Backreaction of the infrared modes of scalar fields on de Sitter geometry

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Motivations

Why de Sitter ?

- It is maximally symmetric
- It is relevant for inflation

For scalar field in dS,

- Large gravitational effects in the infrared (superhorizon scales)
- Infrared modes are amplified
- Interactions cannot be treated perturbatively

A. A. Starobinsky, J. Yokoyama ’94 ; C. P. Burgess et al. ’10 ; N. C. Tsamis, R. P. Woodard ’05

It is interesting to study the **backreaction** of these infrared modes fluctuations to test whether de Sitter space is stable under their effects.

A. M. Polyakov ’10, ’12 ; E. Mottola ’85 ; I. Antoniadis et al. ’86

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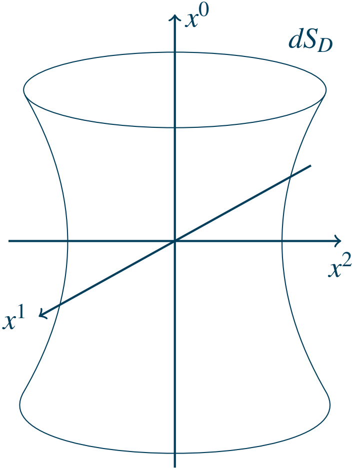
De Sitter space

We consider the **Expanding Poincaré patch**

- $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{X}^2$, $a(t) = e^{Ht}$ with constant H.
- Conformal time, $\mathrm{d}\eta = \frac{\mathrm{d}t}{a(t)}$, $\mathrm{d}s^2 = -a^2(\eta)\left(\mathrm{d}\eta^2 + \mathrm{d}\vec{X}^2\right)$ $\rightarrow$ spatially homogeneous.
- Lemaitre-Painlevé-Gullstrand $\mathrm{d}s^2 = -(1-\vec{x}^2)\mathrm{d}t^2 - 2\vec{x}\cdot\mathrm{d}\vec{x}\mathrm{d}t + \mathrm{d}\vec{x}^2$, $\rightarrow$ stationary.



Free scalar field

With the action

$$S = \int \mathrm{d}^D x \sqrt{-g} \left(\frac{1}{2} \varphi \Box \varphi - \frac{m^2}{2} \varphi^2 \right)$$

We get the Klein Gordon equation $\left(-\Box + m^2\right)\varphi = 0$ where

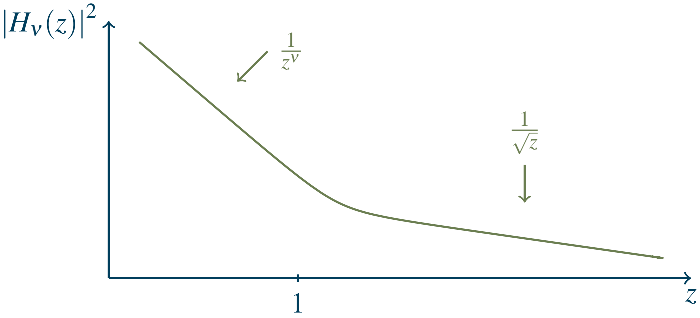
$$\Box = \frac{1}{a(\eta)} \left(-\partial_\eta^2 + \frac{d-1}{\eta} \partial_\eta + \vec{\partial}_X^2 \right)$$

It gives for the mode decomposition of φ

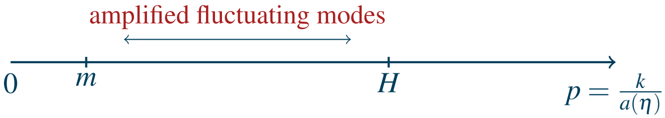
$$\varphi(\eta, \vec{X}) \sim \int \frac{\mathrm{d}^d k}{(2\pi)^d} \left(e^{i\vec{k}\cdot\vec{X}} H_\nu \left(\frac{k}{a(\eta)} \right) a_k + \mathrm{h.c.} \right)$$

