

Quantum corrections to the Classical Statistical Approximation for the longitudinally expanding scalar field

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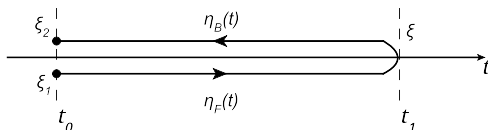
Keldysh contour

- Highly excited initial state (not vacuum) \rightarrow density matrix

$$\hat{\rho}(t) = \sum_i P_i |\psi_i(t)\rangle \langle \psi_i(t)|$$

- Physical observable $\langle F(\hat{\varphi}) \rangle = \text{tr}(F(\hat{\varphi})\hat{\rho})$
- Nonequilibrium evolution from t_0 to t_1 :

$$\hat{\rho}(t) = \hat{U}(t, t_0)\hat{\rho}(t_0)\hat{U}^\dagger(t, t_0)$$



Path-integral representation

Observable

$$\langle F(\hat{\varphi}) \rangle_{t_1} = \int \mathcal{D}\xi \int \mathcal{D}\xi_1 \int \mathcal{D}\xi_2 \langle \xi_1 | \hat{\rho}(t_0) | \xi_2 \rangle$$

$$\times F(\xi) \int_{\eta_F(t_0, \vec{x}) = \xi_1(\vec{x})}^{\eta_F(t_1, \vec{x}) = \xi(\vec{x})} \mathcal{D}\eta_F(t, \vec{x}) \int_{\eta_B(t_0, \vec{x}) = \xi_2(\vec{x})}^{\eta_B(t_1, \vec{x}) = \xi(\vec{x})} \mathcal{D}\eta_B(t, \vec{x}) e^{iS_K[\eta_F, \eta_B]}.$$

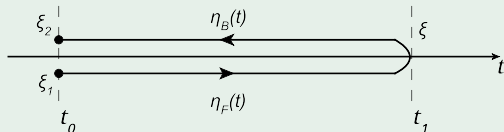
notations

$$\hat{\varphi}(\vec{x})|\xi\rangle = \xi(\vec{x})|\xi\rangle$$

$$\hat{1} = \int \mathcal{D}\xi(\vec{x}) |\xi\rangle \langle \xi|$$

$$S_K[\eta_F, \eta_B] = S[\eta_F] - S[\eta_B]$$

Keldysh contour



General formula

Classical and quantum components are:

$$\phi_c = \frac{\eta_F + \eta_B}{2}, \quad \phi_q = \eta_F - \eta_B.$$

For almost any field theory

$$\begin{aligned} \langle F(\hat{\varphi}) \rangle_{t_1} &= \int \mathcal{D}\chi_1 \int \mathcal{D}\xi_1 \int \mathcal{D}\xi_2 \langle \xi_1 | \hat{\rho}(t_0) | \xi_2 \rangle \\ &\times \int_{\substack{\phi_c(\infty, \vec{x}) = \chi_1(\vec{x}) \\ \phi_c(t_0, \vec{x}) = \frac{\xi_1(\vec{x}) + \xi_2(\vec{x})}{2}}} \mathcal{D}\phi_c \int_{\substack{\phi_q(\infty, \vec{x}) = 0 \\ \phi_q(t_0, \vec{x}) = \xi_1(\vec{x}) - \xi_2(\vec{x})}} \mathcal{D}\phi_q F(\phi_c(t_1)) e^{iS_K[\phi_c, \phi_q]} \end{aligned}$$

Scalar φ^4 theory

QCD $\rightarrow \varphi^4$ for simplicity

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{g^2}{4} \varphi^4 + J\varphi$$

J - auxiliary source

The Keldysh action

$$S_K[\phi_c, \phi_q] = \int d^3x \dot{\phi}_c(t_0, \vec{x}) \phi_q(t_0, \vec{x}) - \int_{t_0}^{\infty} dt \int d^3x [\phi_q] \left(\underbrace{\partial_\mu \partial^\mu \phi_c + g^2 \phi_c^3 - J}_{\text{equation of motion}} \right) - \frac{g^2}{4} \phi_c \phi_q^3$$

