Quarkonium hybrids

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Projects T2.2 and T2.3

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These two projects aim at the computation of correlators and potentials in lattice QCD to study quarkonia, quarkonium exotica and the quarkonium evolution in a medium.

Members:

Gert Aarts (Swansea University)
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Project T2.2: heavy-quark, hybrid and tetraquark potentials

Presentations:

- $\longrightarrow$ this talk,
- $\longrightarrow$ Sinéad Ryan's talk,
- $\longrightarrow$ related Nora Brambilla's talk,
- $\longrightarrow$ related Joan Soto's talk.

Project T2.2: heavy-quark, hybrid and tetraquark potentials

Bibliography (last 2 years):

- N. Brambilla, W. K. Lai, J. Segovia, J. Tarrus and A. Vairo
  Spin structure of heavy-quark hybrids
  Phys. Rev. D99 (2019) 014017 arXiv:1805.07713
- N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan
  The XYZ states: experimental and theoretical status and perspectives
  Phys. Rep. 873 (2020) 1 arXiv:1907.07583
- N. Brambilla, W. K. Lai, J. Segovia and J. Tarrus
  QCD spin effects in the heavy hybrid potentials and spectra
  Phys. Rev. D101 (2020) 054040 arXiv:1908.11699
- J. Soto and J. Tarrus
  Nonrelativistic effective field theory for heavy exotic hadrons
  Phys. Rev. D102 (2020) 014012 arXiv:2005.00552
- Hadron Spectrum Collaboration, S. Ryan et al.
  Excited and exotic bottomonium spectroscopy from lattice QCD
  arXiv:2008.02656

Project T2.3: matrix elements for in medium evolution

Presentations:

→ Viljami Leino's talk.

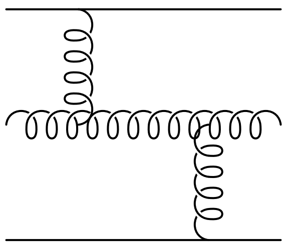
Bibliography (last 2 years):

- N Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend
  Transport coefficients from in medium quarkonium dynamics
  Phys. Rev. D100 (2019) 054025 arXiv:1903.08063
- G. Aarts, C. Allton, J. Glesaaen, S. Hands and B. Jäger
  Properties of the QCD thermal transition with $N_f = 2 + 1$ flavours of Wilson quark
  arXiv:2007.04188
- N. Brambilla, V. Leino, P. Petreczky and A. Vairo
  Lattice QCD constraints on the heavy quark diffusion coefficient
  Phys. Rev. D102 (2020) 074503 arXiv:2007.10078

Quarkonium hybrids

Quarkonium hybrids

Among the many new quarkonium states discovered at the B factories, some may be hybrids. A quarkonium hybrid consists of a heavy quark-antiquark pair in a color octet configuration bound with gluons.



Many approaches have been developed over the years to describe hybrids: constituent gluon picture, Born–Oppenheimer approximation, flux tube model ...

We argue that for hybrids made of heavy quarks combining effective field theories and lattice QCD provides a method to study these systems systematically from QCD. This is because the heavy quark mass m_h is the largest scale in the system:

- $m_h \gg p$
- $m_h \gg \Lambda_{\rm QCD}$

The non-relativistic expansion

- $m_h \gg p$ implies that quarkonia are non-relativistic and characterized by the hierarchy of scales typical of a non-relativistic bound state:

$$m_h \gg p \sim 1/r \sim m_h v \gg E \sim m_h v^2$$

Systematic expansions in the small heavy-quark velocity v may be implemented at the Lagrangian level by constructing suitable effective field theories (EFTs):

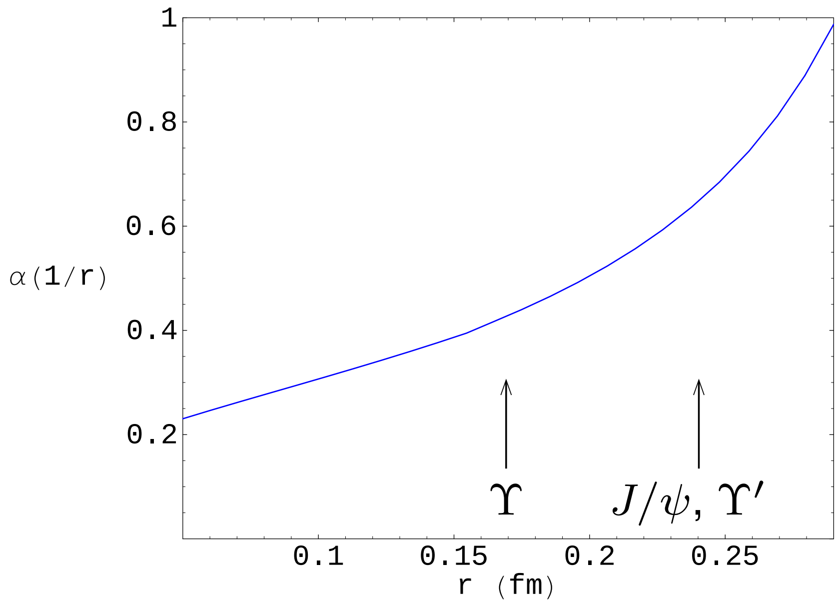
  - expanding QCD in $p, E/m_h$ leads to NRQCD
    ○ Bodwin Braaten Lepage PRD 51 (1995) 1125
  - expanding NRQCD in $E/p, 1/r$ leads to pNRQCD
    d.o.f.: $Q\bar{Q}$ color singlet and color octet fields, low-energy gluons
    ○ Brambilla Pineda Soto Vairo RMP 77 (2004) 1423

The hierarchy of non-relativistic scales makes the very difference of quarkonia with heavy-light mesons, which are just characterized by the two scales m_h and $\Lambda_{\rm QCD}$.

The perturbative expansion

- $m_h \gg \Lambda_{\rm QCD}$ implies $\alpha_{\rm s}(m_h) \ll 1$: phenomena happening at the scale m_h may be treated perturbatively.

  We may further have small couplings if $m_h v \gg \Lambda_{\rm QCD}$ and $m_h v^2 \gg \Lambda_{\rm QCD}$, in which case $\alpha_{\rm s}(m_h v) \ll 1$ and $\alpha_{\rm s}(m_h v^2) \ll 1$ respectively.
  This is likely to happen only for the lowest charmonium and bottomonium states.



