# low-scale resonant leptogenesis at complete leading order[1,2]

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[1] based on collaboration with Jacopo Ghiglieri

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# some neutrino phenomenology

## lots of experimental data to respect[3]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} ,$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$

(NH) : $\theta_{12} = 33.48^{\circ}{}^{+0.78}_{-0.75}\,, \quad \theta_{23} = 42.3^{\circ}{}^{+3.0}_{-1.6}\,, \quad \theta_{13} = 8.50^{\circ}{}^{+0.20}_{-0.21}\,,$

$\Delta m^2_{21} = 7.50^{+0.19}_{-0.17} \times 10^{-5}\text{eV}^2\,, \quad \Delta m^2_{31} = 2.457^{+0.047}_{-0.047} \times 10^{-3}\text{eV}^2\,,$

(IH) : $\theta_{12} = 33.48^{\circ}{}^{+0.78}_{-0.75}\,, \quad \theta_{23} = 49.5^{\circ}{}^{+1.5}_{-2.2}\,, \quad \theta_{13} = 8.51^{\circ}{}^{+0.20}_{-0.21}\,,$

$\Delta m^2_{21} = 7.50^{+0.19}_{-0.17} \times 10^{-5}\text{eV}^2\,, \quad \Delta m^2_{23} = 2.449^{+0.048}_{-0.047} \times 10^{-3}\text{eV}^2\,.$

[3] e.g. M.C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, *Updated fit to three neutrino mixing: status of leptonic CP violation*, 1409.5439

## introduce right-handed fields $\Rightarrow$ majorana masses

$$\begin{aligned} L_E &\equiv L_{\text{old-SM}} + \bar{\nu}_R \partial\!\!\!/ \nu_R \\ &+ \tilde{\phi}^\dagger \bar{\nu}_R h_\nu \ell_L + \bar{\ell}_L h_\nu^\dagger \nu_R \tilde{\phi} \\ &+ \frac{1}{2}(\bar{\nu}_R^c M \nu_R + \bar{\nu}_R M^\dagger \nu_R^c) \end{aligned}$$

singular value decomposition & field rotation

$\Rightarrow M = \text{diag}(M_1, M_2, M_3)$, where $M_I \geq 0$

there are two types of eigenstates, "active" and "sterile" neutrinos

## look first at seesaw in the hierarchical limit

suppose that $M_I$ are hierarchical and "large" and that only one neutrino yukawa coupling contributes to a given mass difference

$$\Rightarrow |\Delta m| = \frac{|h_{Ia}|^2 v^2}{2M_I} \,, \quad v \simeq 246 \text{ GeV} \,.$$

for two different $\Delta m$ need at least two different $M_I$

traditionally: $M_I \overset{\text{GUT?}}{\sim} 10^{15}$ GeV $\Rightarrow h_{Ia} \sim 1$

more recently: $M_I \sim 1...100$ GeV $\Rightarrow h_{Ia} \sim 10^{-7}...10^{-6}$

## general parametrization for the two-flavour situation:[4]

$$M \equiv \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} , \quad R \equiv \begin{pmatrix} \cos z & \sin z \\ -\sin z & \cos z \end{pmatrix} , \quad z \in \mathbb{C} ,$$

$$P_{\rm NH} \equiv \begin{pmatrix} 0 & e^{-i\phi_1} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad P_{\sf IH} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi_1} & 0 \end{pmatrix} .$$

$$\Rightarrow \boxed{h_\nu = -i\sqrt{M}\, R\, P\, \underbrace{\sqrt{m_\nu}\, V^\dagger}_{\text{data}} \frac{\sqrt{2}}{v}}$$

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[4] J.A. Casas and A. Ibarra, *Oscillating neutrinos and* $\mu \to e, \gamma$, hep-ph/0103065; generalization beyond seesaw limit: A. Donini, P. Hernández, J. López-Pavón, M. Maltoni and T. Schwetz, *The minimal 3+2 neutrino model versus oscillation anomalies*, 1205.5230

