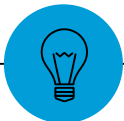


Memory effect in LISA sources: prospects and relevance

Silvia Gasparotto (IFAE)

Based on Phys.Rev.D 107 (2023) 12 and 2406.09228



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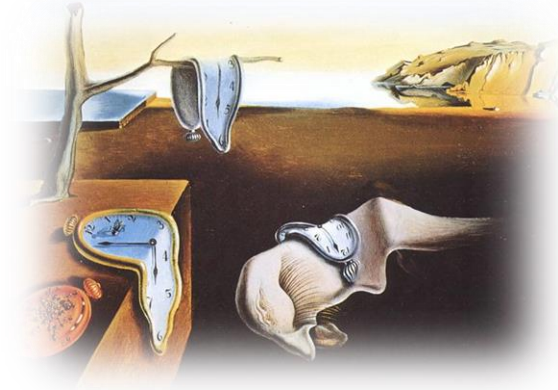
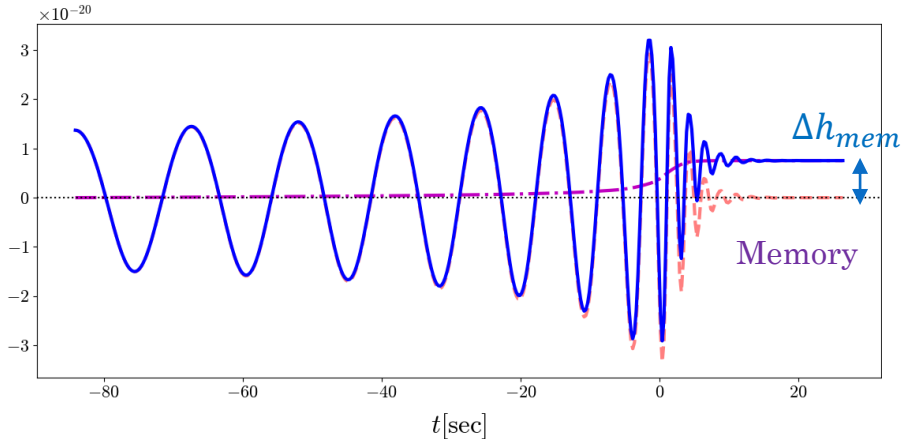
LISA Spain Meeting,
16th October 2024

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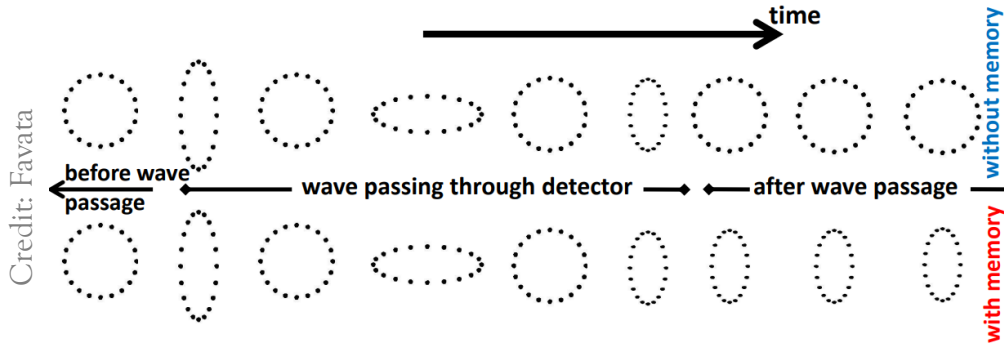
Gravitational Wave Memory



“The Persistence of Memory”
(also known as “The Soft Watches”)
Salvador Dalí, 1931

Persistent off-set of the
GW strain

The effect is a net displacement
between two comoving
observers



Linear and Non-linear Memory

- **Linear memory** (Zeldovich & Polnarev '74, Brangisky & Grischuk '85, Brangisky & Thorne '87)

Motion of unbound objects or radiation to infinity
(ex: SN neutrinos, hyperbolic objects etc)

- **Non-linear memory** (Christodoulou '91, Blanchet & Damour '92, Wiseman & Will '91...)

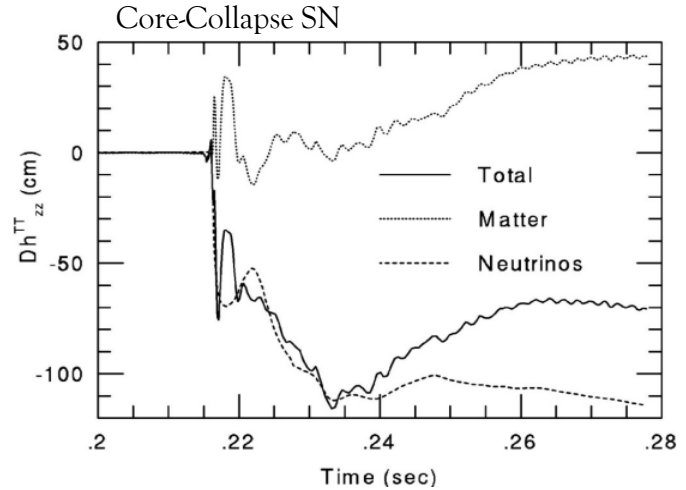
The GW itself sources GWs!

$$\partial^\mu \partial_\mu \bar{h}^{j,k} = 16\pi \left(T_{matter}^{jk} + T_{GW}^{jk} \right)$$

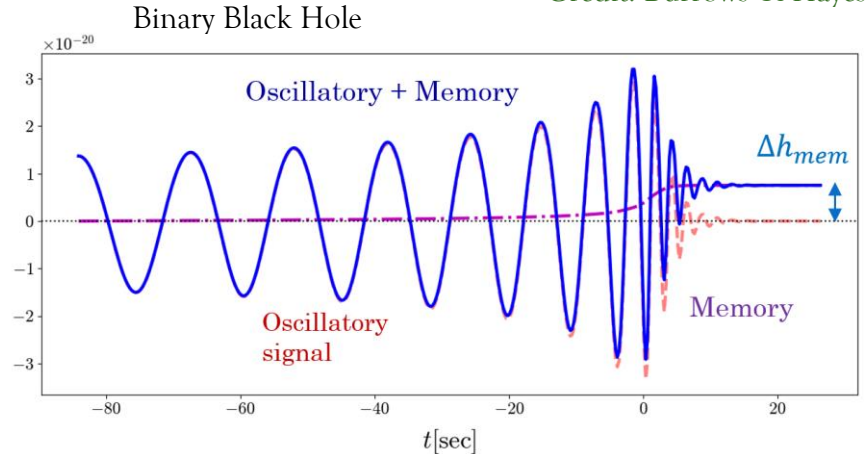
$$T_{GW}^{jk} = \frac{1}{R^2} \frac{dE_{GW}}{dt d\Omega} n_j n_k \sim \mathcal{O}(h^2)$$

Thorne Formula:

$$\delta \bar{h}_{ij}^{TT}(T_R) = \frac{4}{R} \int_{-\infty}^{T_R} dt' \left[\int \frac{dE_{GW}}{dt' d\Omega'} \frac{n'_j n'_k}{|1 - n' \cdot N|} d\Omega' \right]^{TT}$$



Credit: Burrows & Hayes '96



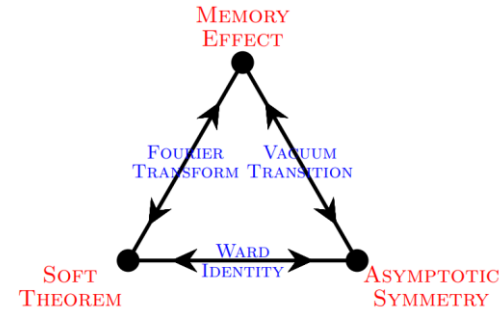
Non-linear GW memory

Why do we care?

- Non-linear prediction of GR, still undetected
- BMS symmetries and the Soft theorem
- “Displacement memory” related to the super-translation symmetry, but new (subdominant) memories from other symmetries

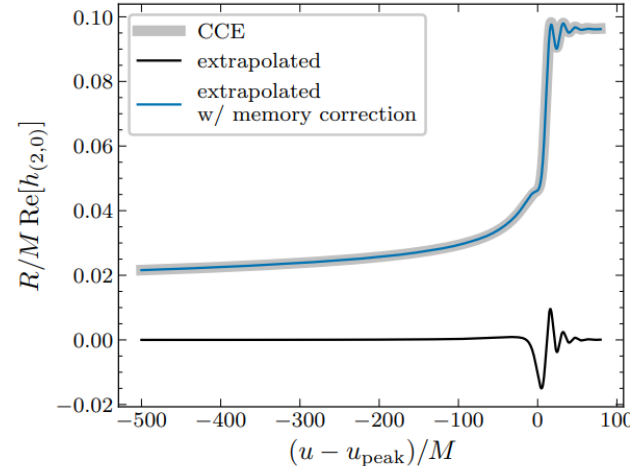
How do we compute it?

