# Global quantum effects on gravitational radiation in the era of the LISA mission

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# Gravitation & Cosmology Group, IEM-CSIC

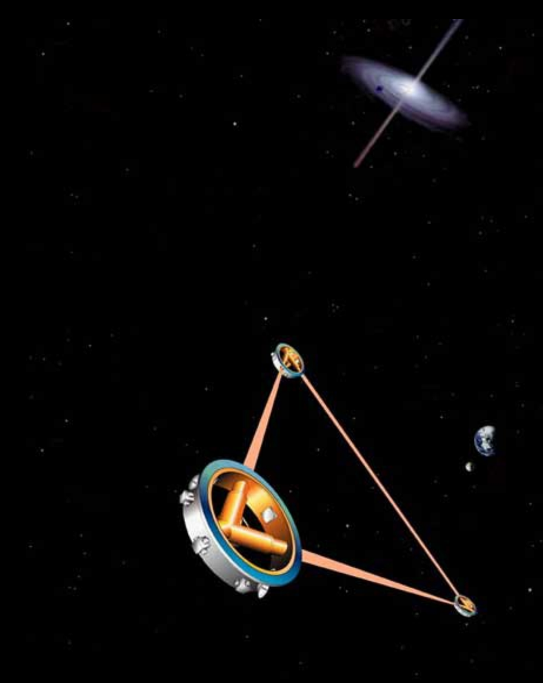
- Faculty members:

- Fernando Barbero,
- Gianluca Calcagni,
- Guillermo Mena,
- Francesca Vidotto (from June 25).

- The Group develops theoretical models of classical and quantum gravity and dark energy models that modify the production and/or propagation of GWs.

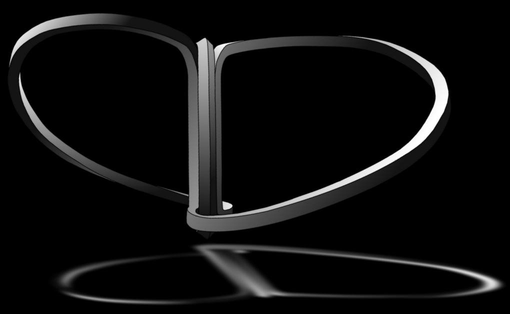
- The Group participates in LISA's Cosmology WG and Fundamental Physics WG.

- G.C. Is member of the Science Group, Membership Management Team and Internal Networking Committee for Science.