with $\nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}$

Free scalar field 2



In the case of light scalar fields $m \ll H$,



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Semiclassical approach

The theory is described by an **effective action** $\Gamma[\varphi, g]$, the Legendre transform of $\mathscr{W}[j, g]$ defined as

$$e^{i\mathscr{W}[j,g]} = \int \mathscr{D}\hat{\varphi} e^{iS[\hat{\varphi},g]+i\int j\hat{\varphi}}, \quad \Gamma[\varphi, g] = \mathscr{W}[j,g] - j \cdot \varphi$$

with $g_{\mu\nu}$ the background metric.
The action S will be typically an $O(N)$ theory with φ^4 interaction.

Non perturbative renormalization group

A. Kaya '13 ; J. Serreau '13 ; M. Guilleux, J. Serreau '15

Add a regulator

$$i\Delta S_\kappa[\hat{\varphi}, g] = i\int_{x,y} R_\kappa(x,y)\hat{\varphi}(x)\hat{\varphi}(y).$$

And define an effective action which interpolates between S ($\kappa \to \infty$) and Γ ($\kappa \to 0$)

$$\Gamma_\kappa[\varphi, g] = \mathscr{W}_\kappa[j, g] - j\cdot\varphi - \Delta S_\kappa[\varphi, g]$$

The **physical values** for g and φ are simultaneously determined at each scale κ through

$$\frac{\delta\Gamma_\kappa}{\delta\varphi} = 0, \quad \frac{\delta\Gamma_\kappa}{\delta g^{\mu\nu}} = 0$$

which we evaluate at constant values of φ.

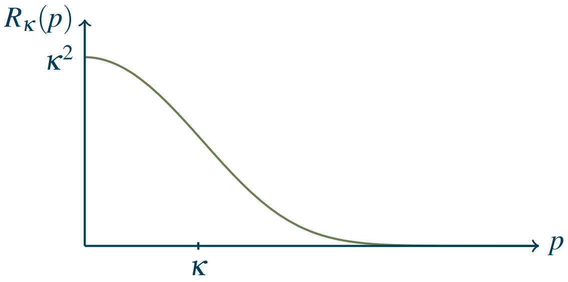
Non perturbative renormalization group 2

We want to solve the flow of Γ_κ : it obeys the **Wetterich equation**

$$\dot{\Gamma}_\kappa = \frac{1}{2}\,\mathrm{tr}\,\dot{R}_\kappa(\Gamma_\kappa^{(2)} + R_\kappa)^{-1}.$$

C. Wetterich '93

This equation is regulated both in the infrared and the ultraviolet.



De Sitter Background

$$\frac{\delta\Gamma_\kappa}{\delta g^{\mu\nu}} = 0 \qquad \Rightarrow \qquad G^\kappa_{\mu\nu} = \langle T^\kappa_{\mu\nu} \rangle$$

- Problem : the regulator only preserve a subgroup of de Sitter symmetries. We have no certitude that de Sitter will be solution for all κ.
- Solution :
  - We still preserve a large subgroup (FLRW and stationarity, the affine subgroup).
  - The flow of the metric is dominated by the flow of the effective potential.
  - This flow is insensitive to the missing isometries : it is computed with good accuracy in our setup.
  - Taking a de Sitter metric as a solution should be a good approximation.

The flow of the metric is reduced to the **flow of its Hubble constant**.

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Local potential approximation (LPA)

We compute the flow equation in the LPA by taking the ansatz

$$\Gamma_{\tilde{\kappa}}[\varphi,h] = -\int \mathrm{d}^D x \sqrt{-\tilde{g}} \left(\frac{Z(h)}{2} \tilde{g}_{\mu\nu} \partial^\mu \varphi_a \partial^\nu \varphi_a + N \tilde{U}_{\tilde{\kappa}}(\varphi_a, h) \right)$$

with $\tilde{g}$ the dS metric with $h = 1$. The h factors are hidden in Z and $\tilde{U}$.

It amounts to **discard higher derivative interactions**, which are expected to be subdominant in the infrared regime ($\kappa \ll h$). For constant φ, we compute the flow of U, the effective potential.

