

# Data analysis efforts at UIB from the ground up

**Jorge Valencia**

on behalf of the UIB Gravitational Wave Group

LISA Spain Meeting  
Barcelona, 15 October 2024



**Universitat**  
de les Illes Balears

**IAC3** Institute of Applied Computing  
& Community Code.

# UIB GRAVITATIONAL PHYSICS GROUP

- +25 people, PI Alicia Sintes
- Members of LVK, ET, LISA Consortium, DDPC, Anna Heffernan of LST
- IMRPhenom waveform models:
  - used routinely by LVK for all events detected,
  - LISA data challenges
- Data analysis:
  - continuous waves and long transient searches
  - lensing searches
  - CB parameter estimation



## MEMBERS

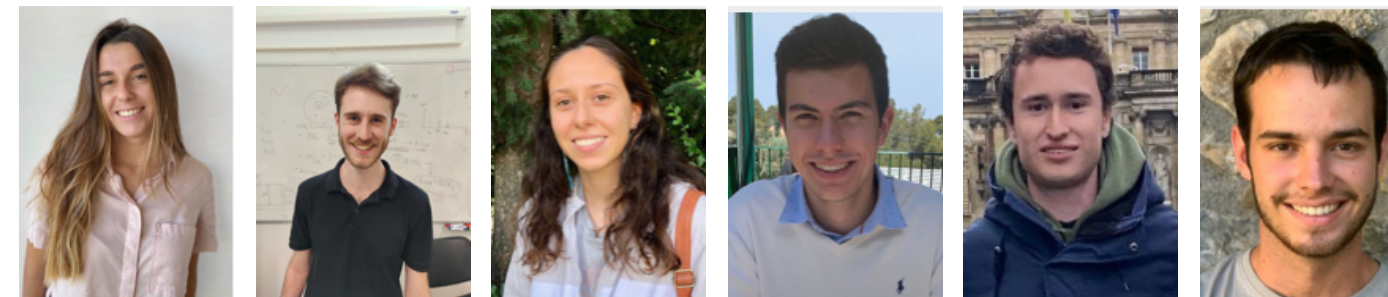
### Faculty



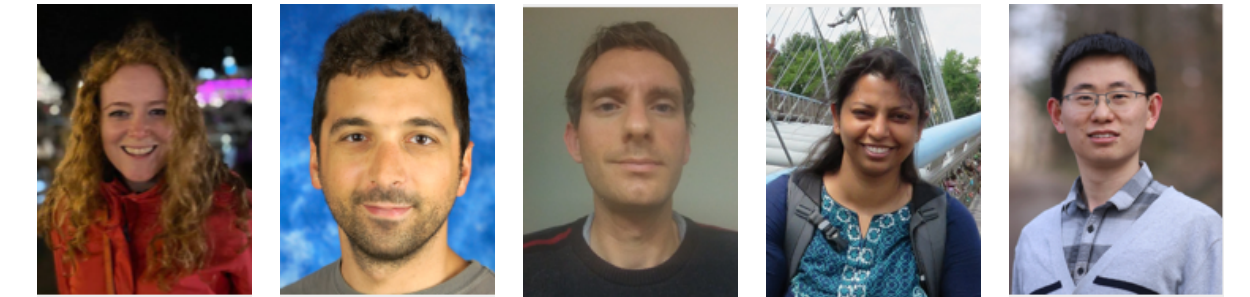
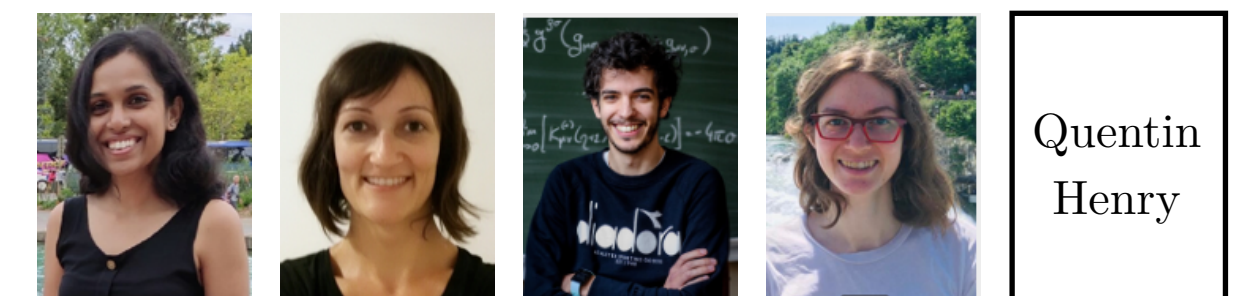
### Collab. Researchers / Tech. Support



### PhD Students



### Postdoc Researchers



Quentin  
Henry

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## Analysis of real events

**Towards the routine use of subdominant harmonics in gravitational-wave inference: re-analysis of GW190412 with generation X waveform models**

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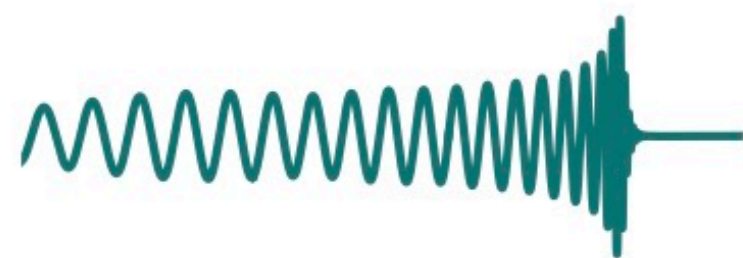
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Justin Janquart<sup>1,2,3,4,\*</sup>, David Keitel<sup>5,6,†</sup>, Rico K. L. Lo<sup>7,‡</sup>, Juno C. L. Chan<sup>7</sup>, Jose María Ezquiaga<sup>7</sup>, Otto A. Hannuksela<sup>8</sup>, Alvin K. Y. Li<sup>9,8,10</sup>, Anupreeta More<sup>11,12</sup>, Hemantakumar Phurailatpam<sup>8</sup>, Neha Singh<sup>5</sup>, Laura E. Uronen<sup>8</sup>, Mick Wright<sup>13</sup>, Naresh Adhikari<sup>14</sup>, Sylvia Biscoveanu<sup>15</sup>, Tomasz Bulik<sup>16</sup>, Amanda M. Farah<sup>17</sup>, Anna Heffernan<sup>5</sup>, Prathamesh Joshi<sup>18,19</sup>, Vincent Juste<sup>20</sup>, Atul Kedia<sup>21</sup>, Shania A. Nichols<sup>22</sup>, Geraint Pratten<sup>23</sup>, C. Rawcliffe<sup>24</sup>, Soumen Roy<sup>4,3,1,2</sup>, Elise M. Sängner<sup>25</sup>, Hui Tong<sup>26,27</sup>, M. Trevor<sup>28</sup>, Luka Vujeva<sup>7</sup>, Michael Zevin<sup>29,15</sup>

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## GW Lensing

**False positives for gravitational lensing: the gravitational-wave perspective**

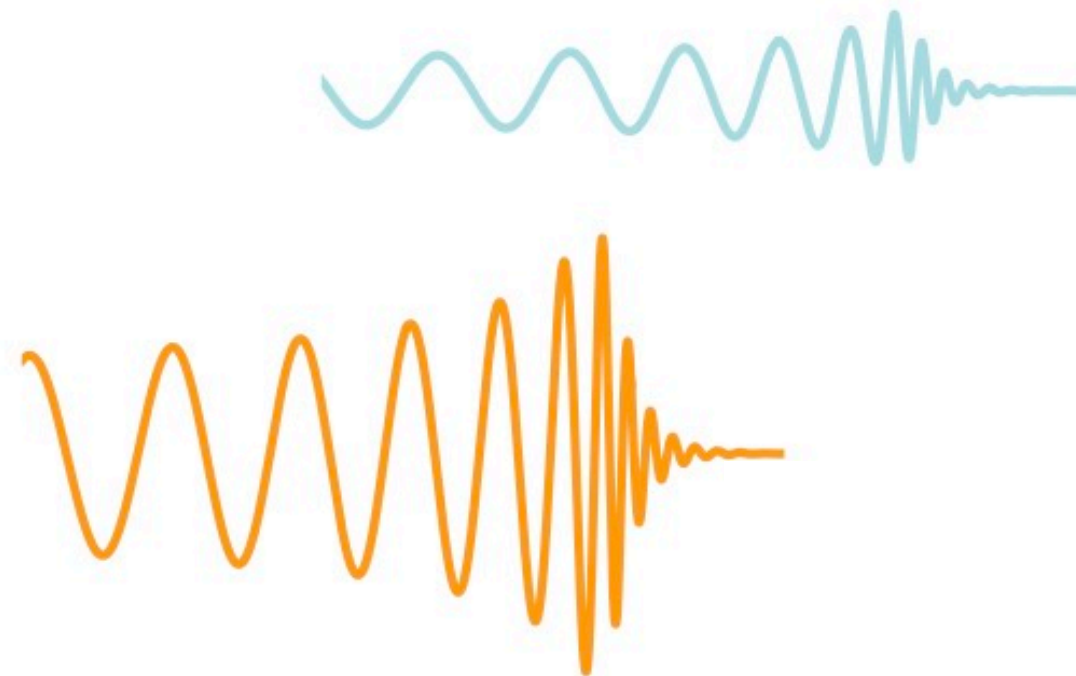
David Keitel<sup>1,2</sup>

**Characteristic Features of Gravitational Wave Lensing as Probe of Lens Mass Model**

Paolo Cremonese,<sup>\*</sup> David Fonseca Mota, and Vincenzo Salzano

**Waveform systematics in identifying strongly gravitationally lensed gravitational waves: posterior overlap method**

Ángel Garrón<sup>2,1</sup> and David Keitel<sup>1</sup>



## Continuous Waves

**Application of a hierarchical MCMC follow-up to Advanced LIGO continuous gravitational-wave candidates**

Rodrigo Tenorio,<sup>\*</sup> David Keitel, and Alicia M. Sintes

**Assessing the Similarity of Continuous Gravitational-Wave Signals to Narrow Instrumental Artifacts**

Rafel Jaume,<sup>\*</sup> Rodrigo Tenorio and Alicia M. Sintes

**Search methods for continuous gravitational-wave signals from unknown sources in the advanced-detector era**

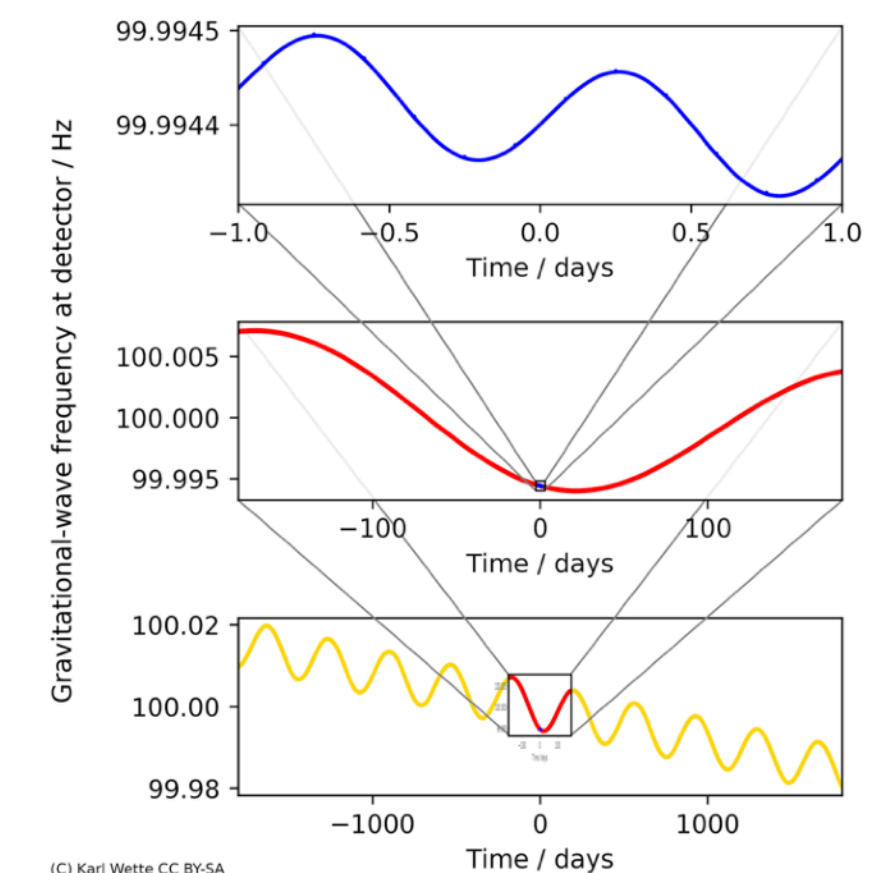
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(C) Karl Wette CC BY-SA

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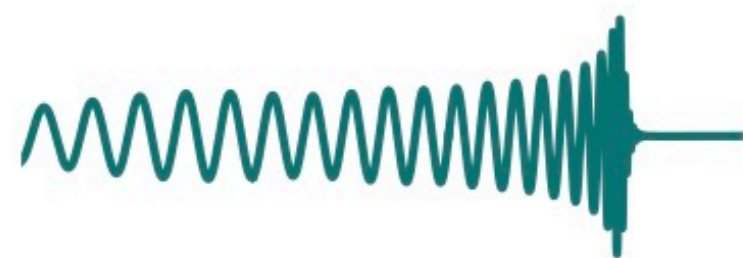
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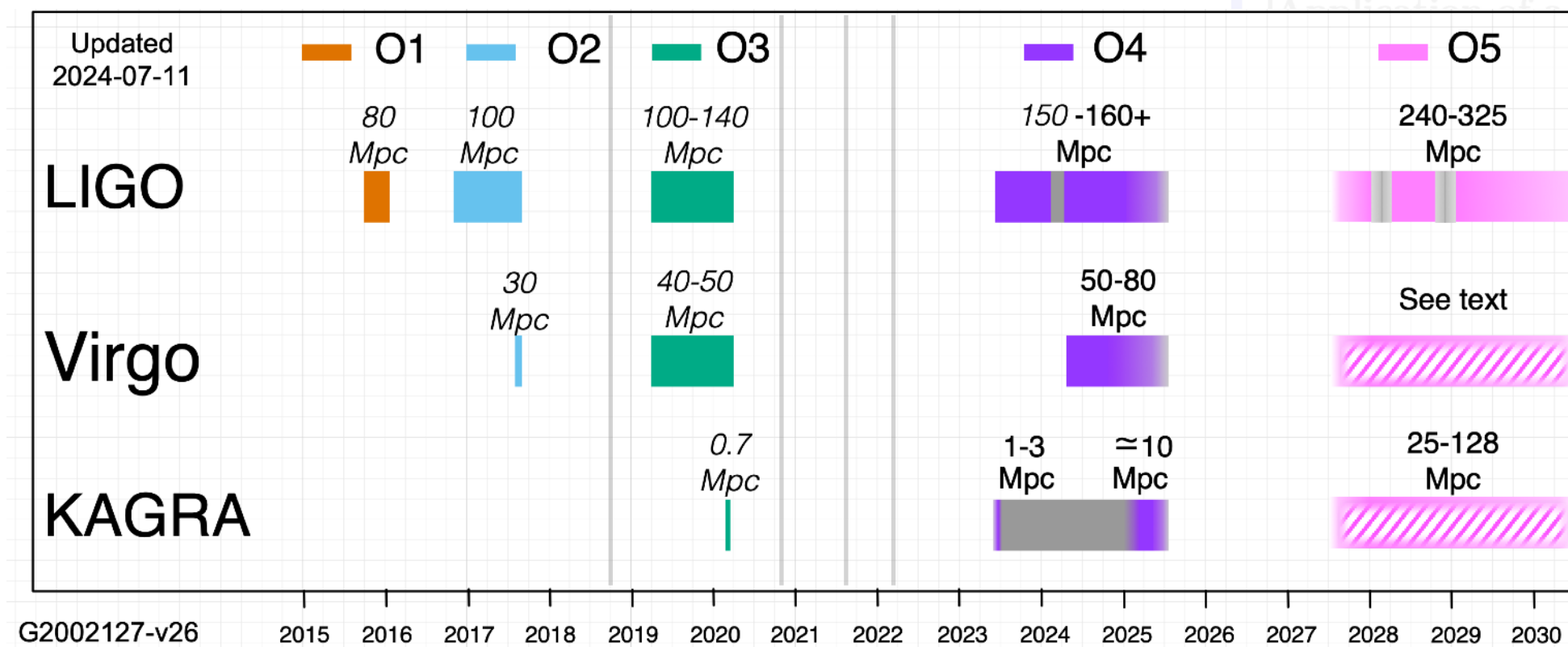
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- Periodic data releases after observing periods → same for LISA

<https://observing.docs.ligo.org/plan/>



## Continuous Waves

GWTC-1, [Phys. Rev. X 9, 031040](#)

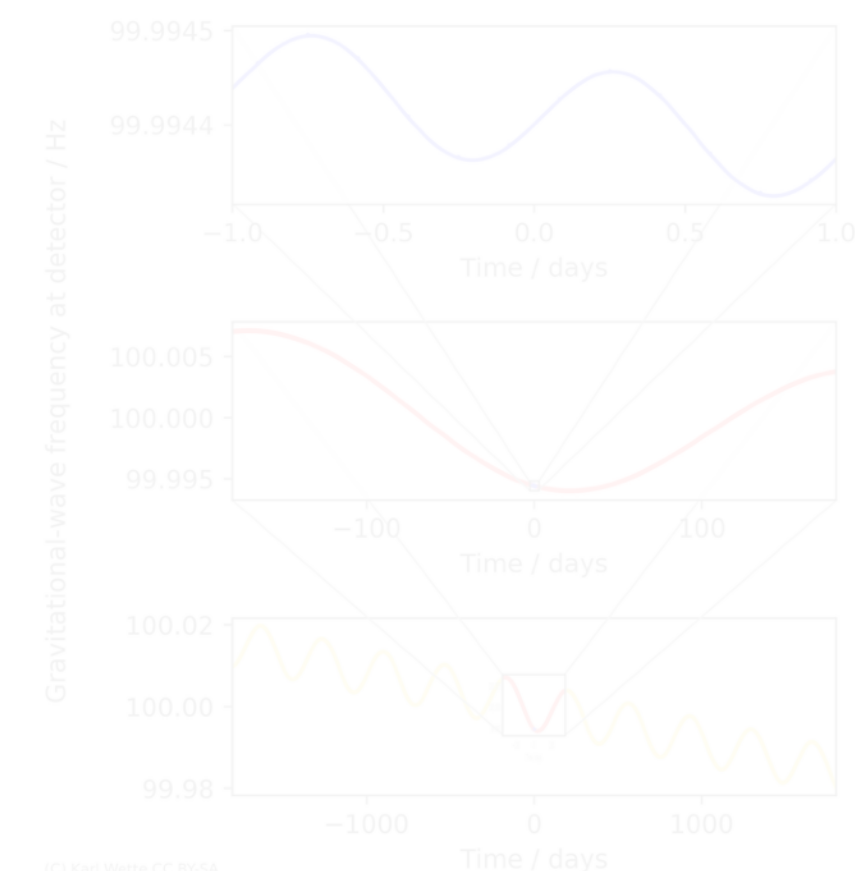
GWTC-2, [Phys. Rev. X 11, 021053](#)

GWTC-2.1, [Phys. Rev. D 109, 022001](#)

GWTC-3, [Phys. Rev. X 13, 041039](#)

