Stability & Electromagnetic Properties of the Magnetic DCDW Phase of Dense QCD

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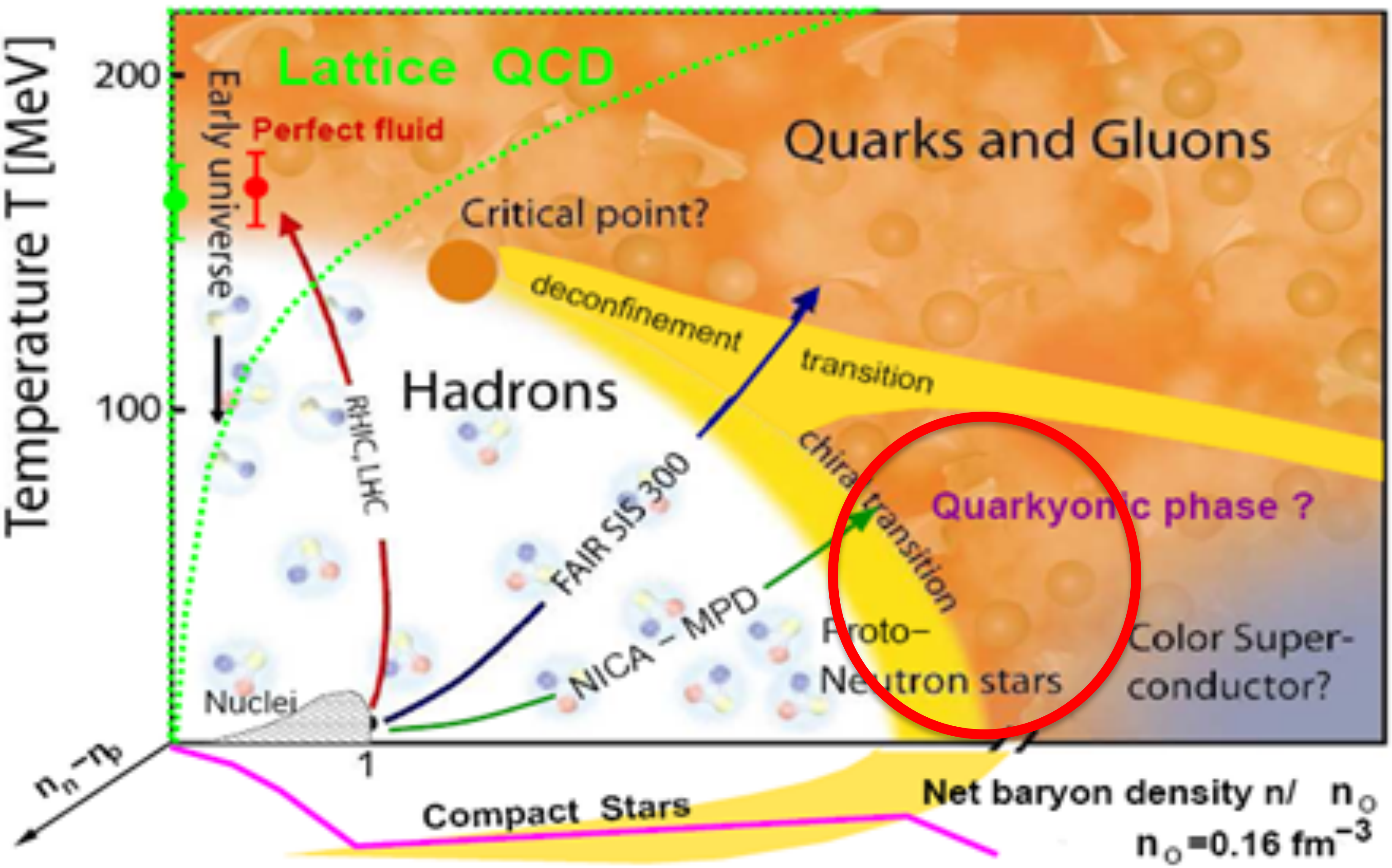
Outline

- Inhomogeneous condensates are unavoidable
- B plus DCDW dense quark matter phase -> Axion electrodynamics
- Anomalous electric transport: Dissipationless Hall Current
- Stability of the Magnetic DCDW Phase
- Analogy with Weyl Semimetals
- Observables?
- Conclusions & Outlook

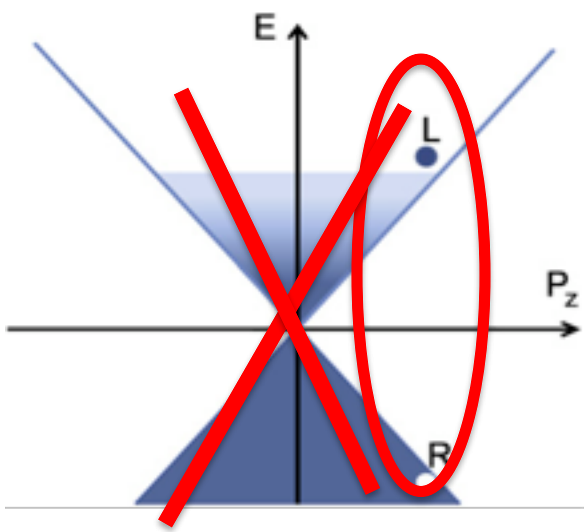
In collaboration with E.J. Ferrer. Based on
Phys.Lett. B769 (2017) 208; Nucl.Phys. B931 (2018) 192

