Hands-on session

Planetary magnetic field measurements

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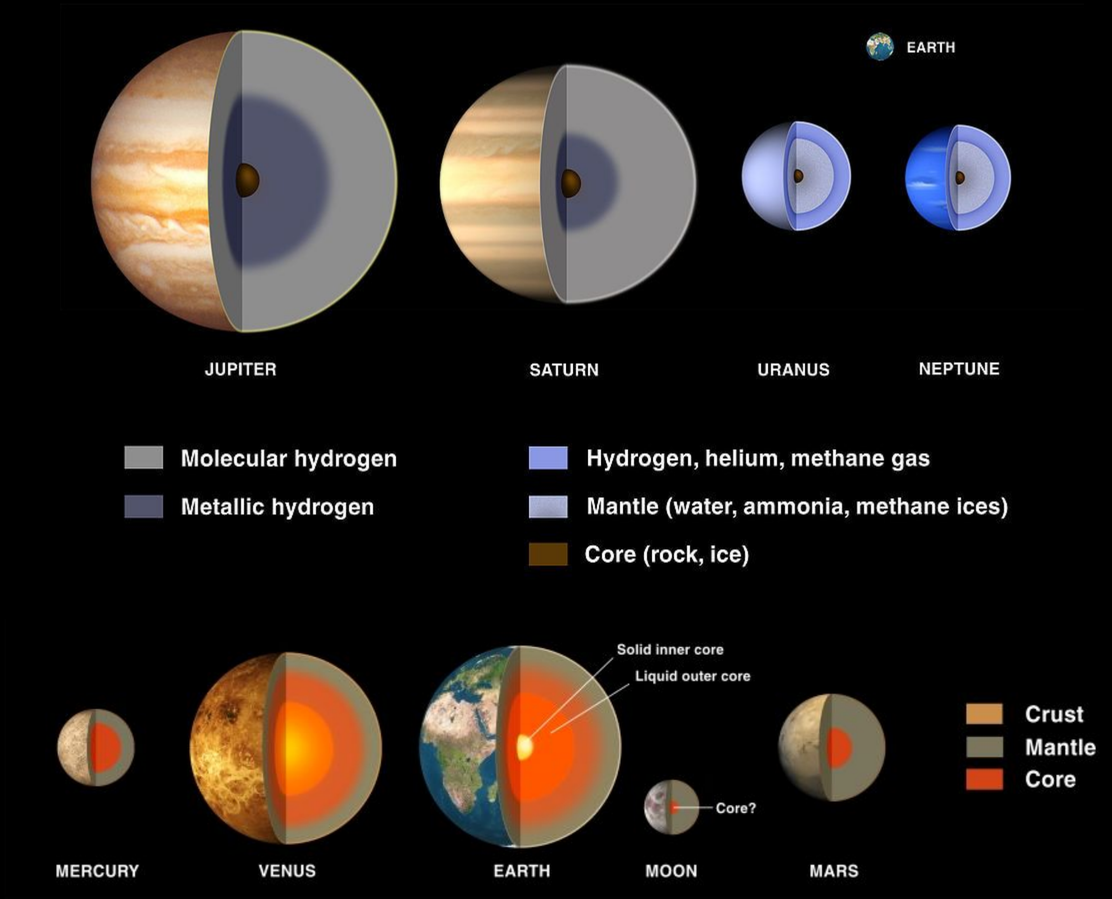
CSIC IEEC

Magnetic fields in the solar system



All currently existing planetary magnetic must be dynamo generated, as they would have decayed by magnetic diffusion if there was no amplification/sustainment mechanism.

The dynamo must be in the **conductive** AND **convective** regions in their interior. The nature of the convective region is different between planets!

How are magnetic fields measured?

What happens to E and B outside the dynamo region?

Let's start from Maxwell equations!

$$\nabla \cdot \mathbf{E} = \frac{\rho_q}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$
$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B}$$

Is we assume a non-relativistic and non-charged conducting fluid, Maxwell equations can be simplified to the classical MHD ones.

If we are outside the dynamo region, i.e. outside the conducting fluid, then there are no currents and thus no magnetic field sources (**J**=0). Then the dynamo-created/sustained magnetic field can be expressed as a potential:

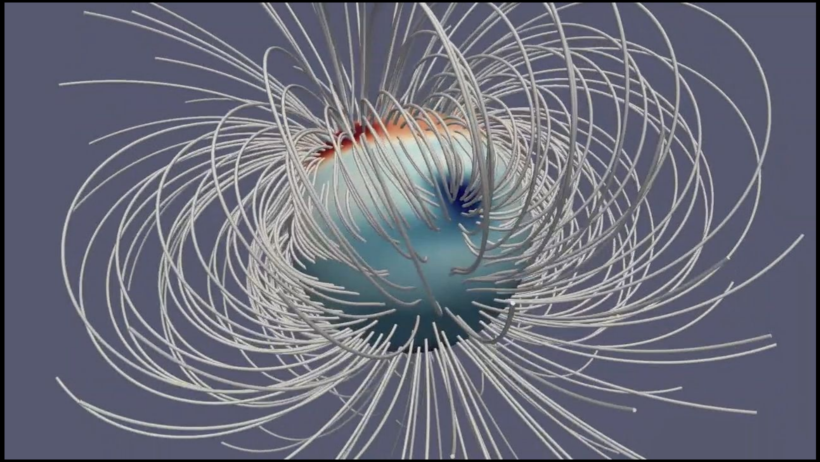
$$\boldsymbol{B} = -\nabla V$$

Magnetic potential

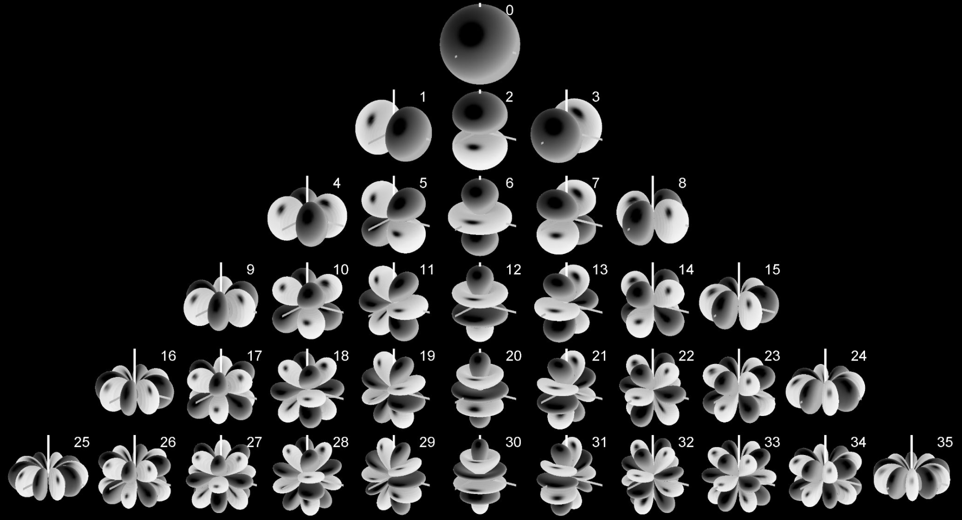
$$\boldsymbol{B} = -\nabla V$$

$$V = a \sum_{n=1}^{n_{max}} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} P_n^m(cos\theta)\,[g_n^m cos(m\phi) + h_n^m sin(m\phi)]$$

As planets are close to a sphere, a logical strategy is to express the magnetic field as an expansion of spherical harmonics.

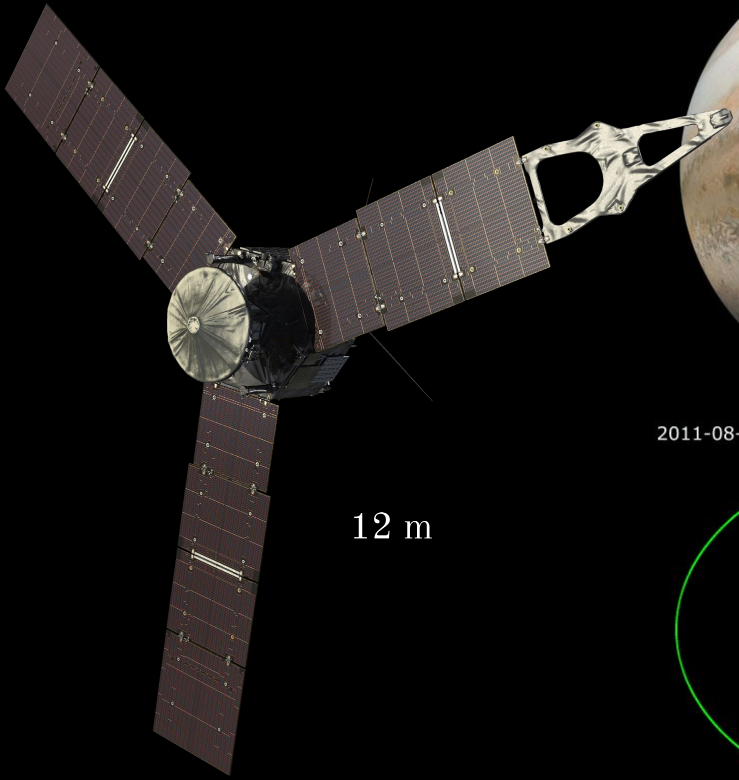


Credits: NASA/JPL-Caltech/Harvard/Moore et al.

