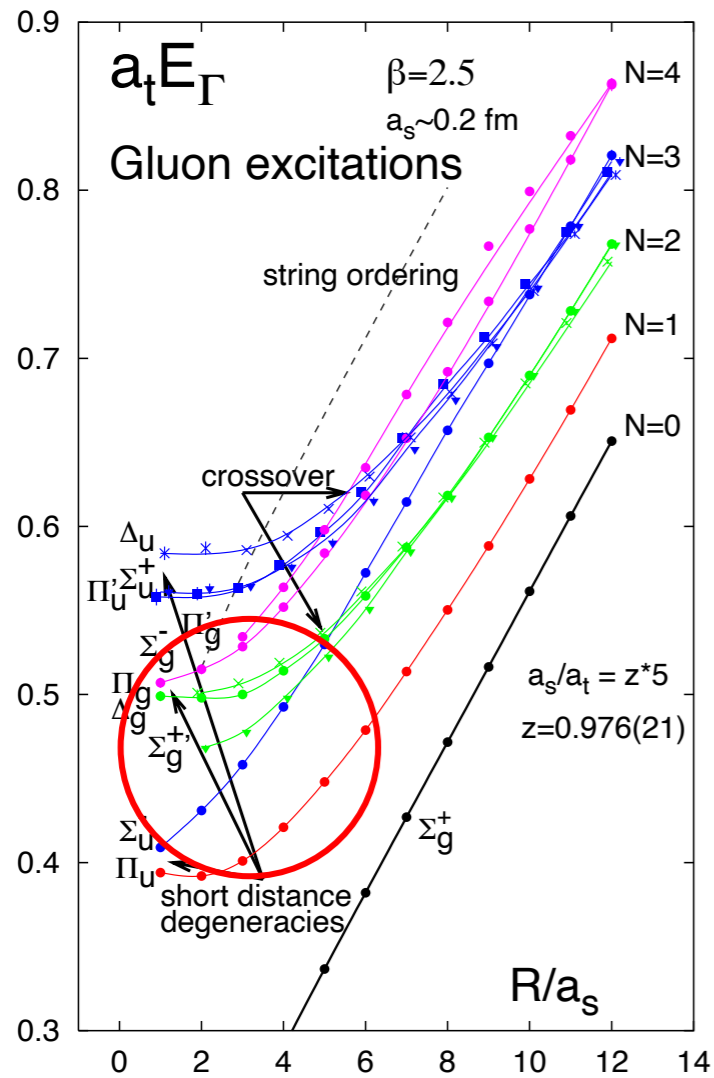
Exotic Quarkonium

N. Brambilla, G. Krein, J. Tarrús, A. Vairo; Phys.Rev. D97 (2018) no.1, 016016

N. Brambilla, W-K. Lai, J. Segovia, J. Tarrús, A. Vairo; Phys.Rev. D99 (2019) no.1, 014017

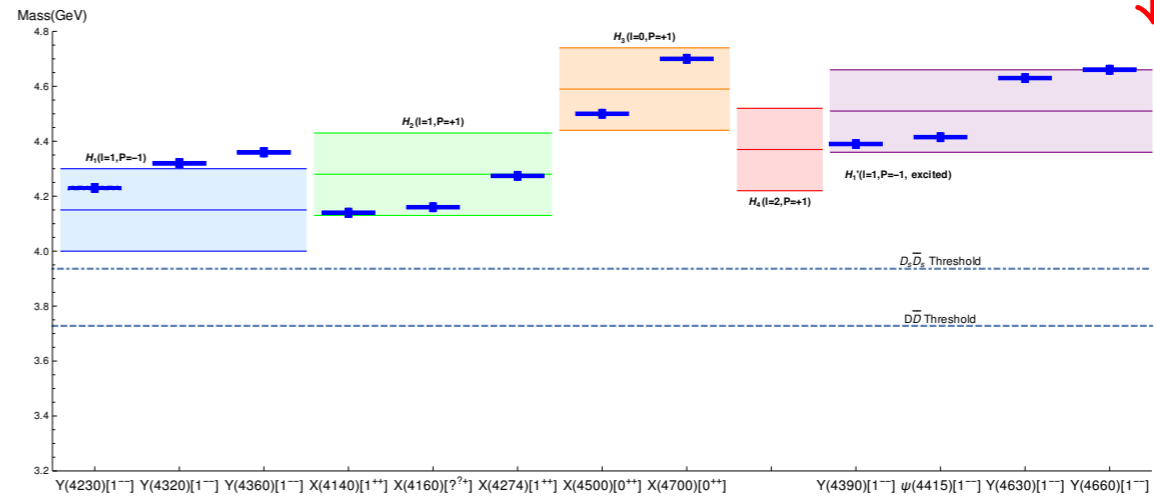
J. Tarrús, G. Krein; Phys.Rev. D98 (2018) no.1, 014029

- Lattice NRQCD static energies:

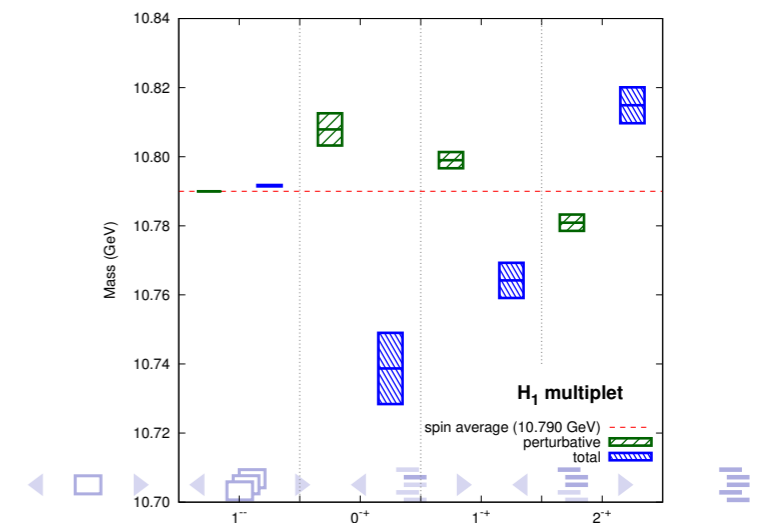
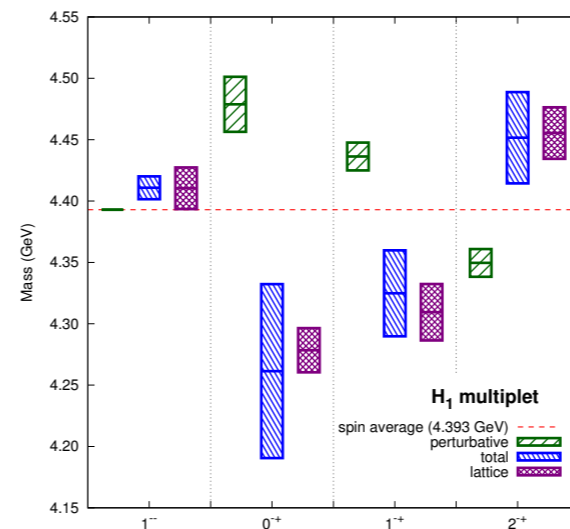


$$L_{BO} = \int d^3 R d^3 r \sum_{\lambda \lambda'} \psi_\lambda^\dagger \left\{ i\partial_t - V_{\lambda\lambda'}(r) - \hat{r}_\lambda^{i\dagger} \frac{\nabla_r^2}{m} \hat{r}_{\lambda'}^i \right\} \psi_{\lambda'}$$

- Quarkonium hybrid spectrum ($cg\bar{c}$) vs experimental exotics:



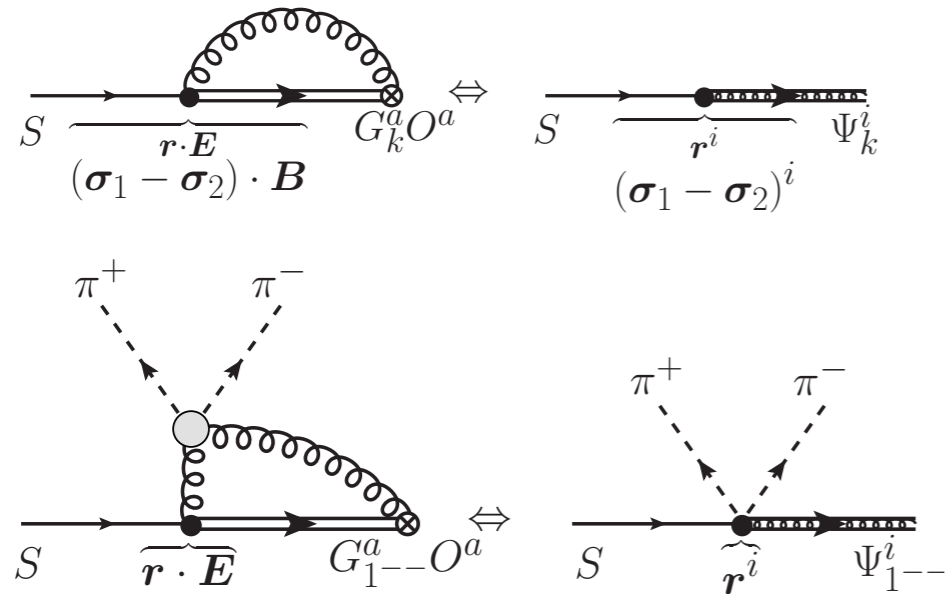
- Including $1/m_Q$ and $1/m_Q^2$ spin dependent potentials: ($cg\bar{c}$ left), ($bg\bar{b}$ right)



