Electroweak in a Quench

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$$\mathcal{L}_{EW} = \mathcal{L}_{Fermion} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Boson}$$

$$\begin{aligned}\mathcal{L}_{Fermion} = & - \bar{l}_L \gamma^\mu D_\mu l_L - \bar{\nu}_R \gamma^\mu D_\mu \nu_R - \bar{e}_R \gamma^\mu D_\mu e_R \\ & - \bar{q}_L \gamma^\mu D_\mu q_L - \bar{u}_R \gamma^\mu D_\mu u_R - \bar{d}_R \gamma^\mu D_\mu d_R\end{aligned}$$

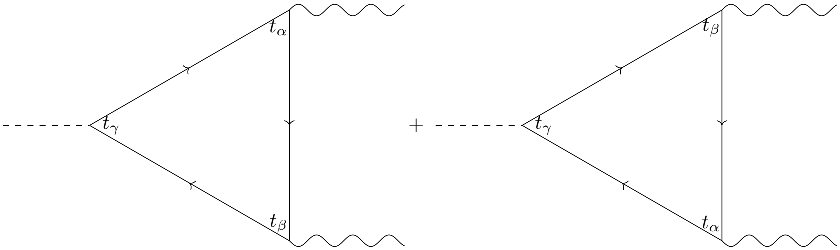
$$\mathcal{L}_{Yukawa} = - G^u \bar{q}_L \tilde{\phi} u_R - G^d \bar{q}_L \phi d_R - G^\nu \bar{l}_L \tilde{\phi} \nu_R - G^e \bar{l}_L \phi e_R + h.c.$$

$$\mathcal{L}_{Boson} = - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - (D_\mu \phi)^\dagger (D^\mu \phi) - \frac{m_H^2}{2v^2} \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2$$

Quantum anomaly in Electroweak theory

$$\mathcal{L}_{Fermion} = -\bar{l}_L\gamma^\mu D_\mu l_L - \bar{\nu}_R\gamma^\mu D_\mu \nu_R - \bar{e}_R\gamma^\mu D_\mu e_R$$
$$- \bar{q}_L\gamma^\mu D_\mu q_L - \bar{u}_R\gamma^\mu D_\mu u_R - \bar{d}_R\gamma^\mu D_\mu d_R$$

Following Weinberg's calculation (*The Quantum Theory of Fields II*),



we can have the equation of anomaly,

$$\langle \partial_\mu j^\mu_\gamma \rangle = \frac{D_{\alpha\beta\gamma}}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}F^\alpha_{\mu\nu}F^\beta_{\rho\sigma}, \quad D_{\alpha\beta\gamma} \equiv \mathrm{Tr}\left[\{t_\alpha, t_\beta\}t_\gamma\right]$$

Quantum anomaly in Electroweak theory

$$\langle \partial_\mu j_b^\mu \rangle = \langle \partial_\mu j_l^\mu \rangle = 3\left(\frac{g_2^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a - \frac{g_1^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma} \right)$$

	weak isospin T_3	hypercharge Y	baryon num	lepton num
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$+1/2$ $-1/2$	$-1/6$	$1/3$	0
$u_R^{\mathcal{C}}$	0	$+2/3$	$-1/3$	0
$d_R^{\mathcal{C}}$	0	$-1/3$	$-1/3$	0
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$	$+1/2$ $-1/2$	$+1/2$	0	1
$e_R^{\mathcal{C}}$	0	-1	0	-1

$$+1 = 2 \times \frac{1}{3} \times 3 \times \frac{1}{2} = 2 \times \frac{1}{2}$$

$$-1 = 2 \times \frac{1}{3} \times 3 \left[2\left(-\frac{1}{6}\right)^2 - \left(\frac{2}{3}\right)^2 - \left(-\frac{1}{3}\right)^2 \right] = 2\left[2\left(\frac{1}{2}\right)^2 - (-1)^2 \right]$$

At first glance, one might believe the many fractional numbers happen to cancel each other, leaving the minus one. But in fact the existence of anomaly is a more general conclusion.

