# QCD phase diagram with heavy quarks: Universal aspects in one-loop models and extension to 2-loop in Curci-Ferrari

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PRD 97 074027; arXiv:1805.10015

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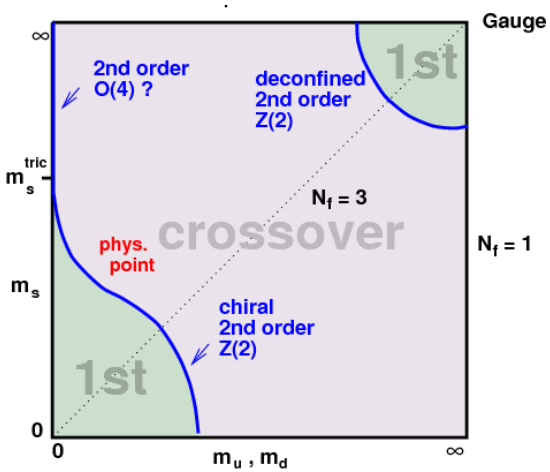
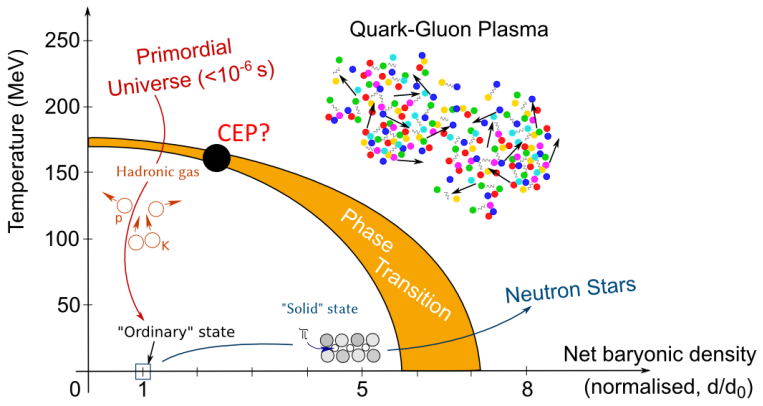
June 28, 2018

# Motivation

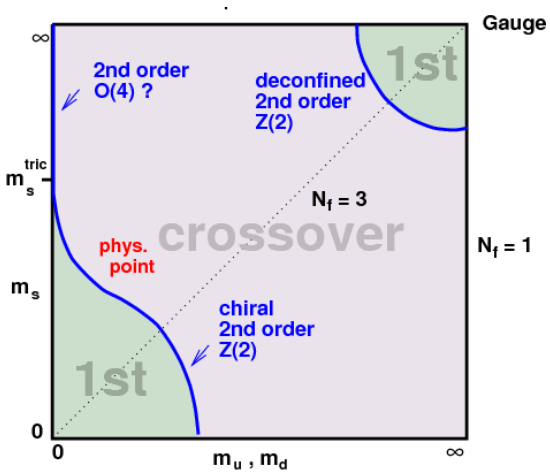



Several approaches on the market:

- Lattice QCD [de Forcrand, Philipsen, Rodriguez-Quintero, Mendes, ...]
- Dyson Schwinger Equations [Alkofer, Fischer, Huber, ...]
- Functional Renormalization Group [Pawlowski, Mitter, Schaefer...]
- Variational Approach [Reinhardt, Quandt, ...]
- Gribov-Zwanziger Action [Dudal, Oliveira, Zwanziger...]
- Matrix-, QM-, NJL-Model,... [Pisarski, Dumitru, Schaffner-B., Stiele, ...]
- Curci-Ferrari Model [Reinosa, Serreau, Tissier, Wschebor, ...]

