Axion Electrodynamics in High-T QCD at B≠0

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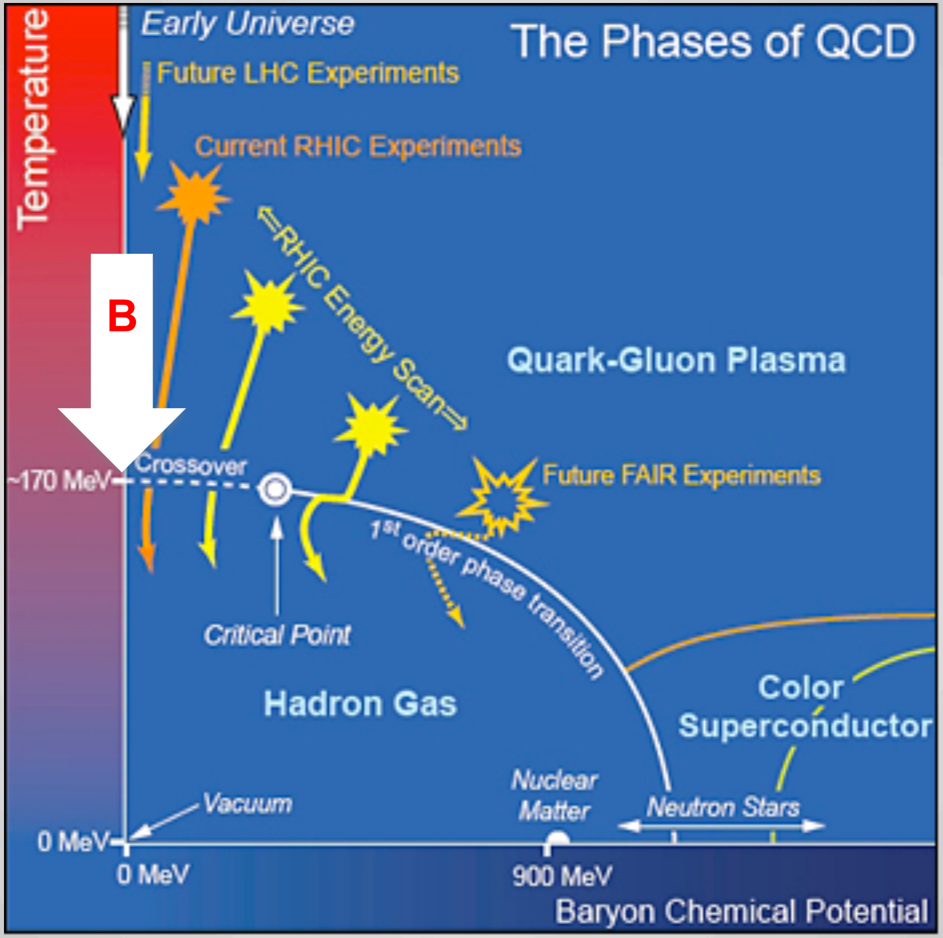
QCD Phase Diagram