- remember about Plank constant
- substitute $\phi_q \rightarrow \hbar \phi_q$
- use semiclassical decomposition

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- remember about Plank constant
- substitute $\phi_q \rightarrow \hbar \phi_q$
- use semiclassical decomposition

Semiclassical decomposition

$$e^{-i\frac{g^2\hbar^2}{4}\int_{t_0}^{\infty} dt \int d^3x \phi_c \phi_q^3} = \underbrace{1}_{LO} - \underbrace{\frac{ig^2\hbar^2}{4}\int_{t_0}^{\infty} dt \int d^3x \phi_c \phi_q^3}_{NLO} + \dots$$

Leading Order

$$\begin{aligned} \langle F(\hat{\varphi}) \rangle_{t_1}^{LO} &= \int \mathcal{D}\chi_{Init} \int_{\substack{\phi_c(\infty, \vec{x}) = \chi_1(\vec{x}) \\ \phi_c(t_0, \vec{x}) = \frac{\xi_1(\vec{x}) + \xi_2(\vec{x})}{2}}} \mathcal{D}\phi_c e^{i\int d^3x \dot{\phi}_c(t_0, \vec{x})(\xi_1(\vec{x}) - \xi_2(\vec{x}))} \\ &\times F(\phi_c(t_1)) \int_{\substack{\phi_q(\infty, \vec{x}) = 0 \\ \phi_q(t_0, \vec{x}) = \xi_1(\vec{x}) - \xi_2(\vec{x})}} \mathcal{D}\phi_q e^{i\int dt d^3x \phi_q (\partial_\mu \partial^\mu \phi_c + g^2 \phi_c^3 - J)} \end{aligned}$$

Notation $\int \mathcal{D}\chi_{Init} = \int \mathcal{D}\chi_1 \int \mathcal{D}\xi_1 \int \mathcal{D}\xi_2 \langle \xi_1 | \hat{\rho}(t_0) | \xi_2 \rangle$

- integration over ϕ_q

Leading Order

calculation

$$\begin{aligned} \langle F(\hat{\varphi}) \rangle_{t_1}^{LO} = & \int \mathcal{D}p(\vec{x}) \int \mathcal{D}\chi_{Init} \int_{\phi_c(t_0, \vec{x}) = \frac{\xi_1(\vec{x}) + \xi_2(\vec{x})}{2}}^{\phi_c(\infty, \vec{x}) = \chi_1(\vec{x})} \mathcal{D}\phi_c e^{i \int d^3x \dot{\phi}_c(t_0, \vec{x})(\xi_1(\vec{x}) - \xi_2(\vec{x}))} \\ & \times F(\phi_c(t_1)) \delta[\partial_\mu \partial^\mu \phi_c + g^2 \phi_c^3 - J] \delta(p(\vec{x}) - \dot{\phi}_c(t_0, \vec{x})) \end{aligned}$$

- integration over ϕ_q
- insert "initial velocity" unity $1 = \int \mathcal{D}p(\vec{x}) \delta(p(\vec{x}) - \dot{\phi}_c(t_0, \vec{x}))$
- perform integration over ϕ_c with help of EoM and initial conditions $\phi_c \rightarrow \phi_d$
- change variables to $\alpha = \frac{\xi_1 + \xi_2}{2}$, $\beta = \xi_1 - \xi_2$