- Traditional waveforms don't present the memory → difficulties in extracting from NR simulations
- Memory can be computed from the energy flux of GW (GWMemory, BMS flux balance laws); the main (2,0)-mode
- First Surrogate model with the memory with new Cauchy Characteristics Extraction (CCE) scheme NRHybSur3dq8_CCE. First IMRPhenom model M. Rosselló-Sastre et al (2405.17302)



(A. Strominger and A. Zhiboedov 2016)

(J. Yoo et al. 2306.03148)



Detecting GW memory

Earth-based interferometers

- **Ligo-Kagra-Virgo:** no detection so far.
Estimated $O(2000)$ sources to claim detection
(1911.12496,2105.02879,2210.16266,2404.11919)
- **Einstein Telescope & Cosmic Explorer:** $O(1) \text{ yr}^{-1}$ (2210.16266)

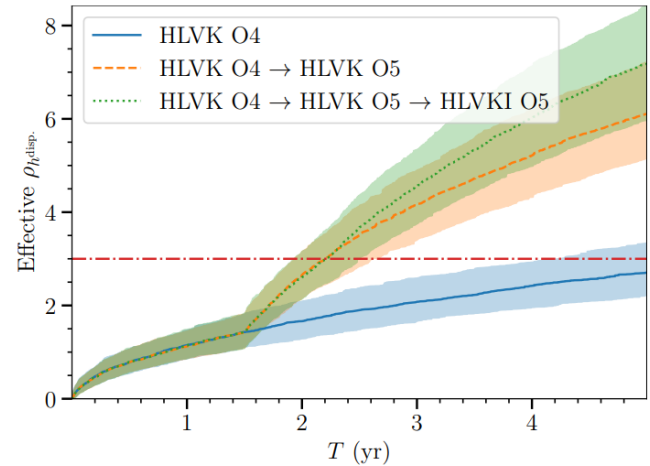
Space-based interferometers

- **LISA:** previous prospects 1906.11936, updates H. Inchauspé & S.Gasparotto et al. 2406.09228
- **TianQin:** 2207.13009,2401.11416

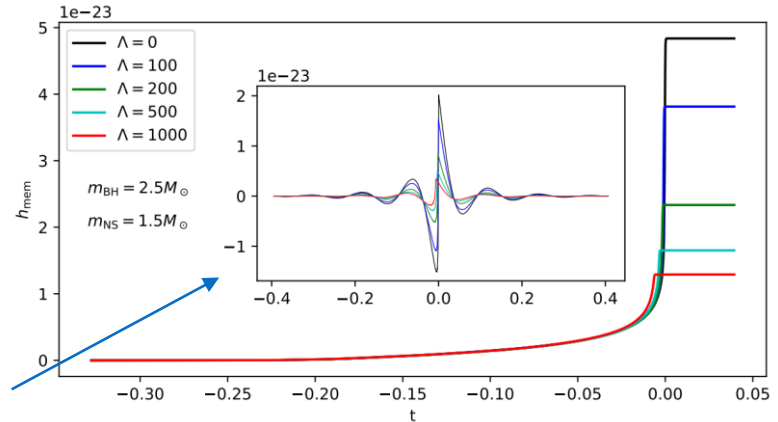
PTA: Search for burst-like signal with memory as mergers of SMBH
 $M \sim 10^8 M_{\odot}$ (0909.0954, 2307.13797)

Others???

In ground-based interferometers, we don't observe the persistent off-set, high-passed signal



Credit: A. M. Grant & D.A.Nichols



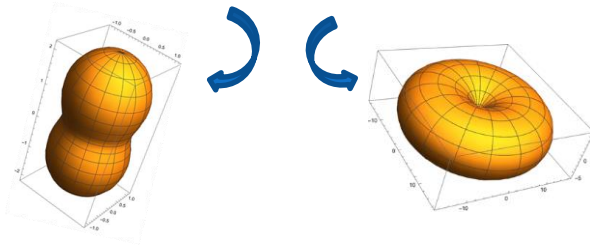
“Leveraging gw memory to distinguish NS-BH binaries from BH binaries” 2110.1117

Enhancing parameter estimation with the memory

- Importance of adding the (2,0)-mode to the waveform

$$h_{+,0PN} = \left[-(1 + \cos^2 i) \cos 2\Phi(t) + \frac{1}{96} \sin^2 i (17 + \cos^2 i) \right] \frac{2\eta M (M\omega(t))^{2/3}}{R}$$

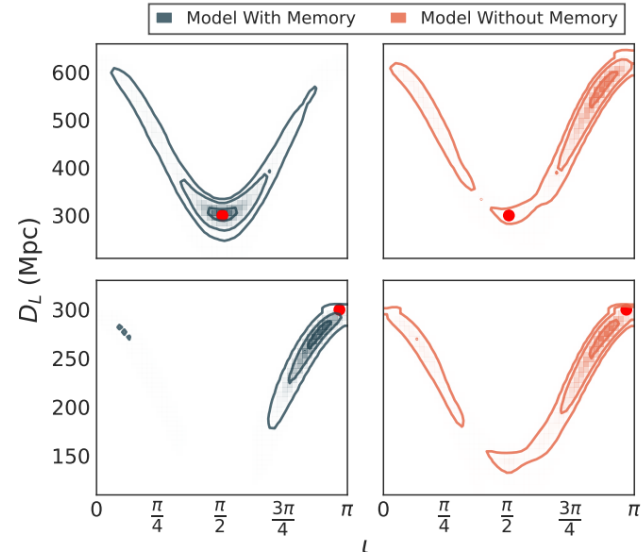
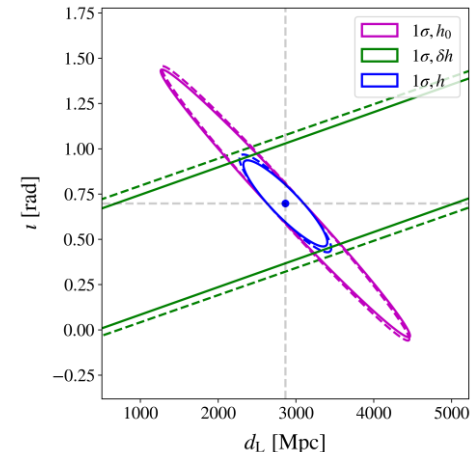
Can the memory break the luminosity distance-inclination degeneracy?



- Results for LISA: Gasparotto et al. 2301.13228 (Fisher matrix)
- Results for (Advanced) LIGO: Yumeng Xu et al. 2403.00441 (Bayesian)

Common outcome:

- Memory extends the signal at a lower frequency, which helps for short inspiral and almost out-of-band sources



Credit: Yumeng Xu et al. 2403.00441

Measuring GW memory with LISA

Based on 2406.09228 H. Inchauspé, SG et al

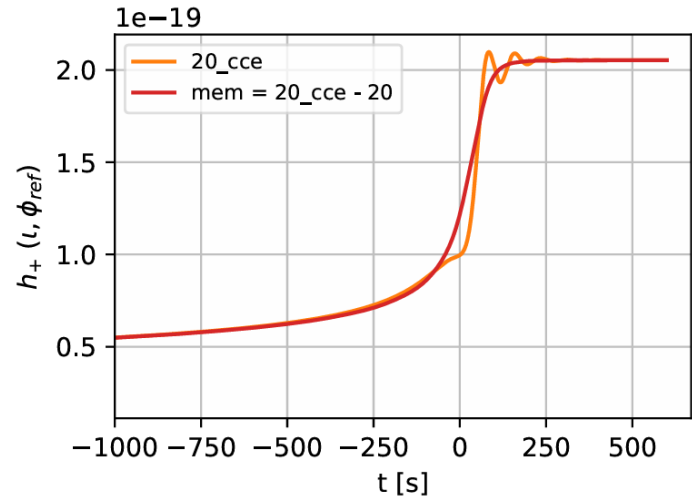
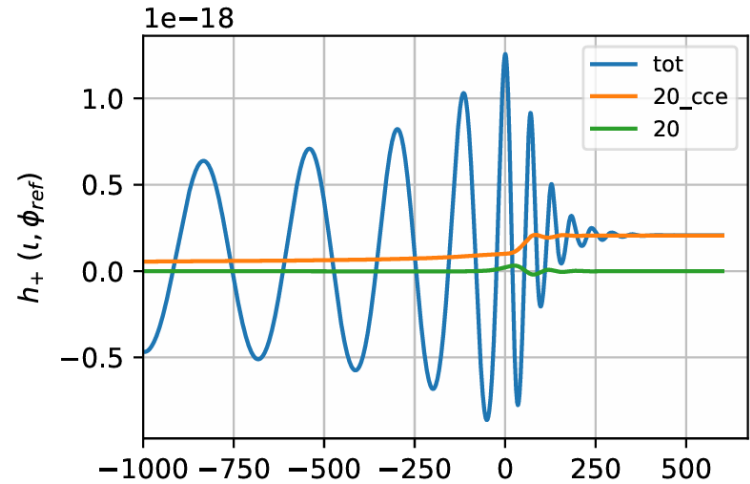
Part of the *Ringdown* collaborative projects of the LISA FPWG.