# Motivation




- Lattice QCD
- Dyson Schwinger Equations
- Functional Renormalization Group
- Variational Approach
- Gribov-Zwanziger Action
- Matrix-, QM-, NJL-Model,...
- Curci-Ferrari Model

**Part 1:**

- generic aspects of the heavy quark region
- common to all approaches at one-loop order

**Part 2:**

- higher order corrections in one particular model
- Curci-Ferrari at two-loop order

# Polyakov loops & effective potentials

At the YM point, a relevant order parameter for the deconfinement transition is the (anti-)Polyakov loop. It is related to the free energy $F_q$ necessary to bring a quark into a "bath" of gluons.

$$\ell \equiv \frac{1}{3}\mathrm{tr}\left\langle P\exp\left(ig\int_0^\beta d\tau A_0^a t^a\right)\right\rangle \sim e^{-\beta F_q} \qquad \bar{\ell} \sim e^{-\beta F_{\bar{q}}}$$

Hence

$$\ell = 0 \leftrightarrow F_q = \infty \leftrightarrow \text{confinement} \qquad \ell \neq 0 \leftrightarrow F_q < \infty \leftrightarrow \text{deconfinement}$$

In all models, for each value of the temperature $T$, one then minimizes an effective potential

$$\boxed{V_{\text{glue}}(\ell, \bar{\ell})}$$

to find the physical position of the system. The particular form of this potential is model-dependent.

# Polyakov loops & effective potentials

Introducing quarks, center symmetry is explicitly broken. For heavy quarks, this breaking is "soft", thus:

$$\ell \approx 0 \leftrightarrow F_q \approx \infty \leftrightarrow \text{confinement} \qquad \ell \not\approx 0 \leftrightarrow F_q < \infty \leftrightarrow \text{deconfinement}$$

Therefore $\ell, \bar{\ell}$ are still approximately good order parameters.

At leading order, the new effective potential is simply found by adding a quark part at one-loop level:

$$\boxed{V_{\text{glue}}(\ell, \bar{\ell}) + V_{\text{quark}}(\ell, \bar{\ell}, \mu)}$$

$\longrightarrow$ Let's look at some particular cases in more detail.

# Explicit Potentials in various Models

**Gribov-Zwanziger:** [JM, U.Reinosa, J.Serreau (2018)]

$$V_{GZ} = \underbrace{-\frac{d}{2}\frac{\sum_\kappa m_\kappa^4}{g^2 C_{\rm ad}} + \frac{d-1}{2}\sum_\kappa \int_Q^T \ln\frac{Q_\kappa^4+m_\kappa^4}{Q_\kappa^2} - \frac{1}{2}\sum_\kappa \int_Q^T \ln Q_\kappa^2}_{V_{\rm glue}} \quad -\mathrm{Tr\,Ln}\,(\not\partial + M)$$

**Curci-Ferrari:** [U. Reinosa, J. Serreau, M. Tissier (2015)]

$$V_{CF} = \underbrace{\sum_\kappa \frac{T}{2\pi^2}\int_0^\infty dq\, q^2 \left\{3\ln\left[1-e^{-\beta\varepsilon q + i r_\kappa}\right] - \ln\left[1-e^{-\beta q + i r_\kappa}\right]\right\}}_{V_{\rm glue}} \quad -\mathrm{Tr\,Ln}\left(\not\partial + M - ig\gamma_0 \bar{A}^k t^k\right)$$

**Matrix-Models:** [K.Kashiwa, R.D.Pisarski and V.V.Skokov (2012)]

$$\begin{aligned} V_M \;=\; & -\frac{4\pi^2}{3}T^2 T_d^2 \left(c_1 \sum_{i,j=1}^N V_1(q_i-q_j) + c_2 \sum_{i,j=1}^N V_2(q_i-q_j) + \frac{(N^2-1)}{60}c_3\right) \\ & \underbrace{-\frac{(N^2-1)\pi^2}{45}T^4 + \frac{2\pi^2}{3}T^4 \sum_{i,j=1}^N V_2(q_i-q_j)}_{V_{\rm glue}} + \ln\det(\gamma^\mu\partial_\mu + q\delta^{\mu 4} + im) \end{aligned}$$

**Lattice:** [M.Fromm, J.Langelage, S.Lottini and O.Philipsen (2012)]

$$Z_{\rm eff} = \int [{\rm d}U_0] \underbrace{\left(\prod_{<ij>}\left[1+2\lambda_1 \mathrm{Re} L_i^* L_j\right]\right)}_{Z_{\rm glue}} \left(\prod_{\vec{x}} \det\left[\left(1+h_1 W_{\vec{x}}\right)\left(1+\bar{h}_1 W_{\vec{x}}^\dagger\right)\right]^{2N_f}\right)$$