When there are n_c colours of quarks

	weak isospin T_3	hypercharge Y	baryon num
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$+1/2$ $-1/2$	y_1	$1/n_c$
$u_R^{\mathcal{C}}$	0	y_2	$-1/n_c$
$d_R^{\mathcal{C}}$	0	y_3	$-1/n_c$

Gauge anomaly cancellation requires:

$$SU(n_c)\text{-}SU(n_c)\text{-}U(1): \qquad 2y_1 + y_2 + y_3 = 0$$

$$SU(2)\text{-}SU(2)\text{-}U(1): \qquad n_c y_1 + \frac{1}{2} = 0$$

$$U(1)\text{-}U(1)\text{-}U(1): \qquad n_c\left(2y_1^3 + y_2^3 + y_3^3\right) + 2\left(\frac{1}{2}\right)^3 + (-1)^3 = 0$$

we obtain, $y_1 = -\frac{1}{2n_c}$, $\quad (y_2, y_3) = \left(\frac{1}{2} + \frac{1}{2n_c}, -\frac{1}{2} + \frac{1}{2n_c}\right)$. And the anomaly is always there

$$-1 = 2 \times \frac{1}{n_c} \times n_c \left[2\left(-\frac{1}{2n_c}\right)^2 - \left(\frac{1}{2} + \frac{1}{2n_c}\right)^2 - \left(-\frac{1}{2} + \frac{1}{2n_c}\right)^2\right]$$

$$\langle \partial_\mu j_b^\mu \rangle = \langle \partial_\mu j_l^\mu \rangle = 3\left(\frac{g_2^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a - \frac{g_1^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma}\right)$$

More general hypercharges

	weak isospin T_3	hypercharge Y	baryon num	lepton num
$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$+1/2$ $-1/2$	$y_1 = -x_1/n_c$	$1/n_c$	0
$u_R^{\mathcal{C}}$	0	$y_2 = x_1/n_c - b$	$-1/n_c$	0
$d_R^{\mathcal{C}}$	0	$y_3 = x_1/n_c + b$	$-1/n_c$	0
$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$	$+1/2$ $-1/2$	x_1	0	1
$\nu_R^{\mathcal{C}}$	0	$x_2 = -x_1 - c$	0	-1
$e_R^{\mathcal{C}}$	0	$x_3 = -x_1 + c$	0	-1

$$SU(n_c)\text{-}SU(n_c)\text{-}U(1): \quad 2y_1 + y_2 + y_3 = 0$$

$$SU(2)\text{-}SU(2)\text{-}U(1): \quad n_c y_1 + x_1 = 0$$

$$U(1)\text{-}U(1)\text{-}U(1): \quad n_c\left(2y_1^3 + y_2^3 + y_3^3\right) + \left(2x_1^3 + x_2^3 + x_3^3\right) = 0$$

$$\text{Graviton-Graviton-}U(1): \quad n_c\,(2y_1 + y_2 + y_3) + (2x_1 + x_2 + x_3) = 0$$

If $x_1 \neq 0$, we find $b^2 = c^2$ and the coefficient of hypercharge term in the anomaly is,

$$-4b^2 = 2 \times \frac{1}{n_c} \times n_c\left[2y_1^2 - y_2^2 - y_3^2\right] = \left[2x_1^2 - x_2^2 - x_3^2\right]$$

$$\langle \partial_\mu j_b^\mu \rangle = \langle \partial_\mu j_l^\mu \rangle = 3\left(\frac{g_2^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma} W_{\mu\nu}^a W_{\rho\sigma}^a - 4b^2 \frac{g_1^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma}\right)$$

When there are Higgs fields

Now consider the Yukawa terms:

$$\mathcal{L}_{Yukawa} = -\,G^u\bar{q}_L\tilde{\phi}u_R - G^d\bar{q}_L\phi d_R - G^\nu\bar{l}_L\tilde{\phi}\nu_R - G^e\bar{l}_L\phi e_R + h.c.$$

Each term has zero hypercharge in total, i.e. $-y_1 - a - y_2 = 0$, $-y1 + a - y_3 = 0$, ..., with hypercharge of Higgs fields $a = -\frac{1}{2}$. Therefore we obtain $b = c = a$, leaving x_1 still a free parameter.

$$\langle\partial_\mu j_b^\mu\rangle = \langle\partial_\mu j_l^\mu\rangle = 3\left(\frac{g_2^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}W_{\mu\nu}^a W_{\rho\sigma}^a - 4\left(-\frac{1}{2}\right)^2\frac{g_1^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}B_{\rho\sigma}\right)$$

• Electromagnetic $\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ is absent in the anomaly equation, when written with $A_\mu,\ Z_\mu,\ W_\mu^\pm$.

