T2.3: Computation of matrix elements for in medium quarkonium evolution

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General Description

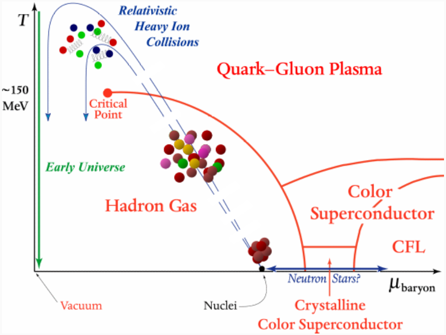
Deal with quarkonium propagation and suppression in the medium formed in heavy-ion collisions. The quarkonium low-energy out of equilibrium dynamics is governed by in-medium correlators, which we aim to compute in lattice QCD.

Related recent publications:

- N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend,
  Transport coefficients from in medium quarkonium dynamics,
  Phys. Rev. D **100** (2019) no.5, 054025 hep-ph/1903.08063.
- G. Aarts, C. Allton, J. Glesaaen, S. Hands, B. Jäger *et al.*
  Properties of the QCD thermal transition with $N_f = 2 + 1$ flavours of Wilson quark,
  hep-lat/2007.04188.
- N. Brambilla, V. Leino, P. Petreczky and A. Vairo,
  Lattice QCD constraints on the heavy quark diffusion coefficient,
  Phys. Rev. D **102** (2020) no.7, 074503 hep-lat/2007.10078.
- N. Brambilla, M. Á. Escobedo, M. Strickland, A. Vairo, P. Vander Griend, J. H. Weber,
  *Bottomonium suppression in an open quantum system using the quantum trajectories method,*hep-ph/2012.01240.

Introduction: QCD and QGP

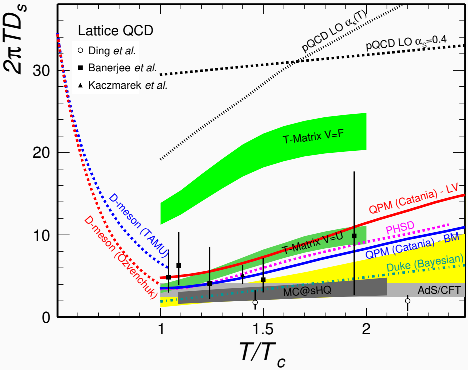
- Heavy quarks and their bound states (Quarkonium) are great probes for the Quark Gluon Plasma (QGP)
- QGP generated relativistic heavy ion collisions.



- Want to predict the experimental quantity: nuclear modification factor R_{AA}
- The QGP can be described in terms of transport coefficients
- R_{AA} related to heavy quark momentum diffusion coefficient κ (and dispersive counterpart γ)

Diffusion Coefficient

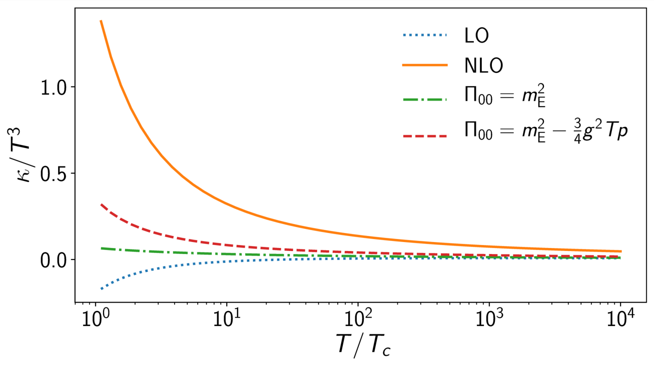
- We don't have R_{AA} and ν_2 directly in our theories, instead different (hydrodynamical) models depend on spatial diffusion coefficient D_x
- Observed ν_2 is larger than expected from kinetic models but in good agreement with hydrodynamic models



- Indicates the medium has fluidlike properties
- R_{AA} tells how heavy quarks see the nuclear medium

Figure from: X. Dong CIPANP (2018)

κ from perturbation theory



- Clearly $m_{\mathrm{E}} \ll T$ is too strict assumption on small T
- Huge perturbative variation
  $\Rightarrow$ needs non-perturbative measurements
- Also huge scale dependence trough $m_{\mathrm{E}} = g(\mu)T$
- Here we have scale from NLO EQCD $\mu \sim 2\pi T$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Heavy Quark in medium