# Commonalities & Assumptions

- Potential $v_{\text{glue}}$ is confining, with a minimum at $\ell = 0$ at zero temperature
- Quarks are added at one-loop level, in form of a Tr Ln

$$V = V_{\text{glue}} - \text{Tr}\,\text{Ln}\,(\not{\partial} + M)$$

Then in the heavy quark limit, the Tr Ln expands and one finds

$$\boxed{\beta^4 V(\ell, \beta, M) = v_{\text{glue}}(\ell, \beta) - 2N_f\, f(\beta M)\, \ell}$$

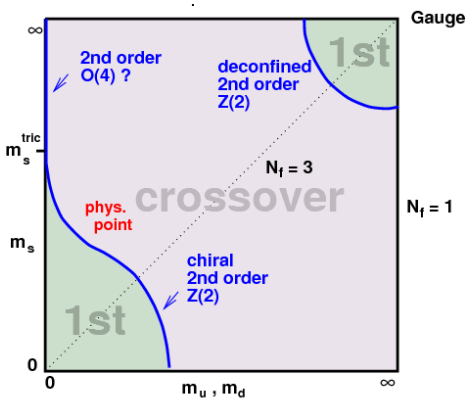
$$f(x) = (3x^2/\pi^2) K_2(x)$$

$K_2(x)$ is the modified Bessel function of the second kind

$\ell$: Polyakov loop
$\beta$: inverse temp.
$M$: deg. quark mass

$\longrightarrow$ How do we find the 2nd order critical line?

# Determination of the critical line




$$\beta^4 V(\ell,\beta,M) = v_{\rm glue}(\ell,\beta) - 2N_f\, f(\beta M)\,\ell$$

For a **fixed** $N_f$:

3 parameters: $\ell$, $\beta$, $\beta M$
3 equations: $\partial_\ell V = \partial_\ell^2 V = \partial_\ell^3 V = 0$

This yields:

$$\underbrace{\partial_\ell v_{\rm glue} = 2N_f f(\beta M)}_{\text{determines model-dep. } \beta M} \quad , \qquad \underbrace{\partial_\ell^2 v_{\rm glue} = \partial_\ell^3 v_{\rm glue} = 0}_{\text{determines } \ell,\beta \text{ , indep. of } \beta M, N_f}$$

$\longrightarrow$ $\boxed{N_f f(\beta M) = N_f' f(\beta M') = f(\beta M_s) + 2f(\beta M_{ud})}$ is const. on the critical line!

# Determination at non-vanishing chemical potential

$$\beta^4 V = v_{\text{glue}}(\ell, \bar{\ell}, \beta) - N_f f(\beta M)(e^{-\beta\mu}\ell + e^{\beta\mu}\bar{\ell})$$

For a **fixed** $N_f$, $\mu$:

4 parameters: $\ell$, $\bar{\ell}$, $\beta$, $\beta M$
4 equations:

$$\partial_\ell V = \partial_{\bar{\ell}} V = 0\,, \tag{1}$$

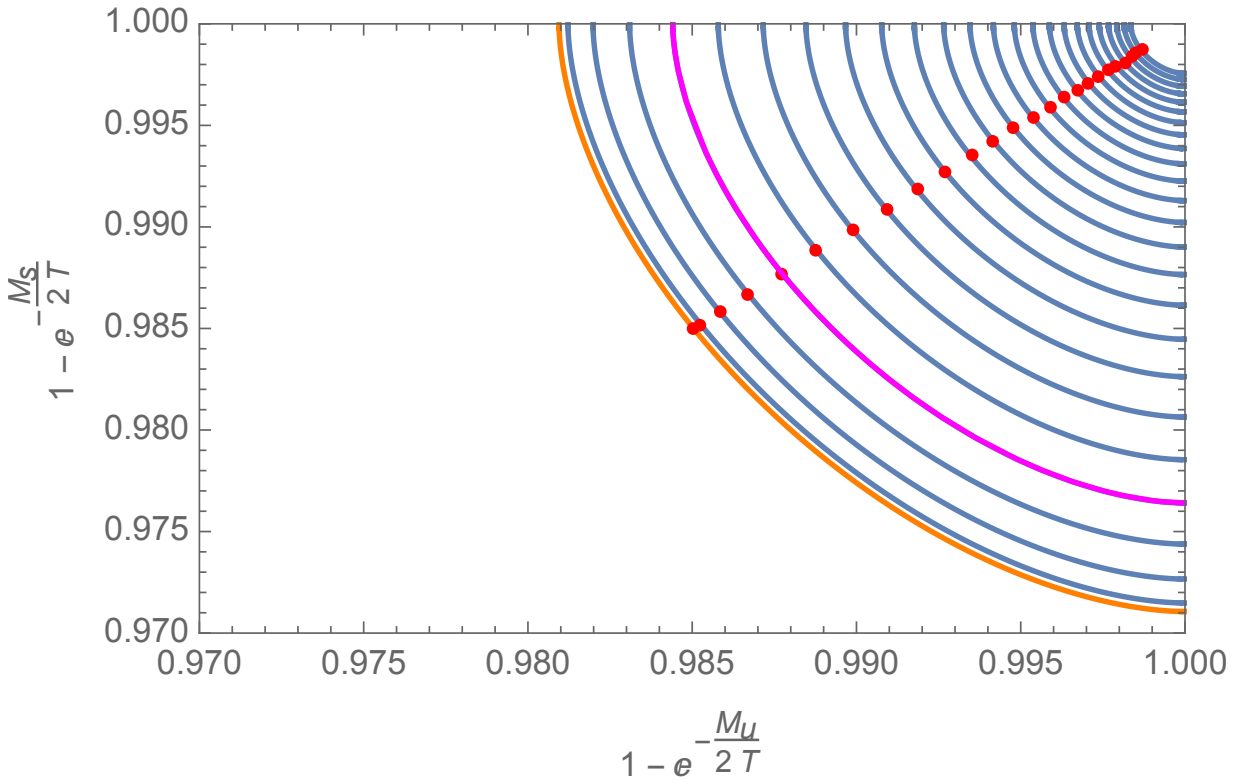
$$\underbrace{\partial_\ell^2 V \partial_{\bar{\ell}}^2 V - (\partial_\ell \partial_{\bar{\ell}} V)^2 = (a\partial_\ell - b\partial_{\bar{\ell}})^3 V = 0}_{\ell(\beta),\bar{\ell}(\beta)\,\text{indep. of}\,N_f,\mu} \tag{2}$$

with $a = \partial_{\bar{\ell}}^2 V|_c$ and $b = \partial_\ell \partial_{\bar{\ell}} V|_c$. The first two equations rewrite

$$N_f f(\beta M) = e^{\beta\mu}\,\partial_\ell v_{\text{glue}} = e^{-\beta\mu}\,\partial_{\bar{\ell}} v_{\text{glue}} \longrightarrow \underbrace{e^{-2\beta\mu} = \partial_\ell v_{\text{glue}}/\partial_{\bar{\ell}} v_{\text{glue}}}_{\ell(\beta(\mu)),\bar{\ell}(\beta(\mu))\,\text{indep. of}\,N_f} \tag{3}$$

$$\longrightarrow \boxed{N_f f(\beta M) = N_f' f(\beta M') = f(\beta M_s) + 2f(\beta M_{ud})} \text{ for each value of } \mu$$

# $N_f f(\beta M) = N'_f f(\beta M')$ So what?




**Red**: Model-dep. $N_f = 3$ input determined from: $\partial_\ell V = \partial_\ell^2 V = \partial_\ell^3 V = 0$

**Blue**: Model-indep. line from: $N_f f(\beta M) = f(\beta M_s) + 2f(\beta M_{ud})$

# $N_f f(\beta M) = N'_f f(\beta M')$ So what?

If expanded in large $R \equiv \beta M$, allows for the simple relation

$$R_{N'_f} - R_{N_f} \approx \ln \frac{N'_f}{N_f} \quad \longrightarrow \quad \boxed{Y_{N_f} \equiv \frac{R_{N_f} - R_1}{R_2 - R_1} \approx \frac{\ln N_f}{\ln 2}}$$

