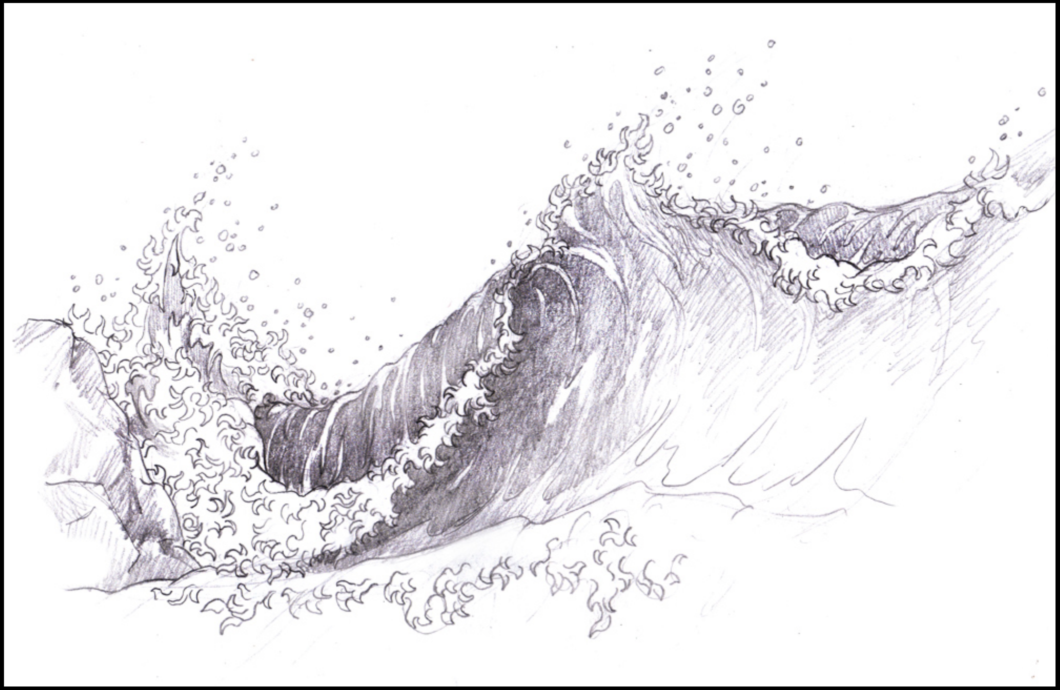
# unstable r-modes and gravitational waves



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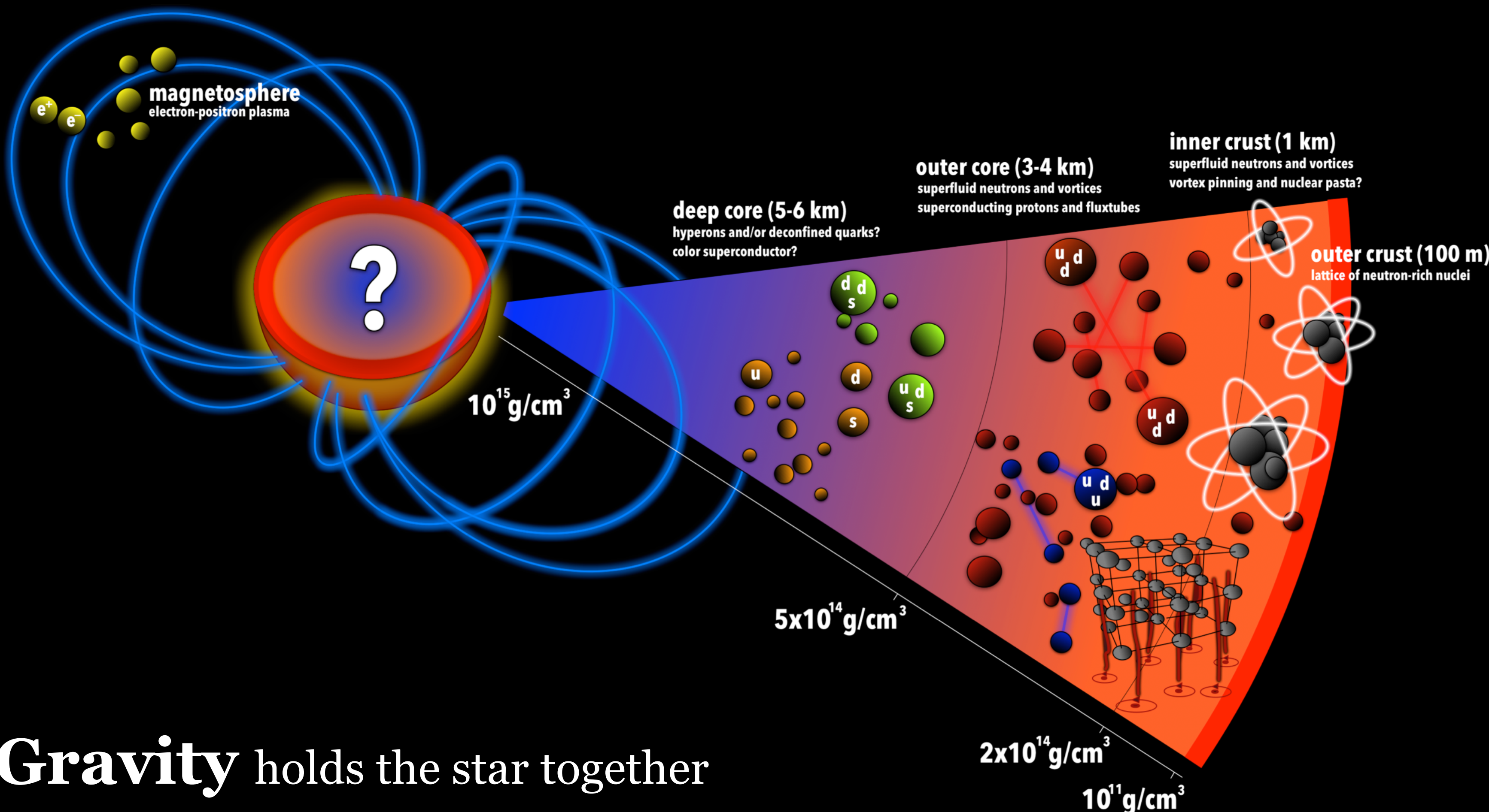


