Institut d'Estudis Espacials de Catalunya

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

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AGENCIA ESTATAL DE INVESTIGACIÓN

Estimate of the magnetic contribution to acceleration noise in LISA Pathfinder

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16-10-2024
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Arxiv:

M Armano et al. "Precision measurements of the magnetic parameters of LISA Pathfinder test masses". 2024, arXiv:2407.04431.

M Armano et al. "Magnetic-induced force noise in LISA Pathfinder free-falling test masses". 2024, arXiv:2407.04427

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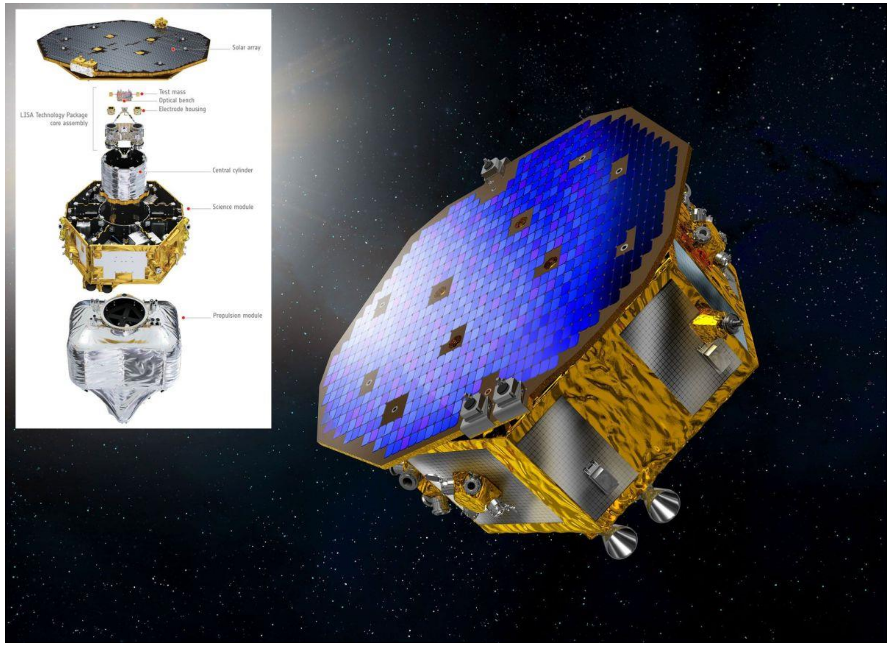
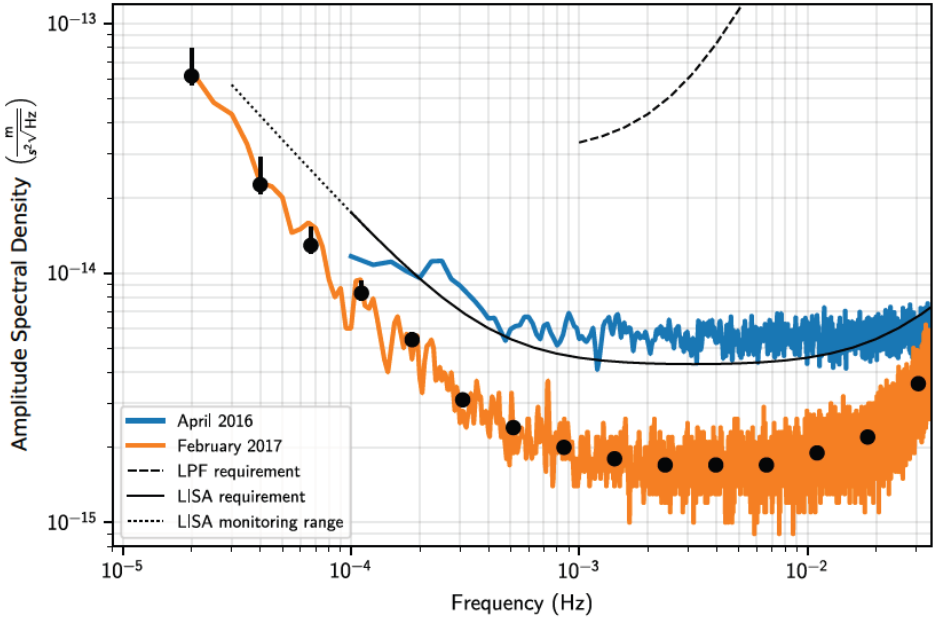
Top image credit: https://spacenews.com/lisa-pathfinders-success-boosts-likelihood-of-future-gravity-wave-observatory/
Bottom image credit: LISA Study Definition Report - Red Book (2024)

LISA Pathfinder

- 2 TMs in free-fall
- Mission from 2015 to 2017
- Beyond LISA requirements
- Δg: residual acceleration between TMs along axis joining them (x axis)





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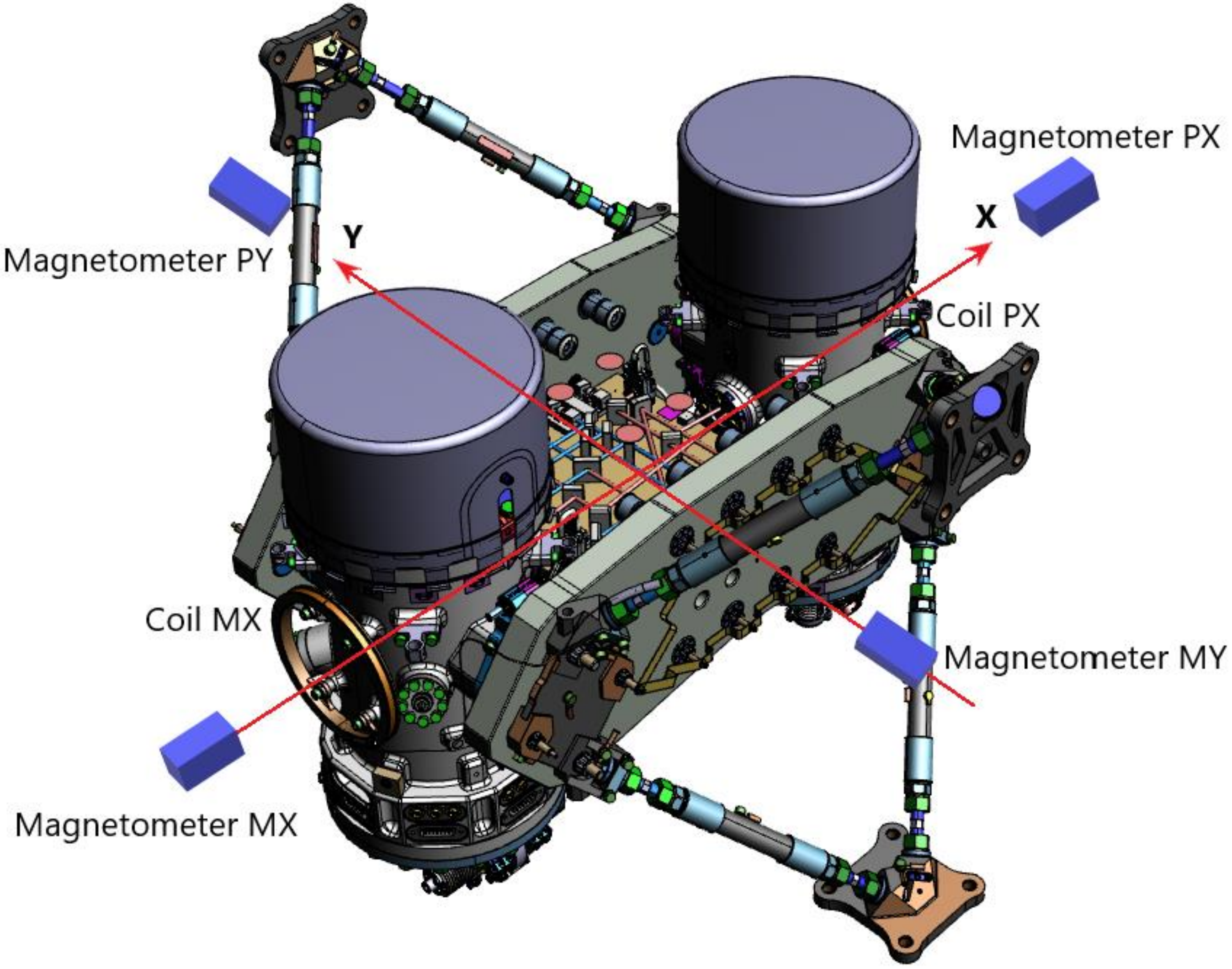
LISA Pathfinder DDS