Sophie Bushwick, News, July 22, 2010

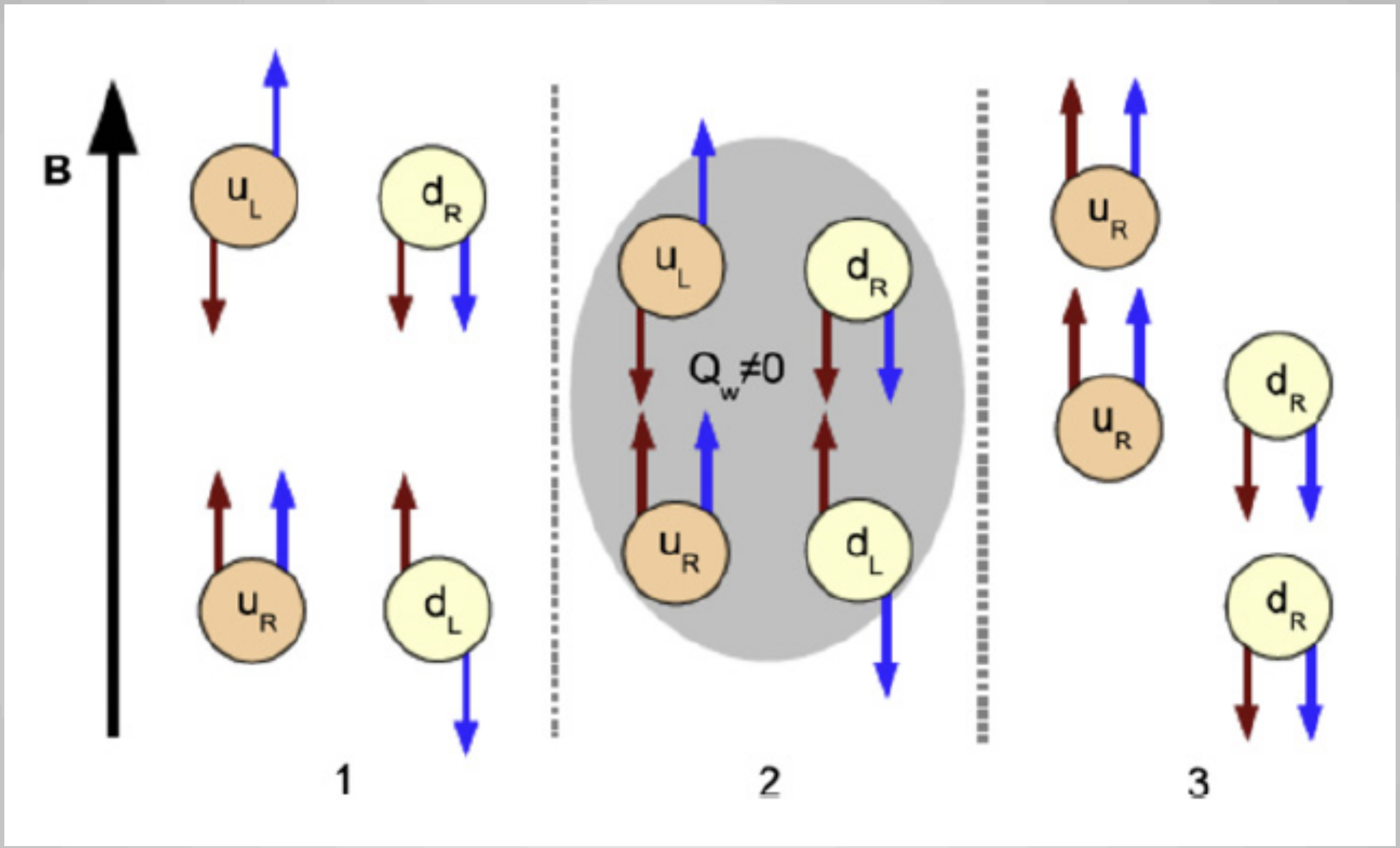
MOTIVATION

Work done in collaboration with V. de la Incera

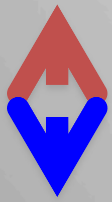
- ***To revise the transport properties of the Chiral Magnetic Effect (CME) in hot QCD.***
- ***Because of the topological origin of CME, this effect should last even at strong coupling.***
- ***Theoretically, the limit of strong coupling is accessible through the holographic correspondence, thus the CME has been studied in this context.***
- ***Following this approach some controversy has been raised regarding the fact that the CME vanishes when using a gauge invariant ultraviolet regularization.***

CME & Anomaly-Induced Transport

Fukushima/Kharzeev/Warringa PRD 78 (2008) 074033



Kharzeev/ Prog. In Part. & Nucl. Phys. 75 (2014) 133



For Q_W=-1, the L quarks will be flipped into R. The u_R will move along the field and the d_R in the opposite direction, producing a charge separation.