Classical Statistical Approximation

- find solution of the classical EoM
- calculate observable on this solution
- average over all initial conditions with weight of the Wigner functional

$$\langle F(\hat{\varphi}) \rangle_{t_1} = \int \mathcal{D}\alpha(\vec{x}) \mathcal{D}p(\vec{x}) f_w[\alpha(\vec{x}), p(\vec{x}), t_0] F(\phi_{cl}(t_1))$$

$$f_w[\alpha(\vec{x}), p(\vec{x}), t_0] = \int \mathcal{D}\beta(\vec{x}) \langle \alpha + \frac{\beta}{2} | \hat{\rho}(t_0) | \alpha - \frac{\beta}{2} \rangle e^{i \int d^3x p(\vec{x}) \beta(\vec{x})}$$

$$\partial_\mu \partial^\mu \phi_{cl} + g^2 \phi_{cl}^3 = 0$$

$$\phi_{cl}(t_0, \vec{x}) = \alpha(\vec{x})$$

$$\dot{\phi}_{cl}(t_0, \vec{x}) = p(\vec{x})$$

- We should define the Wigner function at the initial time
- We do not need the small coupling constant for the CSA
- Where the CSA works?

Notation

We introduce notation for averaging over initial condition with the Wigner functional as

$$\langle \mathcal{O} \rangle_{i.c.} = \int \mathcal{D}\alpha(\vec{x}) \mathcal{D}p(\vec{x}) f_w[\alpha(\vec{x}), p(\vec{x}), t_0] \mathcal{O}$$

So, the LO answer can be written simply as

$$\langle F(\hat{\varphi}) \rangle_{t_1}^{LO} = \langle F(\phi_{cl}(t_1)) \rangle_{i.c.}$$

Auxiliary source at work

Semiclassical decomposition

$$e^{-i\frac{g^2}{4} \int_{t_0}^{\infty} dt \int d^3x \phi_c \phi_q^3} = \underbrace{1}_{LO} - \underbrace{\frac{ig^2}{4} \int_{t_0}^{\infty} dt \int d^3x \phi_c \phi_q^3}_{NLO} + \dots$$

Keldysh action reminder

$$S_K = \int d^3x \dot{\phi}_c(t_0, \vec{x}) \phi_q(t_0, \vec{x}) - \int_{t_0}^{\infty} dt \int d^3x \left[\underbrace{\phi_q (\partial_\mu \partial^\mu \phi_c + g^2 \phi_c^3 - J)}_{\text{auxiliary source term}} - \frac{g^2}{4} \phi_c \phi_q^3 \right]$$

Functional derivative

$$\frac{\delta}{\delta J(t', \vec{x}')} e^{iS_K[\phi_c, \phi_q]} = i\phi_q(t', \vec{x}') e^{iS_K[\phi_c, \phi_q]}$$

Quantum corrections

LO+NLO terms

$$\langle F(\hat{\varphi}) \rangle_{t_1}^{LO+NLO} = \left\langle \underbrace{F(\phi_{cl}(t_1, \vec{x}))}_{LO} + \underbrace{\frac{g^2}{4} \int_{t_0}^{t_1} dt' \int d^3x' \phi_{cl}(t', \vec{x}') \frac{\delta^3 F(\phi_{cl}(t_1, \vec{x}))}{\delta J^3(t', \vec{x}')} \Big|_{J=0}}_{NLO} \right\rangle_{i.c.}$$

Coupling constant is not small

All terms

$$\langle F(\hat{\varphi}) \rangle_{t_1} = \left\langle \bar{T} e^{\frac{g^2}{4} \int d\tau d\vec{y} \phi_{cl}(\tau, \vec{y}) \frac{\delta^3}{\delta J^3(\tau, \vec{y})} F(\phi_{cl}(t_1, \vec{x}))} \right\rangle_{i.c.}$$

Attention: Not for numerical use

For numerical use

Let us define k -th variation of the classical solution over source J as

$$\frac{\delta^k \phi_{cl}(t_1, \vec{x}_1)}{\delta J^k(t_2, \vec{x}_2)} = \Phi_k(t_1, \vec{x}_1; t_2, \vec{x}_2).$$

