HEAVY QUARKONIUM PRODUCTION IN PNRQCD

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Based on
Brambilla, Chung, Müller, Vairo, JHEP 04 (2020) 095
Brambilla, Chung, Vairo, arXiv:2007.07613 [hep-ph]

Theoretical aspects of Hadron Spectroscopy and Phenomenology
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OUTLINE

- Quarkonium production in NRQCD and pNRQCD
- Exclusive electromagnetic production and decay
- Inclusive hadroproduction of quarkonia at the LHC
- Summary and outlook

PRODUCTION OF QUARKONIUM

- Heavy quarkonia can be described from nonrelativistic EFTs, which are based on the hierarchy of scales $m \gg mv \gg mv^2$
- NREFTs provide a factorization formalism for quarkonium production, where cross sections are given by perturbative short-distance coefficients times nonperturbative matrix elements.
- Decay rates and exclusive electromagnetic production provide precision tests of the NREFT formalism.
- Inclusive production processes are expected to be useful probes for various areas of QCD such as the QGP.

NRQCD FACTORIZATION

- NRQCD provides a factorization formalism for decay rates and production cross sections.

NRQCD matrix elements

$$\sigma = \sum_n \sigma_{Q\bar{Q}(n)} \langle \Omega | \mathcal{O}_n | \Omega \rangle$$

Perturbatively calculable *short-distance coefficients*

Bodwin, Braaten, Lepage, PRD51 (1995) 1125

- Both perturbative short-distance cross sections and nonperturbative matrix elements are needed to make predictions.
- The matrix elements have scalings in powers of v.
- Generally, a *limited class of color-singlet matrix elements* have been computed from potential models and quenched lattice QCD.
- We aim to ***compute the matrix elements in potential NRQCD***, which is obtained by integrating out scales above mv^2.

QUARKONIUM IN PNRQCD

- We work in the strong coupling regime where $mv^2 \ll \Lambda_{\rm QCD}$, which is valid for non-Coulombic quarkonia, such as P-wave quarkonia. The degree of freedom is the singlet field $S(x_1,x_2)$, which describe $Q\bar{Q}$ in a color-singlet state.

$$\mathcal{L}_{\rm pNRQCD} = {\rm Tr}\{S^\dagger(i\partial_0 - h)S\}$$

Pineda, Soto, NPB Proc. Suppl. 64 (1998) 428
Brambilla, Pineda, Soto, Vairo, NPB566 (2000) 275
Brambilla, Pineda, Soto, Vairo, Rev. Mod. Phys. 77 (2005) 1423

- Matching to NRQCD is done nonperturbatively in expansion in powers of $1/m$.

- This provides expressions for matrix elements in terms of wavefunctions at the origin and universal gluonic correlators.

DECAY / ELECTROMAGNETIC PRODUCTION IN PNRQCD

NRQCD MATRIX ELEMENTS

- Decay and exclusive electromagnetic production matrix elements are expectation values on quarkonium states.

***Decay into light hadrons*:**

$$\langle \mathcal{Q} | \psi^\dagger \mathcal{K} \chi \chi^\dagger \mathcal{K}' \psi | \mathcal{Q} \rangle$$

***Electromagnetic production and decay*:**

$$\langle \mathcal{Q} | \psi^\dagger \mathcal{K} \chi | \Omega \rangle \langle \Omega | \chi^\dagger \mathcal{K}' \psi | \mathcal{Q} \rangle$$

color/spin matrices and covariant derivatives

- Potential NRQCD provides expressions for these matrix elements in terms of wavefunctions at the origin and universal gluonic correlators.

- The gluonic correlators arise from matching between NRQCD and pNRQCD, and are independent of the heavy quark flavor. This ***reduces the number of nonperturbative quantities.***

P-WAVE MATRIX ELEMENTS IN PNRQCD

- *P*-wave quarkonium : inclusive decay at leading order in v

short-distance coefficients

$\langle \ldots \rangle$: ***expectation value on quarkonium state***

$$\Gamma = \frac{2\,\mathrm{Im} f_1({}^3P_J)}{m^4} \boxed{\frac{1}{3}\langle \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) \chi \chi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) \psi \rangle}$$

Color-singlet matrix element

$$+ \frac{2\,\mathrm{Im} f_8({}^3S_1)}{m^4} \boxed{\langle \psi^\dagger \boldsymbol{\sigma} T^a \chi \cdot \chi^\dagger \boldsymbol{\sigma} T^a \psi \rangle}$$

Color-octet matrix element

$$\boxed{\frac{1}{3}\langle \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) \chi \chi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma}) \psi \rangle = \frac{3N_c}{2\pi} |R'(0)|^2}$$

Radial wavefunction at the origin

$$\boxed{\langle \psi^\dagger \boldsymbol{\sigma} T^a \chi \cdot \chi^\dagger \boldsymbol{\sigma} T^a \psi \rangle = \frac{2T_F}{9N_c m^2} \frac{3N_c}{2\pi} |R'(0)|^2 \mathcal{E}_3}$$

Dimensionless gluonic correlator

$$\mathcal{E}_n = \frac{T_F}{N_c} \int_0^\infty dt\, t^n \langle \Omega | g E^{i,a}(t,\mathbf{0}) \Phi_{ab}(t,0) g E^{i,b}(0,\mathbf{0}) | \Omega \rangle$$ **: scale like** $\Lambda_{\mathrm{QCD}}^{d_n}$, $d_n = 3 - n$

Schwinger line

$\mathcal{E}_3$ **is dimensionless, and has logarithmic scale dependence**

P-WAVE MATRIX ELEMENTS IN PNRQCD

- Electromagnetic decay/production including order-v^2 correction

short-distance coefficients

$\langle \ldots \rangle$: ***expectation value on quarkonium state***

$$\sigma = c_J^{(O_1)} \frac{1}{3}\langle \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\chi|\Omega\rangle\langle\Omega|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\psi\rangle$$

Color-singlet matrix element at LO in v

$$+ \frac{c_J^{(P)}}{m^2}\left[\frac{1}{3}\langle \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\chi|\Omega\rangle\langle\Omega|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\psi\rangle + \text{c.c.}\right]$$

Color-singlet matrix element, order-v^2 correction

$$+ \frac{c_J^{(T)}}{m}\left[\frac{1}{3}\langle \psi^\dagger (-ig\boldsymbol{E} \cdot \boldsymbol{\sigma})\chi|\Omega\rangle\langle\Omega|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\psi\rangle + \text{c.c.}\right]$$