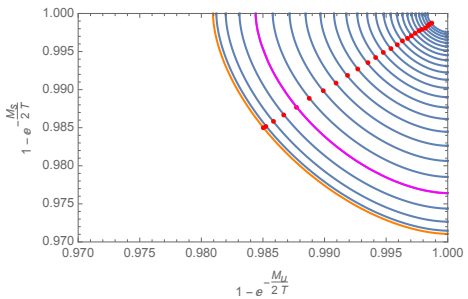
- satisfied both in continuum approaches as well as on the lattice
- robust against higher order corrections in the large $\beta M$ expansion
- independent of chemical potential
- predict $R_{N_f}$ for $N_f > 3$ or $\notin \mathbb{Z}$

| $Y_3$ | $\mu = 0$ | $\mu = i\pi T/3$ |
|---|---|---|
| Lattice | 1.59 | 1.59 |
| GZ1 | 1.58 | 1.57 |
| GZ2 | 1.58 | 1.58 |
| Matrix | 1.59 | 1.56 |
| CF | 1.58 | 1.57 |

$$Y_3 \approx \frac{\ln 3}{\ln 2} \approx 1.58$$

# Intermediate Summary for One-loop Models

- Heavy Quark region exhibits generic features among all one-loop models
- $T_c$ constant along the critical line, whose shape is completely fixed, independently of $\mu$
- Flavor dependence of the critical mass is independent of the gluon dynamics, as predicted by the universal quantity $Y_{N_f}$




$$Y_{N_f} \equiv \frac{R_{N_f} - R_1}{R_2 - R_1} \approx \frac{\ln N_f}{\ln 2}$$

Two assumptions were made:

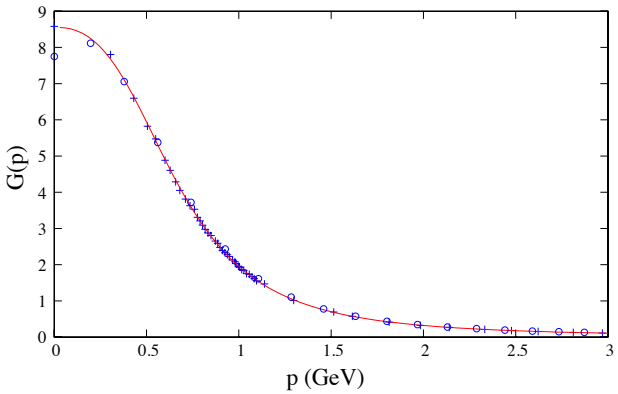
- large quark mass epansion
- quarks contribute at one-loop level

# Curci-Ferrari and gluon mass term

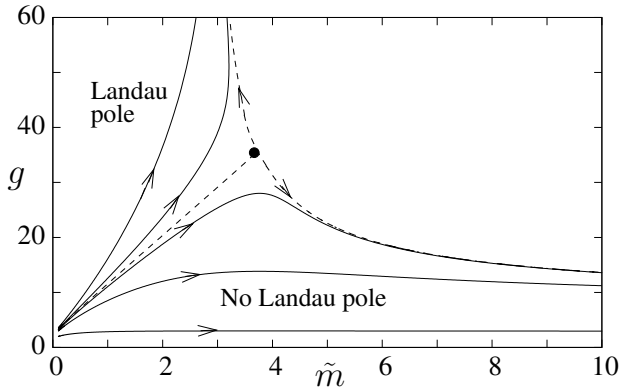
$$S = \int_x \left\{ \frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(\not{D} + M + \mu\gamma_0)\psi \right\} + S_{FP} + \int_x \left\{ \frac{1}{2} m^2 (A^a_\mu)^2 \right\}$$

This gluon mass term can be motivated in several ways

- phenomenologically from lattice data of the Landau gauge gluon propagator saturating in the IR
- Residual ambiguity after non-complete gauge-fixing in Fadeev-Popov procedure due to presence of Gribov copies




one-loop gluon propagator against lattice data,
from [Tissier, Wschebor (2011)]
[Bogolubsky et al. (2009), Dudal, Oliveira, Vandersickel (2010) ]




YM one-loop RG flow,
from [Serreau, Tissier (2012)]

# Landau-DeWitt gauge [Braun, Pawlowski, Gies (2010)]

$$A^a_\mu = \bar{A}^a_\mu + a^a_\mu$$

In practice, at each temperature, the background field $\bar{A}^a_\mu$ is chosen such that the expectation value $\langle a^a_\mu \rangle$ vanishes in the limit of vanishing sources.