Does the anomaly respect spontaneous symmetry breaking? Do Higgs fields determine only the minus sign?

Dynamic of gauge and Higgs fields

$$\mathcal{L}_{Boson} = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu,a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \left(D_\mu \phi\right)^\dagger \left(D^\mu \phi\right) - \frac{m_H^2}{2v^2} \left(\phi^\dagger \phi - \frac{v^2}{2}\right)^2$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu = \partial_\mu - i \frac{g_2}{2} \sigma^a W^a_\mu + i \frac{g_1}{2} B_\mu$$

Special numbers in Electroweak theory

- Chern-Simons number of $SU(2)$ gauge group

$$
\begin{aligned}
N_{cs,SU(2)}(t_f) &= \frac{1}{64\pi^2}\int_{t_0}^{t_f} dt \int d^3x \; \epsilon^{\mu\nu\rho\sigma} g_2^2 W^a_{\mu\nu} W^a_{\rho\sigma} \\
&= \frac{1}{32\pi^2}\int d^3x \; \epsilon^{ijk} g_2^2 \left(W^a_i W^a_{jk} - \frac{g_2}{3}\epsilon^{abc} W^a_i W^b_j W^c_k \right)\Big|_{t=t_0}^{t=t_f}
\end{aligned}
$$

Vacuum is degenerated (without fermions). Pure gauge $W_\mu = \frac{i}{g_2} U\partial_\mu U^\dagger$: Field strength $W_{\mu\nu}$ vanishes, so does the energy. $N_{\mathrm{cs,SU(2)}}$ is integer.

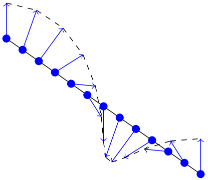
- Chern-Simons number of hypercharge $U(1)$ gauge group

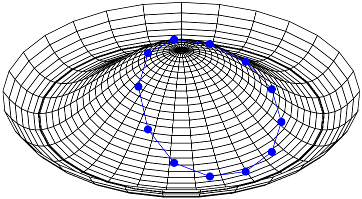
$$
\begin{aligned}
N_{cs,U(1)}(t_f) &= \frac{1}{64\pi^2}\int_{t_0}^{t_f} dt \int d^3x \; \epsilon^{\mu\nu\rho\sigma} g_1^2 B_{\mu\nu} B_{\rho\sigma} \\
&= \frac{1}{32\pi^2}\int d^3x \; \epsilon^{ijk} g_1^2 B_i B_{jk}\Big|_{t=t_0}^{t=t_f}
\end{aligned}
$$

There is no degeneration. Finite $N_{\mathrm{cs,U(1)}}$ requires finite energy.

- Baryon number is not conserved: $\Delta \mathrm{N_b}(\Delta \mathrm{N_l}) = 3\left(\mathrm{N_{cs,SU(2)}} - \mathrm{N_{cs,U(1)}}\right)$.

Special numbers in Electroweak theory





Winding number 1, illustrated for Higgs in 1+1d

- Higgs winding number

$$N_w = -\frac{1}{24\pi^2}\int d^3x \epsilon^{ijk}\mathrm{Tr}\left[\Omega^\dagger\partial_i\Omega\Omega^\dagger\partial_j\Omega\Omega^\dagger\partial_k\Omega\right]$$

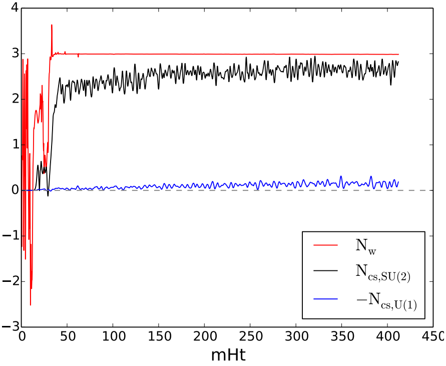
Goldstone modes are represented in the unitary matrix