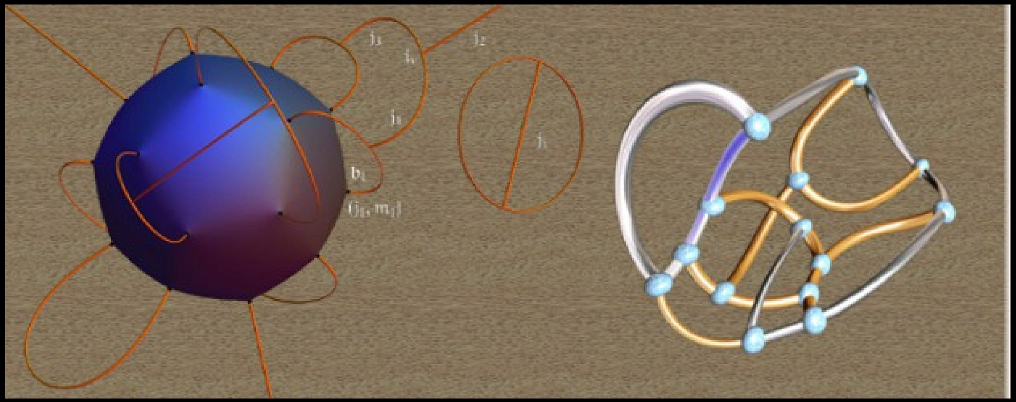


# Loop Quantum Gravity, IEM-CSIC

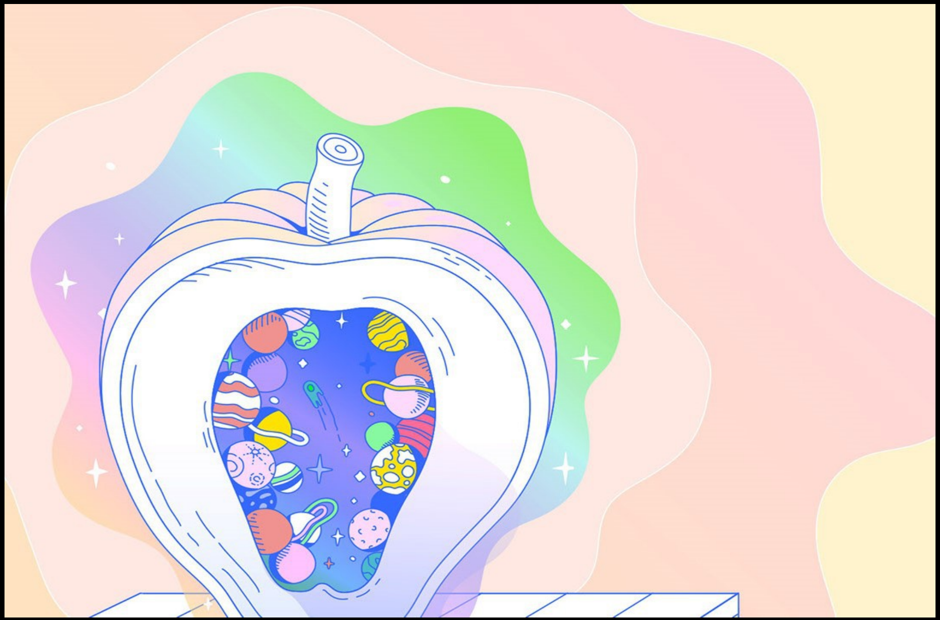
- Extensive activity is devoted to investigations in Loop Quantum Gravity and its application: Loop Quantum Cosmology.



- Loop Quantum Gravity is a background independent and noperturbative quantization of Einstein's gravity, based on the use of triad variables and canonical SU(2) connections.
- Loop Quantum Cosmology cures essential singularities.
- In a top-down approach, leads to predictions about quantum effects for the CMB and the cosmological background of GWs.



# Global quantum effects in GWs



- Many works propose quantum corrections to the  propagation of GWs.
- They might not be significant/ observable in a very classical universe like ours.

- Global quantum effects are more difficult to estimate. In BH radiation, they can influence the global behavior of GWs, affecting the horizon and the **asymptotic regions**.
- Our group has started to study these effects, in an attempt to evaluate their relevance and clarify whether they could be detected by LISA.
- Special attention is paid to the rigdown regime of BH emission after mergers events (of supermasive BHS) .

# Effective BHs in LQG (AOS model)

- Using triadic variables, the metric of the interior of a nonrotating spherically symmetric BH can be described in a Kantowski-Sachs form

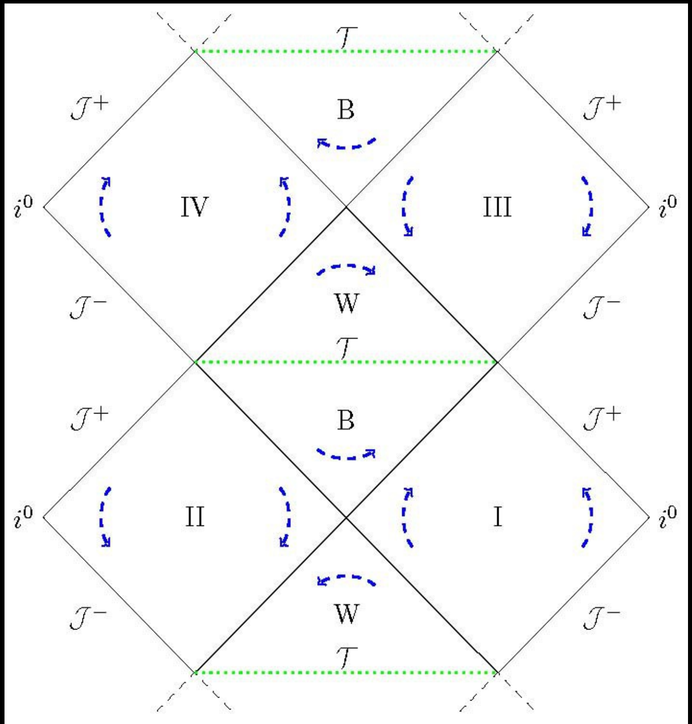
$$ds^2 = p_b^2(\tau)\left[-2\,\underline{N}^2(\tau)\left|p_c(\tau)\right|d\tau^2 + \frac{1}{\left|p_c(\tau)\right|}dx^2\right] + \left|p_c(\tau)\right|\left(d\theta^2 + \sin^2\theta\, d\phi^2\right).$$

  with a densitized lapse function $\underline{N}$ and real variables.

- Correcting GR with LQG modifications, the interior solution becomes regular.