Region of Interest for Inhomogeneous Phases



Approaching Intermediate Densities From Both Sides

Chiral Condensate

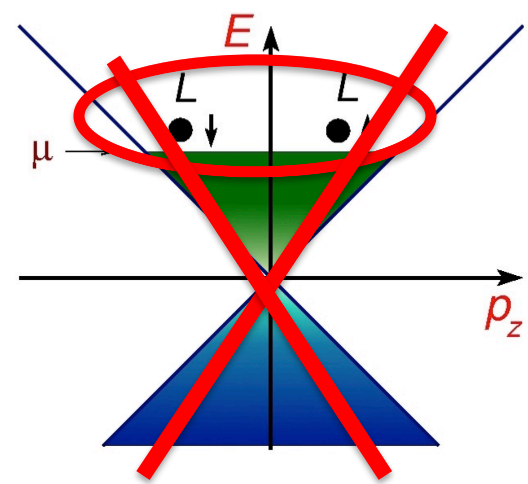


It pairs particle and antiparticle with opposite momentum (homogeneous condensate)

Not favored with increasing density

A Way out: **Spatially Modulated Chiral Condensates**

Cooper Pairing

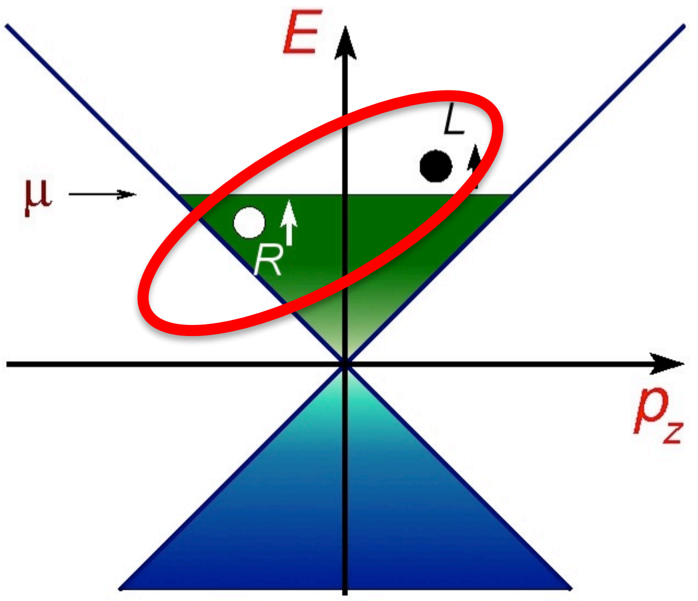


Main channel pairs quarks of different flavors with opposite spins and momenta. Favored at very high densities.

Suffers from Fermi surface mismatch with decreasing densities leading to chromomagnetic instabilities.

A Way out: **Spatially Modulated Quark-Quark Condensates**

Density Wave Pairing



It pairs particle and hole with parallel momenta (nonzero net momentum)

No Fermi surface mismatch

Favored over homogeneous chiral condensate
Favored over CS at large Nc

Dual Chiral Density Wave Phase

Nakano & Tatsumi, PRD71, '05

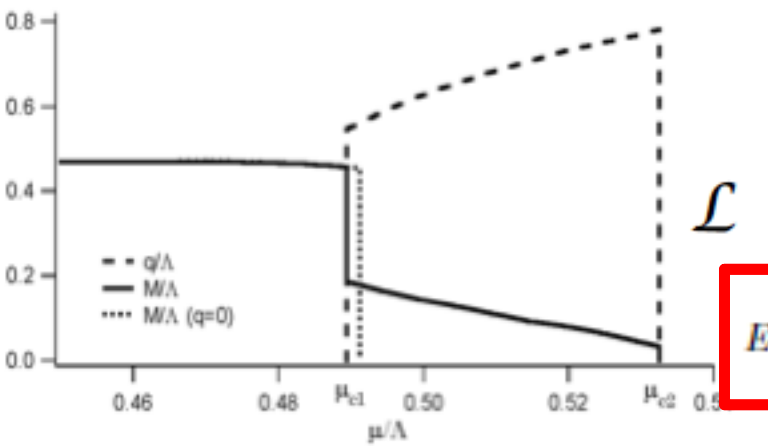
2-flavor NJL model at finite baryon density

$$\mathcal{L} = \bar{\psi}\left(\gamma^\mu\left(i\partial_\mu + \mu\delta_{\mu 0}\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\tau\gamma_5\psi)^2\right]\right.$$

DCDW condensate

$$\begin{aligned}\langle\bar{\psi}\psi\rangle &= \Delta\cos(\mathbf{q}\cdot\mathbf{r}) \\ \langle\bar{\psi}i\gamma_5\tau_3\psi\rangle &= \Delta\sin(\mathbf{q}\cdot\mathbf{r}),\end{aligned}$$

$$\mathcal{L} = \bar{\psi}(r)[i\partial\!\!\!/ + \mu\gamma^0 - m(\cos\mathbf{qr} + i\gamma^5\tau_3\sin\mathbf{qr})]\psi - \frac{m^2}{4G},$$



$$\psi \to U_A\psi = e^{-i\tau_3\gamma_5\frac{qz}{2}}\psi.$$

$$\mathcal{L} = \bar{\psi}(r)[i\partial\!\!\!/ + \mu\gamma^0 - m + \gamma^5\tau_3\gamma b)\psi - \frac{m^2}{4G}.$$

$$\boxed{E^{\pm}(\mathbf{p}) = \sqrt{E_{\mathbf{p}}^2 + |\mathbf{q}|^2/4 \pm \sqrt{(\mathbf{p}\cdot\mathbf{q})^2 + M^2|\mathbf{q}|^2}}, \quad E_{\mathbf{p}} = (M^2 + |\mathbf{p}|^2)^{1/2}}$$

