LISA parameter estimation with time-domain response

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Motivation for using time-domain response in LISA

- Parameter estimation and searches use *matched filtering* defined in Fourier domain

$$(d|h) = 4\mathrm{Re}\int_0^\infty \frac{\tilde{d}(f)\tilde{h}^*(f)}{S_n(f)}df$$

- For ground-based detectors (LIGO), the response is constant in time

$$F_+ h_+(t) + F_\times h_\times(t)$$

- For space-based (LISA), response of each arm is time-dependente dependent

$$y_{slr} = \left(x_+(t)\left(h_+(t_s) - h_+(t_r)\right) + x_\times(t)\left(h_\times(t_s) - h_\times(t_r)\right)\right/(1 - \mathbf{k}\cdot\mathbf{n})$$

- Time delay interferometry (TDI) reduces the laser frequency noise produced by deviations from the equilateral triangular shape

- In Fourier domain, LISA response requires the development of *transfer functions*

  both for amplitude and phase
  - Aligned-spin case EOBNRv2HM (S. Marsat+, 2021)
  - Precessing case IMRPhenomXPHM (G. Pratten+, 2023)

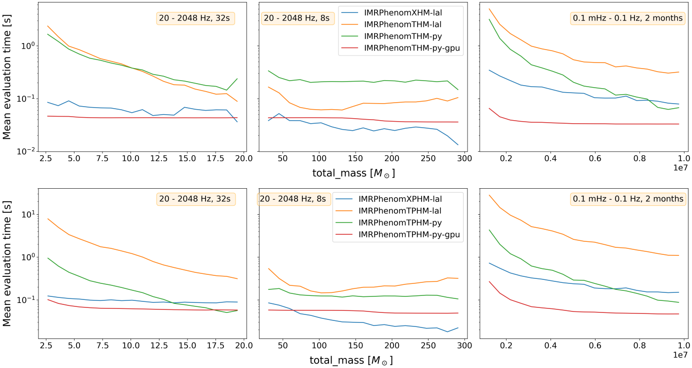
- More complicated effects like eccentricity or memory can be difficult to incorporate in the transfer function formalism

- In time domain, there is no theoretical challenge, but a computational challenge: need for a fine time sampling so the Fast Fourier transform algorithm works

IMRPhenomT family (H. Estellés+ 2021)

- Analytical expressions for amplitude and phase as a function of time
- Include the effects of subdominant harmonics 22, 21, 33, 44, 55
- Precession prescriptions
  - Post-Newtonian Euler angles (NNLO, single-spin)
  - Multi-Scale Analysis (MSA, double-spin)
  - Numerical evolution of spins PN equations (SpinTaylor)

- Reimplemented in python (`phenomxpy`)
  - `numpy` vectorization
  - `numba` just-in-time compilation
  - cupy GPU support
  - Parallelization of time array, for one waveform
  - Efficient calculation of polarizations from rotating frames (M. Boyle+, 2014)
  - 20 mode (M. Roselló, 2024)
  - Extension to eccentricity in progress (M. de Lluc Planas, A. Ramos-Buades)



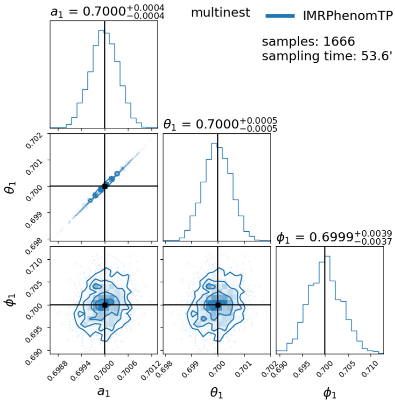
LISA parameter estimation pipeline

Parameter estimation pipeline

- IMRPhenomT $h_{+,\times}(t)$ (GPU `phenomxpy`)
- Time domain LISA arm response & TDI at delayed times (GPU `LISA Data Challenge tools`)
- Fourier transform and likelihood calculation for each TDI channel (AET) (GPU `phenomxpy`)
- Sampling with `Bilby` (multinest, dynesty, ptemcee)