This corresponds to finding the absolute minimum of $\tilde{\Gamma}[\bar{A}] \equiv \Gamma[\bar{A}, \langle a \rangle = 0]$, where $\Gamma[\bar{A}, \langle a \rangle]$ is the effective action for $\langle a \rangle$ in the presence of $\bar{A}$.

Seek the minima in the subspace of configurations $\bar{A}$ that respect the symmetries of the system at finite temperature.
$\longrightarrow$ One restricts to temporal and homogenous backgrounds:

$$\bar{A}_\mu(\tau, \mathbf{x}) = \bar{A}_0 \delta_{\mu 0}$$

$\longrightarrow$ functional $\tilde{\Gamma}[\bar{A}]$ reduces to an effective potential $V(\bar{A}_0)$ for the constant matrix field $\bar{A}_0$.

One can always rotate this matrix $\bar{A}_0$ into the Cartan subalgebra:

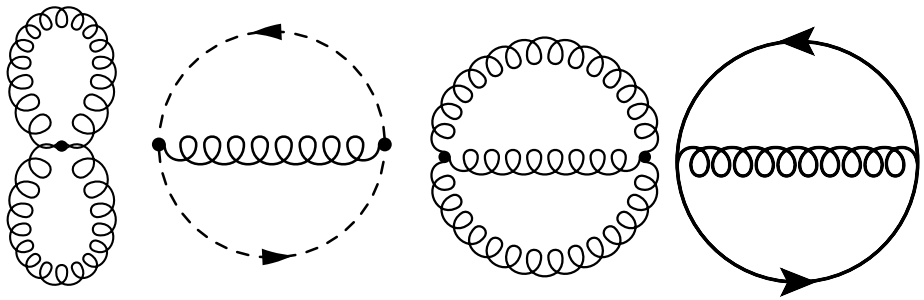
$$\beta g \bar{A}_0 = r_3 \frac{\lambda_3}{2} + r_8 \frac{\lambda_8}{2}$$

| | $r_3$ | $r_8$ |
|---|---|---|
| $\mu = 0$ | $\mathbb{R}$ | $0$ |
| $\mu \in i\,\mathbb{R}$ | $\mathbb{R}$ | $\mathbb{R}$ |
| $\mu \in \mathbb{R}$ | $\mathbb{R}$ | $i\,\mathbb{R}$ |

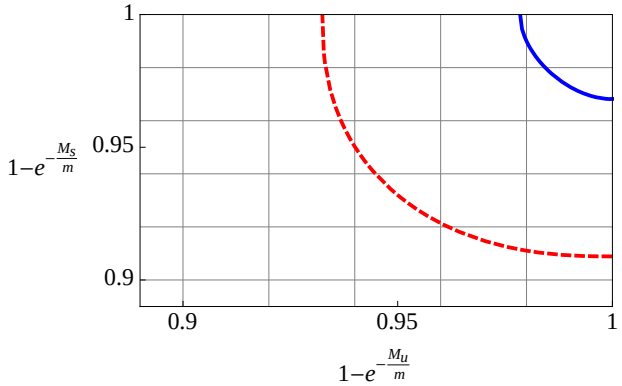
Then $V(\bar{A}_0)$ reduces to a function of 2 components $V(r_3, r_8)$.

# Two-loop Expansion

$$
\begin{aligned}
V(r_3, r_8) \quad &= \quad -\mathrm{Tr}\,\mathrm{Ln}\left(\not{\partial} + M + \mu\gamma_0 - ig\gamma_0 \bar{A}^k t^k\right) \\
&\quad\quad + \frac{3}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2 + m^2\right) - \frac{1}{2}\mathrm{Tr}\,\mathrm{Ln}\left(\bar{D}^2\right) \\
&\quad\quad +
\end{aligned}
$$



# Vanishing chemical potential




$$R_{N_f} \equiv \frac{M_c(N_f)}{T_c(N_f)}$$

$$\mathcal{O}(1): \quad M_{\text{bare}} = M_{\text{ren.}}$$
$$\mathcal{O}(g^2): \quad M_{\text{bare}} = Z_M\, M_{\text{ren.}} + C_M$$

$\longrightarrow$ hard to compare between different approaches!