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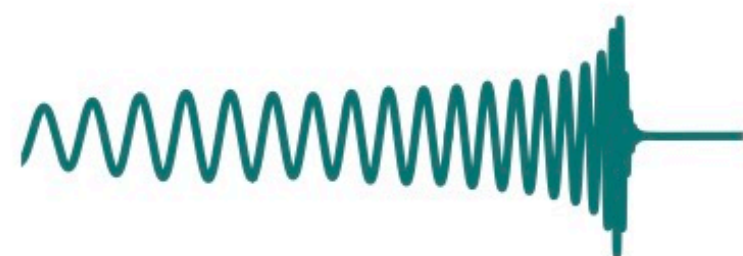
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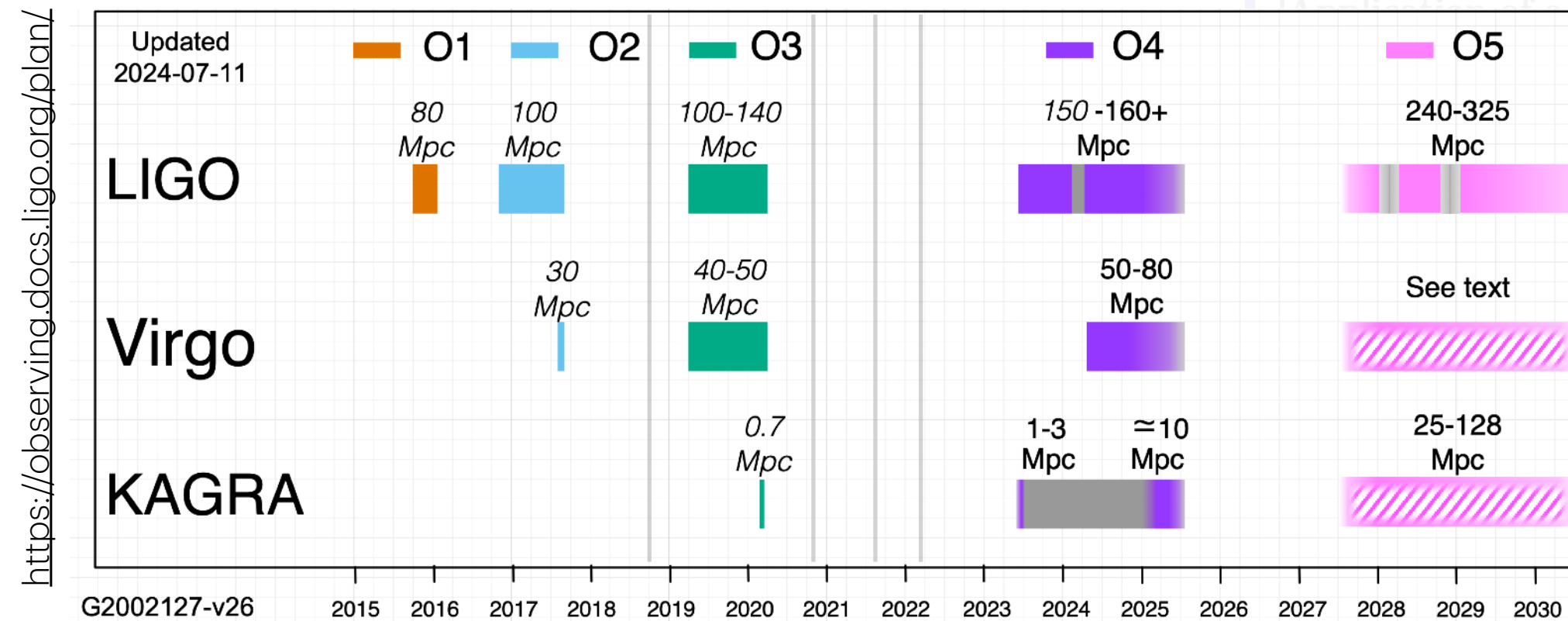
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- Periodic data releases after observing periods → same for LISA



- During observing periods:

- ROTA: PE exploratory runs of detected events
- Public alerts in <https://gracedb.ligo.org/>
- PE investigations of specific events
- Searches: CWs, lensing.
- Waveform code maintenance
- Review of results/catalogs

## Continuous Waves

GWTC-1, [Phys. Rev. X 9, 031040](https://arxiv.org/abs/1606.04868)

GWTC-2, [Phys. Rev. X 11, 021053](https://arxiv.org/abs/1903.04467)

GWTC-2.1, [Phys. Rev. D 109, 022001](https://arxiv.org/abs/1903.04467)

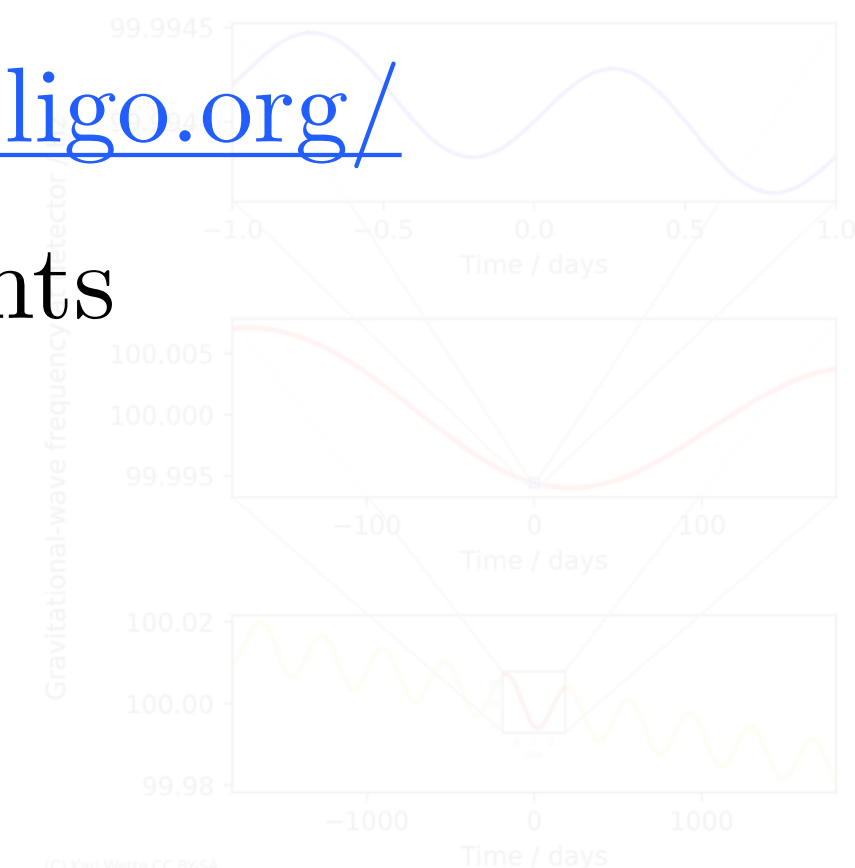
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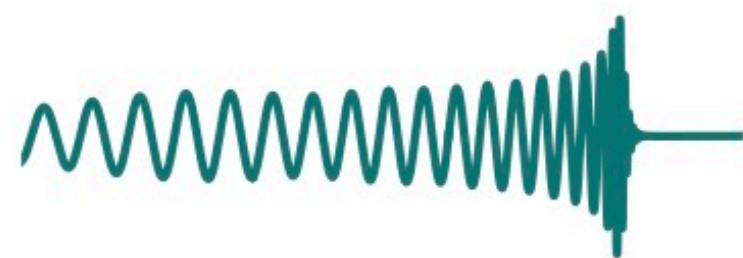
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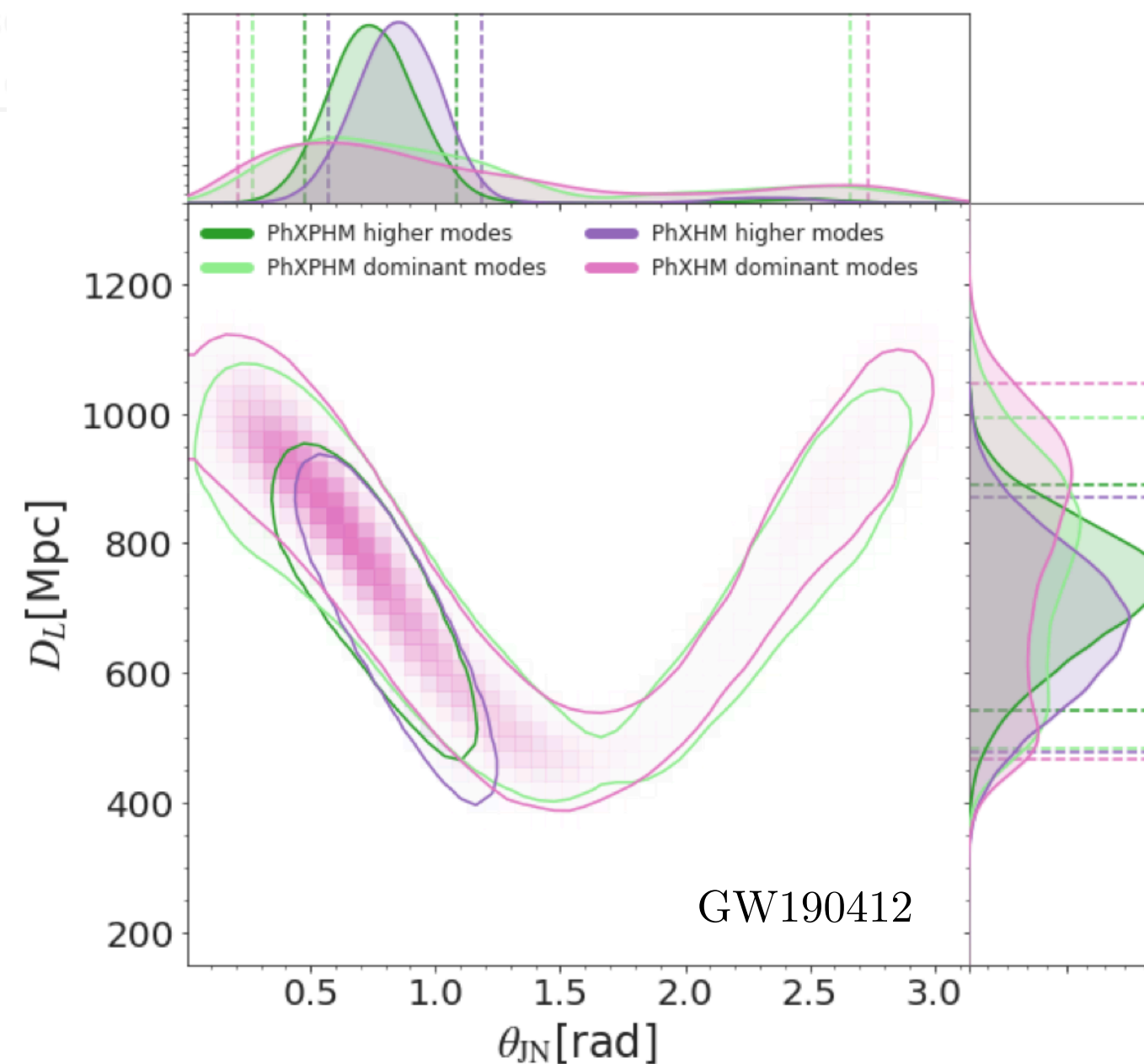
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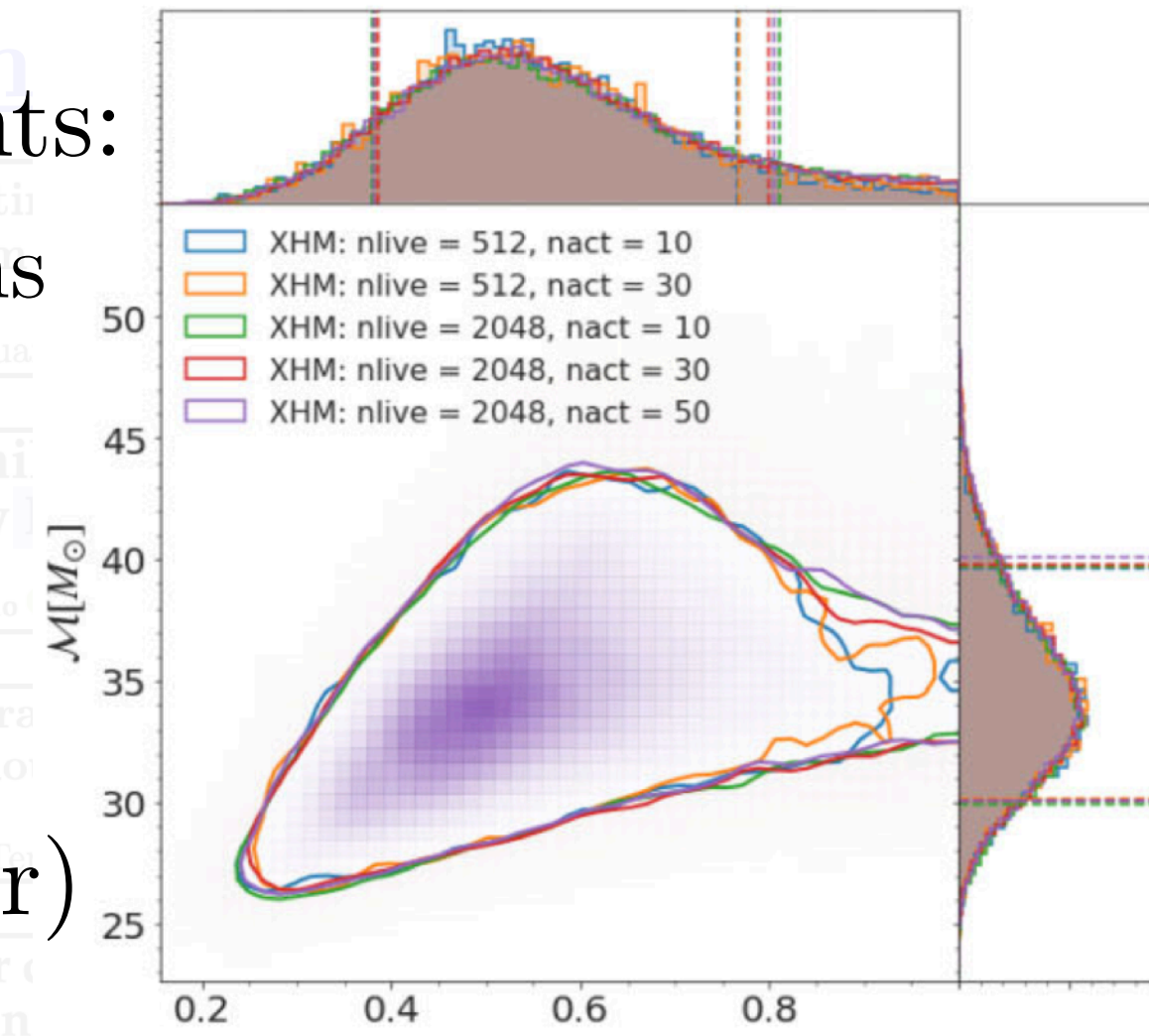
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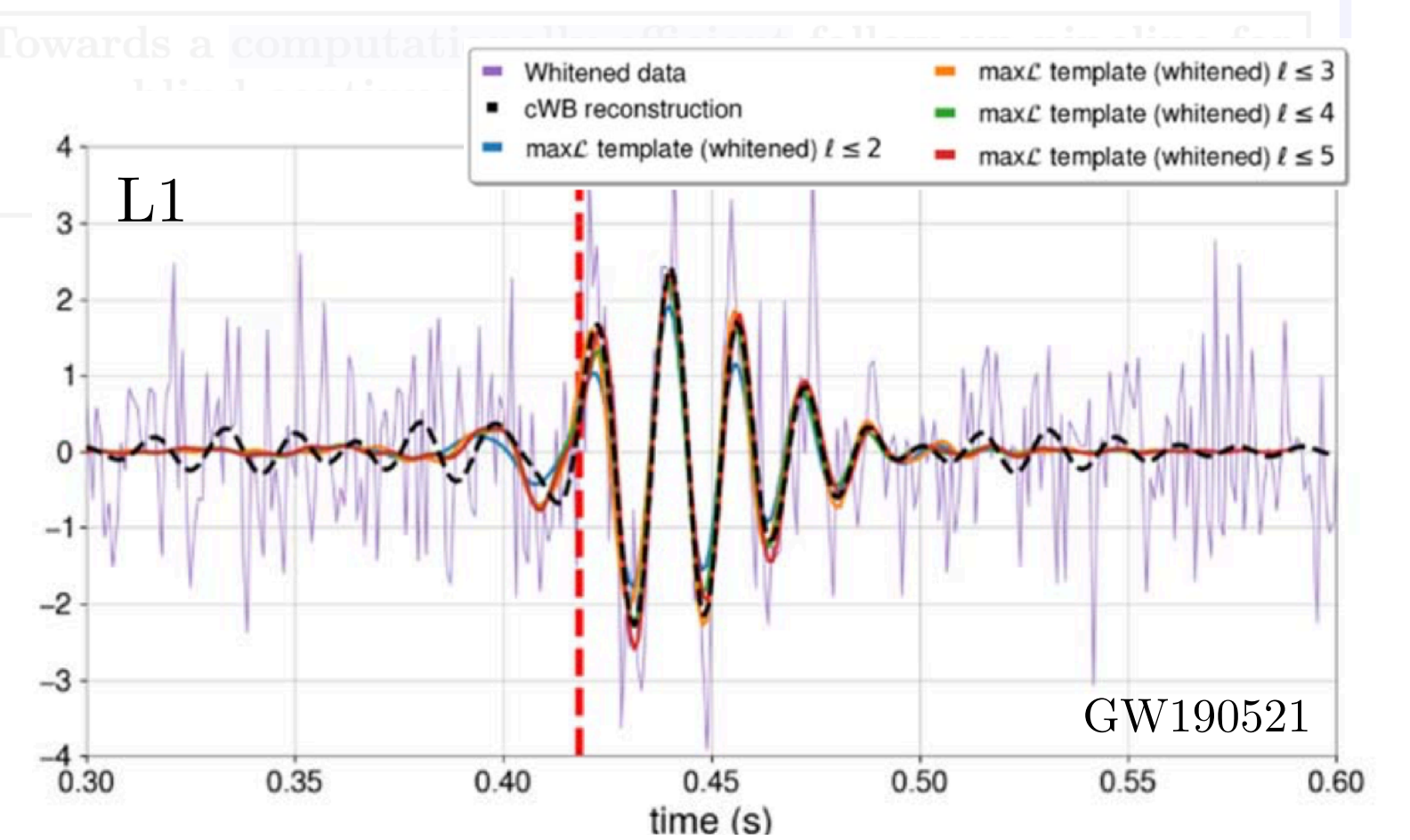
- Very detailed studies of specific LVK events:
  - Sampler convergence for different configurations
  - Higher harmonic content in the recovery of source parameters
  - Reconstructing whitened GW signal
- **LISA**: much more complex studies (see later)



Marta Colleoni + (2021) [[Phys. Rev. D 103, 024029](#)]



Maite Mateu-Lucena + (2022) [[MNRAS 517, 2403-2425](#)]



Héctor Estellés + (2022) [[Astrophys. J. 924 79](#)]

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False positives for gravitational lensing: the gravitational-wave perspective

David Keitel<sup>1,2</sup>

Characteristic Features of Gravitational Wave Lensing as Probe of Lens Mass Model

Paolo Cremonese,<sup>\*</sup> David Fonseca Mota, and Vincenzo Salzano

Waveform systematics in identifying strongly gravitationally lensed gravitational waves: posterior overlap method

Ángel Garrón<sup>2,1</sup> and David Keitel<sup>1</sup>



## Continuous Waves

Application of a hierarchical MCMC follow-up to Advanced LIGO continuous gravitational-wave candidates

Rodrigo Tenorio,<sup>\*</sup> David Keitel, and Alicia M. Sintes

Assessing the Similarity of Continuous Gravitational-Wave Signals to Narrow Instrumental Artifacts

Rafel Jaume,<sup>\*</sup> Rodrigo Tenorio, and Alicia M. Sintes

Search methods for continuous gravitational-wave signals from unknown sources in the advanced-detector era

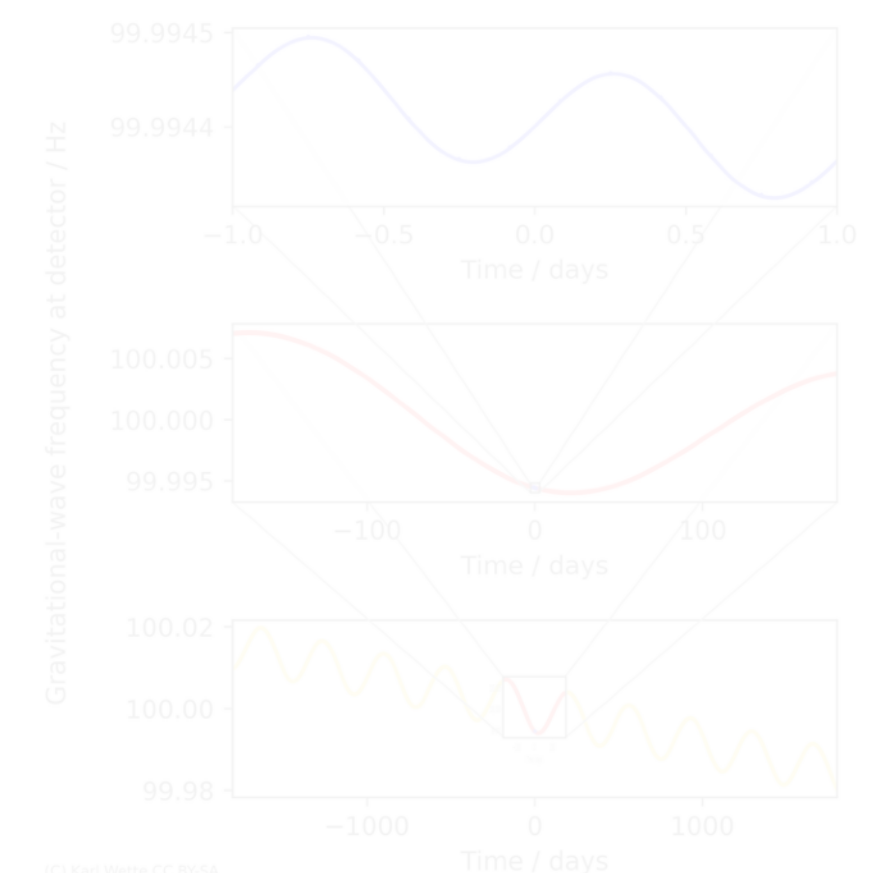
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Lorenzo Mirasola,<sup>1,\*</sup> and Rodrigo Tenorio<sup>2,1</sup>