## summary: free parameters in the two-flavour situation

two masses $M_1$, $M_2$, which we here choose as $\sim 1$ GeV

angle $\mathrm{Re}\, z$ and CP-violating phases $\phi_1$ and $\delta$

absolute magnitude of neutrino yukawas, parametrized by $\mathrm{Im}\, z$

the limit $\Delta M \equiv M_2 - M_1 \to 0, |\,\mathrm{Im}\, z| \to \infty$ is called a "symmetry protected scenario", is arguably "natural", and should be particularly well suited for experimental detection[5]

[5] e.g. M. Shaposhnikov, *A Possible symmetry of the* $\nu$*MSM*, hep-ph/0605047; A. Abada, G. Arcadi, V. Domcke and M. Lucente, *Neutrino masses, leptogenesis and dark matter from small lepton number violation?*, 1709.00415; S. Antusch, E. Cazzato, M. Drewes, O. Fischer, B. Garbrecht, D. Gueter and J. Klarić, *Probing Leptogenesis at Future Colliders*, 1710.03744

**benchmark parameter values from a previous scan:**[6]

$$M_1 = 0.7688\,\text{GeV}\;, \quad M_2 = 0.7776\,\text{GeV}\;,$$
$$z = 2.444 - i3.285\;,$$
$$\phi_1 = -1.857\;, \quad \delta = -2.199\;,$$

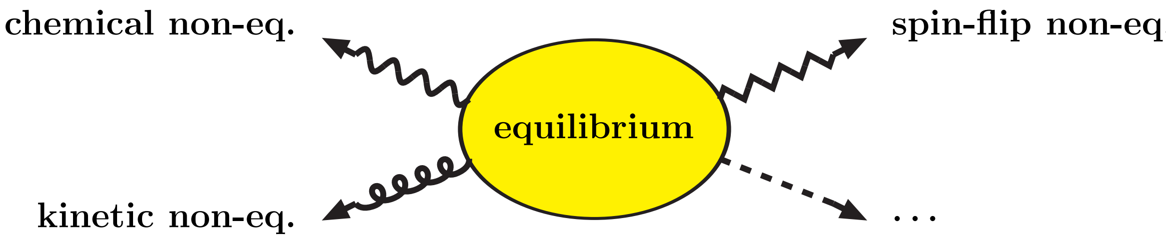
"inverted hierarchy"

not excluded, could be discovered, produces baryon asymmetry!

[6] P. Hernández, M. Kekic, J. López-Pavón, J. Racker and J. Salvado, *Testable Baryogenesis in Seesaw Models*, 1606.06719

# low-scale leptogenesis as a non-equilibrium problem

## there are infinitely many ways to deviate from equilibrium




basic principle: with time, the system relaxes back to equilibrium (just because equilibrium is the most likely configuration)

the relaxation process is characterized by an equilibration rate

## classic leptogenesis: chemical non-eq. of two variables[7]

apart from $n_L$, consider the massive particles species $\nu_\mathrm{R}$ whose energy density (particles + antiparticles, like dark matter) is close to falling out of equilibrium:

$$\begin{aligned}\partial_t(a^3 n_L) &= -\gamma_L\, a^3 n_L - \gamma_{L,R}\,(a^3 n_R - a^3 n_{\text{eq}})\,,\\ \partial_t(a^3 n_R) &= -\gamma_R\,(a^3 n_R - a^3 n_{\text{eq}}) - \gamma_{R,L}\, a^3 n_L\,.\end{aligned}$$

$\Rightarrow n_L \neq 0$ can be generated if $n_R \neq n_{\text{eq}}$ and $\gamma_L$ is not huge

7 M. Fukugita, T. Yanagida, *Baryogenesis Without Grand Unification*, PLB 174 (1986) 45; for current status, see D. Bödeker and M. Wörmann, *Non-relativistic leptogenesis*, 1311.2593; D. Bödeker and M. Sangel, *Lepton asymmetry rate from quantum field theory: NLO in the hierarchical limit*, 1702.02155

