PERTURBATIVE DYNAMICS of MASSIVE GLUONS

Julien Serreau APC Univ. Paris Diderot

- Urko Reinosa
- Marcela Peláez
- Matthieu Tissier
- Nicolás Wschebor

OUTLOOK

- Motivation : the Curci-Ferrari (CF) model

- Perturbative (one-loop) analysis in the infrared safe renormalization scheme

- Application to QCD : spont. chiral sym. breaking ($S\chi SB$)

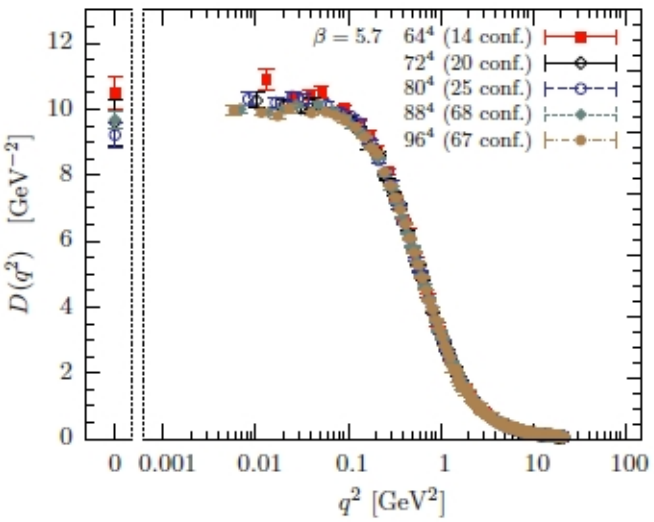
- Conclusions and perspectives

Motivations : general frame

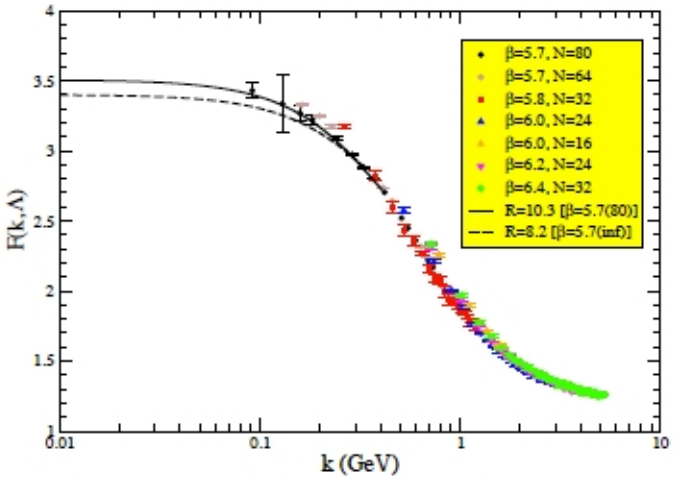
Yang-Mills Green's function (gluon and ghost fields) are the basic ingredients of continuum approaches

- Bound states
- Nonzero T and μ
- etc.

Lattice results (Landau gauge SU(3), d=4)



Massive gluon



Massless ghost

The massive FP (mFP) theory (I)

IR momenta are not accessible from perturbation theory based on the FP Lag. (Landau pole) ➡ Nonperturbative continuum approaches

- DSE [..., Alkofer, Hauck, von Smekal ('97), ...]
- FRG [Ellwanger, Hirsch, Weber ('96) ...]
- HF [Fevchter, Reinhardt ('04) ...]

BUT Not exactly the FP Lagrangian

UV/IR regulator breaks the BRST symmetry which generates quadratic div.

➡ accounted for by suitable subtraction / initial cond's.

Equivalent to a mass (counter) term

➡ Range of parameters for which the mFP provides a viable realization of YM ?

The mFP theory (II)

The FP quantization procedure is not valid / complete : Gribov ambiguities

- Minimal Landau gauge on the lattice (no continuum formulation known)
- (refined) Gribov-Zwanziger approach [Gribov ('78), Zwanziger ('89)...] restrict the path integral to the 1st Gribov regio
- Average of copies [Serreau, Tissier ('12)]

⇒ BRST is (softly) broken

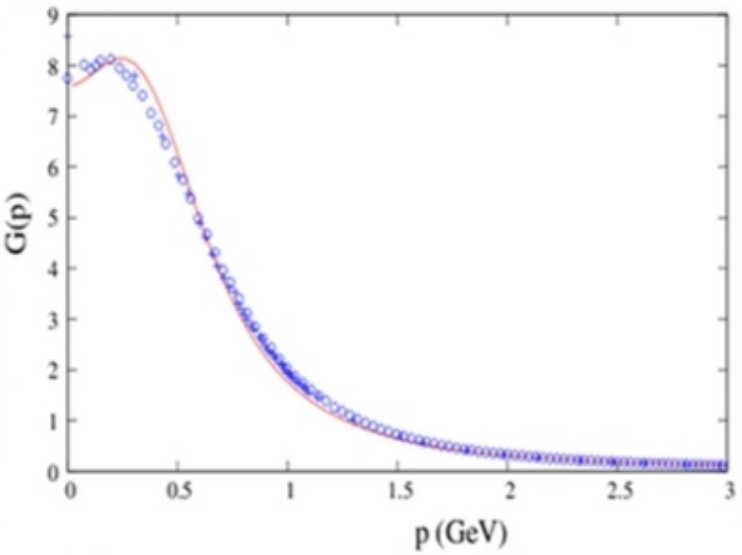
Effective theory : the minimal deformation of the FP theory with the correct UV behavior is mFP

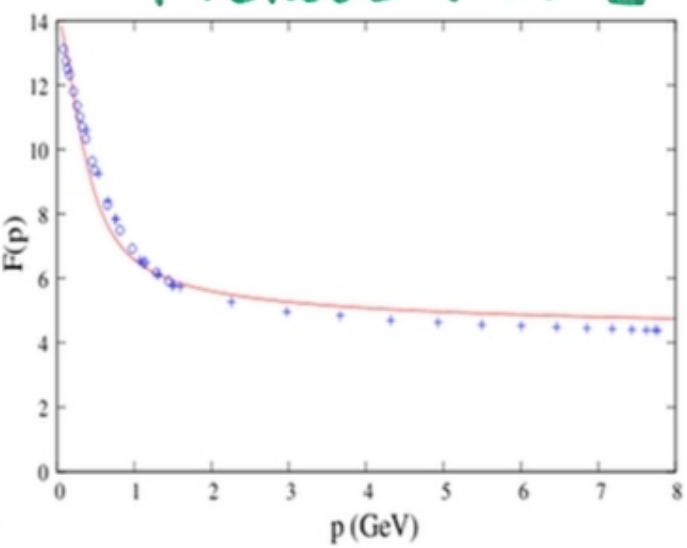
mFP : review

$$\mathcal{L} = \underbrace{\frac{1}{4}F^2 + \partial_\mu \bar{c} D_\mu c + i h \partial_\mu A_\mu}_{\text{FP (Landau gauge)}} + \underbrace{\frac{m^2}{2} A^2}_{\text{mFP}}$$

particular case of the Curci-Ferrari Lagrangian
[Curci, Ferrari ('76)]

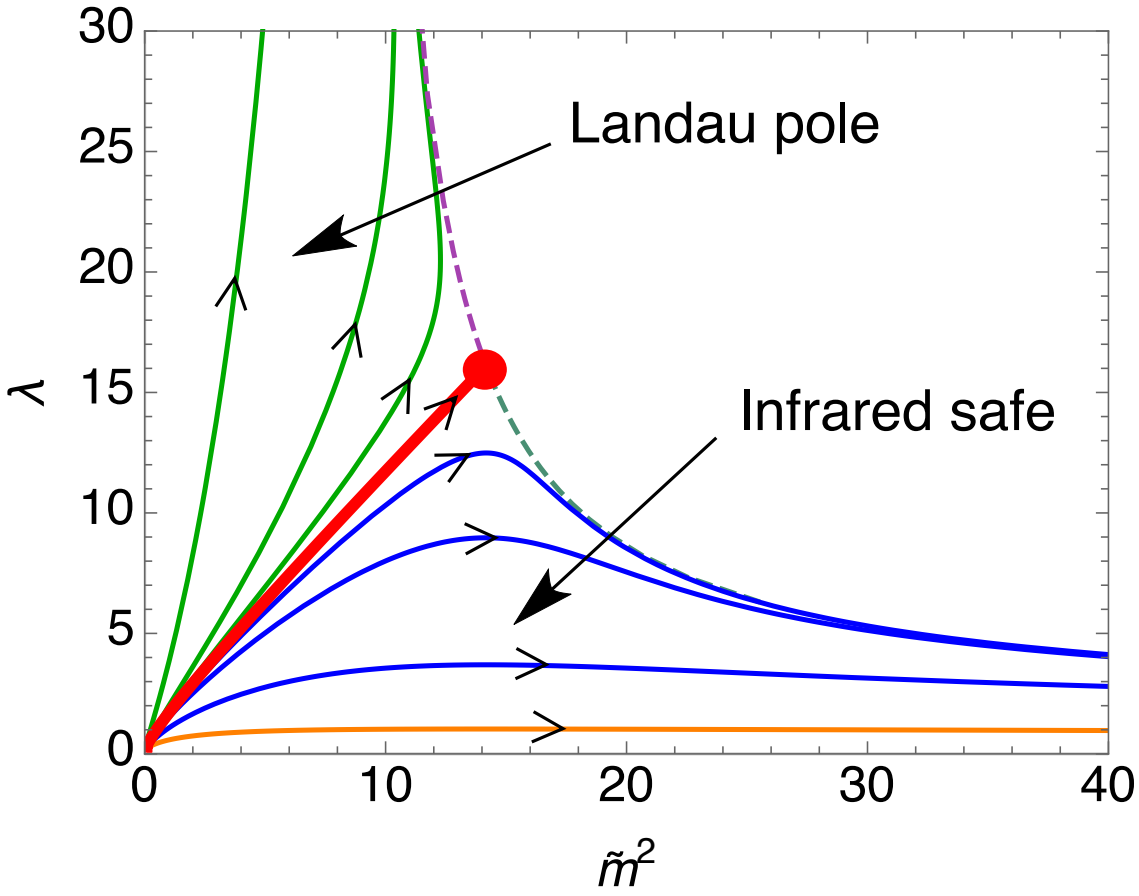
- perturbatively renormalizable ($d \leq 4$)
- IR-safe RG trajectories (no Landau pole)
  [Tissier, Wschebor ('11)]
- Accurate description of vacuum YM correlators
  [Tissier, Wschebor ('10) + Peláez ('13)]