OUTLINE

Work done in collaboration with V. de la Incera

- ***Axion Electrodynamics***
- ***Hot QCD Effective Action at B≠0***
- ***Partition Function & Fujikawa Method in the presence of a chiral vacuum***
- ***Anomalous & Ordinary Charge and Current***
- ***No-Charge Separation in Hot QCD at B≠0***
- ***Concluding Remarks***

Axion Electrodynamics

F. Wilczek, Phys. Rev. Lett. 58 (1987) 1799

In the presence of the axion term in the Maxwell Lagrangian,

$$\frac{\kappa}{4}\theta F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

the Mawxell equations transform into:

$$\begin{aligned} \nabla\cdot\mathbf{E} &= J_0 + J_0^{anom}, \\ \nabla\times\mathbf{B} - \frac{\partial\mathbf{E}}{\partial t} &= \mathbf{J}_V + \mathbf{J}^{anom}, \\ \nabla\cdot\mathbf{B} &= 0, \quad \nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0 \end{aligned}$$

With anomalous charge and current densities given by

$$J_0^{anom} = \kappa\nabla\theta\cdot\mathbf{B}$$

$$\mathbf{J}^{anom} = -\kappa\left(\frac{\partial\theta}{dt}\mathbf{B} + \nabla\theta\times\mathbf{E}\right)$$

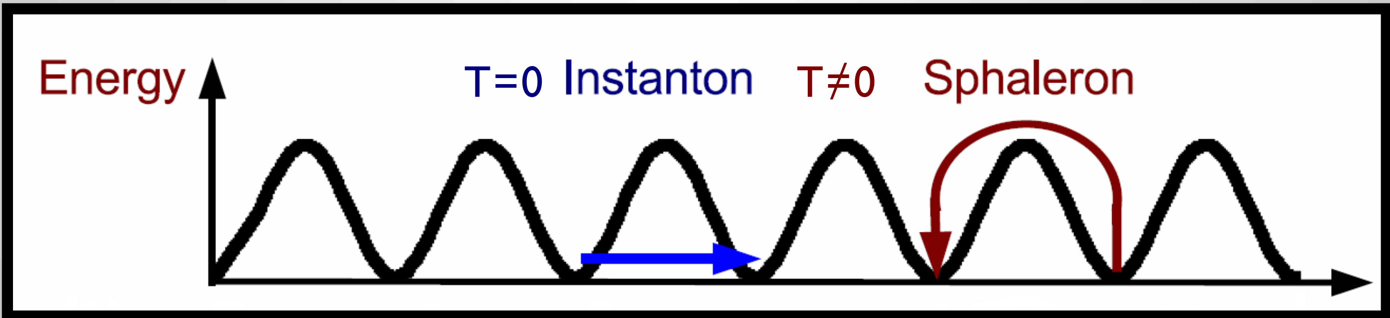
Non-Trivial QCD Vacuum at *T≠0*

QCD Classical vacuum is given by pure gauges so to minimize the energy:

$$G_i(\vec{x}) = ig^{-1}U^{-1}(\vec{x})\partial_i U(\vec{x}),$$

The vacuum configurations are characterized by a topological number:

$$n_w = \frac{1}{24\pi^2}\int d^3x\epsilon^{ijk}tr(U^{-1}\partial_i U)(U^{-1}\partial_j U)(U^{-1}\partial_k U)$$



The superposition of vacuum terms yields to the axion term: $\theta G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a$

Axial Anomaly:

$$(N_R - N_L) = -\frac{g^2 N_f}{16\pi^2}\int d^4x G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a$$

QCD×QED with θ–vacuum term

$$\boxed{\mathcal{L}_{QCD+QED} =}$$

$$\boxed{\begin{aligned} = & -\frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g^2}{32\pi^2}\theta G^{a}_{\mu\nu}\widetilde{G}^{\mu\nu}_{a} \\ & + \bar{\psi}[i\gamma^{\mu}(\partial_{\mu} - igG^{a}_{\mu}\frac{\lambda_a}{2} + iQA_{\mu})]\psi, \end{aligned}}$$

Time-dependent Axion field

$$\boxed{\partial_t \theta = 2\mu_5}$$

The coefficient μ_5 is related to the rate of chirality change.