Then

$$\frac{\delta^3 F(\phi_{cl}(t_1, \vec{x}_1))}{\delta J^3(t_2, \vec{x}_2)} = \frac{\partial F}{\partial \phi_{cl}} \Phi_3 + 3 \frac{\partial^2 F}{\partial \phi_{cl}^2} \Phi_1 \Phi_2 + \frac{\partial^3 F}{\partial \phi_{cl}^3} \Phi_1^3.$$

$\Phi_k(t_1, \vec{x}_1; t_2, \vec{x}_2)$ can be found by variation of the classical EoM

$$\frac{\delta^3}{\delta J^3(t_2, \vec{x}_2)} (\partial_\mu \partial^\mu \phi_{cl}(t_1, \vec{x}_1) + g^2 \phi_{cl}^3(t_1, \vec{x}_1) = J(t_1, \vec{x}_1)),$$

Quantum corrections (numerical)

$$\langle F(\hat{\varphi}) \rangle_{t_1}^{LO+NLO} = \left\langle F(\phi_{cl}(t_1, \vec{x})) + \frac{g^2}{4} \int_{t_0}^{t_1} dt' \int d^3x' \phi_{cl}(t', \vec{x}') \frac{\delta^3 F(\phi_{cl}(t_1, \vec{x}))}{\delta J^3(t', \vec{x}')} \Big|_{J=0} \right\rangle_{i.c.}$$

Variations

$$\frac{\delta^3 F(\phi_{cl}(t_1, \vec{x}_1))}{\delta J^3(t_2, \vec{x}_2)} = \frac{\partial F}{\partial \phi_{cl}} \Phi_3 + 3 \frac{\partial^2 F}{\partial \phi_{cl}^2} \Phi_1 \Phi_2 + \frac{\partial^3 F}{\partial \phi_{cl}^3} \Phi_1^3.$$

$$L_{t_1} \Phi_1(t_1, \vec{x}_1; t_2, \vec{x}_2) = \delta(t_1 - t_2) \delta^{(3)}(\vec{x}_1 - \vec{x}_2)$$

$$L_{t_1} \Phi_2(t_1, \vec{x}_1; t_2, \vec{x}_2) = -6g^2 \phi_{cl}(t_1, \vec{x}_1) \Phi_1^2(t_1, \vec{x}_1; t_2, \vec{x}_2)$$

$$L_{t_1} \Phi_3(t_1, \vec{x}_1; t_2, \vec{x}_2) = -6g^2 \Phi_1^3(t_1, \vec{x}_1; t_2, \vec{x}_2) - 18g^2 \phi_{cl}(t_1, \vec{x}_1) \Phi_1(t_1, \vec{x}_1; t_2, \vec{x}_2) \Phi_2(t_1, \vec{x}_1; t_2, \vec{x}_2)$$

$$L_{t_1} = \partial_{t_1}^2 - \partial_{\vec{x}_1}^2 + 3g^2 \phi_{cl}^2(t_1, \vec{x}_1)$$

Averaging over the initial conditions

$$f_w[\alpha(\vec{x}), \rho(\vec{x}), t_0] = \int \mathcal{D}\beta(\vec{x}) \langle \alpha + \frac{\beta}{2} | \hat{\rho}(t_0) | \alpha - \frac{\beta}{2} \rangle e^{i \int d^3x \rho(\vec{x}) \beta(\vec{x})}$$

$$\partial_\mu \partial^\mu \phi_{cl} + g^2 \phi_{cl}^3 = 0$$

$$\phi_{cl}(t_0, \vec{x}) = \alpha(\vec{x})$$

$$\dot{\phi}_{cl}(t_0, \vec{x}) = \rho(\vec{x})$$

Static box

Spatially homogeneous case $\partial_i \varphi(t, \mathbf{x}) = 0$

$$S = V \int dt \left(\frac{1}{2} \dot{\varphi}^2 - \frac{g^2}{4} \varphi^4 + J\varphi \right), \quad V = \int d^3x$$