**in the "low-scale resonant" case there are more variables**[8]

all rates mediated by neutrino yukawas are slow $\sim |h_{Ia}|^2 g^2 T/\pi$

the flavour oscillation rate $\sim (M_I^2 - M_J^2)/k$ is slow

$\Rightarrow$ $n_L \to 3$-component vector, $n_R \to 2 \times 2$-matrices $\rho(k, s)$

[8] E.K. Akhmedov, V.A. Rubakov and A.Y. Smirnov, *Baryogenesis via neutrino oscillations*, hep-ph/9803255

# rate equations for low-scale resonant leptogenesis

## basic variables

yields ($Y_a \equiv \frac{n_a}{s}$) for lepton asymmetries: $Y_a - Y_B/3$

helicity-(anti)symmetrized density matrices for $\nu_{\rm R}$: $\rho^{\pm}$

cosmological evolution tracked through $x \equiv \ln(\frac{T_{\rm max}}{T})$

redshift tracked through co-moving momentum $k_T \equiv k\,[\frac{s(T)}{s(T_{\rm min})}]^{\frac{1}{3}}$

number densities $\Leftrightarrow$ chemical potentials: $n_a = \partial p/\partial \mu_a$

## evolution equation for lepton asymmetries

$$
\begin{aligned}
Y_a' - \frac{Y_B'}{3} \quad = \quad \frac{4}{s} \int_{\mathbf{k}_T} \mathrm{Tr}\{ & -n_{\mathsf{F}}(k_T)[1 - n_{\mathsf{F}}(k_T)]\, \widehat{A}^{+}_{(a)} \\
& + [\, \rho^{+} - \mathbb{1}\, n_{\mathsf{F}}(k_T)\, ]\, \widehat{B}^{+}_{(a)} \\
& + \rho^{-}\, \widehat{B}^{-}_{(a)} \, \} \; .
\end{aligned}
$$

1st term: washout term ("equilibration") $\propto \{\mu_a\}$

2nd term: source from helicity-symmetric non-equilibrium

3rd term: source from helicity-asymmetric non-equilibrium

# rate coefficients for low-scale resonant leptogenesis

## linear response theory relates rates to 2-point correlators

$$
\begin{aligned}
\Pi_{\rm E}(\tilde K) &\equiv \int_X e^{i\tilde K\cdot X}\langle(\tilde\phi^\dagger\ell_a)(X)\,(\bar\ell_a\tilde\phi)(0)\rangle \\
\rho_a(\mathcal{K}) &\equiv \operatorname{Im}\Pi_{\rm E}(\tilde K)\Big|_{k_n-i\mu_a\to -i[k_0+i0^+]}\;.
\end{aligned}
$$

denoting by $u_{\mathbf{k}\tau I}$ an on-shell spinor for sterile flavour $I$, we need

$$
\Omega_{(\tau)IJ}(k) \;\equiv\; \frac{\bar u_{\mathbf{k}\tau J}\,a_L\,\rho_a(\mathcal{K}_J)\,a_R\,u_{\mathbf{k}\tau I}}{\sqrt{\omega_I^k\omega_J^k}}\;,
$$

where $\tau \equiv \pm$, $\mathcal{K}_J \equiv (\omega_J^k,\mathbf{k})$, $\omega_J^k \;\equiv\; \sqrt{k^2+M_J^2}$

**matrix elements can be split into C-even and odd parts**

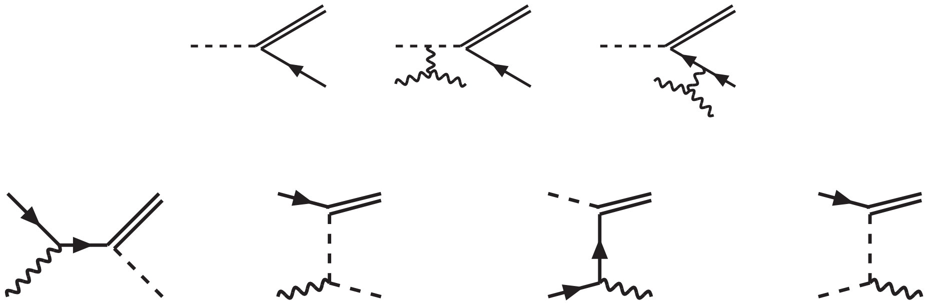
$$Q_{(\tau)IJ} \equiv \frac{1}{2}\Big[\Omega_{(\tau)IJ}\Big|_{\mu} + \Omega_{(\tau)IJ}\Big|_{-\mu}\Big] \; ,$$

$$\bar{\mu}_a R_{(\tau)IJ} + \bar{\mu}_Y S_{(\tau)IJ} \equiv \frac{1}{2}\Big[\Omega_{(\tau)IJ}\Big|_{\mu} - \Omega_{(\tau)IJ}\Big|_{-\mu}\Big] \; .$$