Soliton/Crystalline Chiral Condensate

Nickel, PRL103, 2009; PRD80, 2009

Considered NJL model with discrete chiral symmetry

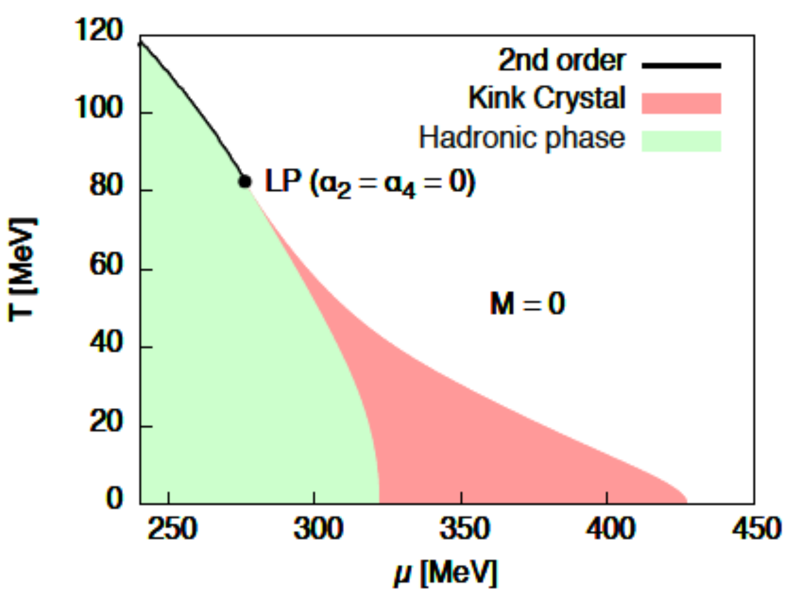
$$\mathcal{L} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + G(\bar{\psi}\psi)^2 \qquad \langle \bar{\psi}(\mathbf{x}) \psi(\mathbf{x}) \rangle = -\frac{1}{2G} M(\mathbf{x})$$

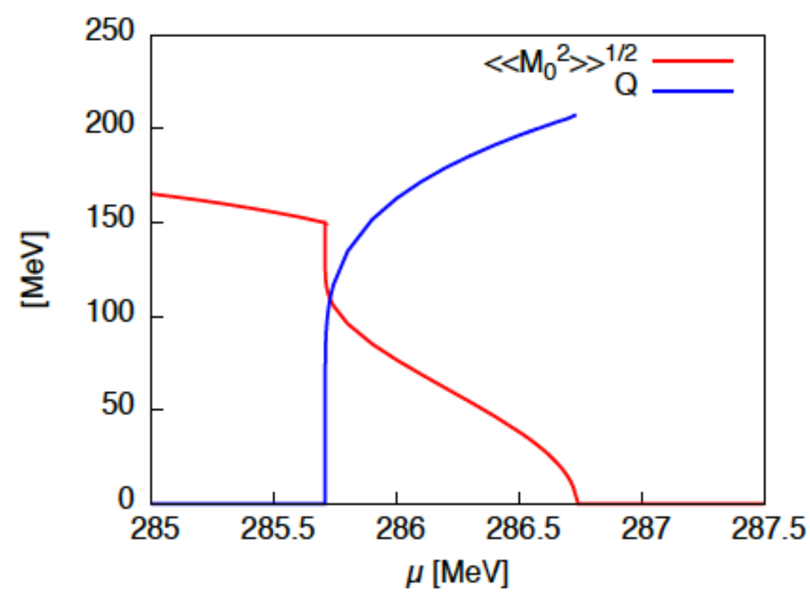
$$M_0(z) = q\sqrt{\nu}\,\mathrm{sn}(qz;\nu)$$

$$L \equiv \frac{4\,\mathrm{K}(\nu)}{q} \quad \text{and} \quad Q \equiv \frac{2\pi}{L}$$

Real kink crystal.

Energetically favored over DCDW





However,

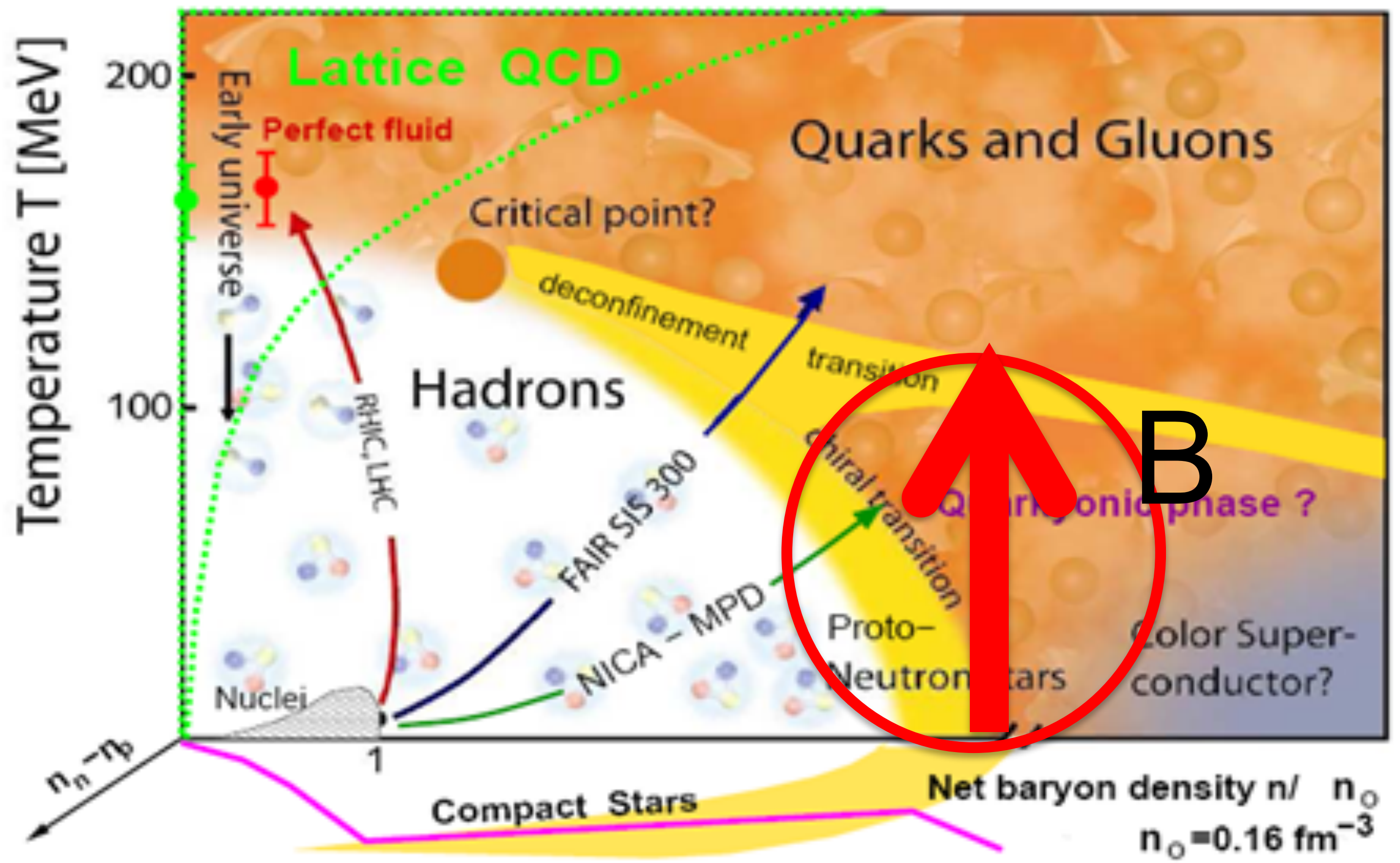
DCDW and the Kink Crystal are both unstable against thermal fluctuations.