- The exterior corresponds to imaginary $p_b$.

- The asymptotically flat behavior in the exterior is not conventional (curvature invariants fall off as the fourth inverse power of the radius).




Credits: Ashtekar, Olmedo & Singh

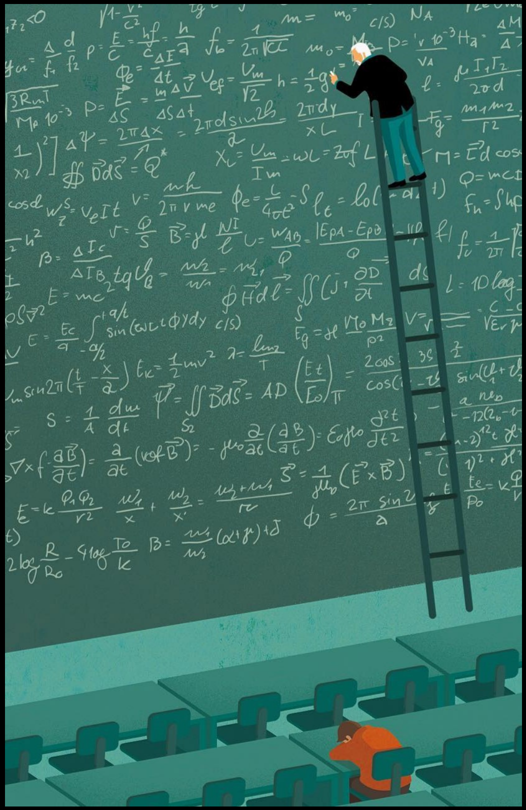
# Exterior/Interior of the BH

- Coordinates $(\tau, x)$ describe the set of orbits of the spherical symmetry. Their timelike or spatial character depends on the complex nature of $p_b$.
- An analytic extension of the phase space variables allows to pass from the interior to the exterior without any coordinate-dependent Wick transformation.

- It is possible to quantize the interior consistently in LQC.
- Moreover, it is possible to perturb the interior and quantize the perturbations with standard methods!

# Perturbations for spherical symmetry

- We introduce metric perturbations (and a perturbative *scalar* field).
- We expand these perturbations in a Regge-Wheeler-Zerilli basis of spherical harmonics.
- We compute the gravitational action and truncate it at second-order in perturbations (*previous work with* J.M. Martín-García and D. Brizuela).
- Perturbations can be divided in axial and polar.



- We pass to a Hamiltonian formulation after expanding also the perturbations in Fourier series in their $x$-dependence.

# Perturbations for spherical symmetry

- We find appropriate canonical variables for the perturbations and the background after correcting the latter with quadratic perturbative terms.

- By appropriate we mean that they are either perturbative gauge invariants or pure gauge.

- Polar gauge invariants, e.g., can be described by the Hamiltonian

$$\boldsymbol{H}^{ax} = \sum_{\nu} b_{\hat{l}} \left\{ (\boldsymbol{P}_1^{\nu})^2 + \left[ k^2 + s_{\hat{l}}(p_b, \Pi_b, p_c, \Pi_c) \right] (\boldsymbol{Q}_1^{\nu})^2 \right\}, \qquad b_{\hat{l}}^2 = p_b^2 \hat{l}^2 + p_c^2 \omega_n^2 / k^2,$$

with mode labels $k^2 = (l+2)(l-1) + \omega_n^2$ and $\hat{l} = \sqrt{(l+2)(l-1)}/k$.

- These gauge invariants are related to the Gerlach-Sengupta one by

$$G^{\nu}(\tau) = \frac{1}{k\sqrt{b_{\hat{l}}}} \sqrt{\frac{(l+2)!}{(l-2)!}} \left[ P_1^{\nu} - 2\frac{\Pi_b p_b}{b_{\hat{l}}} Q_1^{\nu} \right], \qquad G^{(l,m)}(\tau, x) = \sum_n G^{(n,l,m)}(\tau) \frac{e^{i\omega_n x}}{\sqrt{2\pi}}.$$