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# UIB GRAVITATIONAL PHYSICS GROUP

## Analysis of real events

Towards the routine use of subdominant harmonics in gravitational-wave inference: re-analysis of GW190412 with generation X waveform models

Marta Colleoni,<sup>1,\*</sup> Maite Mateu-Lucena,<sup>1</sup> Héctor Estellés,<sup>1</sup> Cecilio García-Quirós,<sup>1</sup> David Keitel,<sup>1</sup> Geraint Pratten,<sup>2</sup> Antoni Ramos-Buades,<sup>3,1</sup> and Sascha Husa<sup>1</sup>

## GW Lensing

### Lensing studies for LISA

Parameter estimation with the current generation of phenomenological waveform models applied to the black hole mergers of GWTC-1

Maite Mateu-Lucena,<sup>1,\*</sup> Sascha Husa,<sup>1,\*</sup> Marta Colleoni,<sup>1</sup> Héctor Estellés,<sup>1</sup> Cecilio García-Quirós,<sup>1,2</sup> David Keitel,<sup>1</sup> Maria de Lluc Planas,<sup>1</sup> and Antoni Ramos-Buades,<sup>1,2</sup>

Matched-filter study and energy budget suggest no detectable gravitational-wave 'extended emission' from GW170817

## Continuous Waves:

- Machine Learning searches
- Galactic binaries
- Almost monochromatic signals

Enhancing Gravitational Wave Parameter Estimation with Non-Linear Memory: Breaking the Distance-Inclination Degeneracy

Yuneng Xu,<sup>1</sup> Maria Rosselló-Sastre,<sup>2</sup> Shubbanshu Tiwari,<sup>1</sup> Michael Ebersold,<sup>3</sup> Eleanor Z Hamilton,<sup>2,1</sup> Cecilio García-Quirós,<sup>1</sup> Héctor Estellés,<sup>1</sup> and Sascha Husa,<sup>1,2</sup>



## GW Lensing

False positives for gravitational lensing: the gravitational-wave perspective

David Keitel<sup>1,2</sup>

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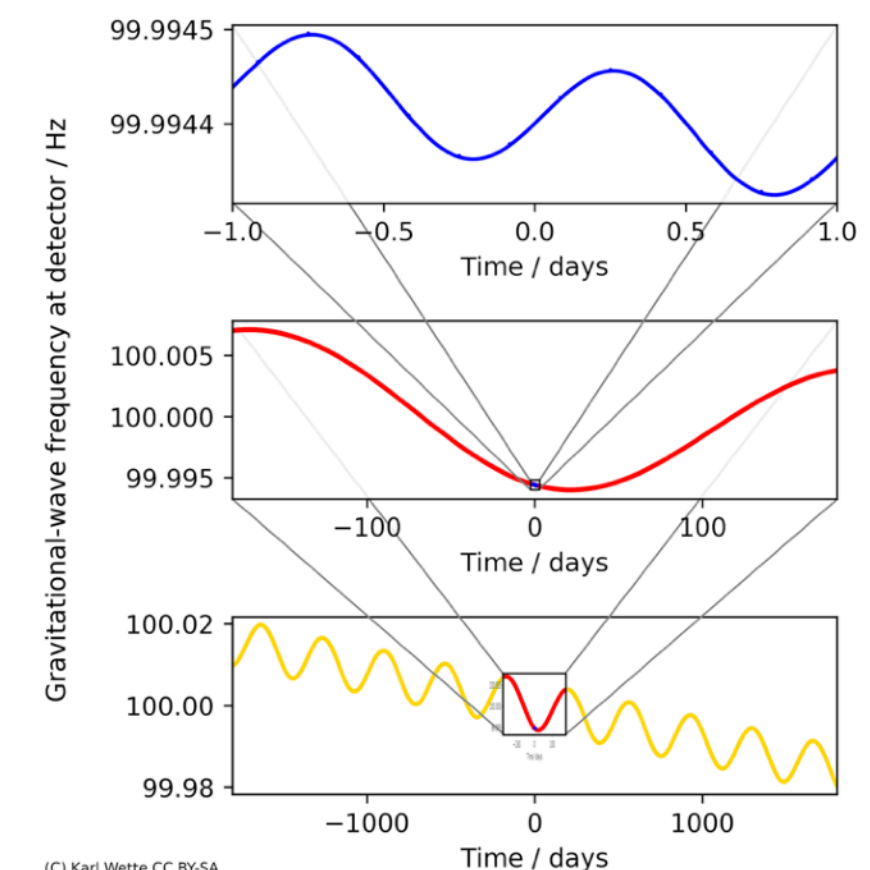
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# UIB GRAVITATIONAL PHYSICS GROUP

## Previous experience with LISA

LISA parameter estimation of supermassive black holes

Miquel Trias<sup>1</sup> and Alicia M Sintés<sup>1,2</sup>

LISA observations of supermassive black holes: parameter estimation using full post-Newtonian inspiral waveforms

Miquel Trias<sup>1</sup> and Alicia M. Sintés<sup>1,2</sup>

Weak lensing effects in the measurement of the dark energy equation of state with LISA

Chris Van Den Broeck,<sup>1,2,\*</sup> M. Trias,<sup>3,†</sup> B. S. Sathyaprakash,<sup>2,‡</sup> and A. M. Sintés<sup>3,§</sup>



Universitat  
de les Illes Balears

MASTER'S THESIS

TESTING WAVEFORM MODELS FOR THE LISA  
AND EINSTEIN TELESCOPE GRAVITATIONAL  
WAVE DETECTORS

Friso Snel

Master's Degree in Advanced Physics and Applied Mathematics

Centre for Postgraduate Studies

Academic Year 2019-20



Universitat  
de les Illes Balears

MASTER'S THESIS

A STUDY OF WAVEFORM  
REQUIREMENTS FOR THE LISA  
SPACE MISSION WITH  
CURRENT INSPIRAL-MERGER-  
RINGDOWN MODELS

Jorge Valencia Gómez

Master's degree in Advanced Physics and Applied Mathematics

Centre for Postgraduate Studies

Academic Year 2022-23



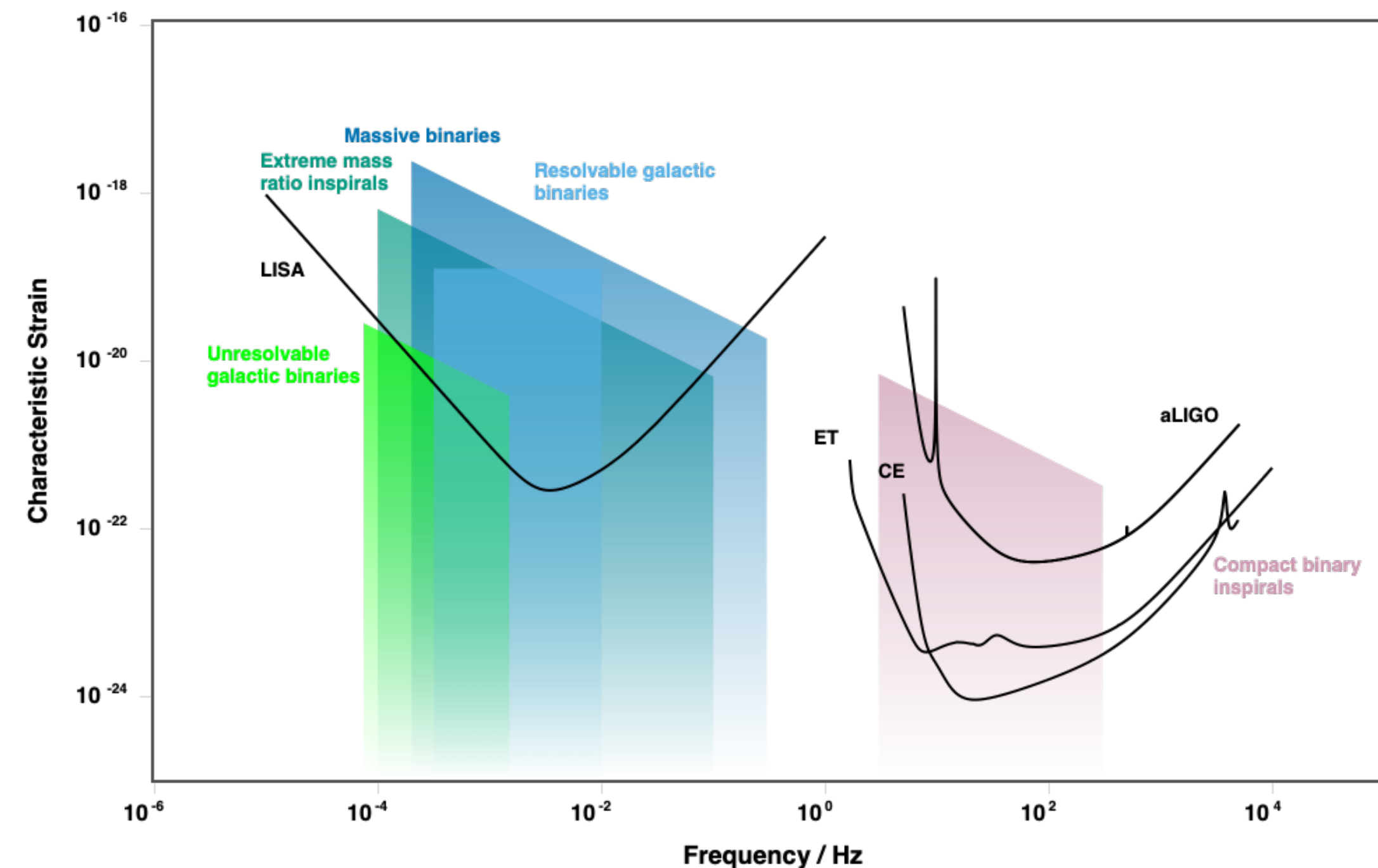
LISA

Detector  
response

Variety of  
new sources

Overlapping  
signals

Really high  
SNRs



Christopher J. Moore + 2014, <http://gwplotter.com/>

# IMRPHENOM MODELS

	Aligned spin	Precessing
Quasi-circular		IMRPHENOMTPHM <a href="#">[Estellés+(2021)]</a>
	IMRPHENOMTHM <a href="#">[Estellés+(2020)]</a>	IMRPHENOMXPHM <a href="#">[Pratten+(2021)]</a>
	IMRPHENOMXHM <a href="#">[García-Quirós+(2020)]</a>	IMRPHENOMXO4A <a href="#">[Thompson,Hamilton+(2024)]</a>
Eccentric		IMRPHENOMXPNR <a href="#">[Colleoni,Hamilton+(in prep.)]</a>

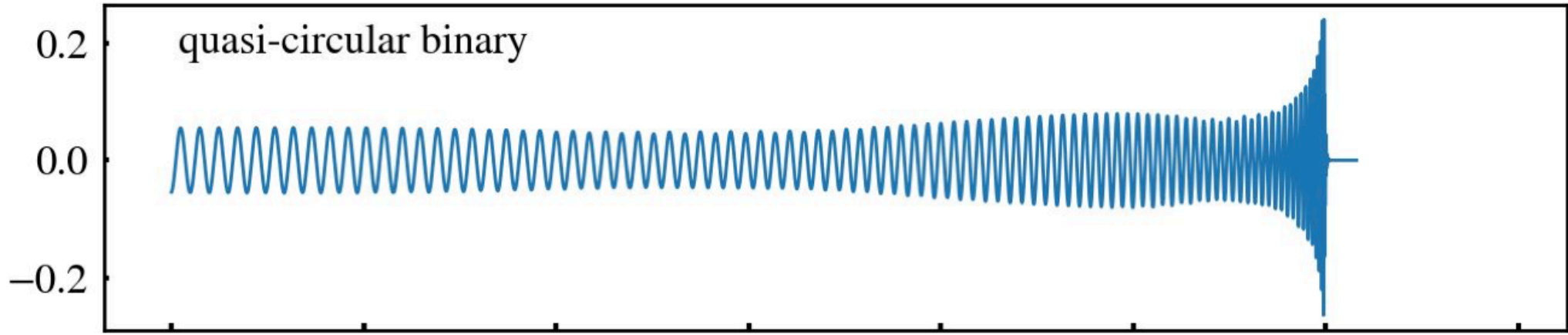


Image taken from Antoni Ramos-Buades [talk](#)  
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Quasi-circular		IMRPHENOMTPHM [Estellés+(2021)]
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	IMRPHENOMTEHM [Planas+(in prep.)]	
	IMRPHENOMXE [Ramos-Buades+(in prep.)]	

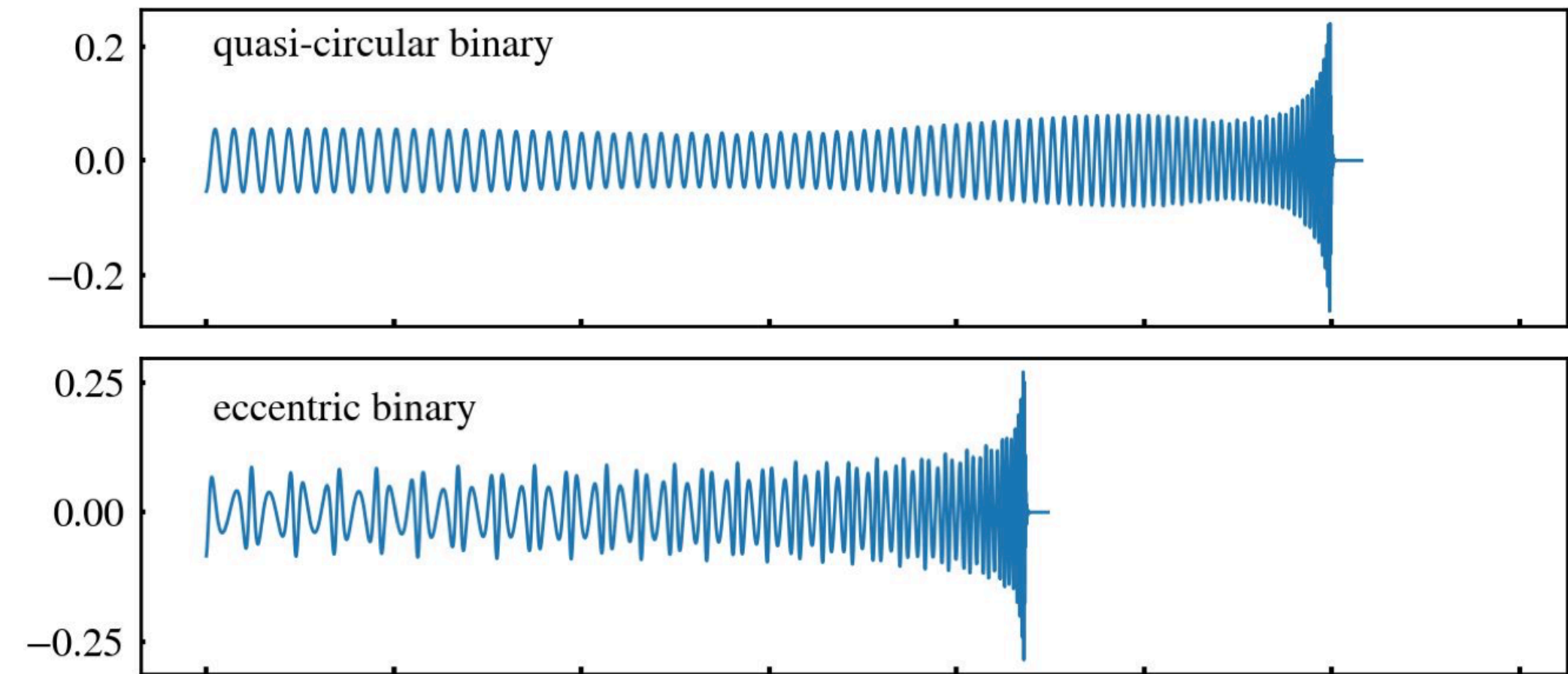


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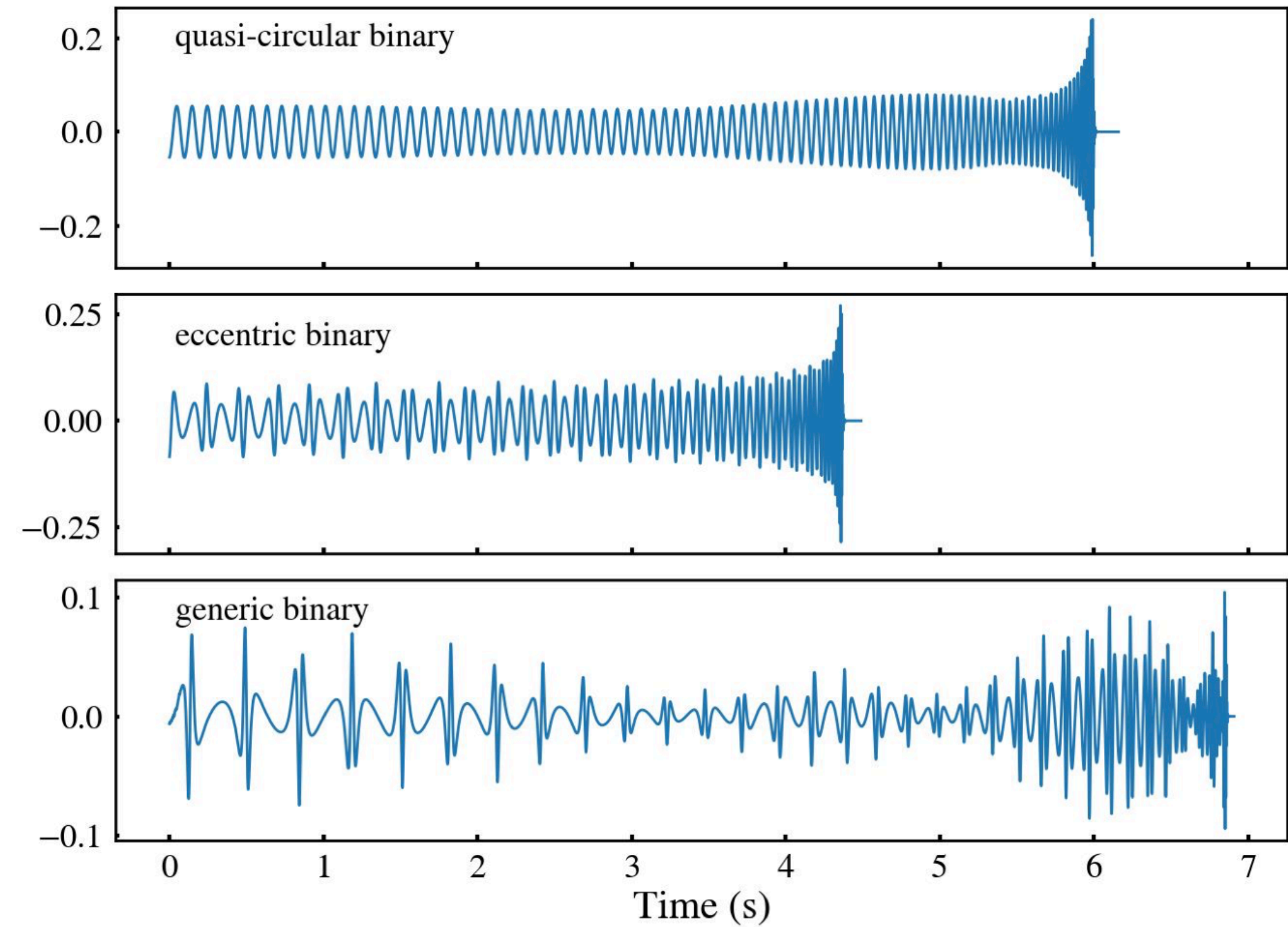