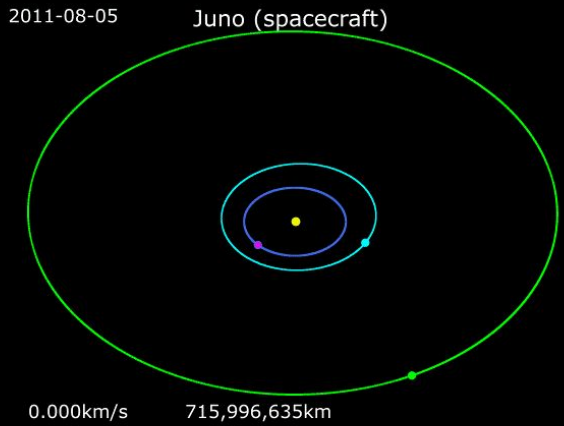


Juno mission

Launched in 2011, still in operation



12 m



Measurement characteristics

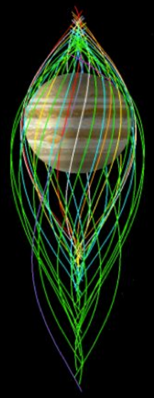
Solve linear equations between observations (B) and parameters (g, h)

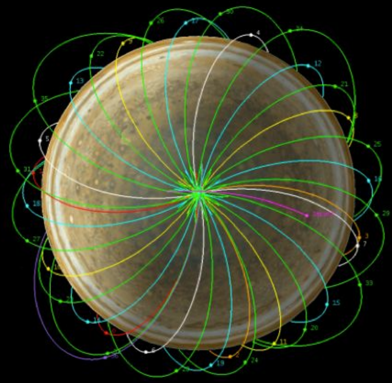
Measurements within 7 R_J m

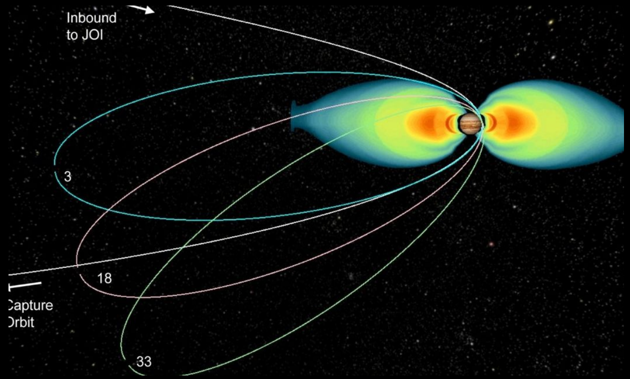
64 samples/s, accuracy of 1 in 10^4

Magnetic field between $1 \cdot 10^3$-$4 \cdot 10^6$ nT

Near the surface ~ 12 G (Earth is ~ 0.5G)







Get the g and h's

$$\boldsymbol{B} = -\nabla V$$

$$V = a \sum_{n=1}^{n_{max}} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} P_n^m(cos\theta)\left[g_n^m cos(m\phi) + h_n^m sin(m\phi)\right]$$

Juno mission

Linear systems between observations (y) and parameters (x)

$$\mathbf{y} = \mathrm{A}\,\mathbf{x}$$

In this case y are the 3D magnetic field measurements and x are the g's and h's.

$$\mathbf{y} = U\Lambda V^{\mathsf{T}}\mathbf{x}, \quad \Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \lambda_M \end{bmatrix}$$

$$\left(U^T\mathbf{y}\right) = \boldsymbol{\beta},$$

$$\mathbf{x} = \sum_{i=1}^{k}\left(\frac{\beta_i}{\lambda_i}\right)\mathbf{v}_i$$

Juno mission magnetic related papers

Check for updates

AGU PUBLICATIONS

Geophysical Research Letters

RESEARCH LETTER

10.1002/2018GL077312

Key Points:
- The Juno spacecraft sampled Jupiter's magnetic field along eight polar passes separated by 45 degrees longitude affording coarse global coverage
- A degree 10 spherical harmonic model of the planetary magnetic field is obtained by partial solution of a degree 20 linear system
- Jupiter's magnetic field exhibits

A New Model of Jupiter's Magnetic Field From Juno's First Nine Orbits

J. E. P. Connerney[1,2], S. Kotsiaros[1,3], R. J. Oliversen[1], J. R. Espley[1], J. L. Joergensen[4], P. S. Joergensen[4], J. M. G. Merayo[4], M. Herceg[4], J. Bloxham[5], K. M. Moore[5], S. J. Bolton[6], and S. M. Levin[7]

[1]NASA Goddard Space Flight Center, Greenbelt, MD, USA, [2]Space Research Corporation, Annapolis, MD, USA, [3]University of Maryland, College Park, MD, USA, [4]Technical University of Denmark (DTU), Kongens Lyngby, Denmark, [5]Harvard University, Cambridge, MA, USA, [6]Southwest Research Institute, San Antonio, TX, USA, [7]Jet Propulsion Laboratory, Pasadena, CA, USA

2018: JRM09

Check for updates

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JGR Planets

RESEARCH ARTICLE

10.1029/2021JE007055

This article is a companion to Bloxham et al. (2022), https://doi.org/10.1029/2021JE007138.

Key Points:
- The Juno spacecraft sampled Jupiter's vector magnetic field along 32 polar passes separated by ~11° longitude at the equator
- A degree 18 spherical harmonic model of Jupiter's magnetic field

A New Model of Jupiter's Magnetic Field at the Completion of Juno's Prime Mission

J. E. P. Connerney[1,2], S. Timmins[2,3], R. J. Oliversen[2], J. R. Espley[2], J. L. Joergensen[4], S. Kotsiaros[4], P. S. Joergensen[4], J. M. G. Merayo[4], M. Herceg[4], J. Bloxham[5], K. M. Moore[6], A. Mura[7], A. Moirano[7], S. J. Bolton[8], and S. M. Levin[6,9]

[1]Space Research Corporation, Annapolis, MD, USA, [2]NASA Goddard Space Flight Center, Greenbelt, MD, USA, [3]ADNET Systems, Inc., Bethesda, MD, USA, [4]Technical University of Denmark (DTU), Kongens Lyngby, Denmark, [5]Harvard University, Cambridge, MA, USA, [6]California Institute of Technology, Pasadena, CA, USA, [7]INAF-IAPS, Rome, Italy, [8]Southwest Research Institute, San Antonio, TX, USA, [9]Jet Propulsion Laboratory (JPL), Pasadena, CA, USA

2021: JRM33

What about other planets?

Mercury	MESSENGER mission	Toepfer et al. EPS 2021, Toepfer et al. Ann. Geo 2022
Earth	IGRF	New model every 5 years
Jupiter	Juno mission	Connerney et al. GRL 2018, Connerney et al. GRL 2021
Ganymede	Juno and Galileo spacecrafts	Weber et al. GRL 2022
Saturn	Cassini mission	Cao et al. 2023
Uranus	Voyager 2	Ness et al. 1989
Neptune	Voyager 2	Connerney et al. 1987

Someone should send a new probe to Uranus or Neptune. There are some plans to go in the 2030s, with an arrival time at 2040s or later...

Magnetic field reconstruction

$$\boldsymbol{B} = -\nabla V$$

$$V = a \sum_{n=1}^{n_{max}} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} P_n^m(cos\theta) \left[g_n^m cos(m\phi) + h_n^m sin(m\phi)\right]$$

You only need take (spherical) derivatives to reconstruct the 3D magnetic field:

$$B_r = -\frac{\partial V}{\partial r} = \sum_{n=1}^{n_{max}} \left(\frac{a}{r}\right)^{n+2} (n+1) \sum_{m=0}^{n} P_n^m(cos\theta) \left[g_n^m cos(m\phi) + h_n^m sin(m\phi)\right]$$

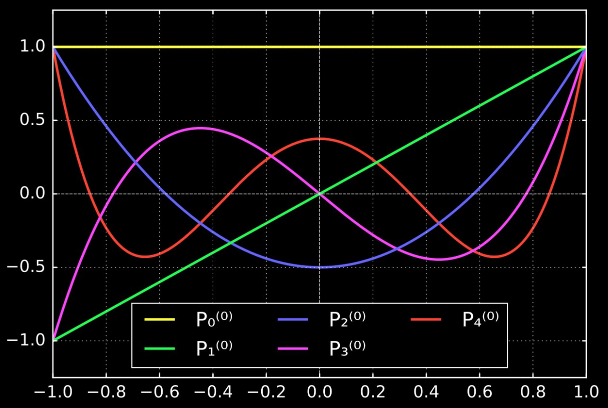
$$B_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \sum_{n=1}^{n_{max}} \left(\frac{a}{r}\right)^{n+2} \sum_{m=0}^{n} \frac{\partial P_n^m(cos\theta)}{\partial \theta} \left[g_n^m cos(m\phi) + h_n^m sin(m\phi)\right]$$

$$B_\phi = -\frac{1}{rsin\theta}\frac{\partial V}{\partial \phi} = -\frac{1}{sin\theta} \sum_{n=1}^{n_{max}} \left(\frac{a}{r}\right)^{n+2} \sum_{m=0}^{n} P_n^m(cos\theta) m \left[-g_n^m sin(m\phi) + h_n^m cos(m\phi)\right]$$

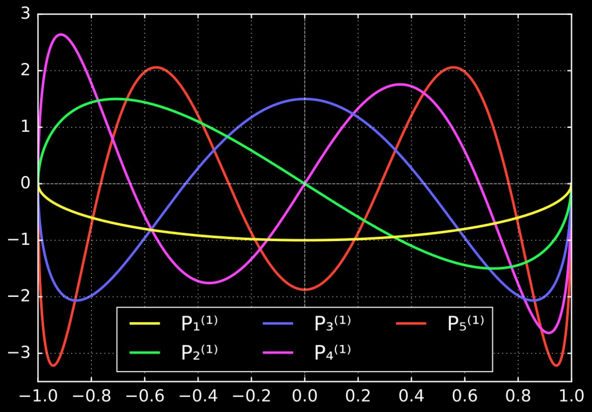
Schmidt quasi-normalized associated Legendre polynomials

$$\boldsymbol{B} = -\nabla V$$

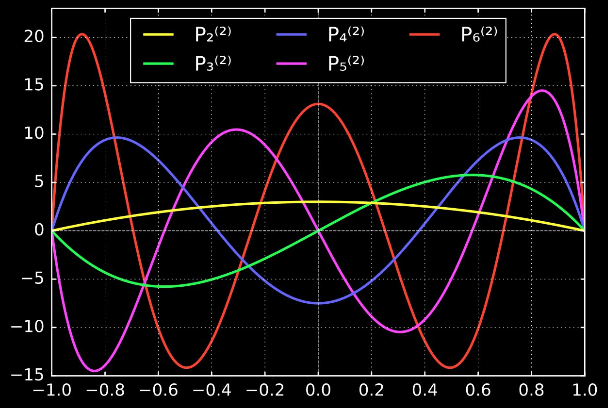
$$V = a \sum_{n=1}^{n_{max}} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} P_n^m(cos\theta)\,[g_n^m cos(m\phi) + h_n^m sin(m\phi)]$$



m=0



m=1



m=2

The latitudinal dependence takes the form of a type of renormalized associated legendre polynomials

Recursive formulas

$$S_{n,m} = \left[\frac{(2-\delta_m^0)(n-m)!}{(n+m)!}\right]^{1/2} \frac{(2n-1)!!}{(n-m)!}$$

Very ugly constant...

$$S_{0,0} = 1$$

$$S_{n,0} = S_{n-1,0}\left[\frac{2n-1}{n}\right]$$

$$S_{n,m} = S_{n,m-1}\sqrt{\frac{(n-m+1)(\delta_m^1+1)}{n+m}}$$

Recursive!