Color-octet matrix element, order-v^2 correction

$$\frac{1}{3}\langle \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\chi|\Omega\rangle\langle\Omega|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\psi\rangle = \frac{3N_c}{2\pi}|R'(0)|^2\left[1 + \frac{2}{3}\frac{i\mathcal{E}_2}{m} + O(v^2)\right]$$

$$\frac{1}{3}\langle \psi^\dagger (-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}})^2\chi|\Omega\rangle\langle\Omega|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\psi\rangle + \text{c.c.} = \frac{3N_c}{2\pi}|R'(0)|^2\left[mE_B - \frac{2}{3}\mathcal{E}_1 + O(v^3)\right]$$

$$\frac{1}{3}\langle \psi^\dagger (-ig\boldsymbol{E} \cdot \boldsymbol{\sigma})\chi|\Omega\rangle\langle\Omega|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}} \cdot \boldsymbol{\sigma})\psi\rangle + \text{c.c.} = \frac{3N_c}{2\pi}|R'(0)|^2\frac{4}{3}\frac{\mathcal{E}_1}{m}$$

Binding energy

$$\mathcal{E}_n = \frac{T_F}{N_c}\int_0^\infty dt\, t^n \langle\Omega|gE^{i,a}(t,\mathbf{0})\Phi_{ab}(t,0)gE^{i,b}(0,\mathbf{0})|\Omega\rangle$$

S-WAVE MATRIX ELEMENTS IN PNRQCD

- *S*-wave quarkonium : electromagnetic decay/production including order-v^2 correction

$\langle \ldots \rangle$: ***expectation value on quarkonium state***

short-distance coefficients

$$\sigma = c_0 \langle \psi^\dagger \boldsymbol{\sigma} \chi | \Omega \rangle \cdot \langle \Omega | \chi^\dagger \boldsymbol{\sigma} \psi \rangle$$

Color-singlet matrix element at LO in v

$$+ \frac{c_2}{m^2} \left[\langle \psi^\dagger \boldsymbol{\sigma} (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}})^2 \chi | \Omega \rangle \cdot \langle \Omega | \chi^\dagger \boldsymbol{\sigma} \psi \rangle + \text{c.c.} \right]$$

Color-singlet matrix element, order-v^2 correction

$$\langle \psi^\dagger \boldsymbol{\sigma} \chi | \Omega \rangle \cdot \langle \Omega | \chi^\dagger \boldsymbol{\sigma} \psi \rangle = \frac{N_c}{2\pi} |R(0)|^2 \left[1 - \frac{E_B}{m} \frac{2\mathcal{E}_3}{9} + O(\Lambda_{\rm QCD}^2/m^2, v^3) \right]$$

$$\langle \psi^\dagger \boldsymbol{\sigma} (-\tfrac{i}{2} \overleftrightarrow{\boldsymbol{D}})^2 \chi | \Omega \rangle \cdot \langle \Omega | \chi^\dagger \boldsymbol{\sigma} \psi \rangle + \text{c.c.} = \frac{N_c}{2\pi} |R(0)|^2 \left[m E_B + O(v^3) \right]$$

Binding energy

$$\mathcal{E}_n = \frac{T_F}{N_c} \int_0^\infty dt\, t^n \langle \Omega | g E^{i,a}(t, \mathbf{0}) \Phi_{ab}(t,0) g E^{i,b}(0, \mathbf{0}) | \Omega \rangle$$

NRQCD MATRIX ELEMENTS IN PNRQCD

- Quarkonium decay and production rates are determined from wavefunctions at the origin and gluonic correlators.
- We compute wavefunctions from potential models, while gluonic correlators can in principle be obtained from lattice QCD.
- Since lattice determinations have not yet been done, we determine $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3$ from measured *P*-wave charmonium decay rates and cross sections.

$$\mathcal{E}_1 = -0.20 \pm 0.14 \pm 0.90 \text{ GeV}^2$$

$$\mathcal{E}_2 = 0.77^{+0.98}_{-0.86} \pm 0.85 \text{ GeV}$$

$$\mathcal{E}_3(1 \text{ GeV}) = 2.05^{+0.94}_{-0.65}$$ **($\overline{\text{MS}}$ scheme)**

P-WAVE CHARMONIUM DECAY RATES

- Electromagnetic decay rates of χ_{c0} and χ_{c2}

pNRQCD

$$\Gamma(\chi_{c0}(1P) \to \gamma\gamma) = 2.80^{+0.12}_{-0.19} \pm 0.52 \text{ keV}$$
$$\Gamma(\chi_{c2}(1P) \to \gamma\gamma) = 0.58^{+0.01}_{-0.00} \pm 0.16 \text{ keV}$$

Experiment

$$\Gamma(\chi_{c0}(1P) \to \gamma\gamma)|_{\text{BESIII}} = 2.33 \pm 0.20 \pm 0.22 \text{ keV}$$
$$\Gamma(\chi_{c2}(1P) \to \gamma\gamma)|_{\text{BESIII}} = 0.63 \pm 0.04 \pm 0.06 \text{ keV}$$

BESIII, PRD 85 (2012) 112008

- Decay rates of χ_{cJ} into light hadrons

pNRQCD

$$\Gamma(\chi_{c0}(1P) \to \text{LH}) = 8.3^{+3.0}_{-3.1} \text{ MeV}$$
$$\Gamma(\chi_{c1}(1P) \to \text{LH}) = 0.42^{+0.06+0.28}_{-0.06-0.22} \text{ MeV}$$
$$\Gamma(\chi_{c2}(1P) \to \text{LH}) = 1.4^{+0.6}_{-0.6} \text{ MeV}$$

Experiment

$$\Gamma(\chi_{c0}(1P) \to \text{LH})|_{\text{from PDG}} = 10.6 \pm 0.6 \text{ MeV},$$
$$\Gamma(\chi_{c1}(1P) \to \text{LH})|_{\text{from PDG}} = 0.552 \pm 0.041 \text{ MeV}$$
$$\Gamma(\chi_{c2}(1P) \to \text{LH})|_{\text{from PDG}} = 1.60 \pm 0.09 \text{ MeV}.$$

PDG, PRD 98 (2018) 030001

P-WAVE BOTTOMONIUM DECAY RATES

- Two-photon decay rates

pNRQCD

$$\Gamma(\chi_{b0}(1P) \to \gamma\gamma) = 46.6^{+11.7}_{-14.0} \pm 6.5 \text{ eV},$$
$$\Gamma(\chi_{b2}(1P) \to \gamma\gamma) = 9.1^{+2.1}_{-2.5} \pm 1.3 \text{ eV},$$
$$\Gamma(\chi_{b0}(2P) \to \gamma\gamma) = 46.8^{+11.8}_{-12.7} \pm 6.5 \text{ eV},$$
$$\Gamma(\chi_{b2}(2P) \to \gamma\gamma) = 9.3^{+2.1}_{-2.3} \pm 1.3 \text{ eV},$$
$$\Gamma(\chi_{b0}(3P) \to \gamma\gamma) = 46.1^{+11.9}_{-11.8} \pm 6.3 \text{ eV},$$
$$\Gamma(\chi_{b2}(3P) \to \gamma\gamma) = 9.2^{+2.2}_{-2.2} \pm 1.3 \text{ eV},$$

- Results are almost independent of radial excitation
- *Sizable uncertainty from potential models*, first-principles calculation desirable