Image taken from Antoni Ramos-Buades [talk](#) in [Fundamental Physics Meets Waveforms With LISA](#)

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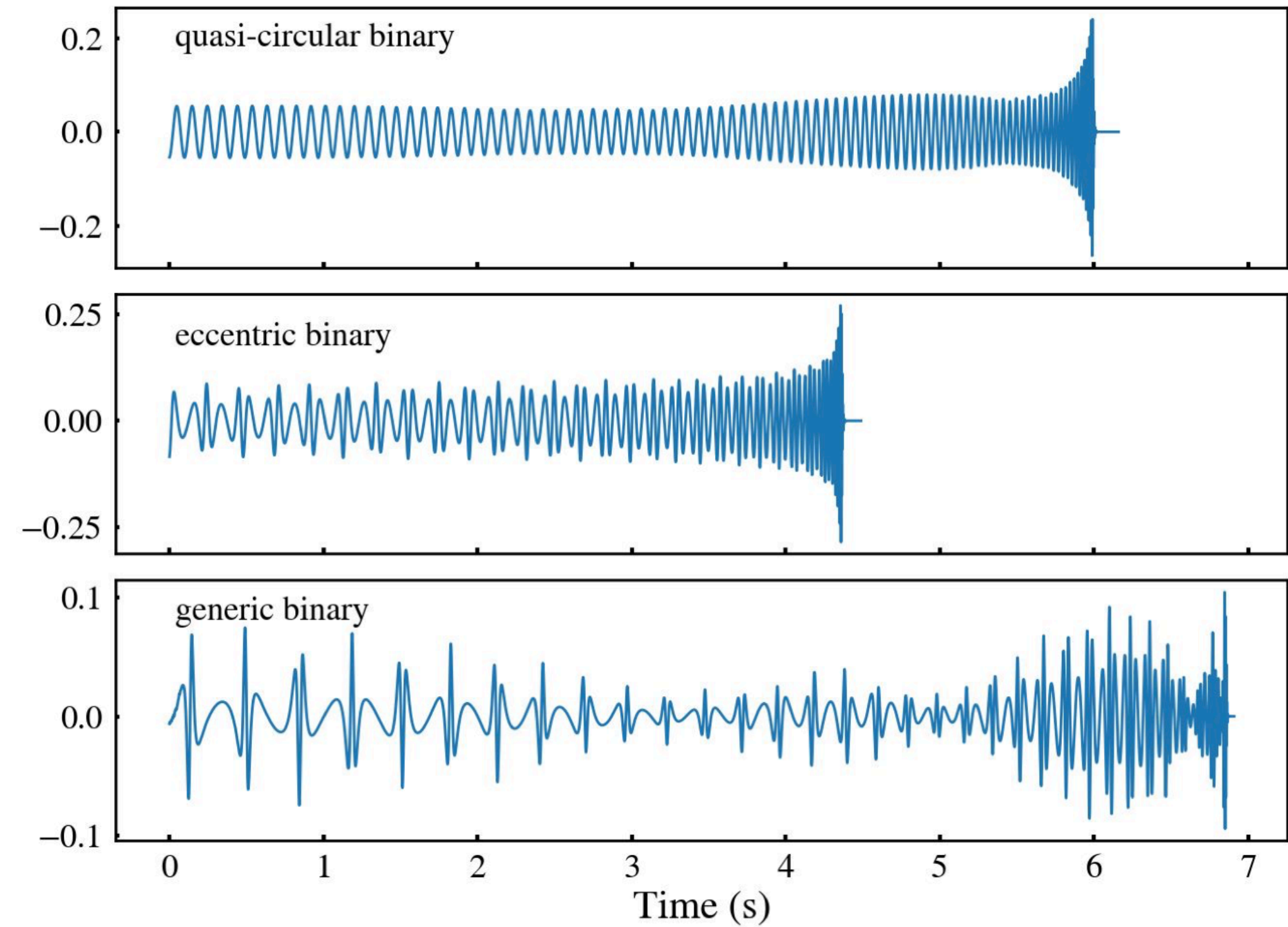


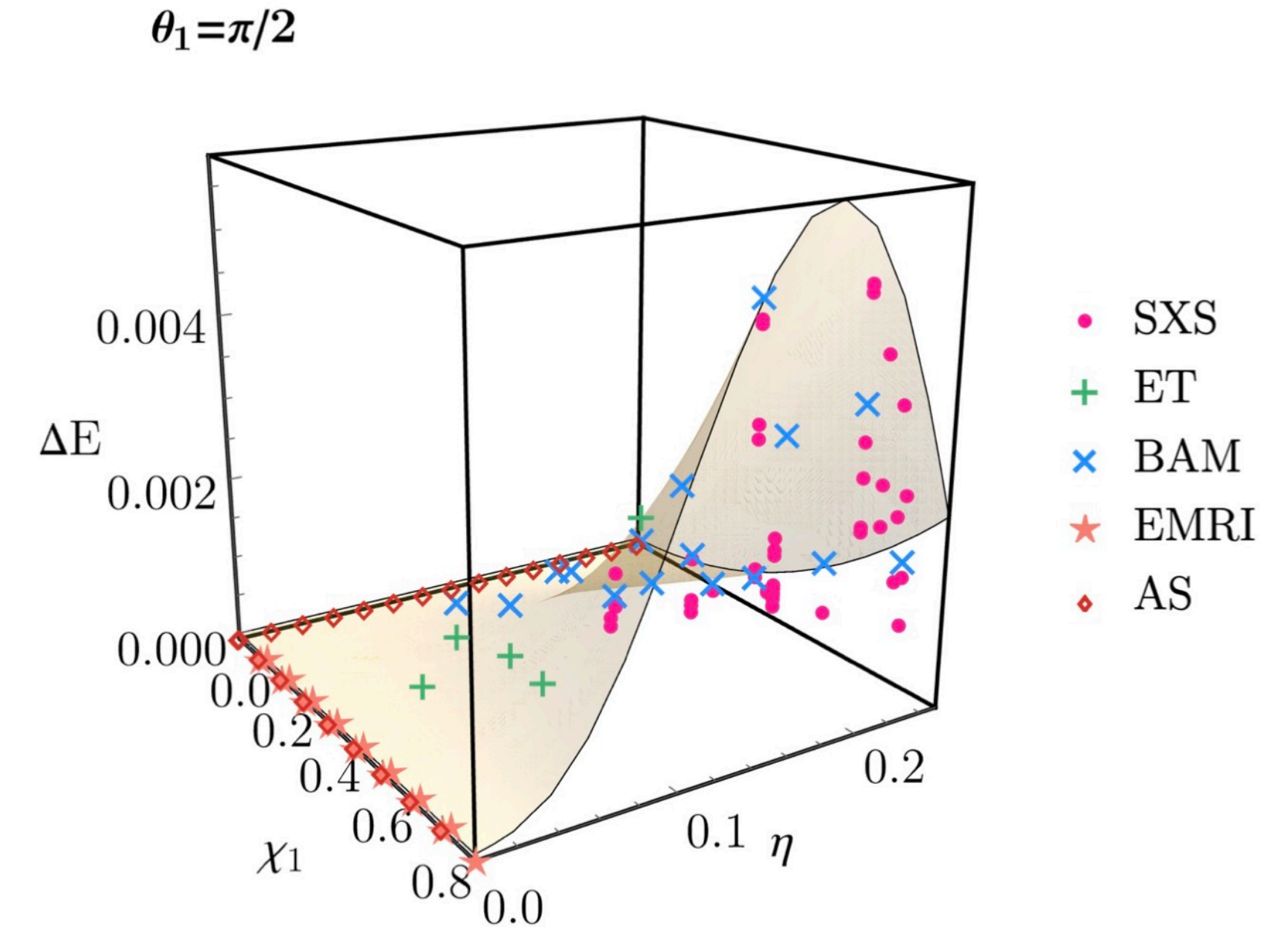
Image taken from Antoni Ramos-Buades [talk](#)  
in [Fundamental Physics Meets Waveforms With LISA](#)

New Python infrastructure for Phenom models:  
**phenomxpy** (GPUs)  
 [see talk by Cecilio García-Quirós]

# RECENT AND SOME ONGOING MODELLING WORK @ UIB: I

## DATA SETS FOR THE ENTIRE PARAMETER SPACE

- Build IMRPhenom models with data from all available NR catalogues.
  - Consistent precessing data: different NR catalogs, NRSurrogate, EMRI waveforms. L. Planas, J. Llobera, S. Husa [[Phys. Rev. D 109, 124028](#)]
  - First application: single spin final state model across all mass ratios.
  - Ongoing: systematic approach to build precessing “complete” hybrids from inspiral waveforms + NR: J. Llobera, S. Husa (in prep.)



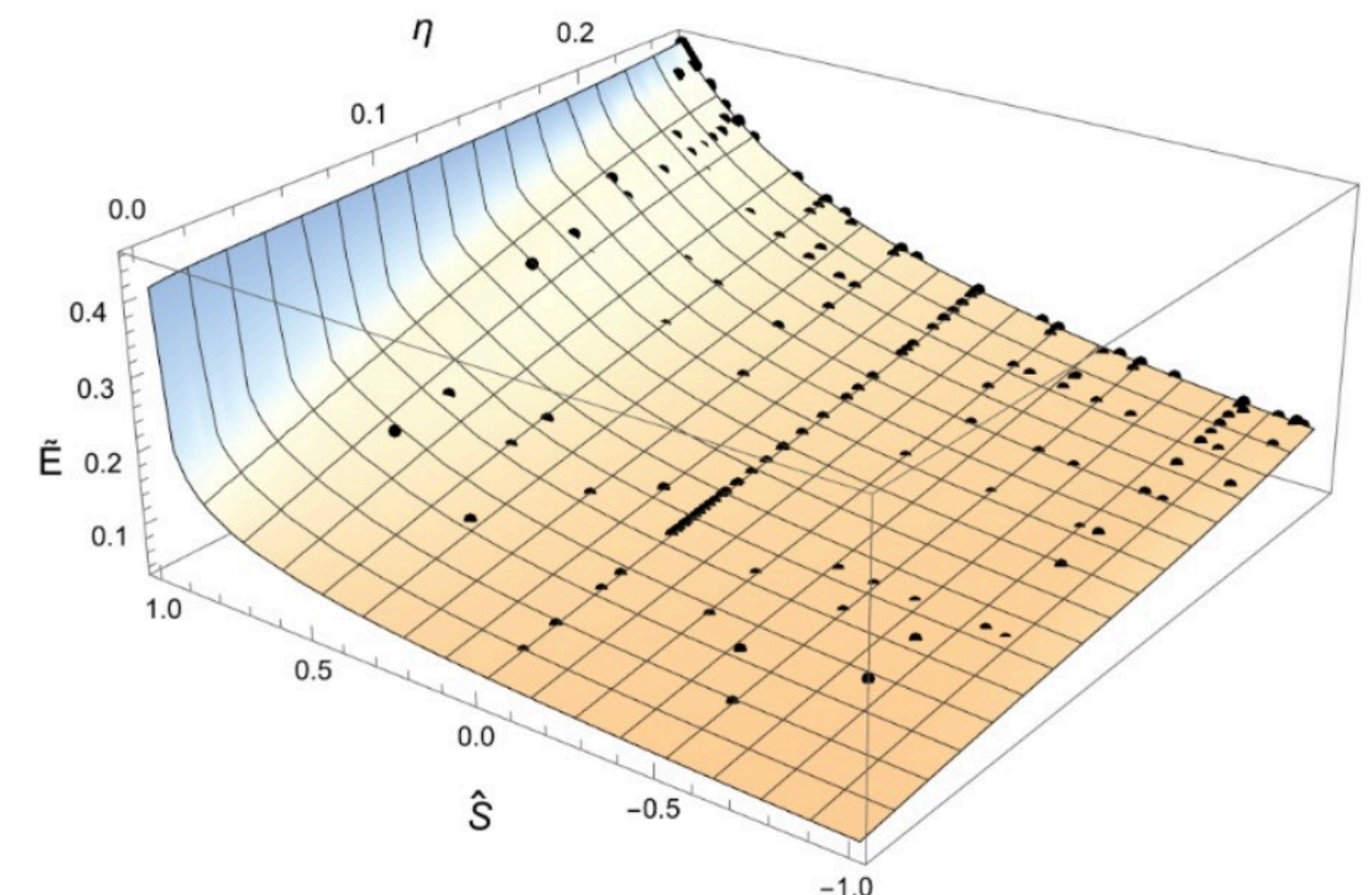
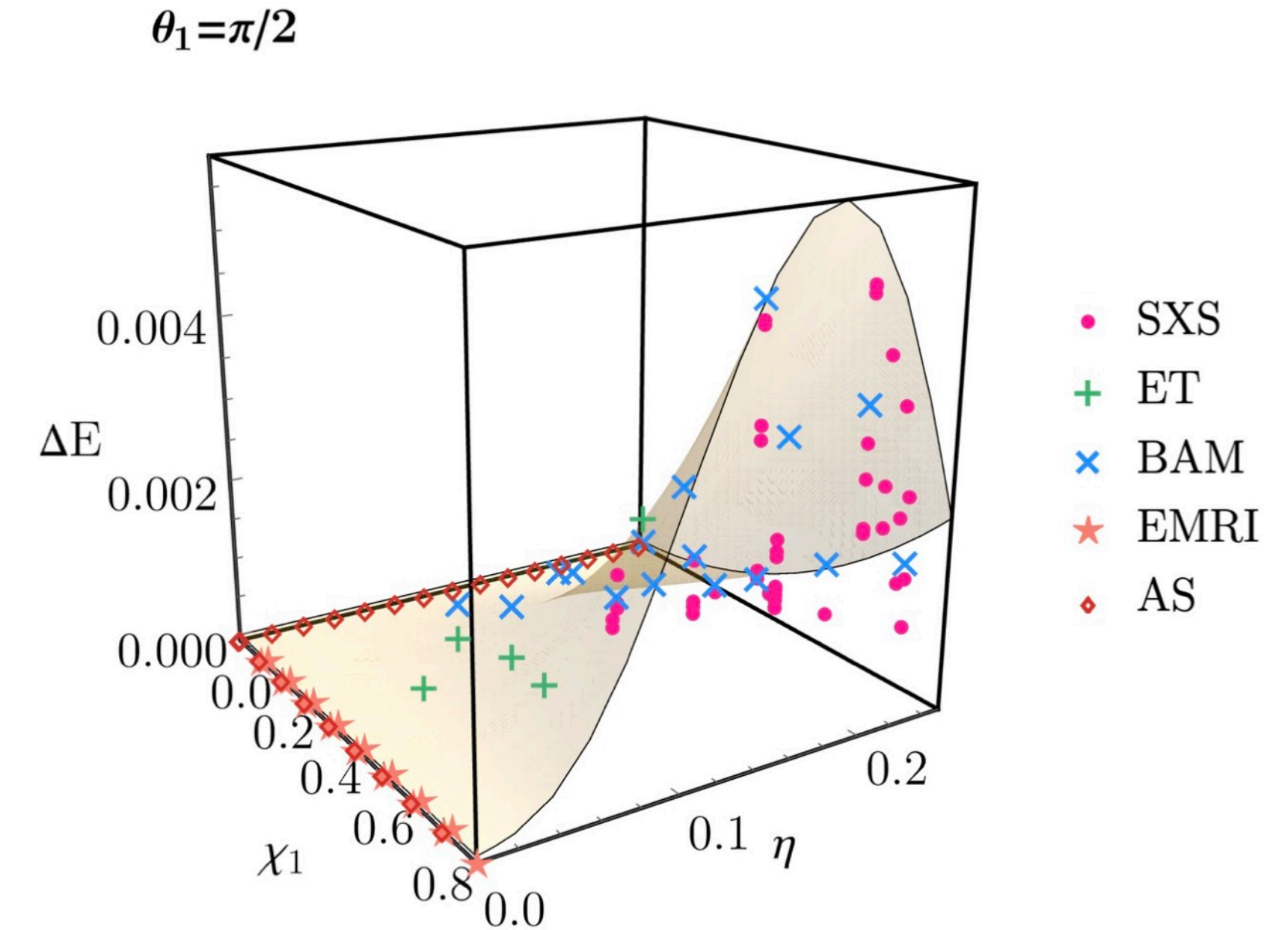
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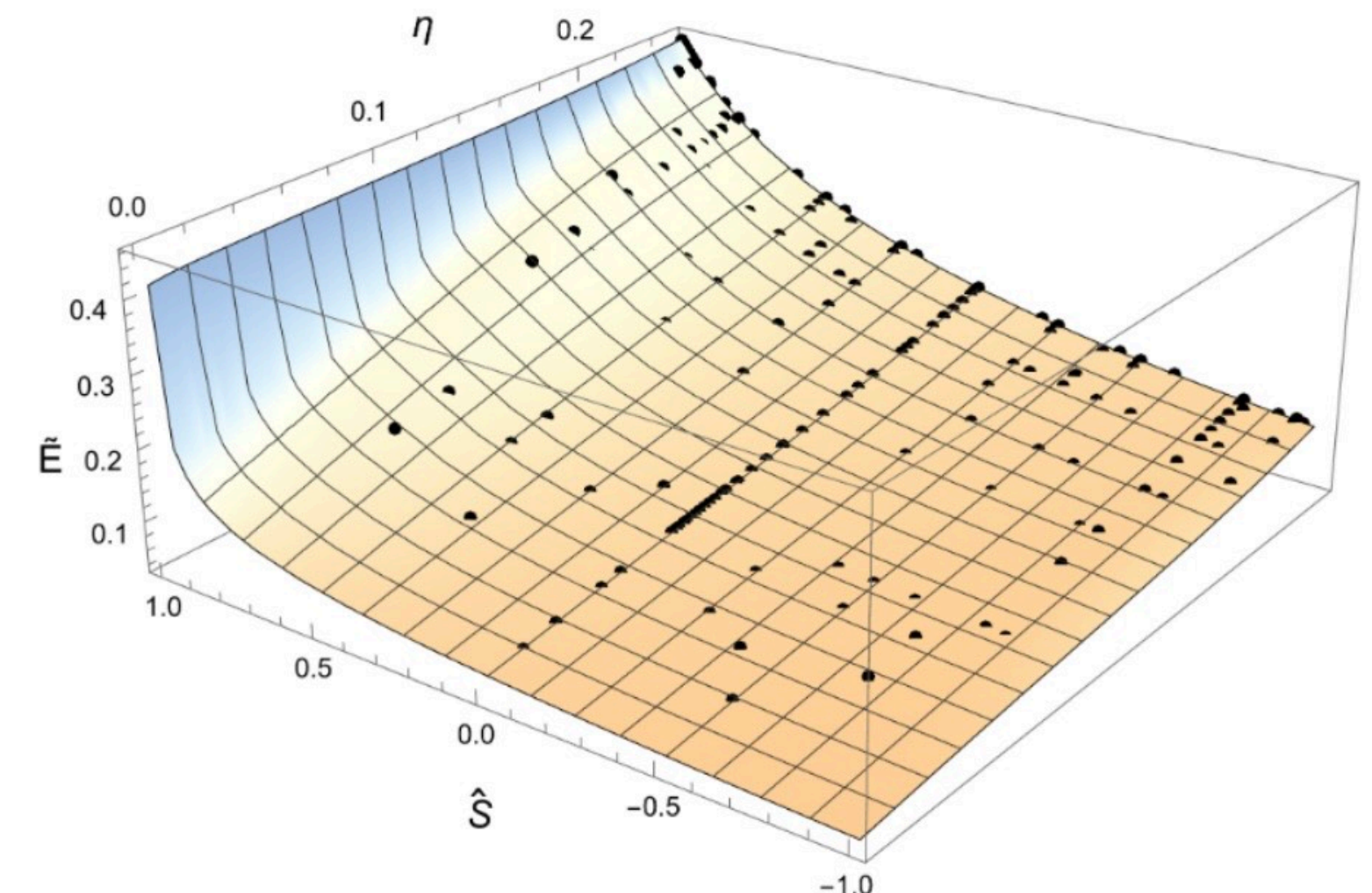
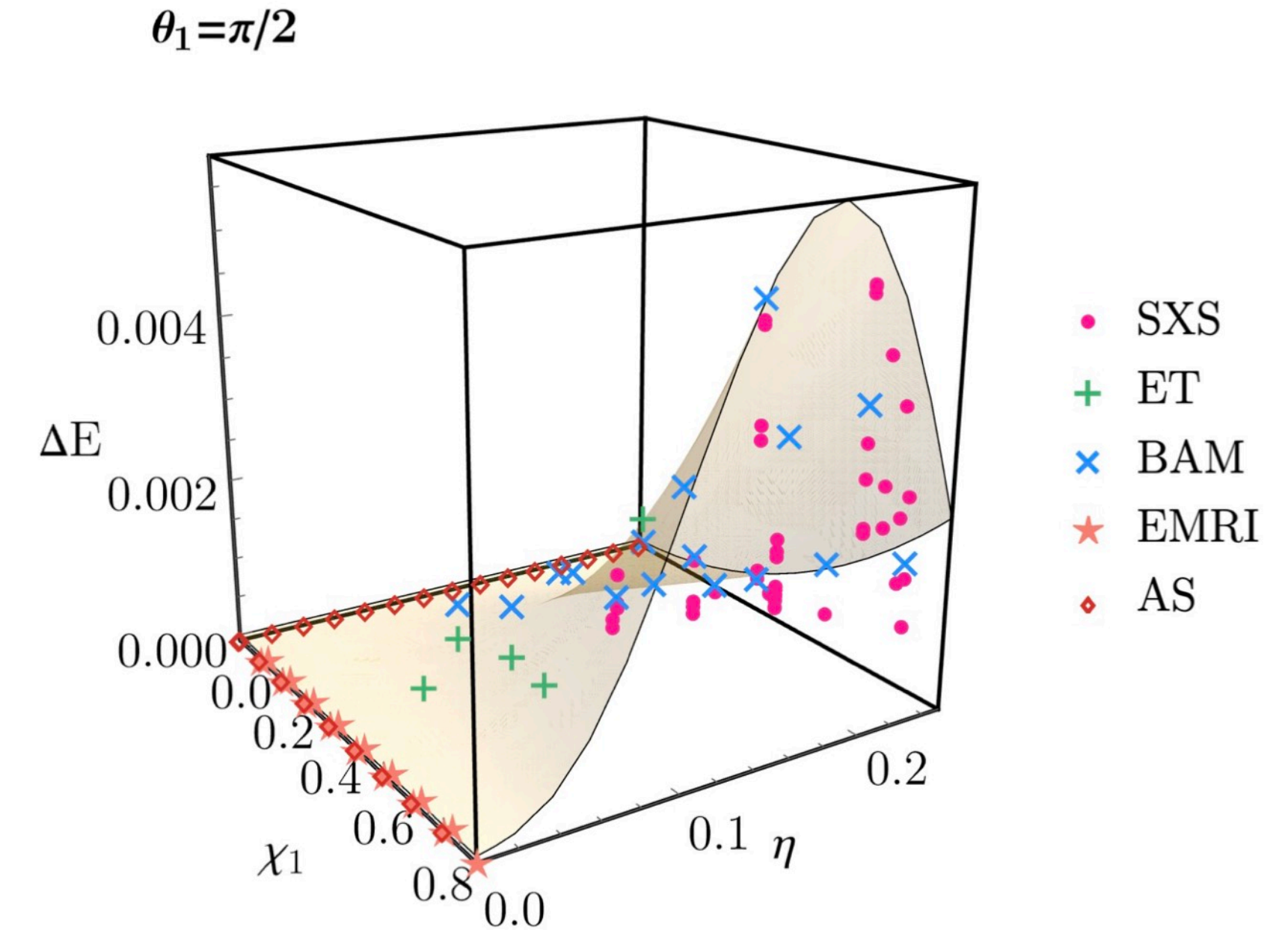
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- Solve by appropriate rescaling: R. Santos, P. Mourier, S. Husa, (in prep.)