- Heavy quark energy doesn't change much in collision with a thermal quark

$$E_k \sim T\,, \quad p \sim \sqrt{MT} \gg T$$

- HQ momentum is changed by random kicks from the medium
  → Brownian motion; Follows Langevin dynamics

$$\frac{dp_i}{dt} = -\frac{\kappa}{2MT} p_i + \xi_i(t)\,, \quad \langle \xi(t)\xi(t') \rangle = \kappa\delta(t-t')$$

- Heavy quark momentum diffusion coefficient κ related to many interesting phenomena

Such as: Spatial diffusion coefficient $D_{\mathrm{s}} = 2T^2/\kappa$,
Drag coefficient $\eta_{\mathrm{D}} = \kappa/(2MT)$,
Heavy quark relaxation time $\tau_{\mathrm{Q}} = \eta_{\mathrm{D}}^{-1}$

Moore and Teaney PRC71 (2005), Caron-Huot and Moore JHEP02 (2008)

Quarkonium in medium

- Quarkonium in strongly interacting medium (environment energy scale πT)

$$M \gg \frac{1}{a_0} \gg \pi T \gg E\,, \quad \tau_R \gg \tau_E \sim 1/\pi$$

- HQ mass M, Bohr radius a_0, binding energy E, correlation time τ_E
- Quarkonium in fireball can be described by Limbland equation

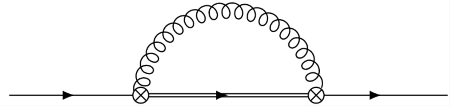
$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \sum_{n,m} h_{nm} \left(L_i^n \rho L_i^{m\dagger} - \frac{1}{2}\{L_i^{m\dagger} L_i^n, \rho\} \right)$$

- All terms depend on two free parameters κ and γ
- κ turns out to be the heavy quark diffusion coefficient
- γ is correction to the heavy quark-antiquark potential

Brambilla *et.al.*PRD96 (2017), Brambilla *et.al.*PRD97 (2018)

The singlet self-energy in pNRQCD

- Studying diagram one can find:



$$\kappa = \frac{1}{6N_c}\int_0^\infty \mathrm{d}t\,\langle\{gE^{a,i}(t,0)gE^{a,j}(0,0)\}\rangle$$
$$\gamma = \frac{-i}{6N_c}\int_0^\infty \mathrm{d}t\,\langle[gE^{a,i}(t,0)gE^{a,j}(0,0)]\rangle$$

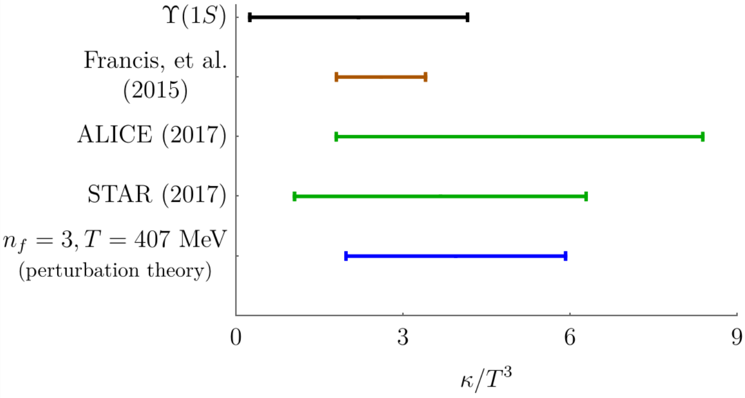
- The self-energies provide the in medium induced mass shifts δM_s and widths Γ_s
- For 1S Coulombic quarkonium state:

$$\Gamma(1S) = 3a_0^2\kappa \qquad \delta M(1S) = \frac{3}{2}a_0^2\gamma$$

- Separate way of lattice measurement Brambilla *et.al.*PRD100 (2019)

Brambilla *et.al.*PRD96 (2017), Brambilla *et.al.*PRD97 (2018)