The LPA give the **exact flow** for the effective potential.

Flow equation

In LPG coordinates, with p the physical momentum,

$$R^{ab}_{\tilde{\kappa}}(x,x';h) = \delta^{ab}\delta(t-t')r_{\tilde{\kappa}}(\vec{x}-\vec{x}',h)$$

$$r_{\tilde{\kappa}}(p,h) = Z(h)(\tilde{\kappa}^2 - p^2)\theta(\tilde{\kappa}^2 - p^2).$$

Then

$$N\dot{\tilde{U}}_{\tilde{\kappa}} = \beta(m^2_{l,\tilde{\kappa}}, \tilde{\kappa}) + (N-1)\beta(m^2_{t,\tilde{\kappa}}, \tilde{\kappa}).$$

With $\kappa = h\tilde{\kappa}$, under the **small curvature** of the potential, in the **infrared** regime,

$$h^D\beta(m^2,\kappa) = \frac{h^D}{\Omega_{D+1}}\frac{\kappa^2}{\kappa^2+m^2}$$

M. Guilleux, J. Serreau ’15

Zero dimensional theory

The solution is a zero dimensional theory

$$e^{h^{-D}\Omega_{D+1}\mathscr{W}_\kappa(j,h)} = \int \mathrm{d}^N\hat{\varphi}\, e^{-h^{-D}\Omega_{D+1}\left(V_{in}(\hat{\varphi},h)+\frac{\kappa^2}{2}\hat{\varphi}^2-j\cdot\hat{\varphi}\right)}$$

with the initial conditions V_{in} that match the microscopic potential,

- It coincides with the equilibrium probability distribution in the stochastic formalism

  A. A. Starobinsky, J. Yokoyama ’94
- It is the effective theory for the scalar field averaged over a Hubble patch at constant values of the field

Flow of the physical quantities

Taking as initial conditions

$$V_{in}(\hat{\varphi}, h) = N\left(\alpha - \frac{\beta}{2}h^2\right) + \frac{m^2 + \xi h^2}{2}\hat{\varphi}_a^2 + \frac{\lambda}{8N}(\hat{\varphi}_a^2)^2.$$

where α is the cosmological constant and βh^2 the Einstein-Hilbert term, evaluated on de Sitter geometry and properly rescaled.

The minimization of the effective action gives

$$\begin{cases} \varphi_\kappa = \langle \hat{\varphi} \rangle \\ h_\kappa^2 = \dfrac{4N\alpha + 2\left(m^2 + \kappa^2\right)\langle \hat{\varphi}^2 \rangle + \frac{\lambda}{2N}\langle \hat{\varphi}^4 \rangle - 2\kappa^2 \varphi^2}{N\beta - \xi \langle \hat{\varphi}^2 \rangle} \end{cases}$$

The expectation values are to be computed in the zero dimensional theory.

Approximations

A summary of our approximations so far :

- Semiclassical regime : $\dfrac{h_\kappa^2}{\beta} \ll 1$
- Infrared regime ($\rightarrow$ LPA) : $\kappa \ll h_\kappa$
- Small curvature : $\dfrac{m_{t/l,\kappa}^2}{h_\kappa^2} \ll 1$

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Gaussian theory

For a Gaussian ($\lambda = 0$) theory, $\varphi_\kappa = 0$ and

$$4\alpha - \beta h_\kappa^2 + \frac{2h_\kappa^4}{\Omega} - \frac{\xi h_\kappa^6}{\Omega\mu(h_\kappa)^2} = 0$$

with $\mu(h)^2 = m^2 + \xi h^2 + \kappa^2$.

- For **minimally coupled** fields ($\xi = 0$), h_κ has **no flow**, as expected.
- For **non zero** ξ, depending on its sign, the Hubble constant is renormalized either **positively** ($\xi < 0$) or **negatively** ($\xi > 0$).