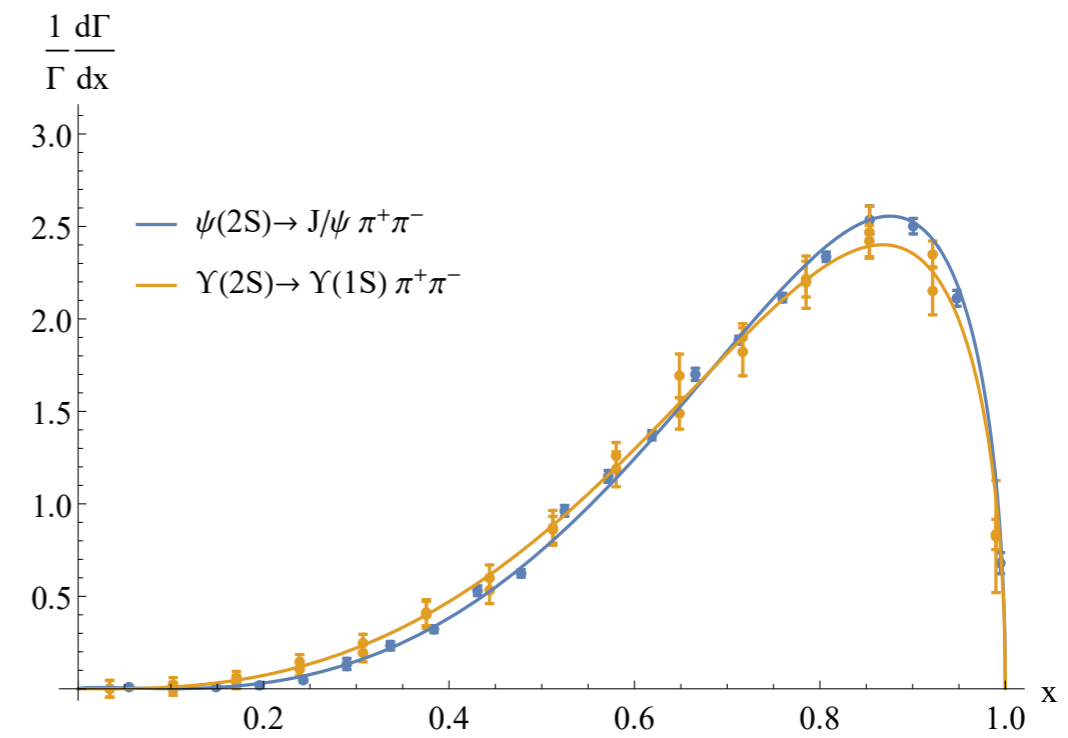
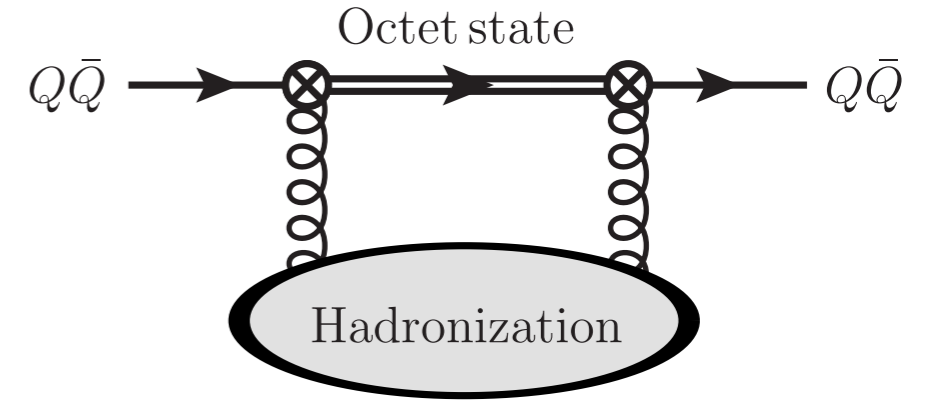
Hadronic transitions in Quarkonium

A. Pineda, J. Tarrús; Phys.Rev. D100 (2019) no.5, 054021

- ▶ **Two step** process (multipole expansion).
- ▶ Traditional approaches based on an OPE of the **octet** propagator are not well justified.
- ▶ Use that the **color-octet state** spectrum corresponds to the **hybrid** quarkonium spectrum.
- ▶ Hadronization using the scale or axial anomaly.



- ▶ We build an EFT with standard (S) and hybrid (Ψ) quarkonium as well as pions.
- ▶ Incorporates m_Q , multipole, chiral and large N_c expansions.



Present and future contributions to HaSP

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— $(g - 2)_\mu$ HLbL physics at IFAE

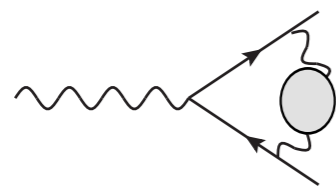
- The $a_\mu = (g - 2)_\mu/2$ is a sensitive probe of new physics

$$a_\mu^{\text{th}} = 116591810(43) \times 10^{-11} \text{ vs. } a_\mu^{\text{exp}} = 116592089(63) \times 10^{-11};$$

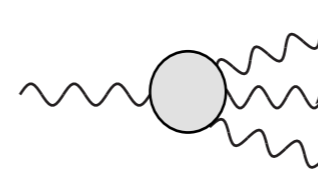
$$a_\mu^{\text{exp}} - a_\mu^{\text{th}} = 279(76) \times 10^{-11} \quad (3.7\sigma)$$

- New experiment at FNAL $\Delta a_\mu^{\text{exp}} = 16 \times 10^{-11} \Rightarrow$ **Needs th error reduction!!**

