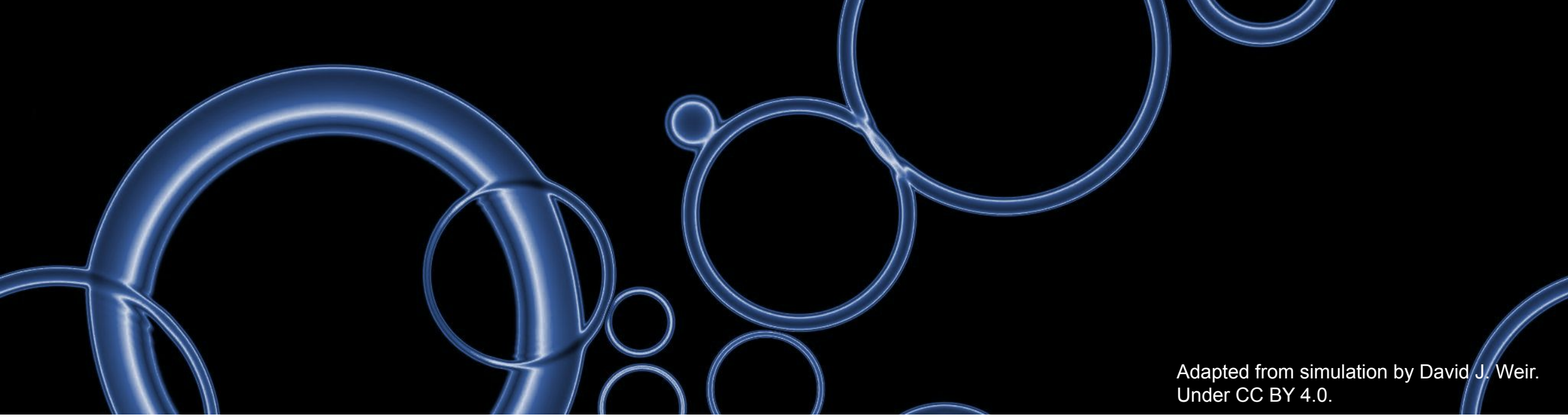


Dimensionally reduced EFTs for cosmological phase transitions

16/10/24 · LISA Spain Meeting 2024 · Institute of Space Sciences (ICE, CSIC and IEEC)

Luis Gil

Based on [2406.02667], by:

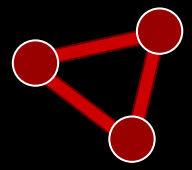
M. Chala, J. C. Criado, LG and J. López Miras

FTAE High Energy Theory

UNIVERSIDAD DE GRANADA

Junta de Andalucía

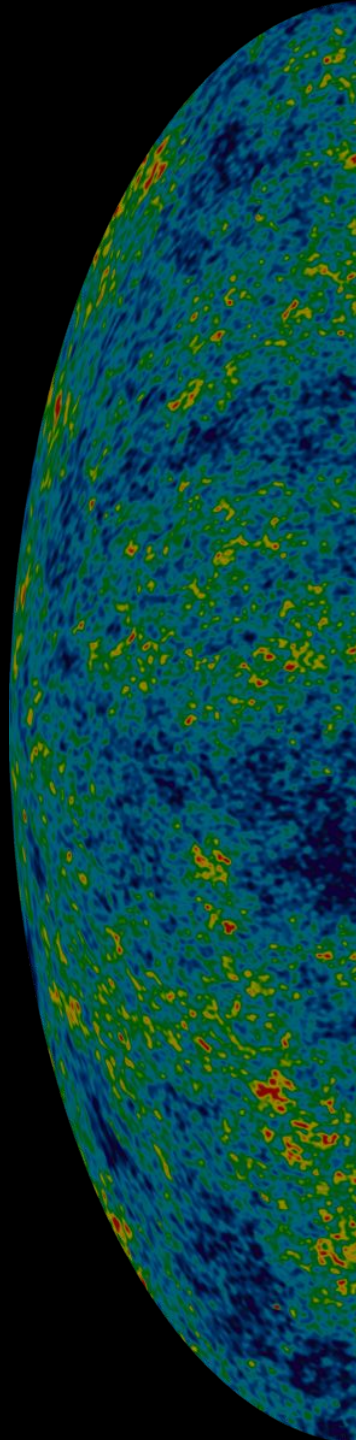
The search for *new physics* in the early universe



Colliders

Cosmic rays

Cosmic Microwave Background



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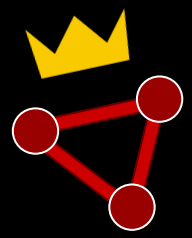

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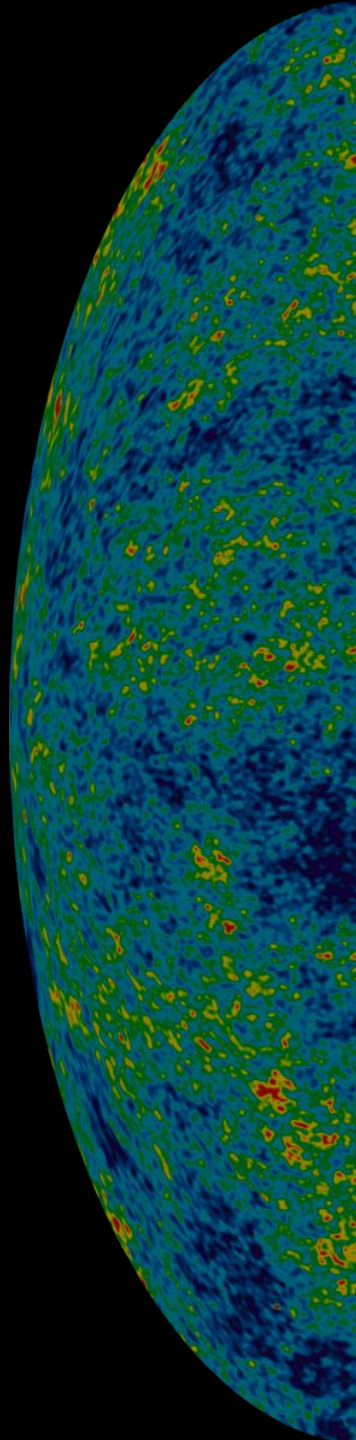