## NR SIMULATIONS FOR ECCENTRIC WAVEFORMS AND LARGE MASS RATIOS

- Einstein Toolkit code, simulations on MN5.

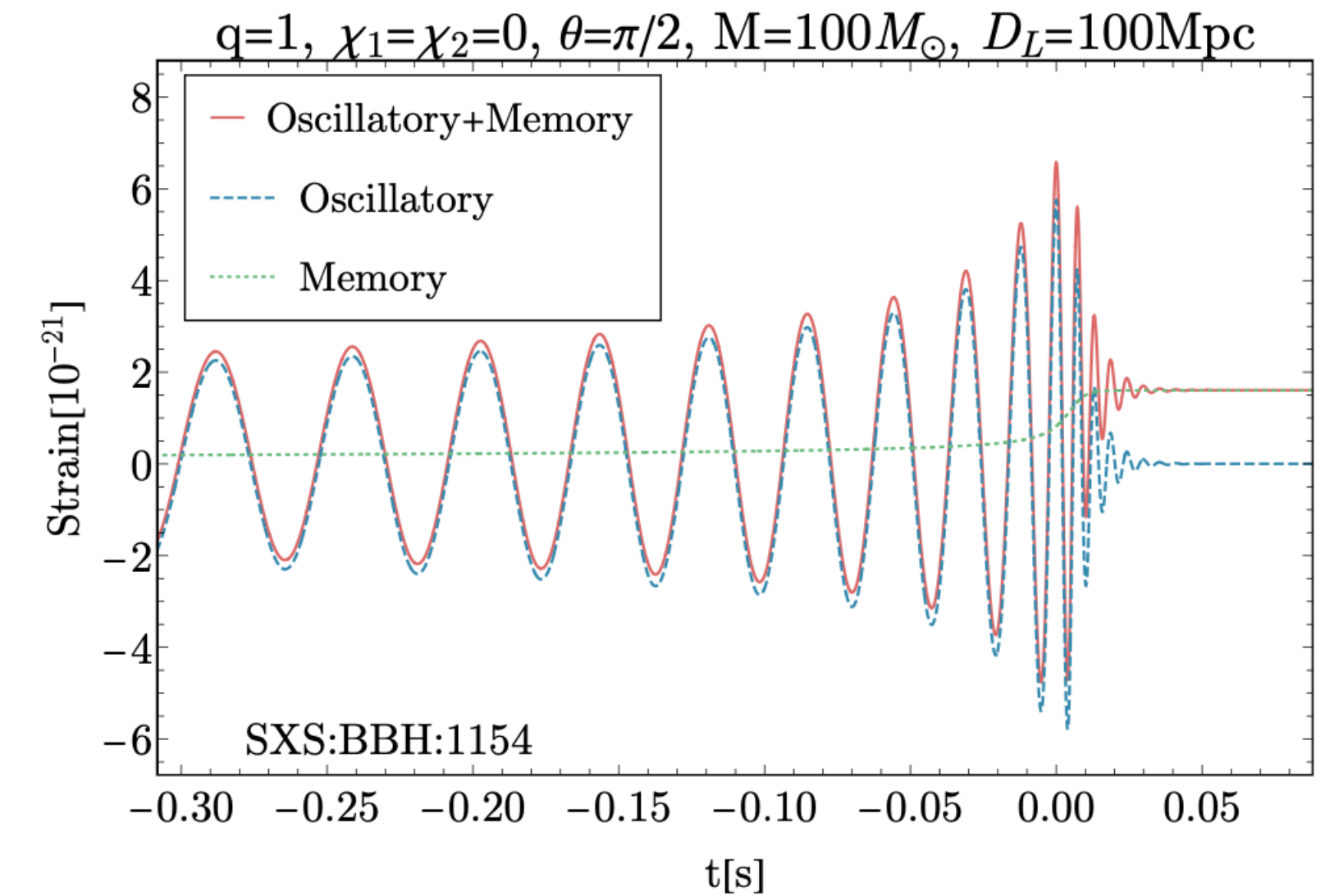




# RECENT AND SOME ONGOING MODELLING WORK @ UIB: II

## GW MEMORY

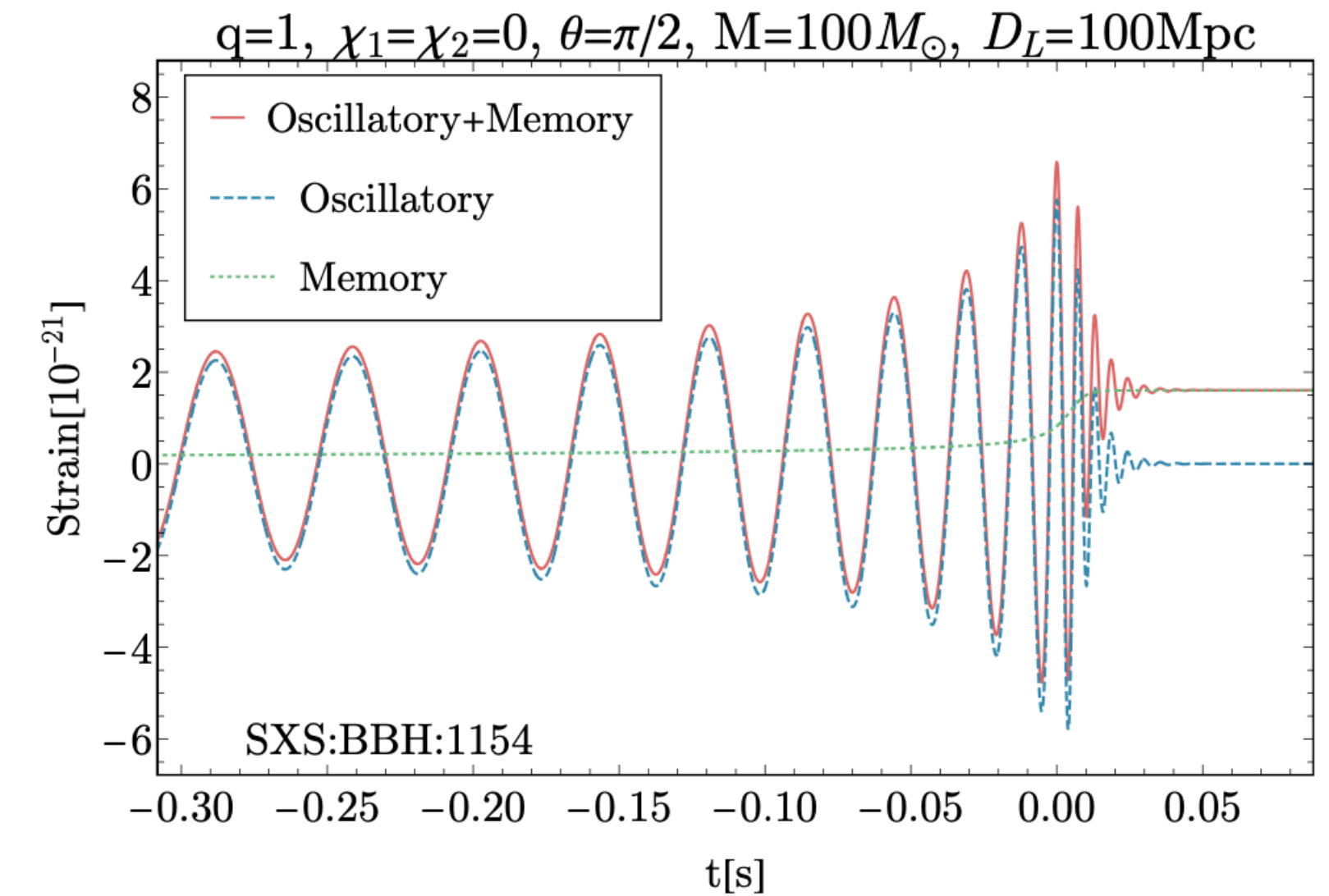
- Developed IMRPhenom model for the **complete (2,0) mode**: memory + oscillatory part.  
M. Rosselló-Sastre, S. Husa, S. Bera (2024) [\[arXiv:2405.17302\]](https://arxiv.org/abs/2405.17302)
- Fast: implemented in IMRPHENOMTHM (LALSuite + phenomxpy)



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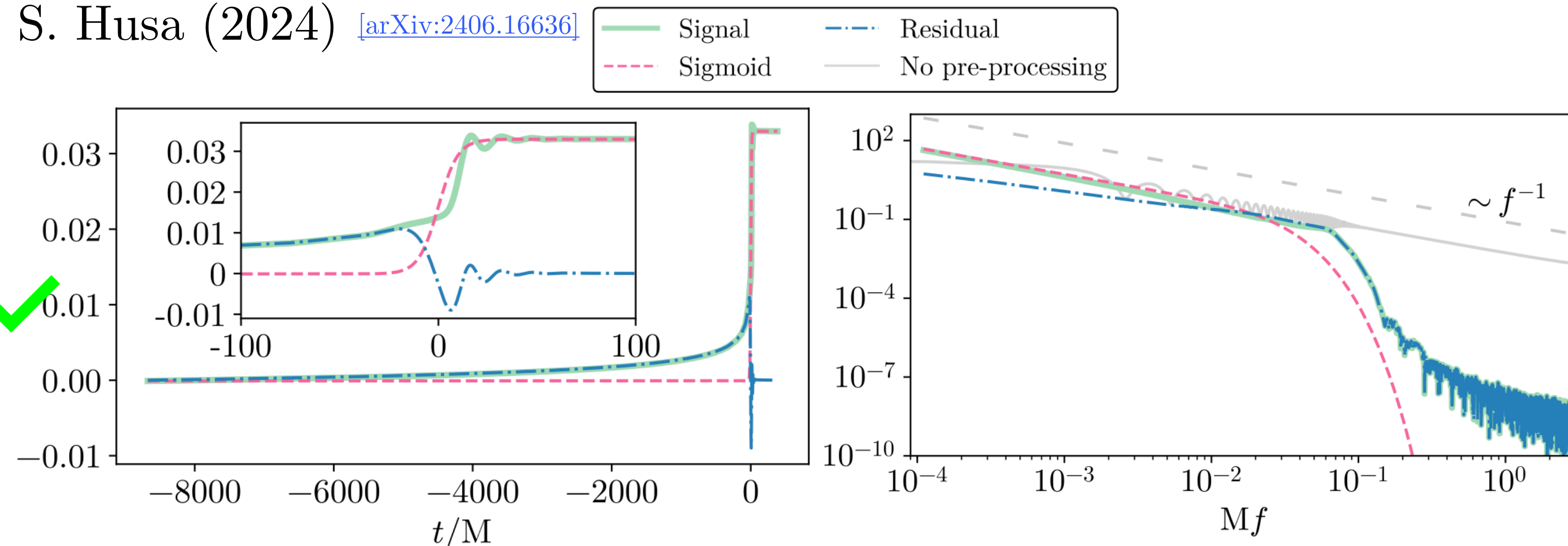


## DATA ANALYSIS WITH STEP-LIKE FUNCTIONS

- Method to compute FT of data with **step-like behaviour**

J. Valencia, R. Tenorio, M. Rosselló-Sastre, S. Husa (2024) [[arXiv:2406.16636](https://arxiv.org/abs/2406.16636)]

- DFTs assume periodic data
- Padding + windowing ❌
- Remove the step and treat it analytically ✅
- Add it back artifact-free

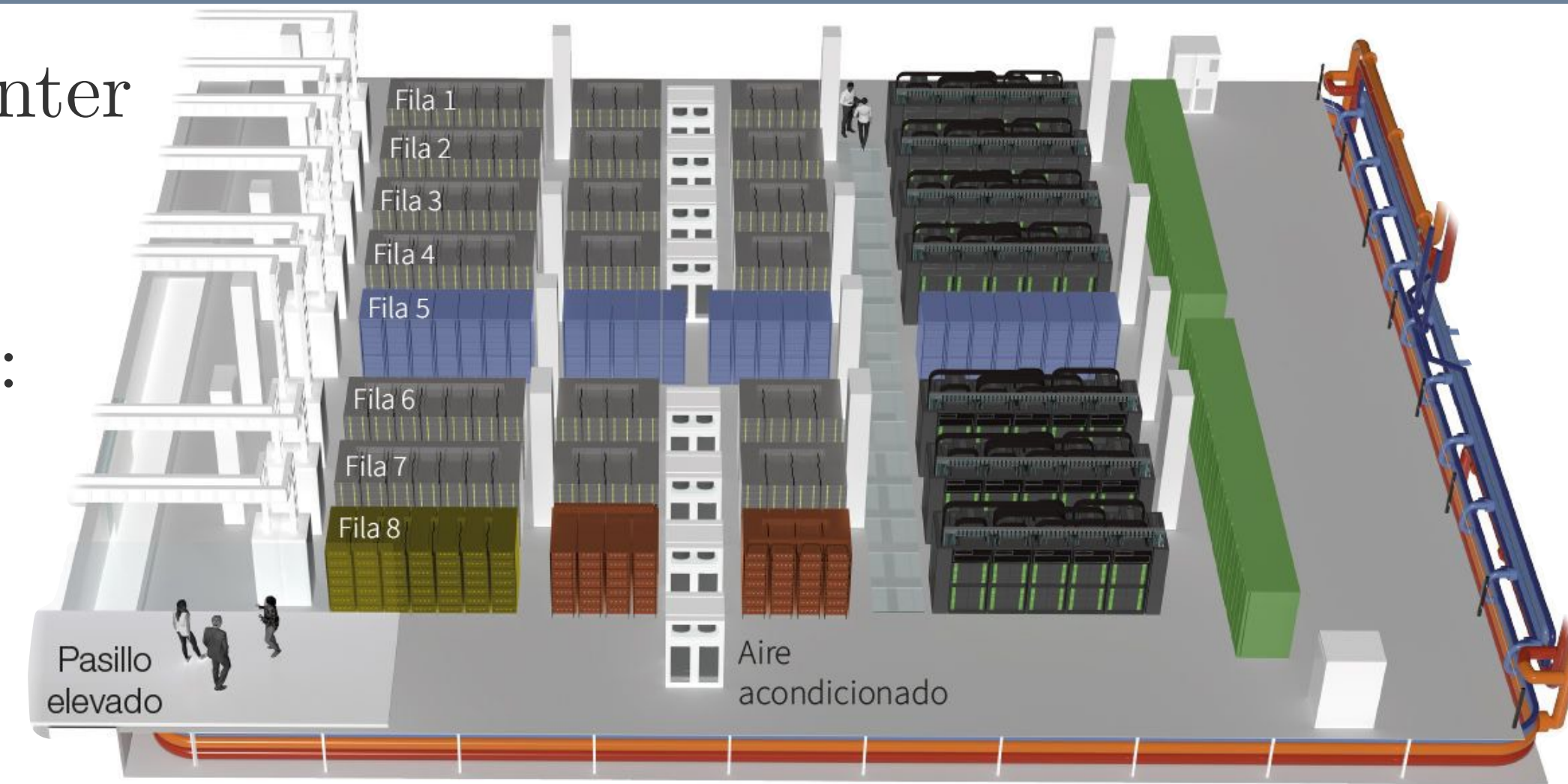


# COMPUTING RESOURCES

**MareNostrum 5** installed at Barcelona Supercomputing Center

- Excellent resource for LISA Spain community
- First allocation toward role of BSC/MareNostrum for DCC:  
“Preparations for the Spanish Contribution to the LISA  
Distributed Data Processing Centre”