$$P^{0,0} = 1$$

$$P^{n,n} = \sin\theta P^{n-1,m-1}$$

$$P^{n,m} = \cos\theta P^{n-1,m} - K^{n,m}P^{n-2,m}$$

Gaussian normalized associated Legendre polynomials recursive formulas

$K^{n,m} = 0$, n=1

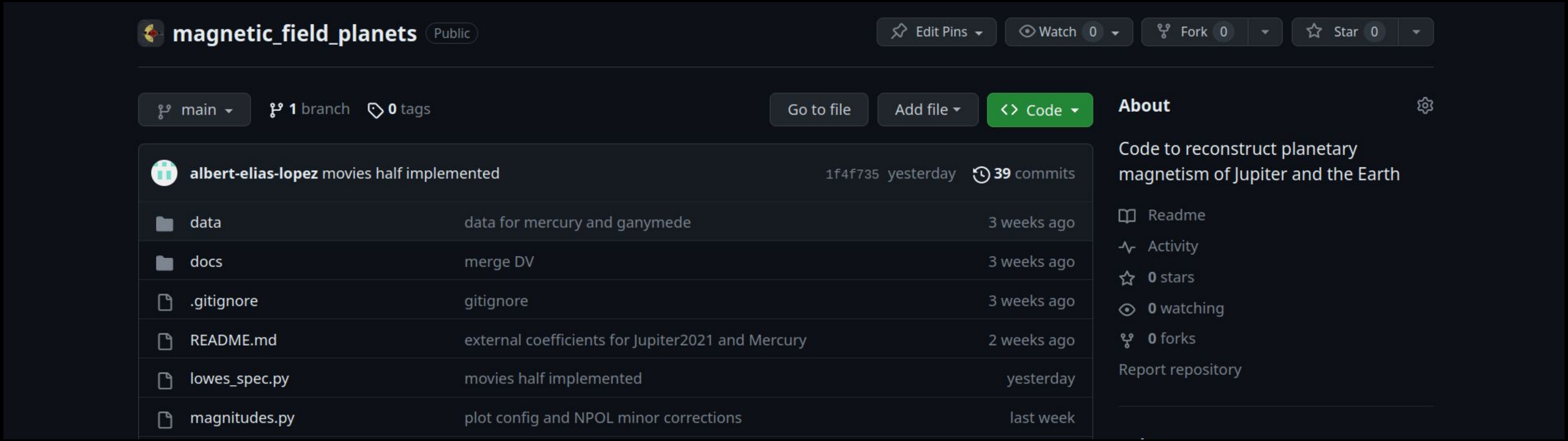
$K^{n,m} = \frac{(n-1)^2 - m^2}{(2n-1)(2n-3)}$, n > 1

$$P_n^m = S_{n,m}P^{n,m}$$

Schmidt quasi-normalized associated Legendre polynomials

Public repository

No need to code this yourself! We made a repository with all the machinery.



https://github.com/csic-ice-imagine/magnetic_field_planets

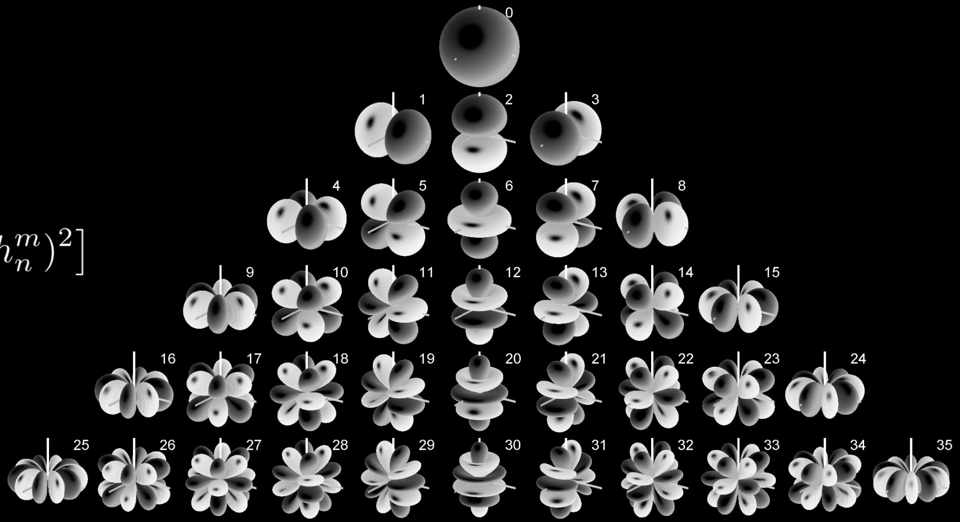
Earth magnetic field inside: Lowes spectrum

$$V = a \sum_{n=1}^{n_{max}} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} P_n^m(cos\theta)\left[g_n^m cos(m\phi) + h_n^m sin(m\phi)\right]$$

$$R_n = (n+1) \sum_{m=0}^{n} \left[(g_n^m)^2 + (h_n^m)^2\right]$$

$$R_n(r) = \left(\frac{a}{r}\right)^{2n+4} R_n = (n+1) \sum_{m=0}^{n} \left[(g_n^m)^2 + (h_n^m)^2\right]$$

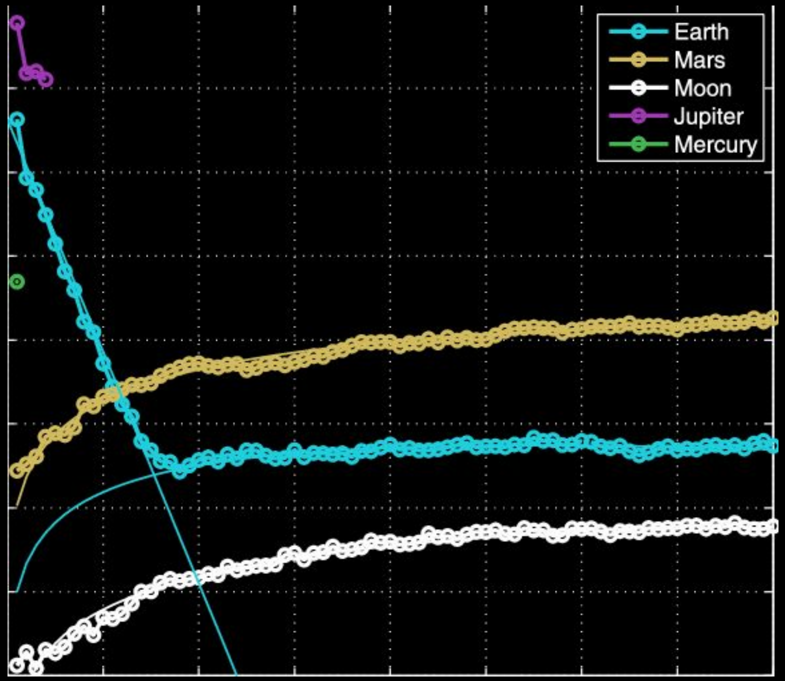
$$2\mu_0 E_B(r) = \sum_{n=0}^{\infty} R_n(r)$$



Lowes spectrum

$$V = a \sum_{n=1}^{n_{max}} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} P_n^m(cos\theta)\left[g_n^m cos(m\phi) + h_n^m sin(m\phi)\right]$$

logE

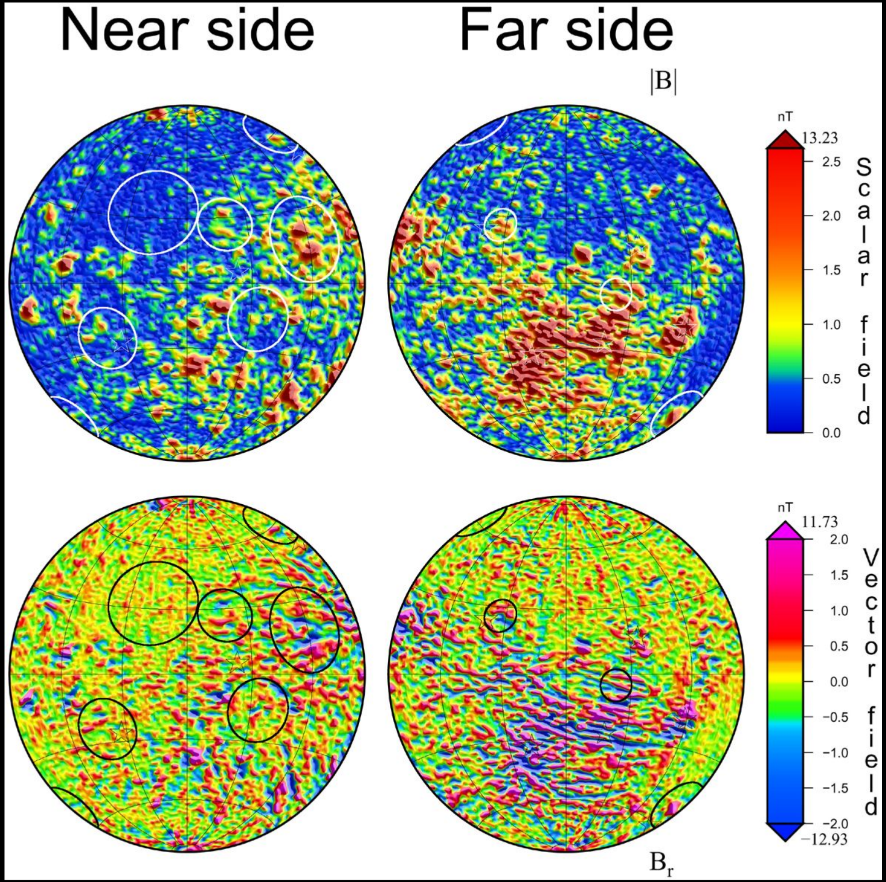


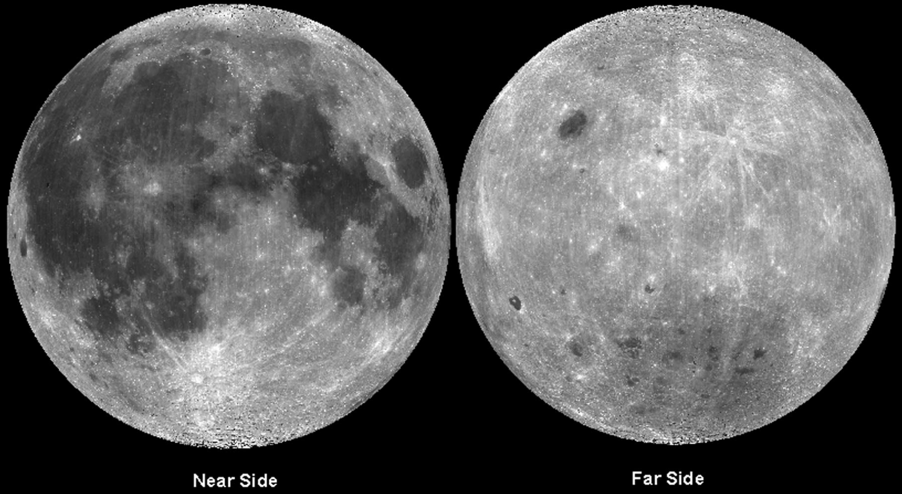
Spherical harmonic degree (l)

Crustal (or rock) magnetization has no dipole or small importance component. All scales remain approximately equal.