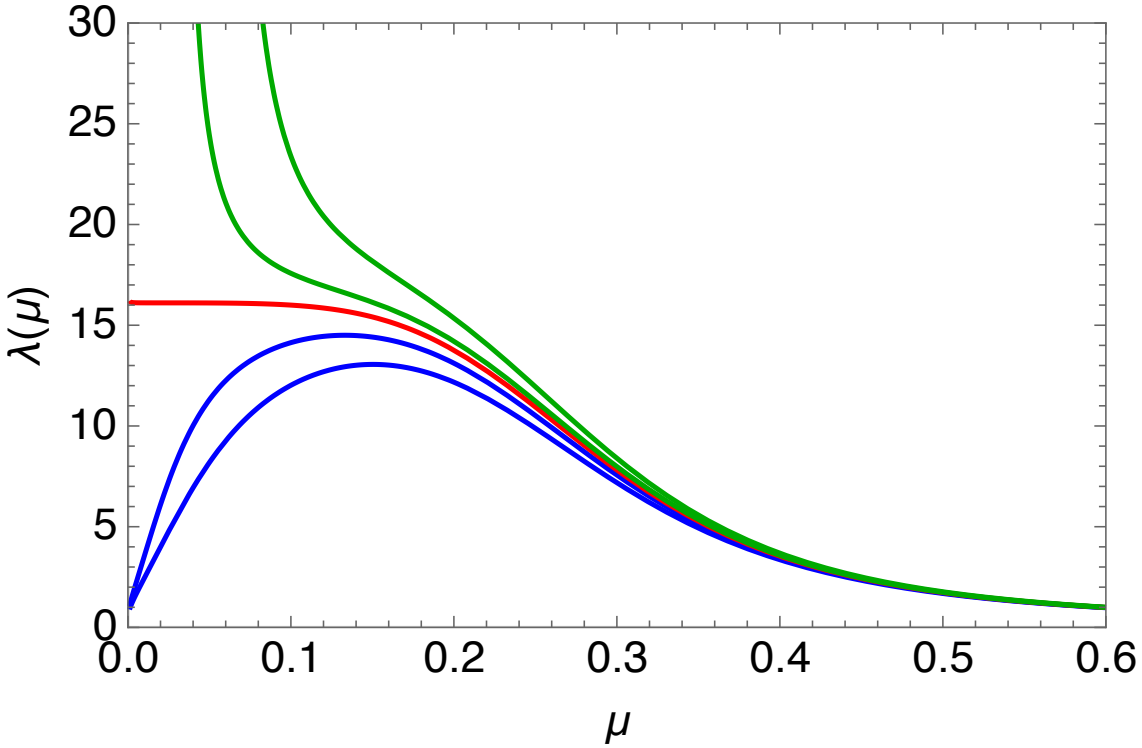


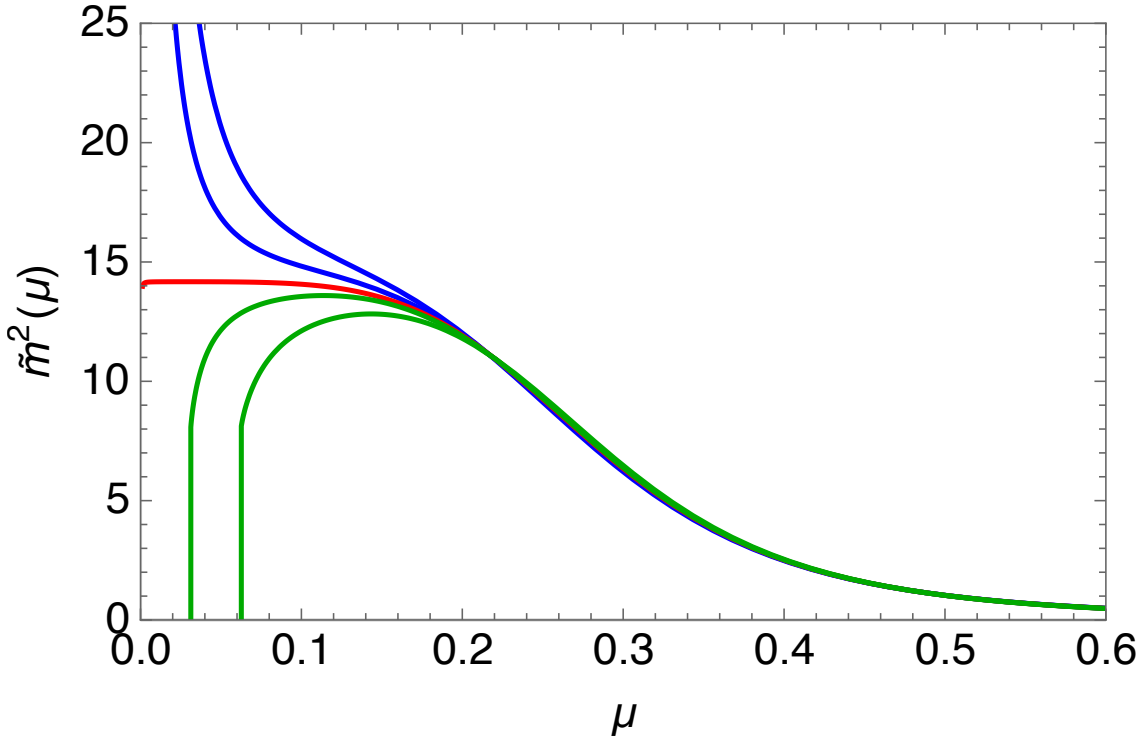
- Successful nonzero T, μ applications
  [Reinosa et al ('14) ...]

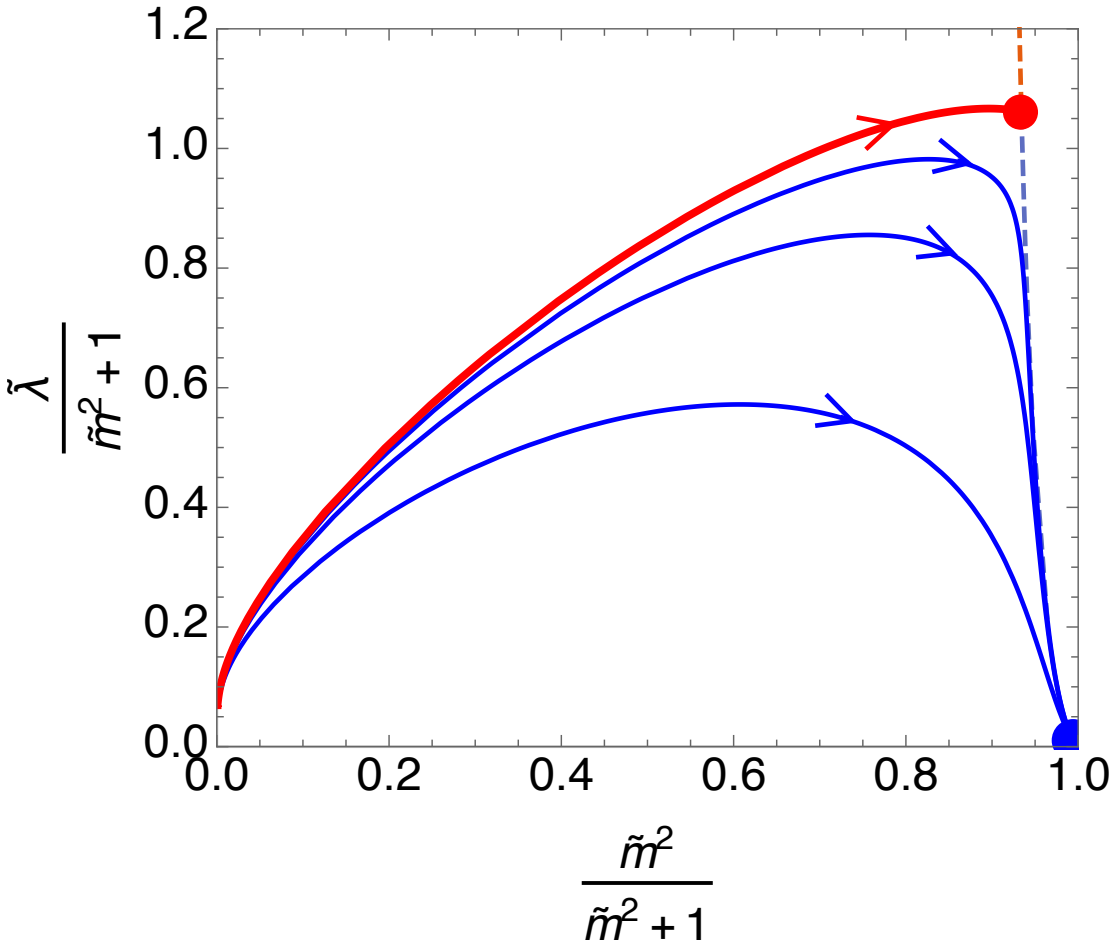
Phase diagram



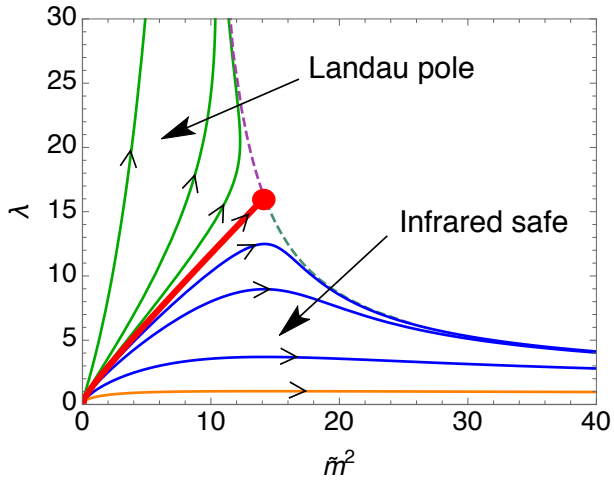
[Reinosa, Serreau, Tissier, Wschebor, PRD 96 (2017)]

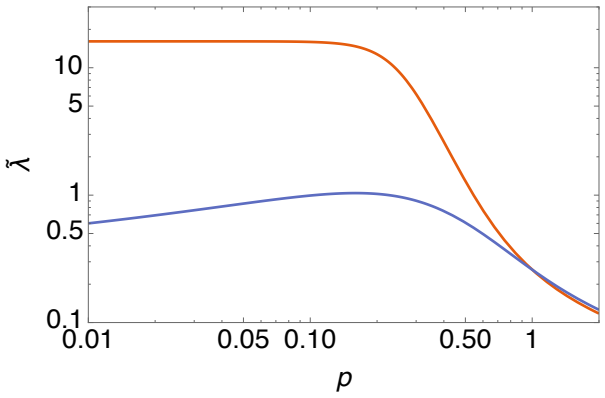

30
25
20
15
10
5
0
λ(μ)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
μ

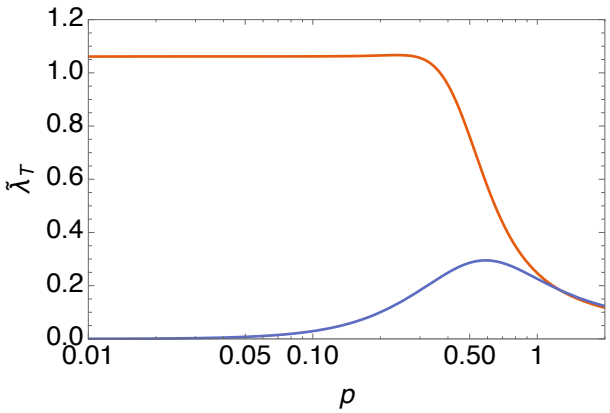

25
20
15
10
5
0
m̃²(μ)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
μ

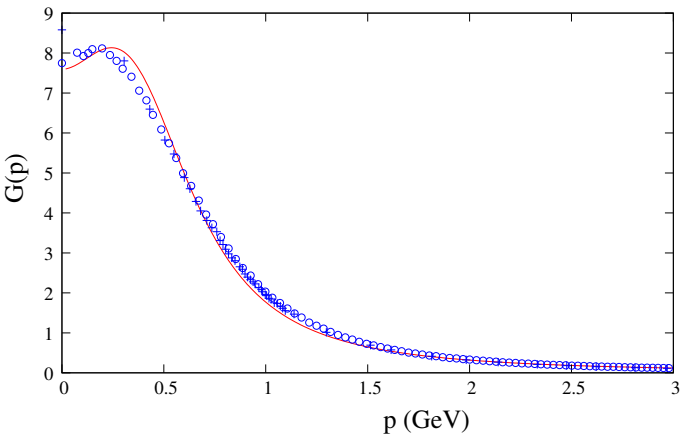

1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
$\frac{\tilde{\lambda}}{\tilde{m}^2+1}$
$\frac{\tilde{m}^2}{\tilde{m}^2+1}$

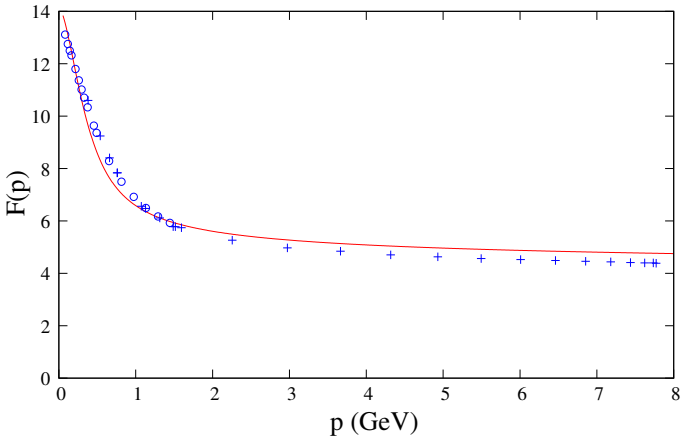
Lattice results: moderate coupling!