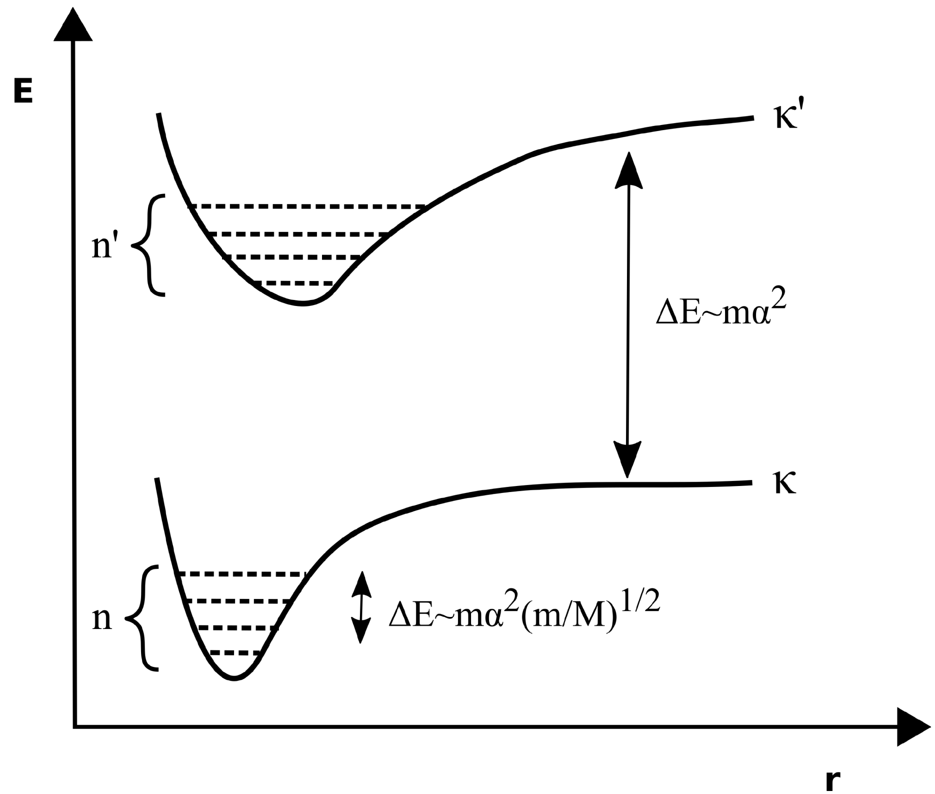
Born–Oppenheimer approximation

- Gluon fields change adiabatically in the presence of heavy quarks.
- The heavy quark interaction may be described at leading order in the non-relativistic expansion by an effective potential E_g between static sources, where g labels different excitations of the light degrees of freedom.
- E_g may be expressed in terms of matrix elements and computed in lattice QCD.
- A plethora of states can be built on each of the potentials E_g by solving the corresponding Schrödinger equation.

This picture goes also under the name of Born-Oppenheimer approximation.

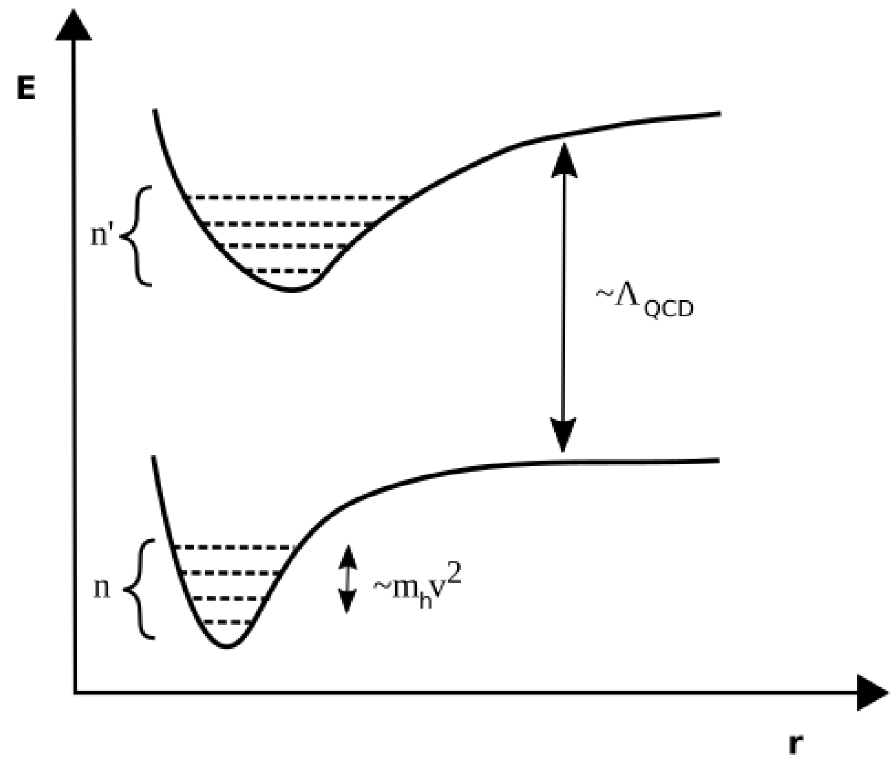
H_2^+-like molecule spectrum

In H_2^+-like molecules excitations of the electronic cloud are separated from each other by a gap of order $m\alpha^2$, while vibrational modes of the nucleus have an energy of order $m\alpha^2\sqrt{m/M}$, which is much smaller than $m\alpha^2$; $m =$ mass of e, $M =$ mass of nucleus.



Hybrid spectrum with a mass gap

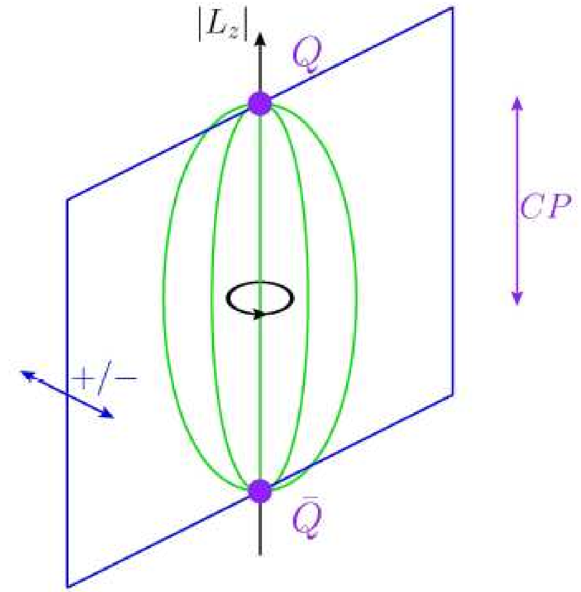
Ideally, gluonic (light-quark pair) excitations are separated from each other by a gap of order $\Lambda_{\rm QCD}$, while vibrational modes of a heavy quark-antiquark pair inside a given potential have an energy of order $m_h v^2$, which is assumed to be smaller than $\Lambda_{\rm QCD}$.



Symmetries

Static states classified by symmetry group $D_{\infty\, h}$
Representations labeled Λ^{σ}_{η}

- Λ rotational quantum number
  $|\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2 \ldots$ corresponds to
  $\Lambda = \Sigma, \Pi, \Delta \ldots$
- η eigenvalue of CP:
  $g \,\hat{=}\, +1$ (gerade), $u \,\hat{=}\, -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states
  (others are degenerate)



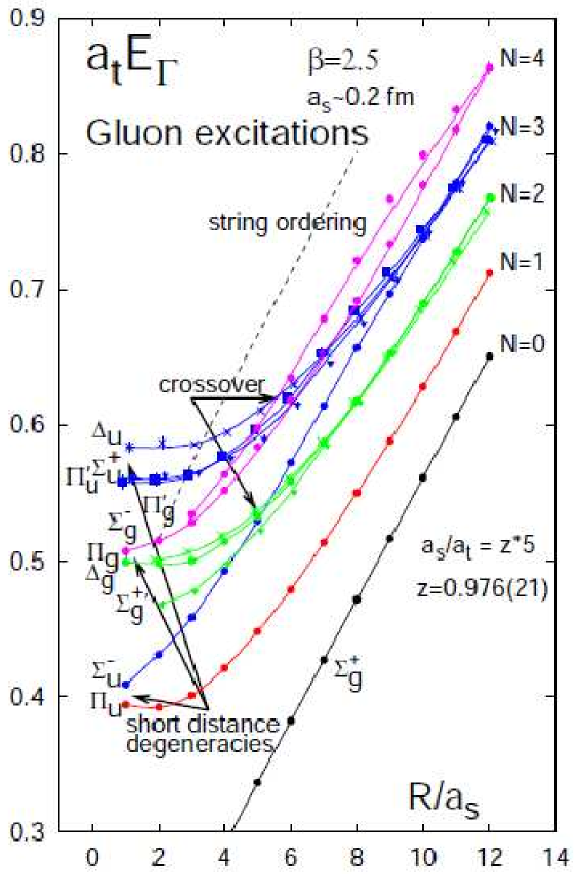
- The static energies correspond to the irreducible representations of $D_{\infty\, h}$.
- In general it can be more than one state for each irreducible representations of $D_{\infty\, h}$, usually denoted by primes, e.g. Π_u, Π'_u, $\Pi''_u \ldots$

In the limit $r \to 0$ more symmetry: $D_{\infty\, h} \;\to\; O(3) \times C$

- Several Λ^{σ}_{η} representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.