**Gravity** holds the star together

**Electromagnetism** makes pulsars pulse/magnetars flare

**Strong interaction** determines the internal composition

**Weak interaction** affects cooling and internal viscosity

# context

The (breakthrough) observation of gravitational waves from a neutron star merger (GW170817) constrains neutron star properties (tidal deformability).

NICER (on the ISS) will soon provide radius constraints for a small number of fast spinning systems.

Suppose we get to a point where we “know” the mass-radius relation: Can we go beyond this and probe the internal composition and state of matter?

# context

The (breakthrough) observation of gravitational waves from a neutron star merger (GW170817) constrains neutron star properties (tidal imprint).

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Suppose we get to a point where we "know" the mass-radius relation: Can we go beyond this and probe the internal composition and state of matter?

**In principle, yes.**

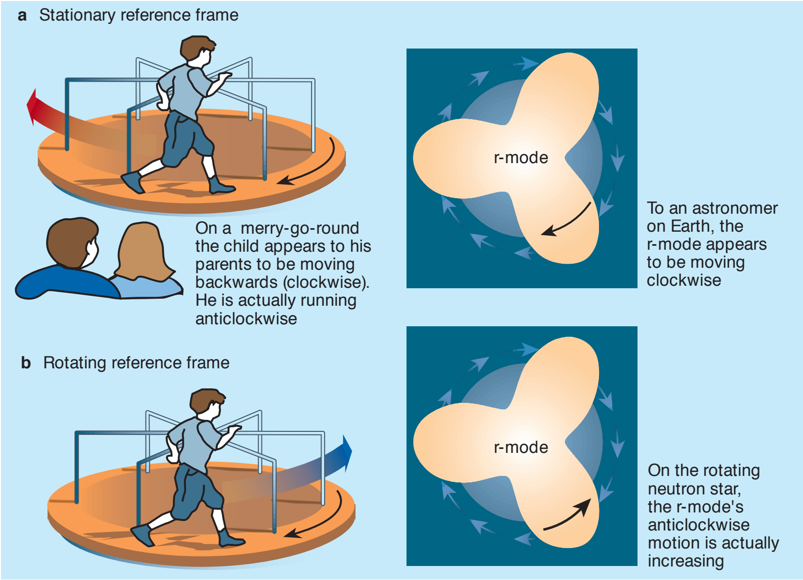
Neutron stars have rich oscillation spectra, with families of modes more or less directly associated with different core physics (cf. Helioseismology).

| | |
|---|---|
| **f-mode:** | scales with average density. |
| **p-modes:** | acoustic modes, depend on sound speed. |
| **g-modes:** | depend on thermal/composition gradients. |
| **w-modes:** | pure spacetime oscillations. |
| **r-modes:** | inertial modes restored by the Coriolis force.<br>Radiate mainly through current multipoles.<br>Driven unstable by GW emission! |

# the CFS instability

Gravitational waves may drive an instability in rotating relativistic stars.