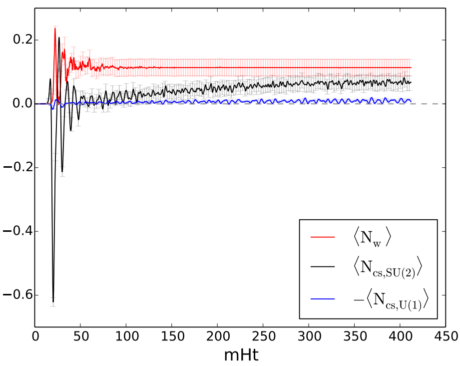
$$\Omega = \frac{\Phi}{|\Phi|}, \quad \Phi = \sqrt{2}(i\sigma^2\phi^*, \phi)$$

The winding number is (small) gauge invariant, perturbation invariant and CP-odd. It is well defined when $|\phi| \neq 0$.

Relationship among the numbers

- On vacuum, $D_\mu \phi = 0 \rightarrow W_\mu = -\frac{i}{g_2}\partial_\mu \Omega \Omega^\dagger \rightarrow N_{\mathrm{cs,SU(2)}} = N_{\mathrm{w}}$.
- Beyond vacuum, a typical output looks like

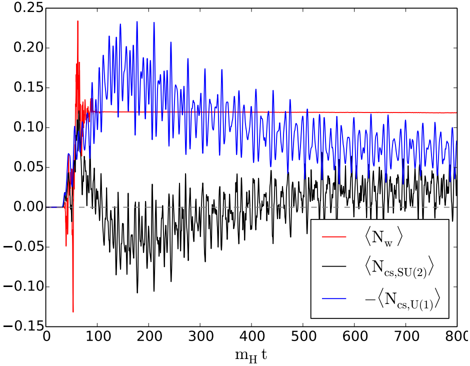


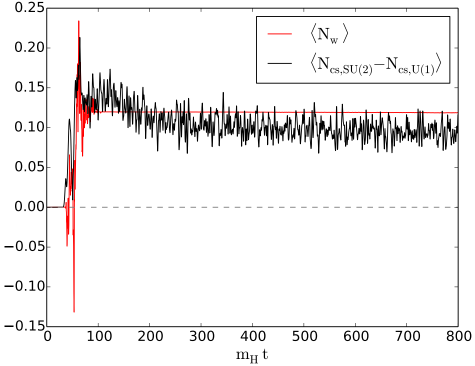


(L): in a single configuration; (R): averaged over hundreds of configurations

- N_w is an integer for a classical field configuration, indicating the Chern-Simons number of the vacuum.

Not small $N_{\mathrm{cs,U(1)}}$





Chern-Simons numbers (L): in individual; (R): in combination.

$$\langle N_w \rangle \cong \langle N_{cs,SU(2)} - N_{cs,U(1)} \rangle$$

Goldstone-Wilczek current in $SU(2) \times U(1)$

We write down a conserved current, whose charge is $-N_{\mathrm{w}}$

$$j^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left[\Omega^\dagger\partial_\nu\Omega\Omega^\dagger\partial_\rho\Omega\Omega^\dagger\partial_\sigma\Omega\right]$$

As Goldstone-Wilczek did, we promote the current into a local, conserved and $SU(2) \times U(1)$ gauge covariant current, with covariant derivative,

$$D_\mu\Omega = \partial_\mu\Omega - ig_2 W_\mu^a\frac{\sigma^a}{2}\Omega - ig_1\Omega\frac{\sigma^3}{2}B_\mu = \partial_\mu\Omega - iW_\mu^M\Omega - i\Omega B_\mu^M$$

$$j^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left[\Omega^\dagger D_\mu\Omega\Omega^\dagger D_\nu\Omega\Omega^\dagger D_\rho\Omega + i3\Omega^\dagger W_{\nu\rho}^M D_\sigma\Omega - i3\Omega^\dagger D_\sigma\Omega B_{\nu\rho}^M\right]$$