$\Rightarrow$ rescaling $\bar{\mu}_i \equiv \mu_i/T$ and $\widehat{Q} \equiv Q/(3c_s^2 H)$, where $H$ is the Hubble rate and $c_s^2$ the speed of sound squared, yields e.g.

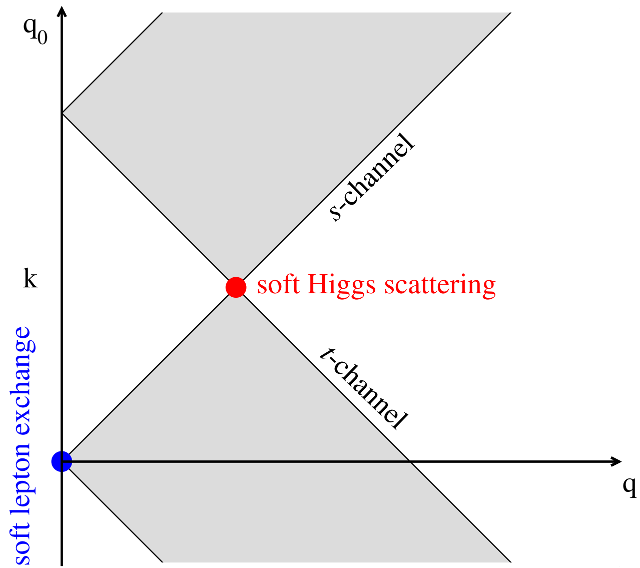
$$\begin{aligned} \widehat{B}^{+}_{(a)IJ} &\equiv -i\,\mathrm{Im}(h_{Ia}h^{*}_{Ja})\,\widehat{Q}^{+}_{\{IJ\}} \\ &+ \mathrm{Re}(h_{Ia}h^{*}_{Ja})\,[\bar{\mu}_a\,\widehat{R}^{+}_{\{IJ\}} + \bar{\mu}_Y\,\widehat{S}_{\{IJ\}}] \end{aligned}$$

## for $\Omega_{(\tau)IJ}$ need $1+n \leftrightarrow 2+n$ and $2 \leftrightarrow 2$ processes, e.g.[9]



[9] originally: A. Anisimov, D. Besak and D. Bödeker, *Thermal production of relativistic Majorana neutrinos: Strong enhancement by multiple soft scattering*, 1012.3784; D. Besak and D. Bödeker, *Thermal production of ultrarelativistic right-handed neutrinos: Complete leading-order results*, 1202.1288; resolved into helicity channels and generalized to broken phase and finite chemical potentials: J. Ghiglieri and ML, *Neutrino dynamics below the electroweak crossover*, 1605.07720; *GeV-scale hot sterile neutrino oscillations: a derivation of evolution equations*, 1703.06087

# new resummation needed for $2 \to 2$ scattering off soft higgs




$$\Omega^{2\leftrightarrow 2}_{(+)IJ} \equiv \Omega^{2\leftrightarrow 2,\,\mathsf{hard}}_{(+)IJ} - \Omega^{2\leftrightarrow 2,\,\mathsf{subtrL}}_{(+)IJ} + \Omega^{2\leftrightarrow 2,\,\mathsf{softL}}_{(+)IJ}$$

$$- \Omega^{2\leftrightarrow 2,\,\mathsf{subtrH}}_{(+)IJ} + \Omega^{2\leftrightarrow 2,\,\mathsf{softH}}_{(+)IJ} .$$