Interesting because the mechanism may limit the spin of neutron stars at the same time as it generates a detectable signal.




**Cartoon explanation:**

A given mode is unstable if the star is losing "negative energy".

A "neutral" mode of oscillation signals the onset of instability.

The modes that are thought to be the most important are the "acoustic" f-modes, and the "Coriolis driven" r-modes.

Instability windows depend sensitively on uncertain physics. Simplest models involve shear- and bulk viscosity.

**Key point:** The problem probes non-equilibrium properties of matter.

# the r-modes

The *l=m=2* r-mode grows (due to current multipole radiation) on a timescale

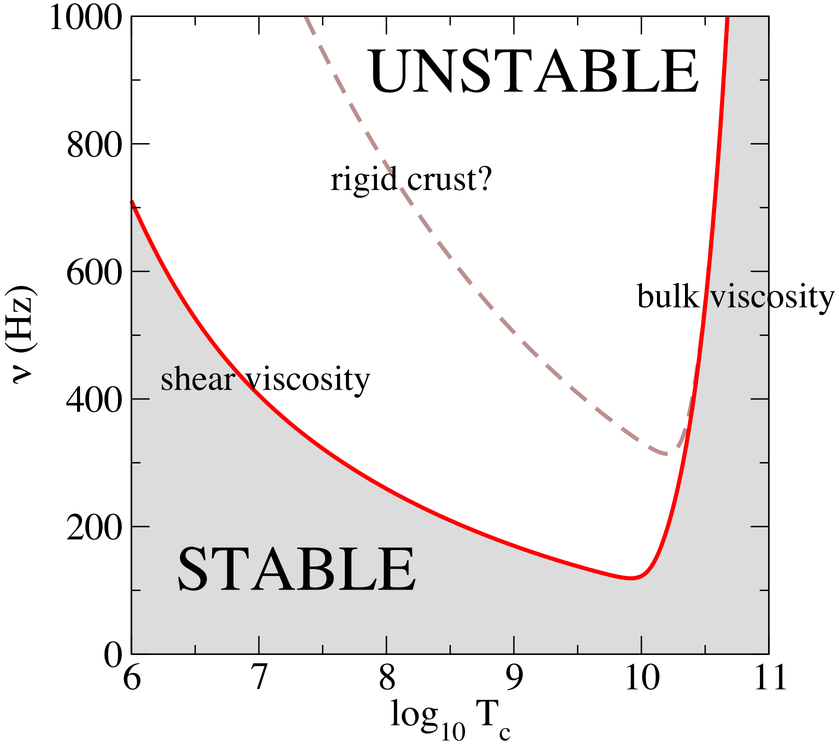
$$t_{\mathrm{gw}} \approx 50 M_{1.4}^{-1} R_{10}^{-4} P_{-3}^{6} \ \mathrm{s}$$

Viscosity may stabilise the star. At low temperature, shear viscosity is expected to dominate. For nn scattering we have

$$t_{sv} \approx 7 \times 10^{7} M_{1.4}^{-5/4} R_{10}^{23/4} T_{9}^{2} \ \mathrm{s}$$

Bulk viscosity is important at high temperatures (requires density perturbation which arises at second order in Ω)

$$t_{bv} \approx 3 \times 10^{11} M_{1.4} R_{10}^{-1} P_{-3}^{2} T_{9}^{-6} \ \mathrm{s}$$