Un-quenched κ Result

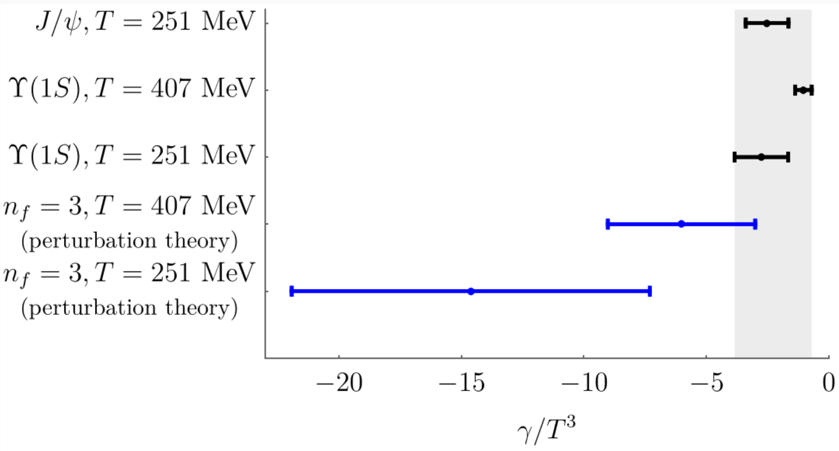


$$0.24 \lesssim \frac{\kappa}{T^3} \lesssim 4.2$$

- Using data from (Kim *et.al.*JHEP11 (2018)) and (Aarts *et.al.*JHEP11 (2011))
- Un-quenched determination of κ from lattice data

Brambilla *et.al.*Phys. Rev. D **100** (2019) no.5, 054025 hep-ph/1903.08063

Un-quenched γ Results



$$-3.8 \lesssim \frac{\gamma}{T^3} \lesssim -0.7$$

- Using data from (Kim *et.al.*JHEP11 (2018))
- First non-perturbative determination of γ

Brambilla *et.al.*Phys. Rev. D **100** (2019) no.5, 054025 hep-ph/1903.08063

Heavy quark diffusion from lattice

- Traditional approach using current correlators has transport peak
- HQEFT inspired Euclidean correlator free of transport peaks

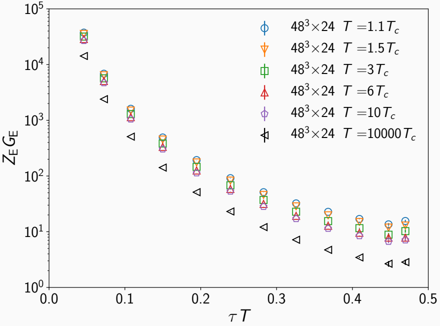
$$G_{\mathrm{E}}(\tau) = -\sum_{i=1}^{3} \frac{\langle \mathrm{Re\,Tr}\,[U(1/T,\tau)E_i(\tau,0)U(\tau,0)E_i(0,0)]\rangle}{3\,\langle \mathrm{Re\,Tr}\,U(1/T,0)\rangle}$$

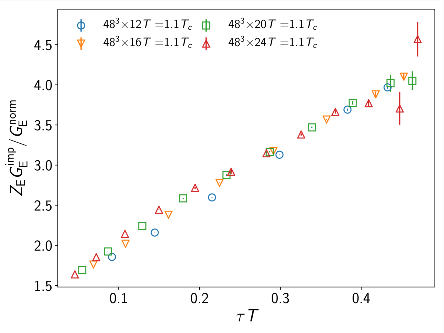
- To get momentum diffusion coefficient κ, a spectral function $\rho(\omega)$ needs to be reversed:

$$G_{\mathrm{E}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\rho(\omega,T)K(\omega,\tau T)\,, \qquad K(\omega,\tau T) = \frac{\cosh\left(\frac{\omega}{T}\left(\tau T - \frac{1}{2}\right)\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

$$\kappa = \lim_{\omega\to 0}\frac{2T\rho(\omega)}{\omega}$$

- Measure using the multilevel algorithm in pure gauge (quenched)
- Compared to earlier studies we measure extremely wide range of temperatures
- Create model $\rho(\omega)$ by matching to perturbation theory at high T
- Invert the spectral function equation by varying the model ρ