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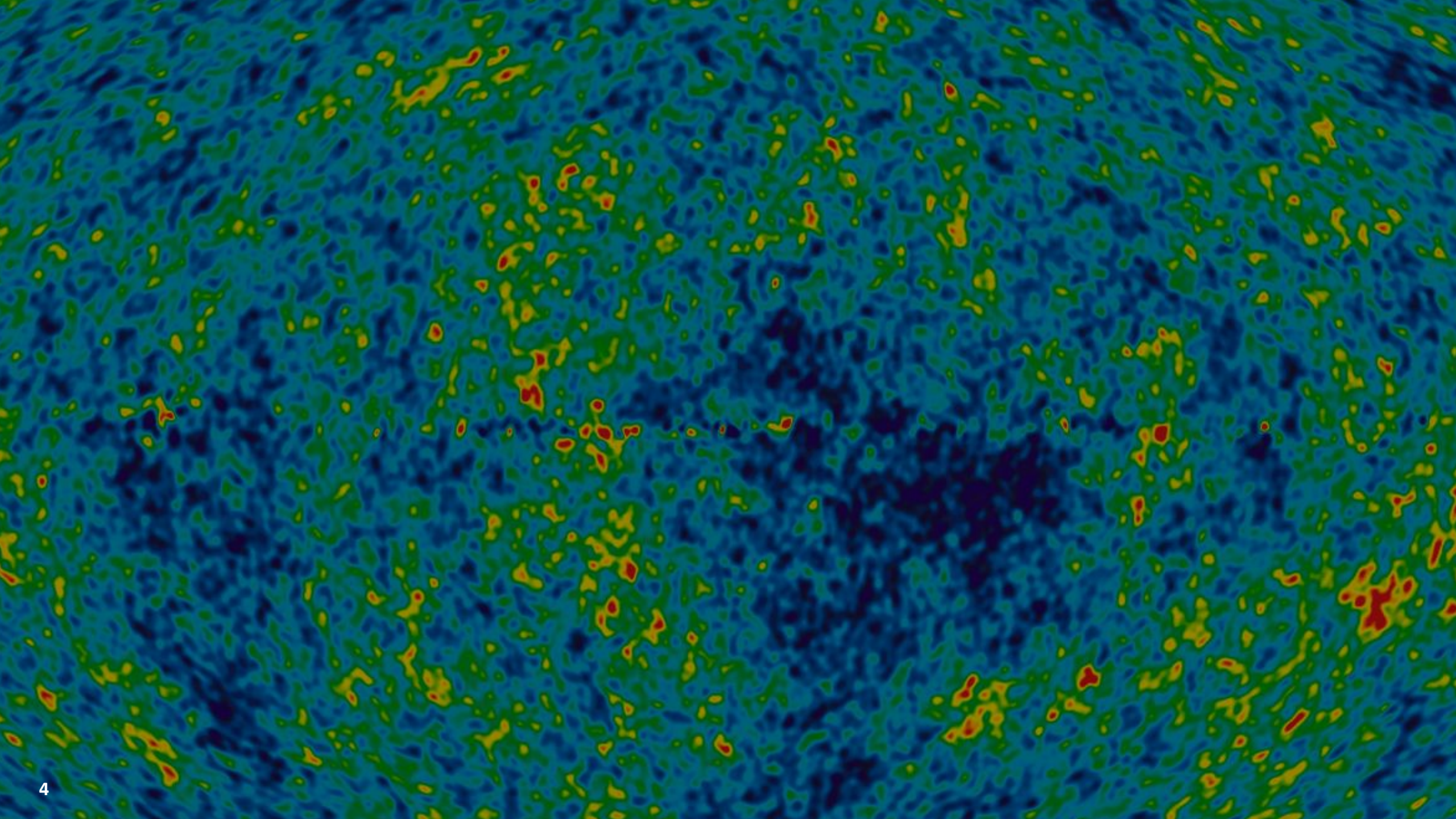
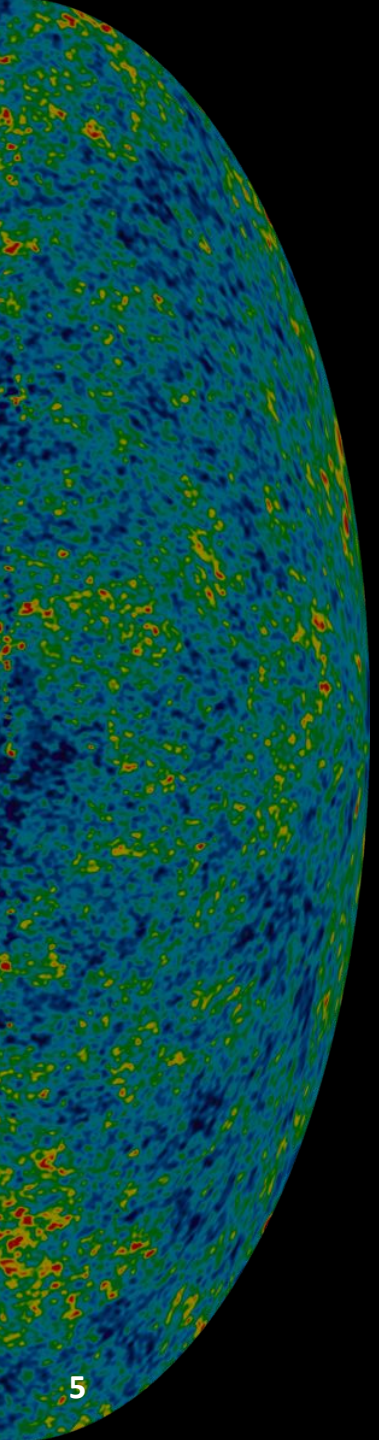
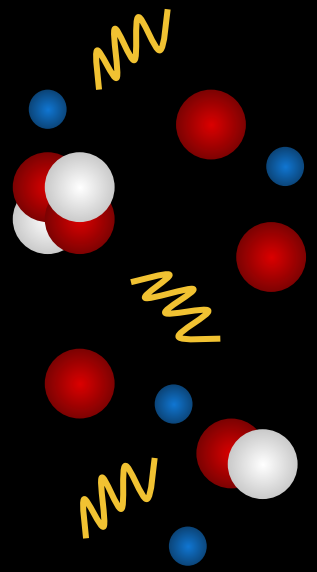
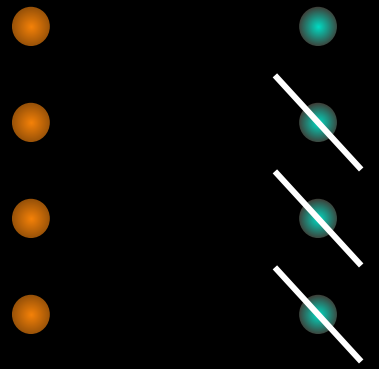