- Foster Michael PRD 59 (1999) 094509

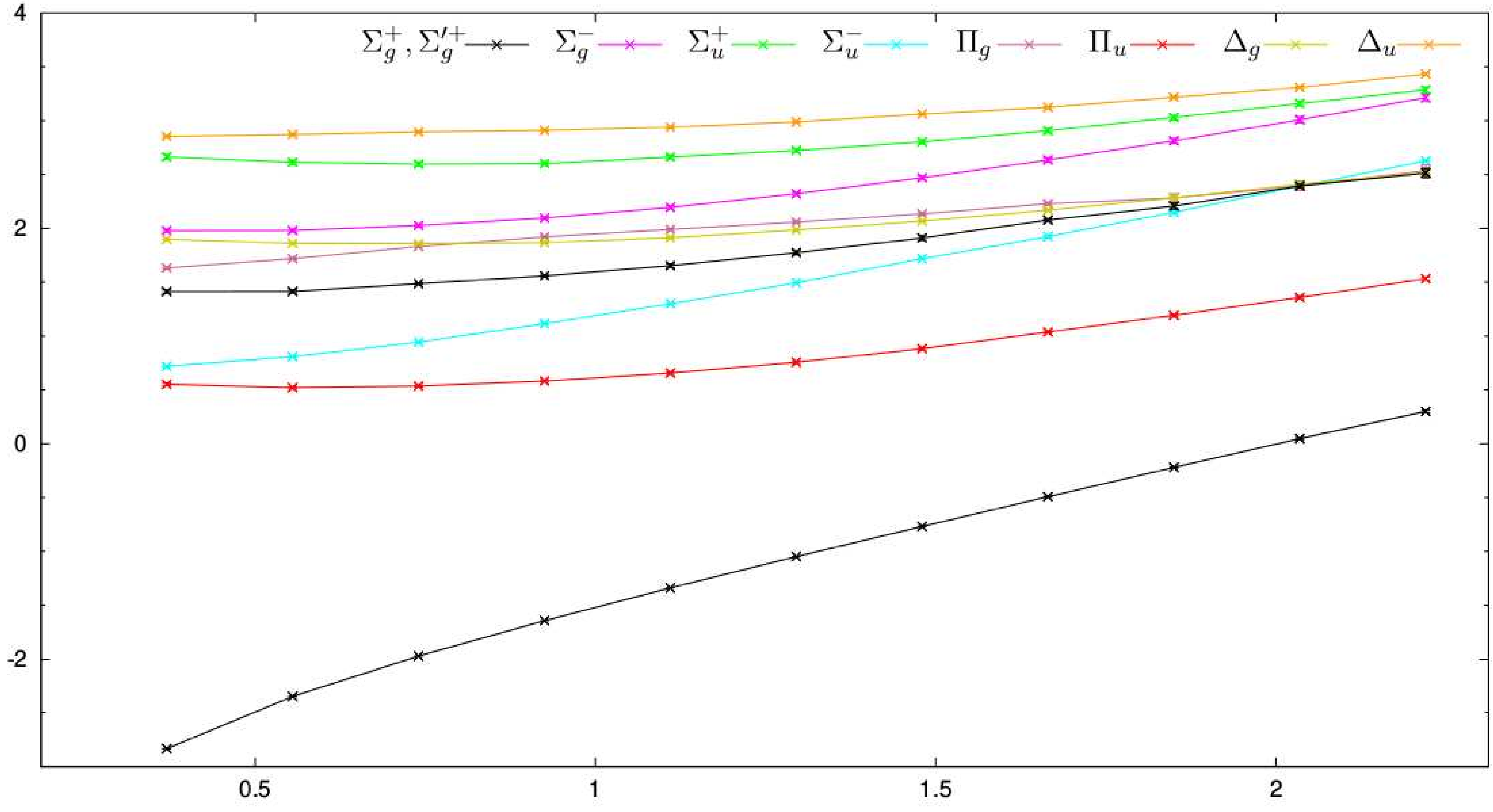
Static quenched lattice energies I



- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.

- Juge Kuti Morningstar PRL 90 (2003) 161601

Static quenched lattice energies II



unit $r_0 \approx 0.5$ fm

- Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 034502

State multiplets

We consider hybrids that are excitations of the lowest lying static energies Π_u and Σ_u^-.
In the $r \to 0$ limit Π_u and Σ_u^- are degenerate and correspond to a gluonic operator with quantum numbers 1^{+-}.

States are organized in spin multiplets.

Multiplet	T	$J^{PC}(S=0)$	$J^{PC}(S=1)$	E_Γ
H_1	1	1^{--}	$(0,1,2)^{-+}$	$E_{\Sigma_u^-}, E_{\Pi_u}$
H_2	1	1^{++}	$(0,1,2)^{+-}$	E_{Π_u}
H_3	0	0^{++}	1^{+-}	$E_{\Sigma_u^-}$
H_4	2	2^{++}	$(1,2,3)^{+-}$	$E_{\Sigma_u^-}, E_{\Pi_u}$

T is the sum of the orbital angular momentum of the quark-antiquark pair and the gluonic angular momentum; $T=0$ state turns out not to be the lowest mass state.

- Braaten PRL 111 (2013) 162003
  Braaten Langmack Smith PRD 90 (2014) 014044

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

$$\mathcal{L}_{\text{BOEFT for }1^{+-}} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{\Psi^{\dagger}_{1^{+-}\lambda}\left(i\partial_0 - V_{1^{+-}\lambda\lambda'}(r) + \hat{r}^{i\dagger}_{\lambda}\frac{\boldsymbol{\nabla}^2_r}{m_h}\hat{r}^{i}_{\lambda'}\right)\Psi_{1^{+-}\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}$.
- For the static potential: $V^{(0)}_{1^{+-}\lambda\lambda'} = \delta_{\lambda\lambda'}V^{(0)}_{1^{+-}\lambda}$, with $V^{(0)}_{1^{+-}0} = E_{\Sigma_u^-}$, $V^{(0)}_{1^{+-}\pm 1} = E_{\Pi_u}$.
- For potentials of higher order in $1/m_h$, there are no available lattice determinations. These potentials have been modeled either using the short-distance multipole expansion or both the short-distance multipole expansion and the long-range effective string theory.

◦ Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
Oncala Soto PRD 96 (2017) 014004
Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

Schrödinger equation for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

The LO e.o.m. for the fields $\Psi^{\dagger}_{1+-\lambda}$ are a set of coupled Schrödinger equations:

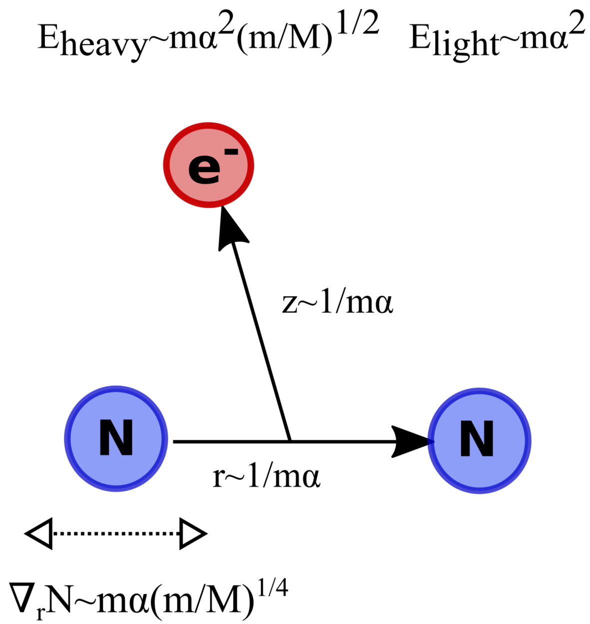
$$i\partial_0 \Psi_{1+-\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}_r^2}{m_h} + V^{(0)}_{1+-\lambda}\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\rm nad}_{1+-\lambda\lambda'}\right]\Psi_{\kappa\lambda'}$$