The fluctuations wash out the long-range order at finite T!

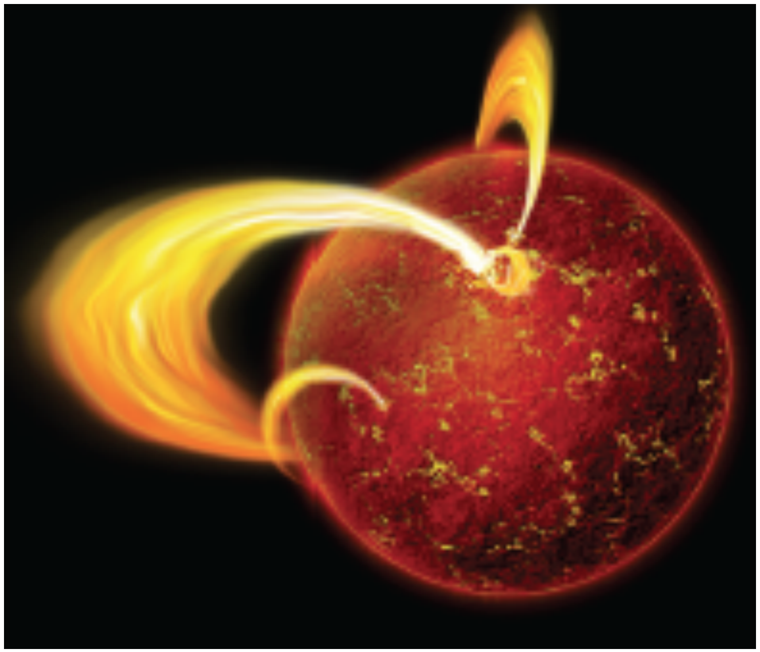
Hidaka, Kamikado, Kanazawa & Noumi, PRD 92, 2015, 034003

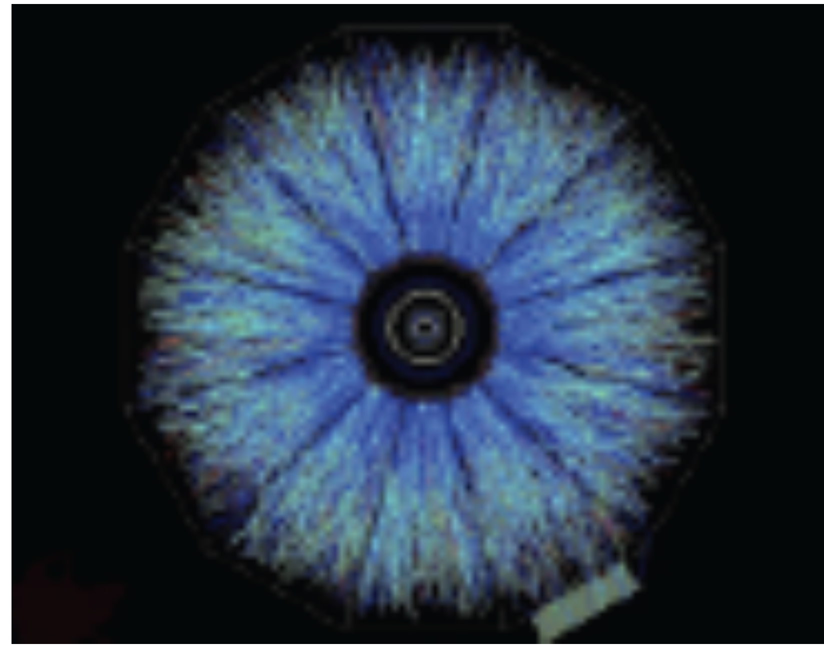
Lee et al. PRD 92, 2015, 0304024

One More Element: Magnetic Field



B is quite pervasive





Pulsar's surface:
B ~ 10^{12}–10^{14}G
Magnetars
surface: B ~ 10^{15}–10^{16}G
Core: B ~ 10^{17}– 10^{18}G