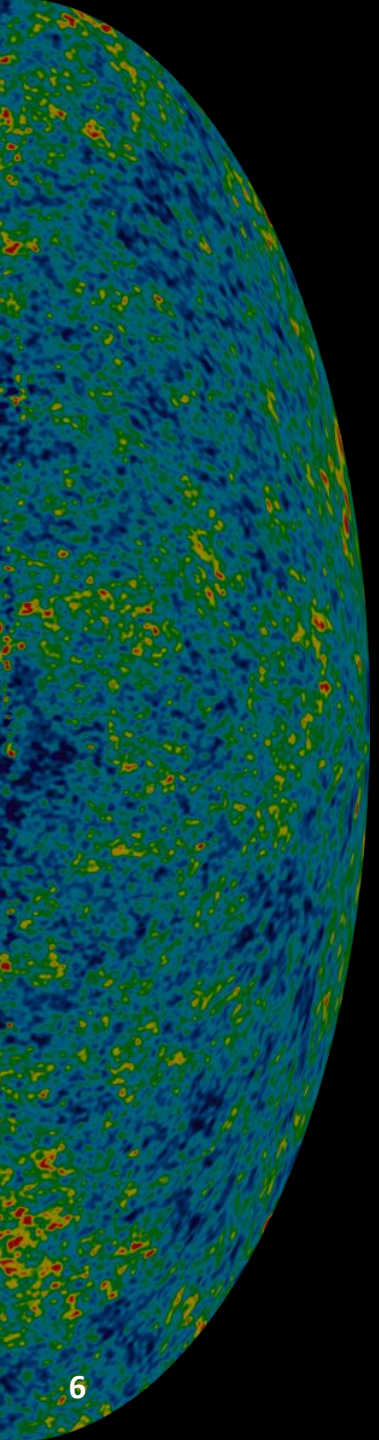
Primordial nucleosynthesis

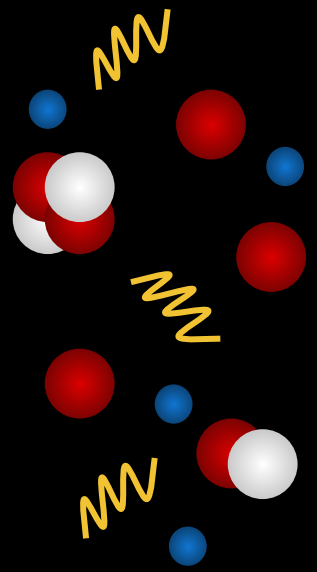
Matter Antimatter



Baryogenesis

Inflation





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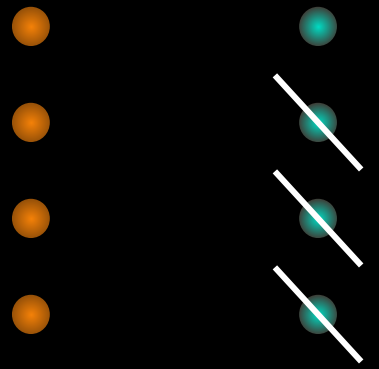
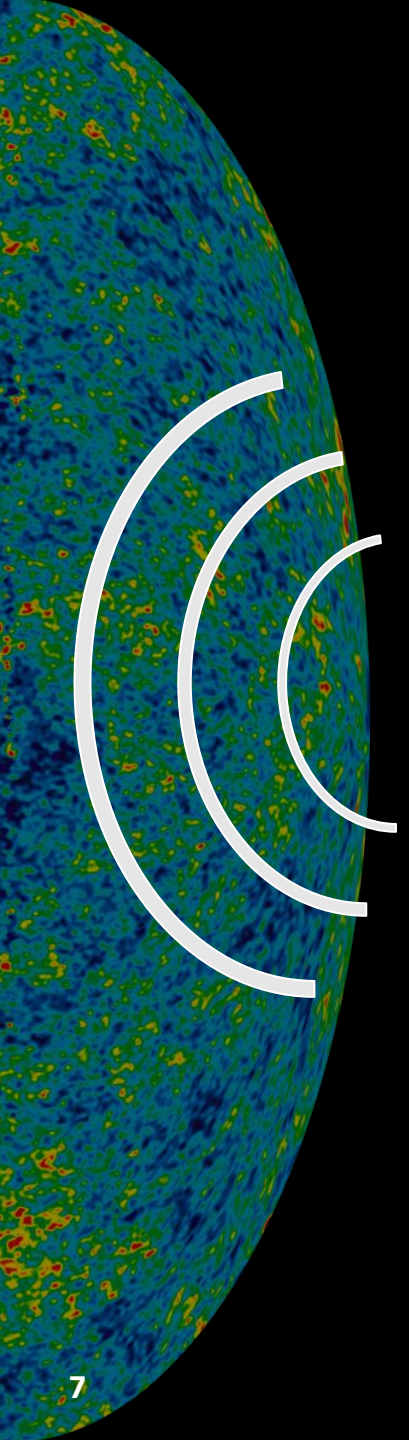


Electroweak (?)
Baryogenesis

Inflation



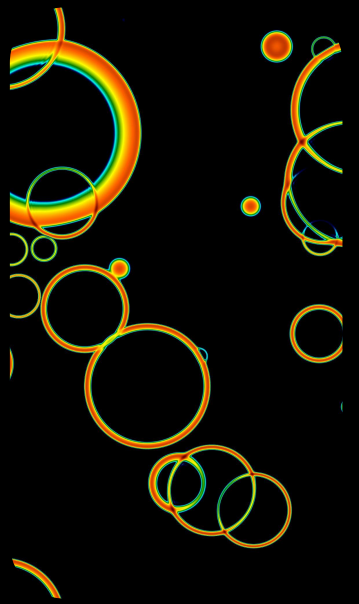
Cosmological phase transition

Stochastic GW background

$$h\Omega_{\rm GW} \simeq h\Omega_{\phi} + h\Omega_{\rm sw} + h\Omega_{\rm turb}$$

Finite temperature QFT

$$T_*, \alpha, \beta/H_*, v_w$$



We need precise theoretical predictions!

Electroweak (?)
Baryogenesis

Inflation

Thermal field theory

Outline of the Matsubara formalism

- **Generating functional** (J=0) in QFT:

$$\mathcal{Z}[0] = \langle q'\ t | q\ 0 \rangle = \langle q'\ 0 | e^{-i\mathcal{H}t} | q\ 0 \rangle = \mathcal{N} \int \mathcal{D}q \exp\left(iS\right)$$

Field correlators

- **Partition function** in quantum statistical mechanics:

$$\mathcal{Z}_{\mathrm{th}} = \mathrm{Tr}\left(e^{-\beta\mathcal{H}}\right) = \sum_q \langle q\ 0 | e^{-\beta\mathcal{H}} | q\ 0 \rangle = \mathcal{N} \int_{q(0)=q(-i\beta)} \mathcal{D}q \exp\left(-S_E\right)$$

QFT at finite temperature	=	Euclidean QFT at zero temperature with periodic time

Thermal field theory

Outline of the Matsubara formalism

- In the path integral formalism, at thermal equilibrium, fields decompose in **Matsubara modes** that live in 3D Euclidean space:

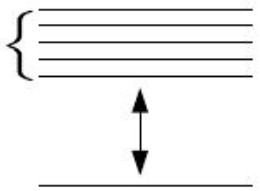
$$\phi(\tau, \mathbf{x}) \equiv T \sum_{n=-\infty}^{\infty} \phi_n(\mathbf{x}) e^{i\omega_n \tau} \qquad \psi(\tau, \mathbf{x}) \equiv T \sum_{n=-\infty}^{\infty} \psi_n(\mathbf{x}) e^{i\omega'_n \tau}$$

$$\omega_n = 2\pi n T \qquad \omega'_n = 2\pi \left(n + \frac{1}{2}\right) T$$

- Each mode acquires a **thermal mass** given by its **Matsubara frequency**.

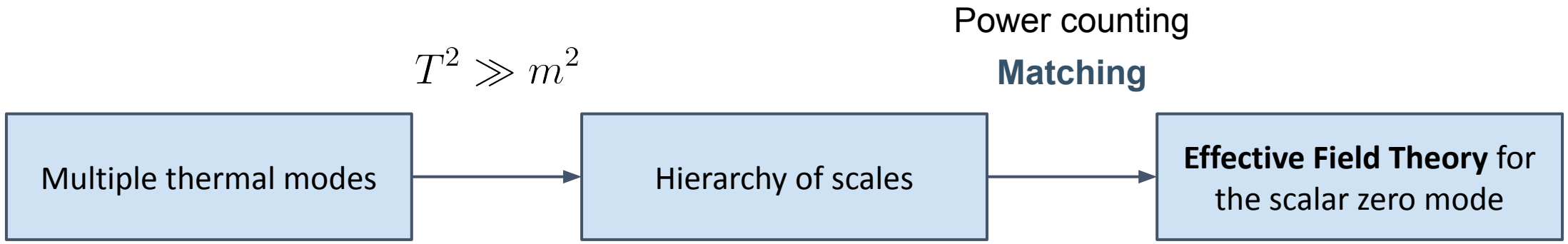
This introduces a **hierarchy of scales**.

$$M \sim \pi T^2$$



Thermal field theory

The 3D EFT approach



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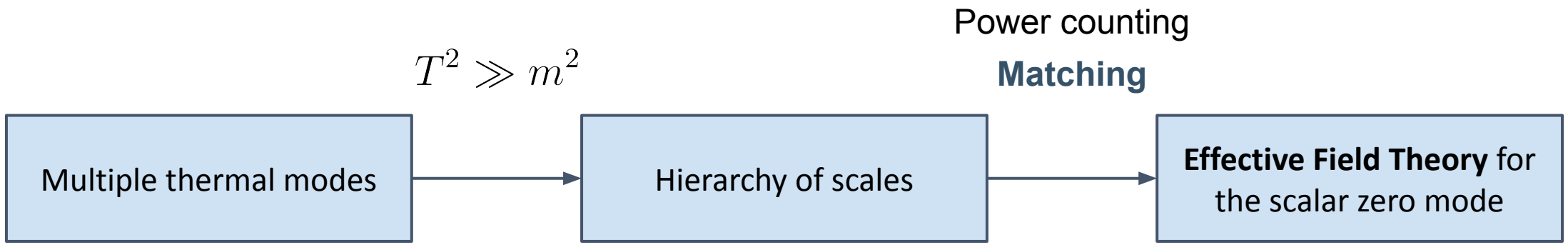


We have a simplified theory to study cosmological phase transitions in the high temperature limit