- Data and Diagnostics Subsystem
  - Temperature subsystem
  - **Magnetic subsystem**
  - Radiation monitor
- Magnetic Diagnostic Subsystem
  - 4 tri-axial fluxgate magnetometers
  - 2 induction coils



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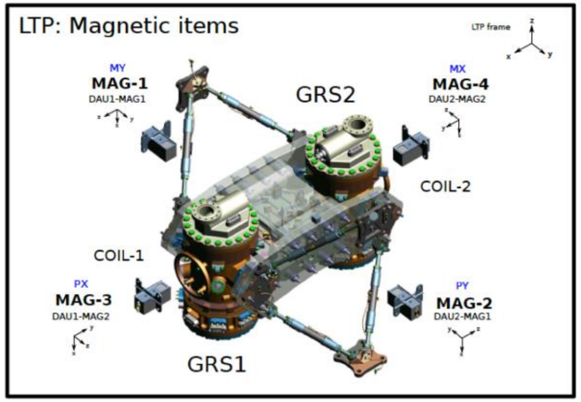
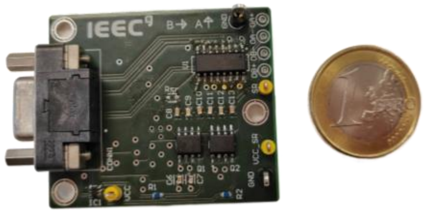
Magnetic Diagnostic Subsystem

DDS → **SDS**

- 4 tri-axial fluxgate magnetometers → 6 bi-axial AMR magnetometers

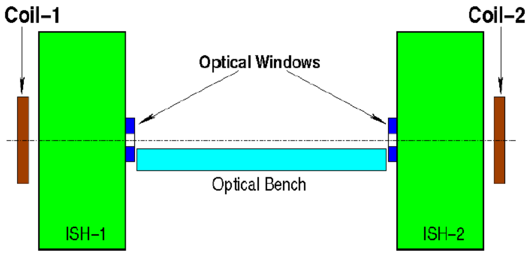


Bulky
Power consuming

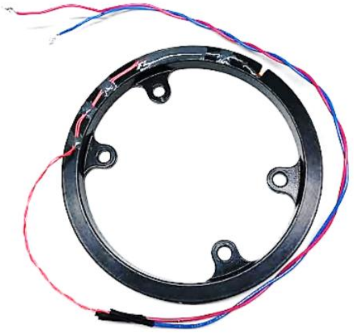


Small remanence
More compact

- **2 injection coils** → 2 audio-band measuring coils



2400 turns of copper wire
5.65 cm radius



Same coils as for LPF
Measuring signals in the 50-500 Hz range

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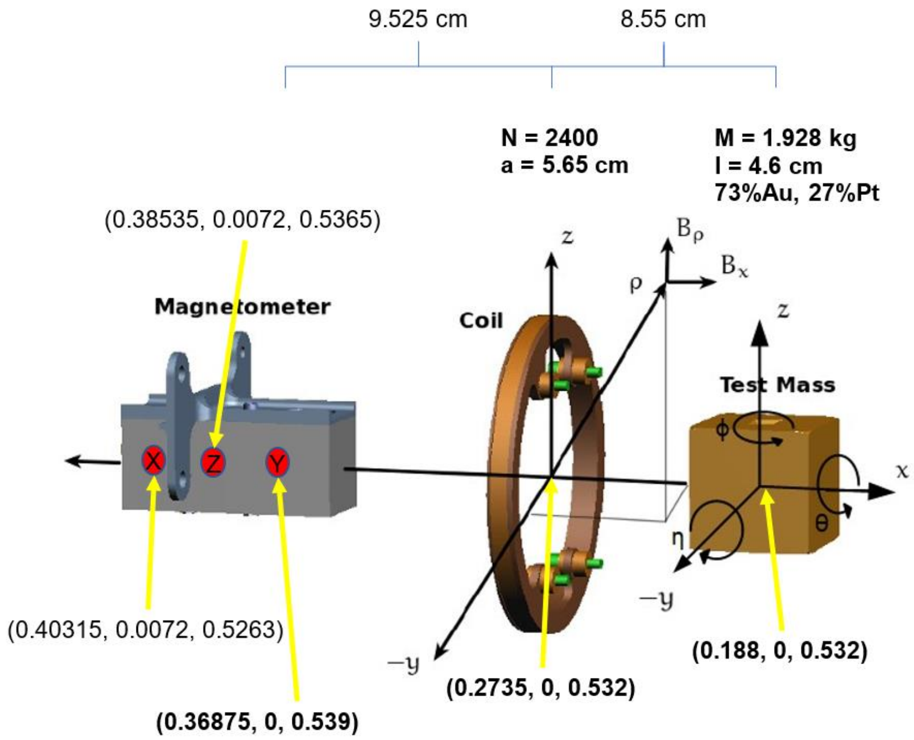
TMs magnetic parameters extraction

- Remanent magnetic moment (Mr)
- Magnetic susceptibility (χ)
- Background magnetic field and gradient at TM location
- Homogeneity and stationarity of magnetic properties