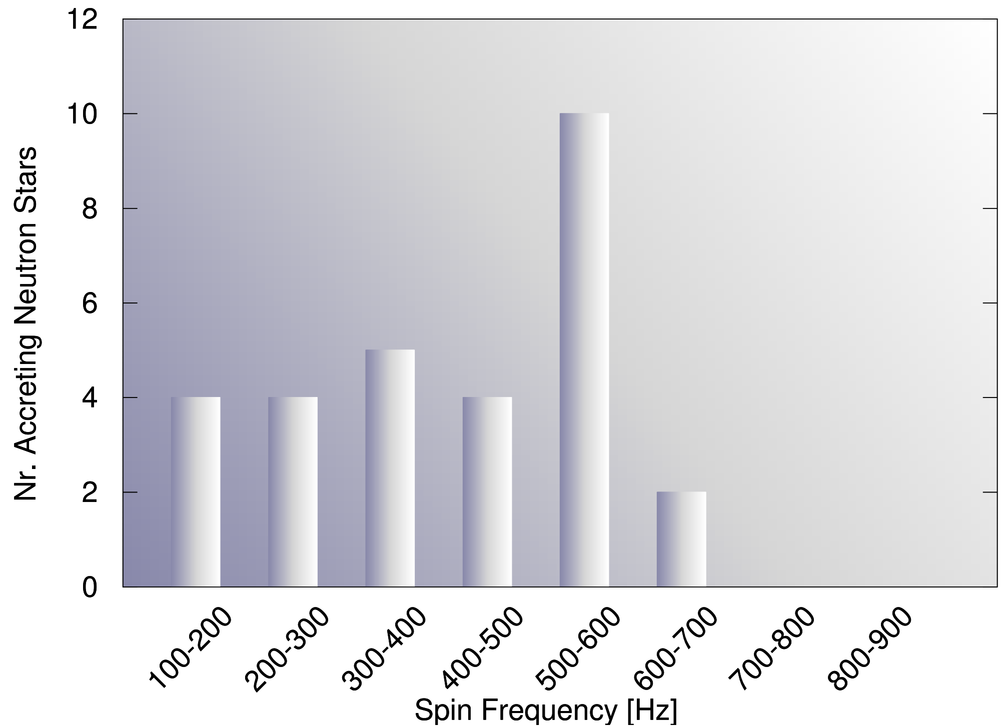



In principle, we should not find any (normal) pulsars inside the instability window.

# LMXBs

Accreting neutron stars in LMXBs are particularly “interesting”.

Observations suggest these systems rotate well below the break-up limit, so some kind of speed-limit seems to be enforced.




X-ray data for accreting systems hint at a possible pile-up of the fastest systems. This would – at least in principle – be consistent with an r-mode instability threshold.

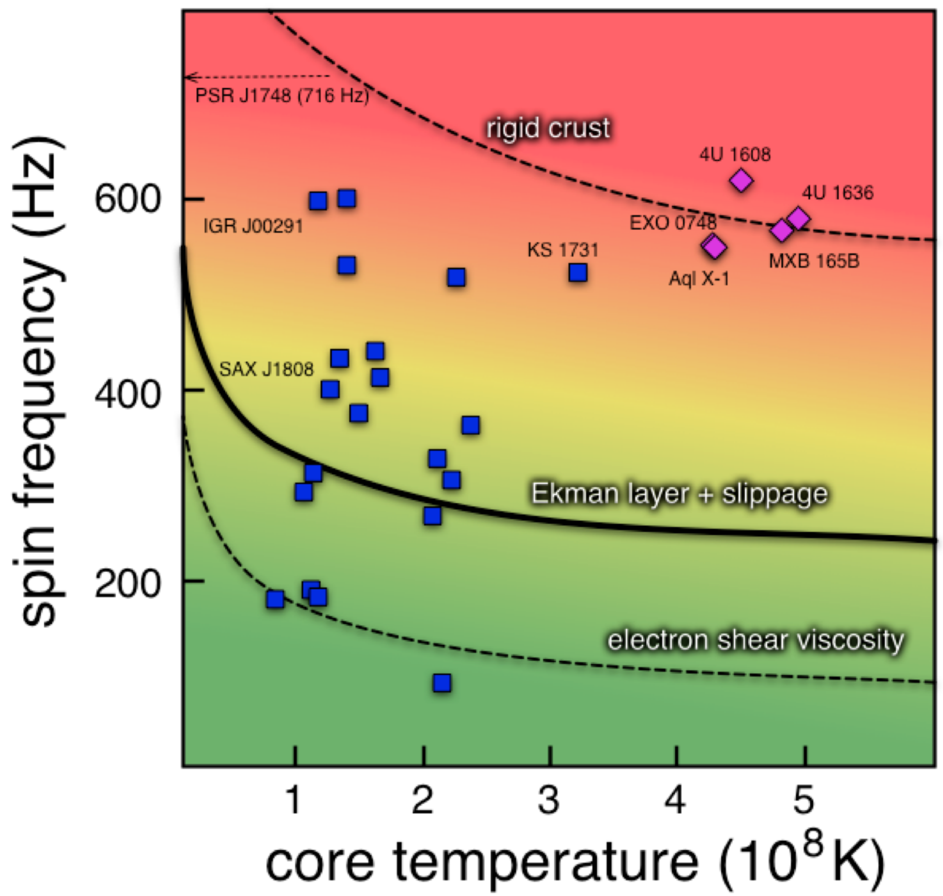
Moreover… many systems lie inside the “conservative” instability window.

Still, this is problematic:

**Rigid crust** with viscous (Ekman) boundary layer would lead to sufficient damping…

…but the crust is more like jelly, so the effect is reduced (“slippage”).