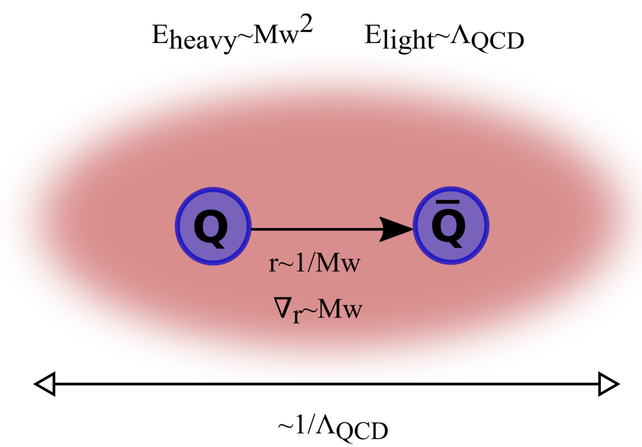
The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m_h + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_{\lambda}\left(\frac{\boldsymbol{\nabla}_r^2}{m_h}\right)\hat{r}^{i}_{\lambda'} = \delta_{\lambda\lambda'}\frac{\boldsymbol{\nabla}_r^2}{m_h} + C^{\rm nad}_{1+-\lambda\lambda'}$$

with $C^{\rm nad}_{1+-\lambda\lambda'} = \hat{r}^{i\dagger}_{\lambda}\left[\frac{\boldsymbol{\nabla}_r^2}{m_h}, \hat{r}^{i}_{\lambda'}\right]$ called the nonadiabatic coupling.

Power counting





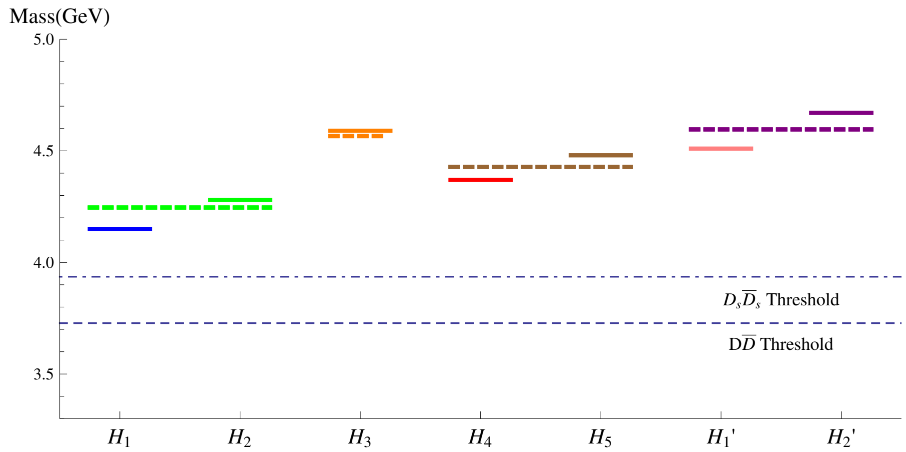
For the different power counting, the adiabatic coupling may be treated as a perturbation in diatomic molecules, while it may contribute at LO to heavy quarkonium hybrids, being of the same order as the temporal derivative, the kinetic energy and the static potential.

Spectrum: general consideration

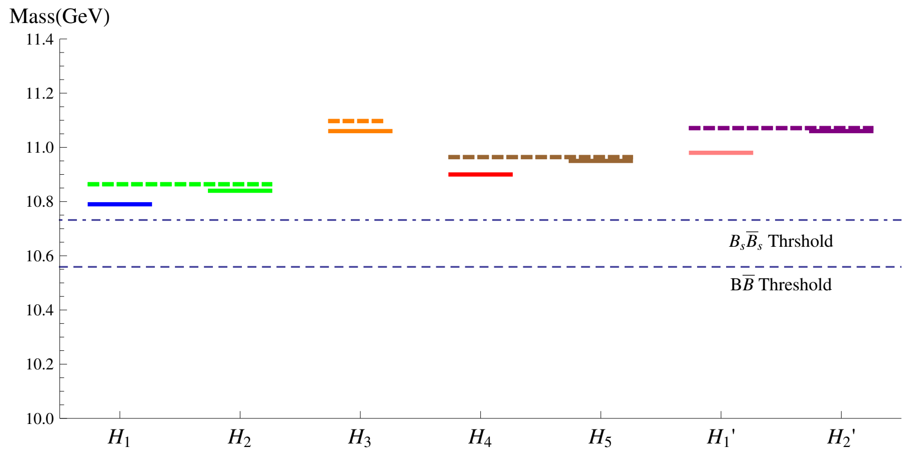
- The Schrödinger equation mixes states with the same parity.
  A consequence is Λ-doubling, i.e., the lifting of degeneracy between states with opposite parity. This happens also in molecular physics, but there Λ-doubling is a subleading effect, while it may be a LO effect in the quarkonium hybrid spectrum.
- The eigenstates are organized in the multiplets H_1, H_2, ... Neglecting off-diagonal terms, the multiplets H_1 and H_2 would be degenerate.
- We compute the spectrum using quark masses in the renormalon subtraction (RS) scheme: $m_{c\,\mathrm{RS}} = 1.477(40)$ GeV and $m_{b\,\mathrm{RS}} = 4.863(55)$ GeV.

  The gluelump masses, which enter in the normalization of the hybrid potentials, have been computed in the same scheme and assigned an uncertainty of ± 0.15 GeV, which is the largest source of uncertainty in the hybrid masses.