DOY	f [mHz]	I^{DC} [mA]	I^{AC} [mA]	duration [s]
170	5	+1.5	1.5	4000
170	5	+1.5	1.0	4000
170	5	+1.5	0.8	4000
170	5	+1.5	0.5	4000
170	5	+0.75	1.5	4000
170	5	+0.75	1.0	4000
170	5	+0.75	0.8	4000
170	5	+0.75	0.5	4000
170	5	0.00	1.5	4000
170	5	0.00	1.0	4000
170	5	0.00	0.8	4000
170	5	0.00	0.5	4000
170	5	-0.75	1.5	4000
170	5	-0.75	1.0	4000
170	5	-0.75	0.8	4000
170	5	-0.75	0.5	4000
170	5	-1.5	1.5	4000
170	5	-1.5	1.0	4000
170	5	-1.5	0.8	4000
170	5	-1.5	0.5	4000

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TMs magnetic parameters extraction

- TM behaves like a magnetic dipole:

$$\vec{F} = (\vec{m} \cdot \vec{\nabla})\vec{B}$$

$$\vec{N} = \vec{m} \times \vec{B} + \vec{r} \times (\vec{m} \cdot \vec{\nabla})\vec{B}$$

$$\vec{m} = \overrightarrow{m_r} + \frac{\chi}{\mu_0}\vec{B}$$

- Magnetic forces and fields dominate the background and other sources during injections

$$\vec{F} = \left\langle (\overrightarrow{m_r} \cdot \vec{\nabla})\vec{B} + \frac{\chi}{\mu_0}(\vec{B} \cdot \vec{\nabla})\vec{B} \right\rangle V \qquad \vec{N} = \left\langle \overrightarrow{m_r} \times \vec{B} + \vec{r} \times \left[(\overrightarrow{m_r} \cdot \vec{\nabla})\vec{B} + \frac{\chi}{\mu_0}(\vec{B} \cdot \vec{\nabla})\vec{B} \right] \right\rangle V$$

$$\vec{B} = \vec{B}_0 + \overrightarrow{B^{AC}} \sin(\omega t) \quad where \quad \vec{B}_0 = \vec{B}_{back.} + \overrightarrow{B^{DC}}$$

$$\vec{F} = \overrightarrow{F_{DC}} + \overrightarrow{F_{1\omega}} + \overrightarrow{F_{2\omega}} \qquad \vec{N} = \overrightarrow{N_{DC}} + \overrightarrow{N_{1\omega}} + \overrightarrow{N_{2\omega}}$$

Terms of interest:

$$\overrightarrow{F_{DC}} = \langle (\overrightarrow{M_r} \cdot \vec{\nabla})\overrightarrow{B_0} \rangle + \frac{\chi V}{\mu_0}\left[\langle (\overrightarrow{B_0} \cdot \vec{\nabla})\overrightarrow{B_0} \rangle + \frac{1}{2}\left\langle \left(\overrightarrow{B^{AC}} \cdot \vec{\nabla} \right) \overrightarrow{B^{AC}} \right\rangle \right]$$

$$\overrightarrow{F_{1\omega}} = \left\{ \left\langle (\overrightarrow{M_r} \cdot \vec{\nabla})\overrightarrow{B^{AC}} \right\rangle + \frac{\chi V}{\mu_0}\left[\left\langle (\overrightarrow{B_0} \cdot \vec{\nabla})\overrightarrow{B^{AC}} \right\rangle + \left\langle \left(\overrightarrow{B^{AC}} \cdot \vec{\nabla} \right) \overrightarrow{B_0} \right\rangle \right] \right\} \sin(\omega t)$$

$$\overrightarrow{F_{2\omega}} = \left\{ -\frac{\chi V}{2\mu_0}\left\langle \left(\overrightarrow{B^{AC}} \cdot \vec{\nabla} \right) \overrightarrow{B^{AC}} \right\rangle \right\} \cos(2\omega t)$$

$$\overrightarrow{N_{1\omega}} = \left\langle \overrightarrow{M_r} \times \overrightarrow{B^{AC}} \right\rangle \sin(\omega t)$$

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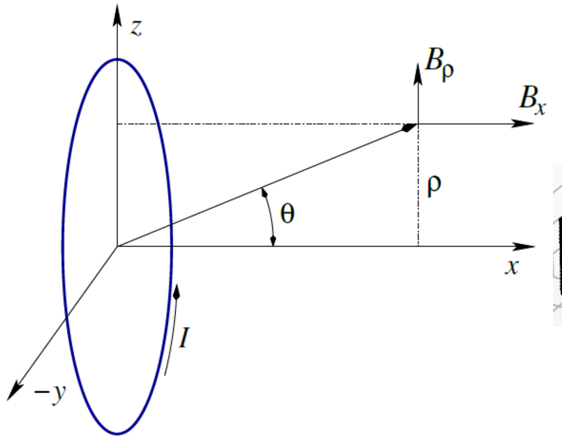
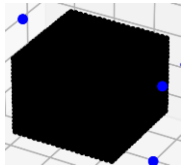
TMs magnetic parameters extraction

- Injected magnetic fields B^{DC} and B^{AC} and their gradients have to be averaged over the TM volume as defined previously by $\langle \dots \rangle$
- Off-axis magnetic field of a coil involves elliptic integrals





$$\rho^2 = y^2 + z^2$$

$$k^2 = \frac{4a\rho}{x^2 + (a+\rho)^2}$$

$$K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2\varphi)^{-1/2} d\varphi$$

$$E(k) = \int_0^{\pi/2} (1 - k^2 \sin^2\varphi)^{1/2} d\varphi$$

$$B_\rho(x,\rho) = A_\rho \frac{x}{\rho^{\frac{3}{2}}} F(k) \qquad B_x(x,\rho) = A_x \rho^{-\frac{3}{2}} G(k) - \frac{\rho}{x} B_\rho(x,\rho)$$

$$A_\rho = \frac{\mu_0}{4\pi} \frac{NI}{a^{1/2}} \qquad A_x = \frac{a}{2} A_\rho \qquad F(k) = k\left[\frac{1 - k^2/2}{1 - k^2} E(k) - K(k)\right] \qquad G(k) = \frac{k^3}{1 - k^2} E(k)$$

When averaged over the TM volume one finds out that thanks to the symmetry of the system :

- Bx is 10 orders of magnitude larger than y and z components
- Equations for all gradients can also be calculated, such that $\partial x Bx$ is 4 orders of magnitude larger than $\partial y, zBx$
- Furthermore, a relationship between Bx and $\partial x Bx$ such that $Bx = \alpha * \partial x Bx$ can be found and it is only dependent on the geometry of the system
- The torque is found not to have a 2ω component

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TMs magnetic parameters extraction