For Earth, crustal magnetic fields dominate over the interior ones from degree 13 onwards.

Moon





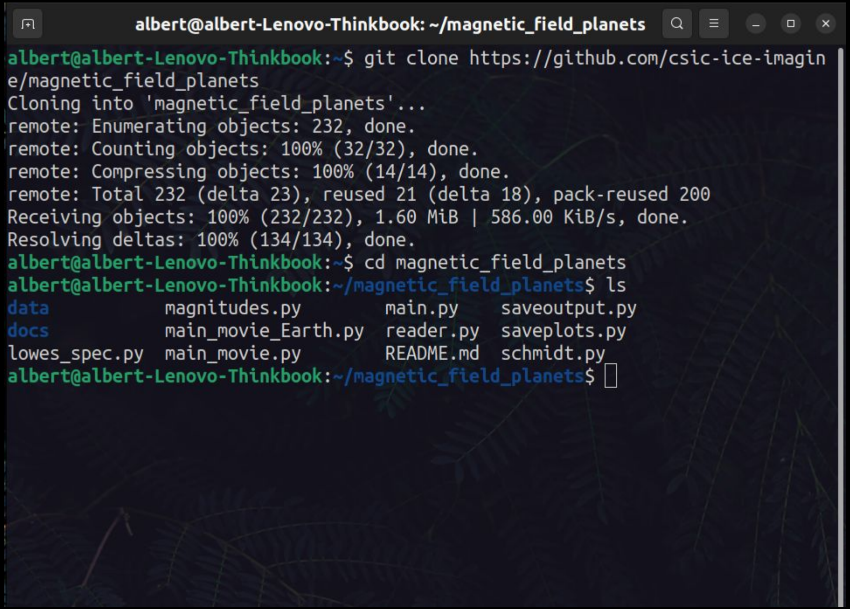
The moon only has residual magnetic field from and old magnetic field.

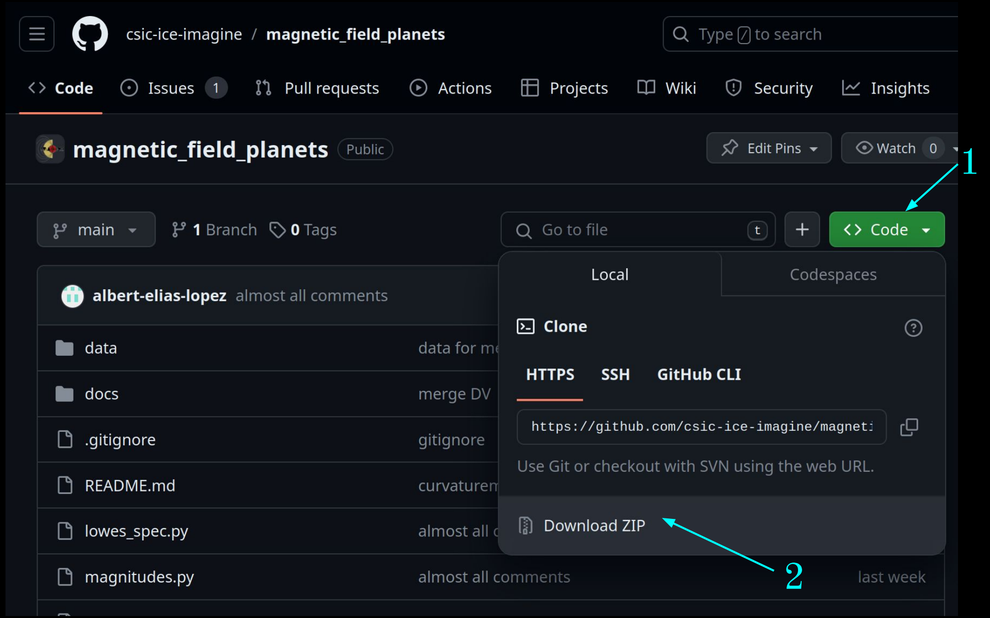
Public repository

0. Have python 3 installed

1. Download the repository (either clone from git or download and decompress directly on the web page)

    git clone https://github.com/csic-ice-imagine/magnetic_field_planets





Public repository

2. Look around the directory. You will only need to use main.py, as it calls the other functions defined in the other files:

lowes_spec.py	Calculates and plot Lowes spectra
magnitudes.py	Calculates and plot the curl, divergence and curvature of **B**
reader.py	Reads the constants defined in tables
saveplots.py	Defines and save the plots
schmidt.py	Recursively calculates constants, Schmidt polynomials and **B**
saveoutput.py	Saves output for 3D visualisation

main_movie.py and main_movie_Earth.py are versions of main.py to recursively plot in radius and Earth data, respectively. All data tables are located in data/, and some pdfs with all the formulae used are located in docs/.

Public repository

```
#---------------------------------------------------------------------------------
# Grid resolution
Ntheta = 50      # Latitudinal points (North-South direction)
Nphi = 2*Ntheta  # Longitudial points (East-West direction)
Nr = 1           # Radial points (change only to generate 3D output)

# Radius considered in the map plot, and name of the corresponding images
# This should be the actual radius in kilometers (6371.2/72492 for
# Earth/Jupiter), but we renormalize to 1, since r/a is what matters.
rc = 1.00

# String used for naming the output files
rc_file = '%.2f'%rc
rc_file = rc_file.replace(".","_")

# Planet (or satellite) to choose. Raw data is located in folder data/
planet, year = "My_own", 2020
# You can choose either Earth, Jupiter, Jupiter_2021, Saturn, Neptune, Uranus,
# Mercury and Ganymede or My_own. Anything else will make the code stop.  If
# you choose Earth, you also need to choose a year, which can only be: 1900,
# 1905, 1910, ..., to 2020.

# Definition of the spherical grid matrices
phi    = np.linspace(0, 2*np.pi, num=Nphi)
theta  = np.linspace(np.pi / Ntheta, np.pi * (1 - 1 /  Ntheta), num=Ntheta)
# To calculate curvature/curl it is recommended to use a fixed value
# theta  = np.linspace(np.pi / 20, np.pi * (1 - 1 / 20), num=Ntheta)
# to avoid doing operations too close to the axis.

#---------------------------------------------------------------------------------
# Switches to save projections in plane and Mollweide projections. Coastlines
#  are included in Earth plots.
planeproj, mollweideproj = True, True
# If you have successfully installed the ccrs library you can put the Earth
# coastline in the Earth plane projections also, using the boolean ccrs_library
ccrs_library = True

# ATTENTION: To plot using the Mollweide projection you need the ccrs library.
# The combination mollweideproj=True, ccrs_library=False will crash if you have
# not installed this library

# Switch to save the Lowes spectrum for the given radius
lowes = True
# Switch to save the Lowes spectrum for a number of radii
multiple_lowes_r, lowes_radii = False, np.array([1.45,1.30,1.15,1.00,0.85,0.70,0.55])
# Switch to plot the curl, divergence and curvature of the magnetic field
plot_magnitudes = False
```

3. Open main.py.

You will only need to play with the 50ish first lines. Things that can be changed:

- Latitude-longitude resolution
- Radius (in corresponding planetary radii units)
- Save plots in plane/Mollweide projections
- Save Lowes spectrum
- Plot curl, divergence, and curvature

Increasing resolution will exponentially increase the computational time. To run the code you will only need to do:

```
python main.py
```

Public repository

4. Before playing with the code, try to install cartopy to enable for the option for Mollweide projection and coastlines (https://scitools.org.uk/cartopy/docs/latest/installing.html). Ideally, these commands should be enough:

```
pip install cartopy
```

or

```
conda install -c conda-forge cartopy
```

In my Ubuntu 22.04 laptop, I had to fight a little...

```
sudo apt-get install libproj-dev proj-data proj-bin
sudo apt-get install libgeos-dev
sudo pip install cython
sudo pip install cartopy
```

If it does not work, do not worry. Your plots will only be a square projection of the sphere (not as aesthetic). In this case you will have to set `ccrs_library = False` and `mollweideproj = False`.

Q1

Up to which multipole degree (n) do each magnetic field models have? Look in each file in data/

Q1

Up to which multipole degree (n) do each magnetic field models have? Look in each file in data/

Earth: 13

Jupiter 2018: 10

Jupiter 2021: 30 + 1 (well resolved until 18)

Saturn: 6

Mercury: 3 + 1

Uranus: 3

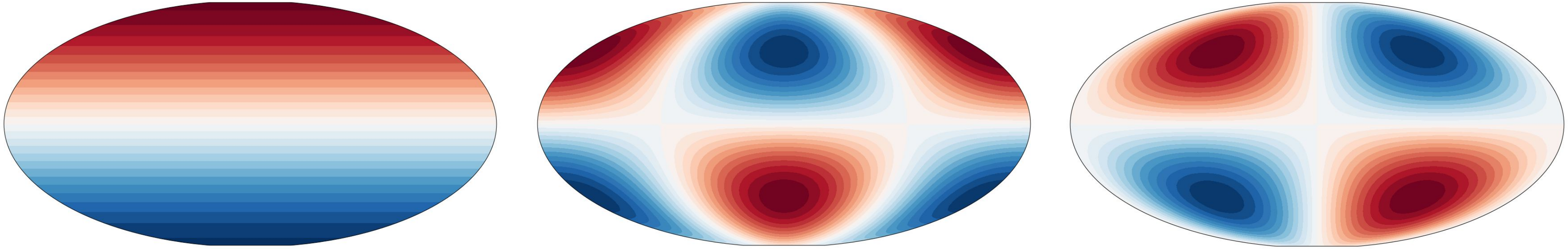
Neptune: 3

Ganymede: 2

Attention: Earth has a set of constant for every 5 years since 1900!

Q2

Using the *"My_own"* option and changing the file in data/my_own_planet.txt play with some multipoles (change some 0's to 1's) to recover plots like g_{10}, g_{21}, or h_{21}, respectively. You should mostly look at the radial field direction.