Driven by HVP & HLbL ; will discuss HLbL



$$a_\mu^{\text{HVP}} = 6845(40) \times 10^{-11}$$



$$a_\mu^{\text{HLbL}} = 92(18) \times 10^{-11}$$

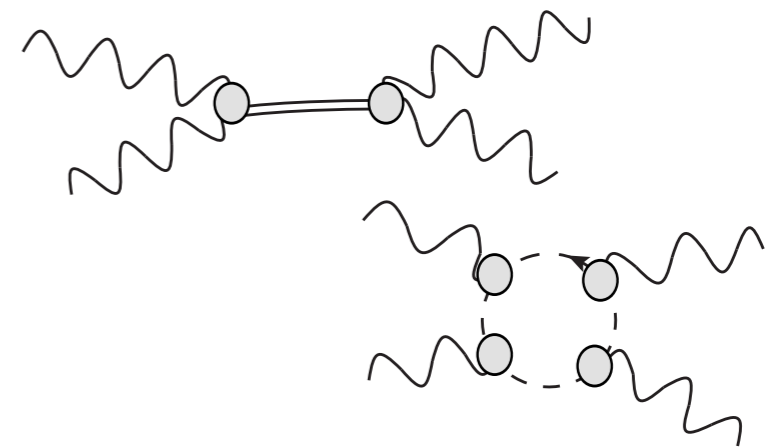
- Among leading HLbL contributions (WP):

$$a_\mu^{\pi, \eta, \eta'} = 94(4) \times 10^{-11}$$

$$a_\mu^{\pi\pi+S\text{-wave}} = -[15.9(2) + 8(1)] \times 10^{-11}$$

$$a_\mu^{\text{axials}} = 6(6) \times 10^{-11}$$

$$a_\mu^{\text{SD}} = 15(10) \times 10^{-11}$$



- Clearly, to meet experimental errors, axials and SD needs be re-examined.

___ Axial-vector meson contributions to a_μ in $R_\chi T$ _____

P. Roig and P. SP, PRD101 (2020) 7, 074019

- Previously $\{2.5(1)[BPP], 22(5)[MV], 6.4(2.0)[J], 7.6(2.7)[PVdH]\} \times 10^{-11}$ demands careful reexamination... so we did!
- $A \rightarrow \gamma^* \gamma^*$: 3 form factors **BUT** uses Schouten Id + EOM ($\varepsilon_A \cdot q_A \rightarrow 0$).
Need HLbL Greens' function: usually via Proca propagator \Rightarrow not transverse!!
- **Our proposal** use $R_\chi T$: good pheno and suitable to study Greens' functions.
High-energy behavior demands 2/3 vector resonances. At LO, antisymmetric FFs; Reconstructing HLbL Green's function we obtain (10^{-11} units)

#V's	a_1	f_1	f'_1
2	1.13(30)	3.14(6)	0.07(4)
3	0.21(4)	0.58(11)	0.015(8)

Symmetric form factor at NLO in $R_\chi T$: $\Delta a_\mu^{\text{axial}NLO} = -0.8 \times 10^{-11}$

$$\text{Final estimate } a_\mu^{\text{axial}} = 0.8^{(+3.5)}_{(-0.8)} \times 10^{-11}$$

- Recently: 22(5) Leutgeb Reban (2020); 28 Capiello et al (2020) (SDs)

Short-Distance constraints to HLbL

P. Roig, PM and P. SP, arXiv:2005.11761

- a_μ sensitive to low energies, but high-energy tails might have an impact, missing something?
- Our work: It is known $\langle VVA \rangle$ has 4 FFs, $\{w_L$ (anomaly) $w_T^{(+)}$, $w_T^{(-)}$, $\tilde{w}_T^{(-)}\}$
Found new relation among w_T 's and the π^0 TFF guaranteeing the anomaly

$$(q_1^2 + q_2^2)w_T^{(+)}(q_1^2, q_2^2, 0) - (q_1^2 - q_2^2)w_T^{(-)}(q_1^2, q_2^2, 0) = 2N_c(1 - \tilde{F}_{\pi\gamma\gamma}(q_1^2, q_2^2))$$

Anomaly enforces $w_T^{(+/-)}(q_1^2, q_2^2, 0) \sim$ subtraction (not a constant π^0 TFF!!)
Remainder piece: $w_T^{(+/-)}(q_1^2, q_2^2, q_{12}^2) - w_T^{(+/-)}(q_1^2, q_2^2, 0)$ (as if subtracted)

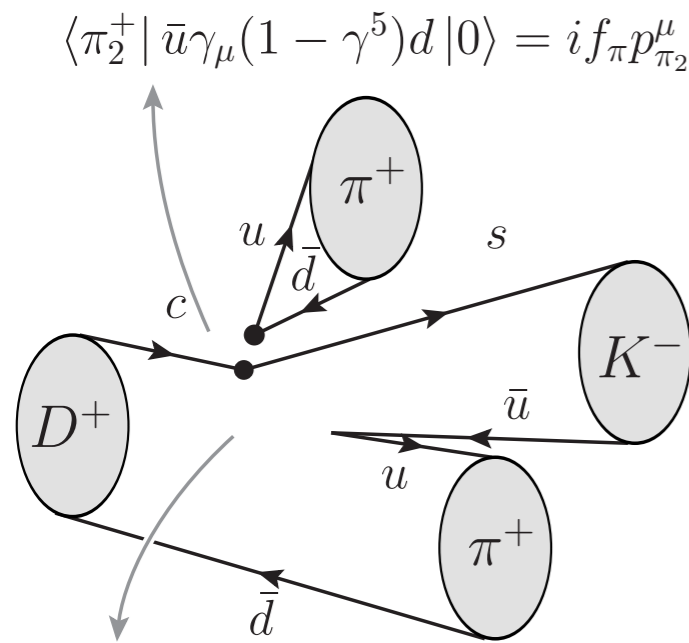
- Back to HLbL, many implications and applications:

- \Rightarrow Axial contributions: trans.+long. = pheno(pole) + anomaly(model)
- \Rightarrow Can estimate heavy axial FFs parameters
- \Rightarrow Outlook (I): consider $m_q \neq 0$ and role of heavy pseudoscalars
- \Rightarrow Outlook (II): model anomaly part

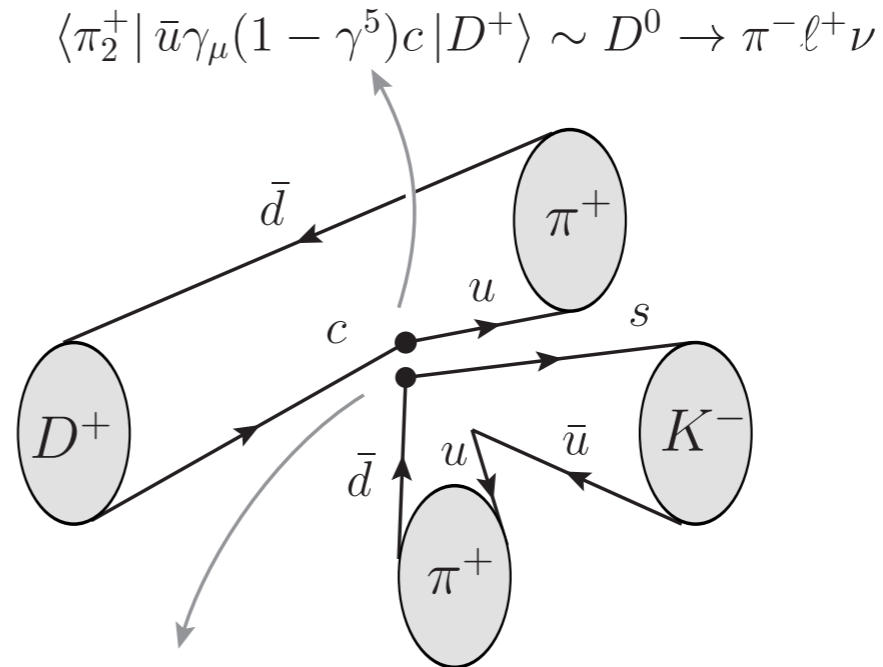
$D^+ \rightarrow K^- \pi^+ \pi^+$ and $D^+ \rightarrow K^- \pi^+ \ell^+ \nu$ decays

RE, PM and PSP, in progress

- Building on Boito&Escribano PRD80, (2009): now combined analysis.



$\langle K^- \pi_1^+ | \bar{s} \gamma_\mu (1 - \gamma^5) c | D^+ \rangle \sim D^+ \rightarrow \pi^+ K^- \ell^+ \nu$



$\langle K^- \pi_1^+ | \bar{s} \gamma_\mu (1 - \gamma^5) d | 0 \rangle \sim f_{0,+}^{K\pi}$

- Challenging part: $f_\pi p_{\pi_2}^\mu \langle K^- \pi_1^+ | \bar{s} \gamma_\mu (1 - \gamma^5) c | D^+ \rangle$ from $D^+ \rightarrow \pi^+ K^- \ell^+ \nu$.

Derivative picks a single FF $\mathcal{O}(m_\ell)$ in semileptonic decays, but use of Ward Id

- Taking exp par BES3(2016)+relative phase in BE ok if rescaling P/S-wave by 1.3(1)/2.2(2) factor