- Equations simplified thanks to symmetry of the system
- Forces obtained by demodulating the Δg signal at the frequencies of interest and multiplying by either the TM mass for the force or by the moment of inertia of a cube for the torque

$$N_{1\omega,\phi} = -M_y \langle B_x^{AC} \rangle$$

$$N_{1\omega,\eta} = M_z \langle B_x^{AC} \rangle$$

$$F_{2\omega,x} = -\frac{\chi V}{2\mu_0} \langle B_x^{AC} \rangle \langle \nabla_x B_x^{AC} \rangle$$

$$F_{1\omega,x} = M_{eff,x} \langle \nabla_x B_x^{AC} \rangle \quad where \quad M_{eff,x} \equiv \left[M_x + 2\frac{\chi V}{\mu_0} \langle B_x^{DC} \rangle \right]$$

$$F_{DC,x} = \left(\frac{\chi V}{\alpha \mu_0}\right) \langle B_x^{DC} \rangle^2 + \left[\frac{M_x}{\alpha} + \frac{\chi V}{\mu_0}\left(\nabla_x B_{back.,x} + \frac{B_{back.,x}}{\alpha}\right)\right] \langle B_x^{DC} \rangle + \left\{ (M_x + M_y + M_z) \nabla_x B_{back.,x} + \frac{\chi V}{\mu_0}\left[3 B_{back.,x} \nabla_x B_{back.,x} + \frac{1}{2} \langle B_x^{AC} \rangle \langle \nabla_x B_x^{AC} \rangle \right] \right\}$$

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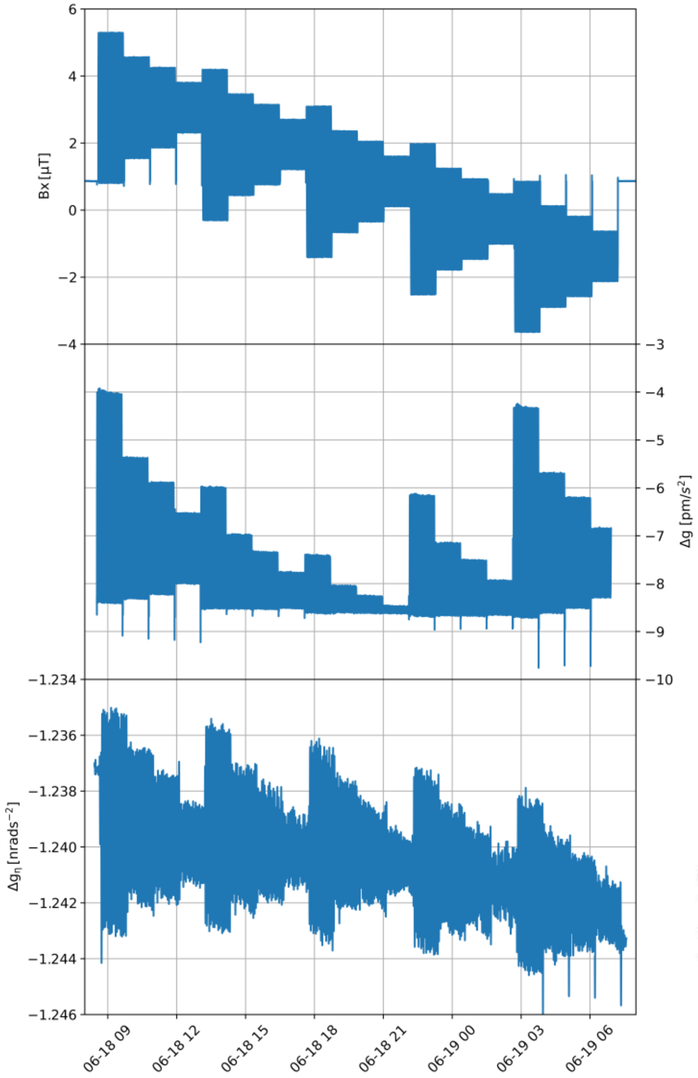
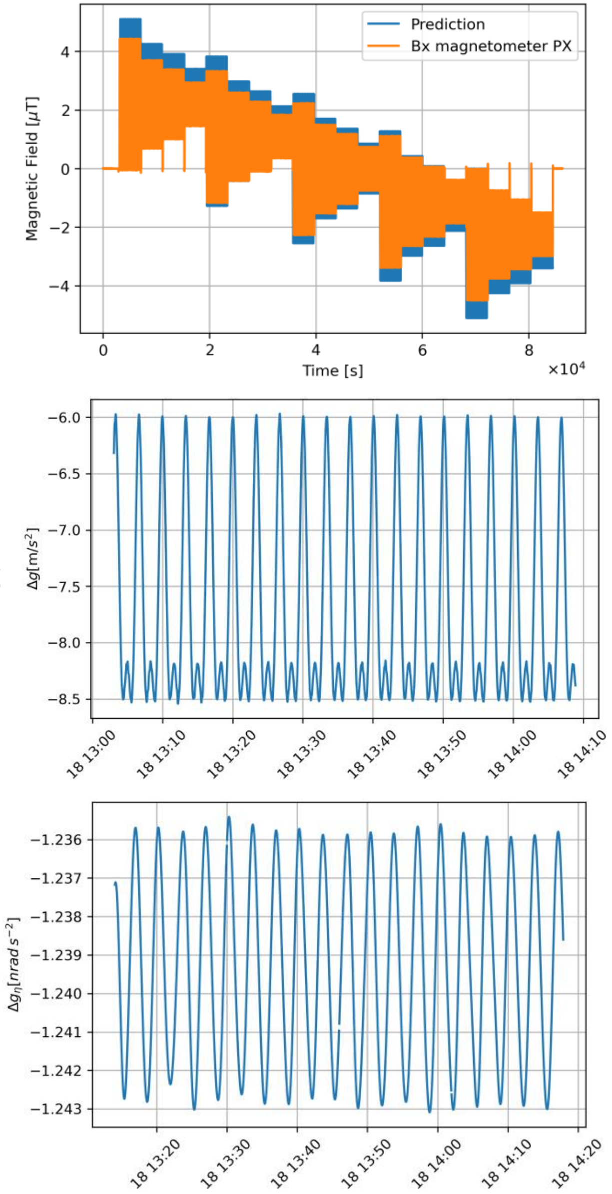
TMs magnetic parameters extraction

- Experimental measurements during injections
- 11% correction to magnetic fields calculated before at the TM location due to magnetometers calibration discrepancy
- Cause: Tolerances during manufacturing and tilts/misalignments during mounting and launch