Off central
collisions:
B~ 10^{17}–10^{19}G

Dual Chiral Density Wave in a Magnetic Field

Frolov, et al PRD82,'10
Tatsumi et al PLB743,'15

DCDW Lagrangian + QED + B

$$\mathcal{L}_{MF} = \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu) + \gamma_0\mu]\psi - m\bar{\psi}e^{i\tau_3\gamma_5 qz}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{4G}$$

Performing the chiral transformation

$$\psi \to U_A\psi = e^{-i\tau_3\gamma_5\frac{qz}{2}}\psi \qquad \bar{\psi} \to \bar{\psi}\bar{U}_A = \bar{\psi}e^{-i\tau_3\gamma_5\frac{qz}{2}}$$

The MF Lagrangian becomes

$$\mathcal{L}_{MF} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu - i\mu\delta_{\mu0} + iQA_\mu - i\tau_3\gamma_5\delta_{\mu3}\frac{q}{2}) - m]\psi - \frac{m^2}{4G}$$

So the fermion spectrum is

$$\boxed{E_k^{LLL} = \epsilon\sqrt{\Delta^2 + k_3^2} + q/2, \quad \epsilon = \pm}$$

LLL mode is Asymmetric!

$$E_k^{l>0} = \epsilon\sqrt{(\xi\sqrt{\Delta^2 + k_3^2} + q/2)^2 + 2e|B|l}, \quad \epsilon = \pm, \xi = \pm, l = 1, 2, 3, \ldots$$

Nontrivial Topology of the MDCDW Phase

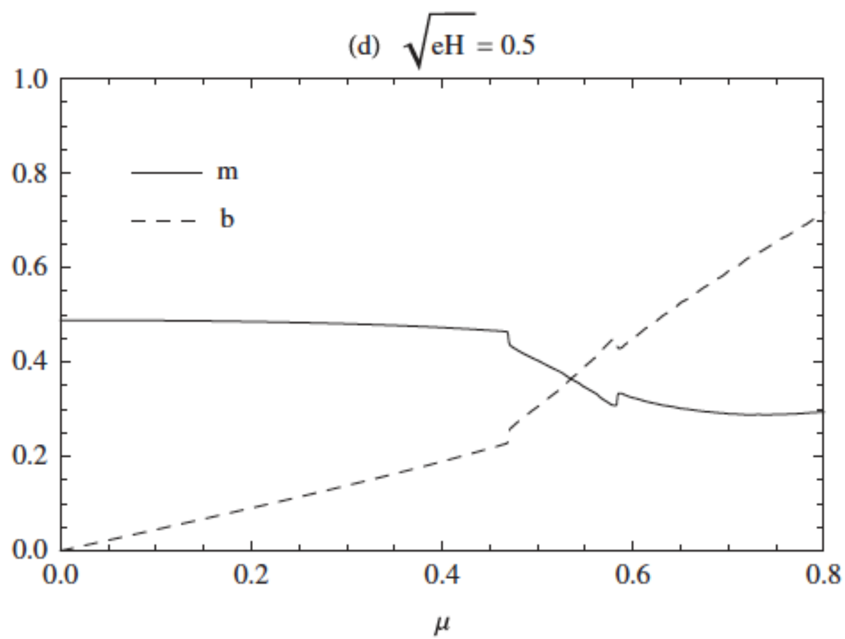
Topology emerges due to the LLL spectral asymmetry

$$\Omega = \Omega_{vac}(B) + \Omega_{anom}(B,\mu) + \Omega_{\mu}(B,\mu) + \Omega_T(B,\mu,T) + \frac{m^2}{4G}.$$

$$\boxed{\Omega^f_{anom} = -\frac{N_c|e_f B|}{(2\pi)^2}q\mu}$$

$$\rho^A_B = 3\frac{|e|}{4\pi^2}qB$$

Anomalous baryon number density



The anomaly makes the DCDW solution energetically favored over the homogeneous condensate

Axion Term

Ferrer & VI, 1512.03972

Key observation: the fermion measure is not invariant under U_A

$$\boxed{D\bar{\psi}D\psi \to (\det U_A)^{-2} D\bar{\psi}D\psi}$$

$$(\det U_A)^{-2} = e^{-2i\int d^4x\theta(x)\delta^{(4)}(0)\mathrm{tr}\tau_3\gamma_5} \quad \text{and} \quad \theta = qz/2$$

The integral is ill-defined and needs regularization.
Can be done in a gauge-invariant way with Fujikawa approach.

Dirac operator of the theory is not Hermitian

$$\boxed{\not{D}(\mu,\theta) = \not{D} + \not{D}^A}$$

$$\not{D} = \gamma_\mu(\partial_\mu + ie_f A_\mu)$$

$$\not{D}^A = \gamma_\mu(i\gamma^5 \mathrm{sgn}(e_f)\partial_\mu\theta - \mu\delta_{\mu 4})$$

Expand in the eigenfunctions of Hermitian operators

$$\not{D}^\dagger(\mu,\theta)\not{D}(\mu,\theta)\phi_n = \lambda_n^2\phi_n \qquad \not{D}(\mu,\theta)\not{D}^\dagger(\mu,\theta)\widetilde{\phi}_n = \xi_n^2\widetilde{\phi}_n$$

$$\boxed{(\det U_A)_R^{-2} = e^{i\int d^4x \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}}} \qquad \kappa = \frac{e^2}{2\pi^2}$$