- Decay rates into light hadrons

pNRQCD

$$\Gamma(\chi_{b0}(1P) \to \text{LH}) = 1.07^{+0.33}_{-0.37} \text{ MeV},$$
$$\Gamma(\chi_{b2}(1P) \to \text{LH}) = 0.27^{+0.08}_{-0.10} \text{ MeV},$$
$$\Gamma(\chi_{b0}(2P) \to \text{LH}) = 1.08^{+0.33}_{-0.35} \text{ MeV},$$
$$\Gamma(\chi_{b2}(2P) \to \text{LH}) = 0.28^{+0.09}_{-0.10} \text{ MeV},$$
$$\Gamma(\chi_{b0}(3P) \to \text{LH}) = 1.06^{+0.33}_{-0.33} \text{ MeV},$$
$$\Gamma(\chi_{b2}(3P) \to \text{LH}) = 0.28^{+0.09}_{-0.10} \text{ MeV}.$$
$$\Gamma(\chi_{b1}(nP) \to \text{LH}) = 0.14 \pm 0.06 \text{ MeV}$$

- Results are almost independent of radial excitation

P-WAVE ELECTROMAGNETIC PRODUCTION

- Electromagnetic production of *P*-wave charmonium has been studied in B factories, and *P*-wave bottomonium can be studied in future lepton colliders

pNRQCD results for charmonium

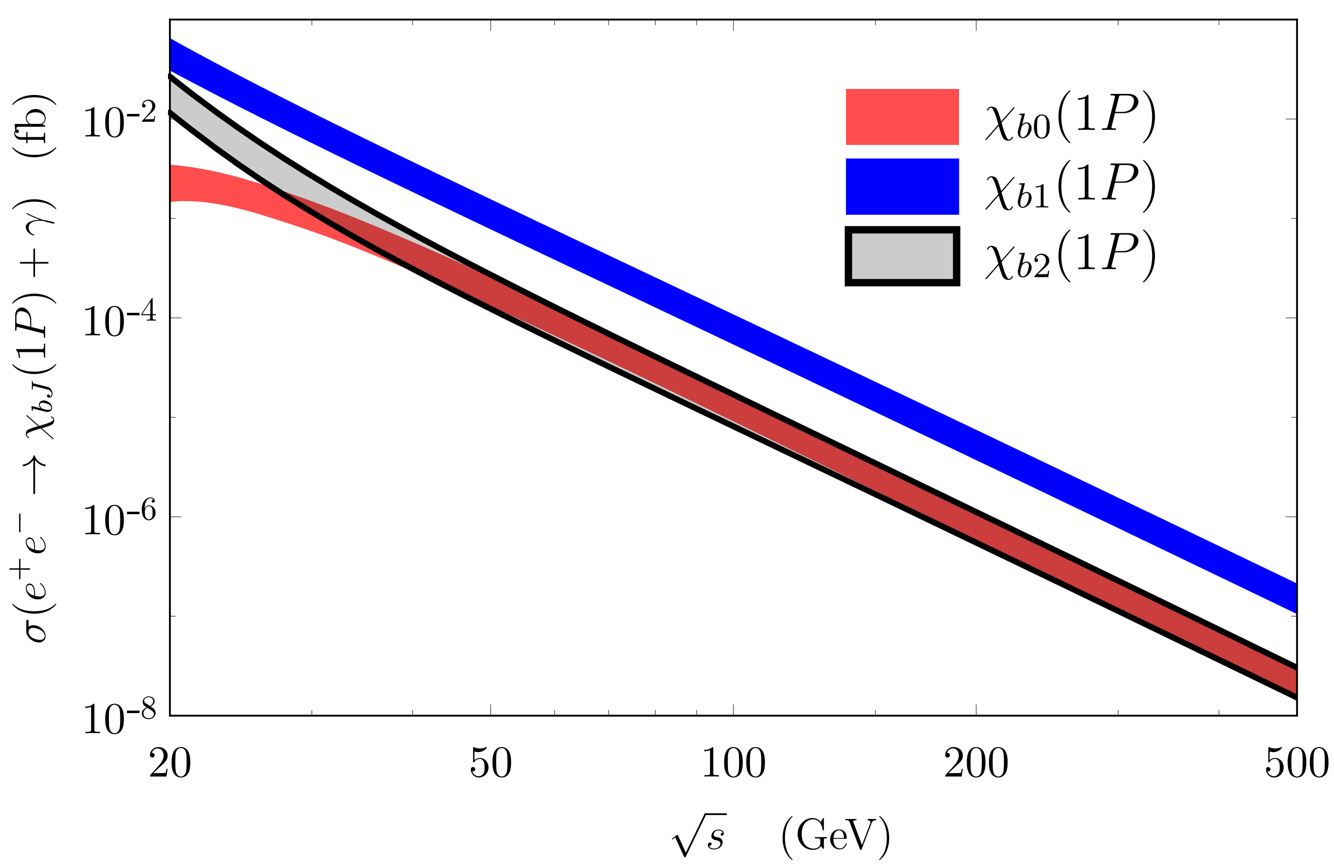
$$\sigma(e^+e^- \to \chi_{c0}(1P)+\gamma) = 1.84^{+0.25}_{-0.26} \pm 0.76\,\mathrm{fb}$$
$$\sigma(e^+e^- \to \chi_{c1}(1P)+\gamma) = 16.4^{+0.2}_{-0.2} \pm 6.4\,\mathrm{fb}\,,$$
$$\sigma(e^+e^- \to \chi_{c2}(1P)+\gamma) = 3.75^{+0.67}_{-0.56} \pm 2.16\,\mathrm{fb}$$

Experiment

$$\sigma(e^+e^- \to \chi_{c1}(1P)+\gamma)\big|_{\mathrm{Belle}} = 17.3^{+4.2}_{-3.9} \pm 1.7\ \mathrm{fb}\,.$$

Belle, PRD 98 (2018) 092015

pNRQCD prediction for bottomonium



S-WAVE BOTTOMONIUM DECAY RATES

- Electromagnetic decay rates of $\Upsilon(2S)$ and $\Upsilon(3S)$

pNRQCD

$$\Gamma(\Upsilon(2S) \to e^+e^-) = 0.63^{+0.28+0.07}_{-0.17-0.07} \text{ keV}$$

$$\Gamma(\Upsilon(3S) \to e^+e^-) = 0.40^{+0.17+0.04}_{-0.11-0.05} \text{ keV}$$

Experiment

$$\Gamma(\Upsilon(2S) \to e^+e^-)\big|_{\mathrm{PDG}} = 0.612 \pm 0.011 \text{ keV}$$

$$\Gamma(\Upsilon(3S) \to e^+e^-)\big|_{\mathrm{PDG}} = 0.443 \pm 0.008 \text{ keV}$$

PDG, PRD 98 (2018) 030001

- *Sizable uncertainty from potential models;* this can be reduced by ***computing the wavefunctions from first principles***, instead of using potential models:

pNRQCD, wavefunctions computed from first principles

$$\Gamma(\Upsilon(2S) \to e^+e^-) = 0.54^{+0.07}_{-0.06} \text{ keV}$$

$$\Gamma(\Upsilon(3S) \to e^+e^-) = 0.41^{+0.05}_{-0.05} \text{ keV}$$

HSC, JHEP12 (2020) 065

INCLUSIVE PRODUCTION IN PNRQCD

NRQCD MATRIX ELEMENTS

- Inclusive production matrix elements are vacuum expectation values with projection onto quarkonium states.