Equation of motion

$$\ddot{\varphi} + g^2 \varphi^3 = J.$$

Solution for $J=0$ is periodic Jacobi elliptic function with period T_{cl} :

$$\phi_{cl}(t) = \phi_m \operatorname{cn} \left(\frac{1}{2}, g\phi_m t + C \right),$$

$$T_{cl} = \frac{4}{g\phi_m} K(1/2), \quad K(1/2) \approx 1.85.$$

Initial conditions:

$$\phi_{cl}(t_0) = \alpha, \quad \dot{\phi}_{cl}(t_0) = p, \quad \phi_m = \phi_m(\alpha, p), \quad C = C(\alpha, p)$$

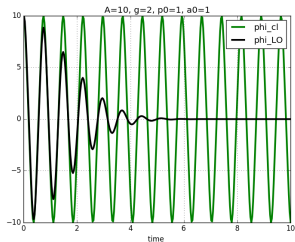
CSA at work

Wigner function

$$f_W(\alpha, p) = \frac{1}{\pi \alpha_0 p_0} e^{-\frac{(\alpha-A)^2}{\alpha_0^2}} e^{-\frac{p^2}{p_0^2}}$$

A - measure of the field excitation

Analytical answer



$$\langle \hat{\varphi} \rangle_{t_1}^{LO} \equiv \int d\alpha \int dp f_W(\alpha, p) \phi_{cl}(t_1) =$$

$$2A \sum_{k=0}^{\infty} u_k e^{-\frac{\pi^2 p_0^2}{g^2 A^4 T^2} k^2} e^{-\frac{\alpha_0^2 \pi^2 g^2}{T^2} k^2 t_1^2} \cos\left(\frac{2Ag\pi k}{T} t_1\right)$$

$$u_m = \frac{1}{T} \int_0^T \text{cn}\left(\frac{1}{2}; t\right) e^{-imt \frac{2\pi}{T}} dt$$

Static T_{μ}^{μ}

Energy-momentum tensor

$$T^{\mu\nu} = \partial^{\mu}\varphi\partial^{\nu}\varphi - g^{\mu\nu}\left(\frac{1}{2}\partial_{\lambda}\varphi\partial^{\lambda}\varphi - \frac{g^2}{4}\varphi^4\right)$$

$$\varepsilon = T^{00} = \frac{1}{2}\dot{\varphi}^2 + \frac{g^2}{4}\varphi^4, \quad p = T^{ii} = \frac{1}{2}\dot{\varphi}^2 - \frac{g^2}{4}\varphi^4$$

Equilibrium stage = well defined equation of state (scale invariance)

$$\langle T_{\mu}^{\mu} \rangle = \varepsilon - 3p = 0$$

Classical level

$$T_{\mu}^{\mu} = \varepsilon - 3p = -\dot{\varphi}_{cl} + g^2\varphi_{cl}^4 \neq 0$$

Quantum level (Keldysh-Schwinger technique)

$$\langle T_{\mu}^{\mu} \rangle_{t_1} = \int d\xi_{init} \int_{\varphi_c(t_0)=\frac{\xi_1+\xi_2}{2}}^{\varphi_c(\infty)=0} \mathcal{D}\varphi_c \int_{\varphi_q(t_0)=\xi_1-\xi_2}^{\varphi_q(\infty)=0} \mathcal{D}\varphi_q e^{iS_K[\varphi_c, \varphi_q]} \left(-\dot{\varphi}_c^2(t_1) + g^2\varphi_c^4(t_1) \right)$$

Static T_{μ}^{μ}

-Consider variation of the Keldysh action over the field φ_q

$$\frac{\delta S_K}{\delta \varphi_q} \Big|_{J=0} = -V_3(\ddot{\varphi}_c + g^2 \varphi_c^3 + \frac{3}{4} g^2 \varphi_c \varphi_q^2).$$

- Integrate by parts
- Neglect surface terms

without semiclassical decomposition

$$\langle T_{\mu}^{\mu} \rangle_{t_1} = -\frac{1}{2} \int d\xi_{init} \int \mathcal{D}\varphi_c \mathcal{D}\varphi_q e^{iS_K[\varphi_c, \varphi_q]} \partial_{t_1}^2 \varphi_c^2(t_1)$$