Effective Action for the DCDW in B

The effective MF Lagrangian acquires an axion term:

$$\mathcal{L}_{eff} = \bar{\psi}[i\gamma^\mu(\partial_\mu + iQA_\mu - i\tau_3\gamma_5\partial_\mu\theta) + \gamma_0\mu - m]\psi - \frac{m^2}{4G}$$
$$- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \boxed{+ \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}}$$

The effective action for the electromagnetic field in the DCDW in B is then

$$\Gamma(A) = V\Omega + \int d^4x \left[-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}\right]$$
$$- \int d^4x A^\mu(x) J_\mu(x) + \cdots,$$

$\Omega = \Omega(\mu, B)$ the thermodynamic potential

$$\Gamma(A) = V\Omega + \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}\right] - \int d^4x A^{\mu}(x) J_{\mu}(x) + \cdots,$$

Notice that

$$\frac{\kappa}{4}\theta F_{\mu\nu}\tilde{F}^{\mu\nu} = -A_{\nu}\kappa\partial_{\mu}\theta\tilde{F}^{\mu\nu} = -A_{\nu}J^{\nu}_{\boldsymbol{anom}}$$

So there are two 4-current contributions

Ordinary: $\boxed{J_{\mu}(x) = (J_0, \mathbf{J})}$ given by the tadpole diagrams

Anomalous: $\boxed{J^{\nu}_{\boldsymbol{anom}} = \kappa\partial_{\mu}\theta\tilde{F}^{\mu\nu}}$

The two current contributions do not cancel each other

QED in MDCDW is Axion QED

$$\nabla \cdot \boldsymbol{E} = J_0 + \boxed{\frac{e^2}{4\pi^2} qB}$$

Anomalous charge

$$\nabla \times \boldsymbol{B} - \frac{\partial \boldsymbol{E}}{\partial t} = \boldsymbol{J}_V + \boxed{\frac{e^2}{4\pi^2} \boldsymbol{q} \times \boldsymbol{E}}$$

$$\nabla \cdot \boldsymbol{B} = 0 \quad \nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{J}_V$$

Dissipationless Hall current
⊥ to both B and E

Anomalous Hall conductivity

$$\boxed{\sigma_{xy}^{anom} = e^2 q / 4\pi^2}$$

Ferrer & VI, Phys.Lett. B769 (2017) 208; Nucl.Phys. B931 (2018) 192

Low Energy Theory of the DCDW Phase

Lee et al. PRD 92 (2015) 034024

$\boxed{B=0}$ Symmetry: SU(2)L x SU(2)R , plus spatial rotations and translation

$$\Omega_{GL}(\phi(x)) = \alpha_2(\phi\cdot\phi) + \alpha_{4,1}(\phi\cdot\phi)^2 + \alpha_{4,2}(\nabla\phi\cdot\nabla\phi) + \alpha_{6,1}(\nabla^2\phi\cdot\nabla^2\phi) + \alpha_{6,2}(\nabla\phi\cdot\nabla\phi)(\phi\cdot\phi) + \alpha_{6,3}(\phi\cdot\phi)^3 + \alpha_{6,4}(\phi\cdot\nabla\phi)^2 + \ldots$$

Consider a general fluctuation of the condensate. The phonon and the axial isospin rotation about the third axis (neutral pion) are locked. There are 3 NG modes.

$$\phi = (\Delta+\delta)\begin{pmatrix} \cos(qz+\beta_3)\cos\beta_2\cos\beta_1 \\ \cos(qz+\beta_3)\cos\beta_2\sin\beta_1 \\ \cos(qz+\beta_3)\sin\beta_2 \\ \sin(qz+\beta_3) \end{pmatrix}$$

Substituting $\phi(x)$ in the free-energy and expanding in powers of the fluctuations and their derivatives, one obtains the low-energy theory of the fluctuations.

$$\mathcal{V}_\delta = M^2\delta^2 + a_{6,4}\Delta^2(\nabla\delta)^2 + 4a_{6,1}q^2(\nabla_z\delta)^2 + a_{6,1}(\nabla^2\delta)^2,$$

$$\mathcal{L} = (\partial_0\delta)^2 + \Delta^2(\partial_0\vec{\beta}_U)^2 + \Delta^2(\partial_0\beta_3)^2 - (\mathcal{V}_\delta + \mathcal{V}_{\delta\beta} + \mathcal{V}_\beta),$$

$$\mathcal{V}_{\delta\beta} = 4q\Delta[a_{6,2}\Delta^2\delta - 2a_{6,1}\nabla^2\delta]\nabla_z\beta_3,$$

$$\mathcal{V}_\beta = a_{6,1}\Delta^2(\nabla^2\vec{\beta}_U + q^2\vec{\beta}_U)^2 + a_{6,1}\Delta^2[(\nabla^2\beta_3)^2 + 4q^2(\nabla_z\beta_3)^2].$$

(Lack of) Stability of the DCDW Phase

The spectra of the pions have soft modes in the transverse directions

$$\omega_-^2 \simeq a_{6,1}[u_{z-}^2 k_z^2 + \boxed{(\vec{k}^2)^2} - A\vec{k}^2 k_z^2 - B k_z^4,$$

Which in turn leads to infrared divergencies in the second order fluctuations

$$\Delta^2 \langle \beta_3^2(x) \rangle \simeq \frac{1}{2} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{T}{\omega_-^2}$$

The DCDW phase is not stable against the thermal fluctuations of the condensate. There is no true long-range order at nonzero temperature

Lee et al. PRD 92 (2015) 034024

Magnetic DCDW Phase

$\boxed{B\neq 0}$ Symmetry: $(U(1)_L \times U(1)_R)_f$, spatial rotation about z and translation

$\phi^T = (\sigma, \pi)$ transforms as a 2-D vector under O(2) rotations

Breaking of symmetry: translation and chiral, but they are locked like in the zero-B case.

$$\phi(x) = \phi_0(z + u(x))e^{i\pi} = \Delta e^{iq(z+u(x))}e^{i\pi} = \phi_0(z)e^{i(qu+\pi)}$$

We can then consider only one, say the phonon u(x).