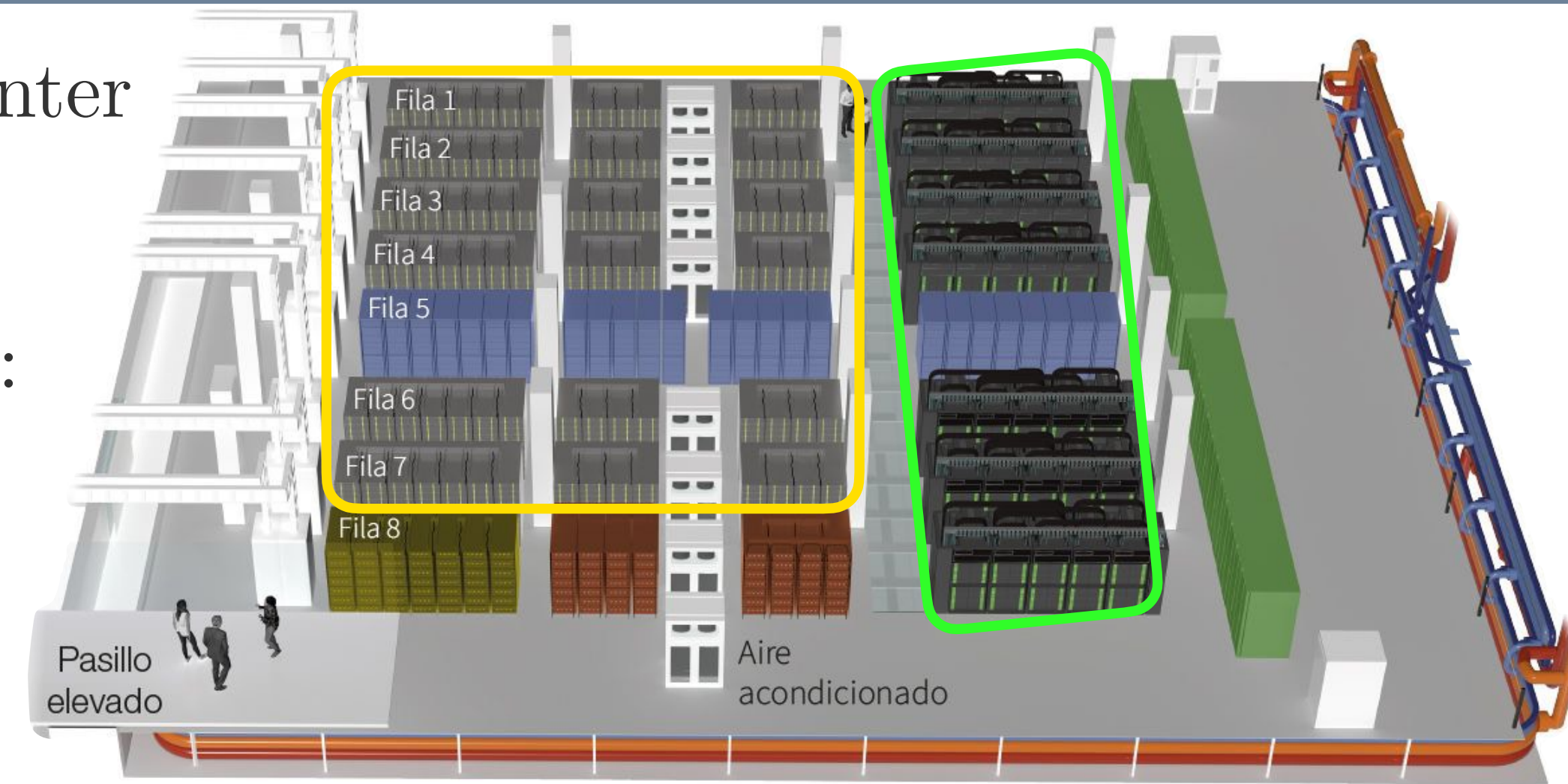
7 active users



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### General Purpose Partition

- 6192 nodes    112 CPUs    256 GB
- 216 nodes    112 CPUs    1024 GB
- 72 nodes    112 CPUs    128 GB

Allocation time: 985 kh

Currently running:

- CBC PE
- High-mass ratio NR simulations  
toward comparison with GSF

### Accelerated Partition

- 1120 nodes
- Total of ~700,000 compute units

Allocation time: 85 kh

Currently running:

- Code benchmarks
- LISA PE exploratory runs

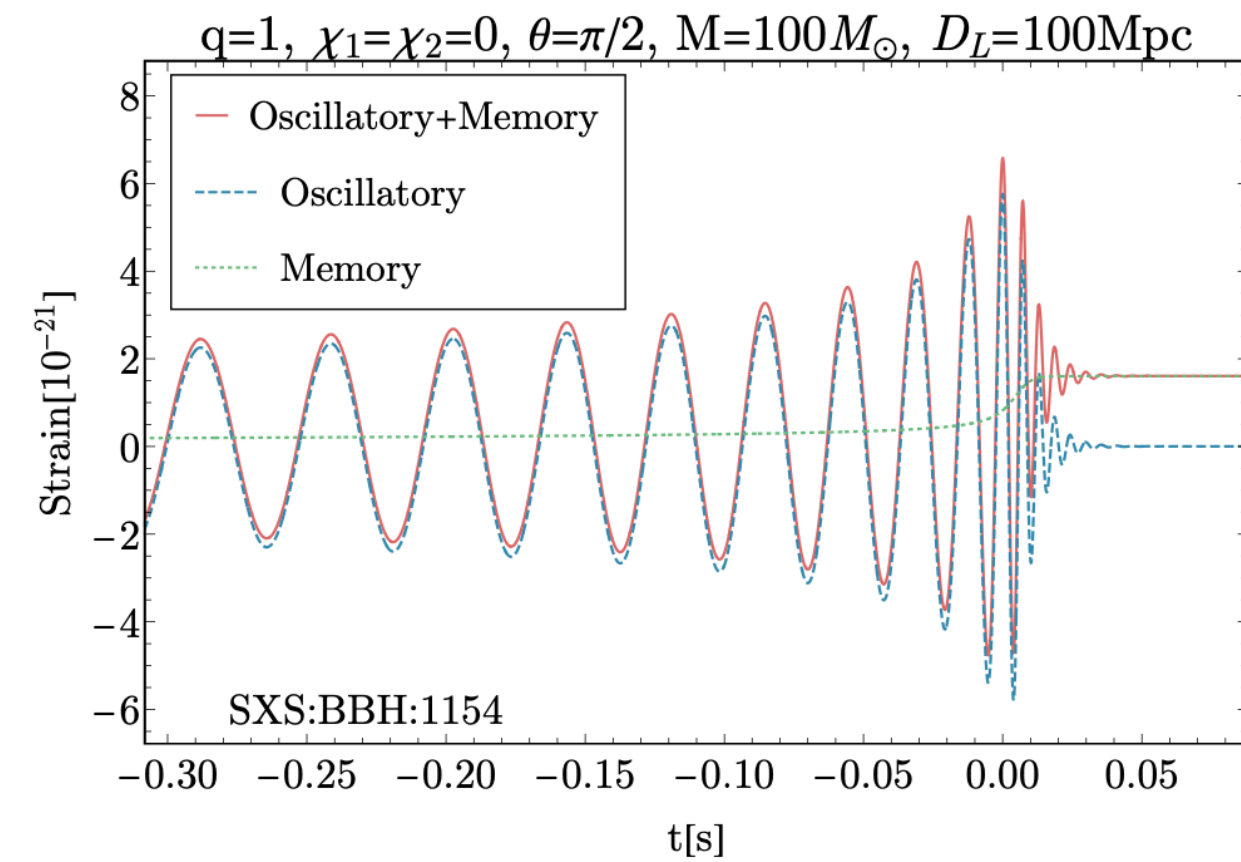
Hardware info taken from: <https://www.bsc.es/supportkc/docs/intro>

Images taken from: <https://www.bsc.es/es/noticias/noticias-del-bsc/arranca-marenostrum-5-el-nuevo-supercomputador-europeo-instalado-en-el-bsc>

# LISA CURRENT RESULTS...

## GW MEMORY

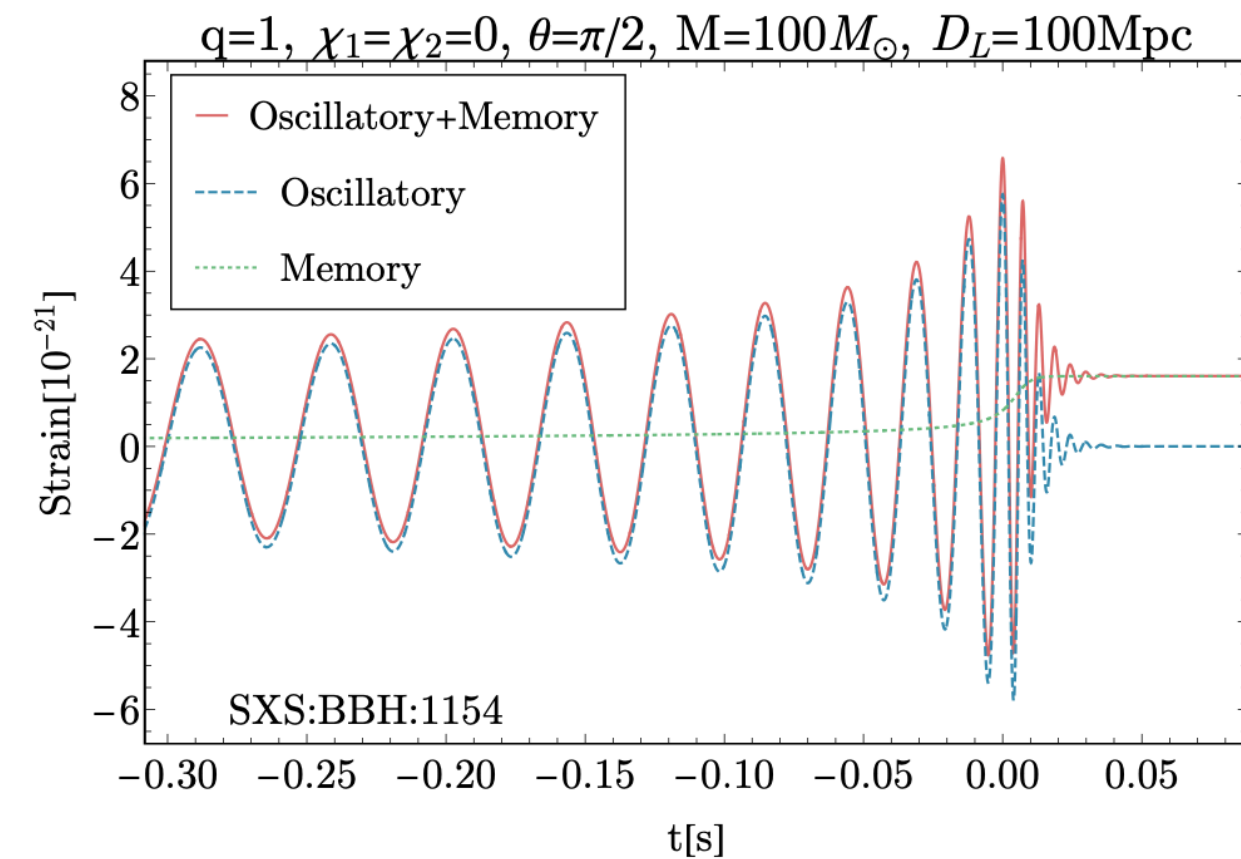
- (2,0) mode implemented in IMRPHENOMTHM by M. Rosselló-Sastre, S. Husa, S. Bera (2024) [\[arXiv:2405.17302\]](https://arxiv.org/abs/2405.17302)



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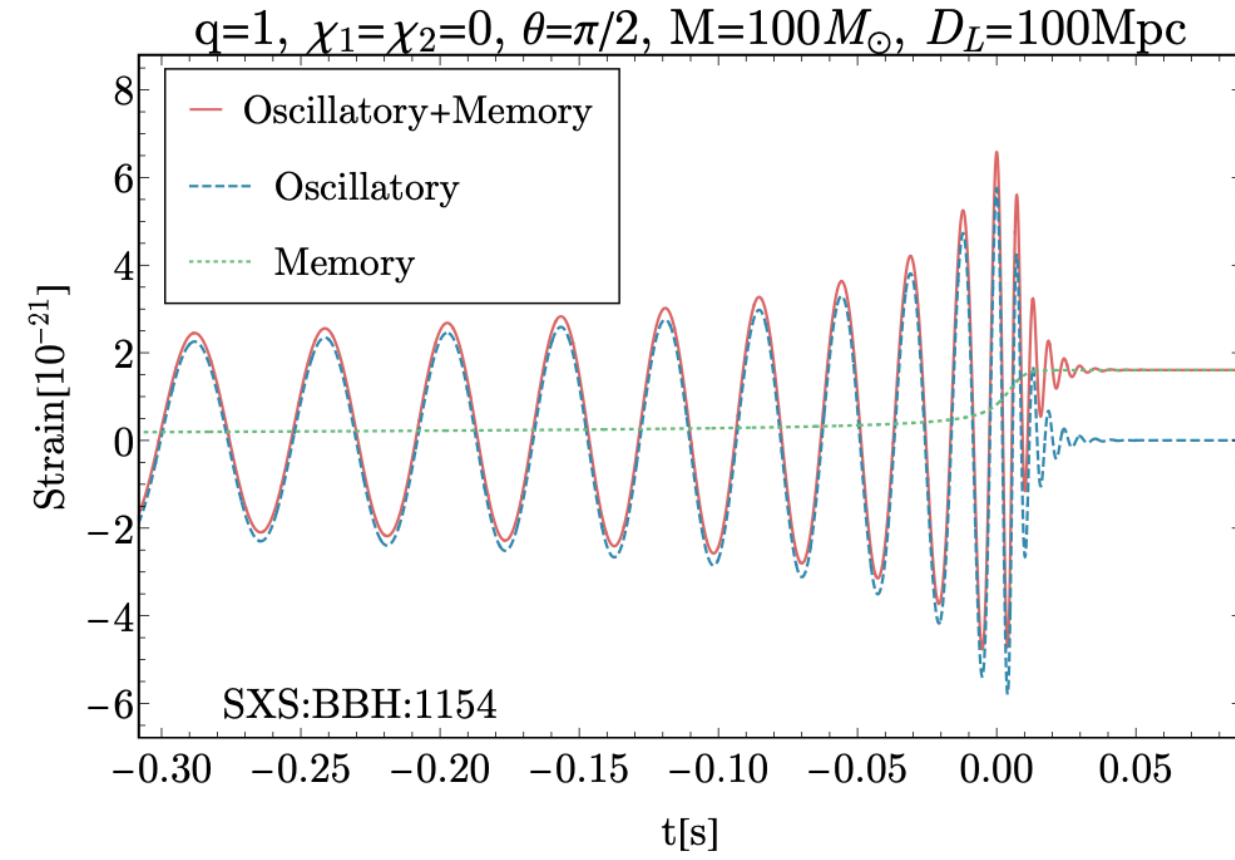
## LISA PARAMETER ESTIMATION LDC + phenomxpy (GPU)

- Zero noise injection with HMs + (2,0)
- Duration  $\sim 70$  days,  $M = 1.8 \times 10^6 M_\odot$ ,  $q = 1.25$
- SNR  $\sim 2500$
- $f_{\text{low}} = 10^{-4}$  Hz
- Sampler ptemcee with Fisher initialization

# LISA CURRENT RESULTS...

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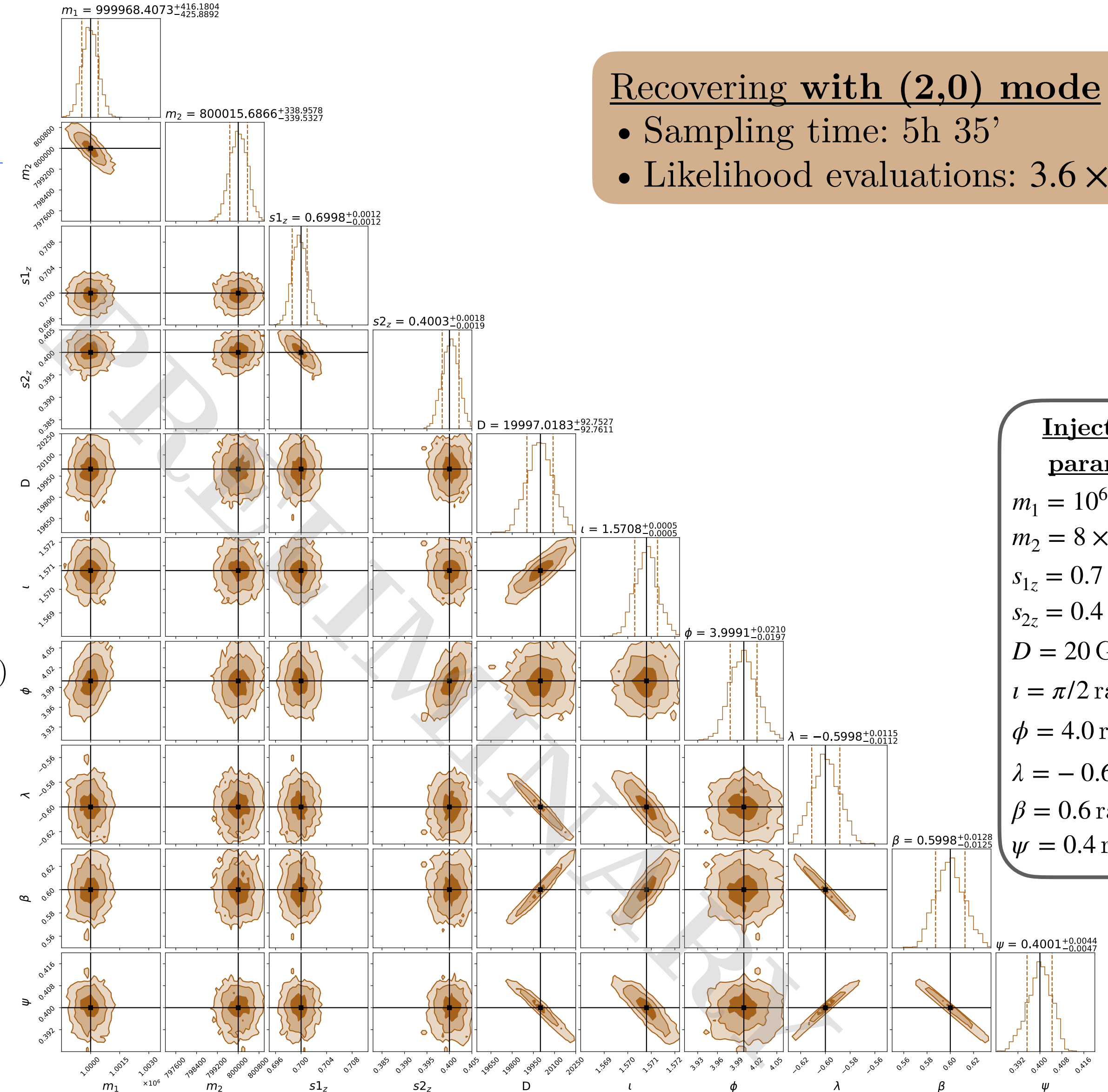
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### Recovering with (2,0) mode

- Sampling time: 5h 35'
- Likelihood evaluations:  $3.6 \times 10^5$

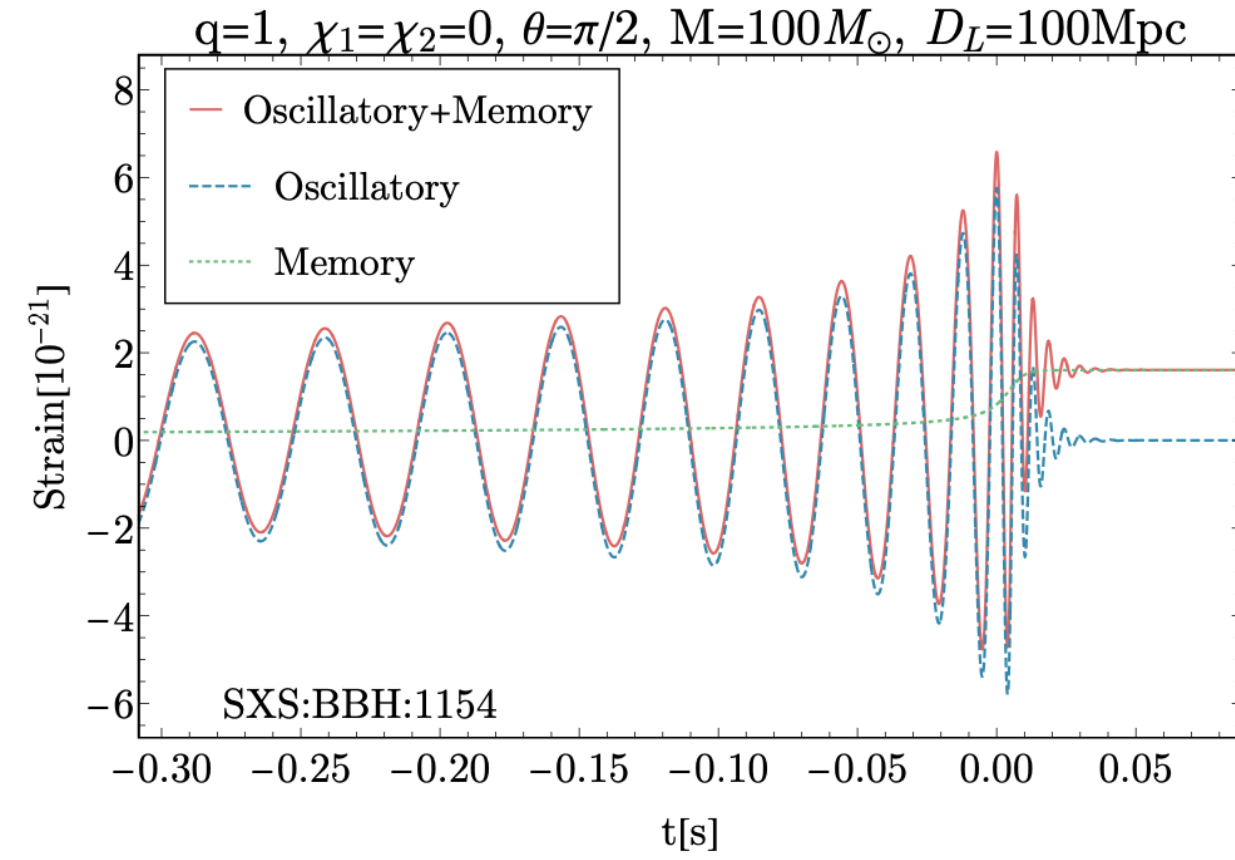
**Injection params.**

- $m_1 = 10^6 M_\odot$
- $m_2 = 8 \times 10^5 M_\odot$
- $s_{1z} = 0.7$
- $s_{2z} = 0.4$
- $D = 20$  Gpc
- $l = \pi/2$  rad
- $\phi = 4.0$  rad
- $\lambda = -0.6$  rad
- $\beta = 0.6$  rad
- $\psi = 0.4$  rad