- i. *Imprint of GW memory*
- ii. *Memory vs non-memory signal*
- iii. *Scientific reach of LISA for the GW memory*

We simulate the full time-domain response of the detector down to the TDI data stream using the new [NRHybSur3dq8_CCE](#) (2306.03148), and the [GWmemory](#) package (1807.00990)

$$h_+^{mem}(t) \equiv h_{+,CCE}^{20}(t) - h_{+,CCE}^{20}(t).$$

Memory + ringdown Ringdown



TDI imprint of GW memory

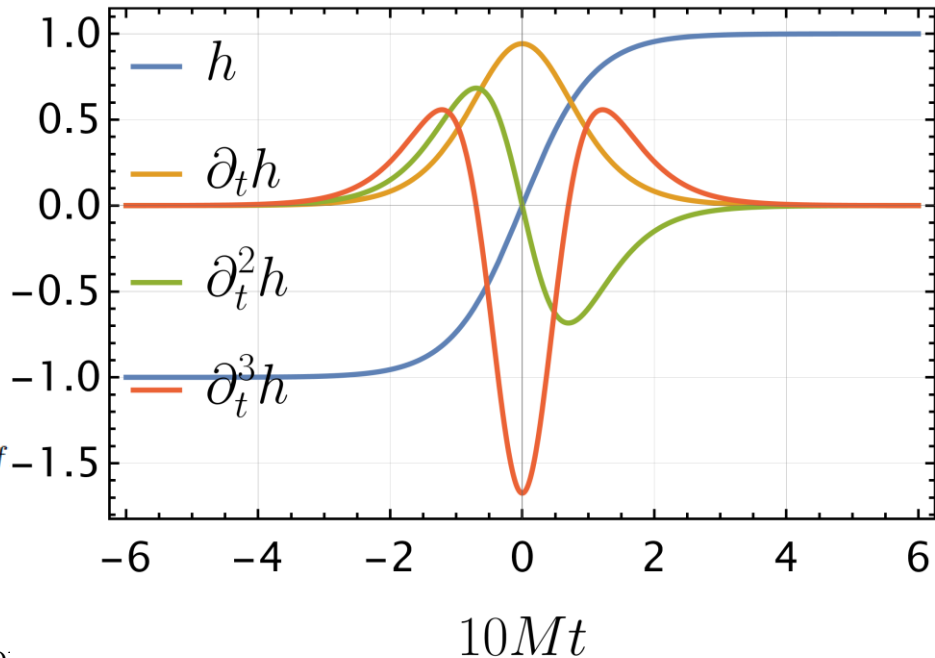
$$\begin{aligned}
 X_2 = & X_{1.5} + \mathbf{D}_{13121}y_{12} + \mathbf{D}_{131212}y_{21} + \mathbf{D}_{1312121}y_{13} \\
 & + \mathbf{D}_{13121213}y_{31} - [\mathbf{D}_{12131}y_{13} + \mathbf{D}_{121313}y_{31} \\
 & + \mathbf{D}_{1213131}y_{12} + \mathbf{D}_{12131312}y_{21}],
 \end{aligned}$$

with

$$\begin{aligned}
 X_{1.5} = & y_{13} + \mathbf{D}_{13}y_{31} + \mathbf{D}_{131}y_{12} + \mathbf{D}_{1312}y_{21} \\
 & - (y_{12} + \mathbf{D}_{12}y_{21} + \mathbf{D}_{121}y_{13} + \mathbf{D}_{1213}y_{31}).
 \end{aligned}$$

$$\mathbf{D}_{ij}x(t) = x(t - L_{ij}(t)) \xrightarrow{FT} \tilde{\mathbf{D}}_{ij}\tilde{x}(f) = \tilde{x}(f) e^{-2\pi i f}$$

Delay operator



Burst-like signal: We don't observe the persistent off-set of the memory, but just its time-variation $X \propto \partial^3 h$

TDI imprint of GW memory

$$\begin{aligned}
 X_2 = & X_{1.5} + \mathbf{D}_{13121}y_{12} + \mathbf{D}_{131212}y_{21} + \mathbf{D}_{1312121}y_{13} \\
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 & + \mathbf{D}_{1213131}y_{12} + \mathbf{D}_{12131312}y_{21}],
 \end{aligned}$$

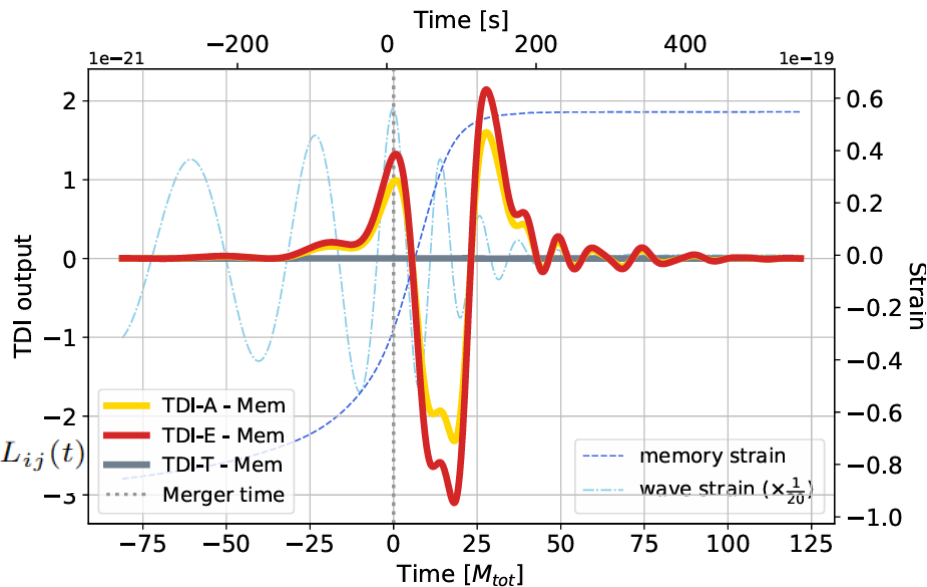
with

$$\begin{aligned}
 X_{1.5} = & y_{13} + \mathbf{D}_{13}y_{31} + \mathbf{D}_{131}y_{12} + \mathbf{D}_{1312}y_{21} \\
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$$\mathbf{D}_{ij}x(t) = x(t - L_{ij}(t)) \xrightarrow{FT} \tilde{\mathbf{D}}_{ij}\tilde{x}(f) = \tilde{x}(f) e^{-2\pi i f L_{ij}(t)}$$

Delay operator

Burst-like signal: We don't observe the persistent off-set of the memory, but just its time-variation $X \propto \partial^3 h$

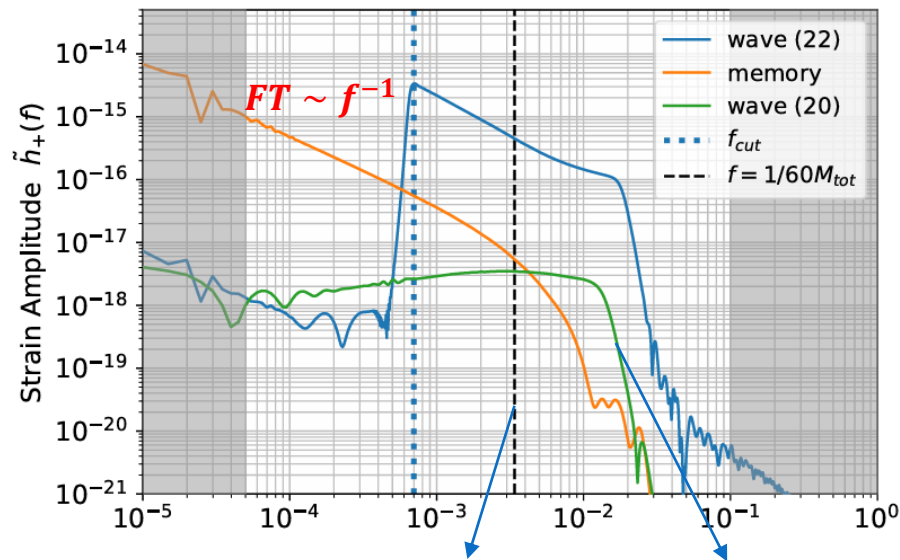


Look at the different scale of the y-axis!

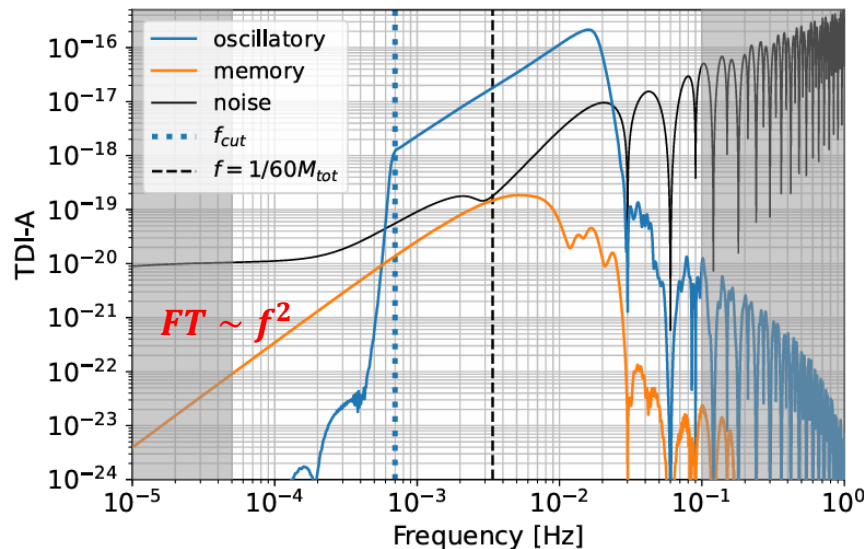
Time domain vs Frequency domain

Fourier Transform of a step like function $FT[\Theta(\tau - \tau_{merger})] = \delta(f) + \frac{1}{i2\pi f}$