Find which multipole (g_{nm} or h_{nm}) creates this Br plot:

Q2

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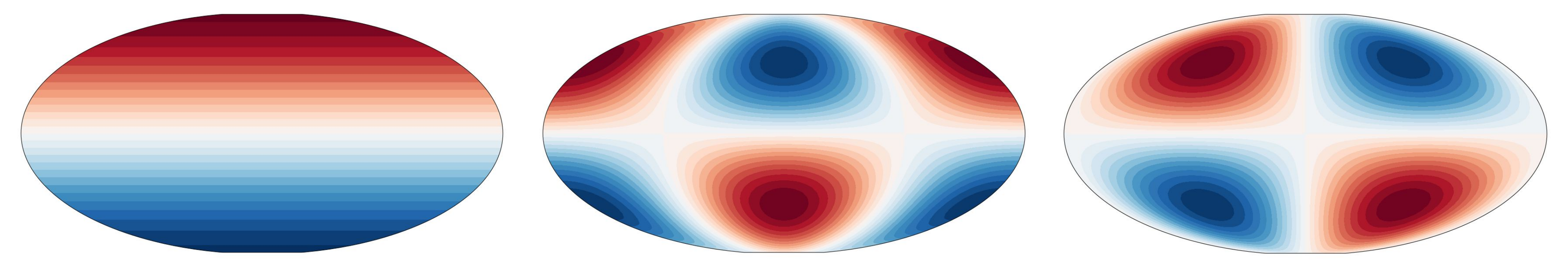
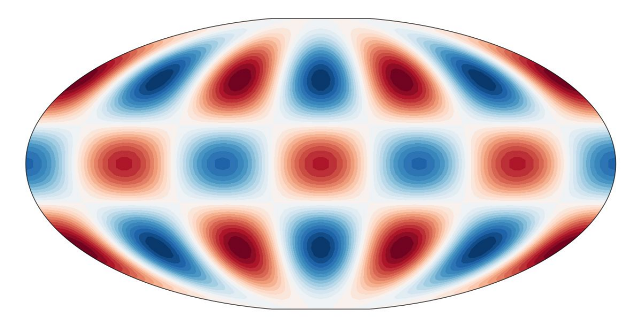


Find which multipole (g_{nm} or h_{nm}) creates this Br plot:



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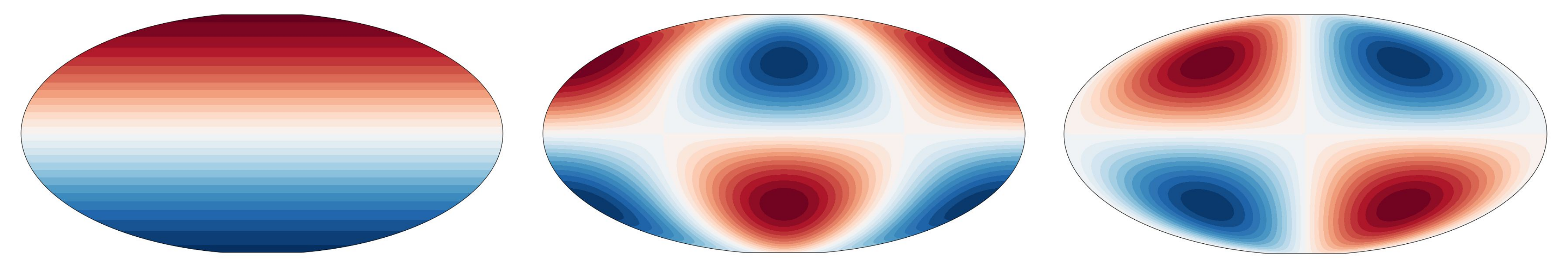
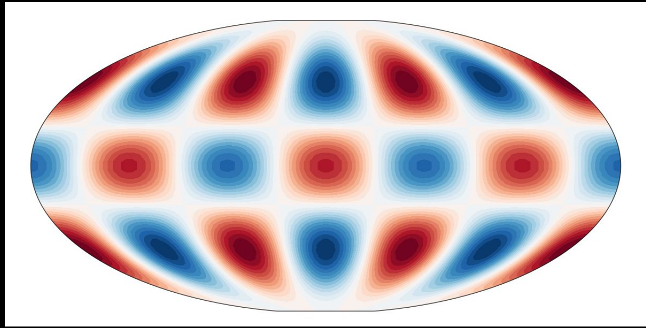


Find which multipole (g_{nm} or h_{nm}) creates this Br plot: g_{53}

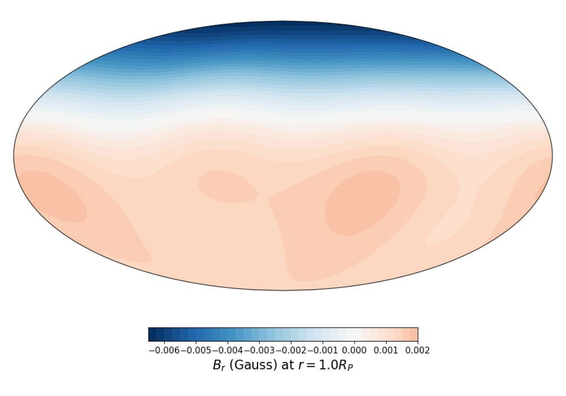


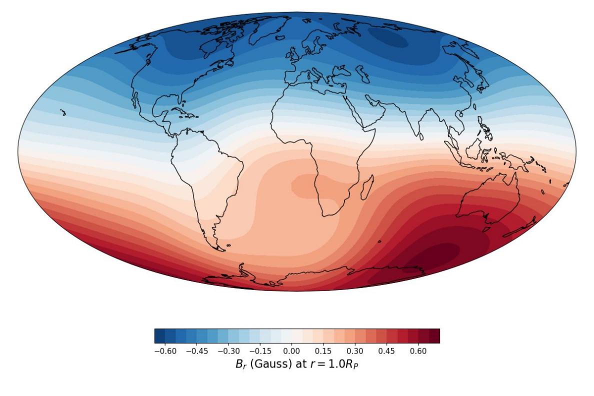
Q3

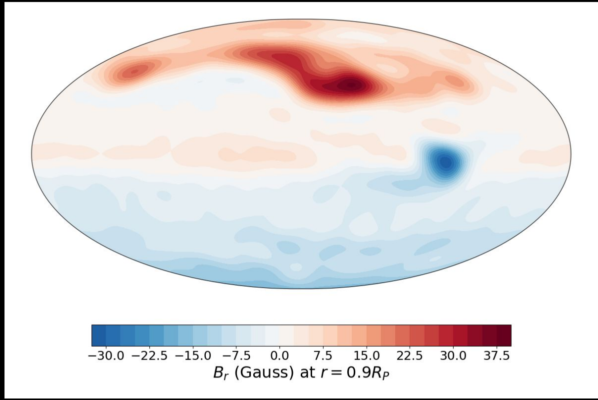
You can start running main.py, use `lowes = True`, to also save spectral plots. Use some small resolution (at most N_θ=50) to plot all planets available and see the differences. Be aware that if you choose Earth you can specify which year you want (from 1900 to 2020 every 5 years). Put True or False:

- Earth has an almost constant magnetic field modulus throughout its surface.
- Earth magnetic inclination has a nearly perfect horizon with 0º tilt.
- Saturn's magnetic field is aligned with the rotation axis.
- Ganymede's magnetic field seems to be very different from the other planets.
- Uranus' magnetic field is aligned with its rotation axis.
- Neptune's magnetic field is aligned with its rotation axis.
- Jupiter magnetic field is measured more accurately than Earth's.
- At the dynamo surface Earth's is better measured than any other planet.
- Mercury's magnetic field is stronger than Earth's.