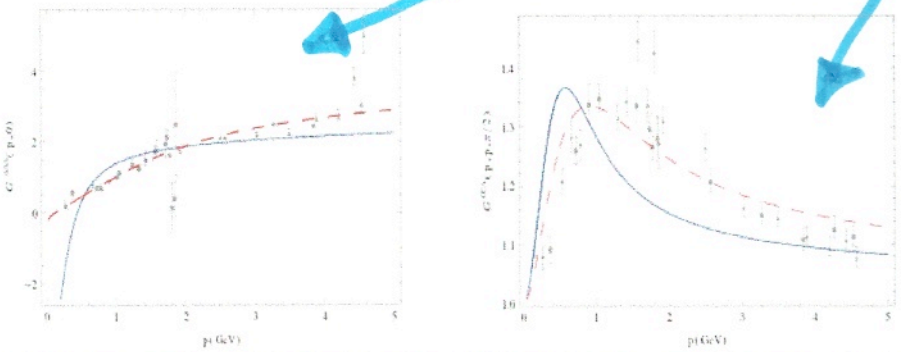
[Tissier, Wschebor PRD84 (2011) 045018]

Data from [Bogolubsky *et al.*, PLB 676 (2009) 69 ; Dudal, Oliveira, Vandersickel, PRD 81 (2010) 074505]

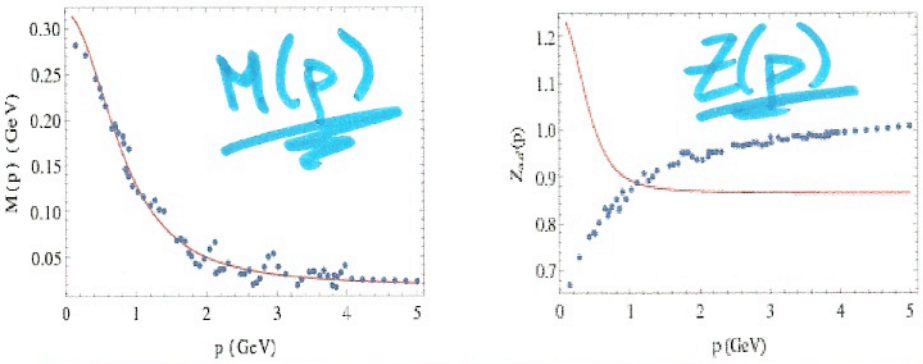
Massive model : more

higher correlators : $\langle AAA \rangle$, $\langle c\bar{c}A \rangle$

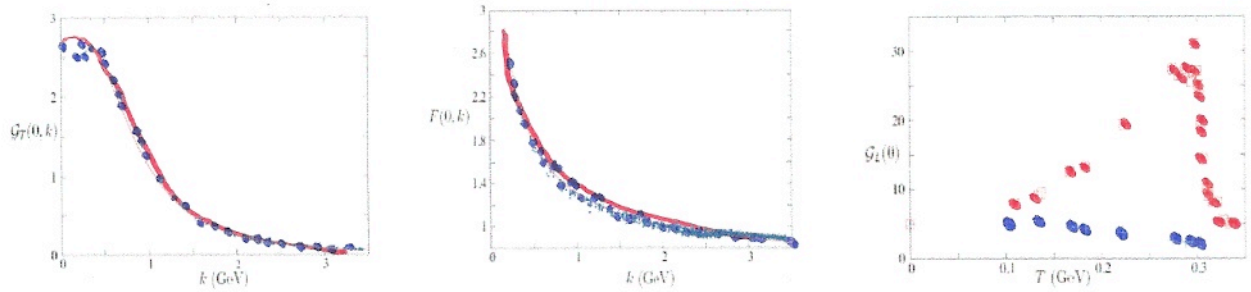
[Peláez, Tissier, Wschebor ('13)]



Dynamical quarks [Peláez et al. ('14)]



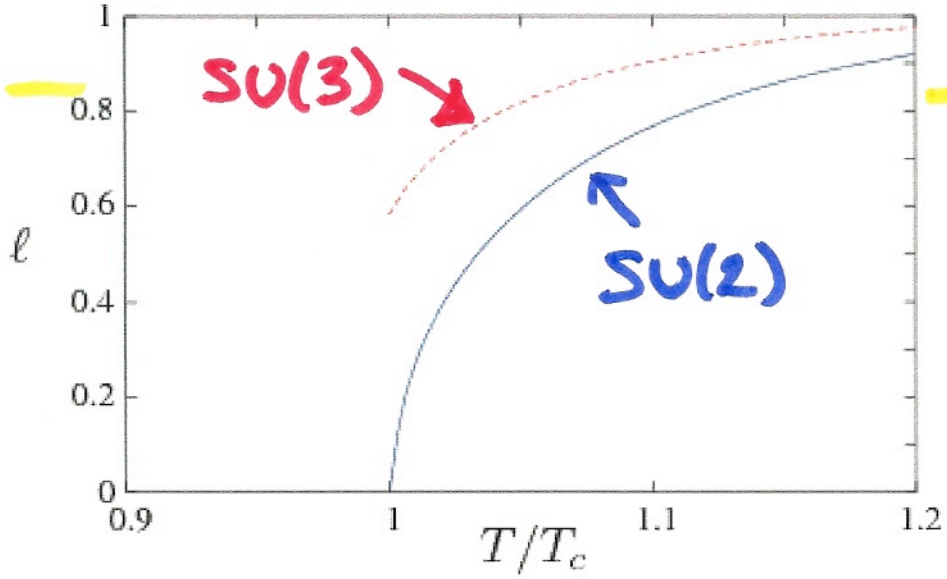
Finite temperature



[Reinosa, Serreau, Tissier, Wschebor ('14)]

Results : Summary

$$\ell(r) = \frac{1}{N} tr\left[e^{i r^a t^a}\right] + O(g^2)$$



	T_c/m	m	T_c	$T_c^{latt.}$ *	T_c^{FRG} **
SU(2)	0.33	710	238	295	230
SU(3)	0.36	510	185	270	275

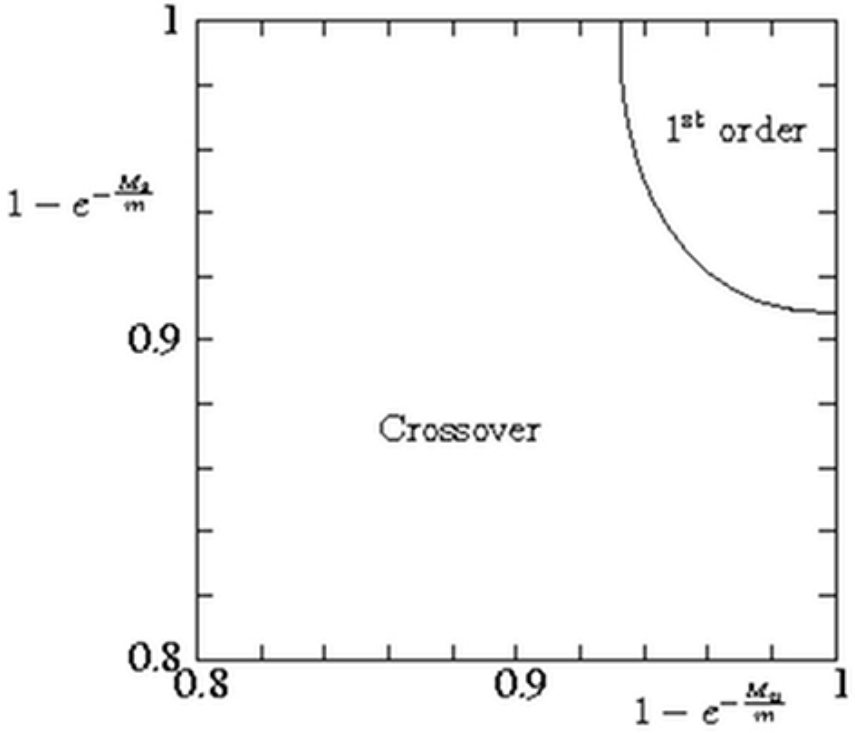
(All in MeV)

* [Lucini, Panero ('13)]

** [Fister, Pawlowski ('13)]

One-loop results : $\mu = 0$

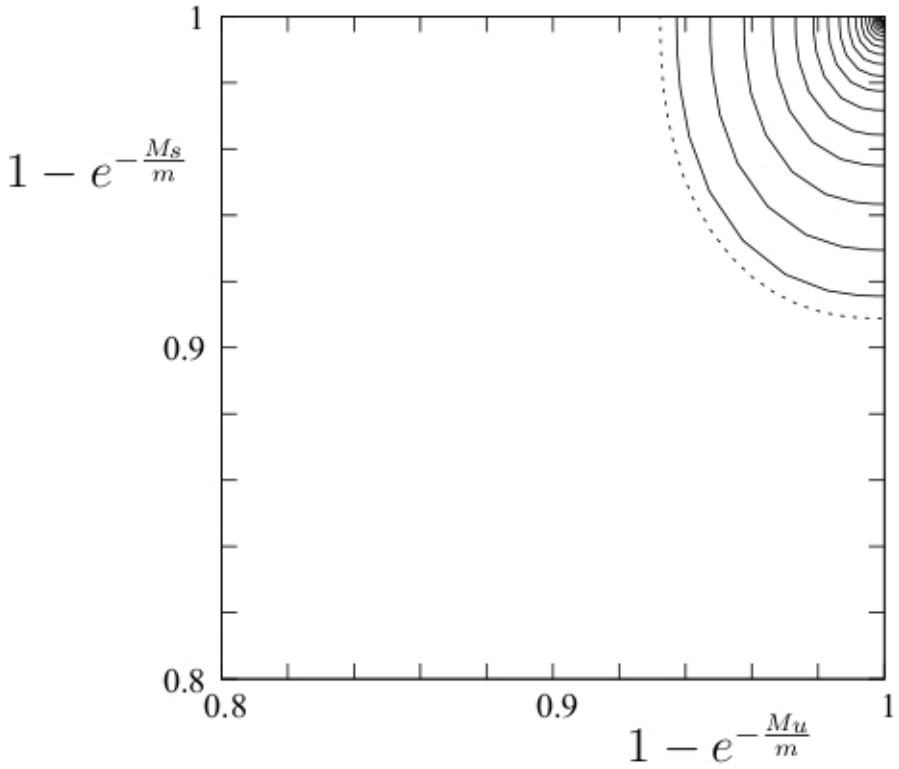
1 + 2 flavors

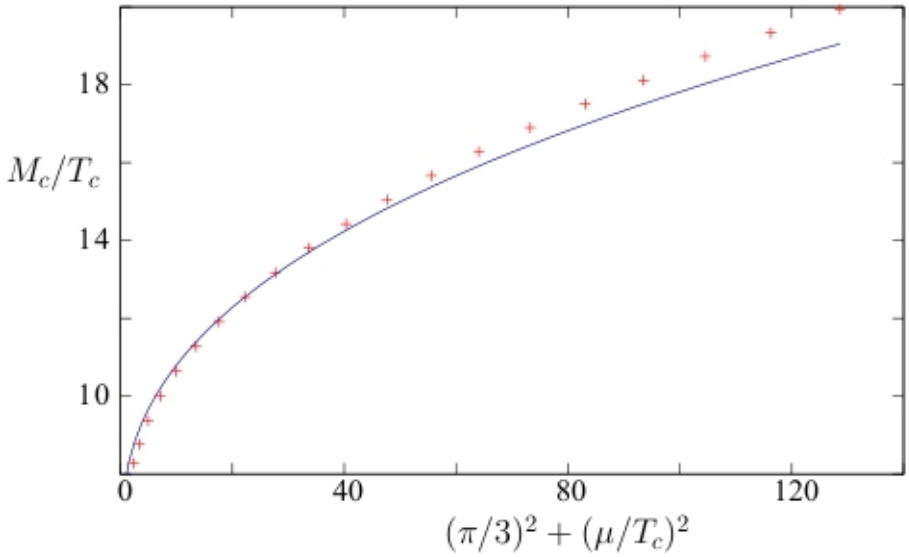


N_f	M_c/m	M_c/T_c	latt.*	matrix**	DSE***
1	2.39	6.74	7.22	8.04	1.42
2	2.69	7.59	7.91	8.85	1.83
3	2.86	8.07	8.32	9.33	2.04

* Fromm et al ('12) ; ** Kashiwa et al. ('12) ;
*** Fisher et al. ('15)