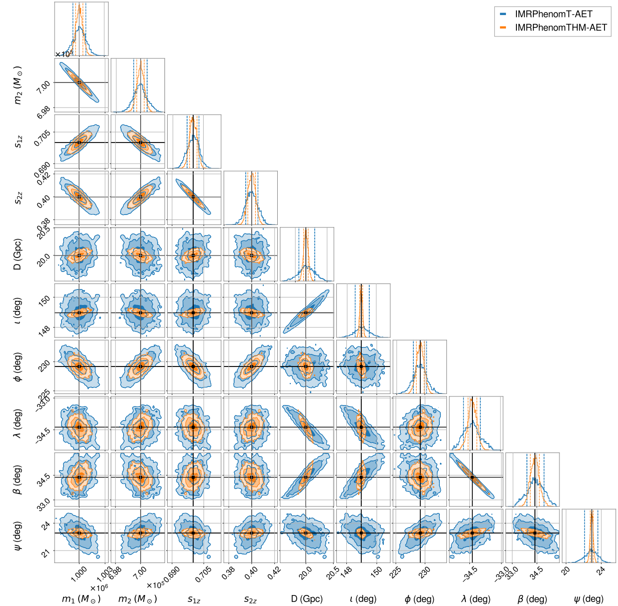
Injection of a Massive Black Hole Binary

- $M_{\rm tot} = 1.7 \cdot 10^6\ M_\odot$
- Mass ratio $= 1.4$
- Spins $a_{1,2} = 0.7, 0.4$
- Distance $= 20\ Gpc$
- $f_{\rm low} = 0.1 mHz$
- Sampling rate $= 0.2$ Hz
- Duration $= 61$ days
- SNR ~ 3800
- Uniform, wide priors
- Zero noise

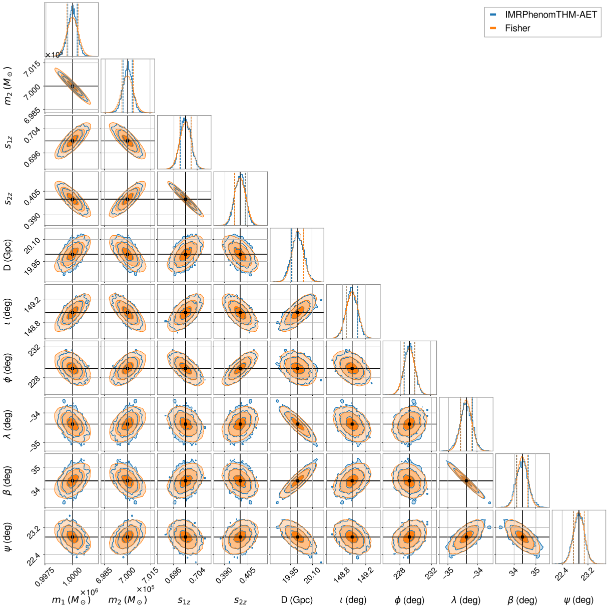


Aligned spins: Effect of subdominant modes

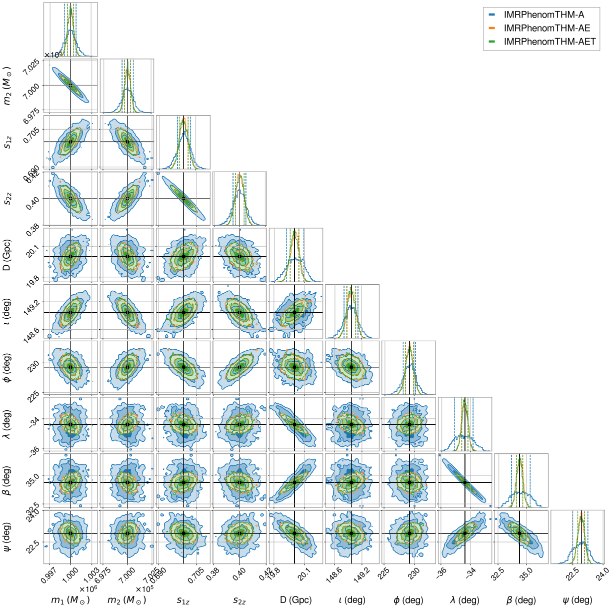
(ptemcee Fisher initialization)