→ Extends the signal at lower frequencies



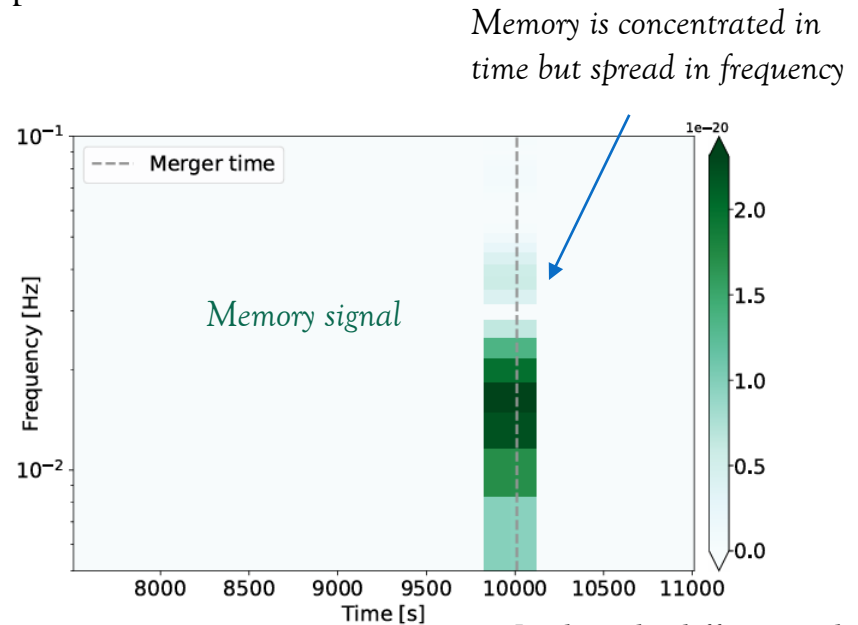
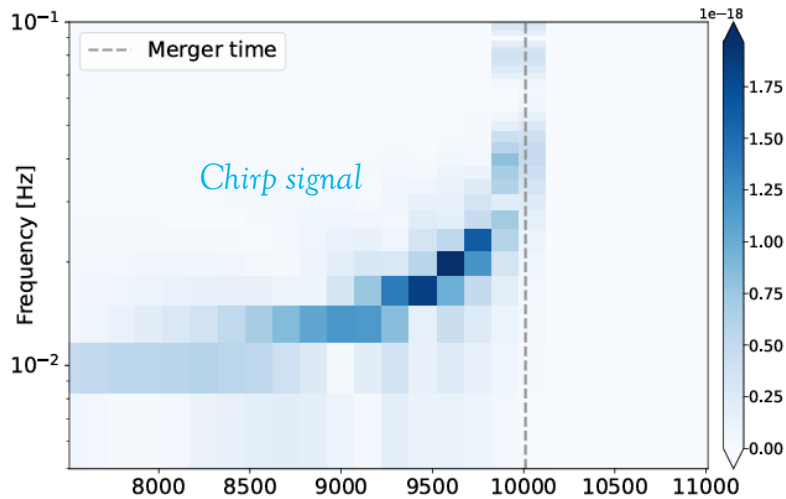
Frequency cutoff related to $f \sim \Delta\tau_{merger}^{-1}$



Ringdown contribution to the (2,0) mode

Time-Frequency representation

Oscillatory and memory signals have very separate time-frequency representation.
Can we use this to separate the two?

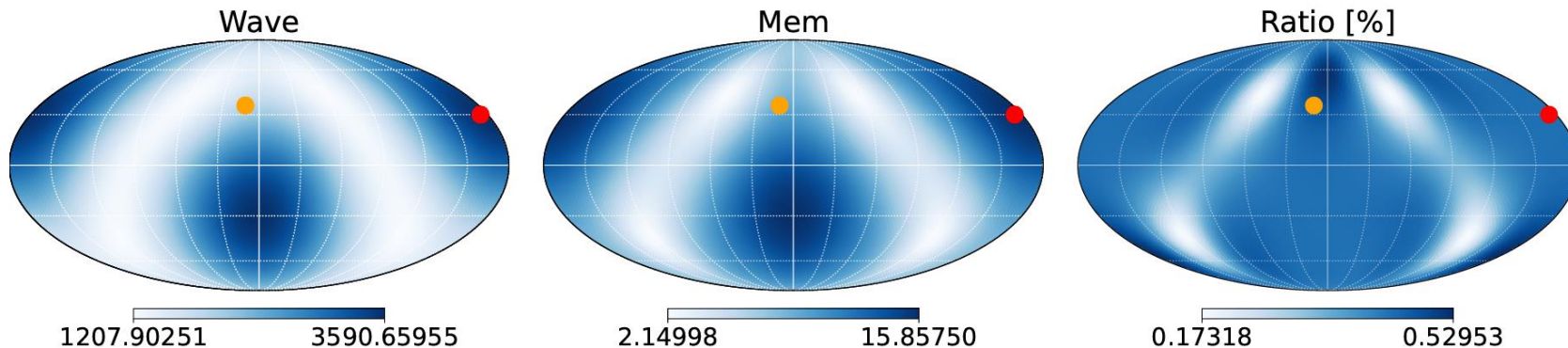


Memory is concentrated in time but spread in frequency

Look at the different scales!

Sky Localization Dependence

Merger with $M = 10^6 M_{\odot}$, $\iota = \frac{\pi}{3}$

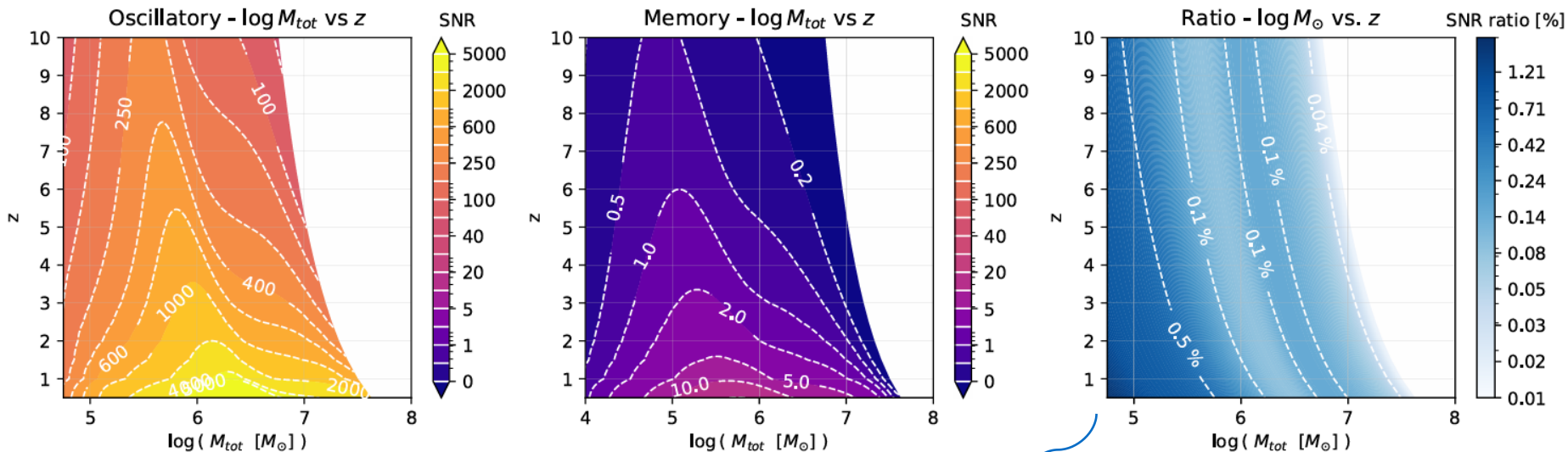


We select the pixel corresponding to the average and the best SNR, **three baselines**:

Baseline	q	χ	inclination ι [rad]	lat. β [rad]	long. λ [rad]	pixel p
1. Conservative	2.5	0.0	1.047	0.62	0.20	145
2. Optimistic	1.0	0.0	1.571	0.52	3.24	192
3. Opt. & Spin.	1.0	0.8	1.571	0.52	3.24	192