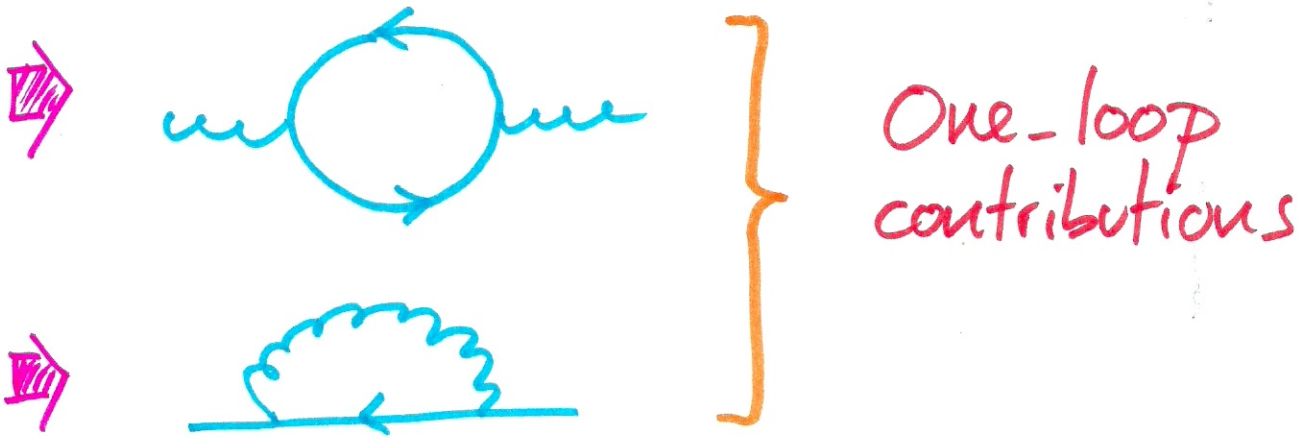
Real μ : Columbia plot and tricritical scaling





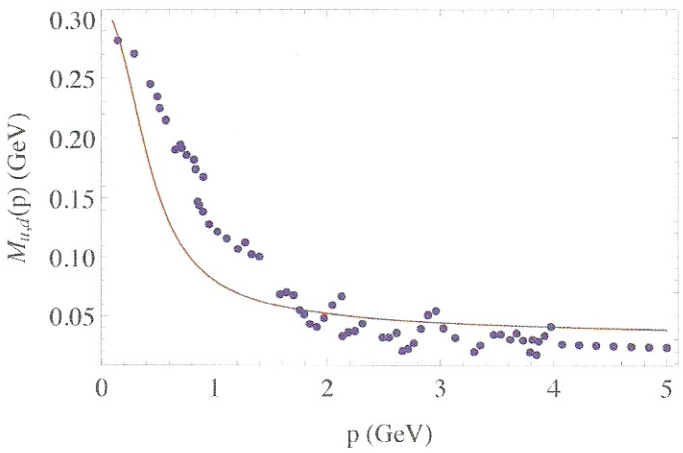
PART II : QCD dynamics

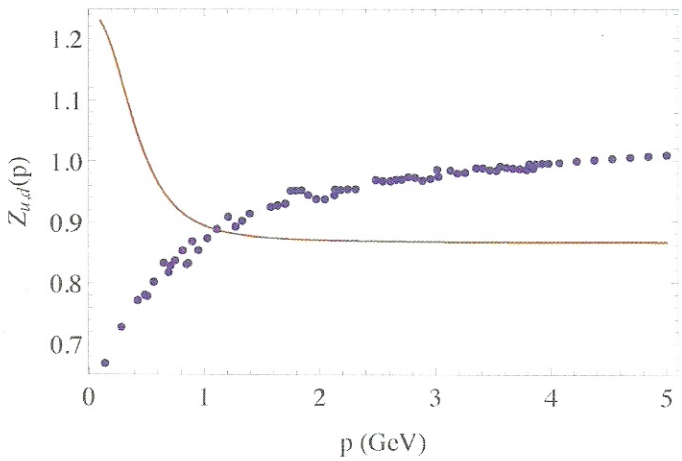
$$\mathcal{L} = \frac{1}{4}F^2 + ih\partial A + \partial\bar{c}Dc + \frac{m^2}{2}A^2 + \bar{\psi}(\not{p}+M)\psi$$



One-loop contributions

$N_f = 2+1$, $g_0 = 4.8$, $m_0 = 0.42$ GeV, $M_0 = 0.08$ GeV

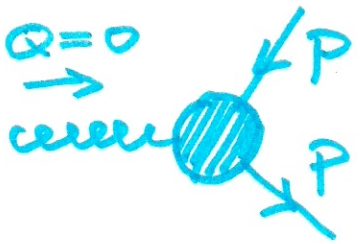




[Peláez, Tissier, Wschebor PRD 90 (2014) 065031]

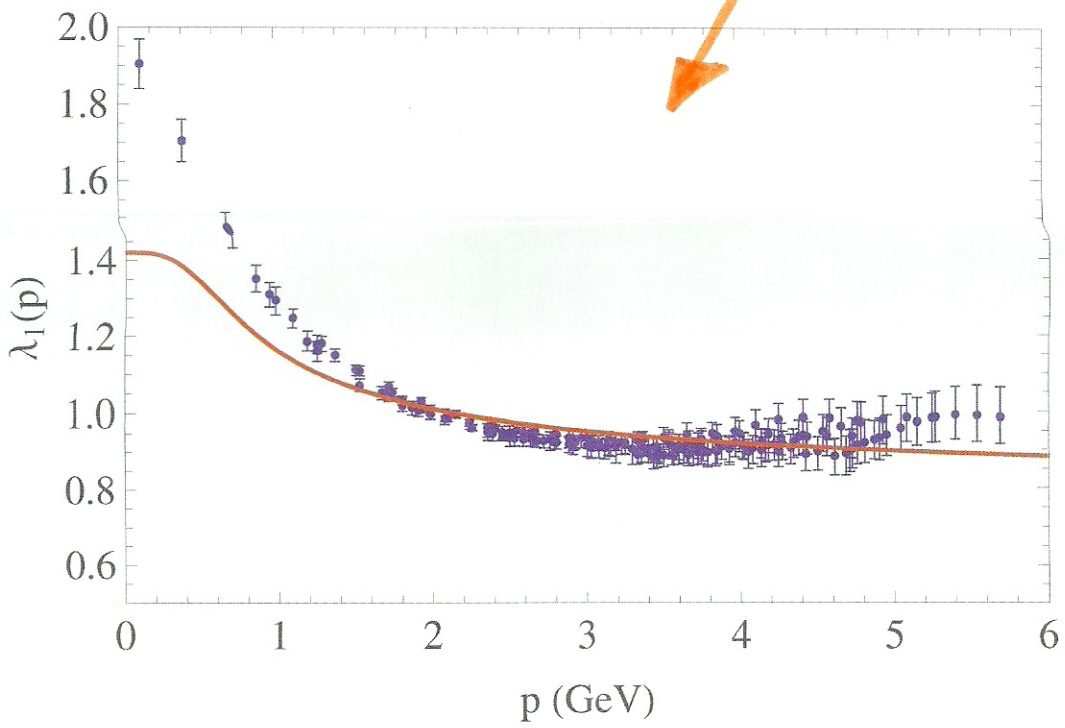
Data from [Bowman *et al.* PRD 71 (2005) 54507]

The quark-gluon coupling



$$= -i\, g_q(p)\, \gamma_\mu + \cdots$$

$$g_q(p) = \underbrace{g_g(p)}_{\text{ghost-gluon}}\, \lambda_1(p)$$

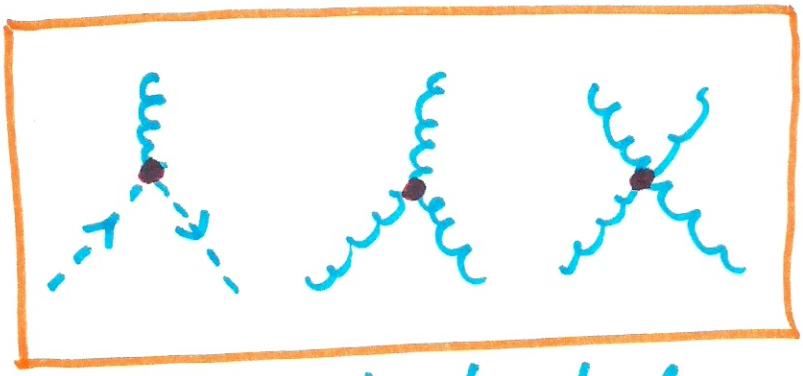


[Peláez, Tissier, Wschebor PRD 92 (2015) 045012]

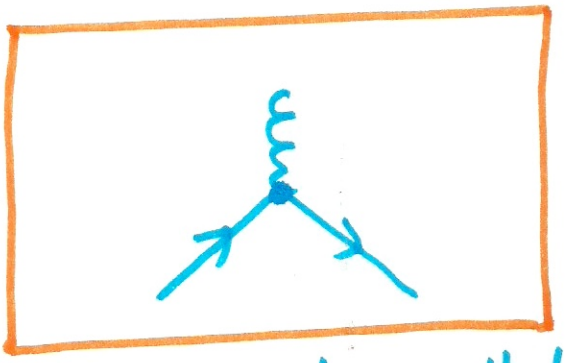
Data from [Skullerud *et al.* JHEP 0304 (2003) 047]

Small parameters in infrared QCD

[Peláez, Reinosa, Serreau, Tissier, Wschebor ('17)]



g_g : can be treated perturbatively



g_q : not small !!

(1) Expand in g_g, not in g_q.

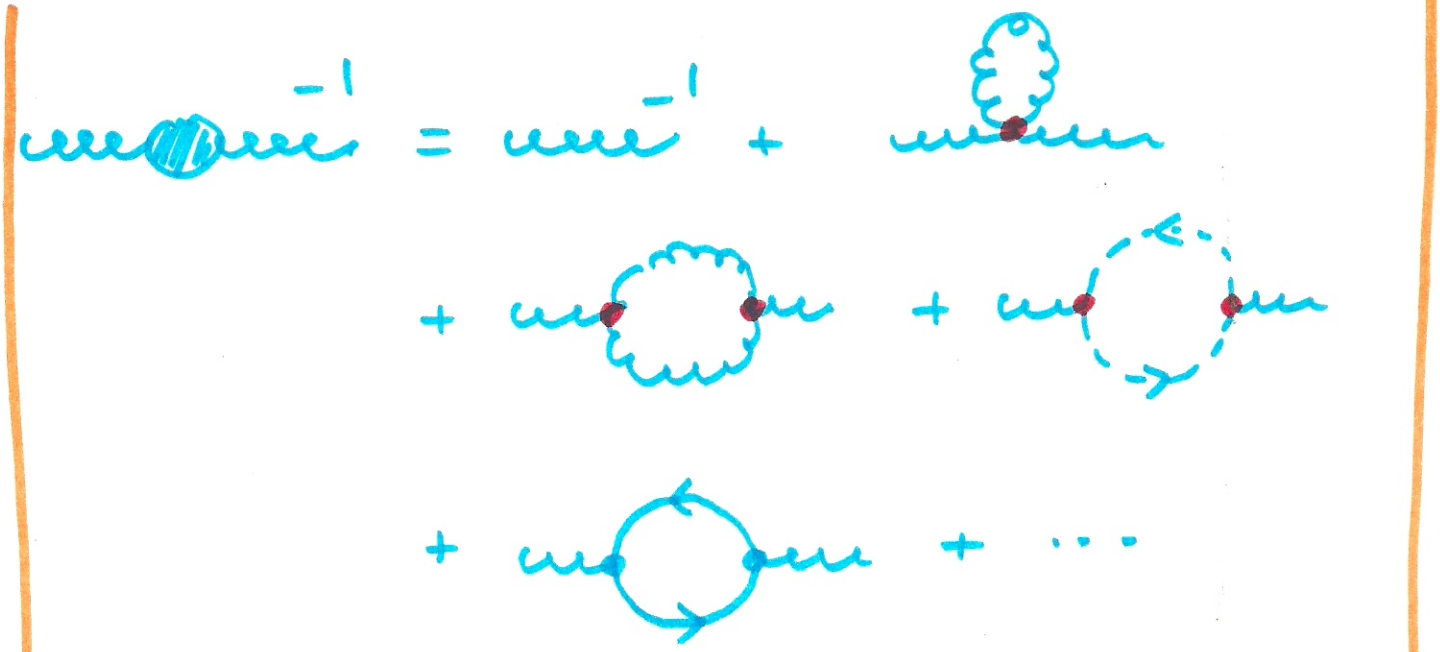
e.g. : only QED-like diagrams survive at L.O. in the quark sector