Projection onto states containing quarkonium

Inclusive production : $\langle \Omega | \chi^\dagger \mathcal{K} \psi \mathcal{P}_{\mathcal{Q}} \psi^\dagger \mathcal{K}' \chi | \Omega \rangle$

color/spin matrices and covariant derivatives

- The projection is onto states that contain a quarkonium + anything. $$\mathcal{P}_{\mathcal{Q}} = \sum_X |\mathcal{Q} + X\rangle\langle \mathcal{Q} + X| = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$$

- Existing pNRQCD formalism describes the state $|\mathcal{Q}\rangle$. We extend the formalism to describe the states $|\mathcal{Q} + X\rangle$, which is valid up to corrections of order $1/N_c^2$.

Brambilla, Chung, Vairo, arXiv:2007.07613 [hep-ph]

INCLUSIVE P-WAVE PRODUCTION

- We apply the pNRQCD formalism for $\chi_{QJ}\,(Q{=}c$ or $b,\ J{=}1,\,2)$
- At leading order in v, the cross section is given by

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}\langle\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle$$
$$+ (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}\langle\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle$$

Bodwin, Braaten, Yuan, Lepage, PRD46, R3703 (1992)
Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

color singlet : $\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]}) = \frac{1}{3}\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\psi\mathcal{P}_{\chi_{Q0}}\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\chi$

color octet : $\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]}) = \chi^\dagger\sigma^iT^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{\chi_{Q0}}\Phi_\ell^{bc}\psi^\dagger\sigma^iT^c\chi$

Nayak, Qiu, Sterman, PLB613, 45 (2005)

- We compute both color singlet and color octet matrix elements in strongly coupled pNRQCD.

P-WAVE MATRIX ELEMENTS

- Color-singlet matrix element: $\langle \mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]}) \rangle = \frac{3N_c}{2\pi} |R^{(0)\prime}_{\chi_{Q0}}(0)|^2$ we reproduce the known result in vacuum-saturation approximation.

- Color-octet matrix element: result is given in terms of a universal gluonic correlator.

$$\langle \mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]}) \rangle = \frac{3N_c}{2\pi} |R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \, \frac{\mathcal{E}}{9N_c m^2}$$

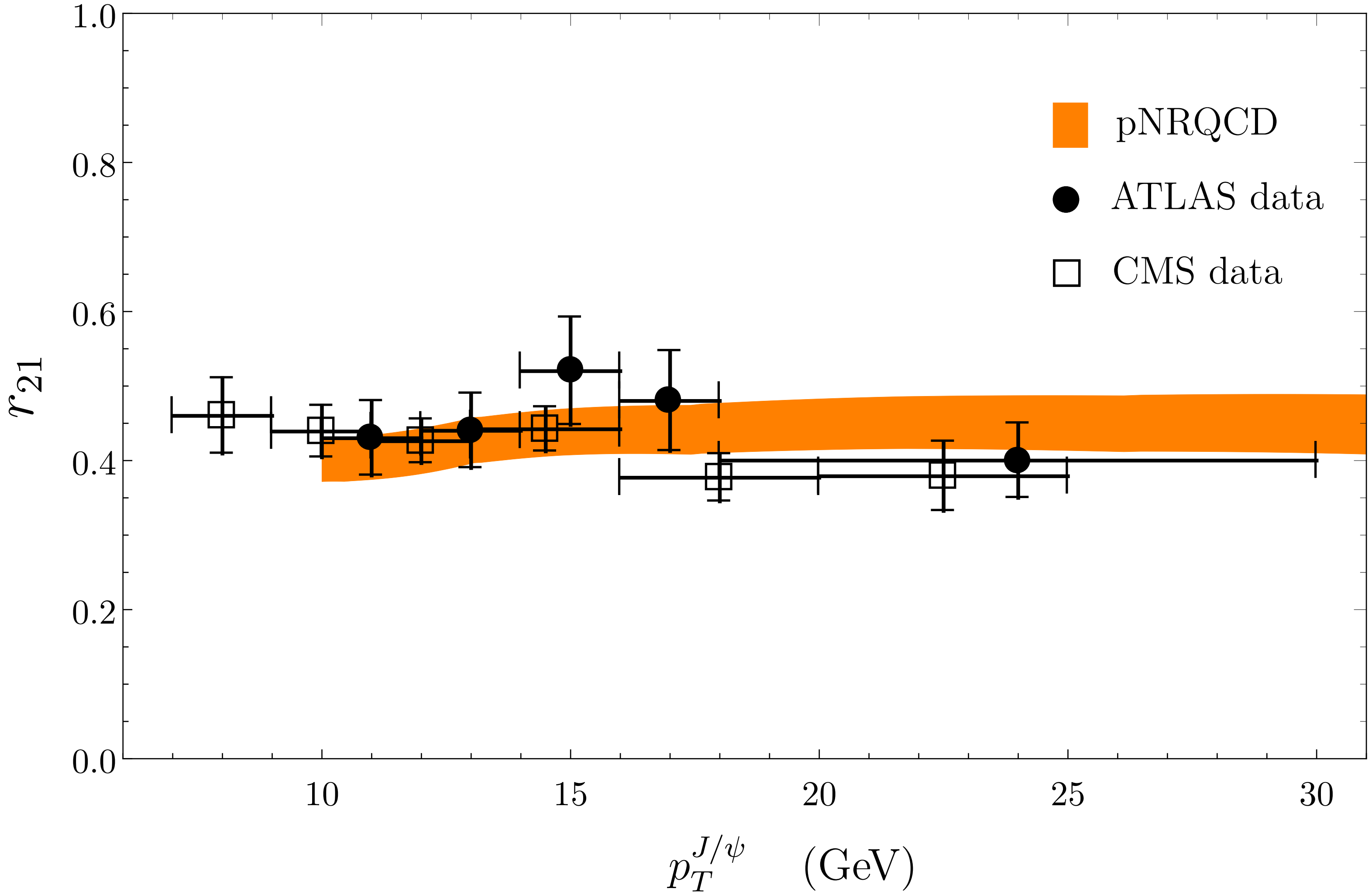
$$\mathcal{E} = \frac{3}{N_c} \int_0^\infty t\, dt \int_0^\infty t'\, dt' \langle \Omega | \Phi_\ell^{\dagger ab} \Phi_0^{\dagger da}(0,t) g E^{d,i}(t) g E^{e,i}(t') \Phi_0^{ec}(t',0) \Phi_\ell^{bc} | \Omega \rangle$$

- $\mathcal{E}$ is a universal quantity that does not depend on quark flavor or radial excitation. Determination of $\mathcal{E}$ leads to determination of ***all P-wave quarkonium cross sections***.