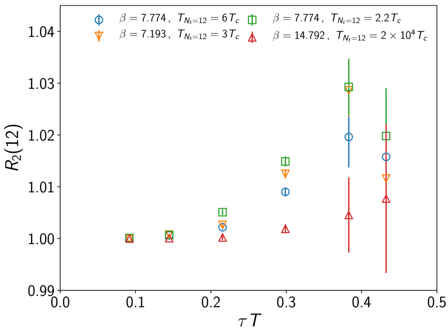


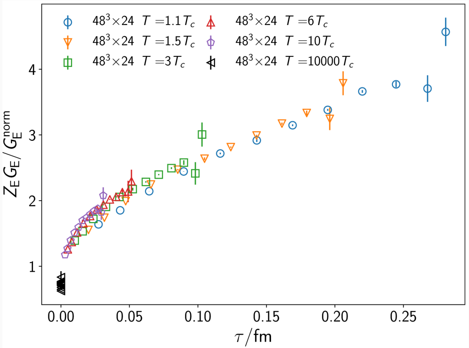
- Normalize lattice data with the LO Perturbative result:

$$G_{\mathrm{E}}^{\mathrm{norm}} = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

- Perform tree-level improvement by matching lattice and continuum perturbation theories

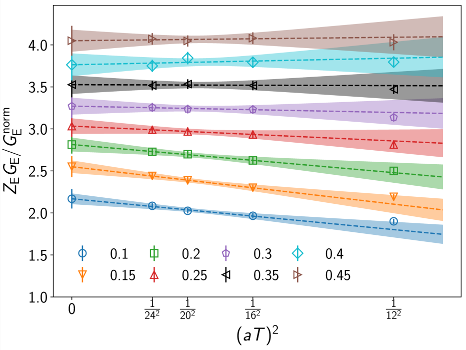
When do thermal effects start

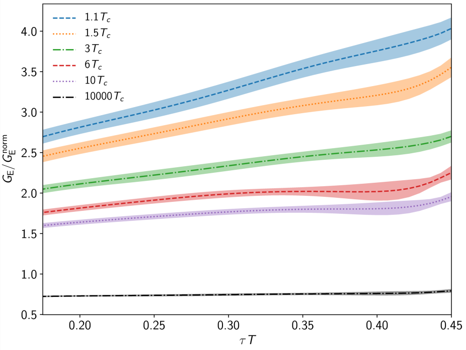




$$R_2(N_t) = \frac{G_E(N_t, \beta)}{G_E^{norm}(N_t)} \Bigg/ \frac{G_E(2N_t, \beta)}{G_E^{norm}(2N_t)}\,.$$

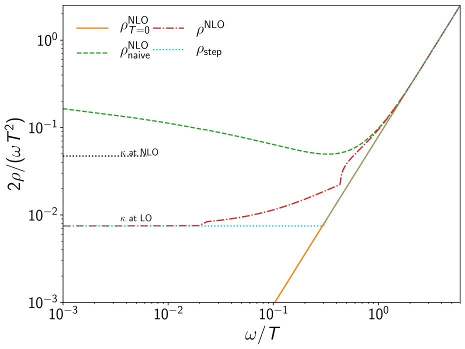
- On small physical separation every T shares a scaling (apart from finite size effects)
- Thermal effect nonexistent for $\tau < 0.10$, then grow

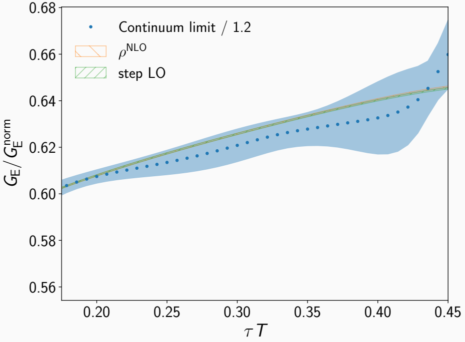




- Use 3 largest lattices for continuum limit
- Systematic include tadpole and extrapolations with and without $N_t = 12$ point or a^4 term.
- Finite size effects are in control, we can go without extrapolation.