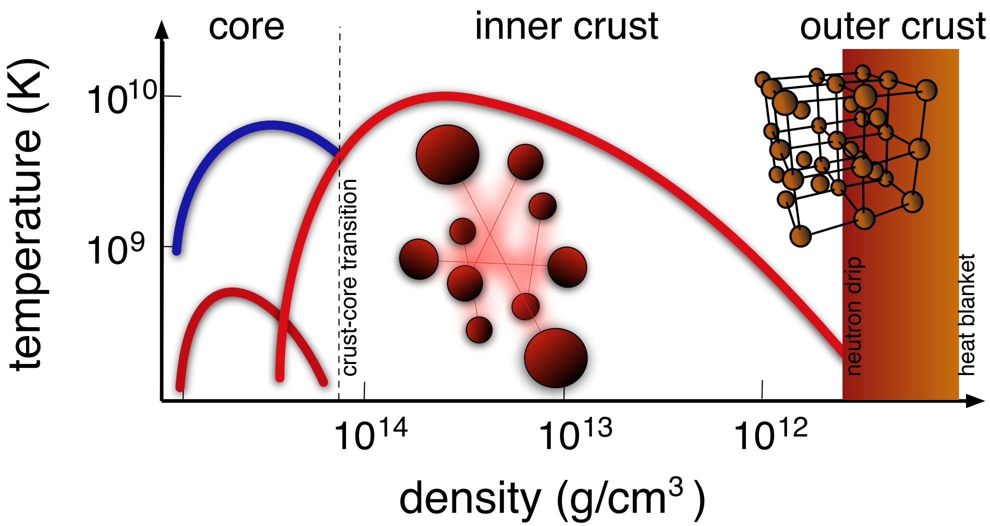
**Saturation amplitude** due to mode-coupling is too large to allow evolution far into instability region.




[Ho, NA & Haskell 2011]

# superfluids

Mature neutron stars are “cold” ($10^8$K<< $T_{Fermi}$=$10^{12}$K) so they **should be** either solid or superfluid.




**Crust** – superfluid neutrons (singlet) coexist with nuclear lattice

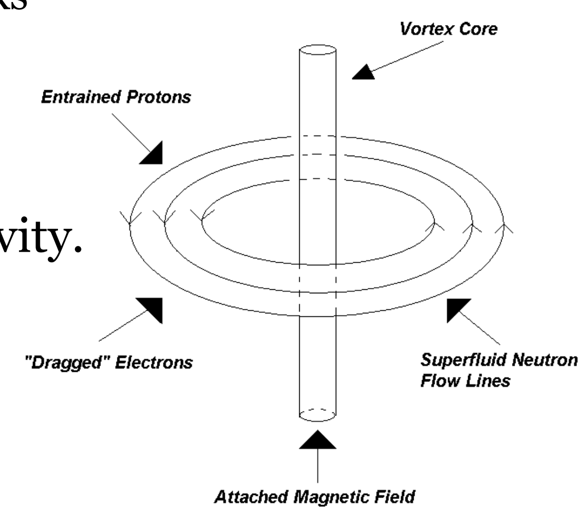
**Outer core** – superfluid neutrons (triplet) coexist with superconducting protons

**Inner core** – possible exotic phases, like colour superconducting quarks

The presence of vortices leads to “mutual friction”.

Standard form balances Magnus force to linear resistivity.