P-WAVE CHARMONIUM PRODUCTION

- Cross section ratio $\sigma(\chi_{c2})/\sigma(\chi_{c1})$ at the LHC compared to ATLAS and CMS data.

CMS, EPJC72, 2251 (2012)
ATLAS, JHEP07, 154 (2014)



- We determine $\mathcal{E}$ from measurements of this ratio:

$$\mathcal{E}(\Lambda = 1.5\ \text{GeV}) = 1.97 \pm 0.06$$

($\overline{\text{MS}}$ scheme)

Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

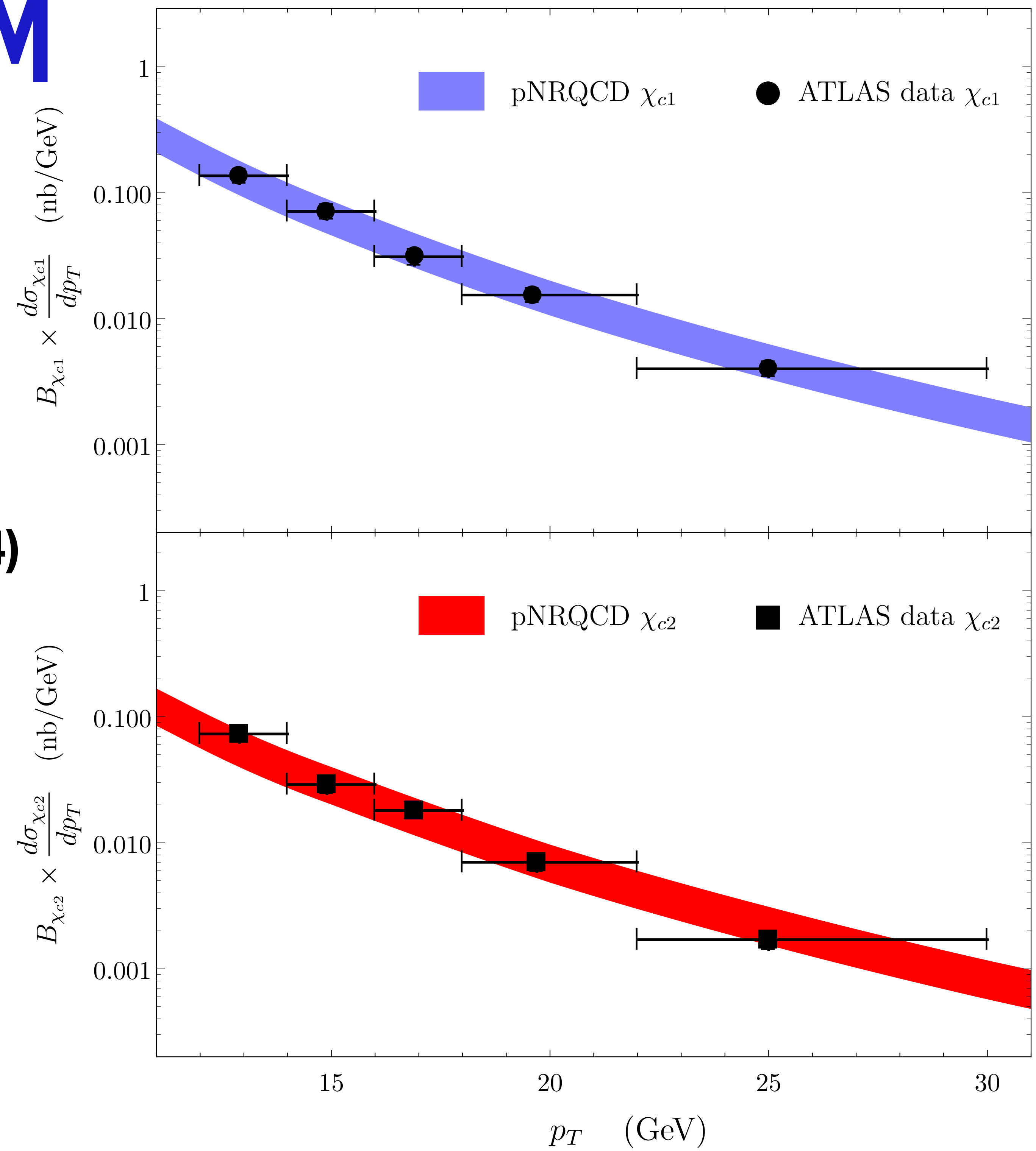
P-WAVE CHARMONIUM PRODUCTION

- χ_{c2} and χ_{c1} cross sections at the LHC, compared to ATLAS data.

ATLAS, JHEP07, 154 (2014)

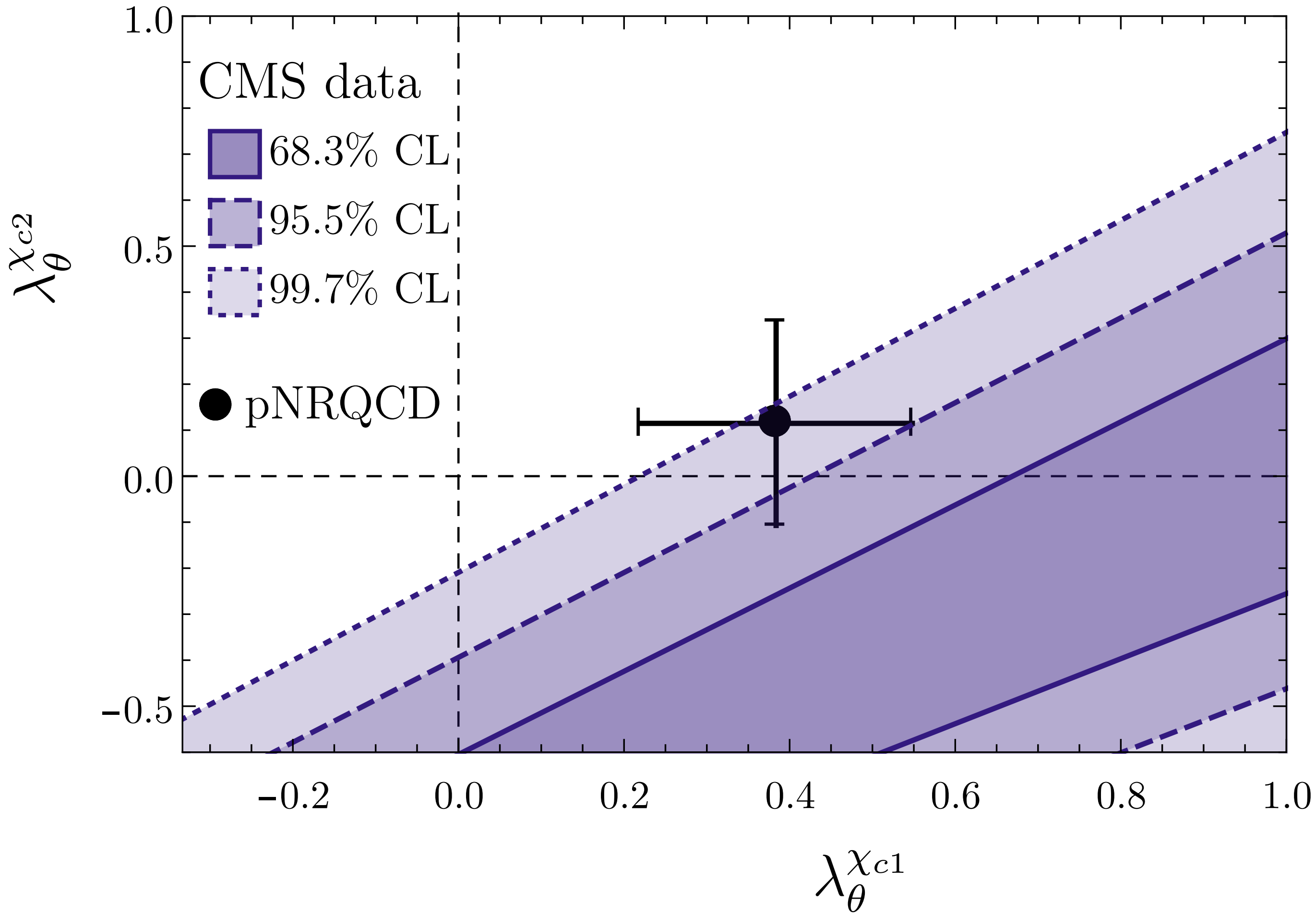
- Wavefunctions at the origin obtained from two-photon decay rates of χ_{c2} and χ_{c0}.

Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)



P-WAVE CHARMONIUM POLARIZATION

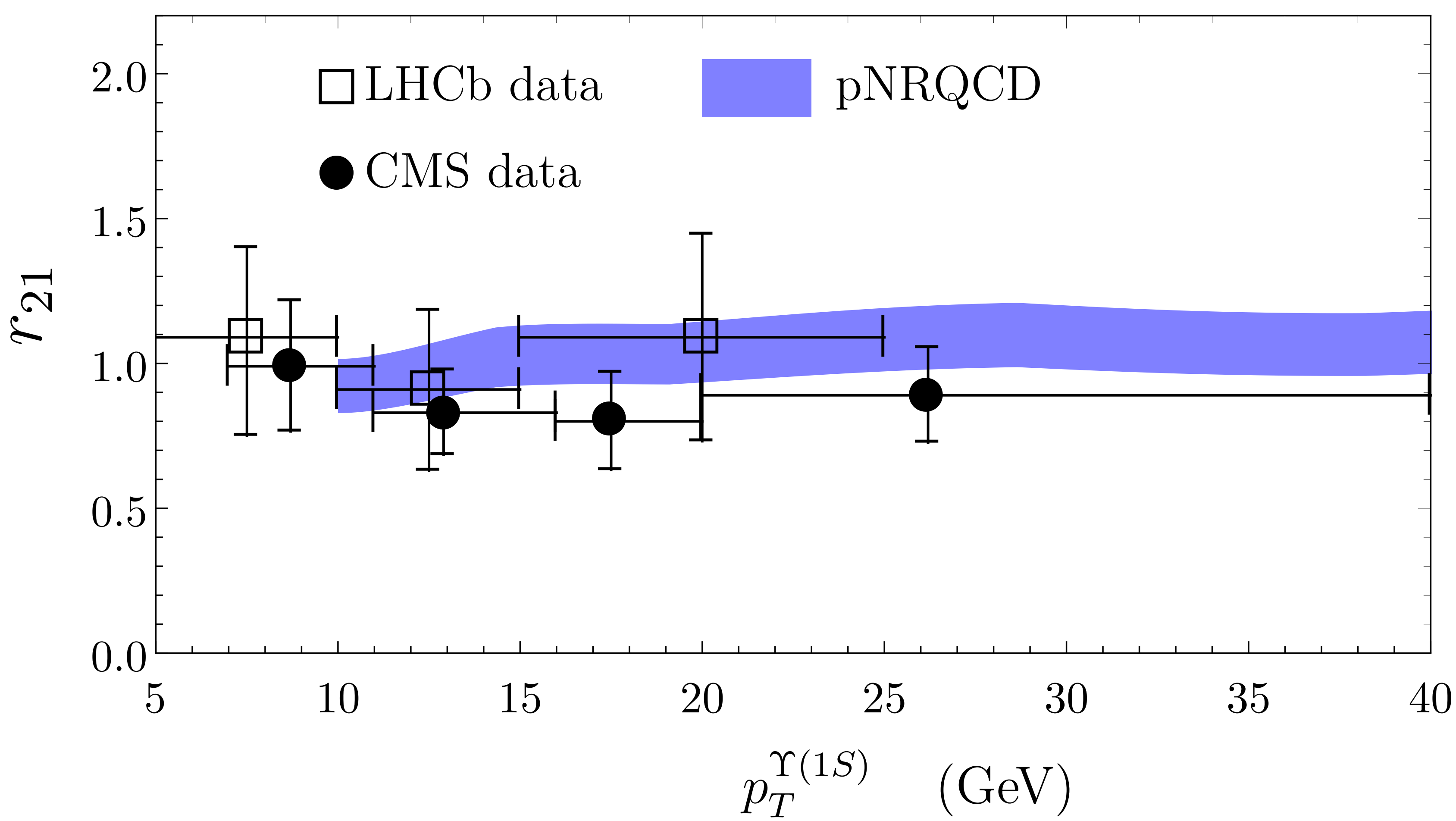
- χ_{c2} and χ_{c1} polarization at the LHC compared to experimental constraints from CMS. **CMS, PRL124, 162002 (2020)**



Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

P-WAVE BOTTOMONIUM PRODUCTION

- Cross section ratio $\sigma(\chi_{b2})/\sigma(\chi_{b1})$ for $1P$ states at the LHC compared to LHCb and CMS measurements.