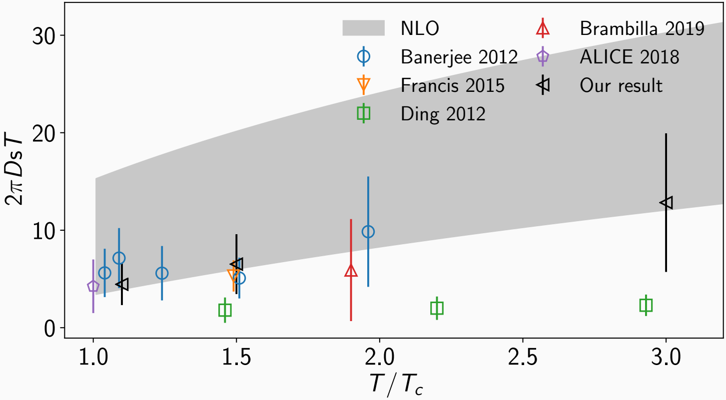
High Temperature





- NLO spectral function works only at very high temperatures
- Different ansatze have different $\omega \sim T$ behavior
- Good matching between perturbation theory and lattice

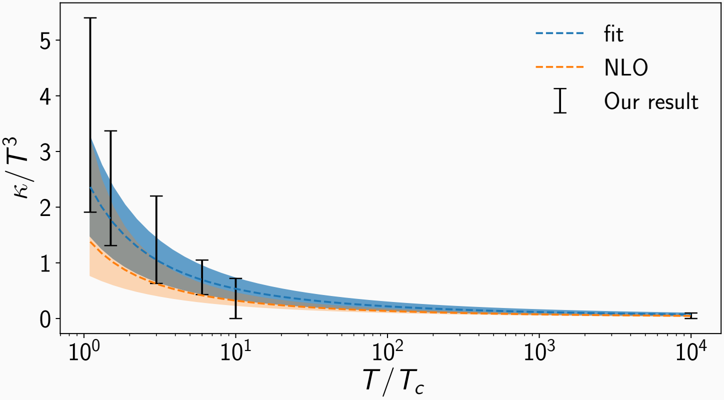
Lattice results for D_s



- On low temperature close to T_c, agreement with other results, including ALICE

Brambilla et al 2020: Accepted to PRD, hep-lat/2007.10078

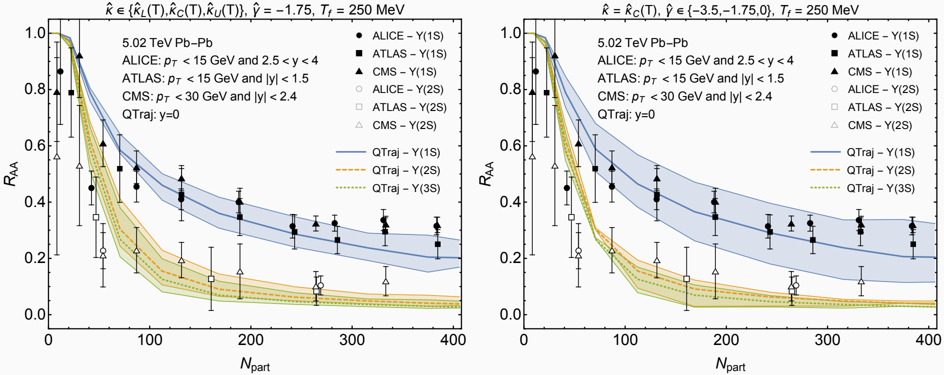
Lattice results for κ



- Unprecedented temperature range: $T = 1.1 - 10^4\,T_\mathrm{c}$

$$\frac{\kappa^{\mathrm{NLO}}}{T^3} = \frac{g^4 C_\mathrm{F} N_\mathrm{c}}{18\pi}\left[\ln\frac{2T}{m_\mathrm{E}} + \xi + C\frac{m_\mathrm{E}}{T}\right].$$

- Can fit temperature dependence $C = 3.81(1.33)$



- Developed a new program for estimating R_{AA}
- Use the temperature dependence of κ from previous slide

Conclusions and Future prospects

- Measurement of unquenched kappa possible indirectly from lattice
- We have measured quenched κ in wide range of temperatures and fitted the temperature dependence
- κ (and γ) are major source of uncertainty for R_{AA}
- Where to go from now:
  - γ needs to be measured from lattice
  - Can unquenched κ be measured directly on lattice $\Rightarrow$ Gradient flow
  - $1/M$ corrections to κ can be calculated

Thank You