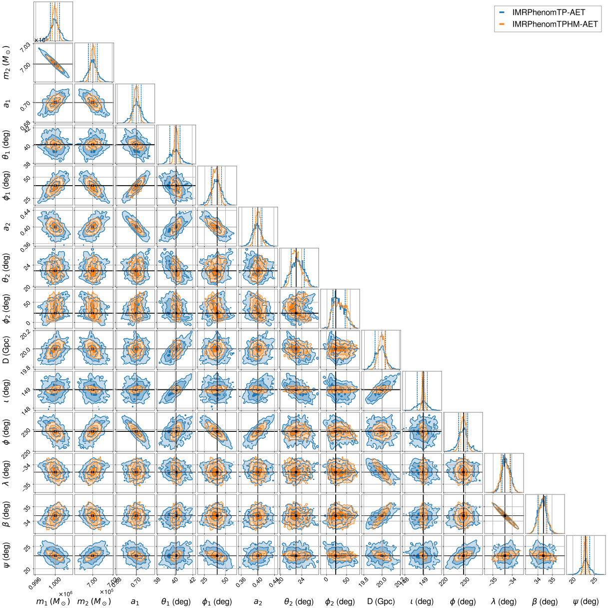
Aligned spins: Comparison with Fisher analysis



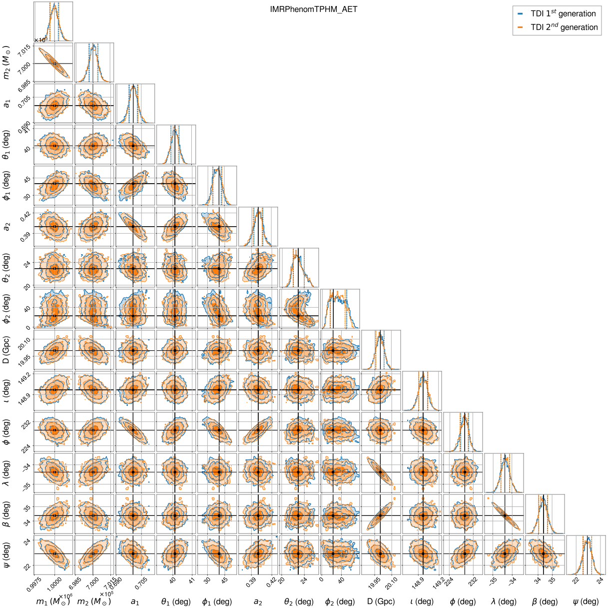
Aligned spins: Comparison between TDI channels



Precessing spins: Effect of subdominant modes



Precessing spins: TDI generations 1^{st} vs 2^{nd}



Conclusions

- Speed-up in waveform generation and LISA response $\rightarrow$ parameter estimation tractable
- First Bayesian analysis of MBBHs for LISA with a time-domain response including the effects of subdominant harmonics and precession
- Mass components recovered at 0.1% accuracy. Spins components at 1%
- Sampling all the parameters requires a smarter initialization of the sampler or more restrictive priors
- Smarter algorithms combining non-uniform time grids and interpolation (Multibanding, Relative binning, FANTA, etc.) will reduce more the computational cost

IMRPhenomT* (H. Estellés+ 2021)

- Analytical expressions for amplitude and phase as a function of time
- Include the effects of subdominant harmonics 22, 21, 33, 44, 55
- Precession
  - Post-Newtonian Euler angles (NNLO, single-spin)
  - Multi-Scale Analysis (MSA, double-spin)
  - Numerical evolution of spins PN equations (SpinTaylor)
  - No mode asymmetries

- Reimplemented in python (`phenomxpy`)
  - `numpy` vectorization
  - `numba` just-in-time compilation
  - cupy GPU support
  - Parallelization of time array, for one waveform
  - Efficient calculation of polarizations from rotating frames (M. Boyle+, 2014)
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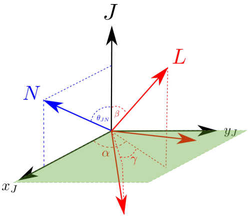
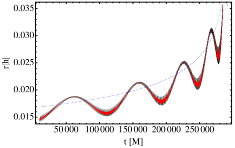
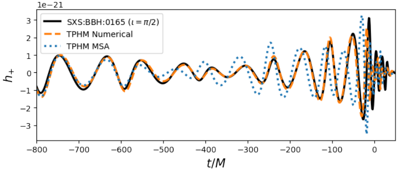