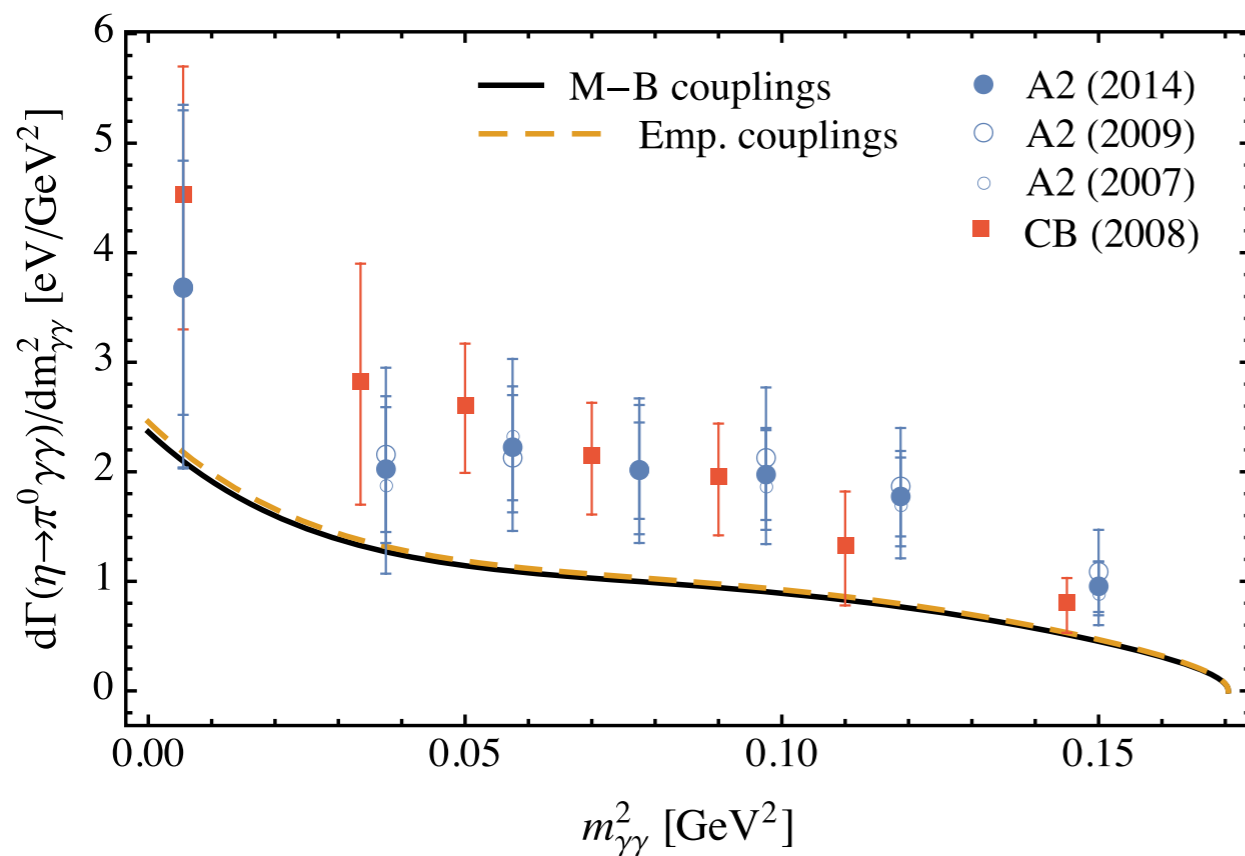
- $D_s^+ \rightarrow K^+ K^+ \pi^-$ straightforward: CKM and $f_\pi \rightarrow f_K$: excellent! fact test!

- Outlook: improve exp. S-wave description based on Bernard et al PLB638 (2006)

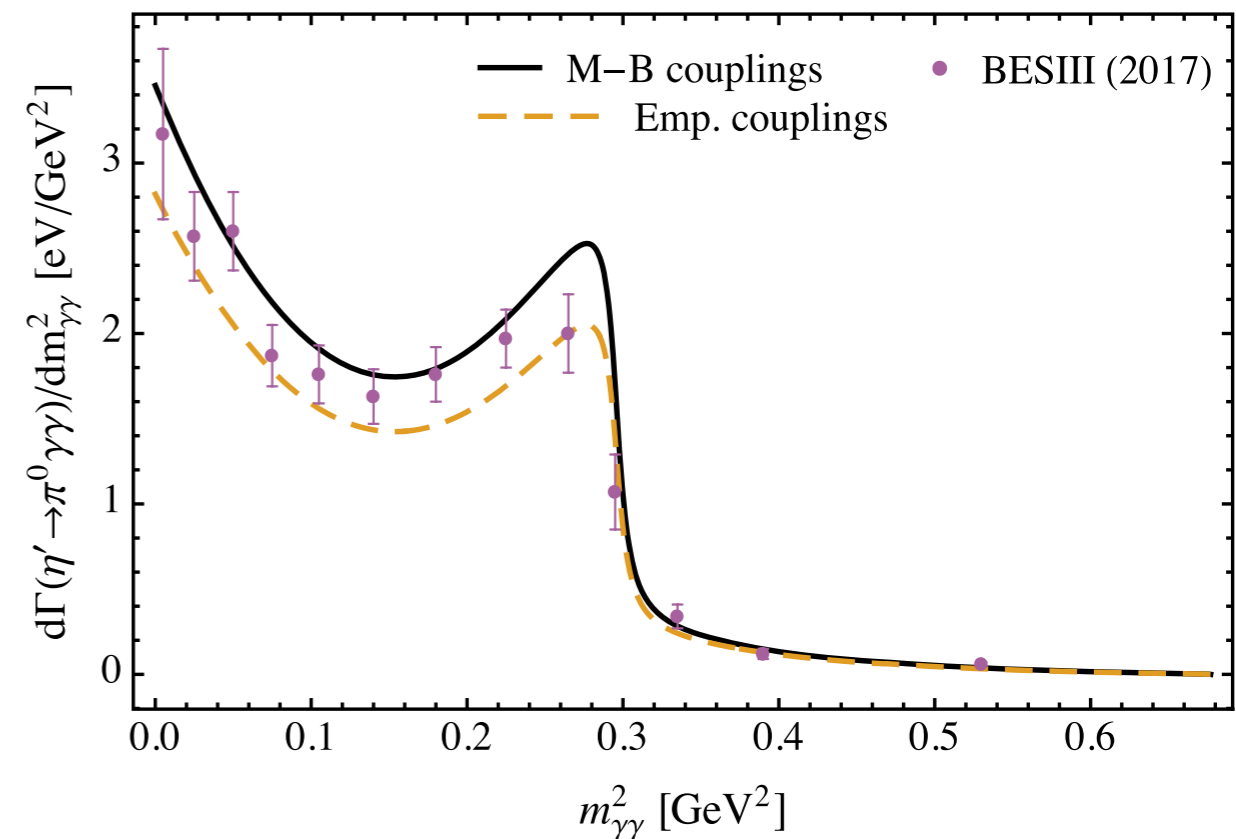
Theoretical analysis of the doubly radiative decays

$$\eta(\eta') \rightarrow \pi^0 \gamma\gamma \text{ and } \eta' \rightarrow \eta \gamma\gamma \quad \text{Phys. Rev. D 102, 034026 (2020)}$$

Relevant for testing the chiral expansion, probing scalar dynamics and searching for a hypothetical B-boson



(a) $\eta \rightarrow \pi^0 \gamma\gamma$ decay.