# LISA CURRENT RESULTS...

## GW MEMORY

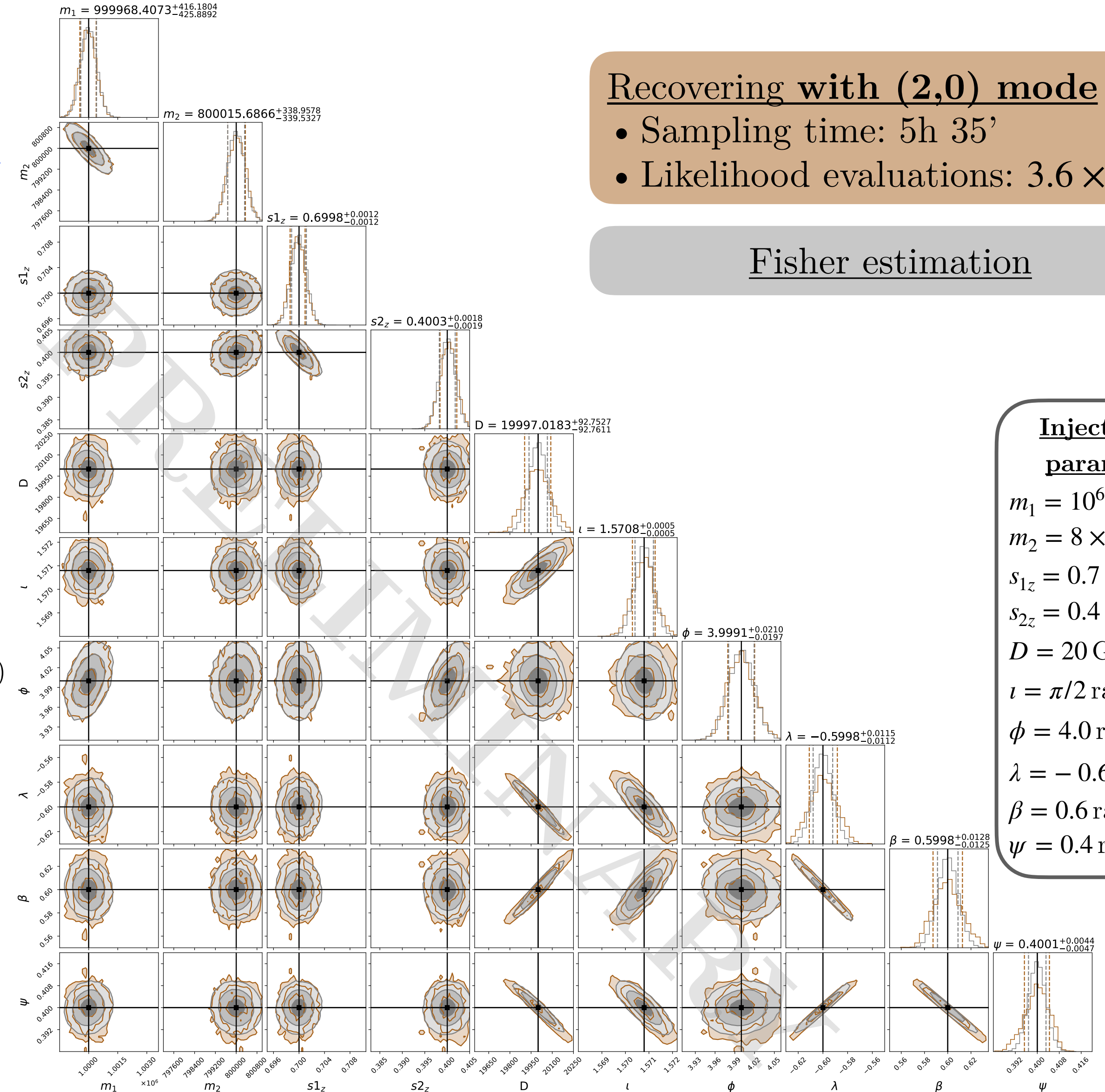
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**Recovering with (2,0) mode**

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**Fisher estimation**

**Injection params.**

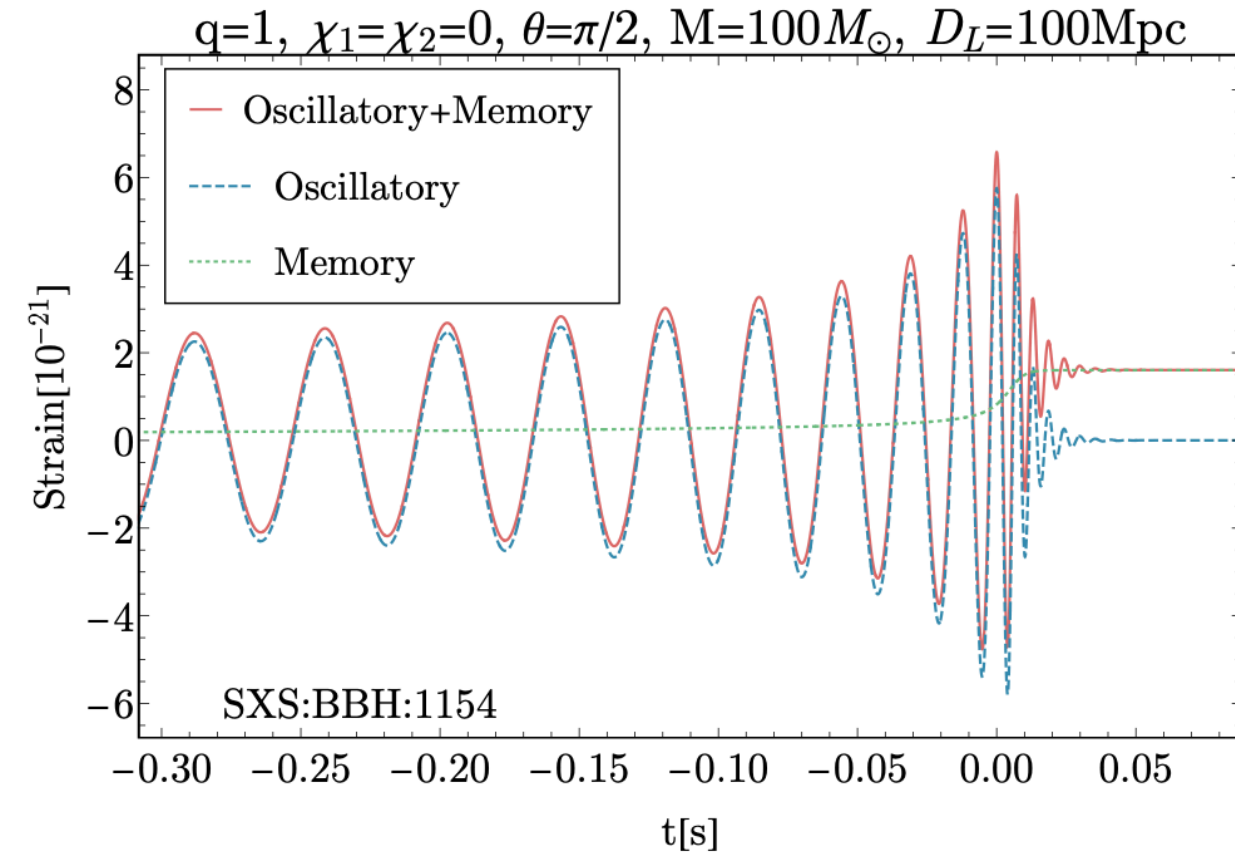
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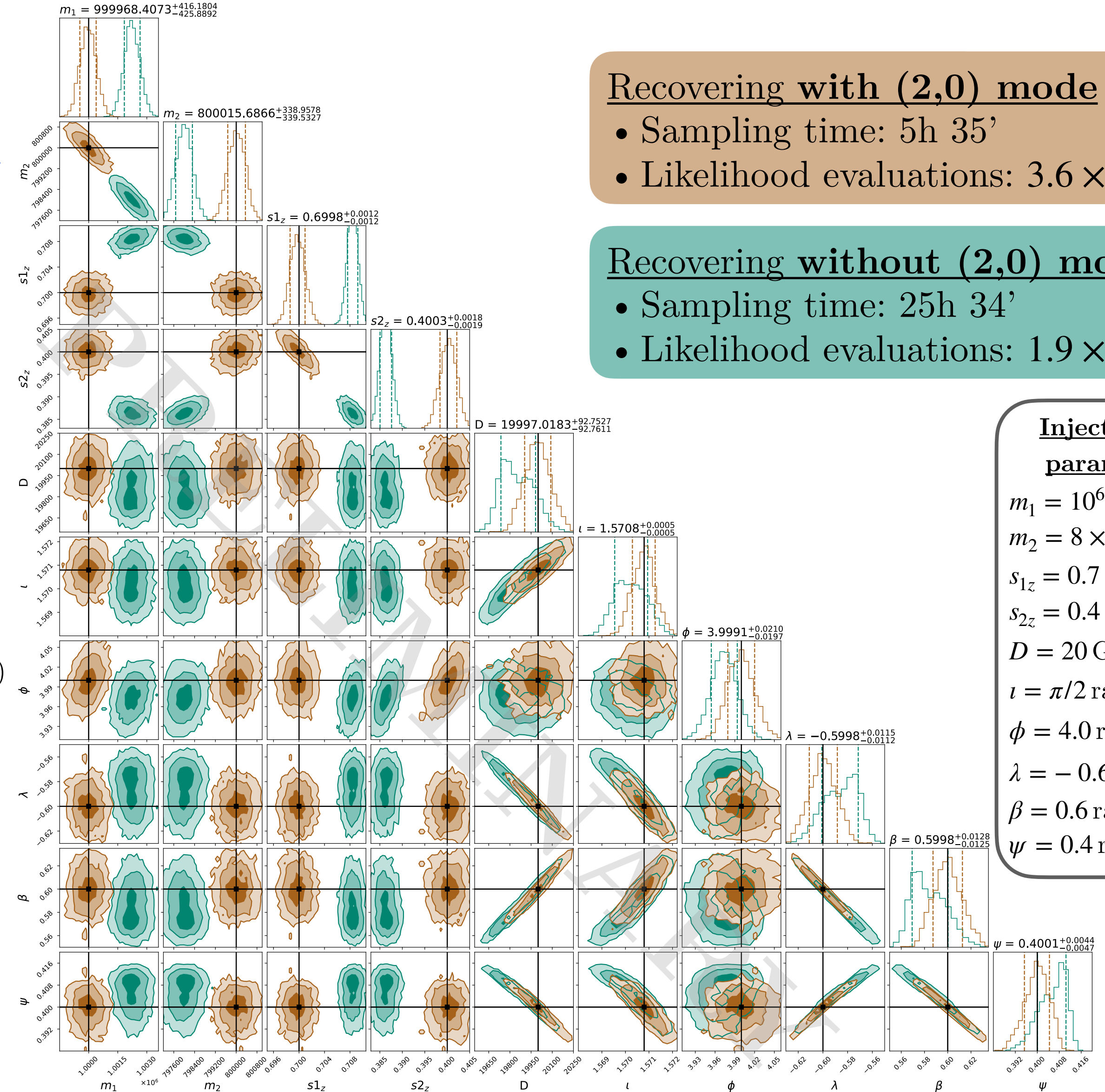
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**Recovering with (2,0) mode**

- Sampling time: 5h 35'
- Likelihood evaluations:  $3.6 \times 10^5$

**Recovering without (2,0) mode**

- Sampling time: 25h 34'
- Likelihood evaluations:  $1.9 \times 10^6$

**Injection params.**

- $m_1 = 10^6 M_\odot$
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# CONCLUSIONS

- Bring experience with different types of data analysis in the context of ongoing observation campaigns, interpretation of challenging events - interpretation of events in the real world can be challenging!
- Bring experience with workflows in a large collaboration: code development, reviews, bylaws, dealing with open data releases, ...
- Group is committed to further develop IMRPhenom models, including precession + eccentricity. Cover larger parameter space, increase speed and accuracy.
  - Current development is based on LALSuite and Cecilio's phenomxpy.
  - How can we make future waveform models run fast, e.g. on GPUs, integrate with fast LISA response.
- **Ongoing:** LISA analyses with the (2,0) mode

# ACKNOWLEDGMENTS

Jorge Valencia Gómez is supported by the Spanish Ministry of Universities (FPU22/02211).

This work was supported by the Universitat de les Illes Balears (UIB); the Spanish Agencia Estatal de Investigación grants PID2022-138626NB-I00, RED2022-134204-E, RED2022-134411-T, funded by MICIU/AEI/10.13039/501100011033, by the ESF+ and ERDF/EU; the MICIU with funding from the European Union NextGenerationEU/PRTR (PRTR-C17.I1); the Comunitat Autònoma de les Illes Balears through the Direcció General de Recerca, Innovació i Transformació Digital with funds from the Tourist Stay Tax Law (PDR2020/11 - ITS2017-006), the Conselleria d'Economia, Hisenda i Innovació grant numbers SINCO2022/18146 and SINCO2022/6719, co-financed by the European Union and FEDER Operational Program 2021-2027 of the Balearic Islands.



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Impost de Turisme Sostenible