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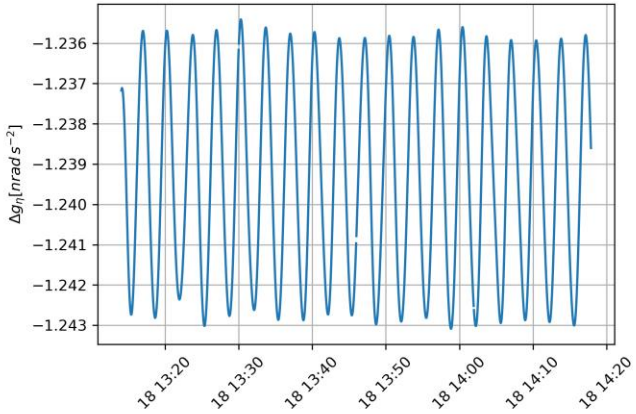
TMs magnetic parameters extraction

$$N_{1\omega,\phi} = -M_y \langle B_x^{AC} \rangle$$

$$N_{1\omega,\eta} = M_z \langle B_x^{AC} \rangle$$

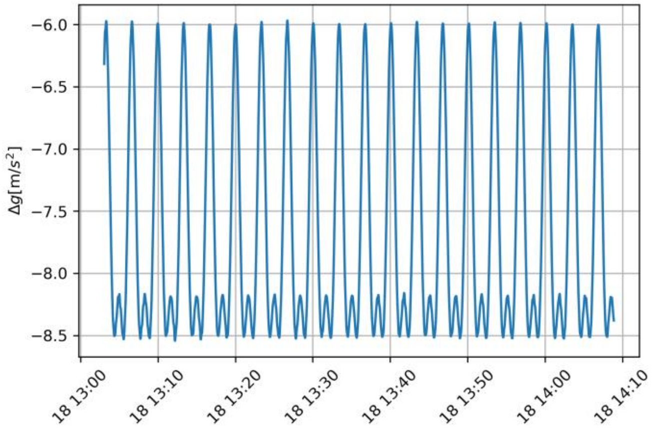


Δg rotations demodulated at the **injected frequency** allow determination of M_y and M_z

$$M_y = (0.178 \pm 0.025)\ nAm^2$$

$$M_z = (0.095 \pm 0.010)\ nAm^2$$

$$F_{2\omega,x} = -\frac{\chi V}{2\mu_0} \langle B_x^{AC} \rangle \langle \nabla_x B_x^{AC} \rangle$$



Δg acceleration demodulated at **twice the injected frequency** provides results of the magnetic susceptibility

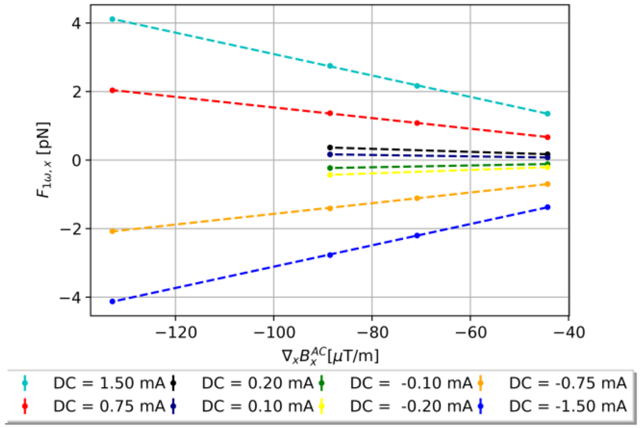
$$\chi_{2mHz} = (-3.43 \pm 0.58) * 10^{-5}$$

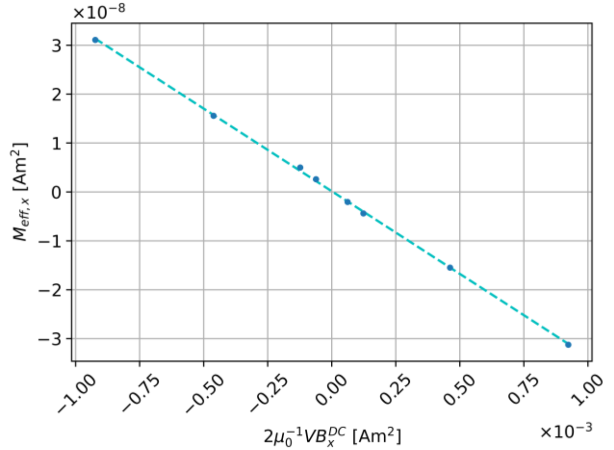
$$\chi_{6mHz} = (-2.65 \pm 0.62) * 10^{-5}$$

$$\chi_{10mHz} = (-3.35 \pm 0.12) * 10^{-5}$$

$$\chi_{30mHz} = (-4.73 \pm 0.34) * 10^{-5}$$

$$F_{1\omega,x} = M_{eff,x} \langle \nabla_x B_x^{AC} \rangle \quad where \quad M_{eff,x} \equiv \left[M_x + 2\frac{\chi V}{\mu_0} \langle B_x^{DC} \rangle \right]$$





Δg acceleration demodulated at **the injected frequency** against the gradient of the AC field have $M_{eff,x}$ as the slope of the fit. $M_{eff,x}$ linear dependence with the DC field determines M_x as the offset and χ as the slope

$$M_x = (0.140 \pm 0.138)\ nAm^2$$

$$\chi_{5mHz} = (-3.3723 \pm 0.0069) * 10^{-5}$$

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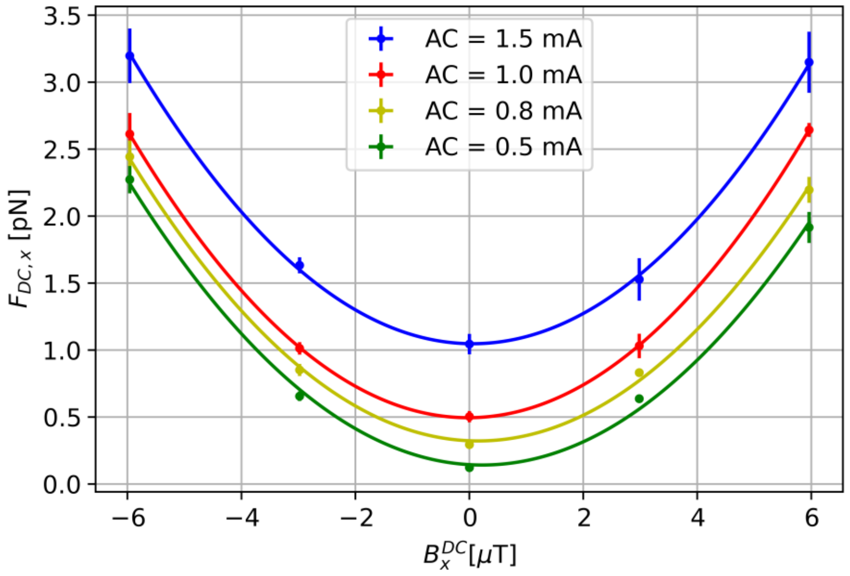
TMs magnetic parameters extraction