Bubbles and gravitational waves

Thermally induced field phase transitions

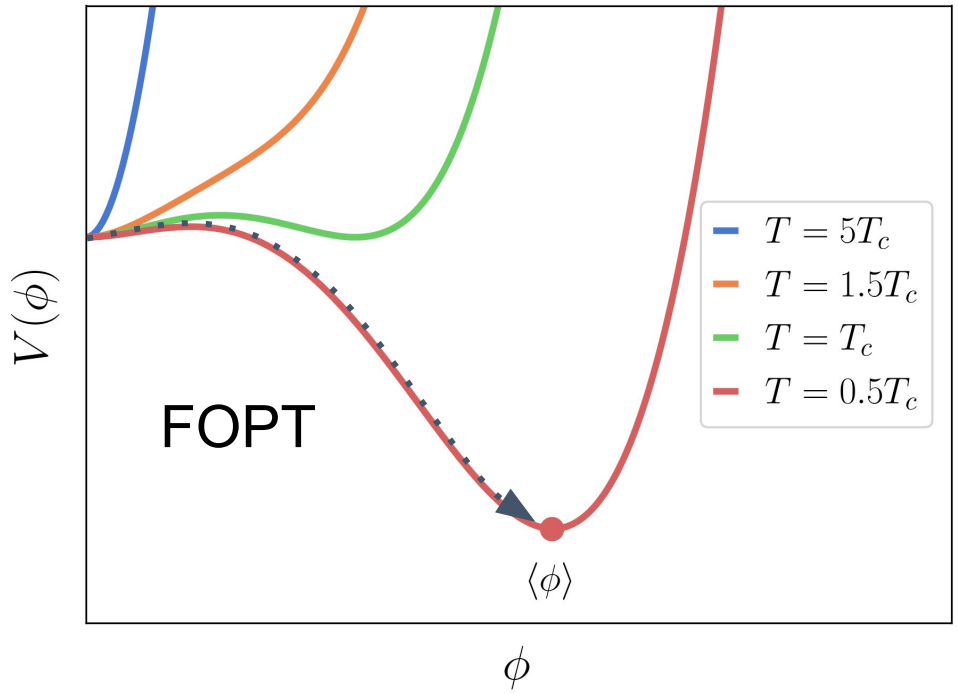


Fig. 1: Temperature evolution of scalar potential

Transition rate: $\Gamma = A(T)e^{-S_E[\varphi_b](T)}$

Depends on static, non-homogeneous solutions to the (Euclidean) EoMs:

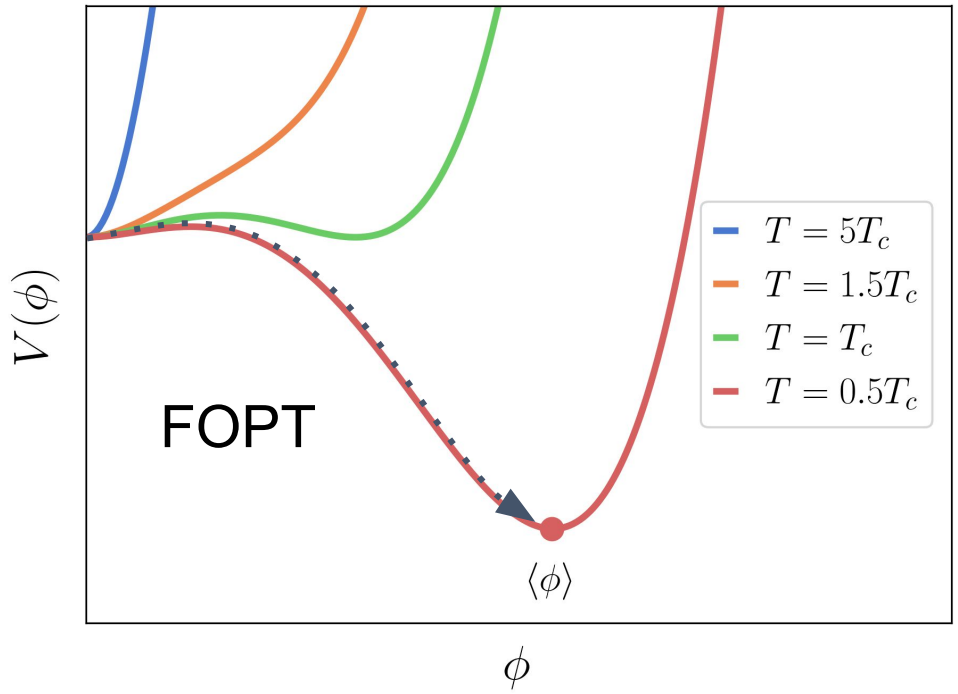
$$\left.\frac{\delta S_E}{\delta \varphi}\right|_{\varphi=\varphi_b} = 0$$

These are the so-called **bounce solutions**.

[Coleman - PhysRevD.15.2929]

Bubbles and gravitational waves

Thermally induced field phase transitions



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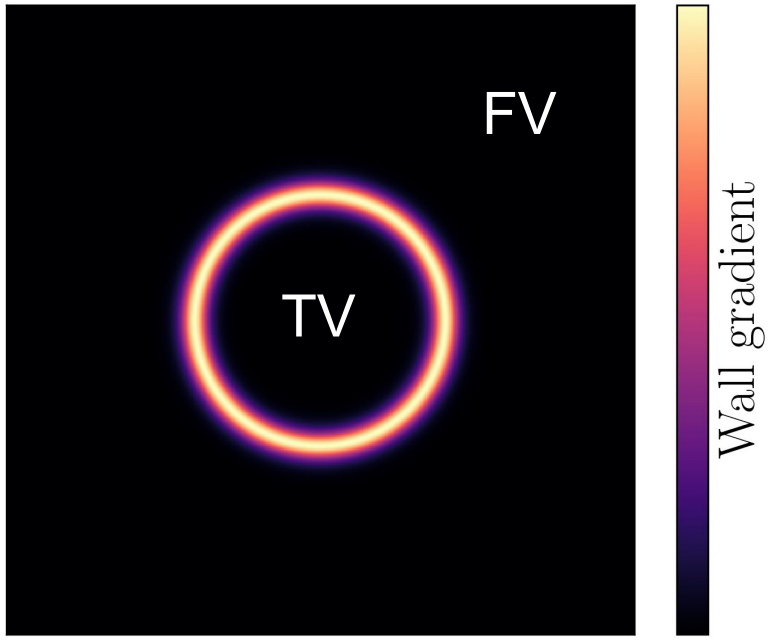

Fig. 2: O(3) symmetric bounce solution

Bubbles and gravitational waves

From PTs to GWs

[Caprini *et al.* - 1512.06239]

As the transition rate grows, in a **first-order PT (FOPT)**:

1) Bubbles of true vacuum nucleate and expand in a hot plasma
2) Bubble fronts collide
3) Sound waves
4) Turbulence

Bubbles and gravitational waves

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Bubbles and gravitational waves

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Probe for NP

How do we connect a QFT model to these GW spectra?

Bubbles and gravitational waves

From PTs to GWs

How do we connect a QFT model to these GW spectra?

Bubbles and gravitational waves

From PTs to GWs

How do we connect a QFT model to these GW spectra?

Nucleation temperature

$$S_3[\varphi_c] \sim 100 - 4 \log \frac{T_*}{100\,\mathrm{GeV}}$$

Strength parameter

$$\alpha = \frac{\Delta \left(V_3(\varphi) - \frac{T}{4} \frac{\mathrm{d}}{\mathrm{d}T} V_3(\varphi) \right) \Big|_{T_*}}{\rho_r(T_*)}$$

Inverse duration of PT

$$\frac{\beta}{H_*} = T_* \frac{\mathrm{d}S_3[\varphi_c]}{\mathrm{d}T} \bigg|_{T_*}$$

Terminal bubble wall velocity

Easy to compute only in some cases

[Lewicki *et al.* - 2111.02393]

3D EFT approach

Building our high-T EFT

UV theory in 4D Minkowski:

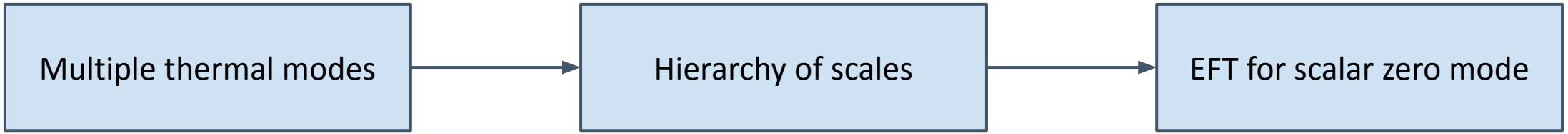
$$\mathscr{L}_4 = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \kappa\phi^3 - \lambda\phi^4 + \overline{\Psi} i\partial\!\!\!/ \Psi - g\phi\overline{\Psi}\Psi$$

[Gould, Xie - 2310.02308]

At temperature T, in Euclidean space:

$$S_0 = (-i)\frac{1}{T}\int \mathrm{d}^3\mathbf{x} \sum_{n=-\infty}^{\infty} \left[\frac{1}{2}(\partial_i \varphi_n)^2 + \frac{1}{2}\left(m^2 + (2\pi n T)^2\right)\varphi_n^2 + \overline{\psi}_n \partial\!\!\!/ \psi_n + \left(2\pi\left(n+\frac{1}{2}\right)T\right)^2 \overline{\psi}_n \psi_n\right]$$

$T^2 \gg m^2$ Power counting



3D EFT approach

Building our high-T EFT

[Kajantie *et al.* - 9508379]
[Criado - 1901.03501]

We determine our 3D EFT by matching correlators to the full theory in the **high-temperature limit**:

$$\mathcal{L}_3 = \frac{K_3}{2}(\partial\varphi)^2 + \frac{1}{2}m_3^2\varphi^2 + \kappa_3\varphi^3 + \lambda_3\varphi^4 + \underbrace{\textbf{higher orders in } \left(\frac{\mathbf{m}}{\mathbf{T}}\right)}$$

Includes:

- New effective operators **(EO)**
- Corrections to operator coefficients

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How important are they?