For large t_1 :

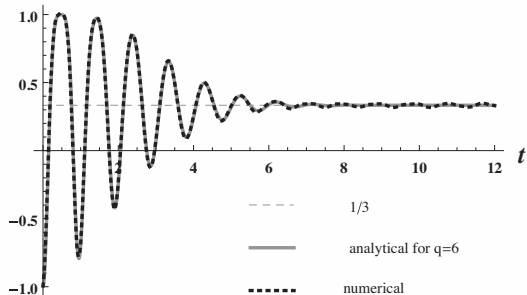
- Field in the static box relaxes to some constant, means $\langle T_{\mu}^{\mu} \rangle_{t_1 \rightarrow \infty} = 0$
- Scale invariance is restored (system forget about initial time)

Static T_{μ}^{μ} LO

$$\langle T_{\mu}^{\mu} \rangle_{t_1}^{LO} \rightarrow 0$$

$$\frac{p_{LO}(t_1 \rightarrow \infty)}{\varepsilon_{LO}} = \left[\frac{1}{3} + 8 \mathcal{I}(2) e^{-\frac{4\pi^2 p_0^2}{g^2 A^4 T^2}} e^{-\frac{4\alpha_0^2 \pi^2 g^2}{T^2} t_1^2} \cos\left(\frac{4\pi A}{T} g t_1\right) + \dots \right]$$

p_{LO}/ε_{LO}



Static T_{μ}^{μ} NLO

$$\langle T_{\mu}^{\mu} \rangle_{t_1}^{NLO} = -\frac{1}{2} \partial_{t_1}^2 \left\langle \frac{1}{2V^2 g^2 \phi_m^4} \left[\psi_0(g\phi_m t_1) + g\phi_m \psi_1(g\phi_m t_1) t_1 + g^2 \phi_m^2 \psi_2(g\phi_m t_1) t_1^2 + g^3 \phi_m^3 \psi_3(g\phi_m t_1) t_1^3 \right] \right\rangle_{i.c.} = 0$$

- $z = g\phi_m t$ - dimensionless variable

- $f_k(z_1, z)$ - variations of the classical solution

- $\psi(t) = \psi(t + T_{cl})$ - periodic functions. Numerical only.

$$t^n \int d\alpha dp f_W(\alpha, p) \sum_{k=-\infty}^{\infty} \psi_n^{(k)} e^{ikt \frac{2\pi}{T_{cl}}} = \sum_{k=-\infty}^{\infty} \psi_n^{(k)} A t^n e^{-Bk^2 t^2} e^{iCkt}, \quad n = 0, 1, 2, 3$$

Finite time evolution!

$$\psi_n(t) = \sum_{k=-\infty}^{\infty} \psi_n^{(k)} e^{ikt \frac{2\pi}{T_{cl}}}$$

Expanding T_{μ}^{μ}

$$\langle T_{\mu}^{\mu} \rangle_{\tau_1} = -\frac{1}{2} \int d\xi_{init} \int \mathcal{D}\varphi_c \mathcal{D}\varphi_q e^{iS_K^{exp}[\varphi_c, \varphi_q]} \left(\partial_{\tau_1}^2 + \frac{1}{\tau_1} \partial_{\tau_1} \right) \varphi_c^2(\tau_1)$$

The system is expanding:

- Asymptotically $\varepsilon = 0$ and $p = o$ due to expansion
- We are looking for intermediate quasi stationary state with definite EoS
- We need to take into account expansion, hence $T_{\mu}^{\mu}/\varepsilon_{LO}$, where $\varepsilon_{LO} \approx \tau^{-4/3}$

$$\frac{\langle T_{\mu}^{\mu} \rangle_{\tau_1}^{NLO}}{\varepsilon_{LO}^e} = -\frac{1}{2\varepsilon_{LO}^e(\tau_1)} \left(\partial_{\tau_1}^2 + \frac{1}{\tau_1} \partial_{\tau_1} \right) \times \left\langle \frac{\tau_1^{-\frac{2}{3}}}{2V_2^2 g^2 \xi_m^4} \left(\int_{z_0^e}^{\bar{z}} dz^e F_{exp}(z_1^e, z^e) + \int_{\bar{z}}^{z_1^e} dz^e F_{static}^e(z_1^e, z^e) \right) \right\rangle_{i.c.}.$$

$$- z^e = \frac{3}{2} g \xi_m \tau^{\frac{2}{3}}$$

- $\xi_m \approx A^{\frac{3}{2}}$ - the first term is suppressed (A -measure of the field excitation)

Expanding T_{μ}^{μ}

$$\langle T_{\mu}^{\mu} \rangle_{\tau_1} = -\frac{1}{2} \int d\xi_{init} \int \mathcal{D}\varphi_c \mathcal{D}\varphi_q e^{iS_K^{exp}[\varphi_c, \varphi_q]} \left(\partial_{\tau_1}^2 + \frac{1}{\tau_1} \partial_{\tau_1} \right) \varphi_c^2(\tau_1)$$

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$$\frac{\langle T_{\mu}^{\mu} \rangle_{\tau_1}^{NLO}}{\varepsilon_{LO}^e} = -\frac{1}{2\varepsilon_{LO}^e(\tau_1)} \left(\partial_{\tau_1}^2 + \frac{1}{\tau_1} \partial_{\tau_1} \right) \times \left\langle \frac{\tau_1^{-\frac{2}{3}}}{2V_2^2 g^2 \xi_m^4} \left(\int_{z_0^e}^{\tilde{z}} dz^e F_{exp}(z_1^e, z^e) + \int_{\tilde{z}}^{z_1^e} dz^e F_{static}^e(z_1^e, z^e) \right) \right\rangle_{i.c.} .$$

$$- z^e = \frac{3}{2} g \xi_m \tau^{\frac{2}{3}}$$

- $\xi_m \approx A^{\frac{3}{2}}$ - the first term is suppressed (A - measure of the field excitation)

Quantum corrections: expanding box

$$\langle F(\hat{\phi}) \rangle_{\tau_1}^{LO+NLO} = \left\langle F(\phi_{cl}(\tau_1)) + \frac{g^2}{4V_2^2} \int_{\tau_0}^{\tau_1} \frac{d\tau_2}{\tau_2^2} \phi_{cl}(\tau_2) \frac{\delta^3 F(\phi_{cl}(\tau_1))}{\delta J^3(\tau_2)} \Big|_{J=0} \right\rangle_{i.c.}$$

Variations

$$\frac{\delta^3 F(\phi_{cl}(\tau_1))}{\delta J^3(\tau_2)} = \frac{\partial F}{\partial \phi_{cl}} \Phi_3(\tau_1, \tau_2) + 3 \frac{\partial^2 F}{\partial \phi_{cl}^2} \Phi_1(\tau_1, \tau_2) \Phi_2(\tau_1, \tau_2) + \frac{\partial^3 F}{\partial \phi_{cl}^3} \Phi_1^3(\tau_1, \tau_2).$$

$$L_{t_1} \Phi_1(\tau_1, \tau_2) = \delta(\tau_1 - \tau_2)$$

$$L_{t_1} \Phi_2(\tau_1, \tau_2) = -6g^2 \phi_{cl}(\tau_1) \Phi_1^2(\tau_1, \tau_2)$$

$$L_{t_1} \Phi_3(\tau_1, \tau_2) = -6g^2 \Phi_1^3(\tau_1, \tau_2) - 18g^2 \phi_{cl}(\tau_1) \Phi_1(\tau_1, \tau_2) \Phi_2(\tau_1, \tau_2)$$

$$L_{\tau_1} = \partial_{\tau_1}^2 + \frac{1}{\tau_1} \partial_{\tau_1} + 3g^2 \phi_{cl}^2(\tau_1)$$