**Illes  
Sostenibles**

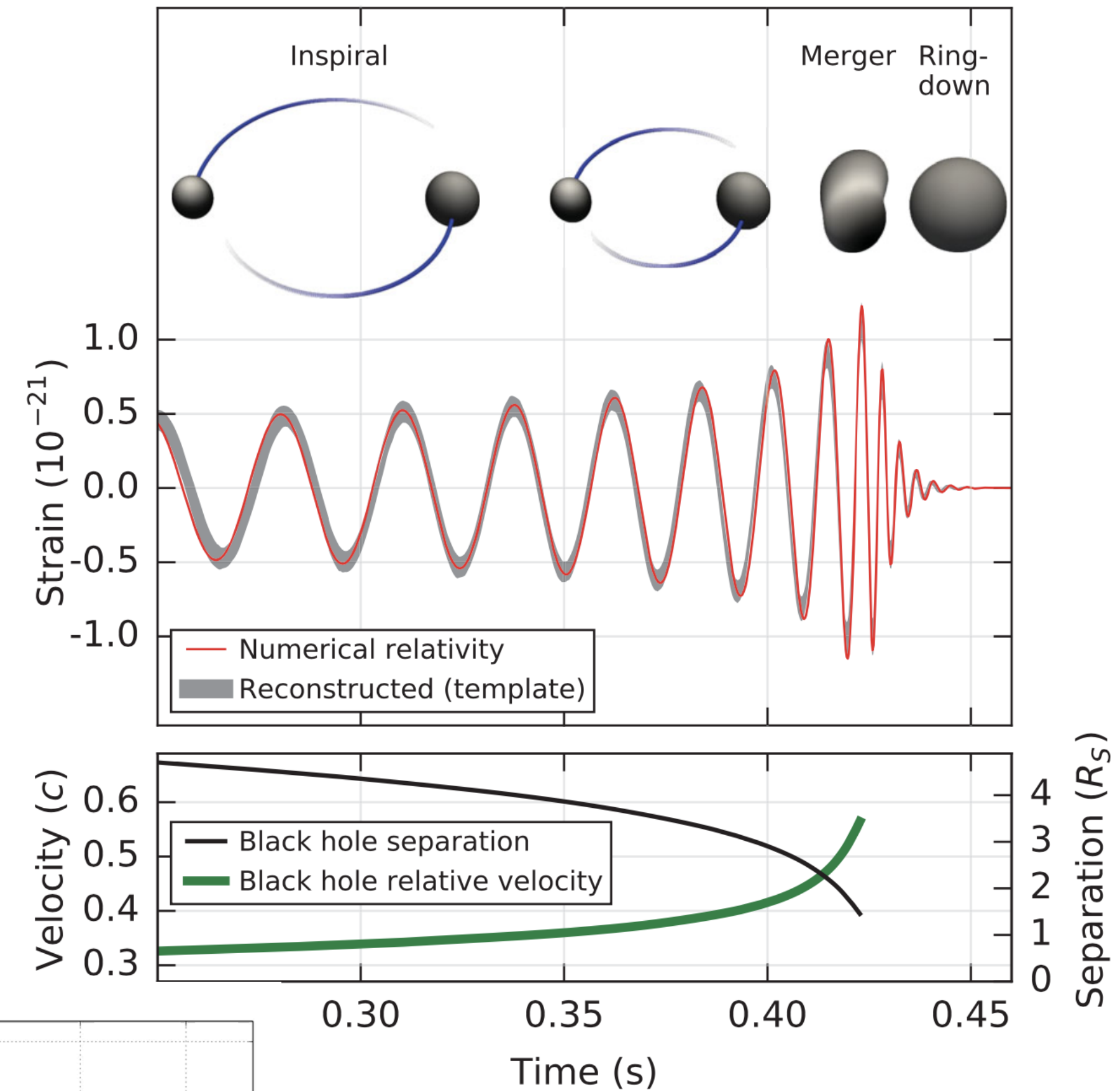


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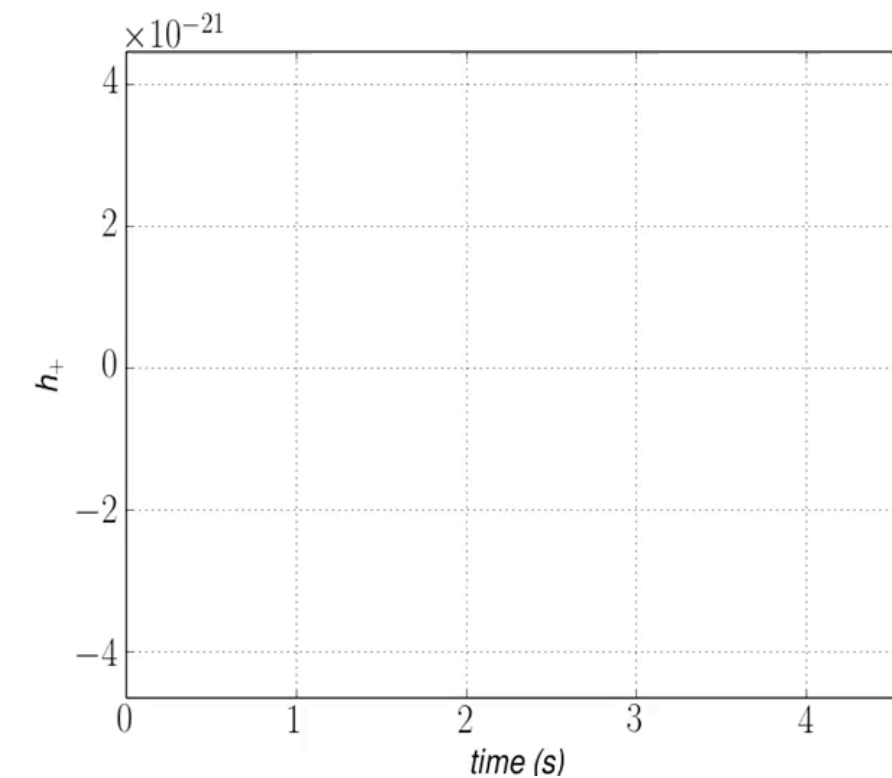
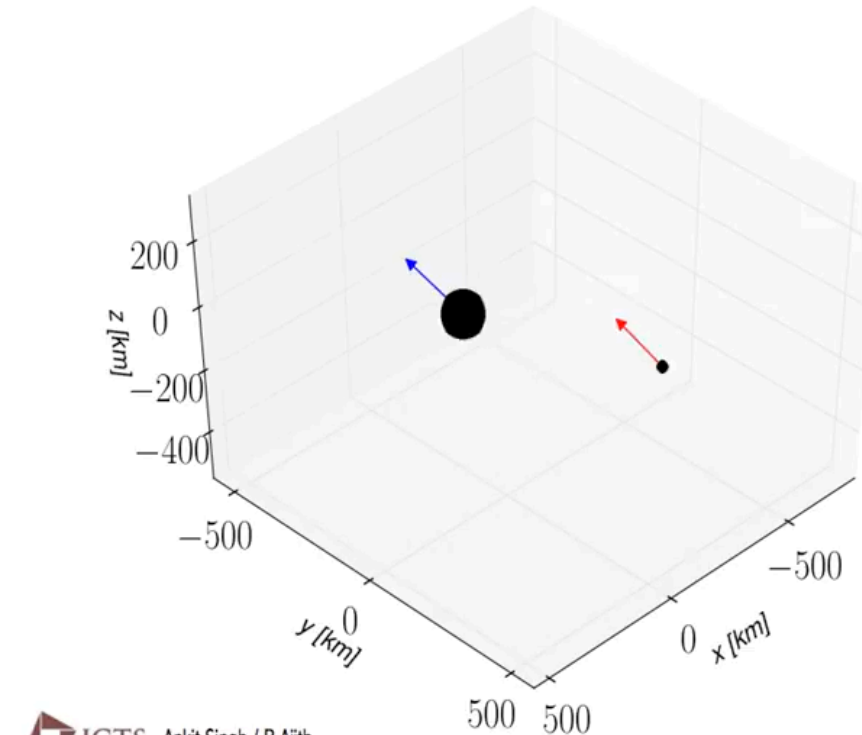
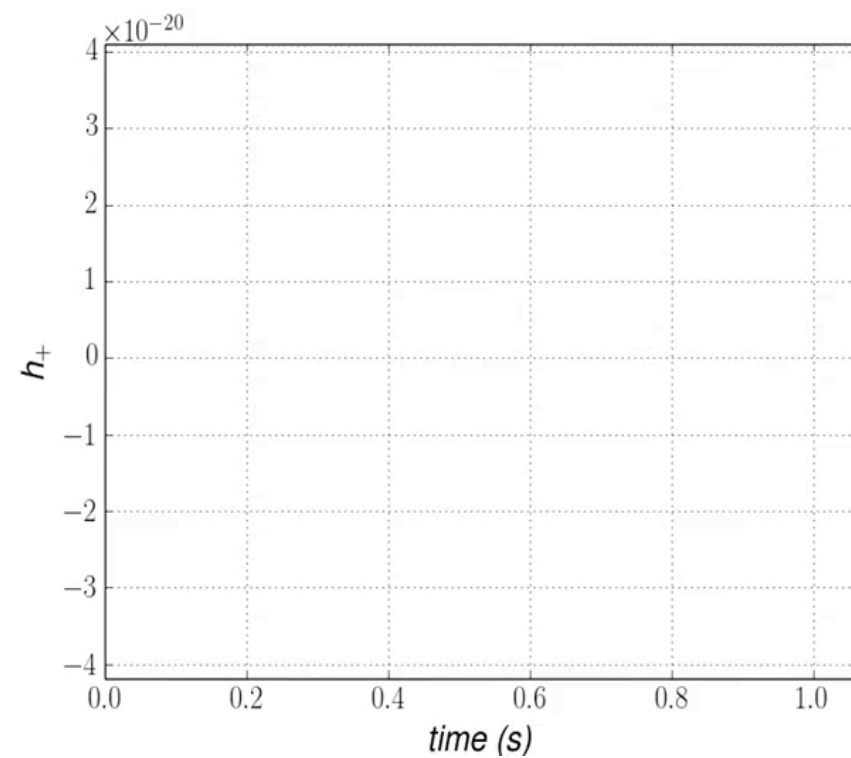
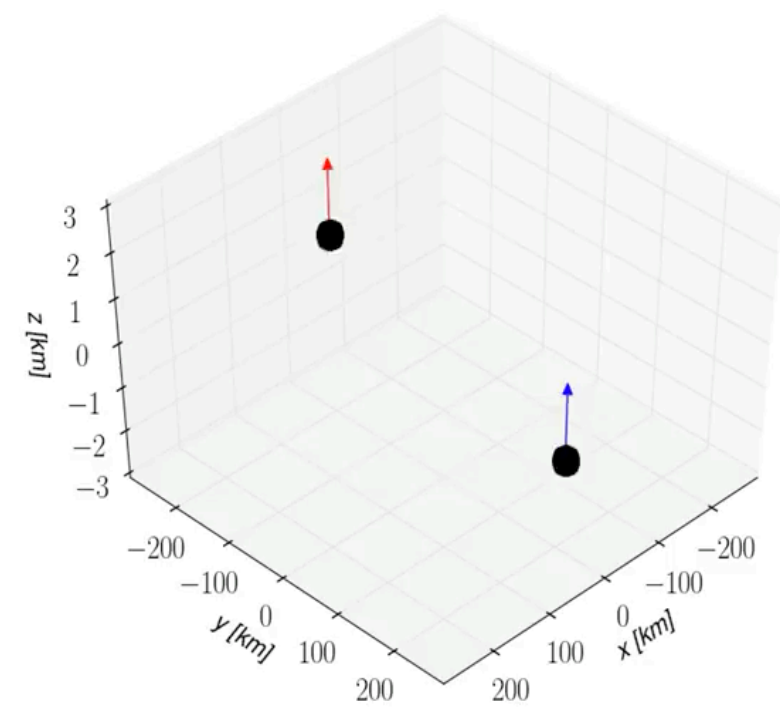
Backup slides

# WAVEFORM MODELLING

- **Binary Black Holes (BBHs)** are so far the **most detected** source of gravitational waves (GWs).
  - The **evolution** of the binary is divided into 3 stages:
    - **Circular orbits: 8 intrinsic parameters:**  $\{m_1, m_2, \vec{\chi}_1, \vec{\chi}_2\}$ , (+2 if eccentric)
    - $0 \leq \chi_{1,2} \leq 1$ ,  $M = m_1 + m_2$ ,  $q = \frac{m_1}{m_2} \geq 1$ ,  $\eta = \frac{q}{(1+q)^2}$ .
    - **7 extrinsic parameters:**  $\{d_L, \lambda, \beta, \psi, \iota, t_c, \phi\}$ .



B. P. Abbott + (2016) [\[Phys. Rev. Lett. 116, 061102\]](https://arxiv.org/abs/1602.03837)

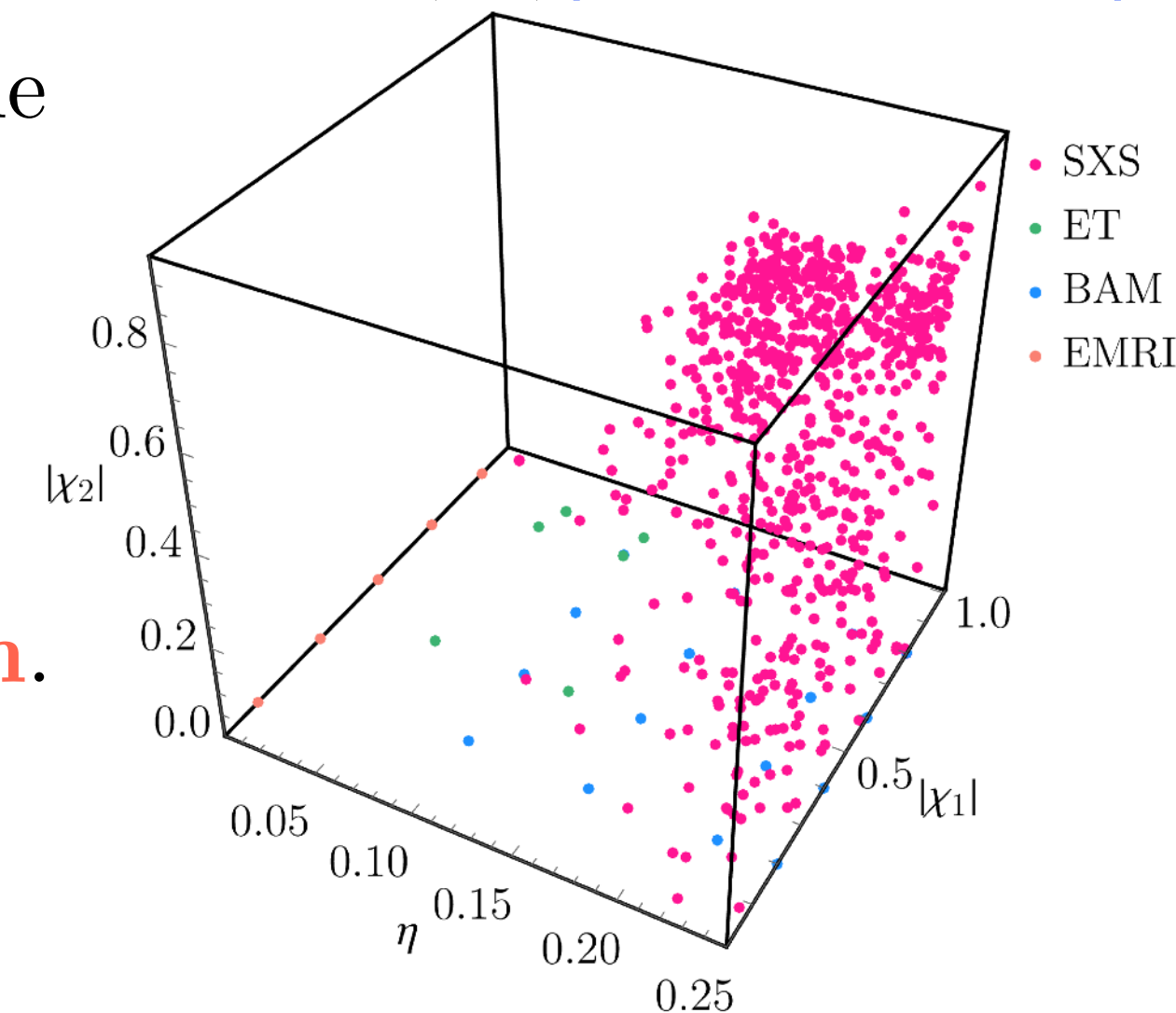


# WAVEFORM MODELLING

**Waveform models** are essential to understand the GW signals emitted from the coalescence of compact binaries.

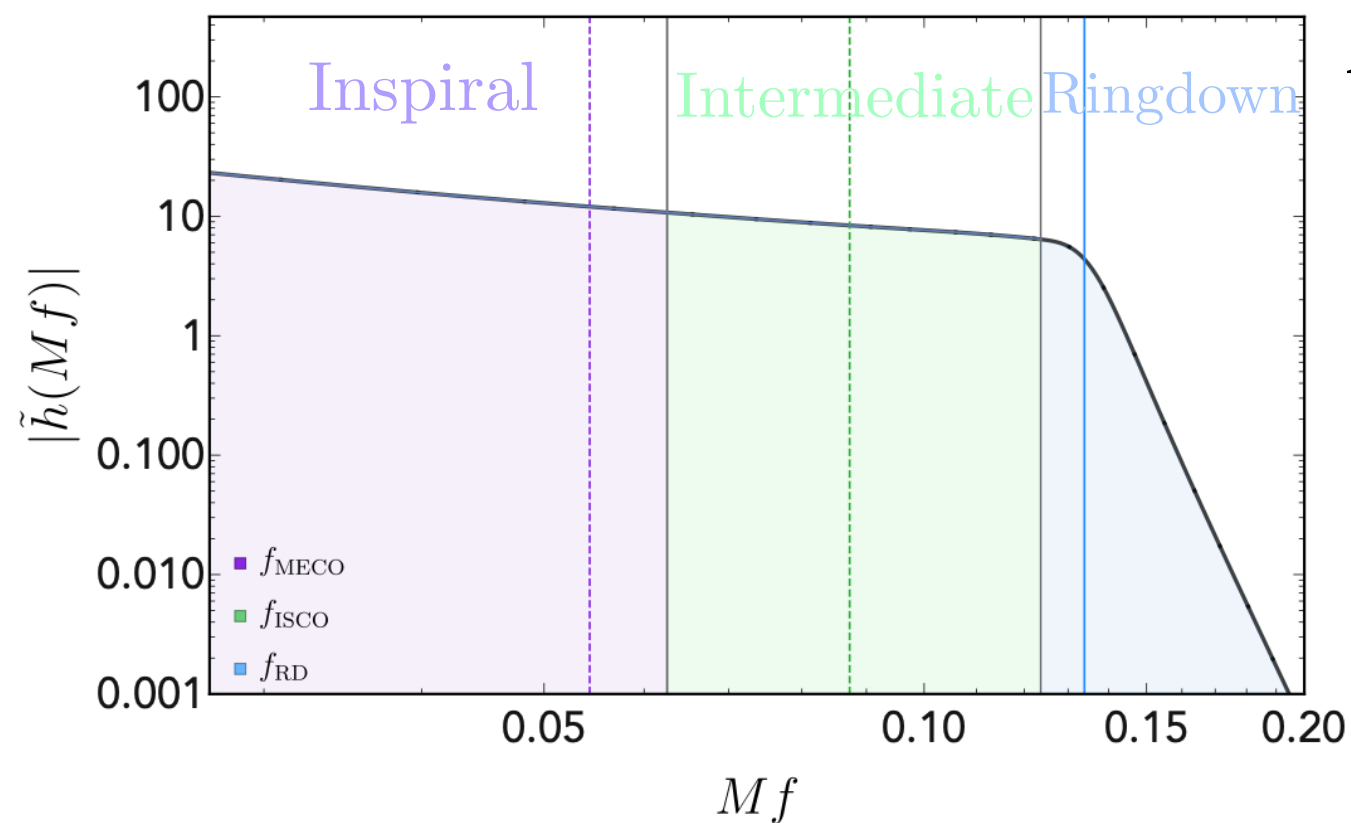
- **Post-Newtonian (PN) theory**: expands Newtonian equations in powers of  $(v^2/c^2)$ . **Less accurate close to the merger.**
- **Numerical relativity (NR)**: crucial to describe the **merger and ringdown**. Currently the most accurate descriptions of GWs. Catalogs: SXS, ET, BAM ...

Lluc Planas + (2024) [[Phys. Rev. D 109, 124028](#)]



**Efficient waveform models** that cover the **full evolution** are needed to reconstruct the source parameters.

- **Effective one-body models (EOB)**: reformulate the two-body problem in GR into an effective one-body problem.
- **Surrogate models (NRSur)**: modern interpolation techniques for datasets of numerical waveforms.
- **Phenomenological models (IMRPhenom)**: use piecewise closed-form analytical expressions to model the three stages.



G. Pratten + (2020) [[Phys. Rev. D 102, 064001](#)]

# FINAL STATE FITS

$$\Delta E = E_{\text{rad}}^{\text{prec}}(\eta, \chi_1, \theta_1) - E_{\text{rad}}^{\text{AS}}(\eta, \chi_1 \cos(\theta_1), \chi_2 \cos(\theta_2)) = 0$$

$$\overline{\Delta E} = \Delta E - \Delta E_{\text{EMR}}$$

$$\begin{aligned} \overline{\Delta E}(\eta, \chi_1, \theta_1) = & \eta^3 \chi_1 [0.759123 \sin(\theta_1) - 2.33392 \sin(2\theta_1)] + \\ & \eta^3 \chi_1^2 [6.51059 \sin(\theta_1) + 7.06906 \sin(2\theta_1)] + \\ & \eta^4 \chi_1 [-11.7873 \sin(\theta_1) + 22.364 \sin(2\theta_1)] + \\ & \eta^4 \chi_1^2 [-37.0594 \sin(\theta_1) - 63.3841 \sin(2\theta_1)] + \\ & \eta^5 \chi_1 [35.0427 \sin(\theta_1) - 51.36 \sin(2\theta_1)] + \\ & \eta^4 \chi_1^2 [-37.0594 \sin(\theta_1) - 63.3841 \sin(2\theta_1)]. \end{aligned}$$

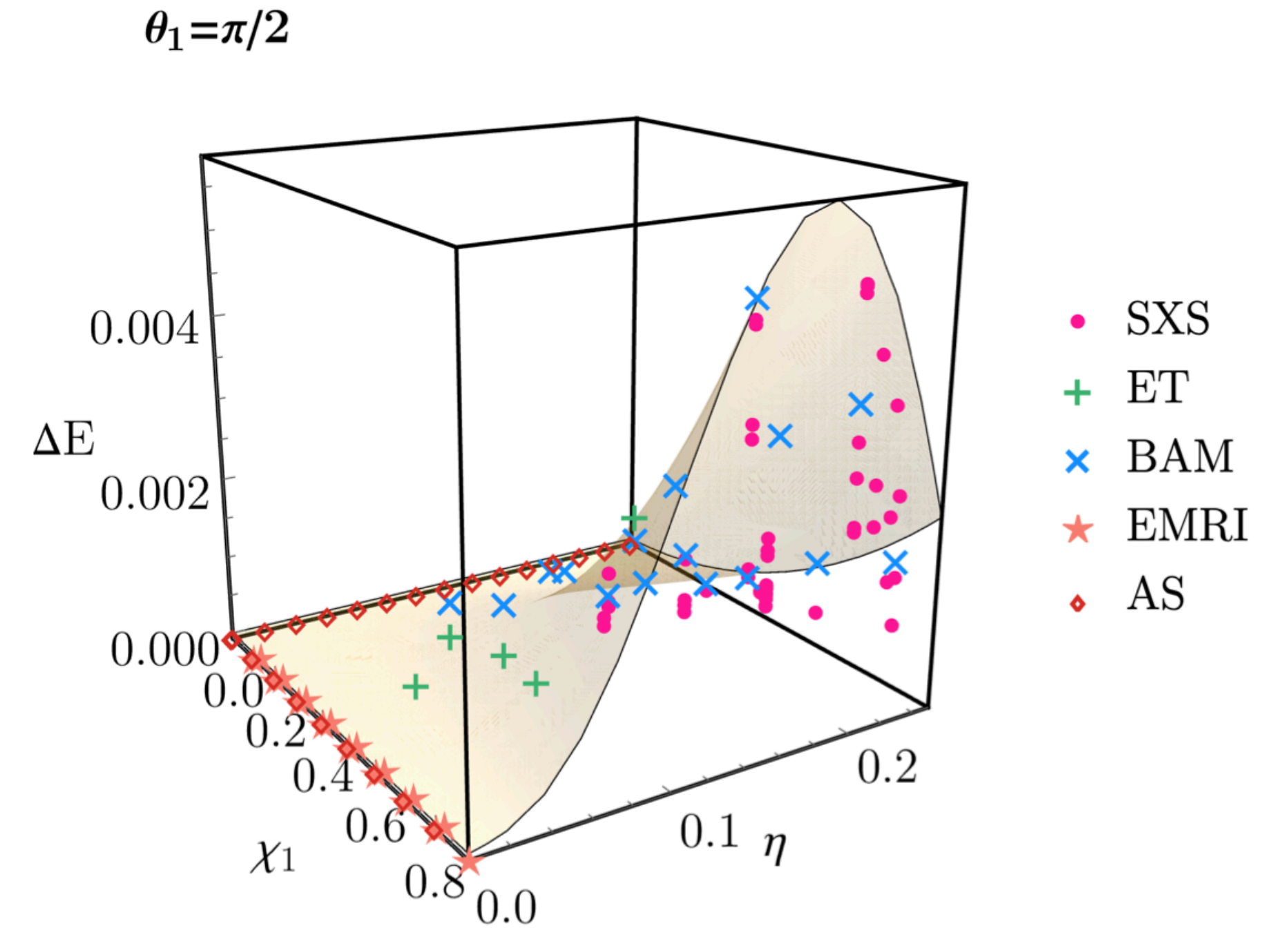


FIG. 10. Numerical evaluation of  $\Delta E$  as defined in Eq. (3.1), obtained from the parameterized fit  $\overline{\Delta E}$  (3.6) and  $\Delta E_{\text{EMR}}$ , at a fixed spin orientation  $\theta_1 = \pi/2$ , while varying the mass ratio  $\eta$  and the spin magnitude  $\chi_1$ . The figure includes the single spin precessing simulations shown in Fig. 2 that fall into this subspace.