Includes:

- New effective operators **(EO)**
- Corrections to operator coefficients

Results

PT magnitudes and effective interactions

[Chala, Criado, LG, López Miras - 2406.02667]

$(m^2, \kappa, \lambda)_A = (20\,000\ \mathrm{GeV}^2, -40\ \mathrm{GeV}, 0.01)$ $(m^2, \kappa, \lambda)_B = (31\,643.5\ \mathrm{GeV}^2, -71.1\ \mathrm{GeV}, 0.045)$

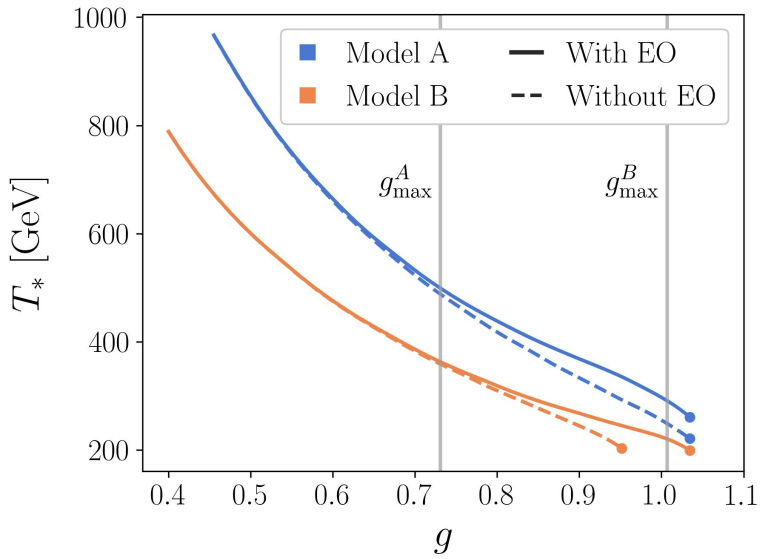

Fig. 5: PT magnitudes in two models, with and w/o effective operators

Results

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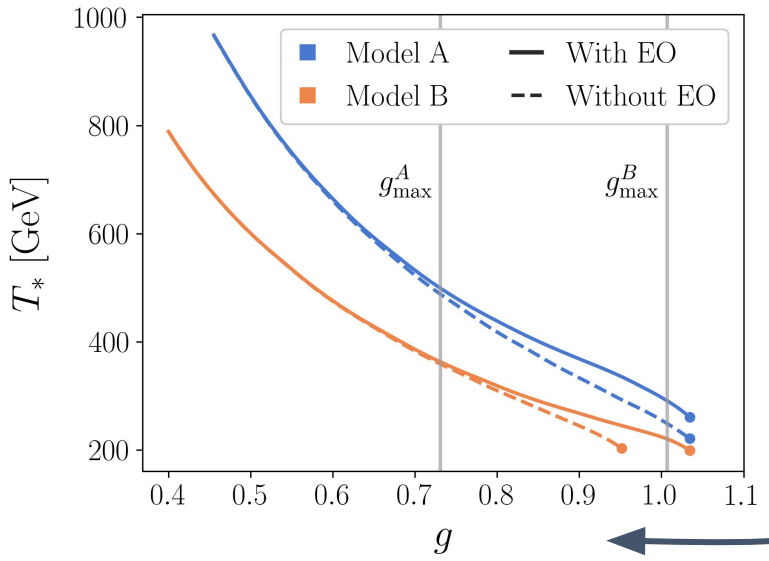

Fixed

Each point is the result for a specific model **(4 parameters)**

Variable

Fig. 5: PT magnitudes in two models, with and w/o effective operators

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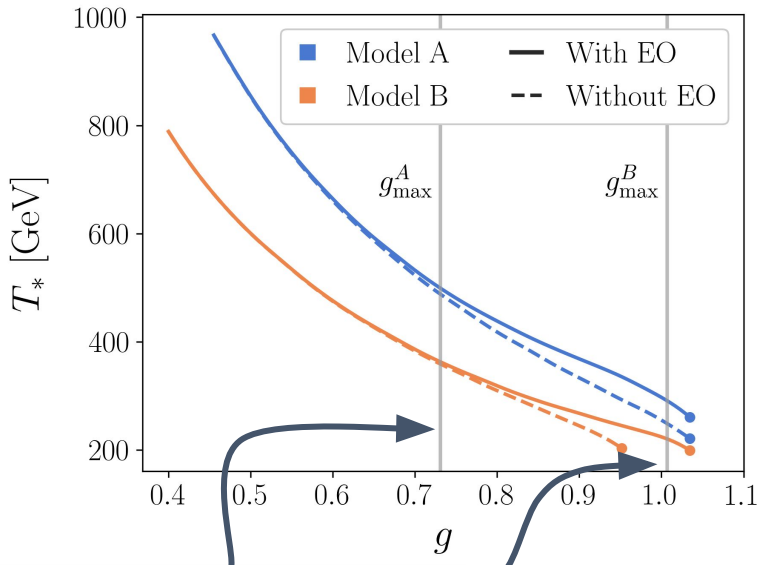

3dEFT not valid beyond the corresponding line!

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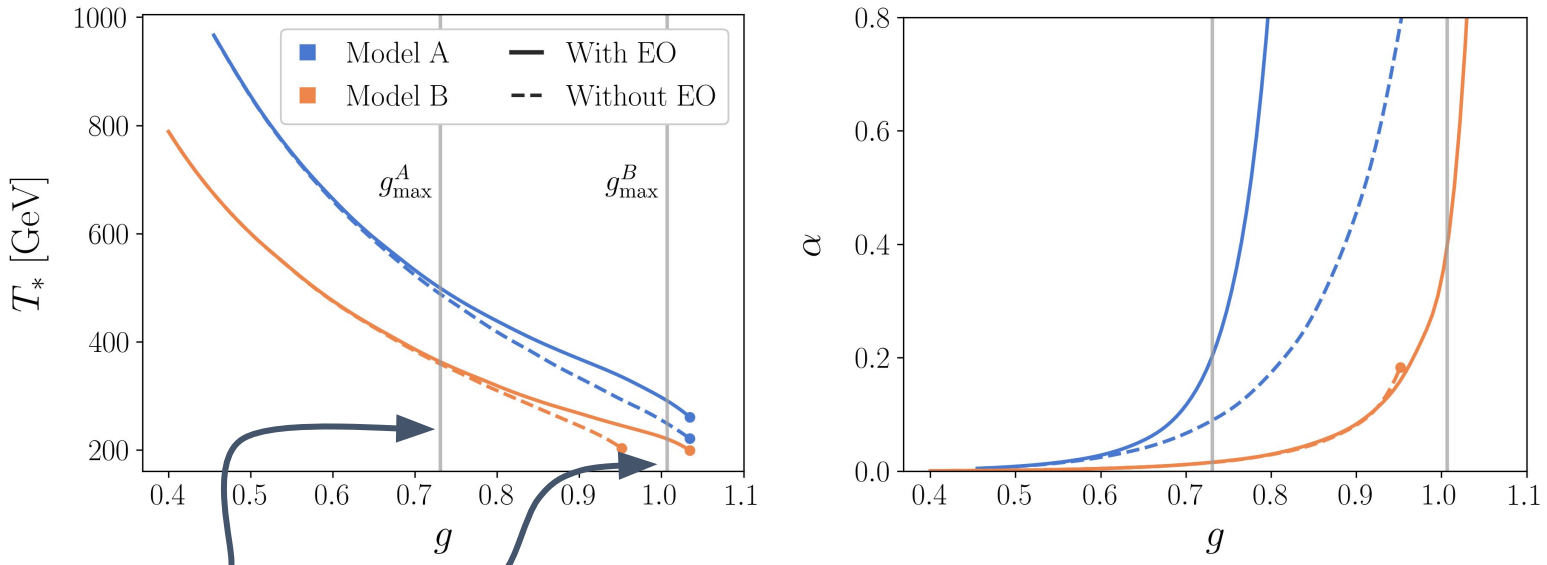