Key observation: because of the magnetic field, there is an external vector

$$n_i = (0, 0, n_z)$$

that can couple to the derivative of the phonon

$$n_i\partial_i(z + u(x)) = n_z + n_z\partial_z u$$

Stability of the Magnetic DCDW Phase

The external vector leads to extra terms in the free-energy of the fluctuations that changes from

$$\boxed{\mathcal{F} = B\left[\partial_z u + \frac{1}{2}(\nabla u)^2\right]^2 + C(\nabla_\perp^2 u)^2}$$

To

$$\boxed{\mathcal{F} = \frac{A}{2}(\nabla u)^2 + B(\partial_z u)^2 + \mathcal{O}(u^3)}$$

No more soft transverse modes. The DCDW phase is stable against fluctuations at nonzero temperature

MDCDW

Described by Dirac Hamiltonian

$$H_f = -i\gamma^0\gamma^i(\partial_i + ie_f A_i + i\frac{e_f}{|e_f|}\gamma_5\partial_i\theta) + \gamma^0 m$$

Axion term in the electromagnetic action

$$S = -\kappa \int d^4x \epsilon^{\mu\alpha\nu\beta} A_\alpha \partial_\nu A_\beta \partial_\mu \theta$$

Topology is associated to asymmetry of the LLL states in the MDCDW.

$$\boxed{\sigma_{xy}^{anom} = e^2 q / 4\pi^2}$$

Anomalous Hall conductivity

Ferrer and VI, '15,'17,'18

Weyl Semimetals

Described by Dirac Hamiltonian

$$H(\mathbf{k}) = \gamma^0\gamma^i(k_i - b_i\gamma^5) + m\gamma^0 + b_0\gamma^5.$$

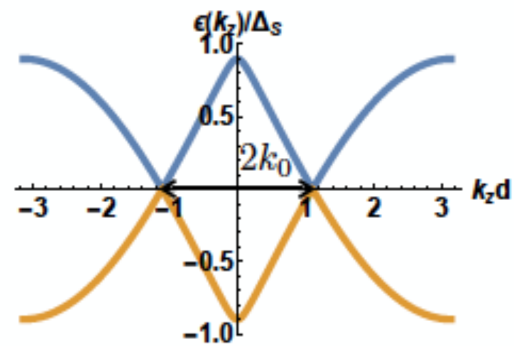
Axion term in the electromagnetic action

$$S = -\frac{e^2}{4\pi^2}\int dt\, d^3r\, b_\mu \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta.$$

Topology is associated to band structure with nodes of opposite chirality separated by 2b in momentum space

$$\boxed{\sigma_{xy} = \frac{e^2}{h}\frac{2|b|}{2\pi^2}}$$

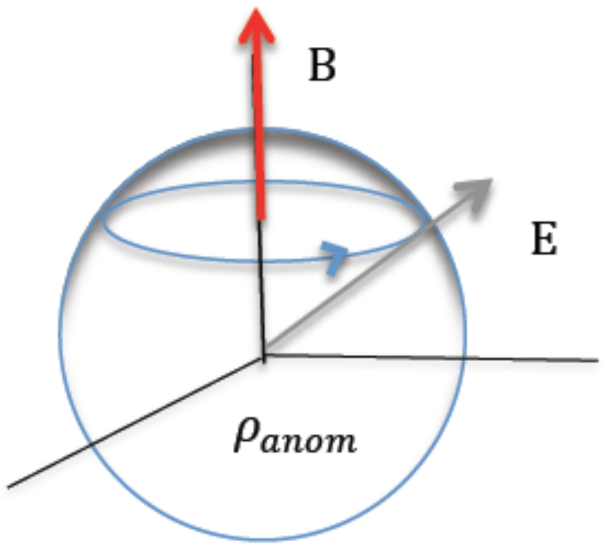
Anomalous Hall conductivity



Burkov, '17

Potential for NS Observables

1.



Solution of the Magnetar Puzzle?

The dissipationless current could sustain a large magnetic field in a magnetar for a long time.

2. Electromagnetic transport affected by robust, topological quantities like σ_{xy}.
   Thermal transport driven by phonons. Can affect NS cooling.
   How to connect to NS observables?
   To NS Mergers?

Summary:

- MDCDW has topological, dissipationless Hall current
- MDCDW is stable against fluctuations, long-range order is not washed out by T
- Anomalous electromagnetic transport similar to WSM

Outlook:

- Need to Connect to Measurable NS observables?
- Observables can confirm/falsify proposed intermediate density candidates: MDCDW, Quarkyonic, CS Phases.
- More studies on consequences of the topological properties

Additional Slides

Low Energy Theory of the DCDW Phase

B=0 Symmetry: $SU(2)_L \times SU(2)_R$, plus spatial rotations and translation

Low-energy theory described by

$$\begin{aligned}\Omega_{GL}(\phi(x)) &= \alpha_2(\phi\cdot\phi) + \alpha_{4,1}(\phi\cdot\phi)^2 + \alpha_{4,2}(\nabla\phi\cdot\nabla\phi) + \alpha_{6,1}(\nabla^2\phi\cdot\nabla^2\phi) \\ &+ \alpha_{6,2}(\nabla\phi\cdot\nabla\phi)(\phi\cdot\phi) + \alpha_{6,3}(\phi\cdot\phi)^3 + \alpha_{6,4}(\phi\cdot\nabla\phi)^2 + \dots\end{aligned}$$

$\phi^T = (\sigma, \vec{\pi})$ transform as a 4-D vector under O(4) rotations

The condensate $\phi_0^T = \Delta(\cos qz, 0, 0, \sin qz)$ minimizes the free energy

and breaks chiral symmetry, rotations about x and y, as well as translation along z.