SNR Waterfall: Oscillatory vs Memory

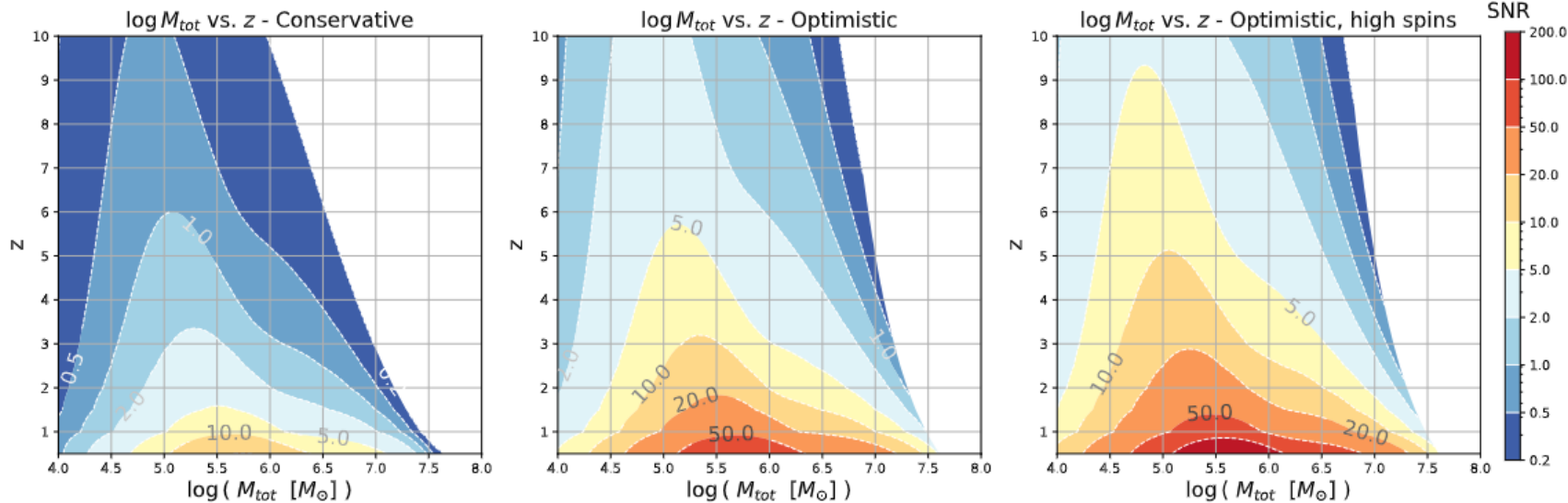


Results for the conservative baseline

The SNR ratio can be up to a few percent for edge-on systems



Scientific Reach of LISA: Memory Waterfall Plots



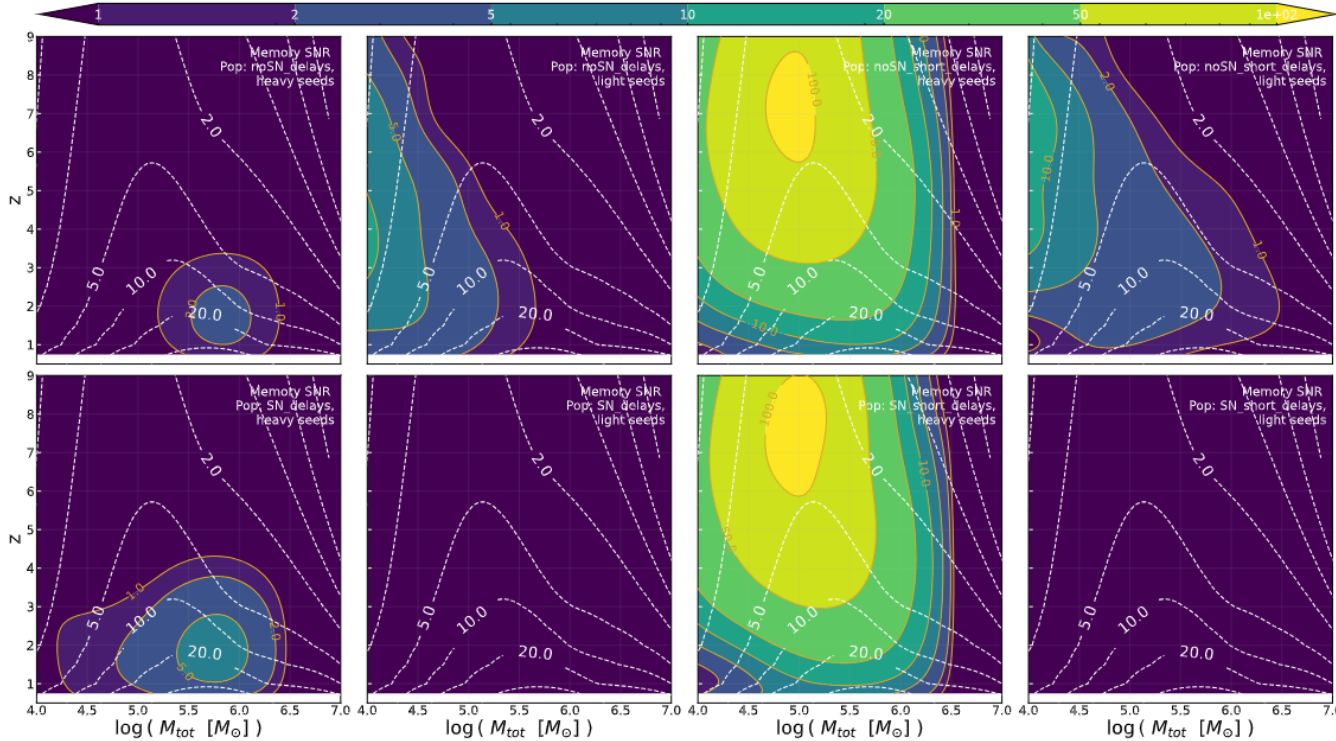
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3. Opt. & Spin.	1.0	0.8	1.571	0.52	3.24	192



Astrophysical population models

Population from E.Barausse et al. 2020

Expected Population (4-years observation)



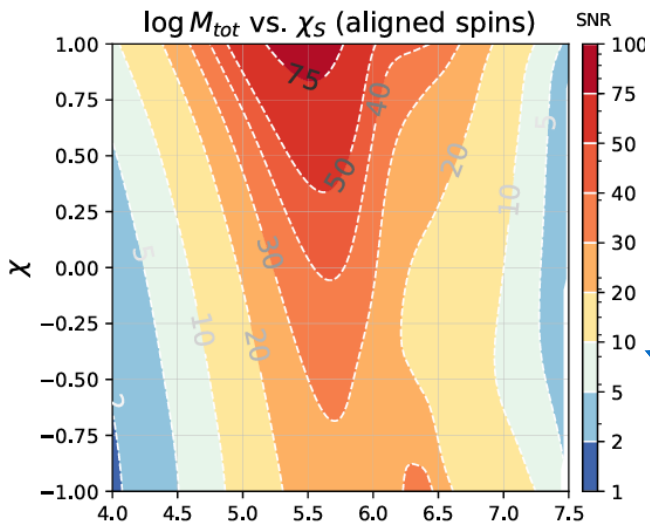
8 different astrophysical models

- Initial Seed: Light vs Heavy
- SN Feedback: Yes or No
- Different delay model on the SMBH merger

Results for the optimistic baseline scenario



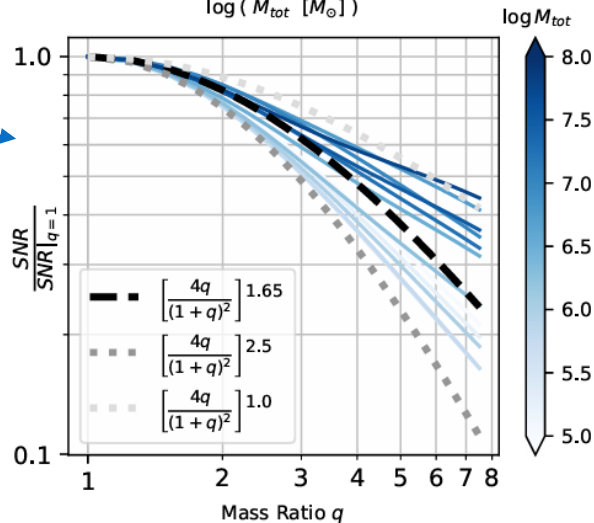
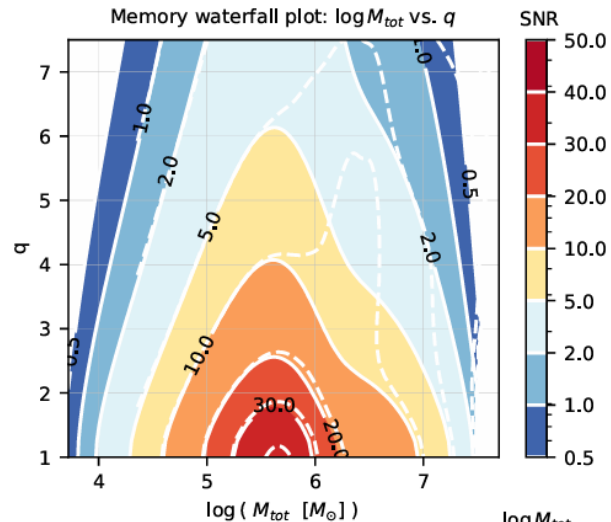
Impact of spin and mass ratio



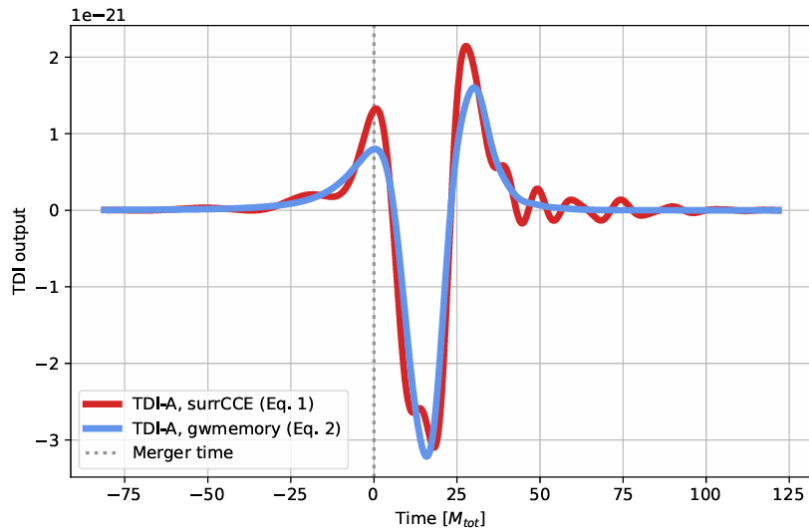
Disagreement between the waveform $h_+^{mem} \equiv h_{+,CCE}^{20} - h_{+,CCE}^{20}$ and that from the GWMemory package