Large N case

In the large N regime, we can solve everything analytically while keeping the main effects.
With

$$\rho = \frac{\varphi_a^2}{2N} \quad \text{and} \quad \mu^2 = m^2 + \xi h^2 + \kappa^2$$

$$U(\rho, h) + \rho\kappa^2 = 4\alpha - \beta h^2 + \frac{\bar{z}^2 - \mu^4}{2\lambda} + \frac{h^4}{2\Omega} \log \frac{\bar{z}\Omega}{2\pi e h^4}$$

$$\bar{z} = m_{t,\kappa}^2 + \kappa^2 = \frac{\mu^2 + \lambda\rho}{2} + \sqrt{\left(\frac{\mu^2 + \lambda\rho}{2}\right)^2 + \frac{\lambda h^4}{2\Omega}}$$

We wish to solve

$$\partial_\rho U_\kappa(\rho, h)\Big|_{\rho_\kappa, h_\kappa} = 0 \quad \text{and} \quad \partial_h \frac{U_\kappa(\rho, h)}{h^4}\Big|_{\rho_\kappa, h_\kappa} = 0$$

Large N : symmetric case

When $\rho = 0$, with $\mu^2 = m^2 + \xi h^2 + \kappa^2$,

$$m_{t,\kappa}^2 + \kappa^2 = \frac{\mu^2}{2} + \sqrt{\frac{\mu^4}{4} + \frac{\lambda h^4}{2\Omega}}, \quad 4\alpha - \beta h_\kappa^2 + \frac{h_\kappa^4}{\Omega}\left(1 + \frac{m^2 + \kappa^2}{m_{t,\kappa}^2 + \kappa^2}\right) = 0$$

We have **finite asymptotic values**

$$h_\infty^2 = \frac{\beta\Omega}{4}\left(1 - \sqrt{1 - \frac{32\alpha}{\beta^2\Omega}}\right) \approx \frac{4\alpha}{\beta} + \frac{2}{\beta\Omega}\left(\frac{4\alpha}{\beta}\right)^2 + \cdots$$

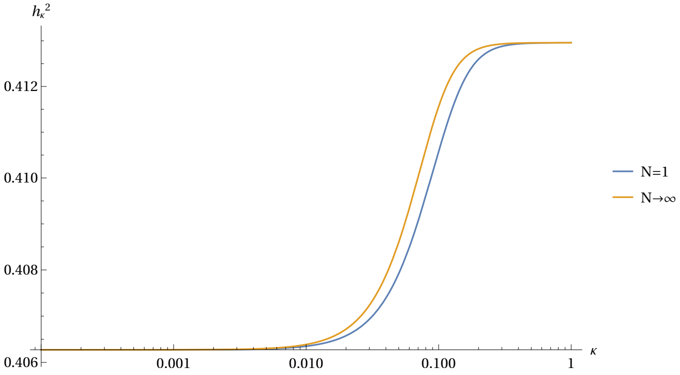
and for example for $m = \xi = 0$

$$h_0^2 = \frac{\beta\Omega}{2}\left(1 - \sqrt{1 - \frac{16\alpha}{\beta^2\Omega}}\right) \approx \frac{4\alpha}{\beta} + \frac{1}{\beta\Omega}\left(\frac{4\alpha}{\beta}\right)^2 + \cdots$$

Large N : interacting massless case

When $m = \xi = 0$, a **mass is generated**

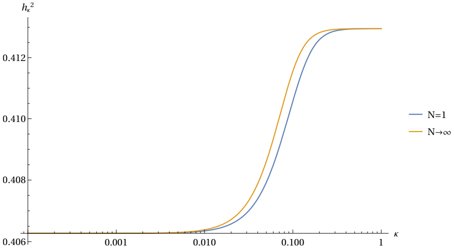
$$m_{t,\kappa}^2 = -\frac{\kappa^2}{2} + \sqrt{\frac{\kappa^4}{4} + \frac{\lambda h^4}{2\Omega}}, \quad 4\alpha - \beta h_\kappa^2 + \frac{h_\kappa^4}{\Omega}\left(1 + \frac{\kappa^2}{m_{t,\kappa}^2 + \kappa^2}\right) = 0$$