The anomaly reappears in a different way

Its charge is in a unique combination of three numbers:

$$N_{cs,SU(2)} - N_{cs,U(1)} - N_w =$$
$$\frac{1}{24\pi^2}\int d^3x \epsilon^{ijk} \mathrm{Tr}\left[\Omega^\dagger D_i \Omega \Omega^\dagger D_j \Omega \Omega^\dagger D_k \Omega + i3\Omega^\dagger W^M_{ij} D_k \Omega - i3\Omega^\dagger D_k \Omega B^M_{ij}\right]$$

- Higgs (winding number) knows there better be a -1.
- This holds true even if hypercharge U(1) is substituted by a SU(2) group, original Goldstone-Wilczek current.
- In the expression above, there is no term of magnetic helicity (electromagnetic $F\tilde{F}$). All terms contain massive fields (W and Z bosons). When $T \ll m_W$

$$N_{cs,SU(2)} - N_{cs,U(1)} - N_w \to 0$$

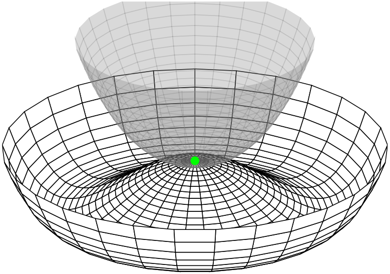
$$\Delta N_b(\Delta N_l) = 3\langle N_{cs,SU(2)} - N_{cs,U(1)}\rangle \cong 3\langle N_w\rangle$$

Baryogenesis in the agent of N_{w}

Creation of Higgs winding number means the creation of net baryon number. In order to generate a net N_{w}, we need

- turbulent process to attain windings
- CP-violation to retain windings

Electroweak phase transition



- Universe cools down as it expands

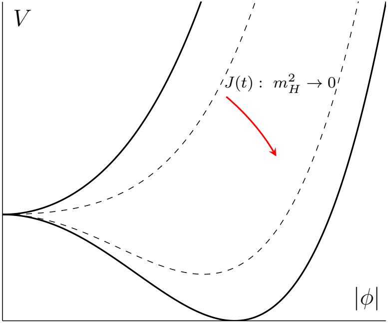
$$t \sim \frac{3.7 \times 10^{18}\ \mathrm{GeV}}{\sqrt{g_*}T^2}$$

$$g_* \sim 100, \quad T_f = 100\ \mathrm{GeV}$$

$$t \sim \frac{3.7 \times 10^{13}}{\mathrm{GeV}} \sim 2 \times 10^{-11}\mathrm{sec}$$

- In comparison, electroweak (energy) time scale is $t \sim 0.01/\mathrm{GeV}$.

Quench: Phase transition in a dramatic way



$\mathcal{L} \to \mathcal{L} - J(t)\phi^\dagger\phi$

We complete the phase transition by changing external current J(t) from m_H^2 to 0 in time interval τ_q.

$$V(\phi) \to -\frac{m_H^2}{2}|\phi|^2 + \frac{m_H^2}{2v^2}|\phi|^4 + J(t)|\phi|^2$$

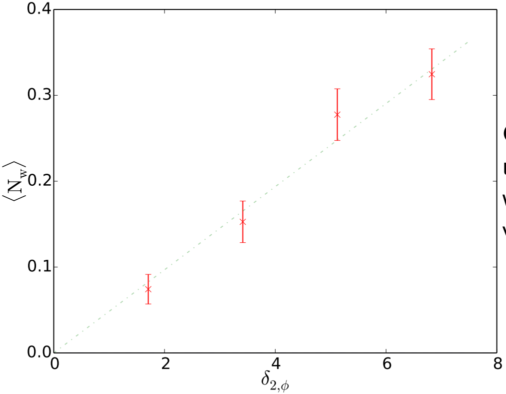
IR modes ($p^2 < m_H^2/2 - J(t)$) grow exponentially:

$$\ddot{\phi}_p + \left(p^2 - \frac{m_H^2}{2} + J(t)\right)\phi_p = 0$$

CP-violating terms

$$\mathcal{L}_{2,\phi} = -\frac{3\delta_{2,\phi} g_2^2}{64\pi^2 m_W^2} \phi^\dagger \phi \epsilon^{\mu\nu\rho\sigma} W^a_{\mu\nu} W^a_{\rho\sigma}$$

- by integrating out SM fermions with CKM-matrix.
- by integrating out degree of freedom beyond SM
- Including the term, we have to solve implicit equation of motion.