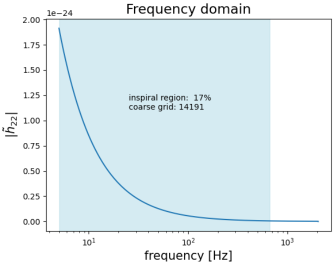
P. Schmidt+ 2012

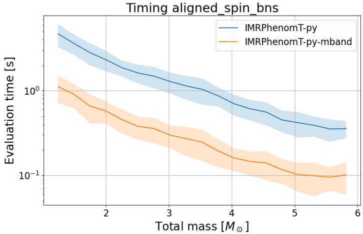


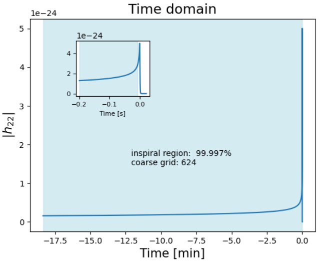
H. Estellés+, 2021

Multibanding for PhenomT (in progress)

- Evaluate model amplitude/phase in a coarse non-uniform grid
- $\Delta x = \sqrt{2R/\phi(x)''}$ (computed on-the-fly)
- Interpolate to final fine grid
- Build waveform
- Speed-up as long as interpolation is cheaper than model evaluation







Efficient calculation of polarizations from rotating frames

$$CP \rightarrow J \rightarrow L_0$$

Traditional Phenom method

$$h^{J}_{\ell m}(t) = \sum_{m'=-\ell}^{\ell} \mathcal{D}^{\ell *}_{mm'}\left(\alpha(t), \beta(t), \gamma(t)\right) h^{CP}_{\ell m'}(t)$$

$$h^{L0}_{\ell m}(t) = \sum_{m'=-\ell}^{\ell} \mathcal{D}^{\ell *}_{mm'}\left(\alpha_0, \beta_0, \gamma_0\right) h^{J}_{\ell m'}(t)$$

$$h_{+} - ih_{\times} = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} h^{L_0}_{lm}(t) {}_{-2}Y_{lm}(\iota, \phi)$$

Wigner-D calculations
PhenomT: 74 + 155 + 32 = 250
Up to l=5: 155 + 155 + 32 = 321
Up to l=8: 959 + 959 + 77 = 1939

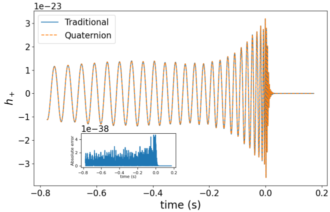
Quaternion method (M. Boyle+, 2014)

$${}_{-2}Y_{lm}(\iota, \phi) = (-1)^{s} \sqrt{\frac{2l+1}{4\pi}} \mathcal{D}^{l}_{m,-s}(R_{\iota,\phi})$$

$$h_{+} - ih_{\times} = e^{2i\hat{\delta}(t)} \sum_{l,m} h^{CP}_{lm}(t) {}_{-2}Y_{lm}(\hat{\iota}(t), \hat{\phi}(t))$$