Large N : interacting massless case

$$h_\infty^2 = \frac{\beta\Omega}{4}\left(1 - \sqrt{1 - \frac{32\alpha}{\beta^2\Omega}}\right)$$

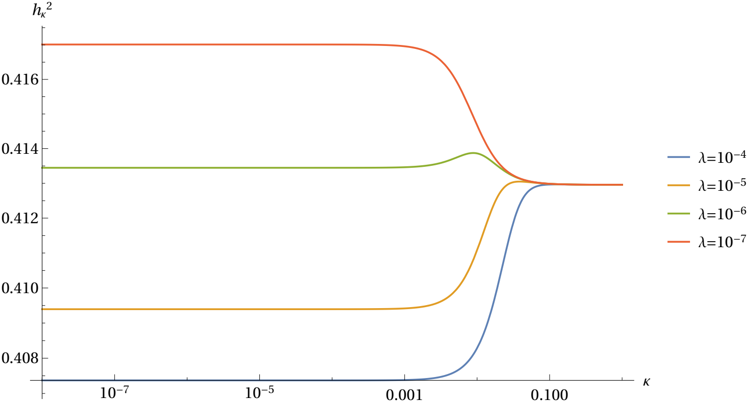
$$h_0^2 = \frac{\beta\Omega}{2}\left(1 - \sqrt{1 - \frac{16\alpha}{\beta^2\Omega}}\right)$$



- The superhorizon modes of the massless scalar fields are greatly enhanced, drawing energy from the gravitational field
- The dynamical generation of a mass screens this effect, leading to a finite renormalization of the Hubble constant
- the asymptotic values can be computed exactly and only depend on α and β

Large N : symmetric phase

Interplay between λ and $\xi < 0$



Large N : (would-be) broken phase

There are initial conditions for which a non zero ρ_κ exists at the beginning of the flow. Note that the **symmetry is always restored** at a finite value of κ.

Now $m^2_{t,\kappa} = 0$, and we get that h_κ has no flow

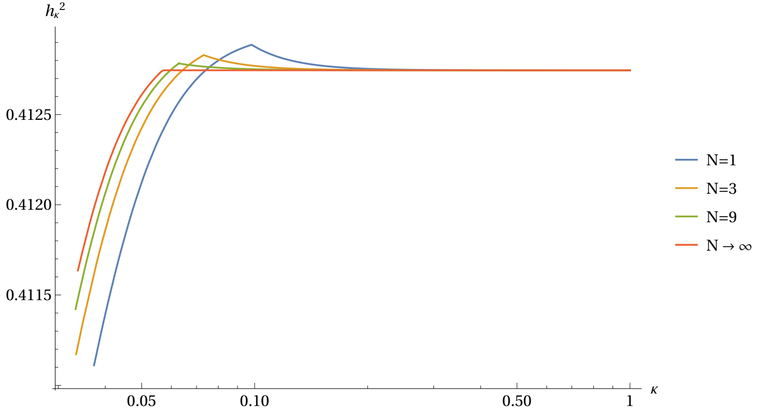
$$\rho_\kappa = -\frac{h_\kappa^4}{2\Omega\kappa^2} - \frac{m^2 + \xi h_\kappa^2}{\lambda}$$

$$4\alpha - \beta h_\kappa^2 - \frac{2m^2\left(m^2 + \xi h_\kappa^2\right)}{\lambda} + \frac{2h_\kappa^4}{\Omega} = 0$$

The Goldstone bosons do not renormalize h_κ!

Large N : (would-be) broken phase

The flow in the (would-be) broken phase is smaller and smaller for increasing N.
The flow of h_κ^2 around the symmetry restoration is as follows



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Conclusion

The backreaction we studied is influenced by several phenomena :

- The mass generation screens the renormalization of the Hubble parameter
- Non minimal coupling between the scalar fields and gravitational field has a non trivial effect on the flow
- Goldstone modes do not contribute

Perspectives :

- Work in a more general FLRW spacetime