CP-violating parameters in use are in a linear regime. So we can extrapolate to small values.

In the name of Cold Electroweak Baryogenesis

- Each real component of Higgs field is initialised

$$\Phi_p = \frac{1}{\sqrt{2\omega_p}} \eta_p^j \sqrt{n_p + \frac{1}{2}}, \quad \Pi_p = \sqrt{\frac{\omega_p}{2}} \xi_p^j \sqrt{n_p + \frac{1}{2}}$$

with random numbers η and ξ, so that ensemble average

$$\langle \Phi_p \Phi_p^\dagger \rangle = \frac{1}{\omega_p} \left(n_p + \frac{1}{2} \right), \quad \langle \Pi_p \Pi_p^\dagger \rangle = \omega_p \left(n_p + \frac{1}{2} \right)$$

  - "Just the half" initialisation (Smit and Tranberg, JHEP 0212 (2002) 020)

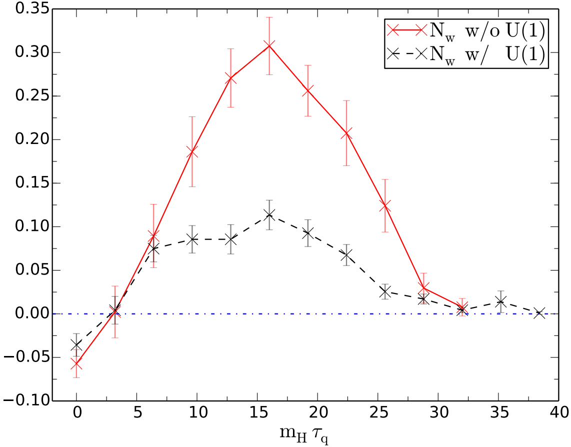
$$p^2 < \frac{m_H^2}{2}, \quad n_p = 0$$

  - Gradient flow to attain zero net Higgs $SU(2) \times U(1)$ charges.

$$\frac{dc}{d\tau} = -\frac{\partial H}{\partial c}, \quad H = \sum \left(\sum_x \rho(x) \right)^2, \quad c = \eta, \xi \ldots$$

- Gauge fields are determined by Gauss's law. $(W_i^a = B_i = 0)$

Results of by-hand quench



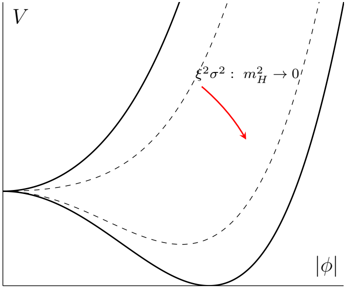
- $N_w = 0$ at very slow transition.
- Hypercharge $U(1)$ makes a difference.
- Optimal quench time $m_H \tau_q \approx 16$.
- N_w changes its sign from fast to slow quenches.

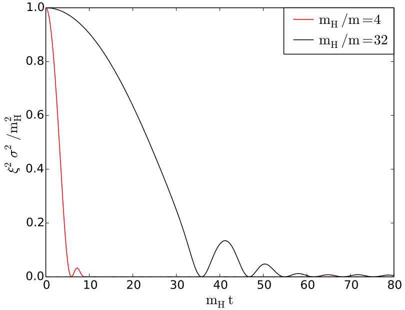
On dynamical quench

Quench in a dynamical model

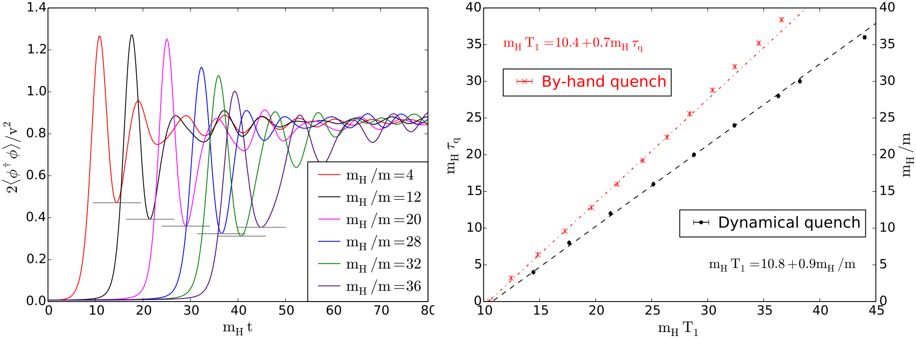
$$\mathcal{L}_\sigma = -\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{m^2}{2}\sigma^2 - \xi^2\sigma^2\phi^\dagger\phi$$