## numerical example

setting $T = 4 \times 10^4$ GeV and $k_T = 3T$, we get

$$
\begin{aligned}
Q_{(+)IJ} &= 5.29 \times 10^{-3}\, T\,, & Q_{(-)IJ} &= 1.16 \times 10^{-3}\, \frac{M_I M_J}{T}\,,\\
R_{(+)IJ} &= -1.76 \times 10^{-3}\, T\,, & R_{(-)IJ} &= -0.37 \times 10^{-3}\, \frac{M_I M_J}{T}\,,\\
S_{(+)IJ} &= 0.87 \times 10^{-3}\, T\,, & S_{(-)IJ} &= 0.04 \times 10^{-3}\, \frac{M_I M_J}{T}\,.
\end{aligned}
$$

$\Rightarrow$ helicity-conserving $Q_{(-)}$, $R_{(-)}$, and $S_{(-)}$ are suppressed by majorana masses, because in the massless limit right-handed neutrinos carry opposite helicity to left-handed leptons

# numerical solution of the non-equilibrium problem

## choice of initial conditions

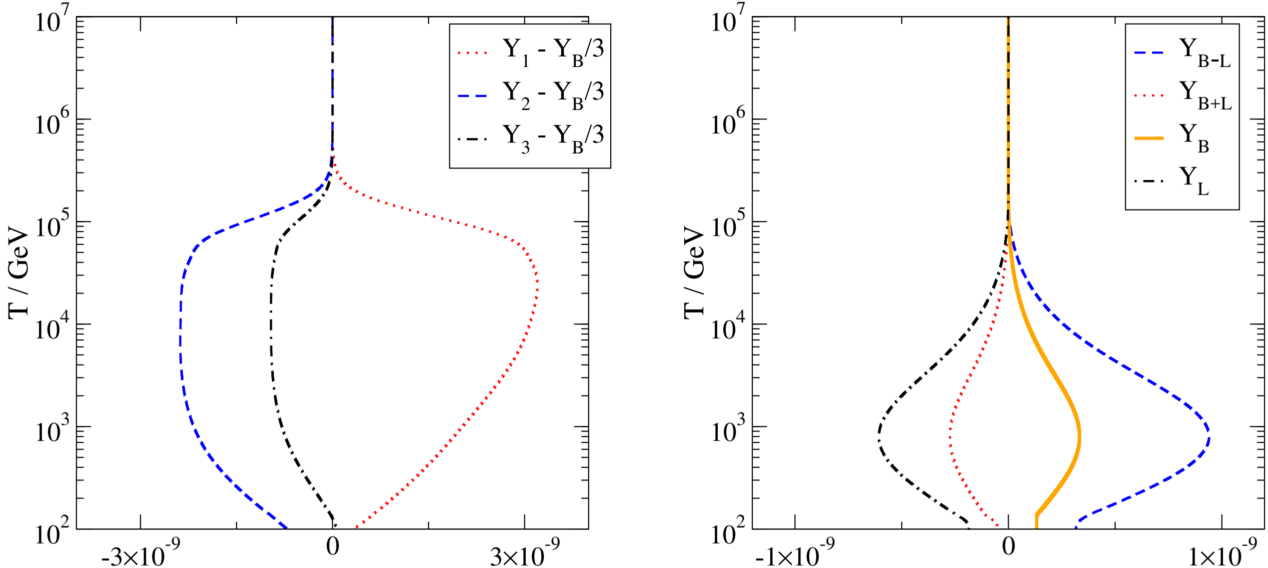
⇒ assume that density matrices and all lepton asymmetries vanish at some $T_{\mathsf{max}}$

⇒ at first not much goes on, but then there is complicated dynamics when first $\nu_{\mathrm{R}}$ oscillations complete around $T \sim T_{\mathsf{osc}}$:[10]