Chiral Gauge Transformation

Local $U_A(1)$ chiral transformation

$$\psi(x) \to e^{i\theta\gamma^5/2N_f}\psi(x)$$
$$\bar{\psi}(x) \to \bar{\psi}(x)e^{i\theta\gamma^5/2N_f}$$

$$\mathcal{Z}[G^a_\mu, A_\mu, \theta] = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}e^{iS_\psi(G^a_\mu, A_\mu, \theta)},$$

$$\mathcal{L}_\psi = \bar{\psi}[i\gamma^\mu(\partial_\mu - igG_\mu + iQA_\mu + i\gamma^5\partial_\mu\theta/2N_f)]\psi,$$

$$\mathcal{D}\bar{\psi}(x)\mathcal{D}\psi(x) \to J_{\bar{\psi}}J_\psi\mathcal{D}\bar{\psi}(x)\mathcal{D}\psi(x)$$

$$J_{\bar{\psi}}J_\psi \neq 1$$

Fujikawa's Method

K. Fujikawa, PRL 42 (1979) 1195; PRD 21 (1980) 2848

$$
\begin{aligned}
(\mathrm{Det} U_A)^{-1} &= e^{-\mathrm{Tr}\ln U_A} = e^{-\int d^4x \langle x|\mathrm{tr}\ln U_A|x\rangle} \\
&= e^{-\int d^4x \delta^4(0)\frac{\theta(x)}{2N_f} i\mathrm{tr}(\gamma_5)},
\end{aligned}
$$

The exponent in the last expression is ill-defined and requires an appropriate regularization. With this goal we follow Fujikawa's method at finite temperature on which the Jacobian is regularized in a gauge invariant way.

Fujikawa's Method

- The Jacobian should be regularized in a gauge invariant way.
- A representation in the eigenfunctions of an Euclidean operator that is gauge invariant and Hermitian (antiHermitian) should be used.
- To ensure the unitarity of the transformation, the chosen representation should diagonalize the fermion action.

Euclidean Dirac Operator

In the present system the Dirac operator in Euclidean space is

$$\not{D}(\theta) = \not{D} + \not{D}^A$$

$$\not{D} = \gamma_\mu(\partial_\mu - igG^a_\mu \frac{\lambda_a}{2} + iQ^f A_\mu)$$ **Hermitian**

$$\not{D}^A = i\gamma_\mu\gamma^5\partial_\mu\theta/2N_f - \mu\gamma_4$$ **Anti-Hermitian**

Hermitian operator

$$\not{D}^\dagger(\theta)\not{D}(\theta)\phi_n = \lambda_n^2\phi_n \qquad \not{D}(\theta)\not{D}^\dagger(\theta)\widetilde{\phi}_n = \widetilde{\lambda}_n^2\widetilde{\phi}_n$$

Fermion field expansion

$$\psi(x) = \sum_n a_n\phi_n(x), \qquad \bar{\psi}(x) = \sum_n \bar{b}_n\widetilde{\phi}_n^\dagger(x)$$

Jacobian in the Φ(x) representation

$$J_\psi^{(f)} J_{\bar{\psi}}^{(f)} = e^{\frac{-N_c}{2N_f}\mathrm{tr}\int d_E^4 x\theta(x)\sum_n[\phi_n^\dagger(x)i\gamma_5\phi_n(x)+\widetilde{\phi}_n^\dagger(x)i\gamma_5\widetilde{\phi}_n(x)]}$$

Heat-Kernel Regularization

M. Nakahara, *Geometry, Topology and Physics*.

In the $\Phi(x)$ representation the Dirac operator is diagonalized,

$$S_F = \int d_E^4 x \bar{\psi} \not{D}(\theta) \psi = \sum_n \lambda_n \bar{b}_n a_n$$

We introduce damping factors with a regulator M that should be taken to infinity at the end.

$$J_\psi^{(f)} J_{\bar{\psi}}^{(f)} = e^{-\frac{N_c}{2N_f}(\mathcal{I}_R + \widetilde{\mathcal{I}}_R)}$$

$$\begin{aligned}
\mathcal{I}_R &= \lim_{M\to\infty} \int d_E^4 x \theta(x) \mathrm{tr} \sum_n \phi_n^\dagger(x) i\gamma_5 e^{-\lambda_n^2/M^2} \phi_n(x) \\
&= \lim_{M\to\infty} \int d_E^4 x \theta(x) \mathrm{tr} \sum_n \phi_n^\dagger(x) i\gamma_5 e^{-\not{D}^\dagger(\theta)\not{D}(\theta)/M^2} \phi_n(x)
\end{aligned}$$