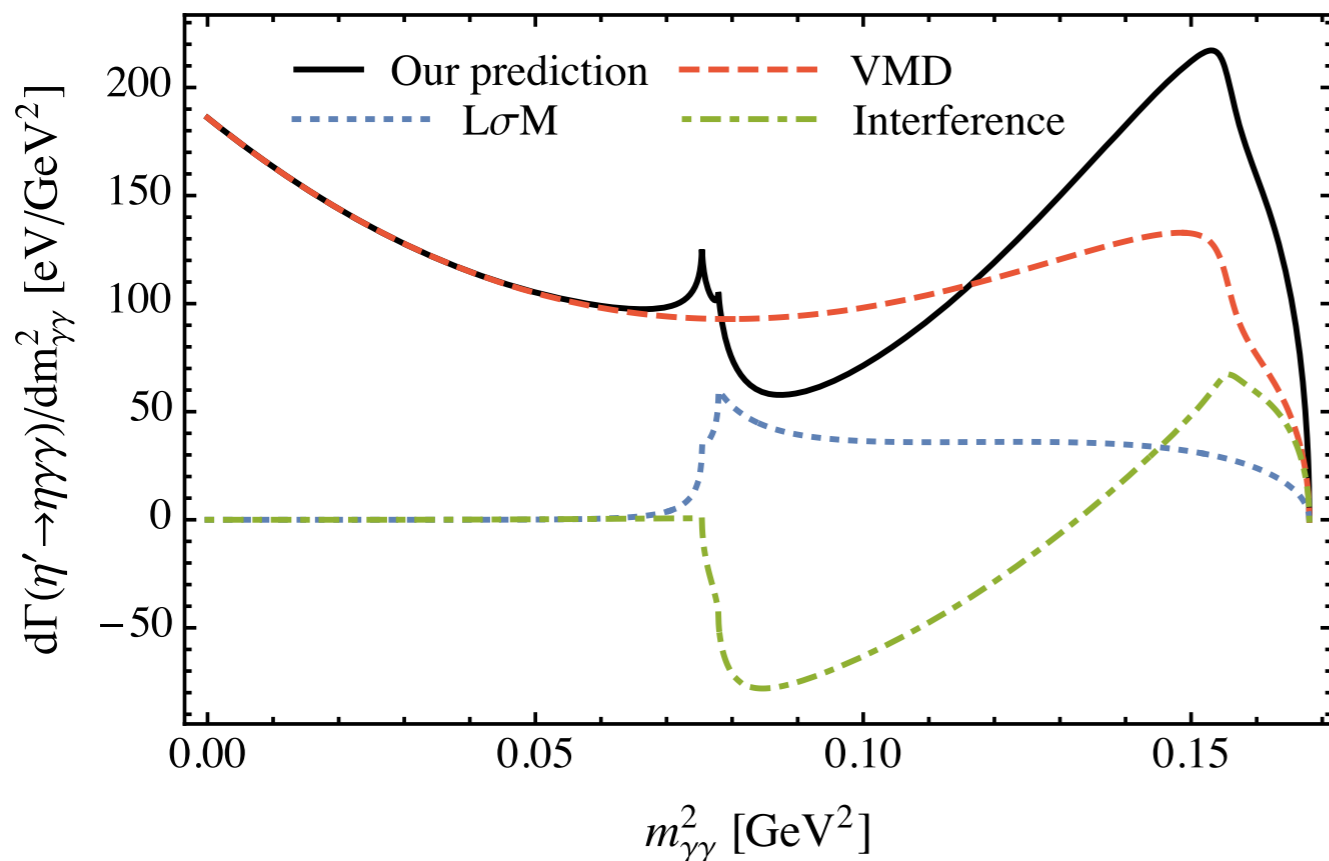
(b) $\eta' \rightarrow \pi^0 \gamma\gamma$ decay.

Not possible to reconcile our predictions for both processes

New experimental analyses welcome!

Theoretical analysis of the doubly radiative decays

$\eta(\eta') \rightarrow \pi^0\gamma\gamma$ and $\eta' \rightarrow \eta\gamma\gamma$



Substantial scalar contribution
of σ and f_0

A first experimental analysis
very welcome!

TABLE II. Chiral-loop, $L\sigma M$, and VMD predictions for the $\eta \rightarrow \pi^0\gamma\gamma$, $\eta' \rightarrow \pi^0\gamma\gamma$, and $\eta' \rightarrow \eta\gamma\gamma$ decays with empirical and model-based VMD couplings. The total decay widths are calculated from the coherent sum of the $L\sigma M$ and VMD contributions.