Q3



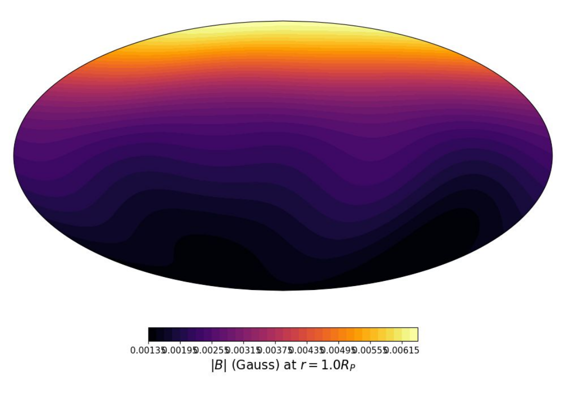


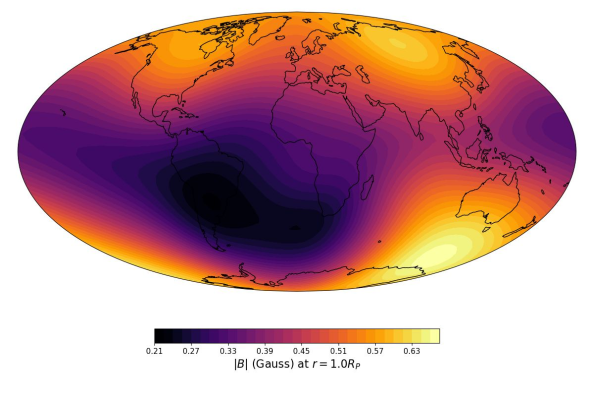


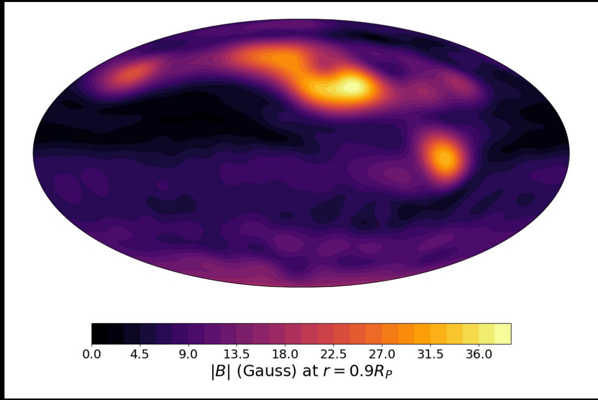
Mercury

Earth

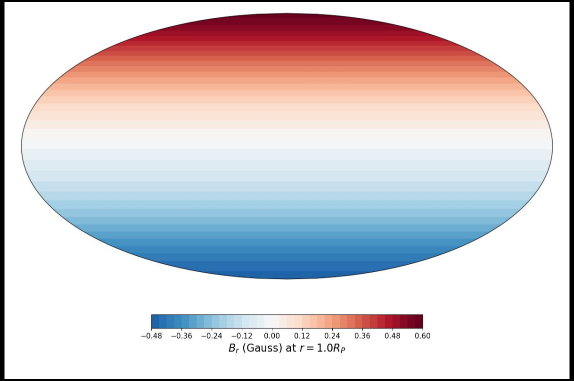
Jupiter

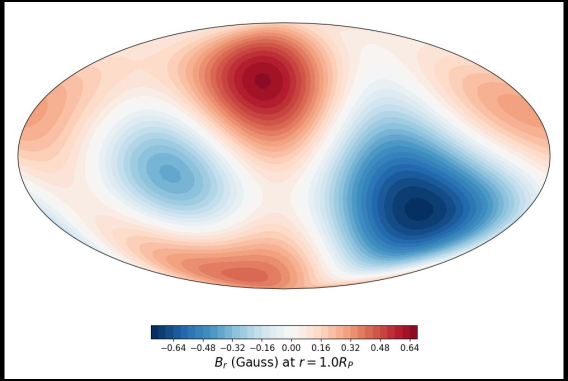


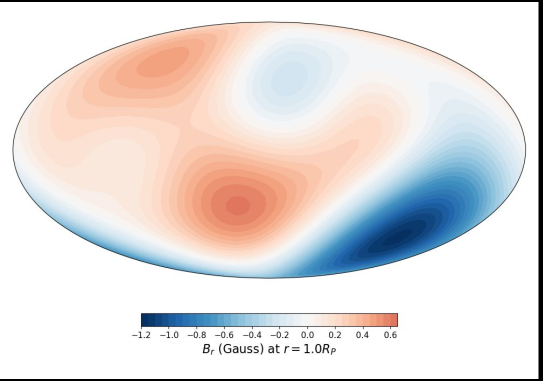




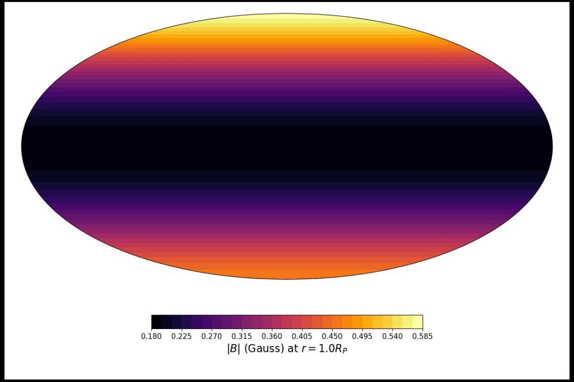
Q3

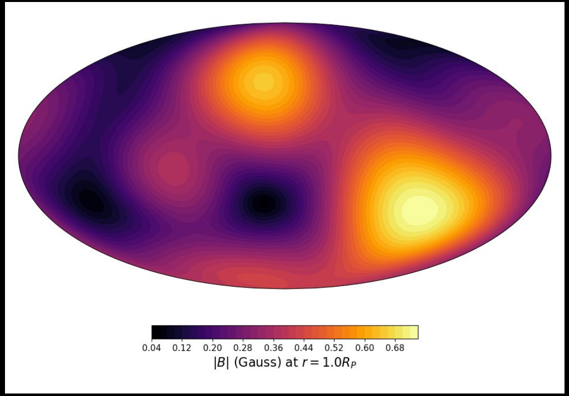


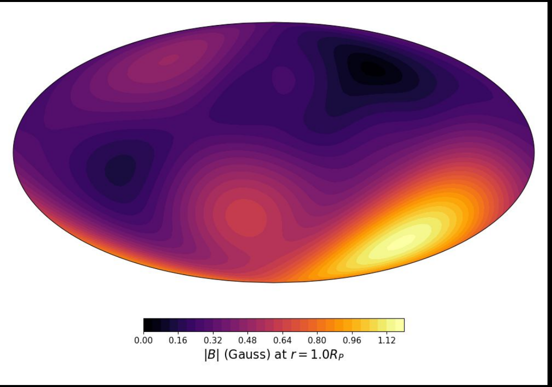




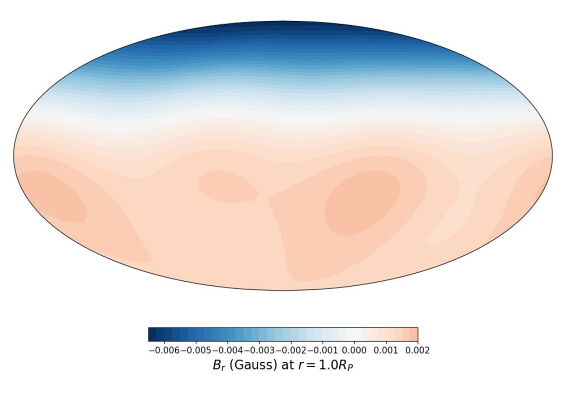
Saturn Neptune Uranus

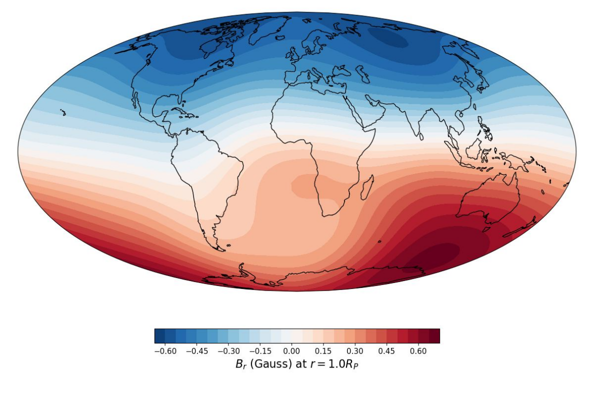


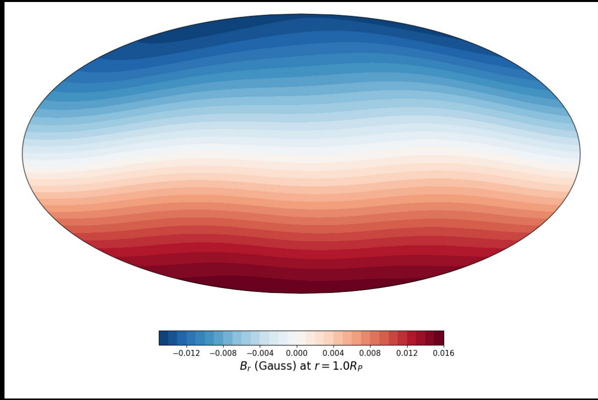




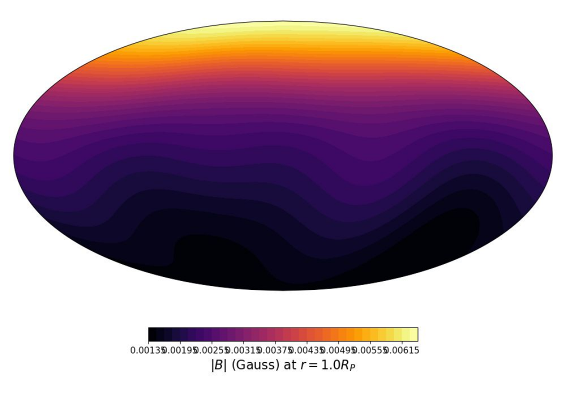
Q3

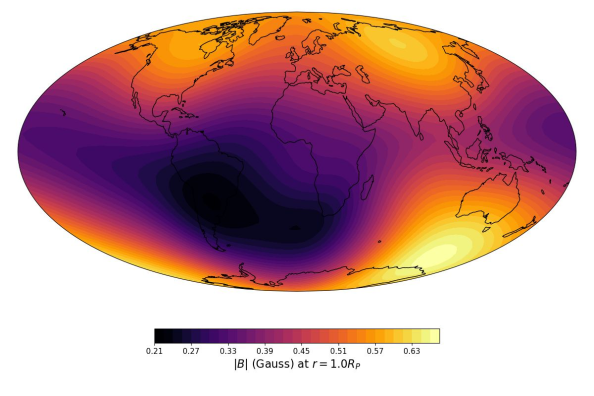


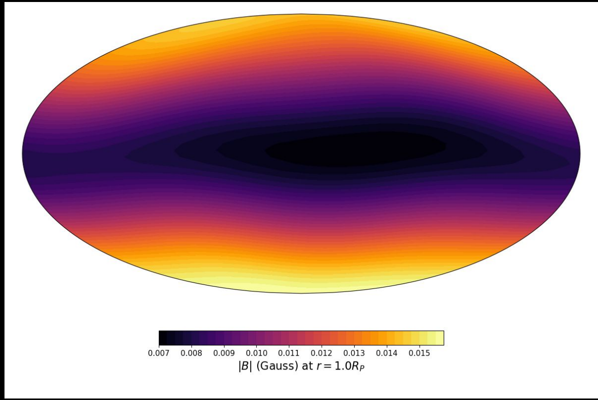




Mercury | Earth | Ganymede

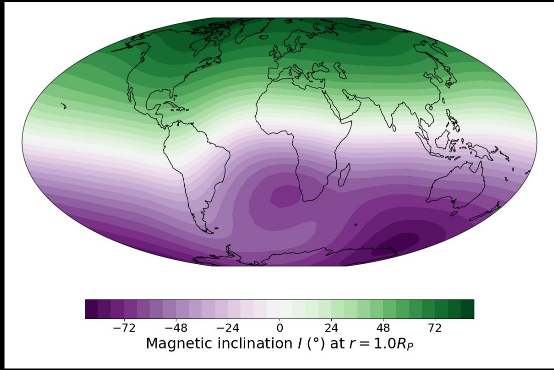






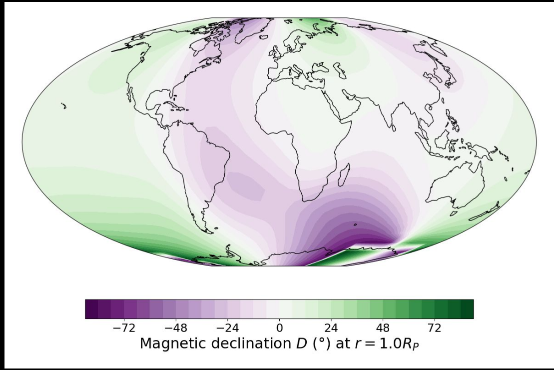
Q3

- Earth has an almost constant magnetic field modulus throughout its surface. F
- Earth magnetic inclination has a nearly perfect horizon with 0° tilt. F
- Saturn’s magnetic field is aligned with the rotation axis. T
- Ganymede’s magnetic field seems to be very different from the other planets. F
- Uranus’ magnetic field is aligned with its rotation axis. F
- Neptune’s magnetic field is aligned with its rotation axis. F
- Jupiter’s magnetic field is measured more accurately than Earth’s. ?
- At the dynamo surface Earth’s is better measured than any other planet. ?
- Mercury’s magnetic field is stronger than Earth’s. F



Inclination

Angle with the Earth’s surface. Positive (negative) means going in (out).



Declination

Deviation from true north. Positive (negative) means towards the east (west)

Q4

Use the main_movie_Earth.py to produce the 5-year frequency images. You can play with the resolution and the radius. Try some other radius other than 1 (not less than 0.5). Is the magnetic field static? Towards which direction does it shift to?