Spectrum: with mixing and Λ-doubling



charmonium hybrids

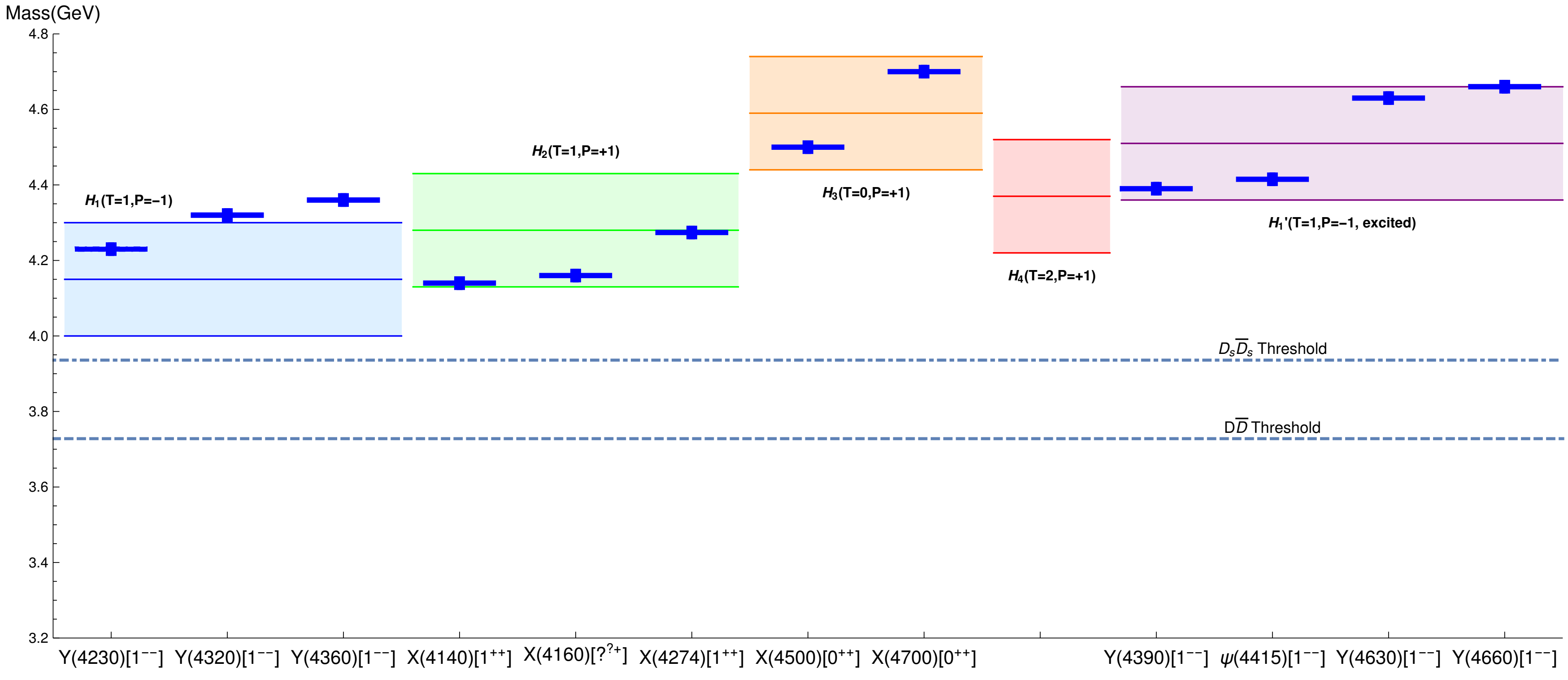


bottomonium hybrids

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019

  data without mixing (dashed) from Braaten et al PRD 90 (2014) 114044

Quarkonium hybrid states vs experiments I

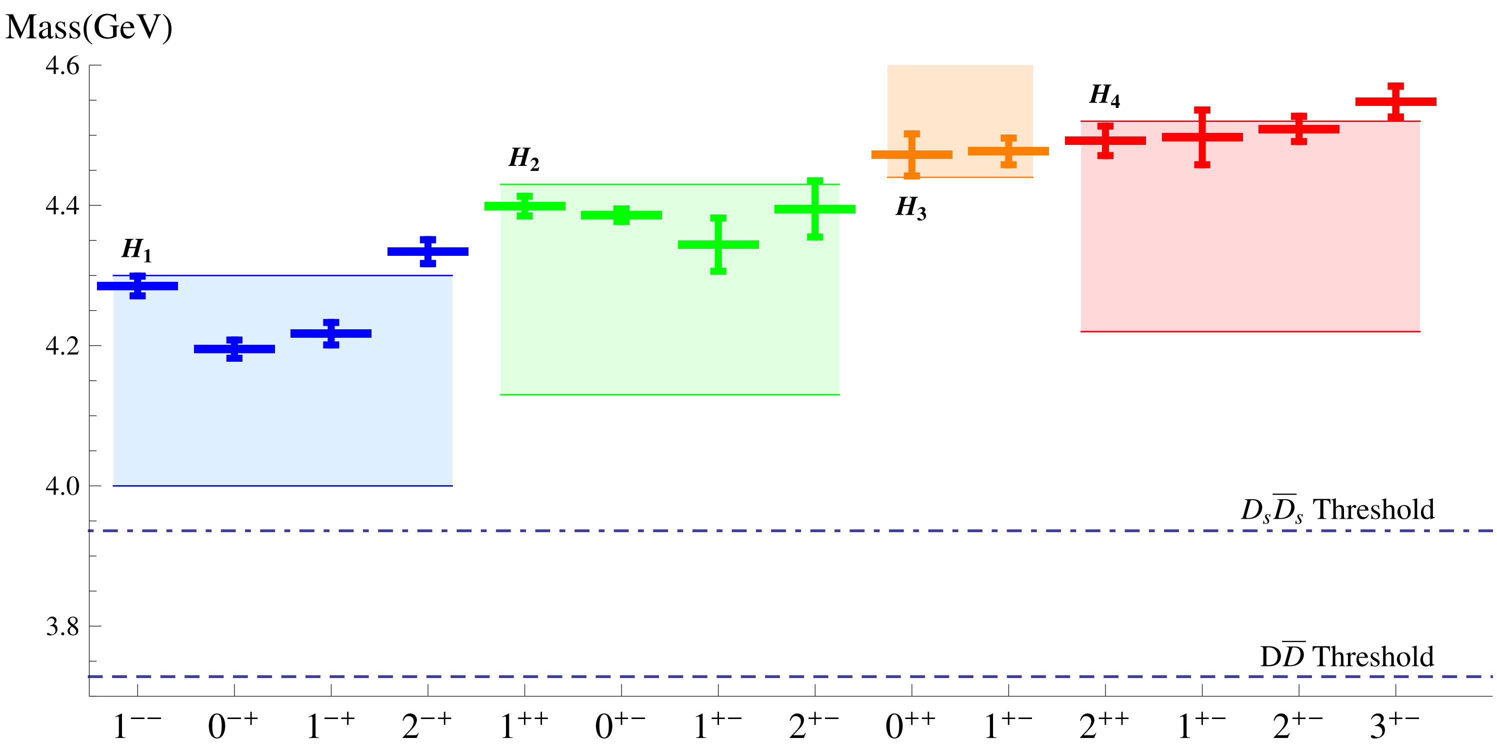


- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019 updated in Brambilla Eidelman Hanhart Nefediev Shen Thomas Vairo Yuan Phys. Rep. 873 (2020) 1