However, $Z_M$, $C_M$ are independent of $N_f$ at $\mathcal{O}(g^2)$, and observing

$$\frac{T_c(N_f = 3) - T_c(N_f = 1)}{T_c(N_f = 1)} \approx 0.2\%$$

allows for:

$$\overbrace{R_{N'_f}/R_{N_f} \approx M_c(N'_f)/M_c(N_f)}^{\text{if } C_M = 0} \qquad \overbrace{Y_{N_f} \equiv \frac{R_{N_f} - R_1}{R_2 - R_1}}^{\text{if } C_M \neq 0}$$

is scheme indep. & comparable to other approaches up to higher order corrections.

# Vanishing chemical potential

| $\mu = 0$ | $R_2/R_1$ | $R_3/R_1$ | $Y_3$ |
|---|---|---|---|
| Matrix [1] | 1.10 | 1.16 | 1.59 |
| GZ1 [2] | 1.12 | 1.19 | 1.58 |
| GZ2 [2] | 1.08 | 1.13 | 1.58 |
| CF 1-loop [3] | 1.13 | 1.20 | 1.58 |
| CF 2-loop [2] | 1.12 | 1.18 | 1.57 |
| Lattice [4] | 1.10 | 1.15 | 1.59 |
| DSE [5] | 1.29 | 1.43 | 1.51 |

$\longrightarrow$ The $Y_3$ values are still satisfied to very good approximation which underlines its importance as a universal quantitiy

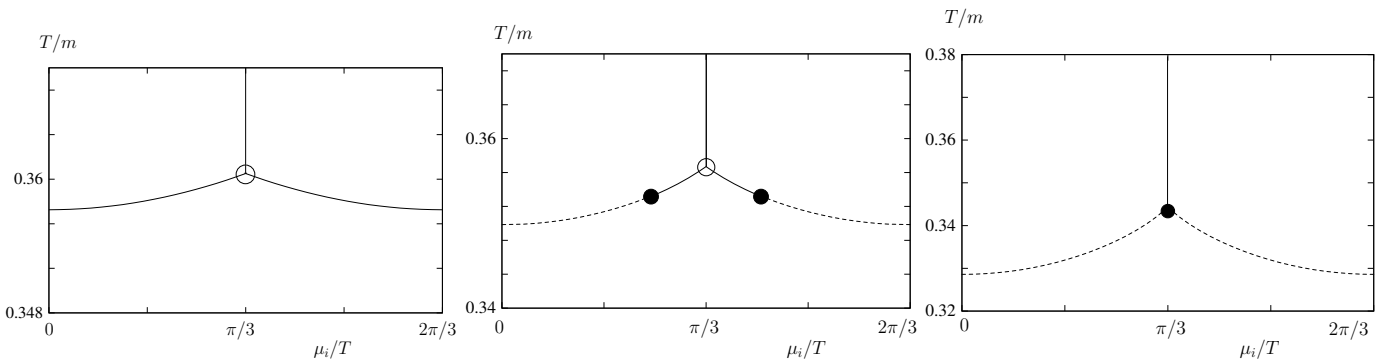
$\longrightarrow$ The overall good agreement seems to suggest that the underlying dynamics is well-described within (Curci-Ferrari) perturbation theory.