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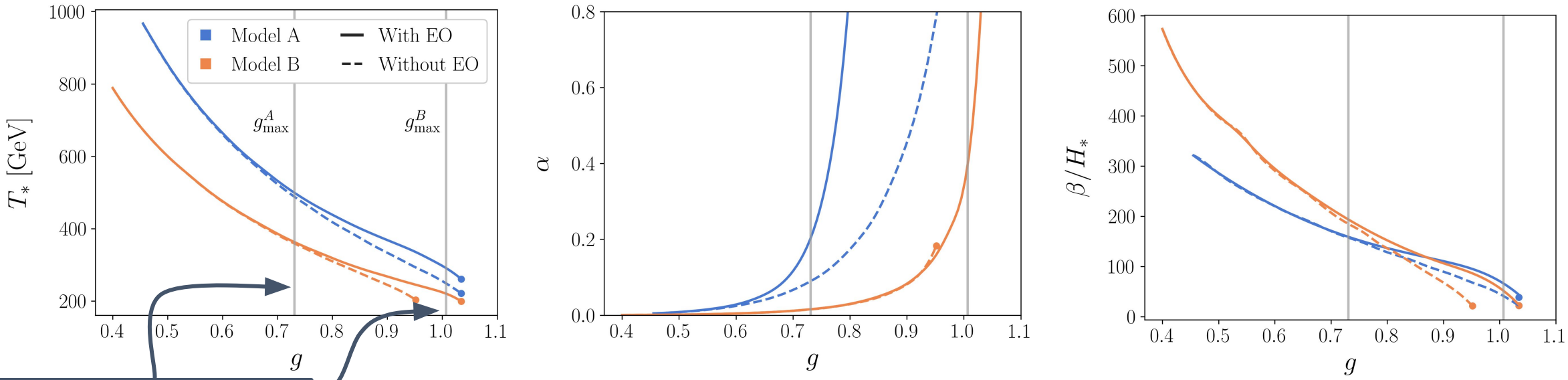

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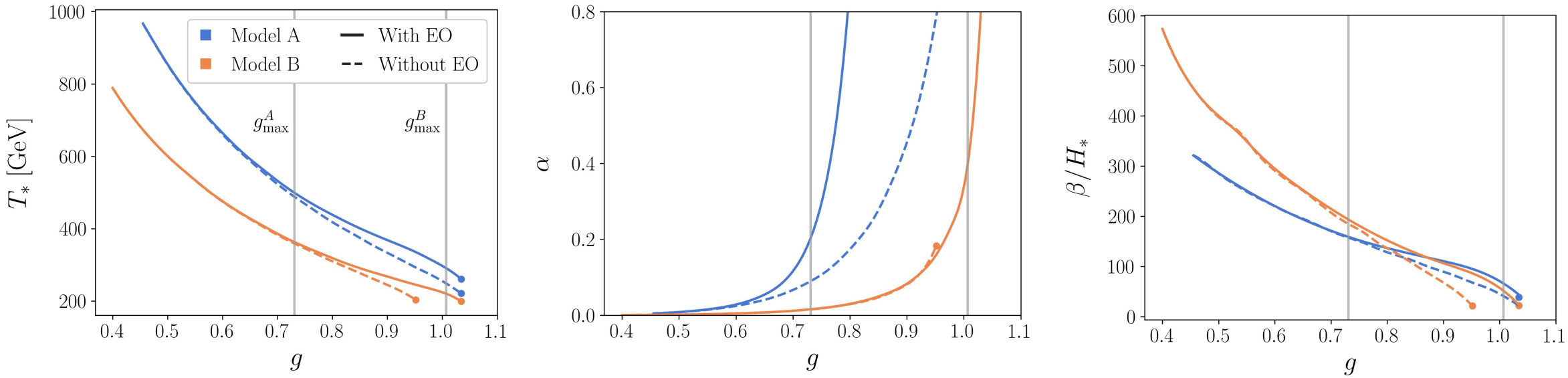

Fig. 5: PT magnitudes in two models, with and w/o effective operators

Two observations

- Eff. ops. **allow for PTs in a wider range** of values of the Yukawa
- Including eff. ops. yields **very different estimations** at large Yukawas

Results

GW power spectra and effective interactions

[Chala, Criado, LG, López Miras - 2406.02667]

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$g \lesssim g_{\max}$

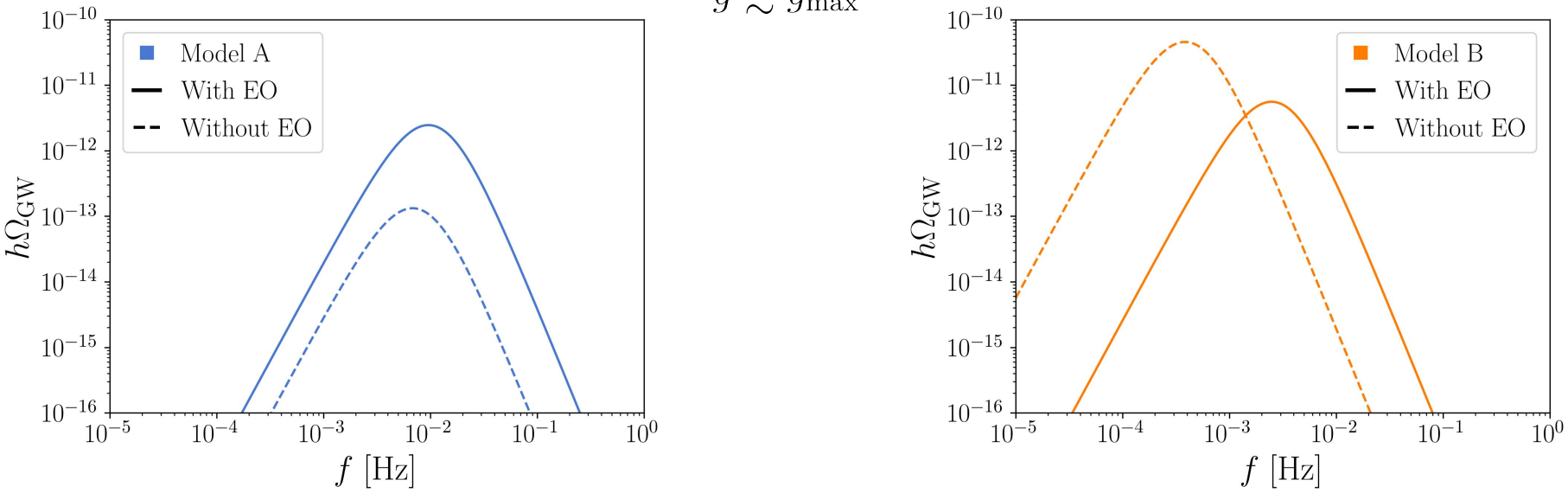

Fig. 6: GW power spectra in two models, with and w/o effective operators

(*) Generated with **PTPlot**
[Caprini *et al.* - 1910.13125]

Results

GW power spectra and effective interactions

[Chala, Criado, LG, López Miras - 2406.02667]