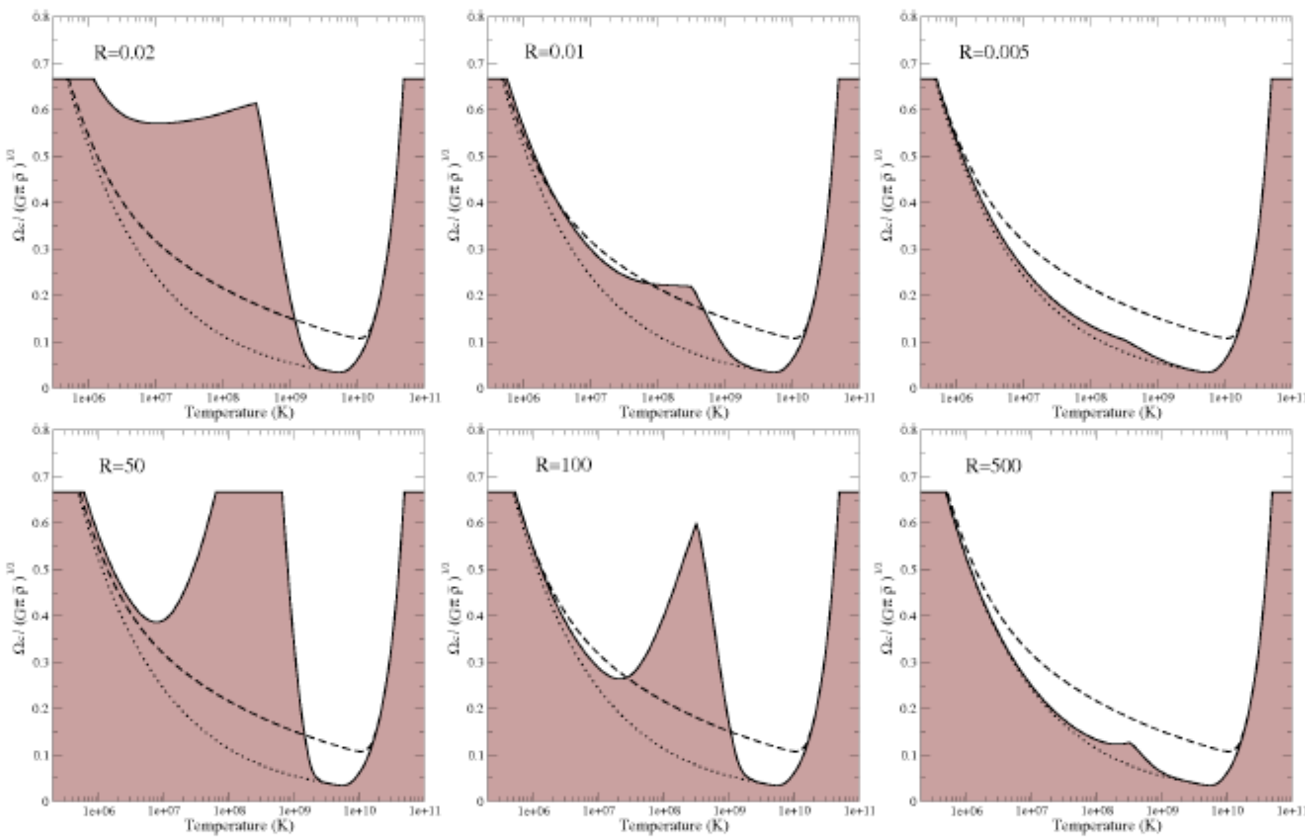
– electron scattering off vortices leads to $R<<1$
– vortex clusters lead to $R>>1$
– vortex/fluxtube interaction?



# variable windows

Mutual friction is an important mechanism in superfluid neutron star dynamics, but has little impact on the r-modes for “expected” parameters.

Would need to be stronger by a factor of about 50 to resolve the problem.



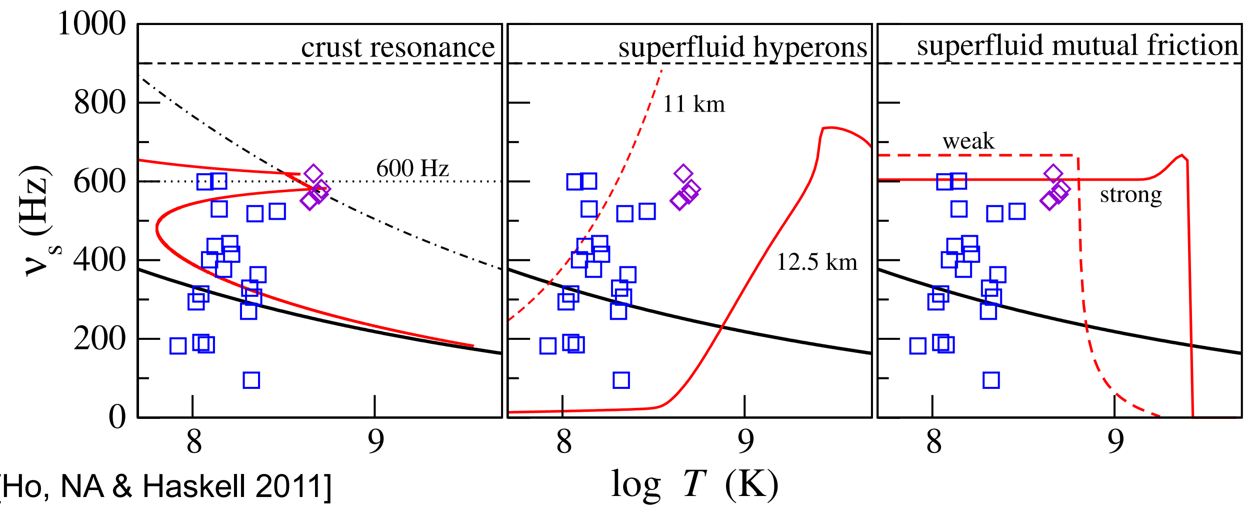

[Haskell, NA & Passamonti 2009]