$$\begin{aligned}
\widetilde{\mathcal{I}}_R &= \lim_{M\to\infty} \int d_E^4 x \theta(x) \mathrm{tr} \sum_n \widetilde{\phi}_n^\dagger(x) i\gamma_5 e^{-\lambda_n^2/M^2} \widetilde{\phi}_n(x) \\
&= \lim_{M\to\infty} \int d_E^4 x \theta(x) \mathrm{tr} \sum_n \widetilde{\phi}_n^\dagger(x) i\gamma_5 e^{-\not{D}(\theta)\not{D}^\dagger(\theta)/M^2} \widetilde{\phi}_n(x)
\end{aligned}$$

Heat-Kernel Regularization

$$J_{\psi}^{(f)} J_{\bar{\psi}}^{(f)} = e^{-\frac{N_c}{2N_f}(\mathcal{I}_R + \widetilde{\mathcal{I}}_R)}$$

The product of Dirac operators entering in the Jacobians are given by

$$\begin{aligned}
\not{D}^{\dagger}(\theta)\not{D}(\theta) = & + \frac{iq_f}{4}[\gamma_\mu, \gamma_\nu]F_{\mu\nu} + \frac{1}{4}[\gamma_\mu, \gamma_\nu]G_{\mu\nu} - isgn(q_f)\gamma_5 [\gamma_\mu, \gamma_4]\, \mu\partial_\mu\theta \\
& + (\partial_\mu\theta)^2 - (D_\mu)^2 + i(\mathrm{sgn}(q_f)[\gamma_\mu, \gamma_\nu]\gamma_5\partial_\mu\theta D_\nu) + \mu^2
\end{aligned}$$

$$\begin{aligned}
\not{D}(\theta)\not{D}^{\dagger}(\theta) = & + \frac{iq_f}{4}[\gamma_\mu, \gamma_\nu]F_{\mu\nu} + \frac{1}{4}[\gamma_\mu, \gamma_\nu]G_{\mu\nu} - isgn(q_f)\gamma_5 [\gamma_\mu, \gamma_4]\, \mu\partial_\mu\theta \\
& + (\partial_\mu\theta)^2 - (D_\mu)^2 - i(\mathrm{sgn}(q_f)[\gamma_\mu, \gamma_\nu]\gamma_5\partial_\mu\theta D_\nu) + \mu^2
\end{aligned}$$

with

$$G_{\mu\nu} = -ig\partial_\mu G_\nu^a \frac{\lambda_a}{2} + ig\partial_\nu G_\mu^a \frac{\lambda_a}{2} - g^2[G_\mu^a \frac{\lambda_a}{2}, G_\nu^b \frac{\lambda_b}{2}],$$

$$D_\mu = \partial_\mu - igG_\mu^a \frac{\lambda_a}{2} + iq_f A_\mu$$

Jacobian of the Chiral Transformation

After taken the trace and the M → ∞ limit

$$\lim_{M\to\infty} (\mathcal{I}_R + \widetilde{\mathcal{I}}_R) = \int \frac{d_E^4 k}{(2\pi)^4} e^{-k^2} \left(-\frac{q_f^2}{32} t_r \gamma^5 [\gamma_\mu, \gamma_\nu][\gamma_\alpha, \gamma_\beta] F_{\mu\nu} F_{\alpha\beta} + \right.$$

$$\left. + \frac{1}{32} t_r \gamma^5 [\gamma_\mu, \gamma_\nu][\gamma_\alpha, \gamma_\beta] G_{\mu\nu} G_{\alpha\beta} \right)$$

After integration in k and using for the Matsubara sum

$$\frac{1}{\beta} \sum_{n=-\infty}^{+\infty} exp\left[-k_4^2\right] = \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} exp\left[-\frac{(2n+1)^2 \pi^2}{\beta^2}\right] = \int_{-\infty}^{\infty} \frac{dk^0}{2\pi} exp\left[-(k^0)^2\right]$$