# Vacuum state of the perturbations

- We have proposed a criterion for the choice of a vacuum based on an asymptotic diagonalization of the Hamiltonian in the ultraviolet.
- This leads to a state that is optimally adapted to the dynamics. It reproduces standard vacua in conventional cases.
- The vacuum gauge invariant *positive-frequency* solutions

$$\mu_{\nu} = \frac{1}{\sqrt{-2\,Im[h_k(\hat{l})]}}\, e^{i\int_{\eta_0}^{\eta} d\tilde{\eta}\, Im[h_k(\hat{l})](\tilde{\eta})}$$

are determined by their asymptotics $kh_k^{-1}(\hat{l}) \sim i\left[1 + 2\sum \left(\frac{-i}{2\mathrm{k}}\right)^{n+2} \gamma_n(\hat{l})\right],$

and the recurrence relations

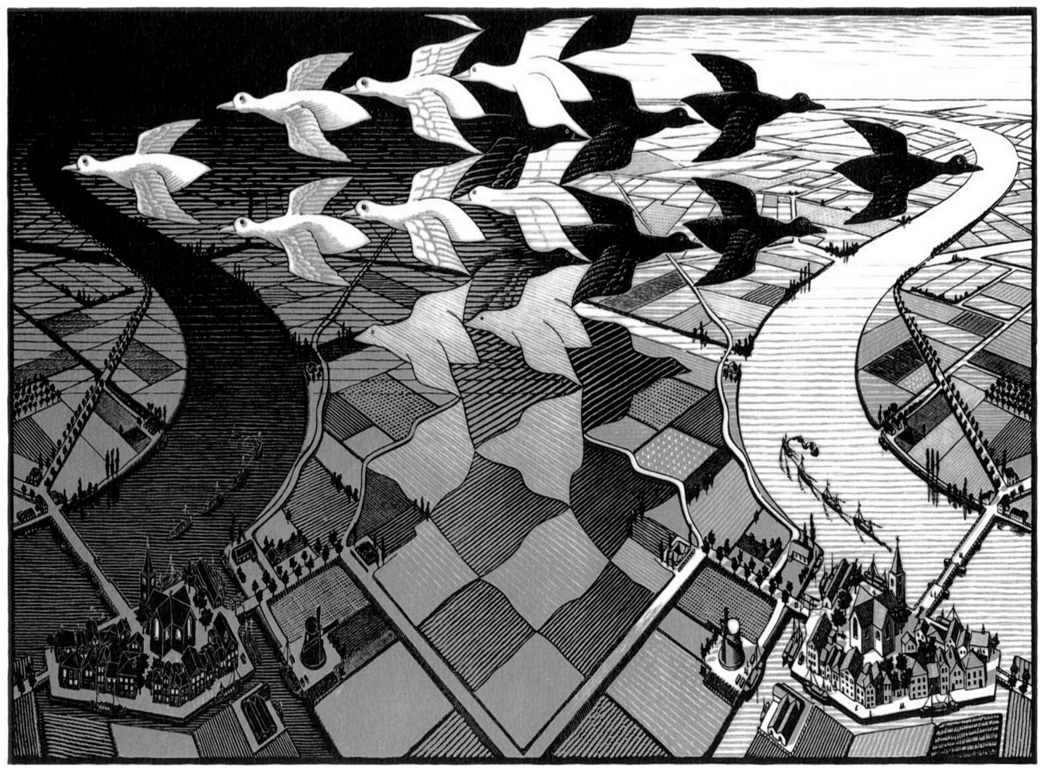
$$\gamma_0 = s_{\hat{l}}, \qquad \gamma_{n+1} = \frac{1}{b_{\hat{l}}}\left\{H^{interior}, \gamma_n\right\} + 4\mathrm{s}_{\hat{l}}\left[\gamma_{n-1} + \sum_{m=0}^{n-3} \gamma_m \gamma_{n-(m+3)}\right] - \sum_{m=0}^{n-1} \gamma_m \gamma_{n-(m+1)}.$$

# Quantum field theory in the exterior

- We can now construct n-point functions in the interior geometry.
- Our construction uses background variables, no any specific coordinate. We can extend our results to the exterior by analytic (complex) continuation of those variables.
- This extended QFT should provide the tools to discuss global quantum effects in quasinormal modes.
- The extension introduces divergences only through $b_{\hat{l}}^2 = p_b^2 \hat{l}^2 + p_c^2 \omega_n^2 / k^2$. But these can be absorbed by redefining the asymptotic expansion in terms of $k\, b_{\hat{l}}^3$.
- We expect no divergence if the interior has a good infrared behavior.

# Transformations between gauge invariants

- With our Hamiltonian formalism, it is easy to find canonical transformations between gauge invariants such that they respect the form of the dynamical equations of the perturbations as generalized wave equations with a *potential*.



- We plan to use them to better understand Darboux covariance, some aspects of isospectroscopy, and the possible unitary equivalence in the quantum theory (*work in progress with collaborators*).