# designer windows

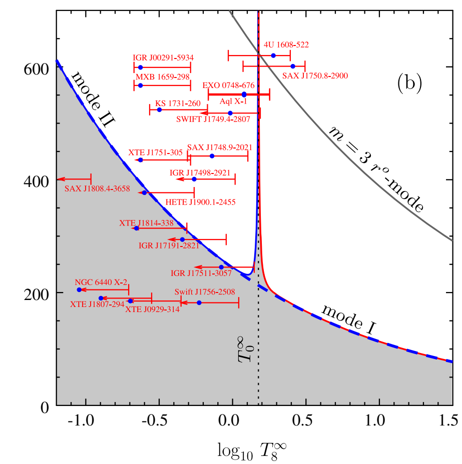
The instability window may have a very different shape due to “resonances”;

– resonant timescale with reactions (hyperon/quark bulk viscosity)

– resonance with other modes (shear modes in crust, other inertial modes in superfluid core)




[Ho, NA & Haskell 2011]




[Gusakov et al 2014]

At the end of the day, the magnetic field may provide the answer…

– slippage at crust-core interface not allowed, but there is still a boundary layer due to discontinuous derivatives (how sharp is the phase transition?)

– damping due to vortex-fluxtube interactions in outer core may be very efficient and could also provide a saturation mechanism.

# J0537-6910

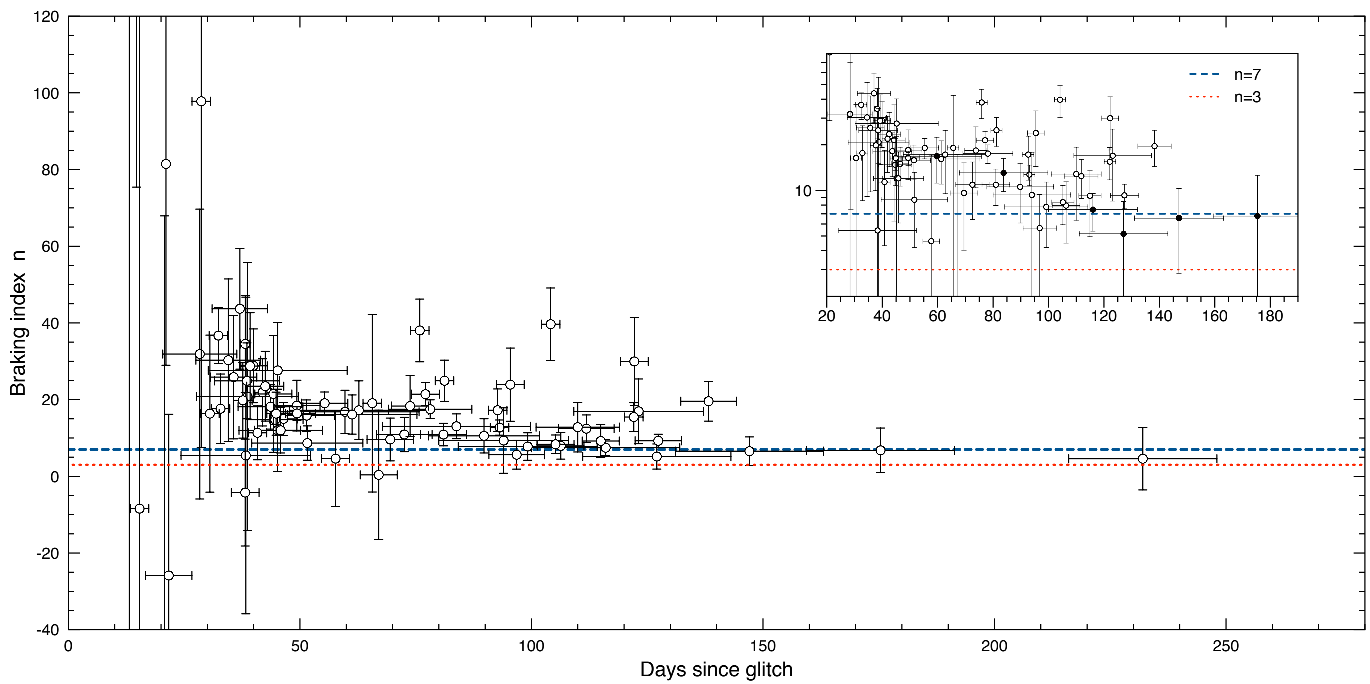
The 16ms x-ray pulsar J0537-6910 is the most energetic young neutron star.