LHCb, JHEP10, 088 (2014)
CMS, PLB743, 383 (2015)

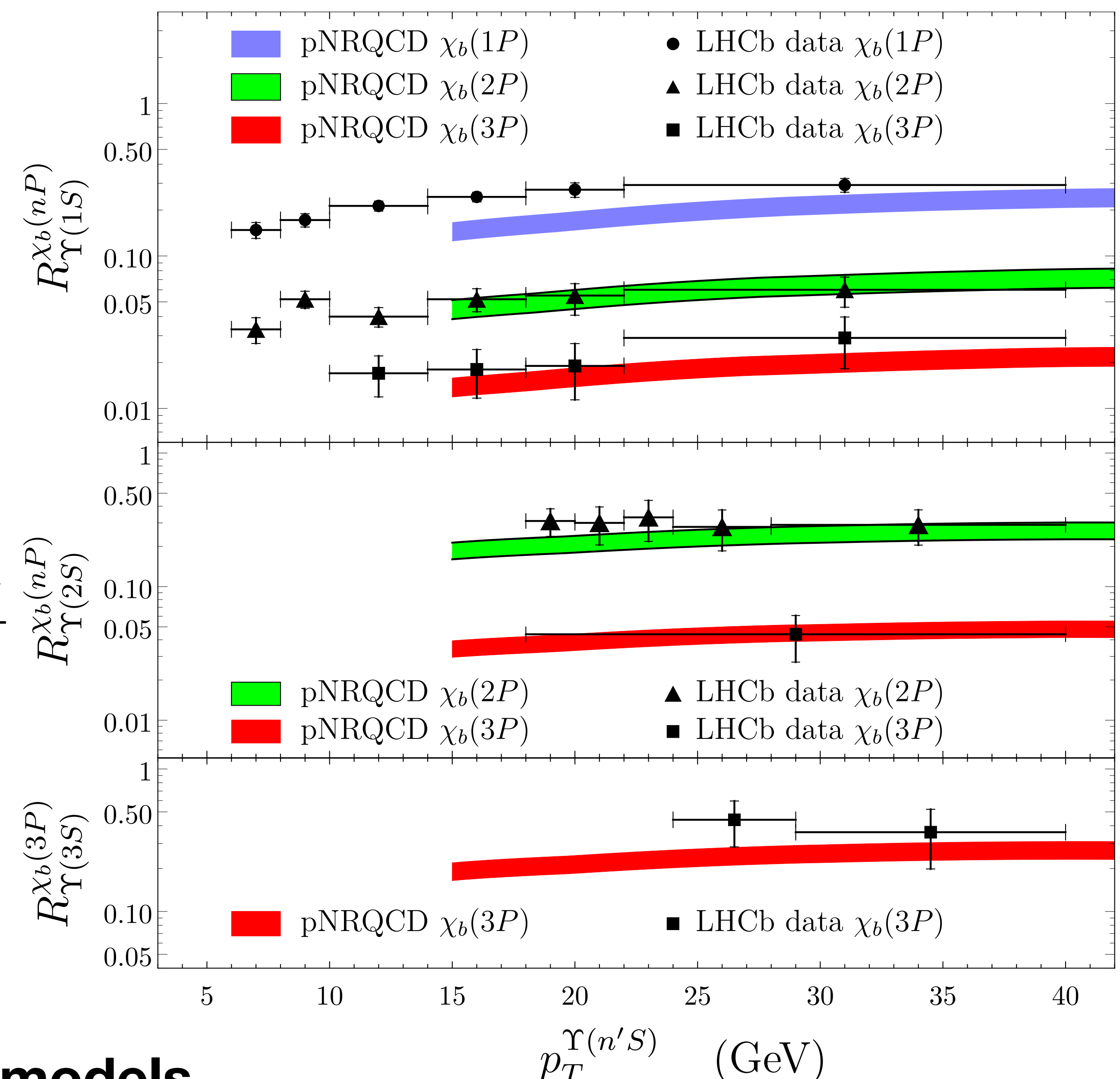
Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)

P-WAVE BOTTOMONIUM PRODUCTION

- $\chi_{bJ}(nP)$ production rates relative to $\Upsilon(n'S)$ cross sections at the LHC compared to LHCb measurement of feeddown fractions.

LHCb, EPJC74, 3092 (2014)

$$R^{\chi_b(nP)}_{\Upsilon(n'S)} = \sum_{J=1,2} \frac{\sigma_{\chi_{bJ}(nP)} \times \mathrm{Br}_{\chi_{bJ}\to\Upsilon(nS)+\gamma}}{\sigma_{\Upsilon(n'S)}}$$



Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)

χ_{bJ} **wavefunctions computed from potential models**

$\Upsilon(nS)$ **matrix elements taken from fits to data in Han, Ma, Meng, Shao, Zhang, Chao, PRD94, 014028 (2016)**

SUMMARY AND OUTLOOK

- We computed quarkonium production cross sections in strongly coupled potential NRQCD, where matrix elements are given in terms of wavefunctions at the origin and gluonic correlators.
- Due to universality of gluonic correlators, the number of independent nonperturbative quantities are reduced.
- We also developed a formalism for inclusive production, and for the first time computed matrix elements for inclusive production from first principles.
- We determine the gluonic correlators from measured P-wave charmonium data, which provide good descriptions of charmonium and bottomonium measurements.
- Lattice QCD determinations of gluonic correlators are desirable.

BACKUP

DECAY MATRIX ELEMENTS IN PNRQCD

- The formalism for computing decay matrix elements in strongly coupled pNRQCD have been developed.

Brambilla, Eiras, Pineda, Soto, Vairo, PRL88 (2002) 012003
Brambilla, Eiras, Pineda, Soto, Vairo, PRD67 (2003) 034018
Brambilla, HSC, Müller, Vairo, JHEP04 (2020) 095

- Decay matrix elements are computed in expansion in powers of $1/m$:

NRQCD Hamiltonian $$H_{\rm NRQCD} = H^{(0)}_{\rm NRQCD} + H^{(1)}_{\rm NRQCD}/m + \ldots$$

Eigenstates $$|\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle = |\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle^{(0)} + \frac{1}{m} |\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle^{(1)} + \ldots$$

- $|\underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ is the ground state, x_1 and x_2 are positions of Q and $\bar{Q}$.

DECAY MATRIX ELEMENTS IN PNRQCD

- A heavy quarkonium in vacuum is given by the ground state $|\underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$, because excited states have energy gaps of $\Lambda_{\rm QCD}$ or more and are integrated out.
- This leads to the following formula

Quarkonium wavefunction

$$\langle \mathcal{Q}|\mathcal{O}|\mathcal{Q}\rangle = \frac{1}{\langle \boldsymbol{P}{=}\boldsymbol{0}|\boldsymbol{P}{=}\boldsymbol{0}\rangle} \int d^3r \int d^3r' \int d^3R \int d^3R' \langle \boldsymbol{P}{=}\boldsymbol{0}|\boldsymbol{R}\rangle \phi^*_{\mathcal{Q}}(\boldsymbol{r}) \times \left[\langle \underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2| \int d^3\xi\, \mathcal{O}(\boldsymbol{\xi}) |\underline{0}; \boldsymbol{x}'_1, \boldsymbol{x}'_2\rangle \right] \langle \boldsymbol{R}'|\boldsymbol{P}{=}\boldsymbol{0}\rangle \phi_{\mathcal{Q}}(\boldsymbol{r}')$$

Contact term, computed order by order in $1/m$

$$\boldsymbol{r} = \boldsymbol{x}_1 - \boldsymbol{x}_2, \quad \boldsymbol{R} = (\boldsymbol{x}_1 + \boldsymbol{x}_2)/2$$

- The contact term is proportional to $\delta^{(3)}(\boldsymbol{x}_1 - \boldsymbol{x}_2)$, so the decay matrix element involves wavefunctions at the origin.