Memory strongly decrease with the mass ratio

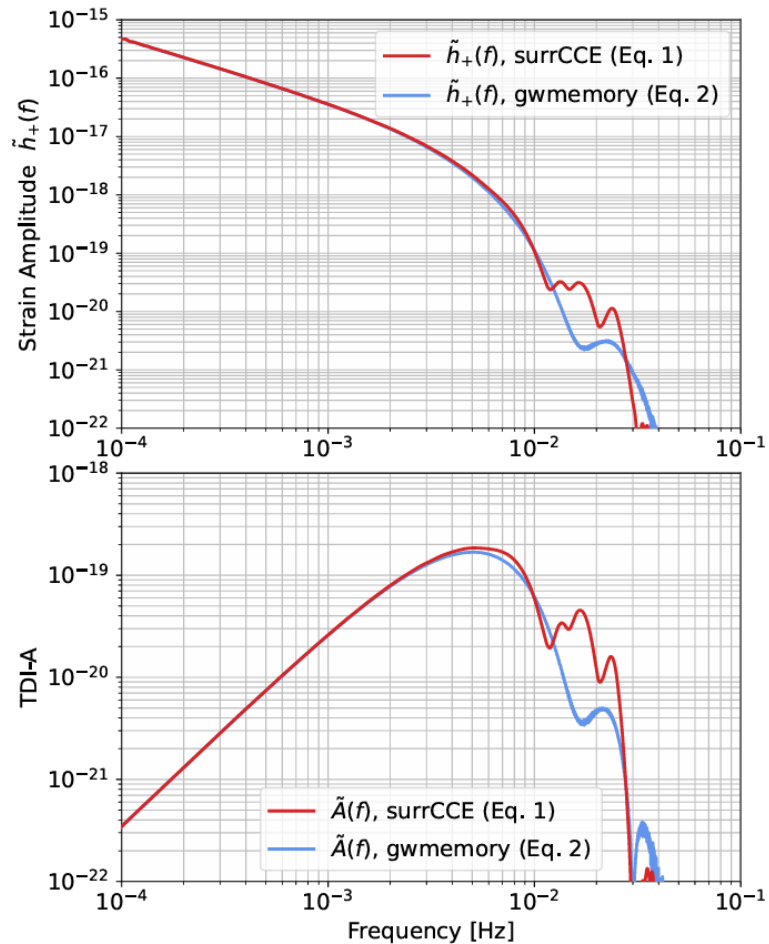
Memory increase for aligned mass spin



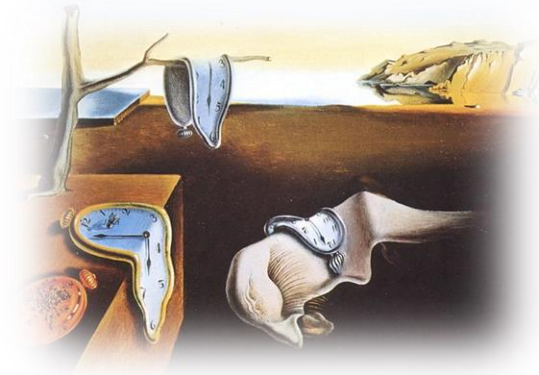
Ringdown Residuals?



Comparison of the two methods of the memory extraction \rightarrow residual difference in the spectrum close to the ringdown frequency: physical or not?



Future Directions

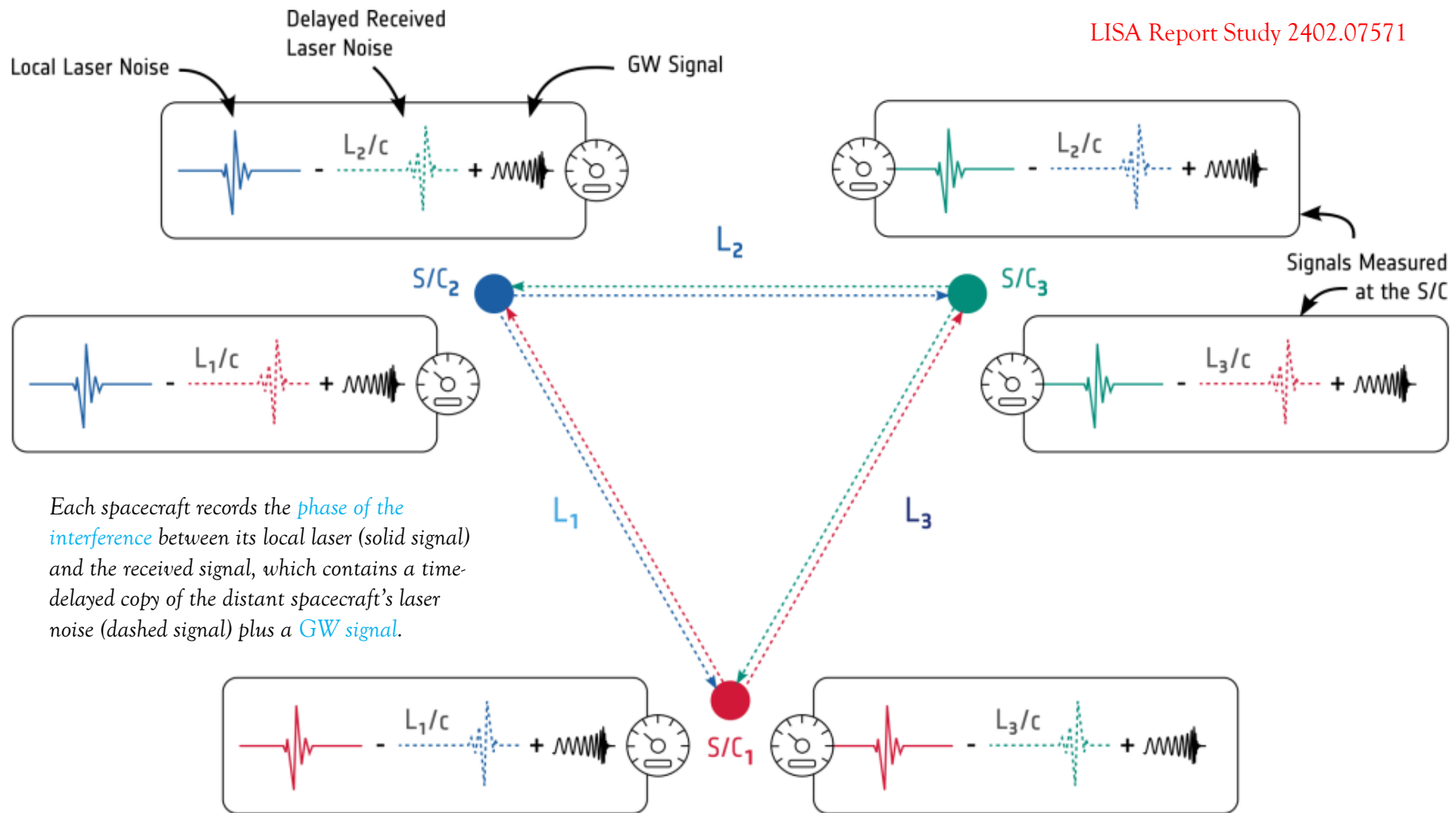


- Extension projects, exploiting the GW memory for
 - Introduce memory component to full Bayesian parameter estimation: mitigate degeneracies, reduce biases... (Jorge's talk)
 - Test of GR and beyond-GR theories in a strong regime: consistency checks between oscillatory and memory components
- Which kind of modification do we expect in Beyond GR?
 - Probing new channels of radiation?
 - Effect on the propagation?

$$\delta h_H^{lm}(u, r) = \frac{1}{r} \sqrt{\frac{(l-2)!}{(l+2)!}} \int_{S^2} d^2\Omega' \bar{Y}^{lm}(\Omega') \times \int_{-\infty}^u du' r'^2 \left\langle |\dot{h}_+|^2 + |\dot{h}_\times|^2 + \sum_{\lambda=1}^N |\dot{\psi}_\lambda|^2 \right\rangle$$

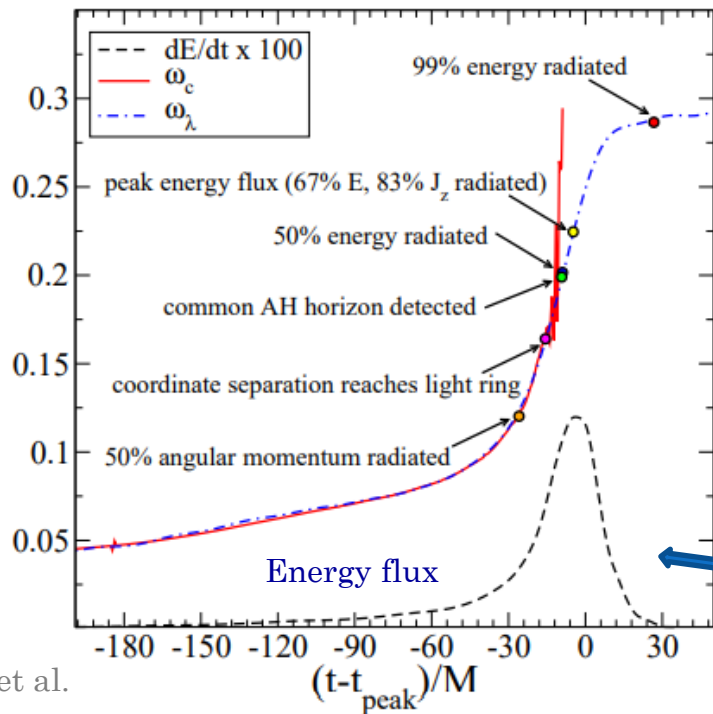
L.Heisenberg et al 2303.02021

Please get in touch with me, or coordinators L. Magaña Zertuche and M. Besançon if you are interested in contributing in extension projects



Growing of the memory

$$h_{+,OPN} = \left[-(1 + \cos^2 i) \cos 2\Phi + \frac{1}{96} \sin^2 i (17 + \cos^2 i) + O(x^{1/2}) \right] \frac{2\eta M (M\omega(t))^{2/3}}{R}, \text{ orbital frequency evolution:}$$