Averaging over the initial conditions

$$f_w[\alpha, p, \tau_0] = \int d\beta \langle \alpha + \frac{\beta}{2} | \hat{\rho}(t_0) | \alpha - \frac{\beta}{2} \rangle e^{i \int V_2 \tau_0 p \beta}$$

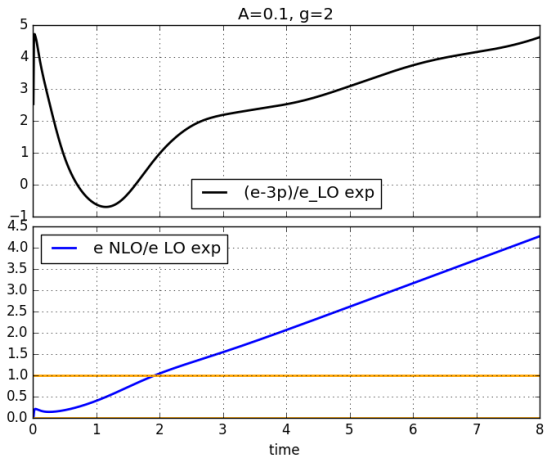
$$\partial_{\tau}^2 \phi_{cl} + \frac{1}{\tau} \partial_{\tau} \phi_{cl} + g^2 \phi_{cl}^3 = 0$$

$$\phi_{cl}(\tau_0) = \alpha$$

$$\partial_{\tau} \phi_{cl}(\tau_0) = p$$

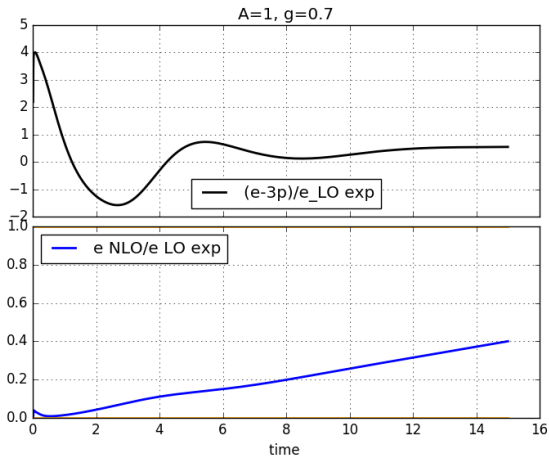
Numerical results

$A = 0.1, g = 2$. CSA does not work



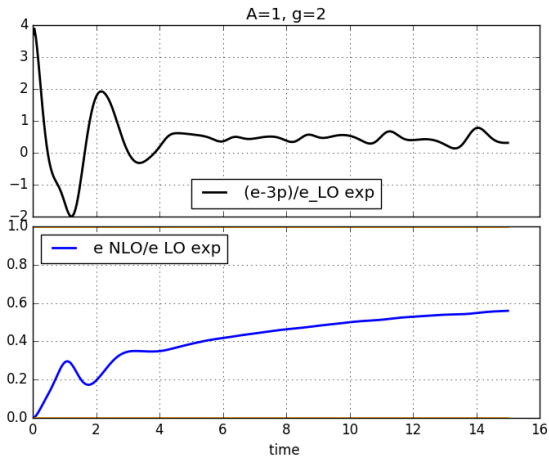
Numerical results

$A = 1, g = 0.7$. CSA works fine. $\varepsilon - 3p = \text{const}$



Numerical results

$A = 1, g = 2$. CSA works fine. $\varepsilon - 3p = \text{const}$



- The systematic procedure for calculation of quantum corrections to the Classical Statistical Approximation is developed.
- Time evolution of the $\langle T_{\mu}^{\mu} \rangle$ is analyzed for homogeneous static and longitudinally expanding models.
- It is shown that quantum corrections can change the CSA predictions.