It is obtained for the Jacobians

$$\begin{aligned} J_\psi^{(f)} J_{\bar{\psi}}^{(f)} &= \exp \int d^4x \frac{N_c \theta(x)}{N_f} \left(\frac{g^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} + \frac{q_f^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \right) \\ &= \exp \int d^4x \left(\theta(x) \frac{g^2 N_c}{N_f 32\pi^2} G_{\mu\nu}^a \tilde{G}_a^{\mu\nu} - \partial_\mu \theta(x) \frac{N_c q_f^2}{N_f 4\pi^2} \epsilon^{\mu\alpha\nu\beta} A_\alpha \partial_\nu A_\beta \right) \end{aligned}$$

Where it was used

$$tr\gamma^5 [\gamma^\mu, \gamma^\nu][\gamma^\alpha, \gamma^\beta] = -16 i \epsilon^{\mu\nu\alpha\beta}$$

Effective QCD×QED Lagrangian

$$\mathcal{L}_{QCD+QED} =$$

Initial

$$= - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{g^2}{32\pi^2} \theta G^a_{\mu\nu} \widetilde{G}_a^{\mu\nu}$$
$$+ \bar{\psi}[i\gamma^\mu(\partial_\mu - igG^a_\mu \frac{\lambda_a}{2} + iQA_\mu)]\psi,$$

$$\mathcal{L}_{QCD+QED} =$$

Final

$$\left[\bar{\psi}(i\gamma^\mu(\partial_\mu - igG_\mu + iq_f A_\mu + i\gamma^5 \partial_\mu \theta / 2N_f)\psi - \frac{N_c q_f^2}{N_f 4\pi^2} \partial_\mu \theta(x) \epsilon^{\mu\alpha\nu\beta} A_\alpha \partial_\nu A_\beta\right]$$

Charge & Currents in Hot QCD at *B≠0*

Photon-Field Effective Action

$$\Gamma(A) = -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu} - \sum_{f=u,d,s}\frac{\mu_5 q_f^2}{2\pi^2}\int d^4x \epsilon^{0\alpha\nu\beta}A_\alpha\partial_\nu A_\beta - i\ln\mathcal{Z}(A)$$

In the photon-field expansion

$$\begin{aligned}\Gamma(A) &= -V\Omega + \int d^4x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \sum_{f=u,d,s}\frac{\mu_5 q_f^2}{2\pi^2}\int d^4x\epsilon^{0\alpha\nu\beta}A_\alpha\partial_\nu A_\beta\right] \\ &+ \sum_{n=1}^{\infty}\int dx_1 ... dx_n \Pi_{\mu_1,\mu_2,...\mu_n}(x_1,x_2,...x_n)A^{\mu_1}(x_1)...A^{\mu_n}(x_n)\end{aligned}$$

Linear-response theory

$$\Gamma(A) \simeq -V\Omega + \int d^4x\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \sum_{f=u,d,s}\frac{\mu_5 q_f^2}{2\pi^2}\int d^4x\epsilon^{0\alpha\nu\beta}A_\alpha\partial_\nu A_\beta\right] - \int d^4x A_\mu(x)J^\mu(x)$$

Anomalous Charge & Currents at $B\neq 0$

$$\Gamma_{anom} = \sum_{f=u,d,s} \frac{q_f^2 \mu_5}{2\pi^2} \int d^4x A_3 B$$

$$J^0_{anom} = 0$$

Anomalous Charge Density

$$J^3{}_{anom} = -\kappa(\frac{\partial\theta}{dt})B$$

Anomalous Current Density

$$= -\sum_{f=u,d,s} \frac{q_f^2 \mu_5}{2\pi^2} B = -\frac{e^2 \mu_5}{3\pi^2} B$$

Anomalous current density as a Polarization current

$$\mathbf{J}_{anom} = \partial_t \mathbf{P} = -\frac{e^2 \mathbf{B}}{6\pi^2} \partial_t \theta, \quad \Rightarrow \quad \mathbf{P} = -\left(\frac{e^2 \theta}{6\pi^2}\right) \mathbf{B}$$