QUARKONIUM PROJECTION OPERATOR

- It is necessary to describe the projection operator in order to compute inclusive production matrix elements.
- The projection operator $\mathcal{P}_{\mathcal{Q}} = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$ is essentially a number operator. If we neglect decay rates, $\mathcal{P}_{\mathcal{Q}}$ and the NRQCD Hamiltonian are simultaneously diagonalizable.
- The matrix elements of $\mathcal{P}_{\mathcal{Q}}$ should involve $|\underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$, because this describes quarkonia in vacuum.
- Matrix elements of $\mathcal{P}_{\mathcal{Q}}$ can also involve $|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ with $n>0$, if this describes a quarkonium + light particles.

QUARKONIUM PROJECTION OPERATOR

- A simultaneous eigenstate of $\mathcal{P}_{\mathcal{Q}}$ and the Hamiltonian is

$$|\mathcal{Q}(n)\rangle = \int d^3x_1 d^3x_2\, \phi_{\mathcal{Q}(n)}(\boldsymbol{x}_1, \boldsymbol{x}_2)|\underline{n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$

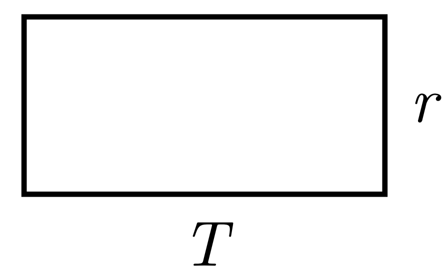
- For $n=0$, this is just the quarkonium in vacuum and ϕ is the usual quarkonium wavefunction.
- For $n>0$, the "wavefunctions" ϕ are in general unknown.
- The projection operator is then $\mathcal{P}_{\mathcal{Q}} = \sum_n |\mathcal{Q}(n)\rangle\langle\mathcal{Q}(n)|$.

  The sum is restricted to states that reduce to color-singlet $Q\bar{Q}$ at leading order in $1/m$ and at $x_1=x_2$.

QUARKONIUM WAVEFUNCTIONS

- Nonperturbatively, quarkonium wavefunctions are determined from a Schrödinger equation, where the potential is a vacuum expectation value of a Wilson loop: at leading order in $1/m$ (static potential for $n=0$),

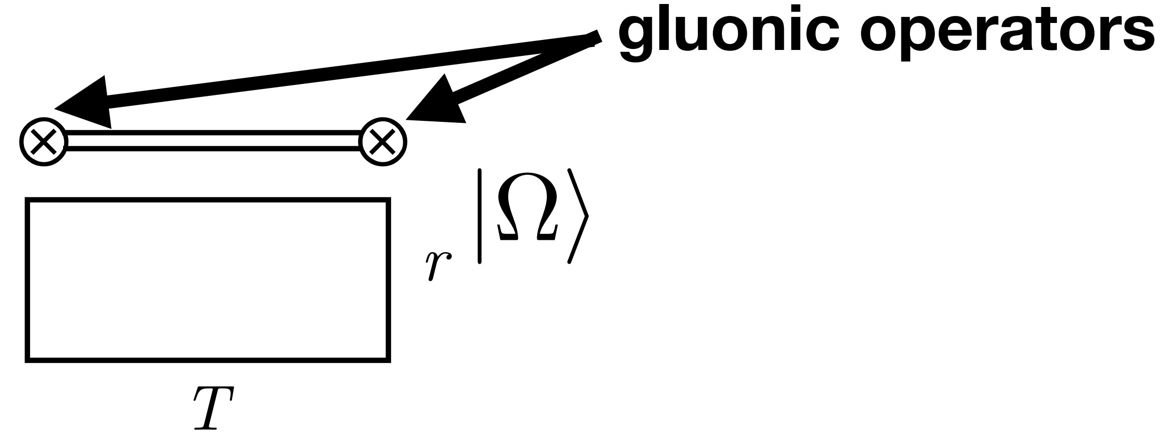
$$V(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle\Omega|$$



$$|\Omega\rangle$$

- For the potential for the $n>0$ states, the light excitations in the $|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ states should be included.

$$V(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle\Omega|$$



$$|\Omega\rangle$$

QUARKONIUM WAVEFUNCTIONS

- In general, VEVs of products of color-singlet operators factorize into products of VEVs of individual operators.

$$\langle\Omega|AB|\Omega\rangle = \langle\Omega|A|\Omega\rangle\langle\Omega|B|\Omega\rangle[1+O(1/N_c^2)]$$

Makeenko, Migdal, PLB88 (1979) 135
Witten, NATO Sci. Ser. B 59 (1980) 403

- So the $n>0$ potentials reduce to

$$V(r) = \lim_{T\to\infty}\frac{i}{T}\log\langle\Omega|\,\underset{T}{\overset{\otimes\!=\!=\!=\!=\!\otimes}{\boxed{\qquad\qquad}}}\,{}_r|\Omega\rangle$$

$$= \lim_{T\to\infty}\frac{i}{T}\log\left(\langle\Omega|\,\underset{T}{\boxed{\qquad\qquad}}\,{}_r|\Omega\rangle \times \langle\Omega|\otimes\!=\!=\!=\!=\!\otimes|\Omega\rangle\right)$$

$$= \lim_{T\to\infty}\frac{i}{T}\log\langle\Omega|\,\underset{T}{\boxed{\qquad\qquad}}\,{}_r|\Omega\rangle + \text{constant}$$

QUARKONIUM PROJECTION OPERATOR

- Hence, the $n>0$ potentials are just the $n=0$ potential, plus constants that have no effect to the wavefunctions.
- Therefore, the wavefunctions ϕ are independent of n, and the projection operator is just

$$\mathcal{P}_{\mathcal{Q}} = \sum_n |\mathcal{Q}(n)\rangle\langle\mathcal{Q}(n)|$$

$$|\mathcal{Q}(n)\rangle = \int d^3x_1 d^3x_2\, \phi_{\mathcal{Q}}(\boldsymbol{x}_1, \boldsymbol{x}_2)|\underline{n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$

- These are valid up to corrections of relative order $1/N_c^2$.

PRODUCTION MATRIX ELEMENTS

- Now we can compute the production matrix elements

$$\langle\Omega|\chi^\dagger\mathcal{K}\psi\mathcal{P}_{\mathcal{Q}}\psi^\dagger\mathcal{K}'\chi|\Omega\rangle = \int d^3x_1 d^3x_2 \int d^3x'_1 d^3x'_2 \phi(\boldsymbol{x}_1,\boldsymbol{x}_2)\phi(\boldsymbol{x}'_1,\boldsymbol{x}'_2) \sum_n \langle\Omega|\chi^\dagger\mathcal{K}\psi|\underline{\mathrm{n}};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle\langle\underline{\mathrm{n}};\boldsymbol{x}'_1,\boldsymbol{x}'_2|\psi^\dagger\mathcal{K}'\chi|\Omega\rangle$$

Contact term,
computed order by order in $1/m$

- This allows calculation of production matrix elements in strongly coupled pNRQCD, by computing the contact terms in the same way as the decay matrix elements.