- $J(t) \to \xi^2\sigma^2$
- σ oscillates and decreases.
- backreaction of Higgs fields
- σ could be inflaton, curvaton, ...

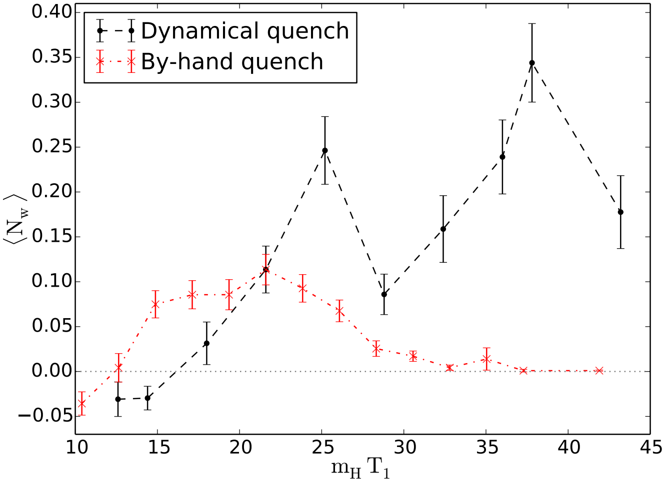




Time of first Higgs rolling back



$$\frac{v\xi}{2m} = 8$$

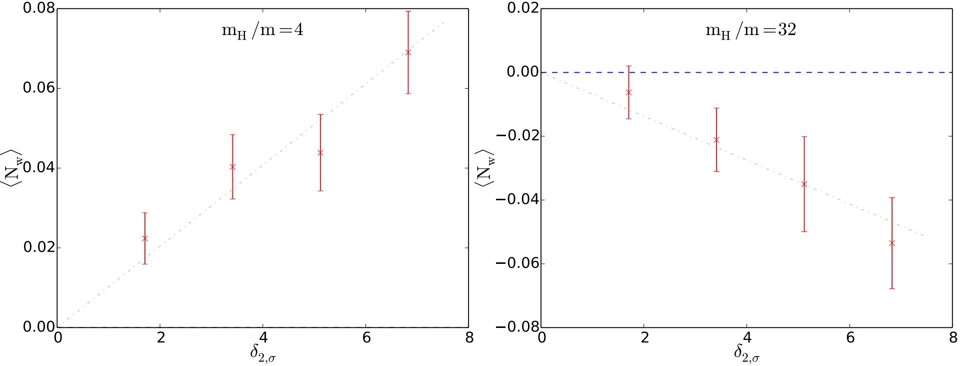


- Resonate at the second bump.
- N_w changes its sign from fast to slow quenches.

On more CP-violating sources

CP-violating terms

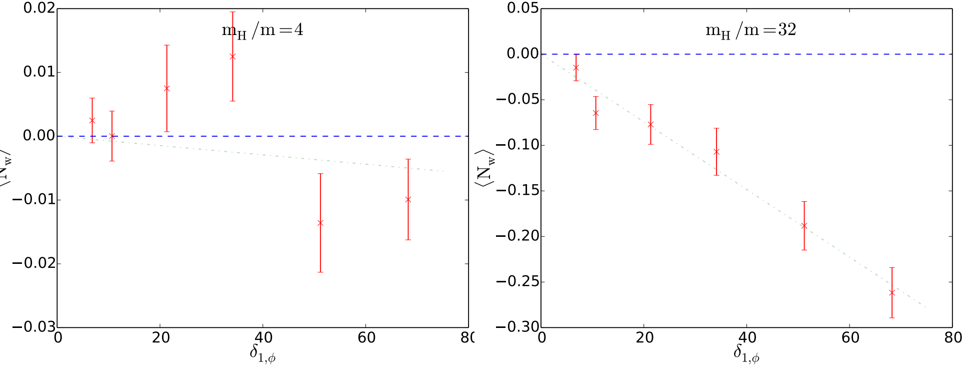
$$\mathcal{L}_{2,\sigma} = -\frac{3\delta_{2,\sigma} g_2^2}{64\pi^2 m_W^2} \xi^2 \sigma^2 \epsilon^{\mu\nu\rho\sigma} W^a_{\mu\nu} W^a_{\rho\sigma}$$