Q4

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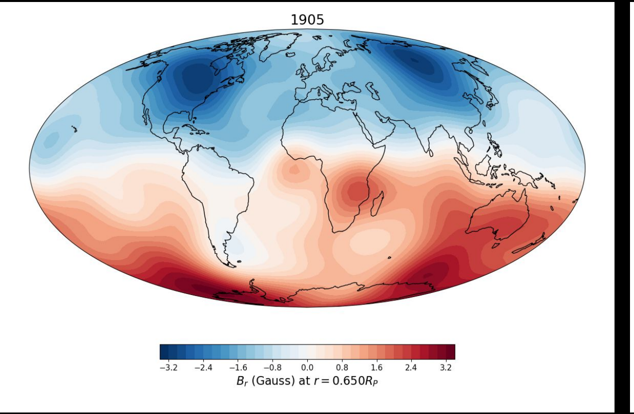
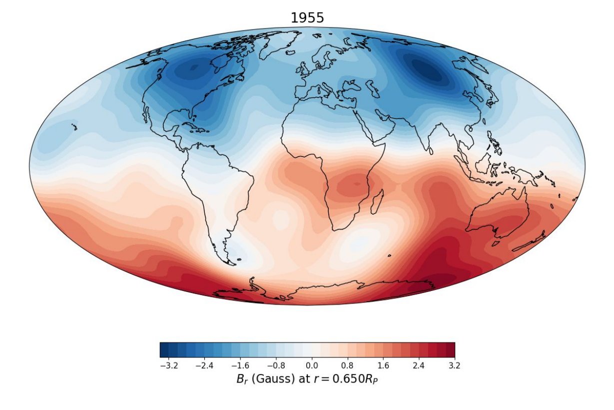
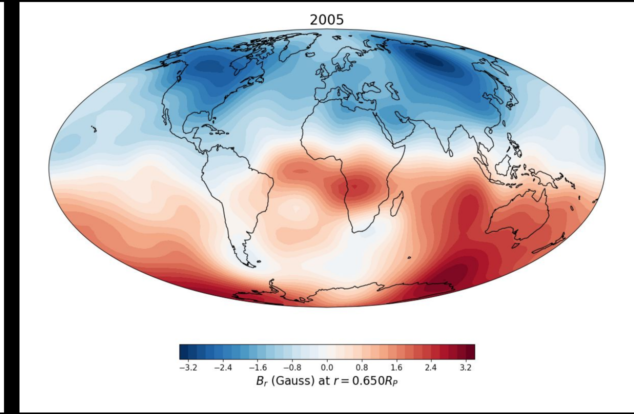




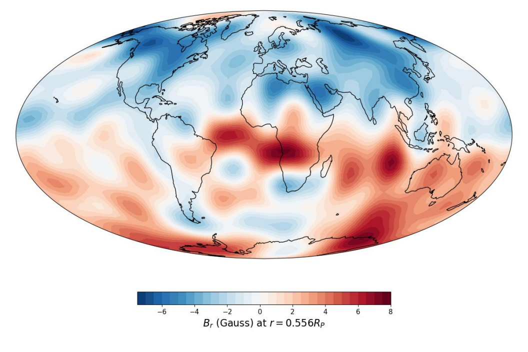


This change in known as secular variation, and in case of Earth is seen as a West-ward drift of the field. This procedures are used to obtain the velocity (tangential to the sphere) at the base of the dynamo. For Jupiter it has also by comparing the 2018 and 2021 models. We have not been able to measure any other planet.

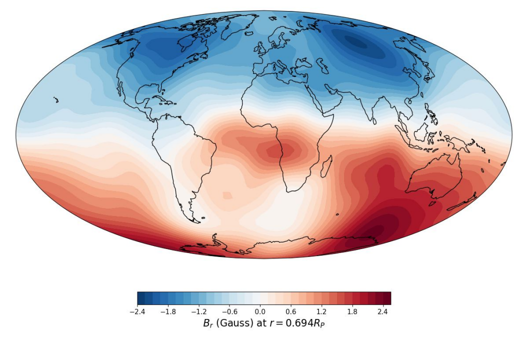
Q5

Use the main_movie.py for some planets to produce different plots at different radii. Which are the radii that correspond to a flat magnetic spectrum? Why are we not able to find the same for planets other than Earth and Jupiter?

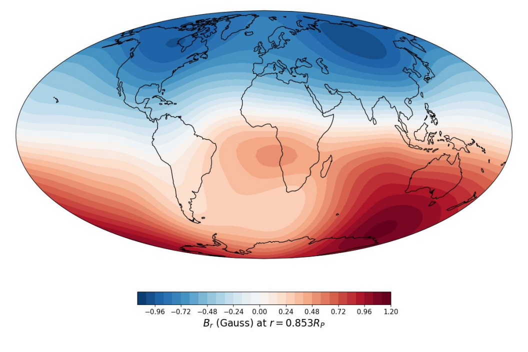
Q5



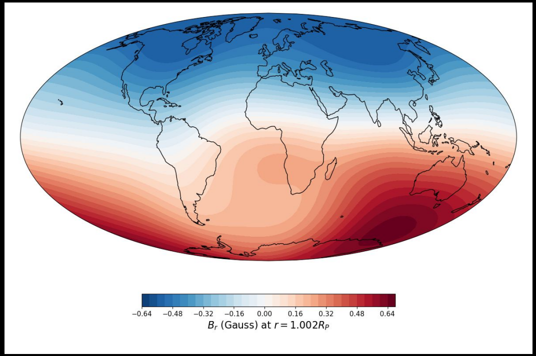
r=0.55



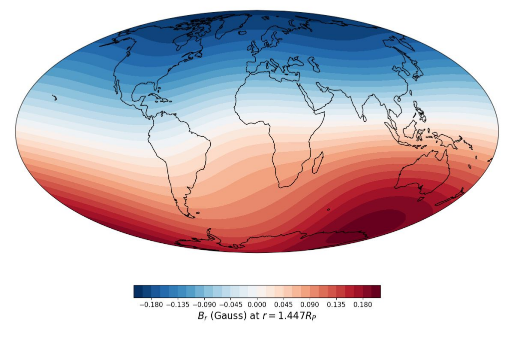
r=0.70



r=0.85

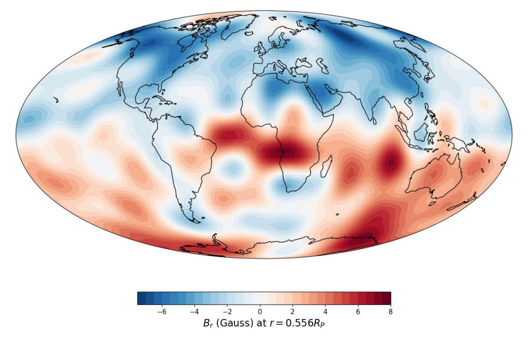


r=1.00



r=1.45

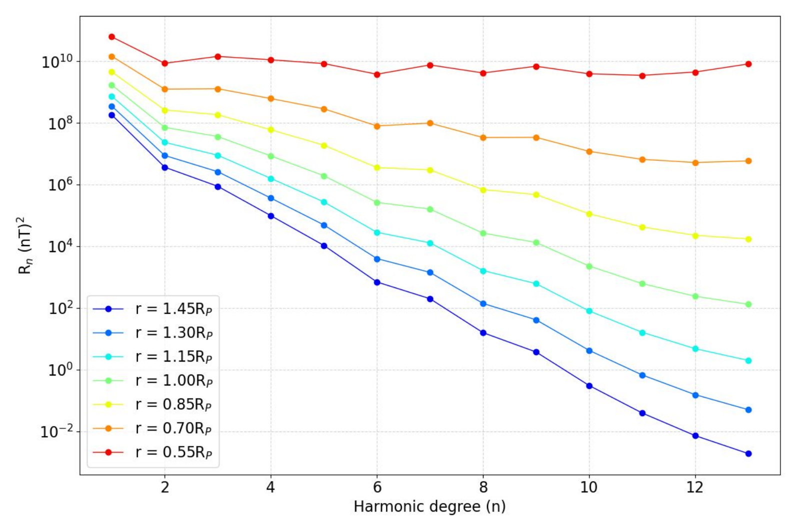
Q5

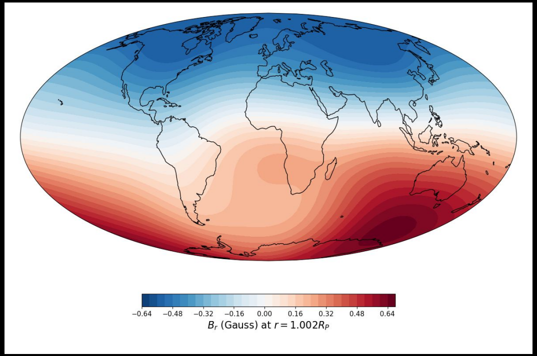


r=0.55

Apart from the dipole, the other multiples are predicted to lead to a flat spectrum at the dynamo.

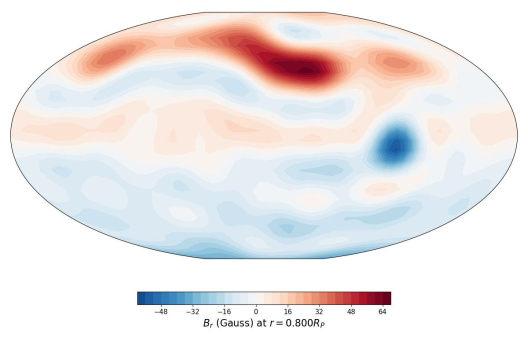
Far away, other multipoles lose importance.