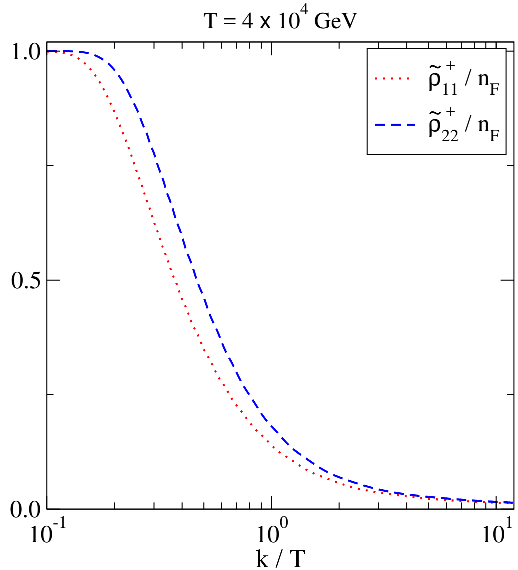
$$T_{\mathsf{osc}} \sim 7 \times 10^4\,\mathsf{GeV}\left(\frac{M}{\mathsf{GeV}}\,\frac{|\Delta M|}{\mathsf{MeV}}\,\frac{T}{k}\right)^{1/3}$$

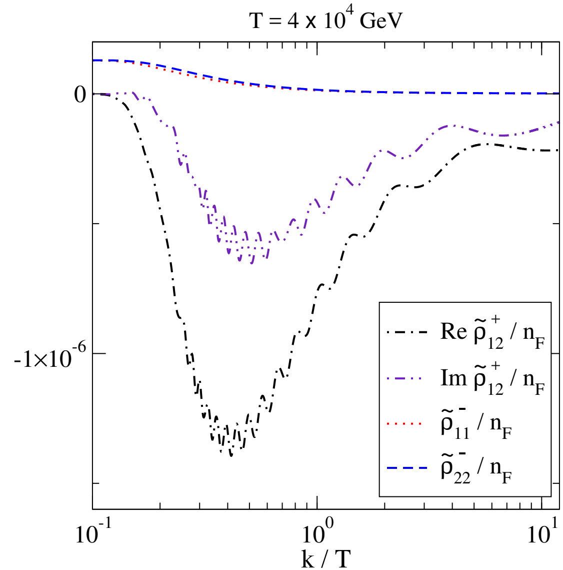
[10] T. Asaka and M. Shaposhnikov, *The $\nu$MSM, dark matter and baryon asymmetry of the universe*, hep-ph/0505013

# evolution of asymmetries



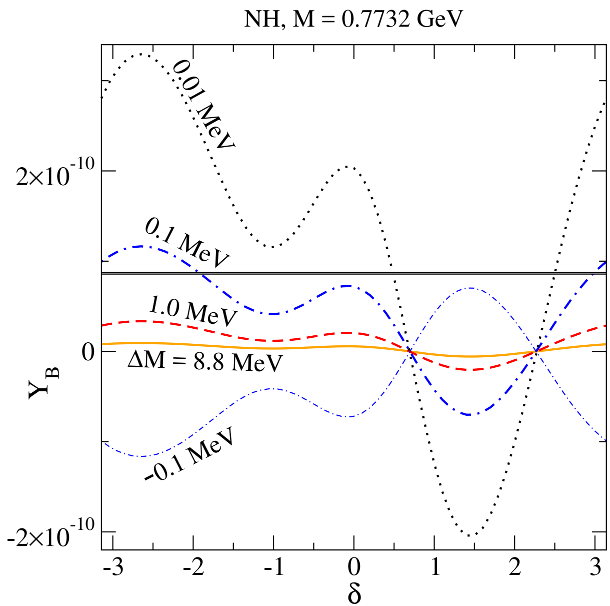
# shapes of density matrices show kinetic non-equilibrium




accounting for this we find $\sim 50\%$ increase over a previous study

# final baryon asymmetry compared with observation




more at http://www.laine.itp.unibe.ch/leptogenesis/

# summary & what's next

$\Rightarrow$ low-scale resonant leptogenesis works in principle

$\Rightarrow$ theoretical uncertainties currently at $\sim 10 - 50\%$ level

$\Rightarrow$ would be nice to fully map the viable parameter space

$\Rightarrow$ detection of $\nu_{\rm R}$ at $0\nu\beta\beta$ and $B$-factory-type experiments??