Ordinary Charge & Currents at *B≠0*

Green's Function in the lowest Landau level

$$G^{-1}_{LLL}(p) = \gamma^{\mu}_{\|}(\tilde{p}^{\|}_{\mu} + \bar{\mu}_5 \delta_{\mu 0}\gamma^5) - m,$$

$$G^{f}_{LLL}(p) = G_{LLL}(p)\Delta(\mathrm{sgn}\,(q_f)),$$

$$G_{LLL}(p) = \frac{\gamma^{\|}_{\mu}\tilde{p}^{\mu}_{+} + m}{(\tilde{p}_0)^2 - \varepsilon^2_{+}}\Delta(+) + \frac{\gamma^{\|}_{\mu}\tilde{p}^{\mu}_{-} + m}{(\tilde{p}_0)^2 - \varepsilon^2_{-}}\Delta(-)$$

$$\tilde{p}^{\nu}_{\pm} \;=\; (p^0 - \mu, 0, 0, p^3 \pm \bar{\mu}_5)$$

$$\gamma^{\|}_{\nu} \;=\; (\gamma_0, 0, 0, \gamma_3)$$

$$\bar{\mu}_5 = \mu_5/N_f$$

$$\varepsilon_{\pm} \;=\; \sqrt{(p_3 \pm \bar{\mu}_5)^2 + m^2}$$

$$\Delta(\pm) = (I \pm i\gamma^1\gamma^2)/2$$

Tadpole Contribution

$$J^{\mu}_{LLL}(\mathrm{sgn}\,(q_f)) = -\frac{q_f|q_f B|N_c}{(2\pi)^2\beta}\sum_{p_4}\int_{-\infty}^{\infty} dp_3 tr\left[\gamma^{\mu}\frac{\gamma^4(p^4+i\mu)+\gamma^3(p^3+\bar{\mu}_5)-m}{(p^4+i\mu)^2+\varepsilon^2_{\mathrm{sgn}(e_f)}}\Delta(\mathrm{sgn}\,(e_f))\right]$$

$$\Delta(\mathrm{sgn}\,(e_f)) = (I + \mathrm{sgn}\,(q_f)\, i\gamma^1\gamma^2)/2$$

Ordinary Charge

$$J^{0}_{LLL}(\mathrm{sgn}\,(q_f)) = \frac{q_f|q_f B|N_c}{2\pi^2}\int_{-\infty}^{\infty} dp_3\left[n_F\left[\beta(\varepsilon_{sgn(q_f)}+\mu)\right] - n_F\left[\beta(\varepsilon_{sgn(q_f)}-\mu)\right]\right]$$

Ordinary Current

$$J^{3}_{LLL}(\mathrm{sgn}\,(q_f)) = \frac{q_f|q_f B|N_c}{2\pi^2\beta}\sum_{p_4}\int_{-\infty}^{\infty} dp_3\frac{p^3+\mathrm{sgn}\,(q_f)\,\bar{\mu}_5}{(p_4+i\mu)^2+\varepsilon^2_{sgn(q_f)}},$$

$$J^{3}_{LLL}(\mu_5) = \sum_{f=u,d,s} J^{3}_{LLL}(sgn(q_f)) = N_c\frac{e^2\bar{\mu}_5}{3\pi^2}B = \frac{e^2\mu_5}{3\pi^2}B$$

No Charge Separation

- The two currents cancel out, hence there is no CME

$$\mathbf{J}^{Total} = \mathbf{J}_V + \mathbf{J}^{anom} = 0$$

- No net current ➔ No charge separation
- The anomalous current is produced by a time-dependent medium polarization generated by topological-charge-changing transitions in the non-trivial QCD vacuum. This topological term produces an effective current opposite to the field direction.
- On the other hand, the ordinary current is produced by the motion of the LLL quarks in a chirally unbalanced medium that leads to a net motion of positively charged particles in the field direction and negative charges in the opposite direction. This has been considered as the hallmark of the CME effect.

Concluding Remarks

1. In equilibrium, there is no net charge separation in high T QCD in a magnetic field.
2. The ordinary current of the LLL quarks is cancel out by the anomalous current coming from the axion term in the action.
3. The non trivial topology in high T QCD comes from the gluon ground state, so the spectrum of the fermions remains symmetric and as a consequence, no net topological effect is manifested in the electromagnetic response of the system .
4. This differs from the MDCDW phase of high dense QCD in a magnetic field, where the non trivial topology is associated to the asymmetry of the LLL fermion spectrum, which in turn leads to an anomalous (topological) electromagnetic response.