Typical **step function** shape FT $\sim i/f$

Support close to **merger phase**

Sensitive to **higher modes**



GWMemory package

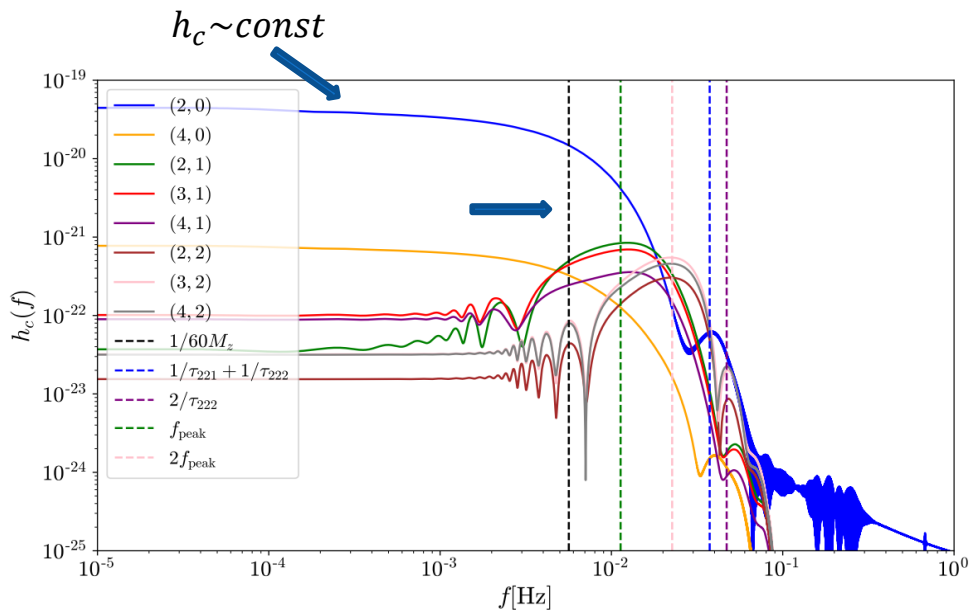
(C. Talbot, E. Thrane, and P. D. Lasky & F. Lin 1807.00990)

Merger phase
 $\tau \sim 60 M$

Memory in the Fourier domain

$h = h_0 + \delta h$ NRHybSur3dq8 (spin-aligned model) to generate $h_0 \Rightarrow \delta h$

Memory in the frequency domain and contribution from **different spherical harmonics**



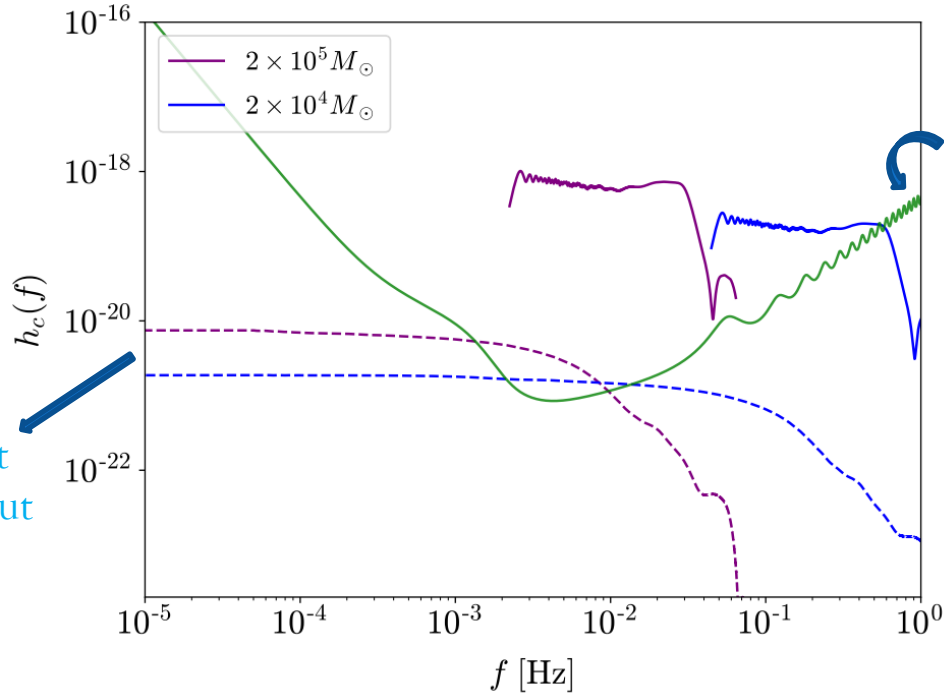
$$\delta h = \sum_{\ell \geq 2} \sum_{|m| \leq \ell} \delta h^{\ell, m} {}_{-2}Y_{\ell m}$$

- (2,0) dominant
- $m \neq 0$ excited close to merger

Characteristic strain

$$h_c(f) = 2f |\tilde{h}(f)|$$

Primary vs Memory



Extends the signal at lower frequencies, but subdominant

Sensitivity curve of the detector (LISA)

Population Forecasts I

N_{th} number of events with detectable memory, i.e. $SNR \geq 1$ (or $SNR \geq 5$), in 4 years

	Astrophysical Catalogues		
	Light seeds	Heavy seeds	
SN-delays	$N_{tot} = 47$ $N_{th} = 0.4 (0.1)$ $\langle \rho \rangle = 0.04$ $\rho_{max} = 7$	$N_{tot} = 27.3$ $N_{th} = 21.2 (10)$ $\langle \rho \rangle = 6$ $\rho_{max} = 97$	75-78% of events with detectable memory
noSN-delay	$N_{tot} = 191$ $N_{th} = 6 (1)$ $\langle \rho \rangle = 0.17$ $\rho_{max} = 11.64$	$N_{tot} = 10$ $N_{th} = 7.5 (4)$ $\langle \rho \rangle = 6.9$ $\rho_{max} = 68.7$	
SN-short Delays	$N_{tot} = 149$ $N_{th} = 1 (1)$ $\langle \rho \rangle = 0.04$ $\rho_{max} = 5.01$	$N_{tot} = 1245$ $N_{th} = 418 (33)$ $\langle \rho \rangle = 1$ $\rho_{max} = 43$	31-33% of events with detectable memory
noSN-short Delays	$N_{tot} = 1203$ $N_{th} = 12 (2)$ $\langle \rho \rangle = 0.06$ $\rho_{max} = 17$	$N_{tot} = 1251$ $N_{th} = 392 (29)$ $\langle \rho \rangle = 1.1$ $\rho_{max} = 51$	

How many events are we going to see with LISA?

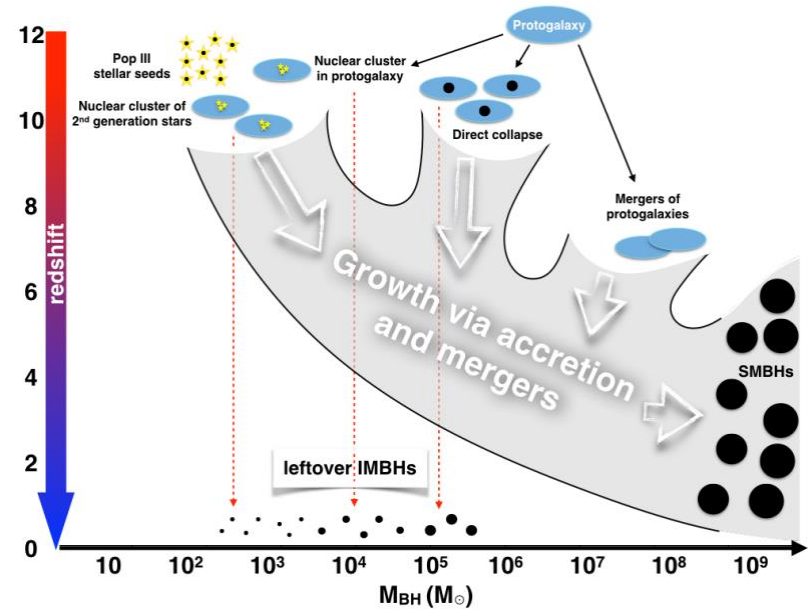
Barausse & Lapi (2020),
Barausse et al (2020)

8 different population models of massive binary black hole mergers, main uncertainties:

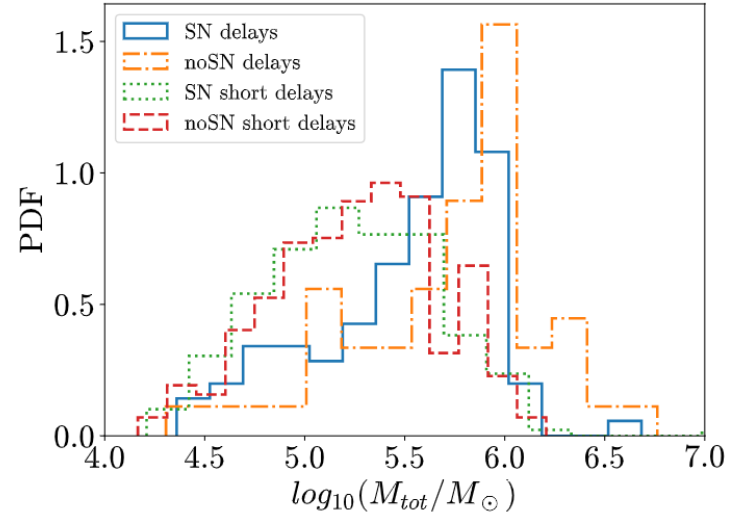
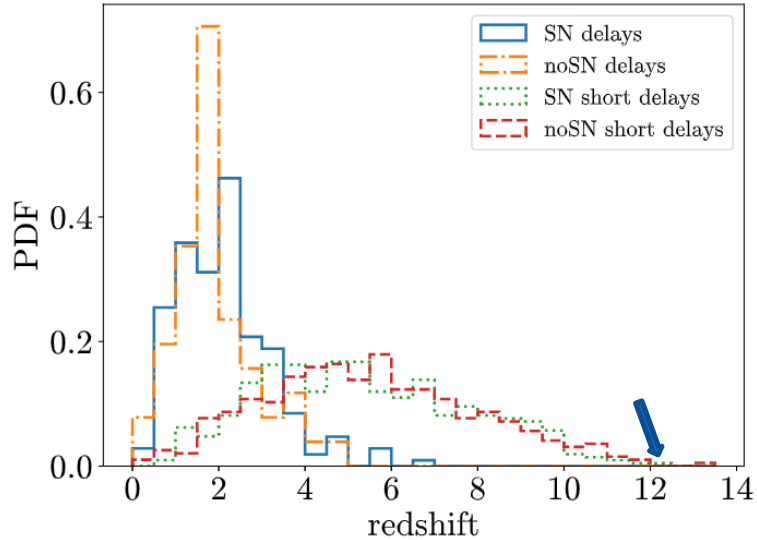
- Initial seeds
 - Light
 - Heavy
- SN feedback
 - Yes
 - No
- Delay in SMBH merger
 - Yes
 - No