(2) Expand in $1/N_c$ with $g_q^2 N_c$ fixed

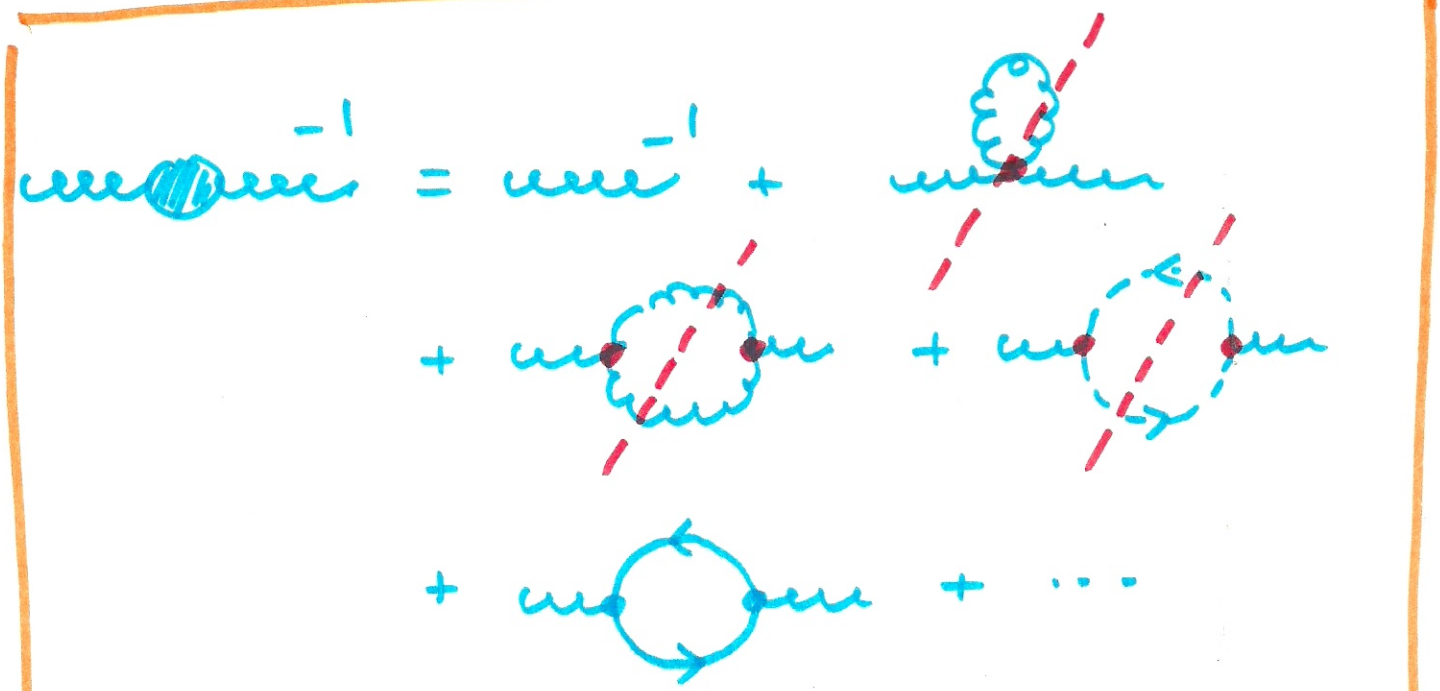
['t Hooft ('74) ...]

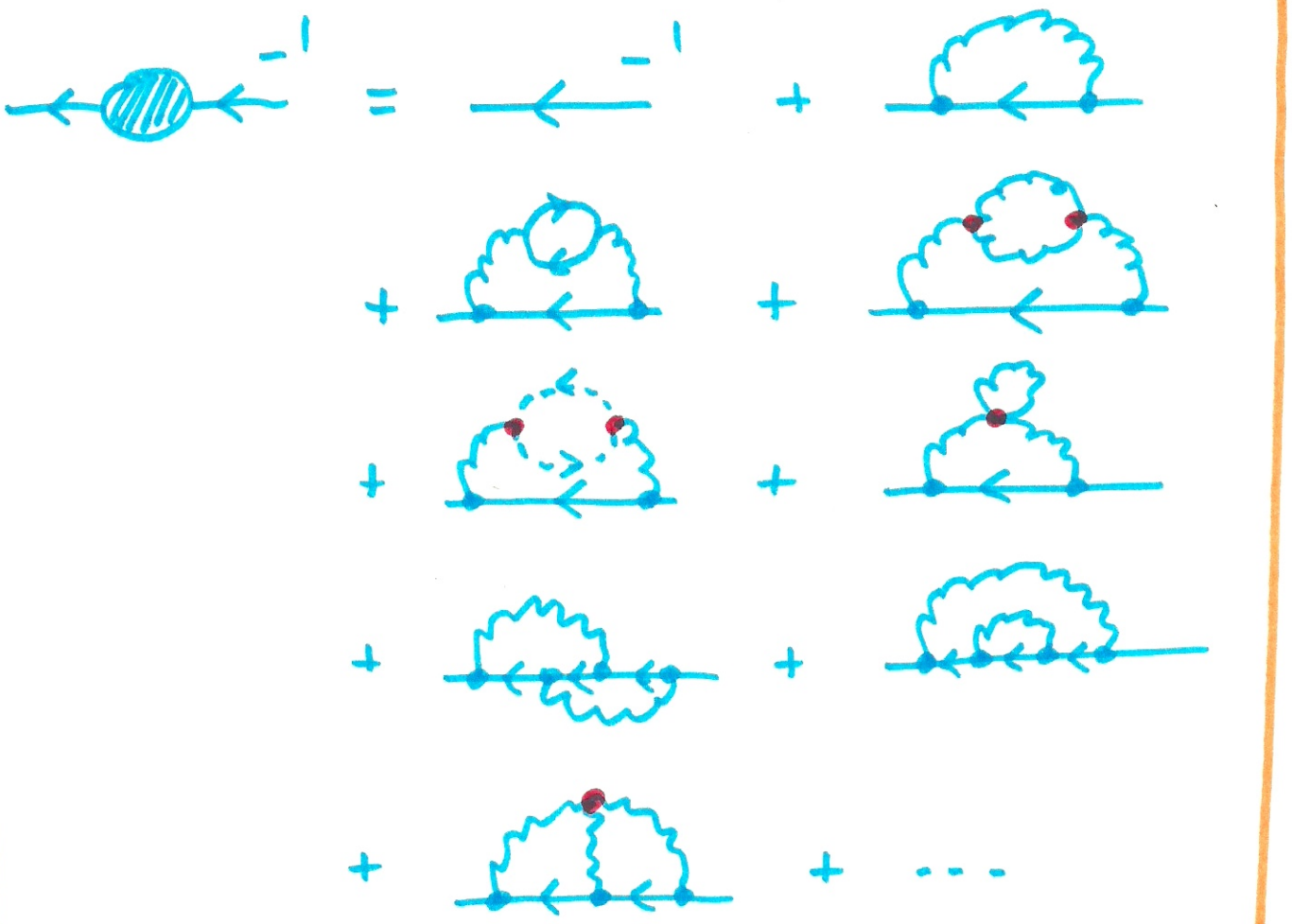
↳ Captures essential aspects of QCD dynamics