(L) fast quench; (R) slow quench.

CP-violating terms

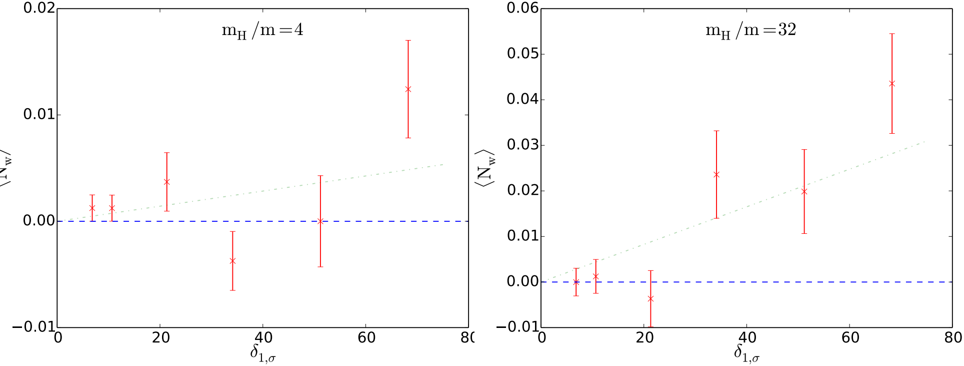
$$\mathcal{L}_{1,\phi} = -\frac{3\delta_{1,\phi} g_1^2}{64\pi^2 m_W^2} \phi^\dagger \phi \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} B_{\rho\sigma}$$



(L) fast quench; (R) slow quench.

CP-violating terms

$$\mathcal{L}_{1,\sigma} = -\frac{3\delta_{1,\sigma} g_1^2}{64\pi^2 m_W^2}\xi^2\sigma^2\epsilon^{\mu\nu\rho\sigma}B_{\mu\nu}B_{\rho\sigma}$$



(L) fast quench; (R) slow quench.

Summary

- $\Delta N_b(\Delta N_l) = 3\langle N_{cs,SU(2)} - N_{cs,U(1)}\rangle \cong 3\langle N_w\rangle$
  This makes more sense when we include hypercharge $U(1)$.
- If we can put SM into a quench, we would likely observe a net baryon creation ($\delta_{cp} \geq 10^{-5}$ to account for the observed baryon asymmetry in the Universe of $\eta \approx 6 \times 10^{-10}$). The sign of N_w can be different from fast to slow quenches.

Thanks for your attention!

Magnetic helicity

Electromagnetic field in general gauge fixing

$$A_\mu = \left[n^a W_\mu^a - \frac{i}{g_2} \left(\varphi^\dagger \partial_\mu \varphi - \partial_\mu \varphi^\dagger \varphi \right) \right] \sin\theta - B_\mu \cos\theta$$

$$\varphi = \frac{\phi}{\sqrt{\phi^\dagger \phi}}, \quad n^a = -\varphi^\dagger \sigma^a \varphi$$

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\mu - \partial_\nu A_\mu \\ &= \left(n^a W_{\mu\nu}^a - \frac{2i}{g_2} \left[\left(D_\mu \varphi \right)^\dagger \left(D_\nu \varphi \right) - \left(D_\nu \varphi \right)^\dagger \left(D_\mu \varphi \right) \right] \right) \sin\theta - B_{\mu\nu} \cos\theta \end{aligned}$$

Magnetic helicity

$$h = \frac{1}{2} \int d^3x \epsilon^{ijk} A_i F_{jk},$$