It exhibits frequent (fairly predictable) glitches, roughly every 100 days.

Ideal system for exploring the glitch phenomenon (RXTE 1999-2011).

The overall "braking index" is negative (most likely due to the glitch "reversals"), but one may also consider the inter-glitch behaviour.

Suggests (perhaps!) a trend towards an effective n=7.



A braking index of n=7 could be explained by gravitational waves from an unstable r-mode:

$$\dot{\nu} \approx -4 \times 10^{-7} \alpha_s^2 \left(\frac{M}{1.4 M_\odot}\right) \left(\frac{R}{10\text{ km}}\right)^4 \left(\frac{\nu}{100\text{ Hz}}\right)^7 \text{ s}^{-2}$$

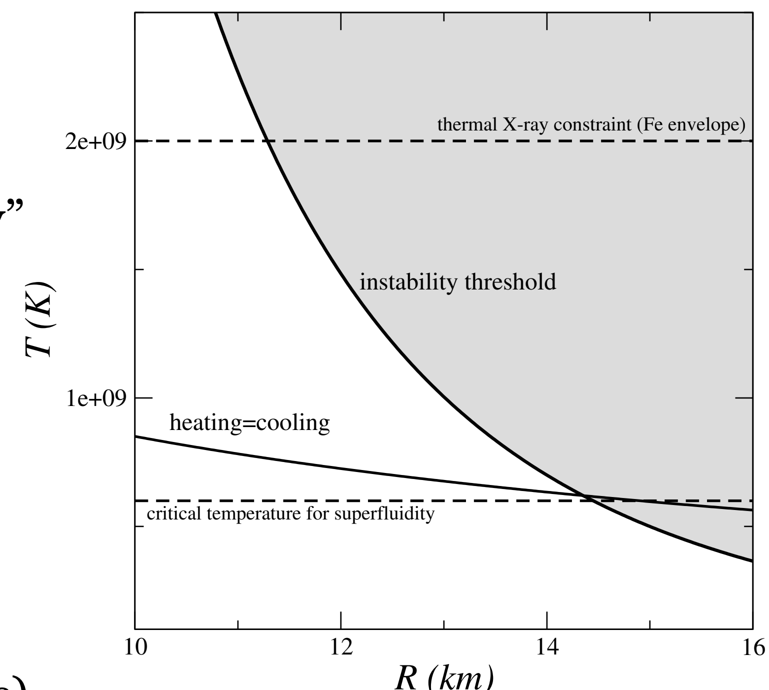
☐ Requires a fixed "saturation amplitude"

$$\alpha_s \approx 0.12 \left(\frac{M}{1.4 M_\odot}\right)^{-1/2} \left(\frac{R}{10\text{ km}}\right)^{-2}$$

This is larger than expected from "theory" (nonlinear mode coupling=messy).

☑ The spin-down age would be consistent with the supernova remnant.

☑ Fairly consistent with the largest predicted instability window (note: LMXBs become more problematic)



Still, the idea may be “testable”…

The gravitational-wave amplitude follows from the observed spin+spindown. We get

$$h_0 \approx 7.5 \times 10^{-25} \alpha_s \left( \frac{M}{1.4 M_\odot} \right) \left( \frac{R}{10 \text{ km}} \right)^3 \left( \frac{\nu}{100 \text{ Hz}} \right)^3 \left( \frac{50 \text{ kpc}}{d} \right)$$

Assuming radius in the range 10-14 km;

$$h_0 \approx 2 - 3 \times 10^{-26}$$

Rough comparison to LIGO O1 sensitivity suggests the detectors are almost at this level. Advanced LIGO at design sensitivity should “detect” this kind of signal after a 2 month integration.

But… this assumes a targeted search with a known timing solution. This would require new x-ray observations, suggesting a joint campaign with NICER.

Note: A “directed” search is a factor of 3-5 or so less sensitive so the integration time increases by a factor of 9-25 = not so easy.

# 20 years later…

Two decades after the "discovery" of the r-mode instability – and despite a fair amount of scrutiny – the r-modes remain a "viable" GW source.

This could be the mechanism that limits neutron star spin, but… the instability window depends on core physics (composition/state of matter/transport coefficients).

The key questions remain;

1. Are the r-modes unstable in a realistic neutron star model (magnetic field)?
2. Why does the growth of an unstable mode saturate and what is the achieved amplitude?
3. How does a star with an active instability actually evolve (differential rotation)?