Quarkonium hybrid states vs experiments II

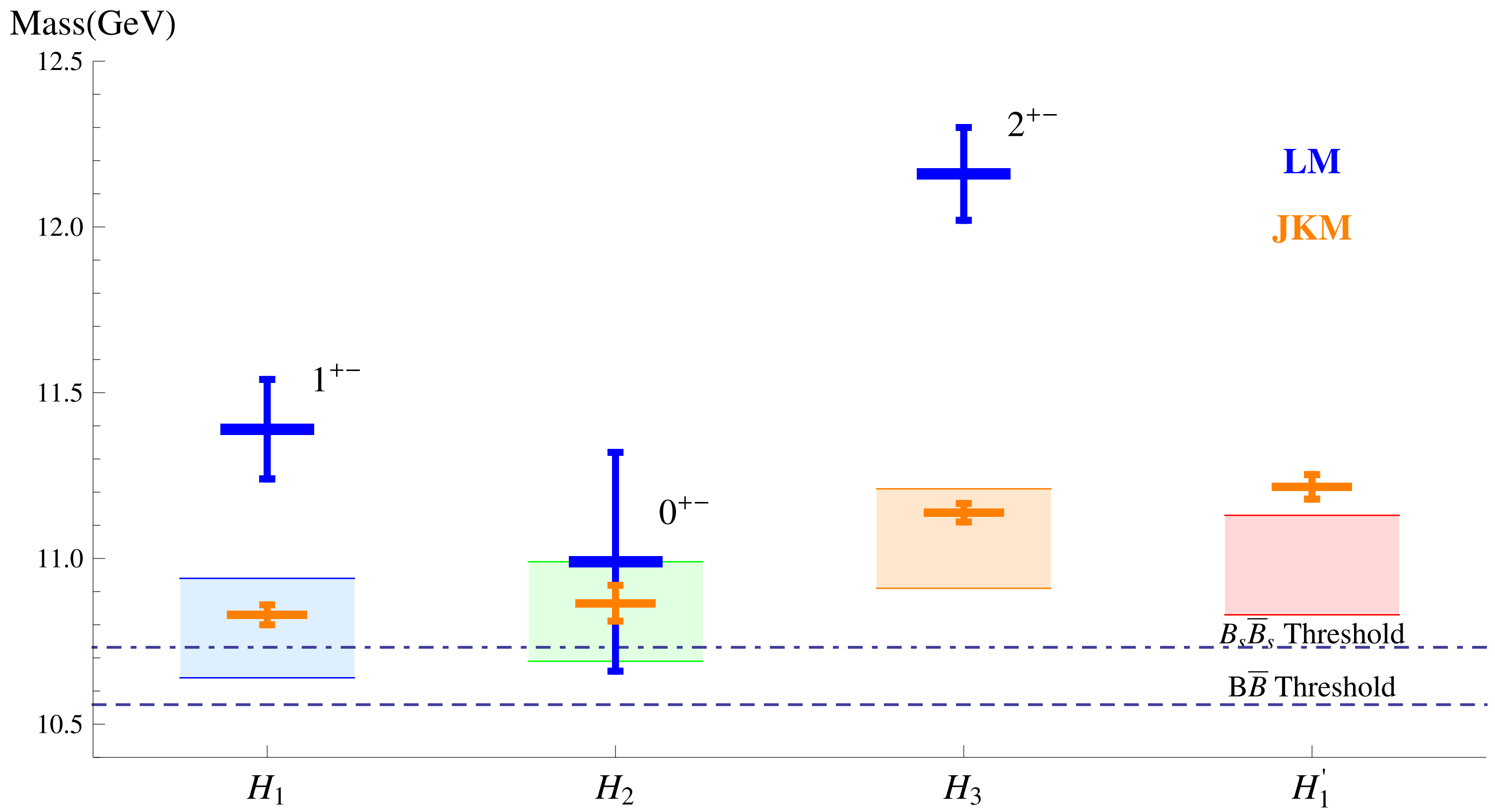
- Promising candidates for charmonium hybrids or for states with a large hybrid component are the $Y(4230)$ and $Y(4390)$ because of their significant width into $\pi^+\pi^- h_c$. This decay does not need spin flipping of the heavy quark-antiquark pair, which is in a spin zero state. Spin-flipping terms are suppressed in the heavy quark limit. Nevertheless, mixing with spin one quarkonium states happens already at order $\Lambda^2_{\text{QCD}}/m_h$. This possibly large mixing may allow for significant widths also into final states with spin one quarkonia, in particular $\pi^+\pi^- J/\psi$.
  ○ `Oncala Soto PRD 96 (2017) 014004`
- From the experimental side, candidate states of bottomonium hybrids in the H_1 or H_1' multiplets are
  the $\Upsilon(10860)$ $[1^{--}]$, with a mass of $M_{\Upsilon(10860)} = (10891.1 \pm 3.2^{+0.6}_{-1.7})$ MeV and
  the $\Upsilon(11020)$ $[1^{--}]$, with a mass of $M_{\Upsilon(11020)} = (10987.5^{+6.4}_{-2.5}{}^{+9.0}_{-2.1})$ MeV.
  ○ `Belle coll PRD 93 (2016) 011101`
  To these we can add the recently observed signal by Belle with a mass of $M_{\Upsilon(10750)} = (10752.7 \pm 5.9\,^{+0.7}_{-1.1})$ MeV, which may also qualify as an H_1 multiplet bottomonium hybrid candidate.
  ○ `Belle coll JHEP 10 (2019) 220`

Charmonium hybrid states vs direct lattice data



- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  lattice data from the Hadron Spectrum coll JHEP 1207 (2012) 126
  [2+1 flavors, $m_\pi = 400$ MeV]

Bottomonium hybrid states vs direct lattice data



- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019

  lattice data from:

  Juge Kuti Morningstar PRL 82 (1999) 4400 [quenched NRQCD]

  Liao Manke PRD 65 (2002) 074508 [quenched QCD]

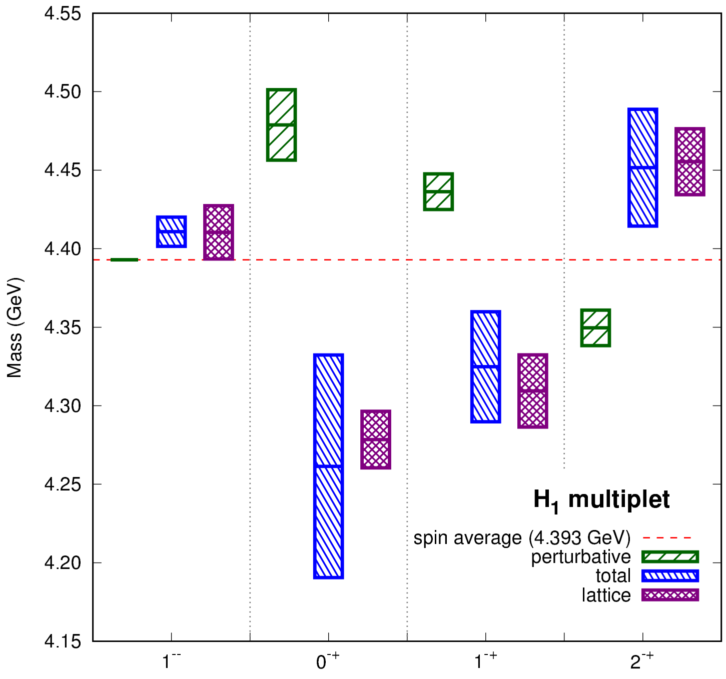
Hybrid spin-dependent potentials at order $1/m_h$ and $1/m_h^2$

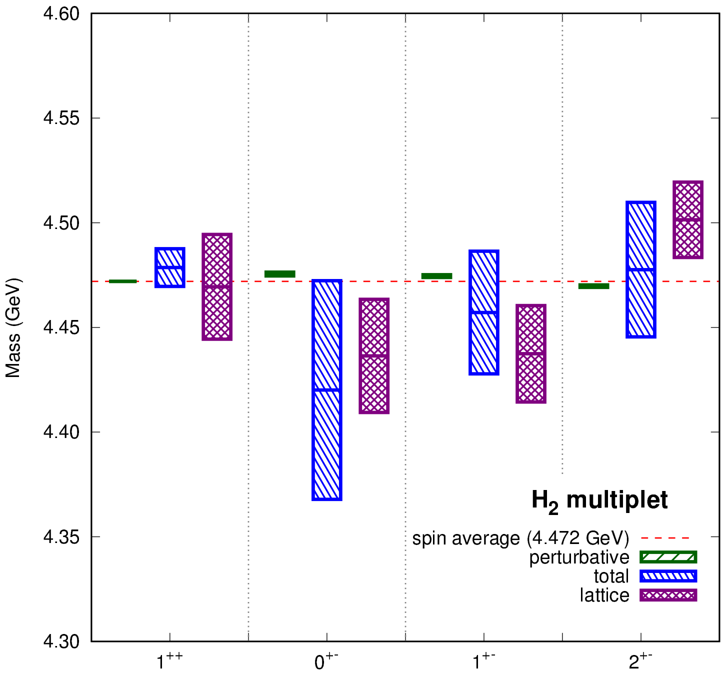
$$V^{(1)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S}$$
$$+ V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$
$$V^{(2)}_{1^{+-}\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j} + S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'}$$
$$+ V^{(2)}_{LS\,c}(r)\left[\hat{r}_{\lambda}\cdot\boldsymbol{r}\,(\boldsymbol{p}\times\boldsymbol{S})\cdot\hat{r}_{\lambda'} + \hat{r}_{\lambda}\cdot(\boldsymbol{p}\times\boldsymbol{S})\,\hat{r}_{\lambda'}\cdot\boldsymbol{r}\right]$$
$$+ V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2} + S^{i}_{2}S^{j}_{1}\right)$$

$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons
and $\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m_h$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