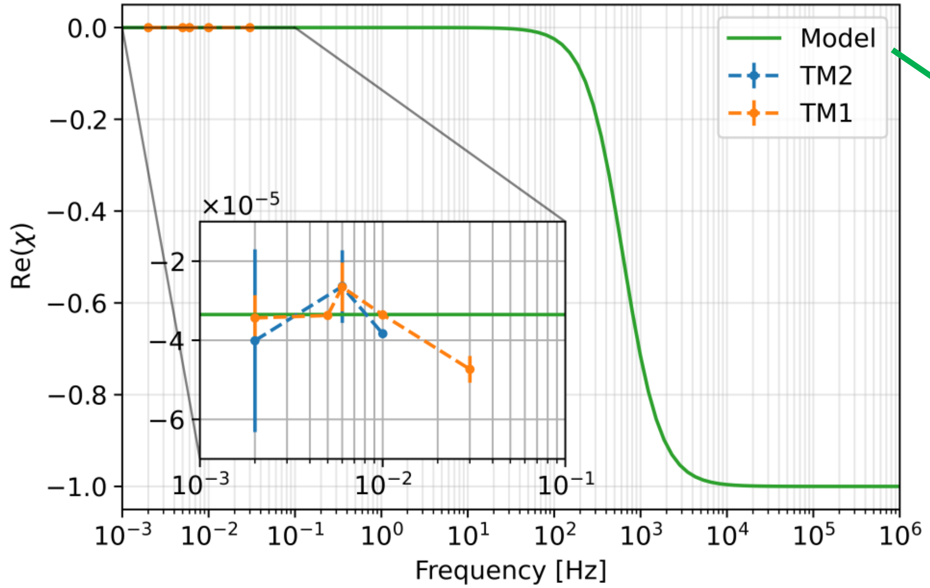
$$F_{DC,x} = \left(\frac{\chi V}{\alpha\mu_0}\right)\langle B_x^{DC}\rangle^2 + \left[\frac{M_x}{\alpha} + \frac{\chi V}{\mu_0}\left(\nabla_x B_{back.,x} + \frac{B_{back.,x}}{\alpha}\right)\right]\langle B_x^{DC}\rangle + \left\{(M_x + M_y + M_z)\nabla_x B_{back.,x} + \frac{\chi V}{\mu_0}\left[3B_{back.,x}\nabla_x B_{back.,x} + \frac{1}{2}\langle B_x^{AC}\rangle\langle\nabla_x B_x^{AC}\rangle\right]\right\}$$

A quadratic fit of the dependence of $F_{DC,x}$ with the injected DC field allows the determination of χ_{DC}, $B_{back.,x}$ and $\nabla_x B_{back.,x}$.
The latter two are of great importance as this is the **only way** to determine the precise values of both within the TM location (magnetometers are located too far away)

$$\chi_{DC} = (-3.35 \pm 0.15) * 10^{-5}$$

$$B_{back.,x} = (414 \pm 74)\ nT$$

$$\nabla_x B_{back.,x} = (-7400 \pm 2100)\ nT/m$$



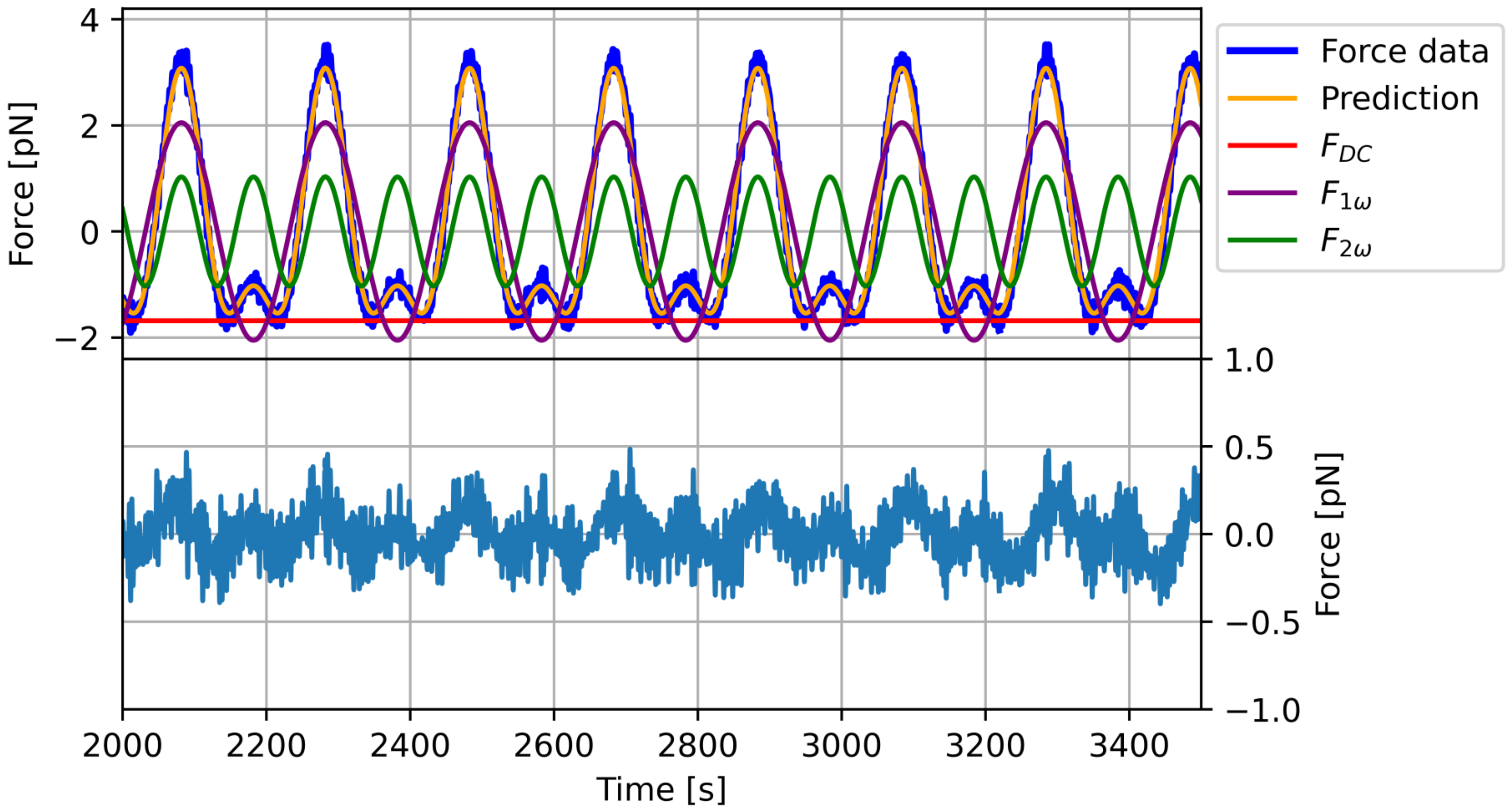
$$\chi(\omega) \approx \chi_{DC} + \frac{-i\omega\tau_e}{1 + i\omega\tau_e}$$

$$\tau_e = (2\pi 630)^{-1} Hz^{-1}$$

S. Vitale, Effect of Eddy currents on down-conversion of magnetic noise., Tech. Rep. Memo LTP package (University of Trento, 2007).