“Last parsec problem”

Credit: MEZCUA



More optimistic: Heavy Seeds with delays



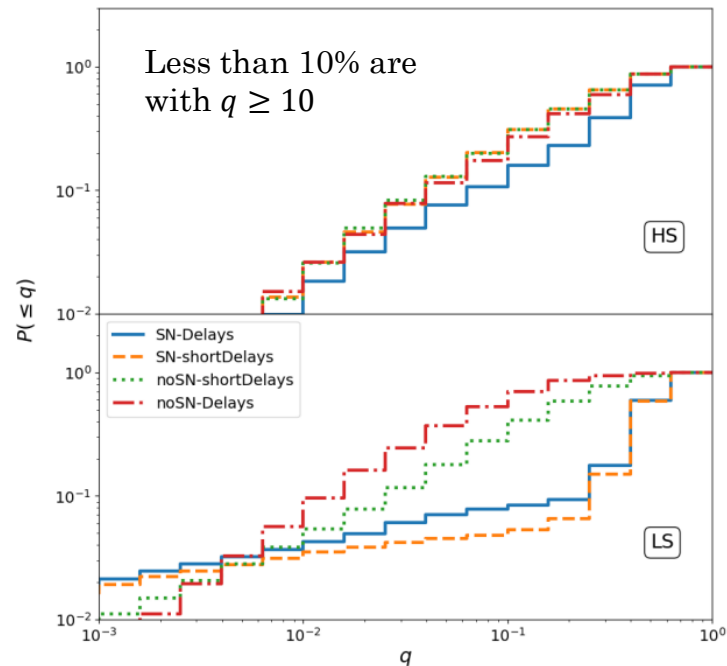
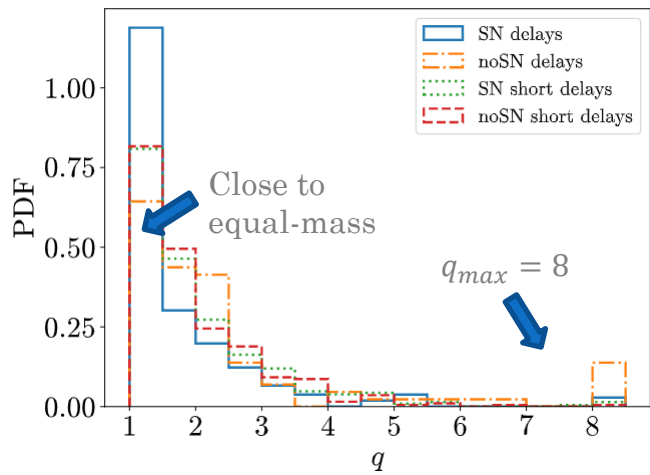
Distribution of single memory events with $SNR \geq 1$ for **heavy-seeds models**

More pessimistic: Light Seeds

Credit: E. Barausse & A.Lapi

- Lighter binaries \rightarrow **smaller SNR**
- More events with **larger mass-ratio**

Drop of detectable events with higher mass-ratio q :

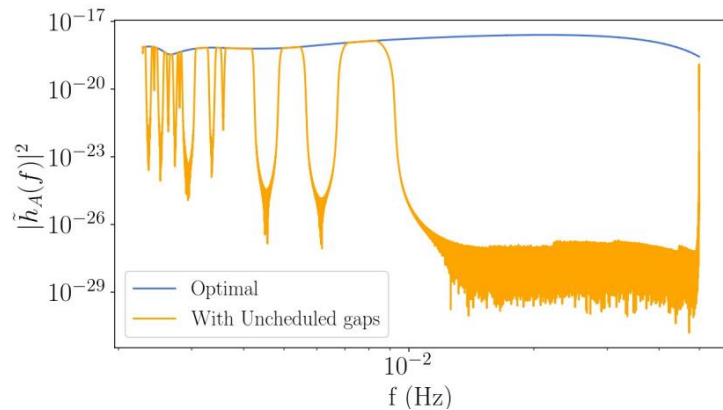


SN feedback has a greater impact for LS models

What's the realistic impact for LISA with gaps in the data?

Q: Can the memory be useful in the presence of **gaps** in the data?

The gaps can truncate the signal prior merger...



Credit: K.Dey

Types of gaps

- scheduled gaps* (regular maintenance, 3.5 h every week)
- unscheduled gaps* (3 days with $p(\Delta T) = \lambda e^{-\lambda \Delta T}$ and $\lambda = 1/9$ days)
~30 gaps per year → **greater degradation of the signal!** (P.A.Seone et al 2021, K.Dey et al 2021)

In the optimistic model (HS SN-short $N_{th} \sim 400$) for **only ~ 0.14 events the memory improves σ_{d_L} by > 5%**

Not likely to help in standard scenarios, but the model's uncertainty is **BIG!**

How do we compute it?

Christodoulou '91, Blanchet & Damour '92
Wiseman & Will '91, Marc Favata '09-'11

To compute the waveform we need to solve this equation:

$$\bar{h}_{ij}^{TT}(t, \mathbf{x}) = 4 \int \frac{(-g) [T_{matter}^{jk}(t', \mathbf{x}') + T_{GW}^{jk}(t', \mathbf{x}')] }{|\mathbf{x} - \mathbf{x}'|} \delta(t' - t - |\mathbf{x} - \mathbf{x}'|) dx'^4$$

The contribution from the energy-momentum tensor of the GW is:

$$\delta \bar{h}_{ij}^{TT} = \frac{4}{R} \int_{-\infty}^{T_R} dt' \left[\int \frac{dE_{GW}}{dt' d\Omega'} \frac{n'_j n'_k}{|1 - \mathbf{n}' \cdot \mathbf{N}|} d\Omega' \right]^{TT}$$

The memory depends on the whole history of the binary

$$T_{GW}^{jk} \sim \frac{1}{R^2} \frac{dE_{GW}}{dt' d\Omega'} = \frac{c^3}{16\pi G} |\dot{h}(t, \Omega)|^2$$

Substituting post-Newtonian waveforms one finds:

$$h_+ = \frac{2\eta M \chi}{R} \left[-(1 + \cos^2 \iota) \cos 2\Phi + \frac{1}{96} \sin^2 \iota (17 + \cos^2 \iota) + O(x^{1/2}) \right]$$

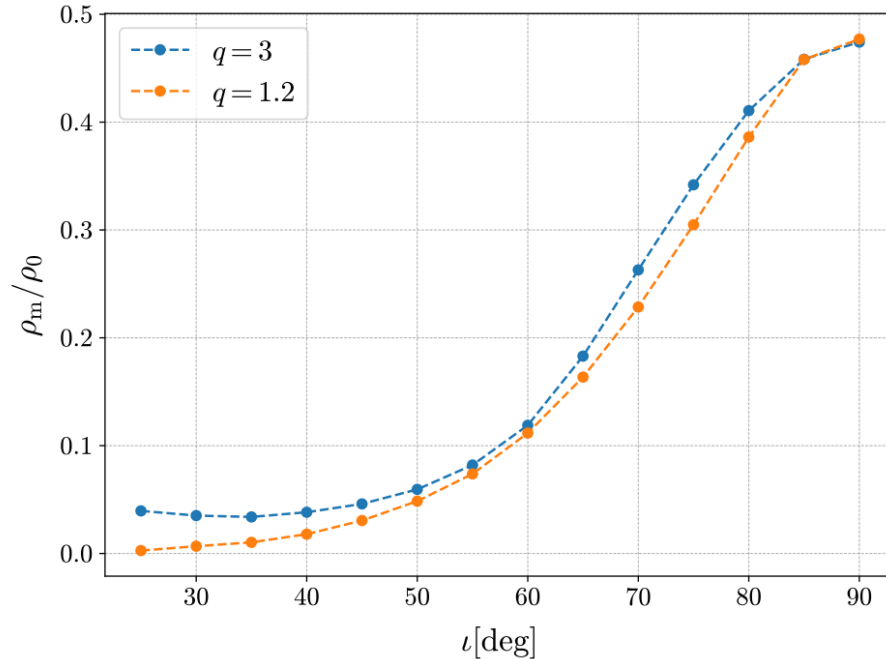
The memory is present only in h_+

Oscillatory

Memory

Mass and mass-ratio dependence

Values of ratio corresponding to $\sigma_{d_L,wm}/\sigma_{d_L} = 0,9$ (10% improvement) \rightarrow do not depend on the mass-ratio q



Light	$h_{0,DM}$	$h_{0,HM}$	δh_{DM}	δh_{HM}
SNR	20.5	20.6	1.9	2.3
	$h_{0,DM}$	$h_{0,HM}$	h_{DM}	h_{HM}
σ_{d_L}/d_L	0.56	0.55	0.20	0.18
σ_l [rad]	0.75	0.73	0.27	0.23
Heavy	$h_{0,DM}$	$h_{0,HM}$	δh_{DM}	δh_{HM}
SNR	1001.4	1006.3	3.5	4.3
	$h_{0,DM}$	$h_{0,HM}$	h_{DM}	h_{HM}
σ_{d_L}/d_L	0.0116	0.0108	0.0115	0.0107
σ_l [rad]	0.0158	0.0140	0.0157	0.0139