Lee et al. PRD 92 (2015) 034024

LLL Contribution to the Ordinary four-current

$$\boxed{J^{\mu}_{LLL}(\mathrm{sgn}\,(e_f)) = (-ie_f)\frac{|e_f B|N_c T}{(2\pi)^3}\sum_{p_4}\int_{-\infty}^{\infty} dp_3 tr\left[i\gamma^{\mu} G^{\mathrm{sgn}(e_f)}_{LLL}(p)\right]}$$

$$\boxed{G^{\mathrm{sgn}(e_f)}_{LLL}(p) = D(p,q)\Delta(\mathrm{sgn}\,(e_f))} \qquad \boxed{\Delta(\mathrm{sgn}\,(e_f)) = (1 + \mathrm{sgn}\,(e_f)\, i\gamma^1\gamma^2)/2}$$

$$\boxed{D(p,q) = \frac{\gamma^{\mu}_{\parallel}\tilde{p}^{-}_{\mu} + m}{(\tilde{p}_0 - q/2)^2 - \varepsilon^2}\Delta(+) + \frac{\gamma^{\mu}_{\parallel}\tilde{p}^{+}_{\mu} + m}{(\tilde{p}_0 + q/2)^2 - \varepsilon^2}\Delta(-)}$$

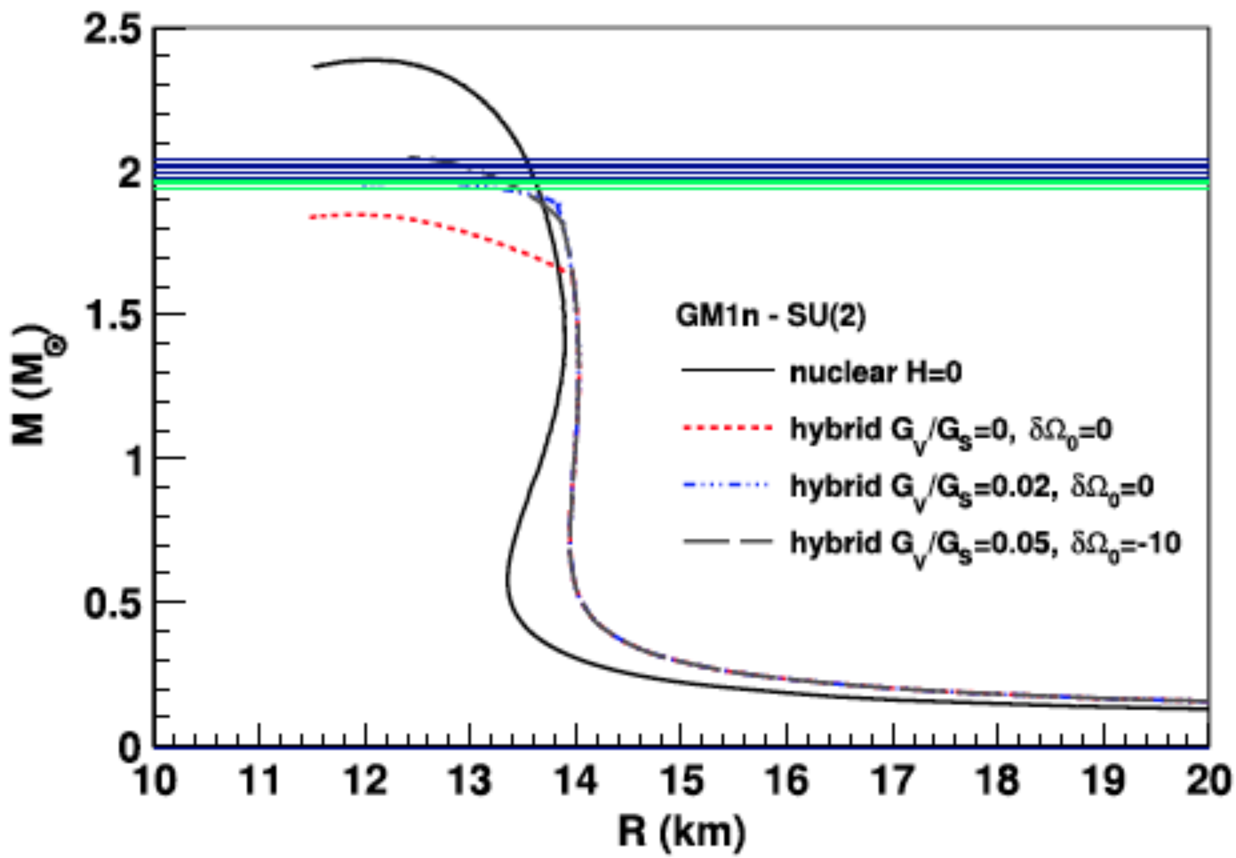
Hence, the LLL contribution to the ordinary 4-current is

$$J^0_{LLL} = \frac{e^2 B}{2\pi^2}\sqrt{(\mu - q/2)^2 - m^2}[\theta(\mu - q/2 - m) - \theta(q/2 - \mu - m)]$$

$$\sigma^{ord}_{xy} = \frac{e^2}{2\pi^2}\sqrt{(\mu - q/2)^2 - m^2}[\Theta(\mu - q/2 - m) - \Theta(q/2 - \mu - m)]$$

Ordinary current/charge cannot eliminate
the anomalous charge/current!

Isospin Asymmetric Magnetic DCDW Compatible with 2 M☉



Carignano, Ferrer, VI, Paulucci, Phys.Rev. D92 (2015) no.10, 105018