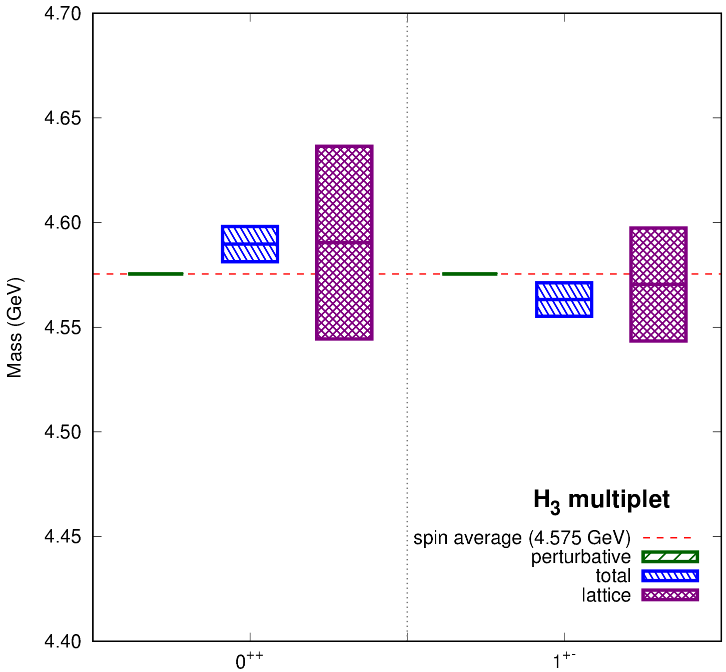
Charmonium hybrid spin splittings

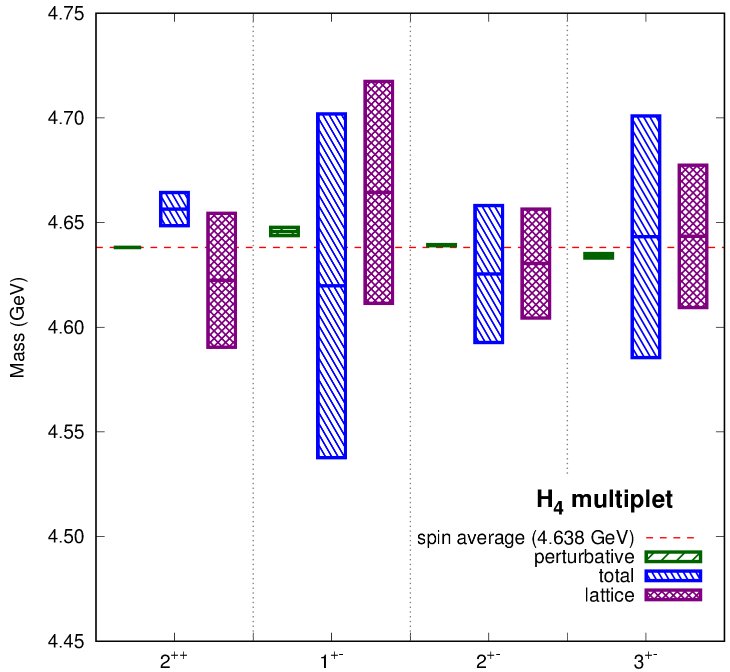




- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data from Liu et al JHEP 1612 (2016) 089
  [2+1 flavors, $m_\pi = 240$ MeV]