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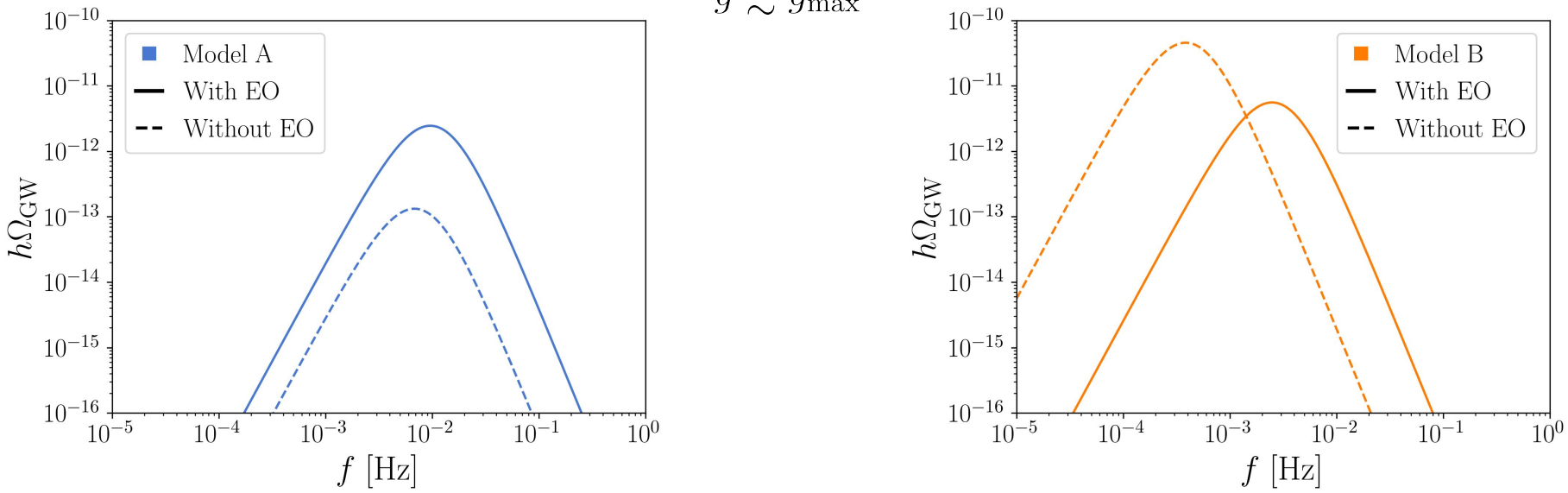


Fig. 6: GW power spectra in two models, with and w/o effective operators

Including eff. ops. can change the peak amplitude and frequency by **one order of magnitude**

Take home messages

We have learned

- Strong FOPTs occur in regions of parameter space **close the limit of validity** of the 3D EFT
- Higher-order-operator corrections in the 3dEFT **are thus relevant in the estimation of PT-related magnitudes** (order of magnitude differences in GW spectra)

Future work

- Application to well-motivated BSM models
- Full order corrections to assess dependence on unphysical parameters

Thank you for your attention!

¡Gracias por vuestra atención!

Bubbles and gravitational waves

What is the state of the art?

This list is NOT exhaustive!

- Several studies of FOPTs in BSM extensions and in the SMEFT:
  - N⁴LO Higgs potential at finite T **[Ekstedt *et al.* - 2405.18349]**
  - EWPT in dim-6 SMEFT **[Camargo-Molina *et al.* - 2103.14022]**
  - EWPT in dim-6 XSM **[Oikonomou *et al.* - 2403.01591]**
  - EWPT in ΣSM **[Niemi *et al.* - 1802.10500]**
- Tools for PT-related computations:
  - *CosmoTransitions* **[Wainwright - 1109.4189]**
  - *FindBounce* **[Guada *et al.* - 2002.00881]**
  - *DRalgo* **[Ekstedt *et al.* - 2205.08815]**
  - *BubbleDet* **[Ekstedt *et al.* - 2308.15652]**

Effective operators with derivatives **not included**

- Theoretical uncertainties persist from the particle physics side:
  - LISA **[Caprini *et al.* - 1910.13125]**

3D EFT approach

Building our high-T EFT

[Kajantie *et al.* - 9508379]
[Criado - 1901.03501]

We find an off-shell basis of operators up to (4D) dim-8 with the help of *BasisGen* and match at 1-loop (fermion only loops)[1]:

$$\mathscr{L}_3 = \frac{1}{2}K_3(\partial\varphi)^2 + \frac{1}{2}m_3^2\varphi^2 + \kappa_3\varphi^3 + \lambda_3\varphi^4$$
$$+ \alpha_{61}\varphi^6 + \beta_{61}\partial^2\varphi\partial^2\varphi + \beta_{62}\varphi^3\partial^2\varphi$$
$$+ \alpha_{81}\varphi^8 + \alpha_{82}\varphi^2\partial_\mu\partial_\nu\varphi\partial^\mu\partial^\nu\varphi + \beta_{81}\varphi\partial^6\varphi + \beta_{82}\varphi^3\partial^4\varphi + \beta_{83}\varphi^2\partial^2\varphi\partial^2\varphi + \beta_{84}\varphi^5\partial^2\varphi$$

[1] Scalar loop contributions to effective operators are subleading (see Appendix A in [2406.02667]).

3D EFT approach

Building our high-T EFT

[Kajantie *et al.* - 9508379]
[Criado - 1901.03501]

We find an off-shell basis of operators up to (4D) dim-8 with the help of *BasisGen* and match at 1-loop (fermion only loops)[1]:

$$\mathscr{L}_3 = \frac{1}{2}K_3(\partial\varphi)^2 + \frac{1}{2}m_3^2\varphi^2 + \kappa_3\varphi^3 + \lambda_3\varphi^4$$

$$+ \alpha_{61}\varphi^6 + \beta_{61}\partial^2\varphi\partial^2\varphi + \beta_{62}\varphi^3\partial^2\varphi$$

$$+ \alpha_{81}\varphi^8 + \alpha_{82}\varphi^2\partial_\mu\partial_\nu\varphi\partial^\mu\partial^\nu\varphi + \beta_{81}\varphi\partial^6\varphi + \beta_{82}\varphi^3\partial^4\varphi + \beta_{83}\varphi^2\partial^2\varphi\partial^2\varphi + \beta_{84}\varphi^5\partial^2\varphi$$

dim-6
Measure eff. ops. relevance

dim-8
Control EFT validity

[1] Scalar loop contributions to effective operators are subleading (see Appendix A in [2406.02667]).

3D EFT approach

Computing relevant PT magnitudes

Eff. ops. depend solely on the Yukawa (g)

Fix all other UV parameters in scan

[Chala *et al.* - 1905.00911]

&

Study all PT magnitudes as functions of the **Yukawa**

m^2, κ, λ

$T_*, \alpha, \beta/H_*, v_w$

Bounce solution must be computed perturbatively!

3D EFT approach

Matching

[Chala, Criado, LG, López Miras - 2406.02667]

$$K_3 = 1 + \frac{g^2}{12\pi^2}\,, \qquad m_3^2 = m^2 + \frac{g^2 T^2}{6}\,, \qquad \kappa_3 = \kappa\sqrt{T}\,, \qquad \lambda_3 = \lambda T\,;$$

$$\alpha_{61} = -\frac{7\zeta(3)g^6}{192\pi^4}\,, \qquad \beta_{61} = -\frac{7\zeta(3)g^2}{384\pi^4 T^2}\,, \qquad \beta_{62} = \frac{35\zeta(3)g^4}{576\pi^4 T}\,;$$