"Rainbow-improved" loop expansion

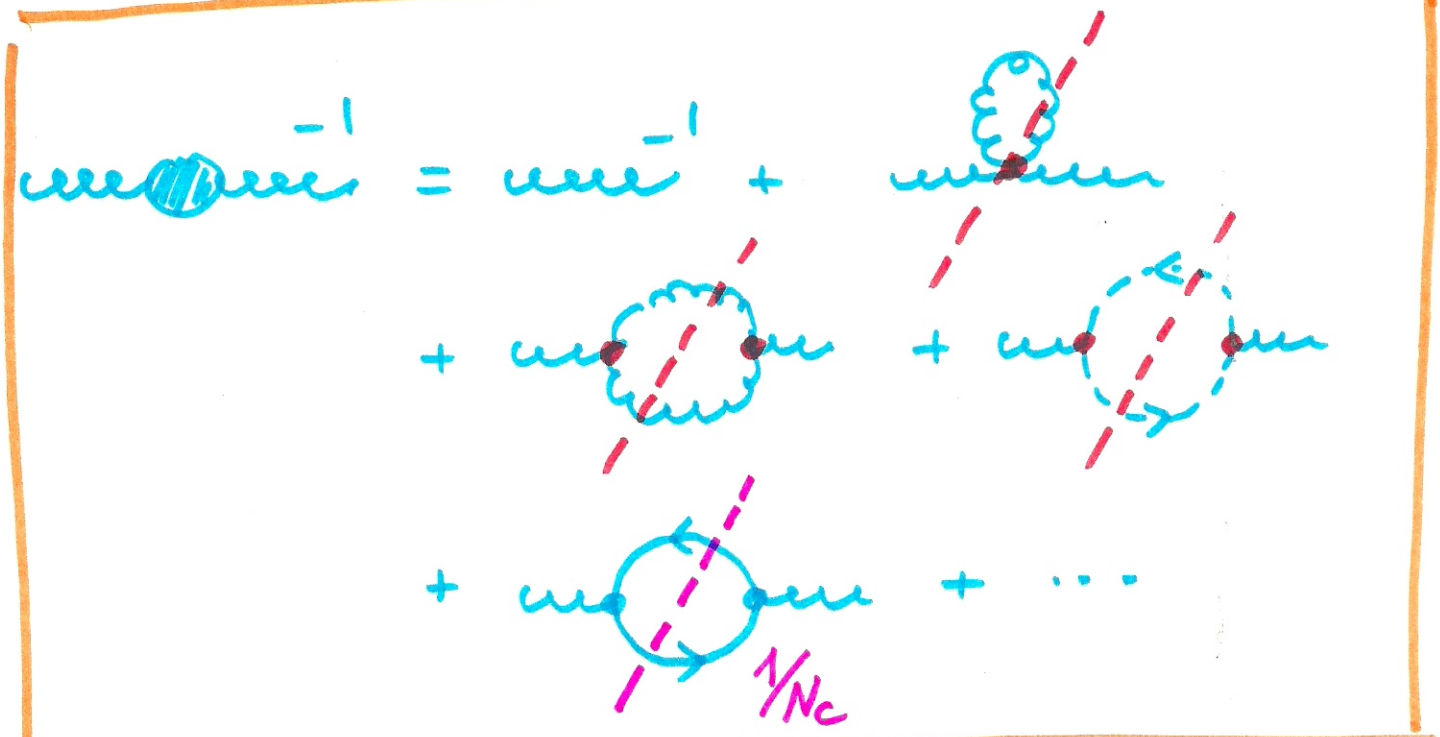


"Rainbow-improved" loop expansion

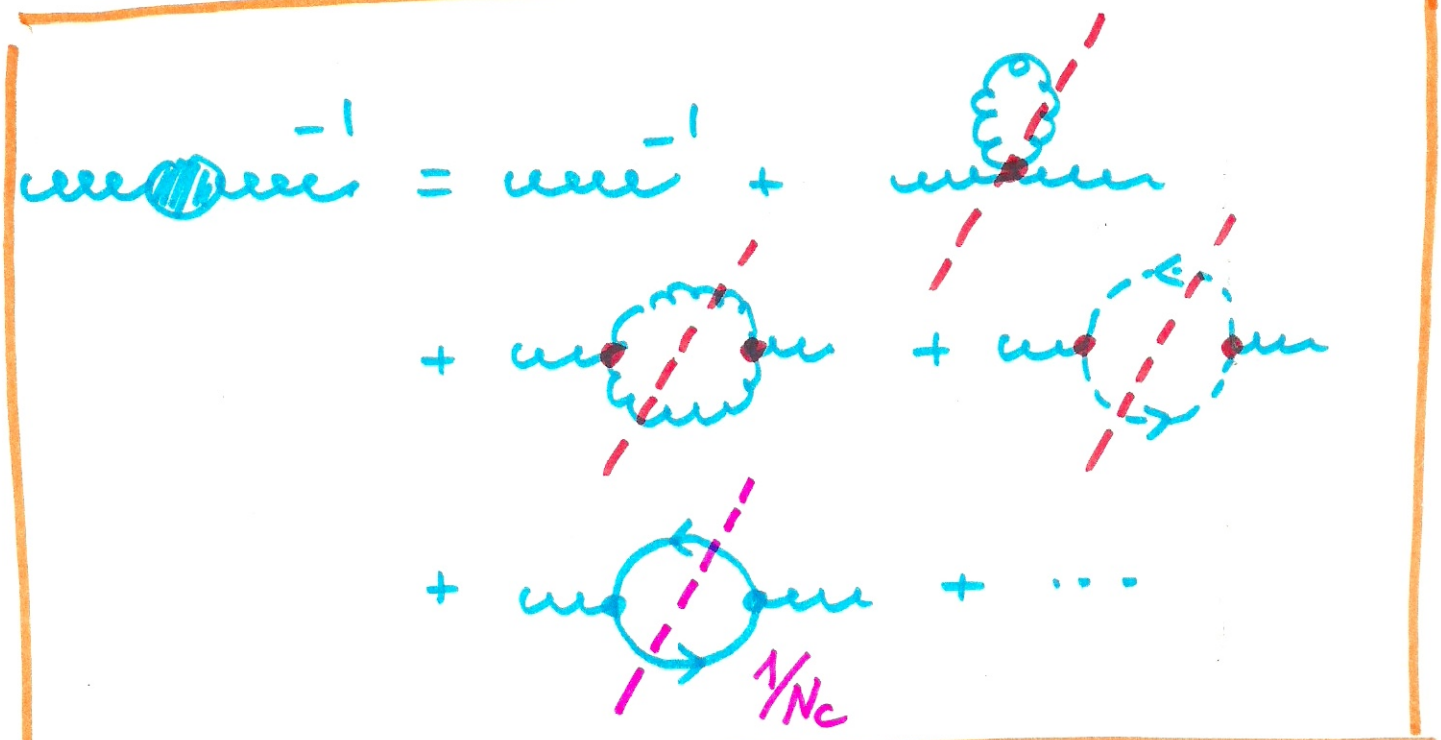


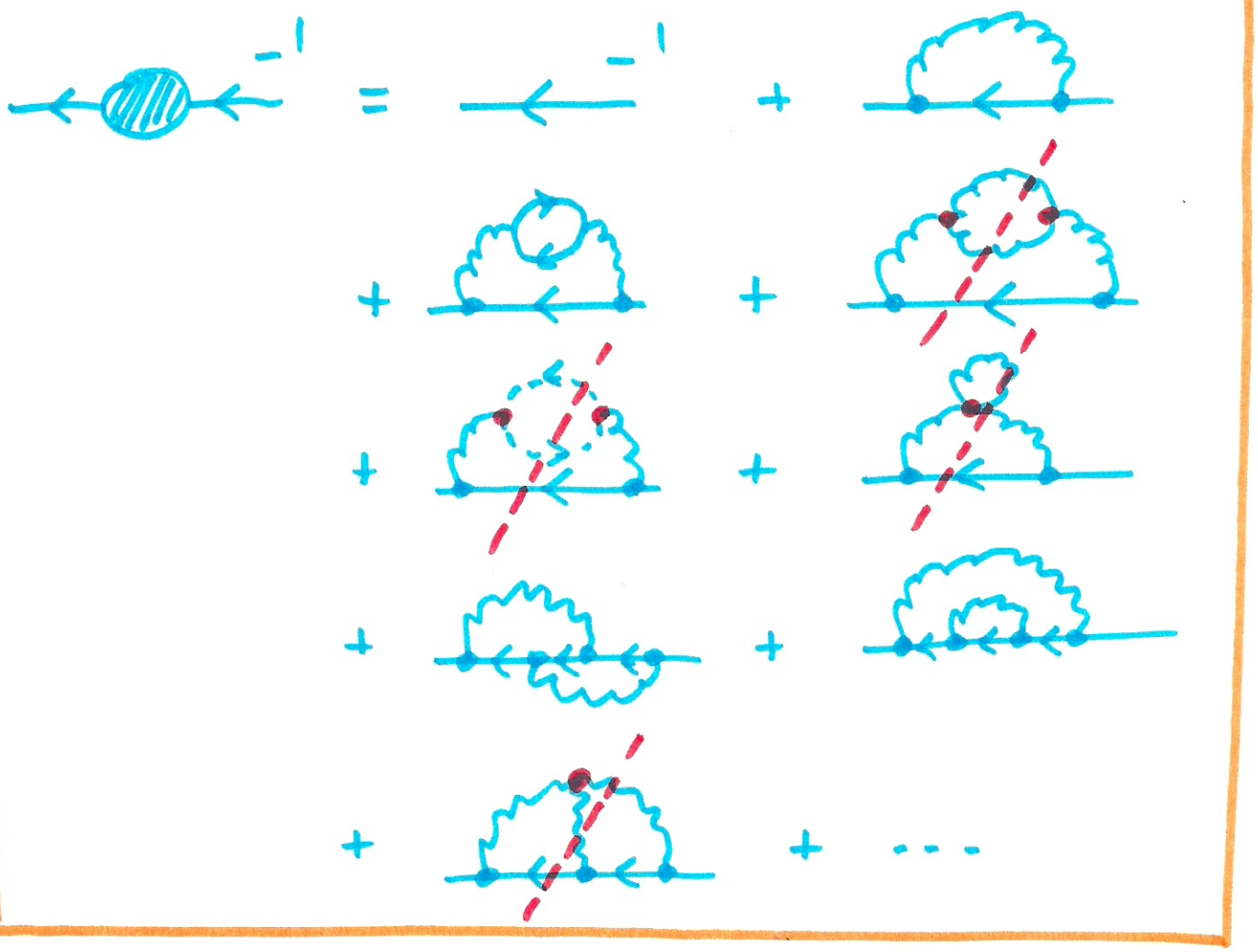


"Rainbow-improved" loop expansion

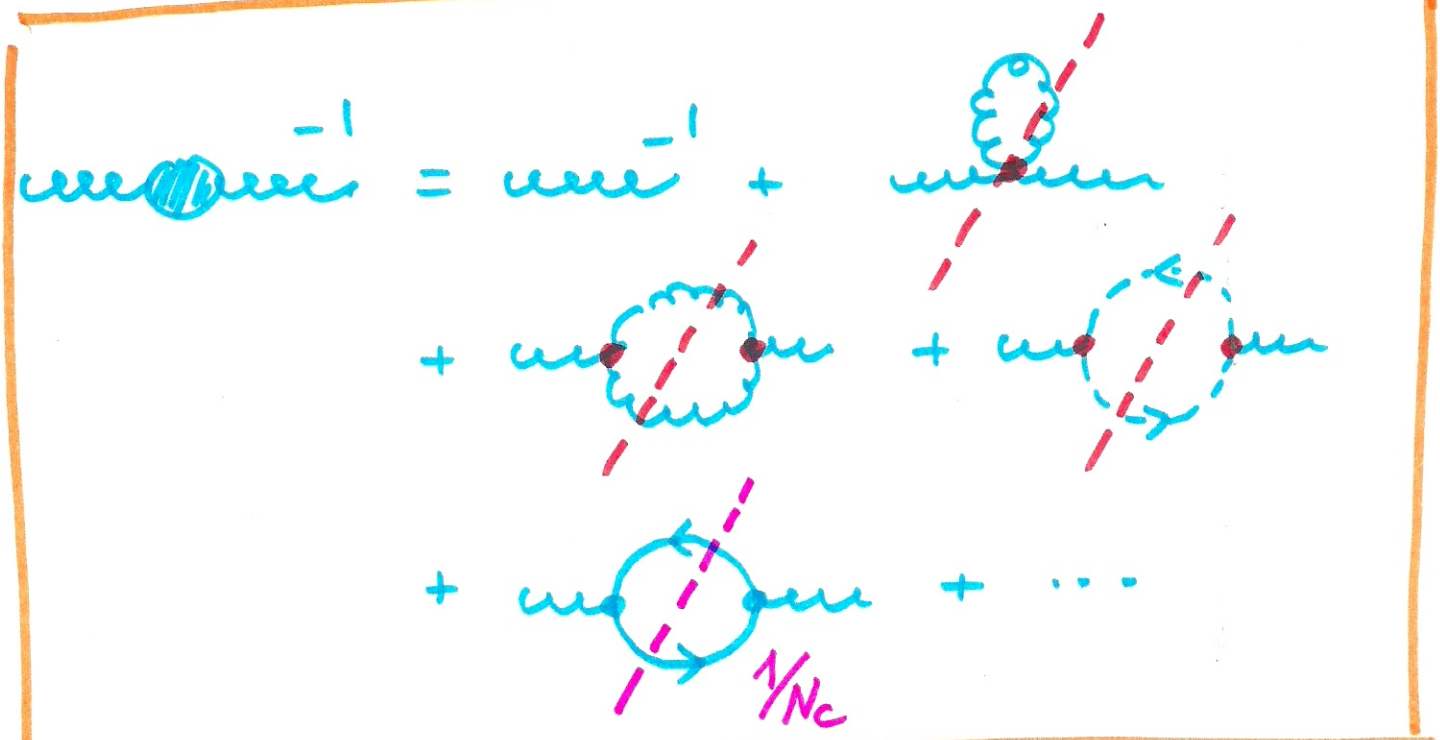


"Rainbow-improved" loop expansion

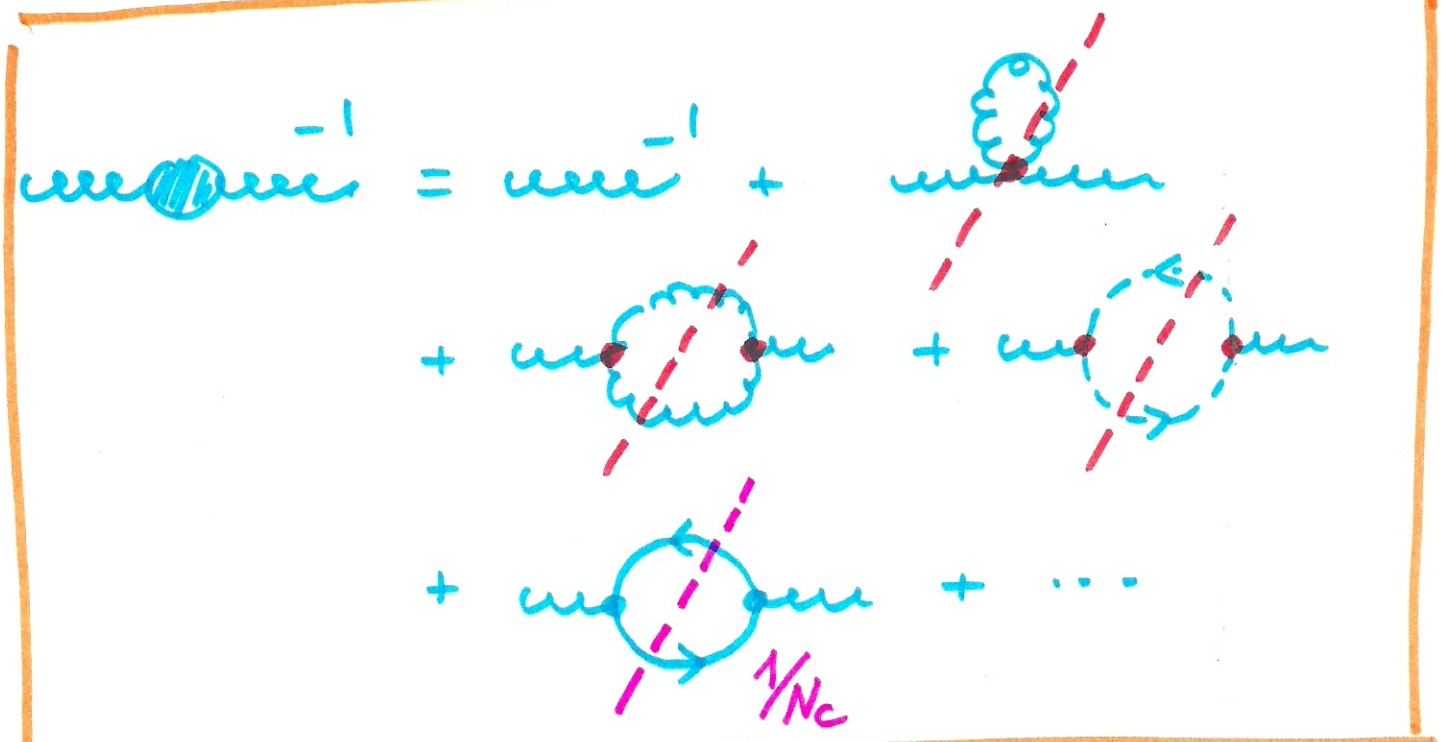


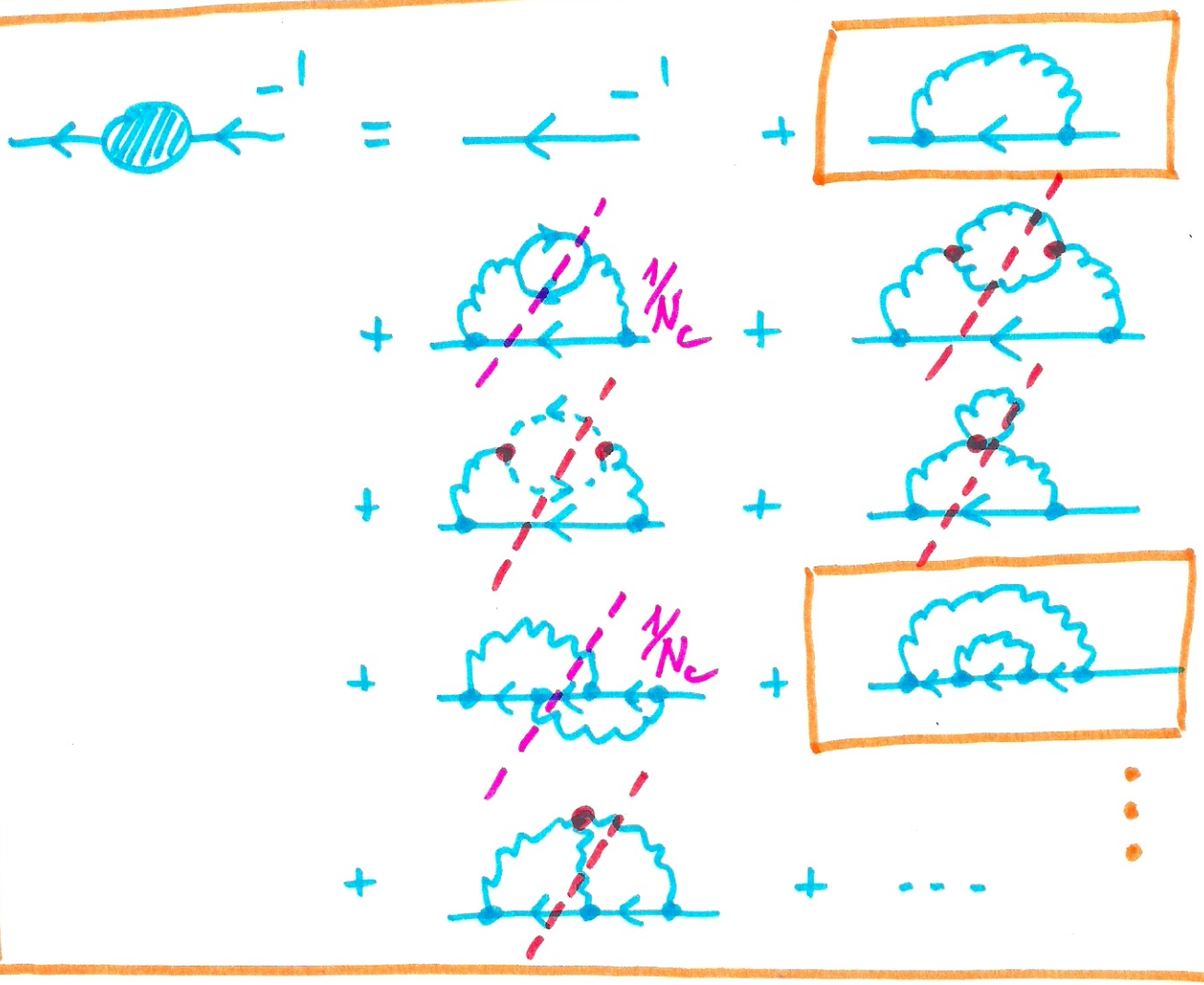


"Rainbow-improved" loop expansion



"Rainbow-improved" loop expansion





Rainbow-improved : Leading Order (RiLO)

L.O. : only rainbow diagrams survive in the quark propagator

$$S(p) = \left(-iA(p)\not{p} + B(p)\right)^{-1} = -i\tilde{A}(p)\not{p} + \tilde{B}(p)$$

$$\frac{\tilde{A}}{A} = \frac{\tilde{B}}{B} = \frac{1}{A^2p^2 + B^2}$$

$$\tilde{A}(p) = 1 - g_q^2 C_F \int_q \frac{f(q,p)\,\tilde{A}(q)}{(p+q)^2 + \underline{m}^2}$$

$$\tilde{B}(p) = M + 3g_q^2 C_F \int_q \frac{\tilde{B}(q)}{(p+q)^2 + \underline{m}^2}$$

CF mass

$$f(p,q) = \frac{2p^2q^2 + 3(p^2+q^2)(p\cdot q) + 4(p\cdot q)^2}{p^2(p+q)^2}$$

RiLO : Renormalization

(1) $A \to \sqrt{Z_A}\, A \quad ; \quad \psi \to \sqrt{Z_\psi}\, \psi$

$m^2 \to Z_{m^2} m^2 \; ; \; M \to Z_M M \; ; \; g_q \to Z_g g_q$

(2) $S^{-1}(p = \mu_0) = -i \not{\mu}_0 + M(\mu_0)$

(3) $Z_A = Z_{m^2} = Z_g Z_\psi \sqrt{Z_A} = 1$ at L.O.

$$\tilde{A}(p) = \underline{Z_\psi} - \underline{g_q^2} C_F \int_q \frac{f(q,p)\, \tilde{A}(q)}{(p+q)^2 + \underline{m^2}}$$

$$\tilde{B}(p) = \underline{M} + 3\underline{g_q^2} C_F \int_q \left\{ \frac{\tilde{B}(q)}{(p+q)^2 + \underline{m^2}} - (p \leftrightarrow \mu_0) \right\}$$

RiLO : RG improvement