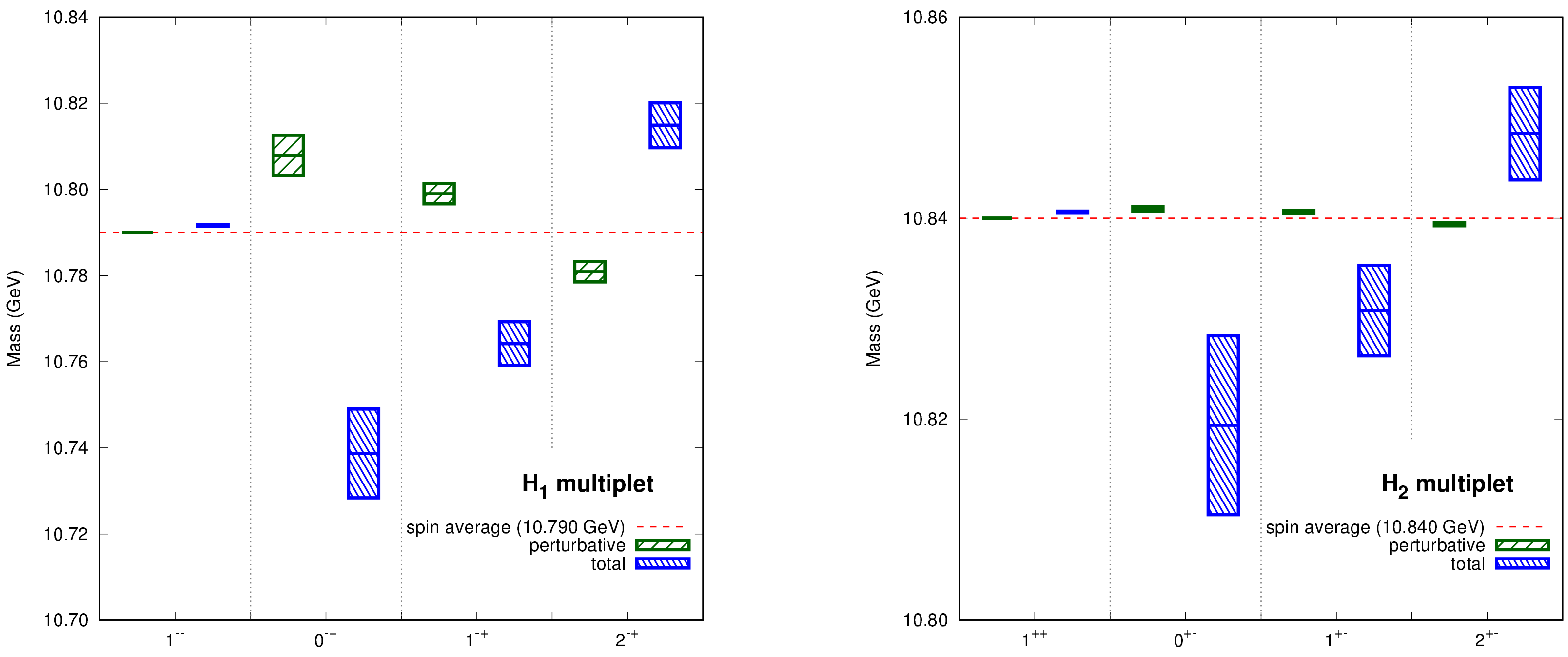
Charmonium hybrid spin splittings





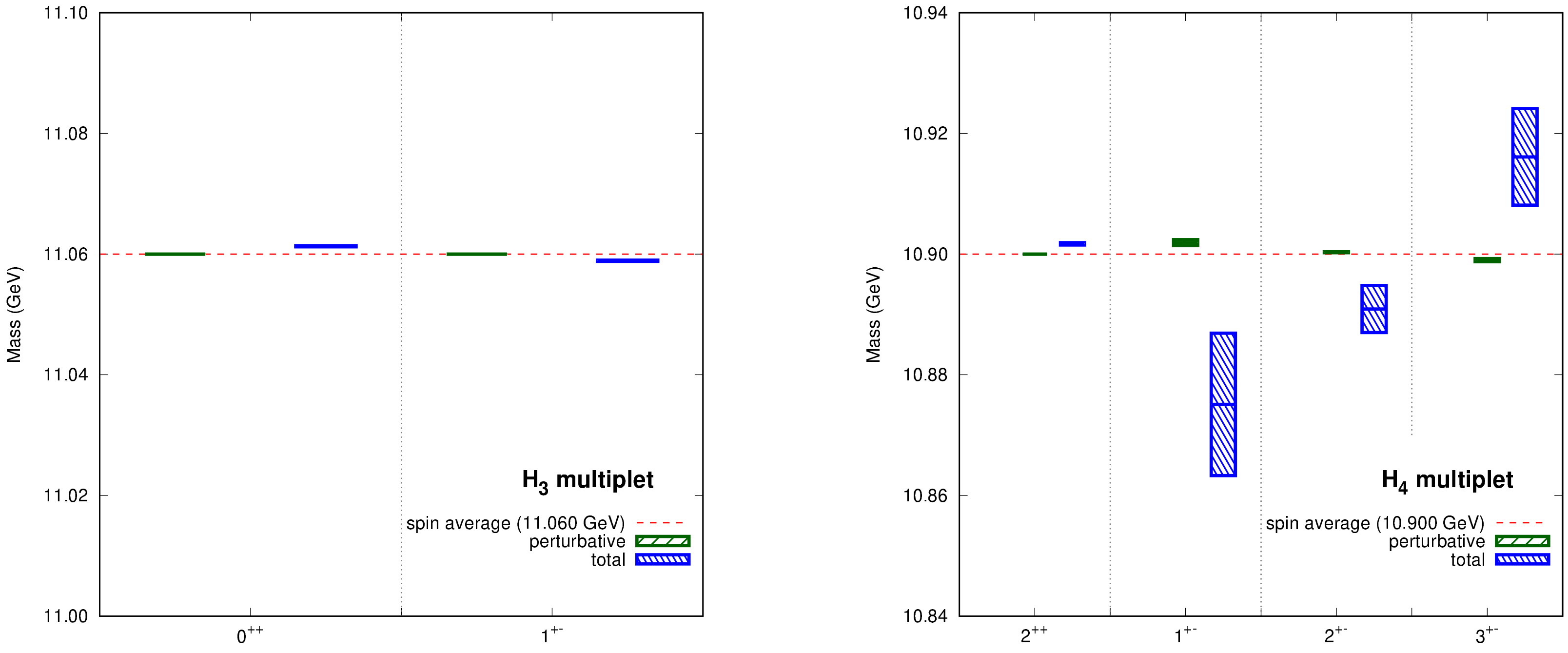
- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data from the Hadron Spectrum coll JHEP 1612 (2016) 089
  [2+1 flavors, $m_\pi = 240$ MeV]

Bottomonium hybrid spin splittings



- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

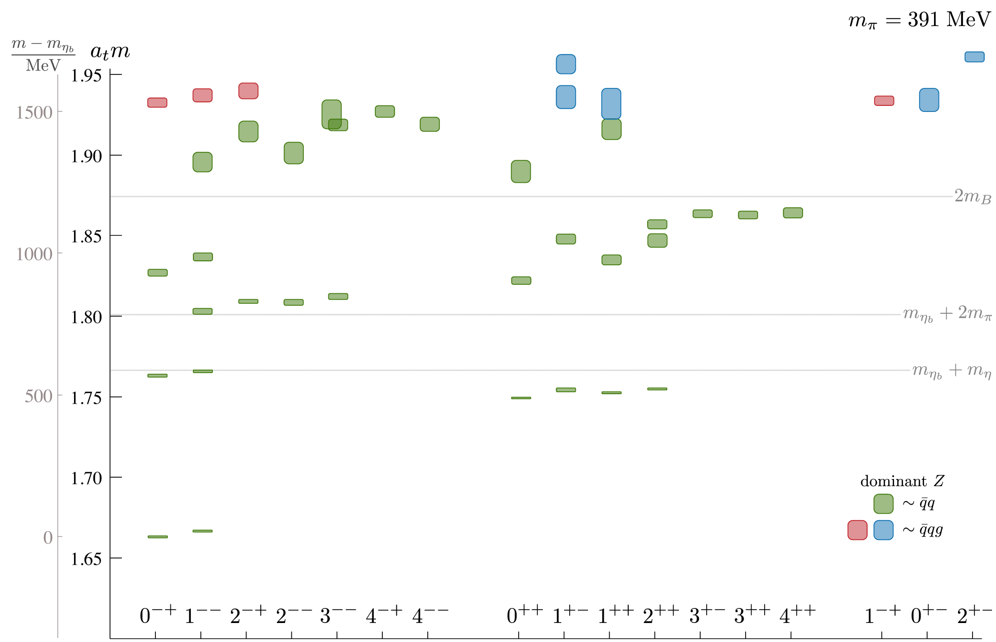
Bottomonium hybrid spin splittings



- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

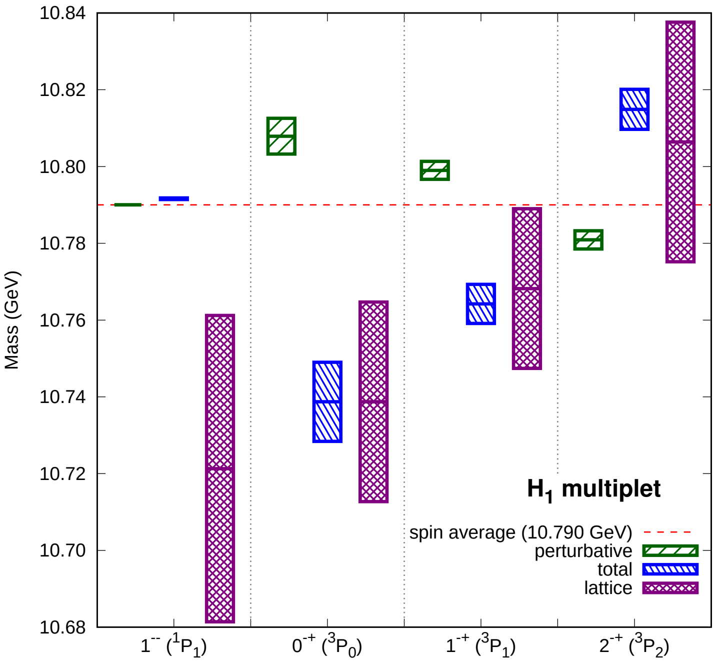
$N_f = 2 + 1$ relativistic bottomonium spectrum from lattice QCD

For the first time the excited and exotic bottomonium spectrum (including hybrids) has been determined by a fully relativistic and unquenched [2+1 flavors, $m_\pi = 391$ MeV] lattice calculation.



- Hadron Spectrum coll, Ryan et al, arXiv:2008.02656

Bottomonium hybrid spin splittings vs 2 + 1 flavors lattice QCD



- preliminary plot by J. Segovia

Conclusions

Conclusions

- Hybrids are one of the most specific exotics expected from QCD.
  Their experimental hunt and theoretical study has a history characterized for long time by few and difficult progresses. This may change with heavy quarkonium hybrids.
- The quarkonium spectrum above threshold has been extensively investigated by experiments in the last decade providing an enormous wealth of new and unexpected states and many new data. Some of these are undoubtedly exotic. Some may indeed be hybrids made of two heavy quarks.
- Effective field theories for heavy quarkonia combined with lattice QCD offer a systematic and rigorous tool (based on the non-relativistic hierarchy of energy scales) to study many of the quarkonium hybrid properties from QCD: spectra, splittings, decays.

Outlook

Several correlators and potentials may be computed in lattice QCD that would integrate the EFT description of hybrids and similarly for other exotica like tetraquarks.

If one had to mention just two quantities, these would be

- the leading hybrid spin potential V_{SK} that requires the computation on the lattice of a Wilson loop with three chromomagnetic field insertions (one each on the two spatial Wilson lines and one on a temporal Wilson line);
- the leading mixing term between hybrids and quarkonia that requires the computation on the lattice of a Wilson loop with two chromomagnetic field insertions (one each on a spatial Wilson line and a temporal Wilson line).