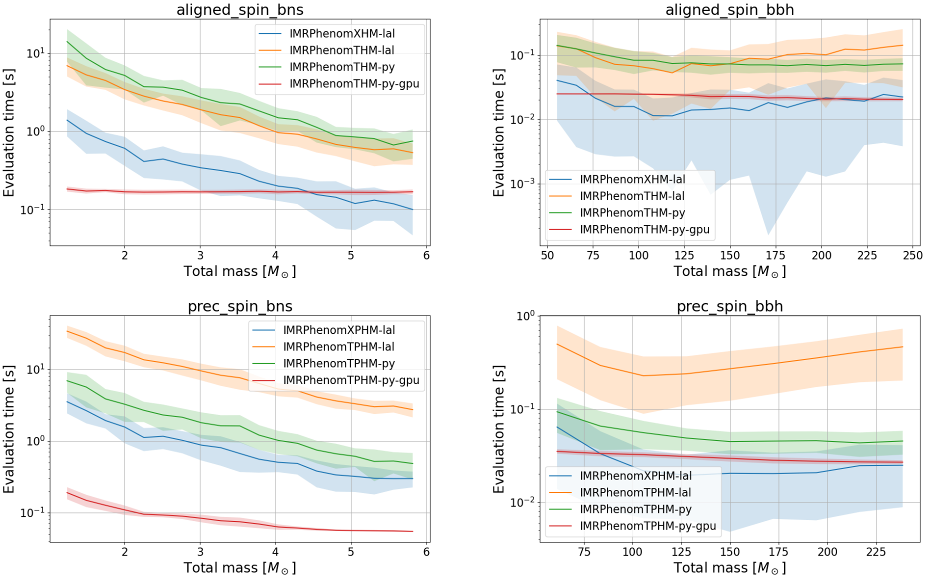
$$h_{+} - ih_{\times} = \sum_{l,m} h^{CP}_{lm}(t) \sqrt{\frac{2l+1}{4\pi}} \mathcal{D}^{l}_{m,2}(R_{CP-J} R_{J-L_0} R_{\iota,\phi})$$

Wigner-D calculations
PhenomT: 10
Up to l=5: 32
Up to l=8: 77



Timing waveform generation (LIGO-like sources)

Catalogs of 1000 BBH, BNS, $f_{low} \sim 20$ Hz



Waveform families

Effective-One-Body (time-domain)

- Solve Hamiltonian dynamics (ODEs)
- Very accurate inspiral (better than PN)
- Free Hamiltonian coefficients calibrated to Numerical Relativity (NR)
- Slow to evaluate $\rightarrow$ Reduced Order Models (ROMs)

Surrogate (time-domain)

- Interpolate NR waveforms
- Most faithful to NR
- Needs high density of NR waveforms
- Reduced parameter space
- Reduced waveform length

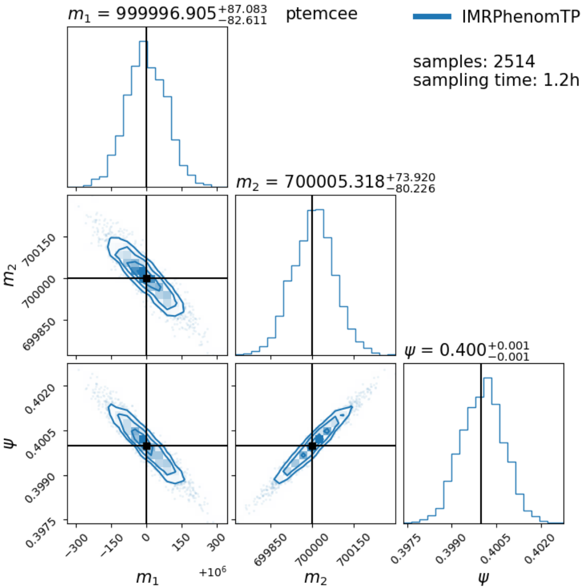
Other approximants

- Taylors (PN analytical)
- SpinTaylor (PN numerical evolution)
- NRTidal (matter effects)

Phenom (Fourier or time domain)

- Closed-form expressions for amplitude and phase as functions of frequency and intrinsic parameters
- "Phenomenological" ansatzes calibrated to NR
- Fast to evaluate
- Large parameter space coverage
- Arbitrary low frequency
- Easily parallelizable
- Easier to implement non-uniform frequency grids speed-up algorithms (multibanding, relative binning, etc.)
- IMRPhenomX (Fourier), IMRPhenomT (time)

Only masses



$m_1 = 999996.905^{+87.083}_{-82.611}$
ptemcee
IMRPhenomTP
samples: 2514
sampling time: 1.2h
$m_2 = 700005.318^{+73.920}_{-80.226}$
$\psi = 0.400^{+0.001}_{-0.001}$
m_2
700150
700000
699850
ψ
0.4020
0.4005
0.3990
0.3975
−300
−150
0
150
300
$+10^6$
m_1
699850
700000
700150
m_2
0.3975
0.3990
0.4005
0.4020
ψ