[1] Kashiwa, Pisarski, Skokov (2012) [2] JM, Reinosa, Serreau (2017+18)

[3] Reinosa, Serreau, Tissier (2015) [4] Fromm, Langelage, Lottini, Philipsen (2012)

[5] Fischer, Luecker, Pawlowski (2015)

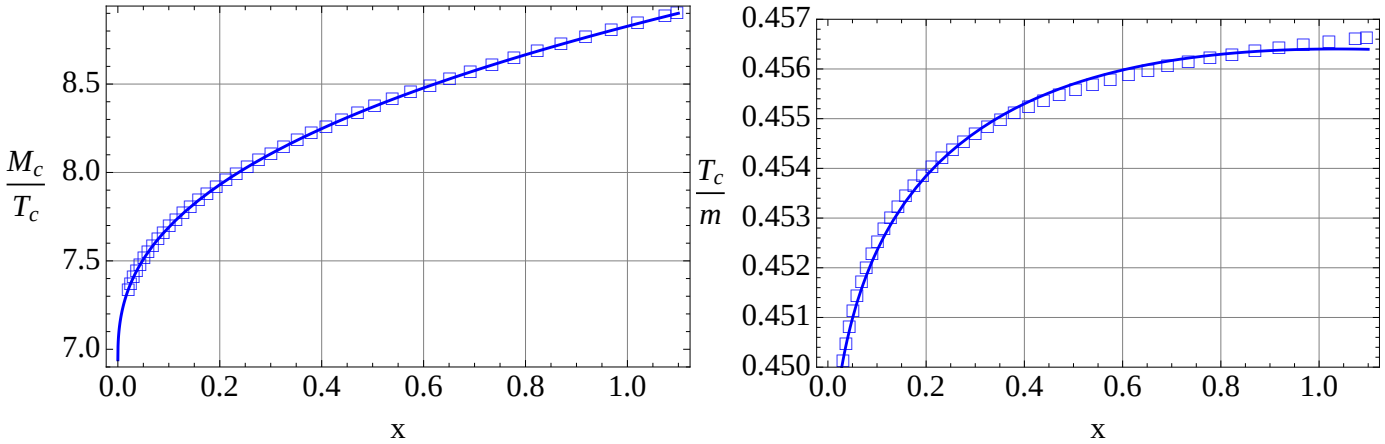
# Imaginary chemical potential $\mu = i\mu_i$




The vicinity of the tricritical point is approximately described by the mean field scaling behavior

$$\frac{M_c(\mu_i)}{T_c(\mu_i)} = \frac{M_{\text{tric.}}}{T_{\text{tric.}}} + K\left[\left(\frac{\pi}{3}\right)^2 - \left(\frac{\mu_i}{T_c}\right)^2\right]^{\frac{2}{5}}$$

[de Forcrand, Philipsen (2010); Fischer, Luecker, Pawlowski (2015)]




$$x \equiv (\pi/3)^2 + (\mu/T_c)^2 = (\pi/3)^2 - (\mu_i/T_c)^2$$

# Imaginary chemical potential $\mu = i\mu_i$

| $\mu = i\pi T/3$ | $R_2/R_1$ | $R_3/R_1$ | $Y_3$ |
|---|---|---|---|
| Matrix [1] | 1.18 | 1.28 | 1.56 |
| GZ1 [2] | 1.18 | 1.28 | 1.57 |
| GZ2 [2] | 1.11 | 1.17 | 1.58 |
| CF 1-loop [3] | 1.19 | 1.30 | 1.57 |
| CF 2-loop [2] | 1.17 | 1.27 | 1.57 |
| Lattice [4] | 1.12 | 1.20 | 1.59 |
| DSE [5] | 2.07 | 2.70 | 1.59 |

$\longrightarrow$ The $Y_3$ values are in overall very good agreement between all cases, one loop models and higher order ones.


[1] Kashiwa, Pisarski, Skokov (2012) [2] JM, Reinosa, Serreau (2017+18)

[3] Reinosa, Serreau, Tissier (2015) [4] Fromm, Langelage, Lottini, Philipsen (2012)

[5] Fischer, Luecker, Pawlowski (2015)

# Conclusion

**One-loop:**

- Heavy Quark region exhibits generic features among all one-loop models
- $T_c$ constant along the critical line, whose shape is completely fixed, independently of $\mu$
- Flavor dependence of the critical mass is independent of the gluon dynamics, as predicted by the universal quantity $Y_{N_f}$

**Higher order:**

- updated $Y_3$ values still agree with one-loop predictions
- suggests that the perturbative description of the phase diagram within the CF model is robust

**Outlook:**

- Can we describe the chiral transition in the lower left part of the Columbia plot?