TMs magnetic parameters extraction residual

Parameter	Value
$\chi(*\,10^{-5})$	(-3.3723 ± 0.0069)
$M_x\ [nAm^2]$	(0.140 ± 0.138)
$M_x\ [nAm^2]$	(0.178 ± 0.025)
$M_x\ [nAm^2]$	(0.095 ± 0.025)
$\lvert\vec{M}\rvert\ [nAm^2]$	(0.245 ± 0.081)
$B_{back.,x}\ [nT]$	(414 ± 74)
$\nabla_x B_{back.,x}\ [nT/m]$	(-7400 ± 2100)



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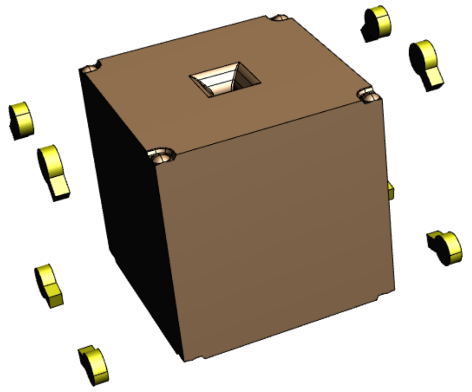
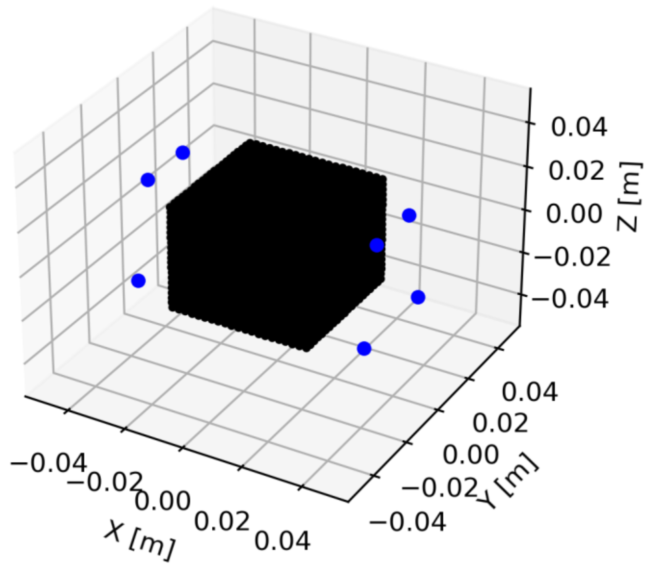
Magnetic contribution to acceleration noise

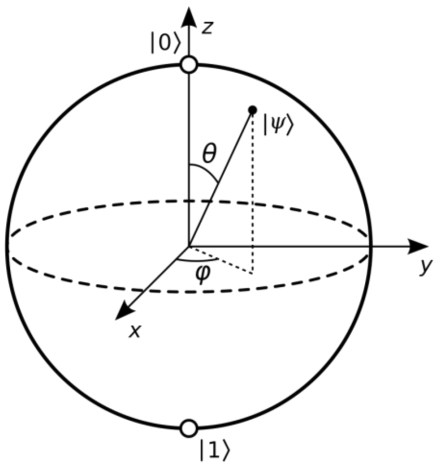
- From the general formula of the force of a dipole we can derive the amplitude spectrum in acceleration

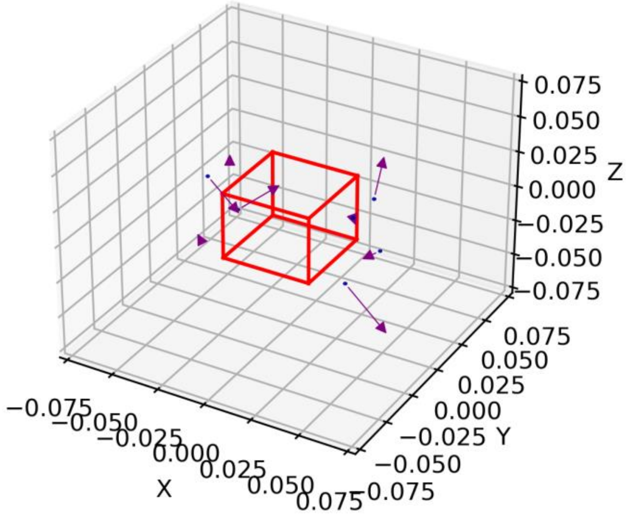
$$S_{\Delta g}^{1/2} = \frac{\chi V}{M_{TM}\mu_0} \langle \vec{\nabla} B_x \rangle S_{\vec{B}}^{1/2}$$

- We have only obtained $\nabla_x B_x = (-7400 \pm 2100)$ nT/m. This value can be attributed to NTCs thermistors at the EH
- Rest of gradients were found by a Monte-Carlo simulation of the NTCs surrounding the TM









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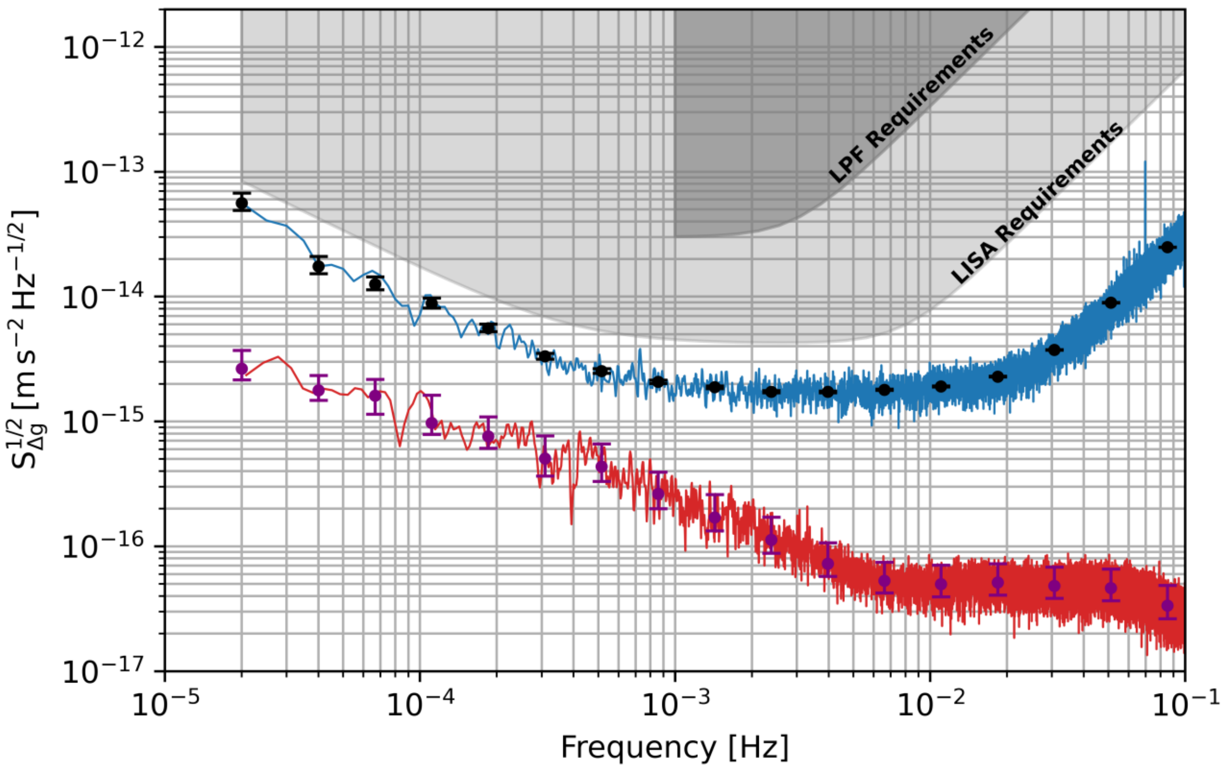
Magnetic contribution to acceleration noise