(1) $$\left(\frac{\partial}{\partial \ln\mu} - \gamma_\psi + \beta_X \frac{\partial}{\partial X}\right) S^{-1}(p) = 0$$

$$X = m, M, g_q$$

$$\gamma_\psi = \frac{\partial \ln Z_\psi}{\partial \ln \mu} \; ; \; \beta_X = \frac{\partial X}{\partial \ln \mu}$$

(2) $$A(p,\mu_0) = \frac{Z_\psi(\mu_0)}{Z_\psi(p)}$$

$$B(p,\mu_0) = A(p,\mu_0)\, M(p)$$

$$Z_\psi(p) = 1 + \frac{g_q^2(p) C_F}{Z_\psi(p)} \int_q \frac{Z_\psi(q)}{q^2 + M^2(q)} \frac{\not{f}(qp)}{(p+q)^2 + m^2(p)}$$

$$p M'(p) = \gamma_\psi(p) M(p) - 3 \frac{g_q^2(p) C_F}{Z_\psi(p)} \times \int_q \frac{Z_\psi(q) M(q)}{q^2 + M^2(q)} \frac{2p^2 + 2p\cdot q}{(p+q)^2 + m^2(p)}$$

RiLO : UV limit

The large p regime is dominated by $\boxed{m \lesssim q \lesssim p}$

$$Z_\psi(p) \approx 1$$

$$p\,M'(p) \approx -\frac{3 g_q^2(p) C_F}{8\pi^2 p^2} \int_{m^2}^{p^2} dq^2\, M(q)$$

$$\oplus \quad g_q^2(p) = \frac{1}{\beta_0 \ln p^2/\mu_0^2} \; ; \; \beta_0 = \frac{\frac{11}{3}N_c - \frac{2}{3}N_f}{16\pi^2}$$

➡ Massive solution

$$M(p) \sim \left(1 + \beta_0 g_0^2 \ln p^2/\mu_0^2\right)^{-\alpha}$$

➡ SxSB solution

$$M(p) \sim p^{-2} (\ln p)^{\alpha - 1}$$

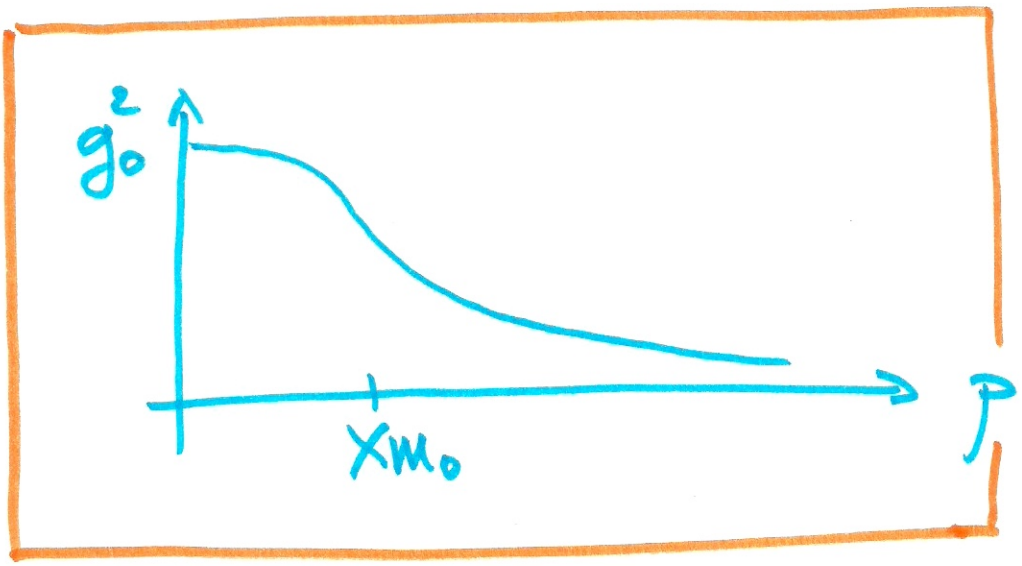
$$\alpha = \frac{3C_F}{16\pi^2 \beta_0}$$

[Miransky ('85); Atkinson et al. ('88); Fischer et al. ('03) ...]

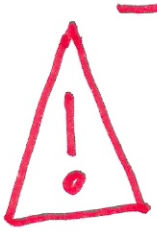
RiLO : Results

Toy model for the flow of g_q^2 :

$$g_q^2(p) = \frac{g_0^2}{1 + \beta_0 g_0^2 \ln\left(\frac{p^2 + x^2 m_0^2}{x^2 m_0^2}\right)}$$



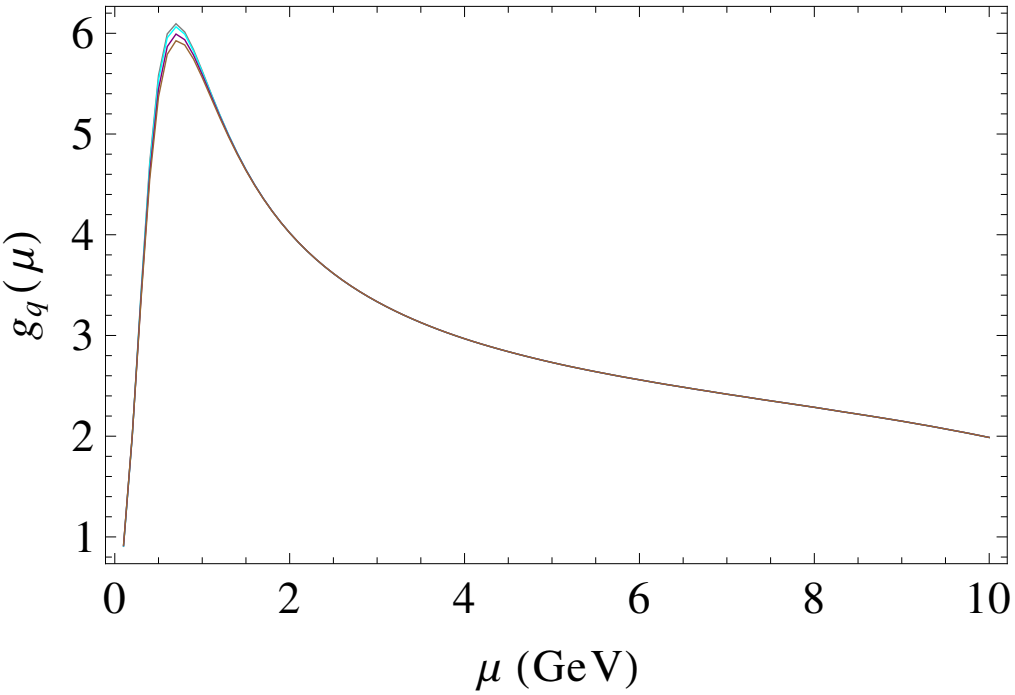
- $m_0 = 0.4$ GeV (from fits of YM propags)
- vary $M(10\,\text{GeV})$, g_0^2, x