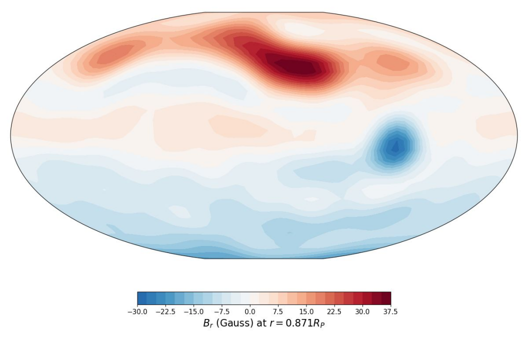


r=1.00

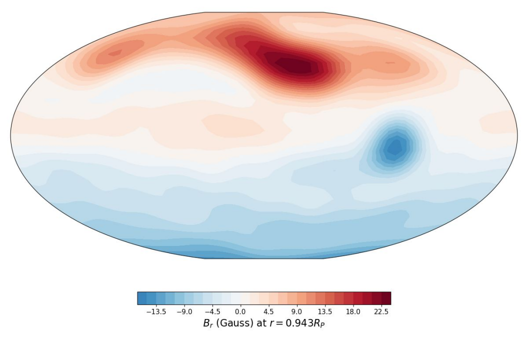
Q5



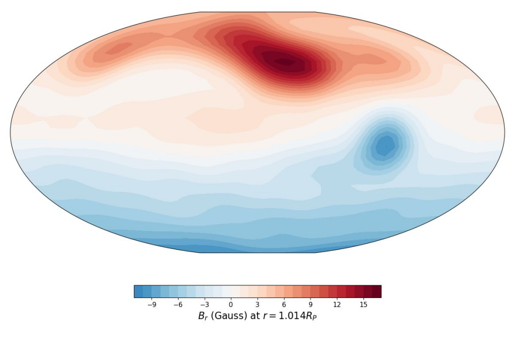
r=0.80



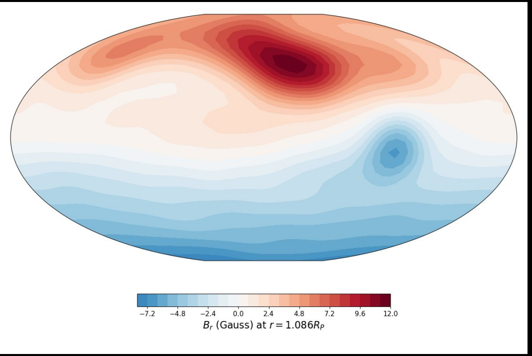
r=0.87



r=0.94

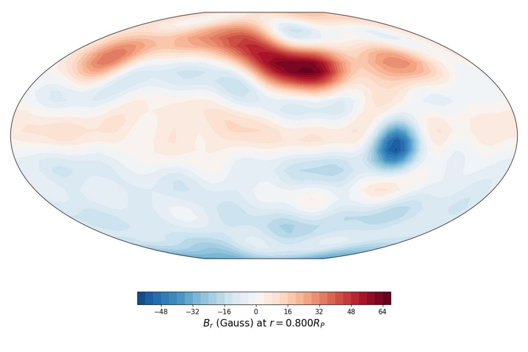


r=1.01



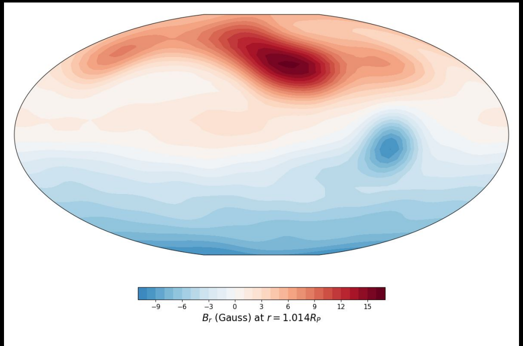
r=1.45

Q5



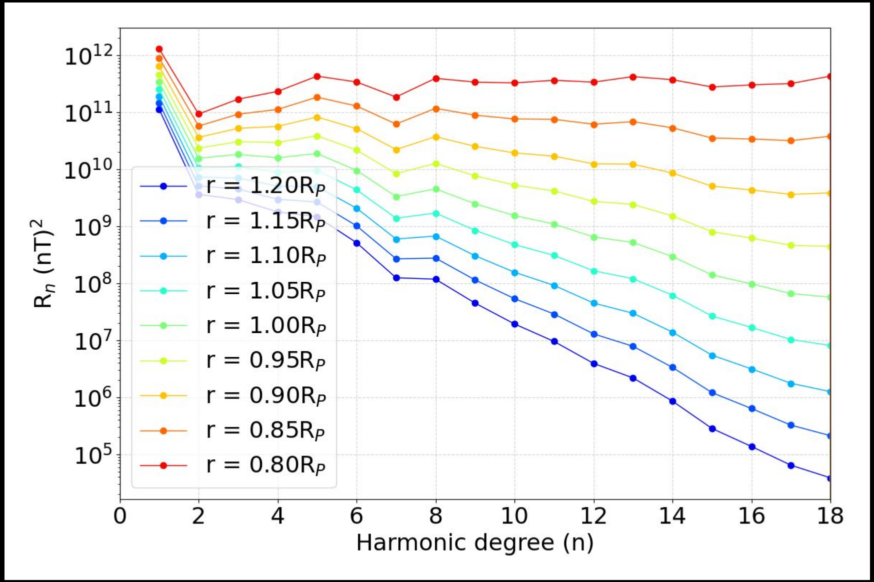
−48 −32 −16 0 16 32 48 64
B_r (Gauss) at $r = 0.800R_P$

r=0.80



−9 −6 −3 0 3 6 9 12 15
B_r (Gauss) at $r = 1.014R_P$

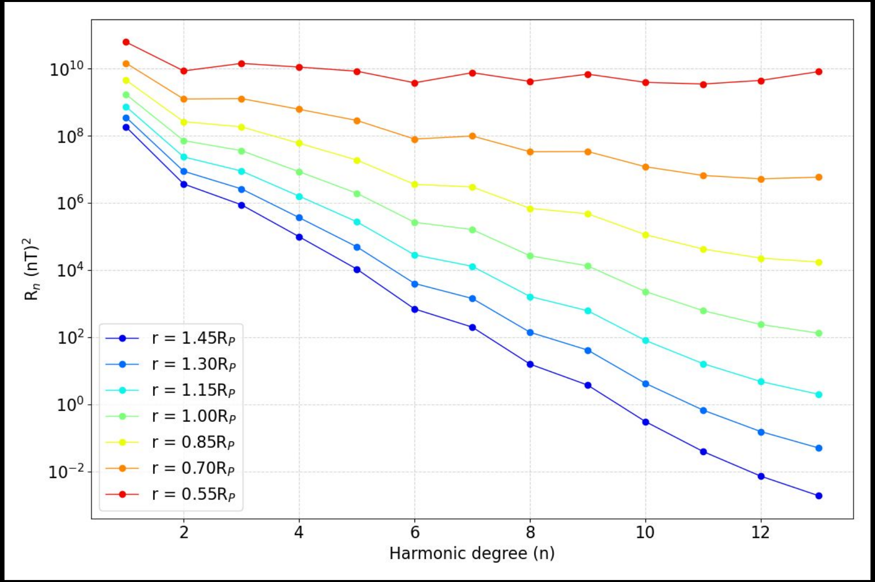
r=1.01



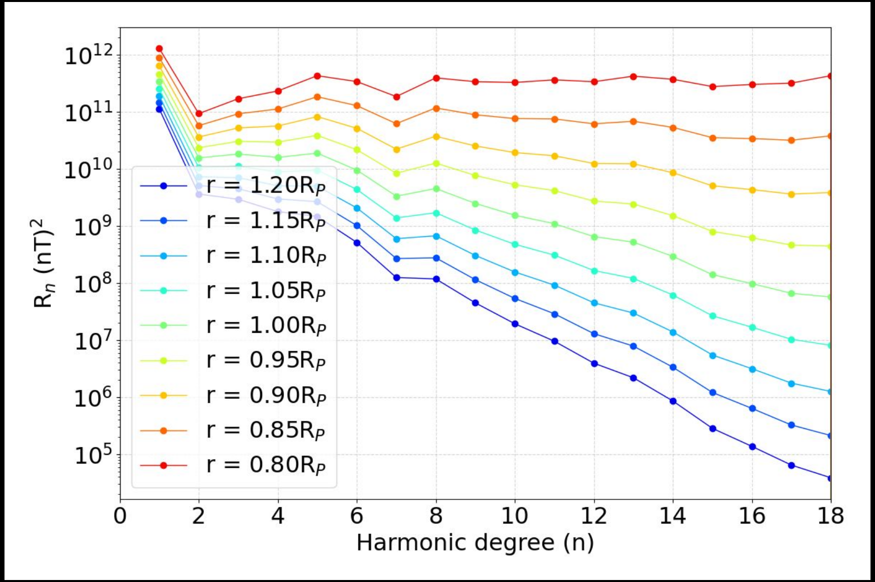
R_n $(nT)^2$
10^{12}
10^{11}
10^{10}
10^9
10^8
10^7
10^6
10^5
0 2 4 6 8 10 12 14 16 18
Harmonic degree (n)
r = 1.20R_P
r = 1.15R_P
r = 1.10R_P
r = 1.05R_P
r = 1.00R_P
r = 0.95R_P
r = 0.90R_P
r = 0.85R_P
r = 0.80R_P

Q5

Earth



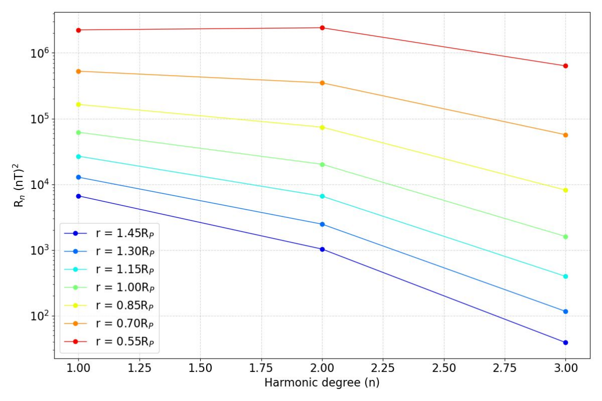
Jupiter



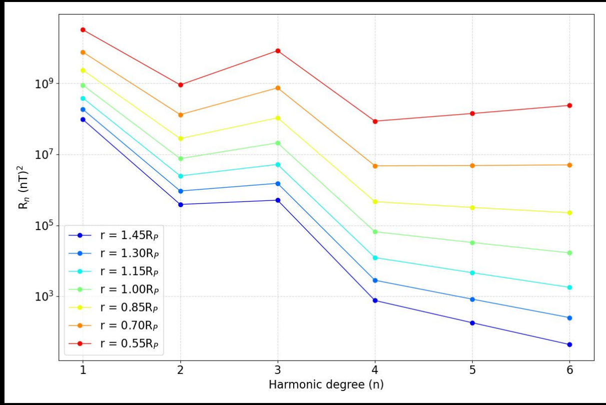
Earth has more multipoles measured (more than 500), but at l=14 and higher the crustal magnetization dominates. Jupiter does not have any other internal sources, therefore all multipoles are attributed to the internal dynamo.

Q5

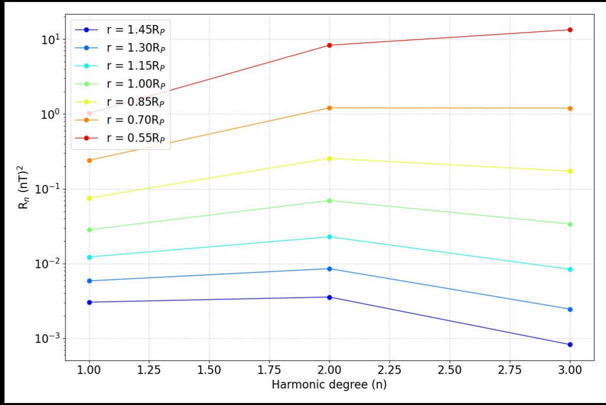
Mercury



Saturn



Uranus



For other planets this does not work, too little multipoles have been accurately measured.

Other planets?

Earth	Yes
Moon	Past
Jupiter	Yes
Saturn	Yes
Uranus	Yes
Neptune	Yes
Venus	No?
Mars	Past
Mercury	Yes
Ganymede	Yes
Other moons	No?
Exoplanets	Yes?