- Fluctuations of magnetic field ($S_{\vec{B}}^{1/2}$) originated by interplanetary magnetic field
- Amplitude Spectrum Density (ASD) during February noise run, 2017



Contribution at 0.1 mHz: $1.46^{+3.73}_{-0.77}$ % (in noise power)

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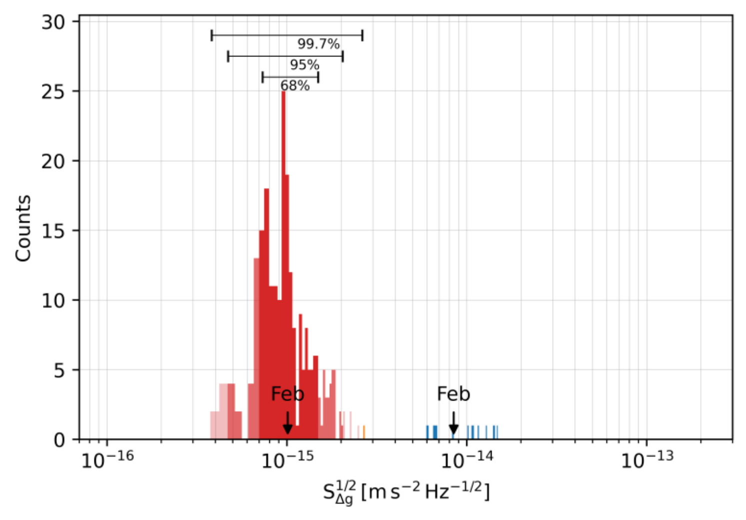
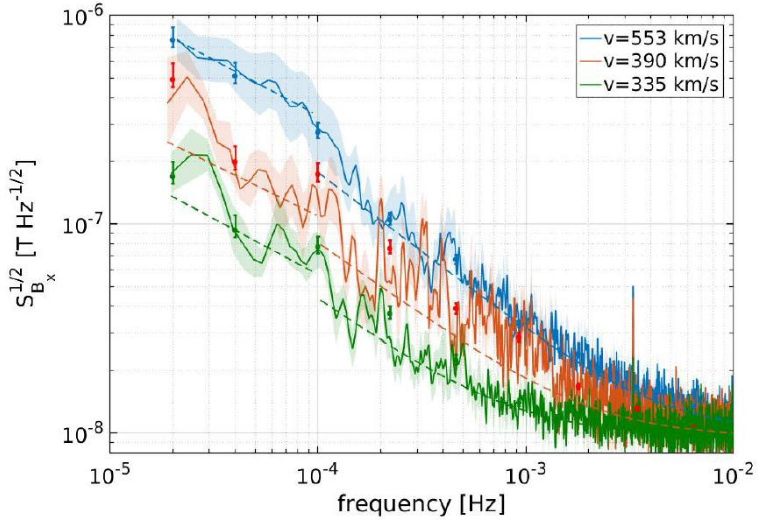
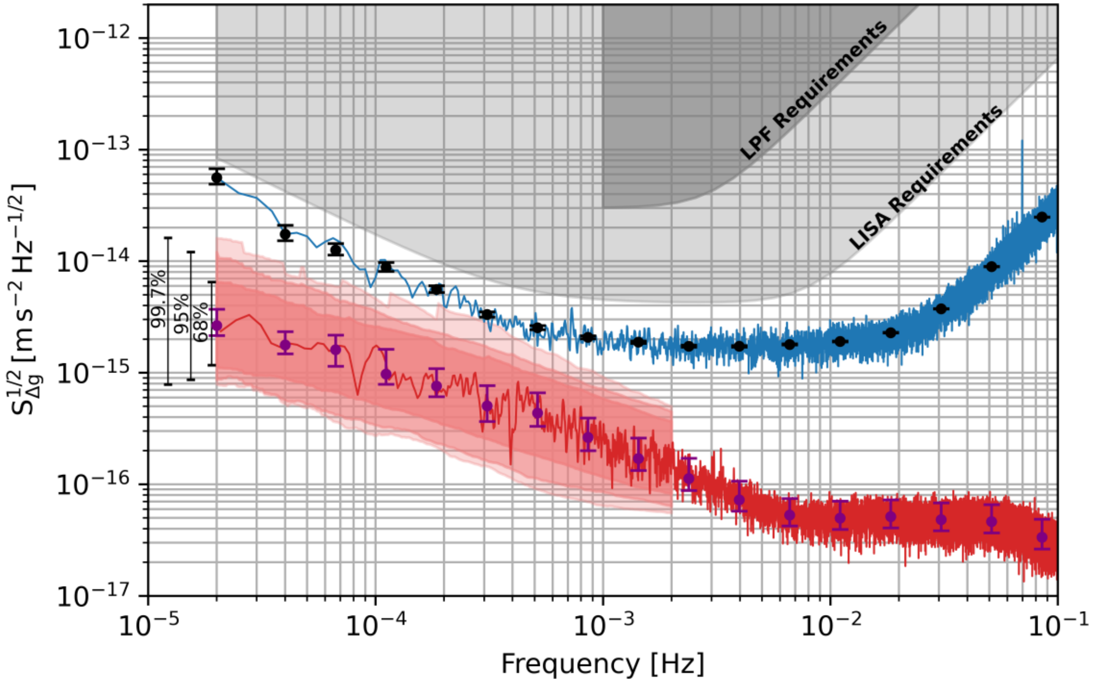
Magnetic contribution to acceleration noise

- Magnetic fluctuations show non-stationarities related to solar wind speed variations



M. Armano et al. Spacecraft and interplanetary contributions to the magnetic environment on-board LISA Pathfinder. Monthly Notices of the Royal Astronomical Society, 494(2):3014–3027, 04 2020.





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Conclusions

TMs magnetic parameters

- $|\vec{M}| = (0.245 \pm 0.081)$ nAm^2 < 10 nAm^2
- $B_{back.,x} = (414 \pm 74)$ nT
- $\nabla_x B_{back.,x} = (-7400 \pm 2100)$ nT/m
- $\chi = (-3.3723 \pm 0.0069) * 10^{-5}$ at 5 mHz

Magnetic induced acceleration noise contribution to Δg

- At 1 mHz: $0.25^{+0.15}_{-0.08}\ fms^{-2}Hz^{-1/2} < 12 fms^{-2}Hz^{-1/2}$
- **Non-stationarities** increase contribution by a factor of 4.6

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Thanks for you attention!

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