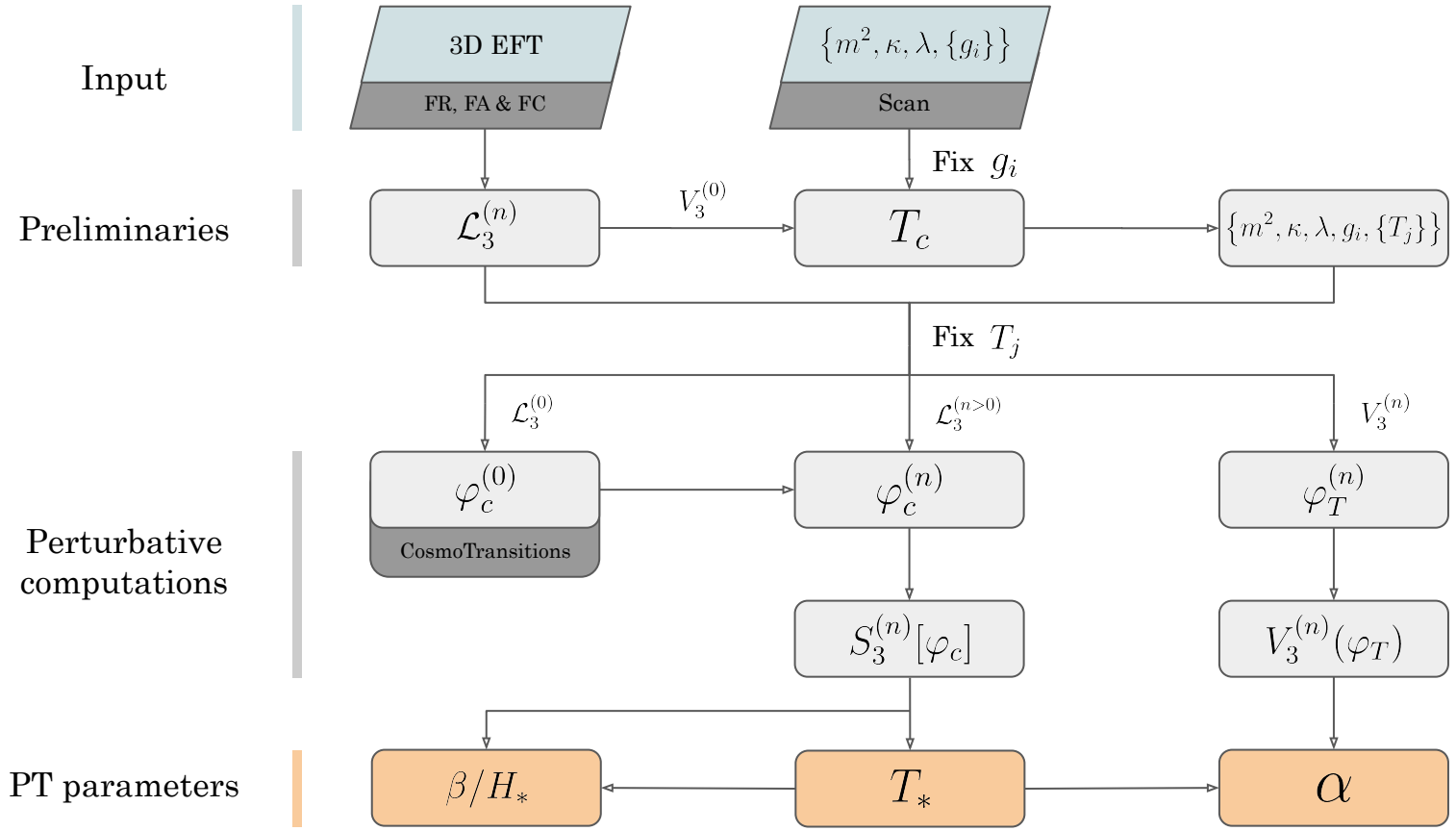
$$\alpha_{81} = \frac{31\zeta(5)g^8}{2048\pi^6 T}\,, \qquad \alpha_{82} = -\frac{31\zeta(5)g^4}{10240\pi^6 T^3}\,, \qquad \beta_{81} = -\frac{31\zeta(5)g^2}{10240\pi^6 T^4}\,,$$

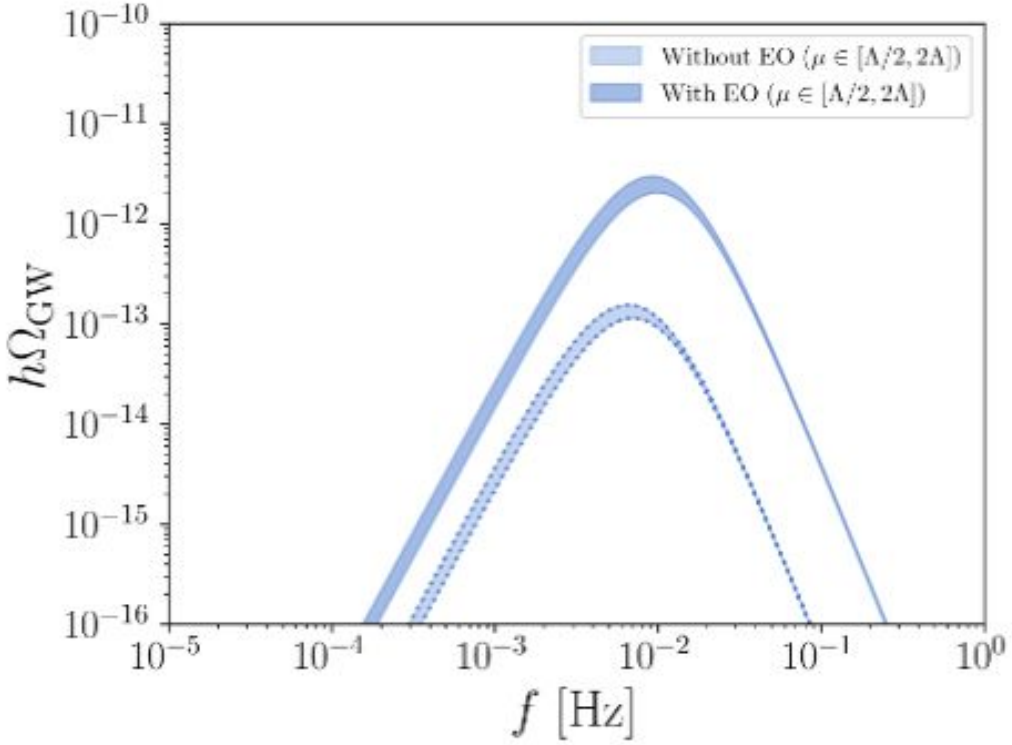
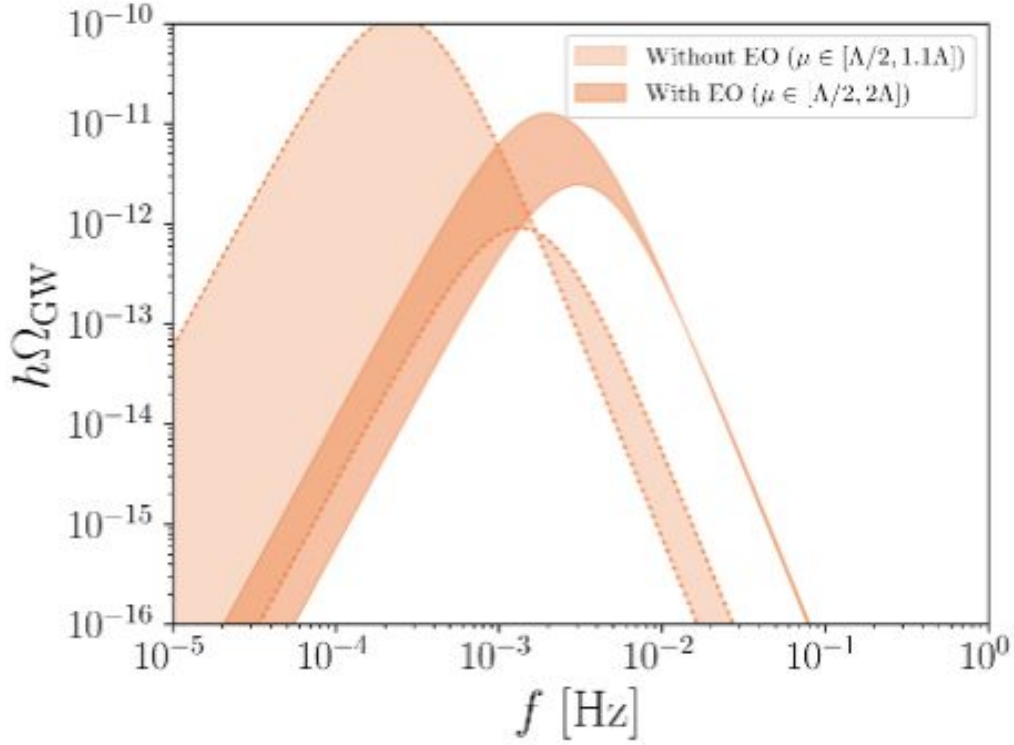
$$\beta_{82} = \frac{217\zeta(5)g^4}{20480\pi^6 T^3}\,, \qquad \beta_{83} = \frac{279\zeta(5)g^4}{20480\pi^6 T^3}\,, \qquad \beta_{84} = -\frac{217\zeta(5)g^6}{5120\pi^6 T^2}\,.$$

3D EFT approach

The algorithm

[Chala, Criado, LG, López Miras - 2406.02667]



Results

GW power spectra and matching scale dependence