Decay	Couplings	Chiral loop	$L\sigma M$	VMD	Γ	BR_{th}	BR_{exp} [14]
$\eta \rightarrow \pi^0\gamma\gamma$ (eV)	Empirical	1.87×10^{-3}	5.0×10^{-4}	0.16(1)	0.18(1)	$1.35(8) \times 10^{-4}$	$2.56(22) \times 10^{-4}$
	Model-based	1.87×10^{-3}	5.0×10^{-4}	0.16(1)	0.17(1)	$1.30(1) \times 10^{-4}$	
$\eta' \rightarrow \pi^0\gamma\gamma$ (keV)	Empirical	1.1×10^{-4}	1.3×10^{-4}	0.57(3)	0.57(3)	$2.91(21) \times 10^{-3}$	$3.20(7)(23) \times 10^{-3}$
	Model-based	1.1×10^{-4}	1.3×10^{-4}	0.70(4)	0.70(4)	$3.57(25) \times 10^{-3}$	
$\eta' \rightarrow \eta\gamma\gamma$ (eV)	Empirical	1.4×10^{-2}	3.29	21.2(1.2)	23.0(1.2)	$1.17(8) \times 10^{-4}$	$8.25(3.41)(0.72) \times 10^{-5}$
	Model-based	1.4×10^{-2}	3.29	19.1(1.0)	20.9(1.0)	$1.07(7) \times 10^{-4}$	

A theoretical analysis of the semileptonic decays

$$\eta(\eta') \rightarrow \pi^0 l^+ l^- \text{ and } \eta' \rightarrow \eta l^+ l^- \quad 2007.12467 [\text{hep-ph}] (\text{tbp EPJC})$$

Relevant for testing fundamental symmetries

C-conserving processes in the SM (2-photon intermediate state)

C-violating processes via single-photon exchange

Important as a background for BSM searches

Decay	Γ_{th}	BR_{th}	BR_{exp}
$\eta \rightarrow \pi^0 e^+ e^-$	$2.8(2)(3) \times 10^{-6} \text{ eV}$	$2.1(1)(2) \times 10^{-9}$	$< 7.5 \times 10^{-6}$ (CL=90%) [10]
$\eta \rightarrow \pi^0 \mu^+ \mu^-$	$1.6(1)(2) \times 10^{-6} \text{ eV}$	$1.2(1)(1) \times 10^{-9}$	$< 5 \times 10^{-6}$ (CL=90%) [17]
$\eta' \rightarrow \pi^0 e^+ e^-$	$9.0(0.5)(1.3) \times 10^{-4} \text{ eV}$	$4.6(3)(7) \times 10^{-9}$	$< 1.4 \times 10^{-3}$ (CL=90%) [17]
$\eta' \rightarrow \pi^0 \mu^+ \mu^-$	$3.5(2)(4) \times 10^{-4} \text{ eV}$	$1.8(1)(2) \times 10^{-9}$	$< 6.0 \times 10^{-5}$ (CL=90%) [17]
$\eta' \rightarrow \eta^0 e^+ e^-$	$7.6(4)(8) \times 10^{-5} \text{ eV}$	$3.9(3)(4) \times 10^{-10}$	$< 2.4 \times 10^{-3}$ (CL=90%) [17]
$\eta' \rightarrow \eta^0 \mu^+ \mu^-$	$3.1(2)(3) \times 10^{-5} \text{ eV}$	$1.6(1)(2) \times 10^{-10}$	$< 1.5 \times 10^{-5}$ (CL=90%) [17]

TABLE I: Decay widths and branching ratios for the six C-conserving decays $\eta^{(\prime)} \rightarrow \pi^0 l^+ l^-$ and $\eta' \rightarrow \eta l^+ l^-$ ($l = e$ or μ). First error is experimental and second is due to numerical integration.

π^0 - η - η' mixing from $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ decays

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Relevant for testing possible isospin-breaking contributions

$$|\pi^0\rangle = |\pi_3\rangle + \epsilon|\eta\rangle + \epsilon'|\eta'\rangle$$

Precise and exhaustive set of experimental data

$$g_{\phi\eta\gamma} = g \left\{ \left[\left(\frac{z_{NS}}{3} - \epsilon_{12} \right) c\phi_{23} + \epsilon_{13} s\phi_{23} \right] s\phi_V + \frac{2}{3} z_S \frac{\bar{m}}{m_s} s\phi_{23} c\phi_V \right\},$$

$$g_{\phi\eta'\gamma} = g \left\{ \left[\left(\frac{z_{NS}}{3} - \epsilon_{12} \right) s\phi_{23} - \epsilon_{13} c\phi_{23} \right] s\phi_V - \frac{2}{3} z_S \frac{\bar{m}}{m_s} c\phi_{23} c\phi_V \right\},$$

$$g = 0.69 \pm 0.01 \text{ GeV}^{-1}, m_s/\bar{m} = 1.17 \pm 0.06,$$

$$\phi_{23} = (41.5 \pm 0.5)^\circ, \quad \phi_V = (4.0 \pm 0.2)^\circ,$$

$$\epsilon_{12} = (2.4 \pm 1.0)\%, \quad \epsilon_{13} = (2.5 \pm 0.9)\%,$$

$$z_{NS} = 0.89 \pm 0.03, \quad z_S = 0.77 \pm 0.04,$$

$$z_K = 0.90 \pm 0.03,$$

$$\epsilon = \epsilon_{\pi\eta} = (0.1 \pm 0.9)\%$$

$$\epsilon' = \epsilon_{\pi\eta'} = (3.5 \pm 0.9)\%$$