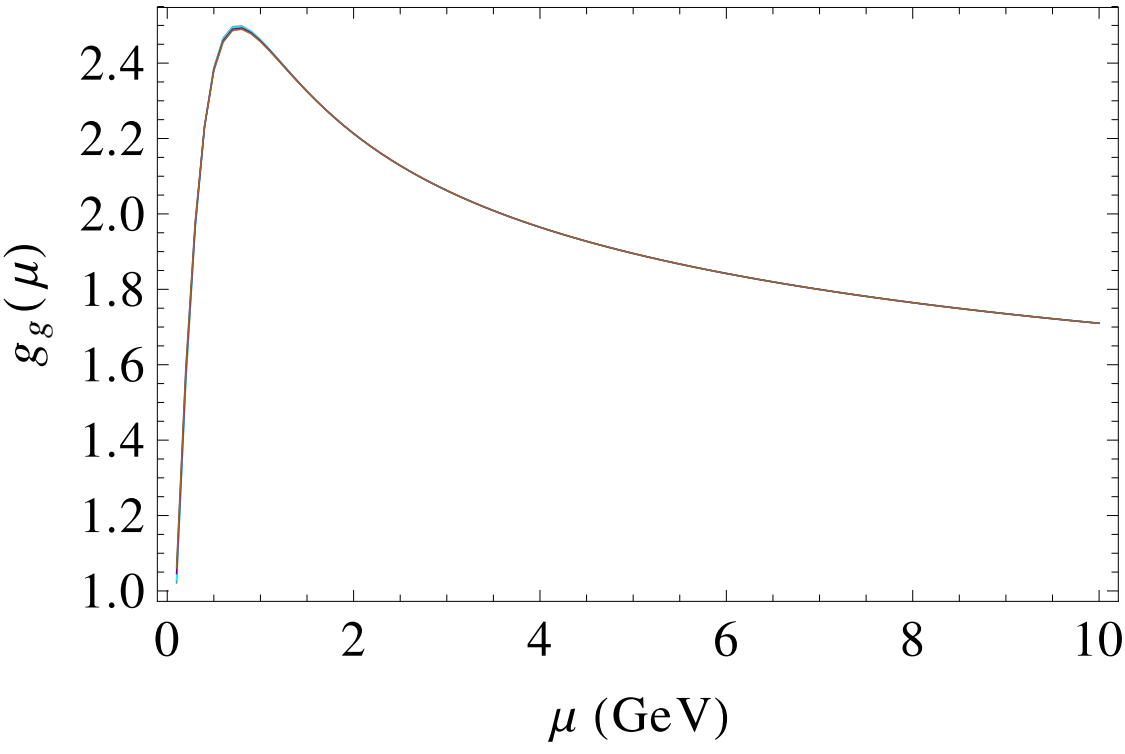


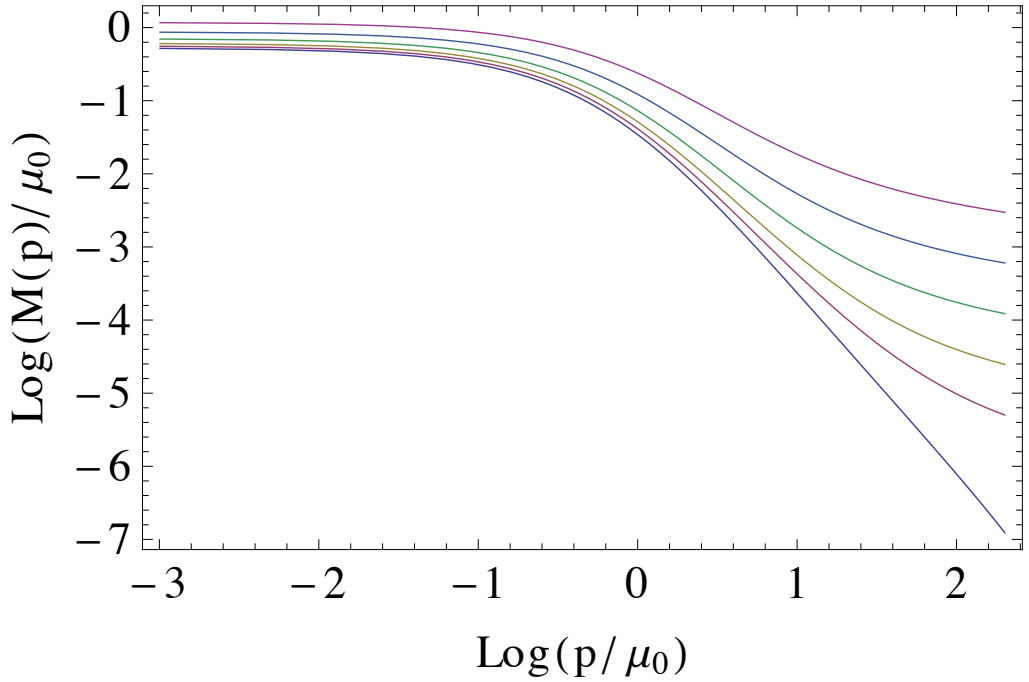
Same quality with the fully consistent flow computed at RiLO (no other parameters than m_0, g_0)

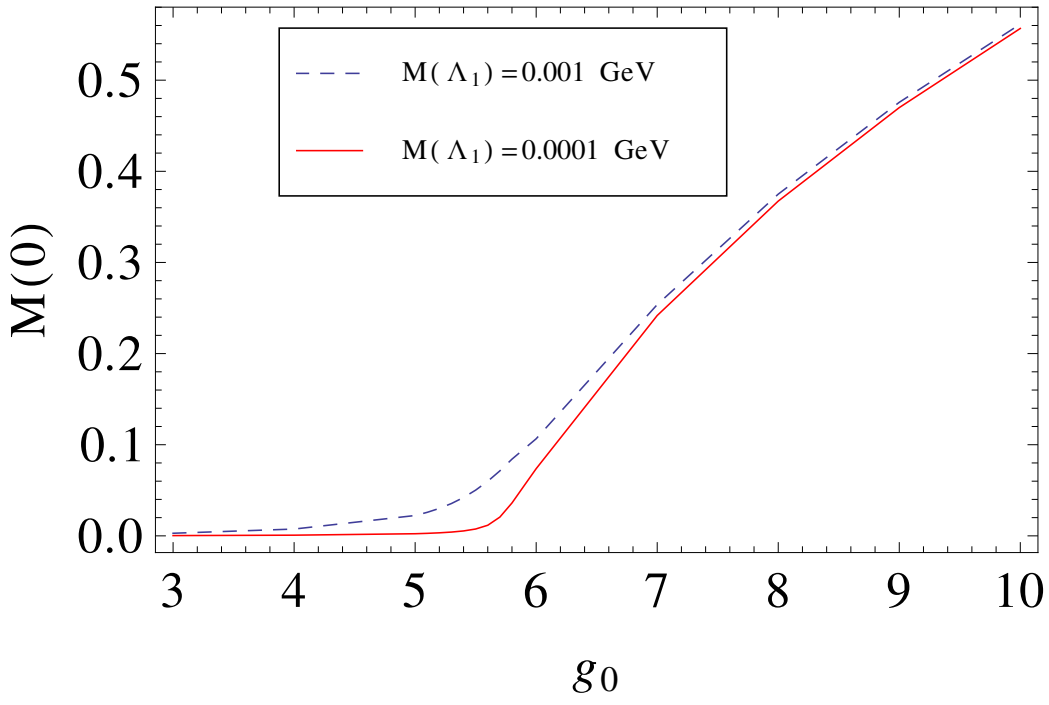
[in preparation]

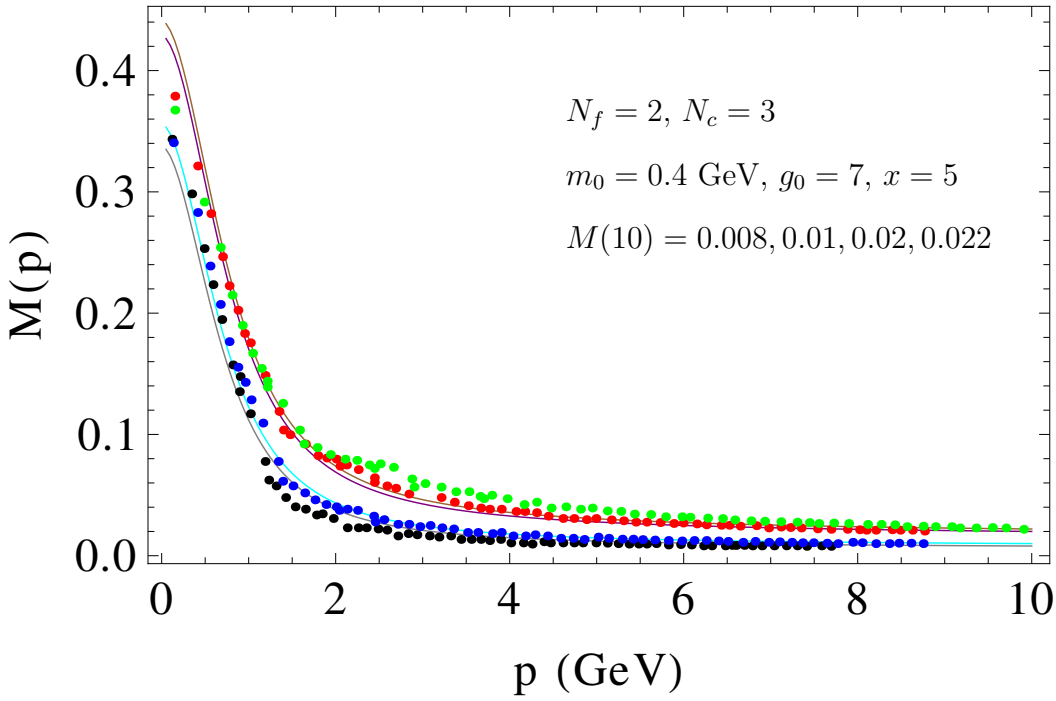
RILO: running couplings



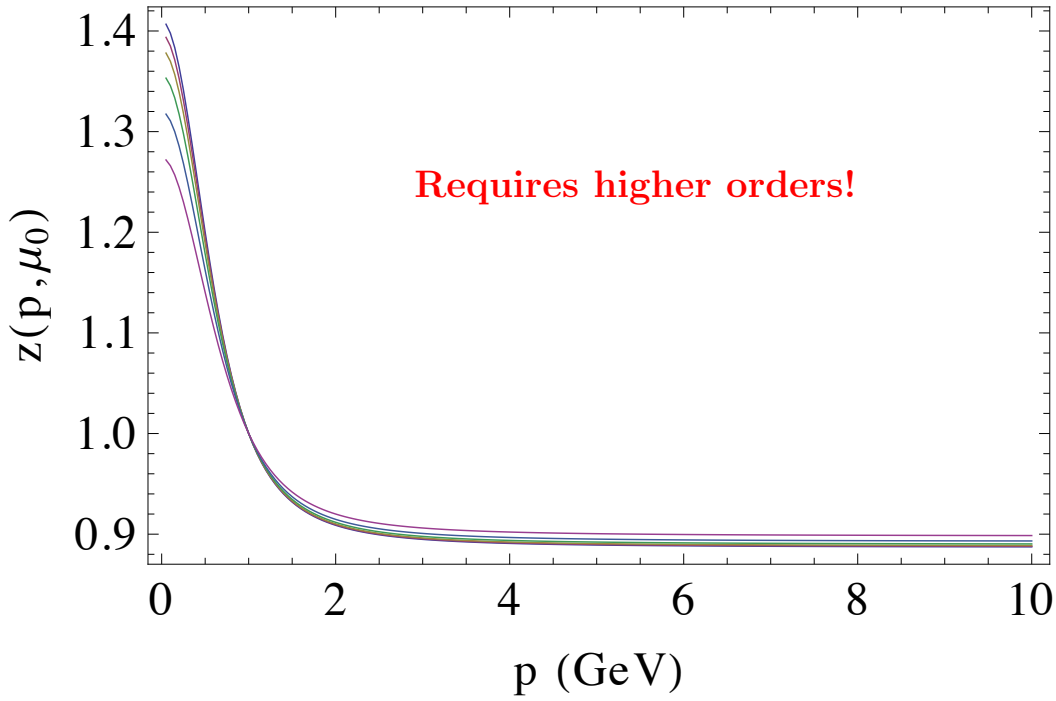



0
−1
−2
−3
−4
−5
−6
−7
−3
−2
−1
0
1
2
Log(M(p)/μ0)
Log(p/μ0)


M(Λ1) = 0.001 GeV
M(Λ1) = 0.0001 GeV
0.5
0.4
0.3
0.2
0.1
0.0
3
4
5
6
7
8
9
10
M(0)
g0



Data from [Oliveira et al. arXiv:1605.09632]



CONCLUSIONS

- The CF model provides a satisfying description of various aspects of the dynamics of nonabelian gauge fields in perturbation theory.

- The dynamics of (light) quarks and, in particular, χSB is well described by a systematic expansion in g_s, $1/N_c$

  ⇒ rainbow resummation of fermion lines

PERSPECTIVES

- Higher (two) loop calculations in Yang-Mills theories and in QCD

- Full RiLO :
  - running couplings m, g_g, g_q
  - gluon / ghost / quark propagators
  - quark-gluon vertex

- Meson properties in RiLO
  e.g. : m_π, f_π as functions of m_0, g_0

- Finite T, μ

...