How does relativistic kinetic theory remember about initial conditions?

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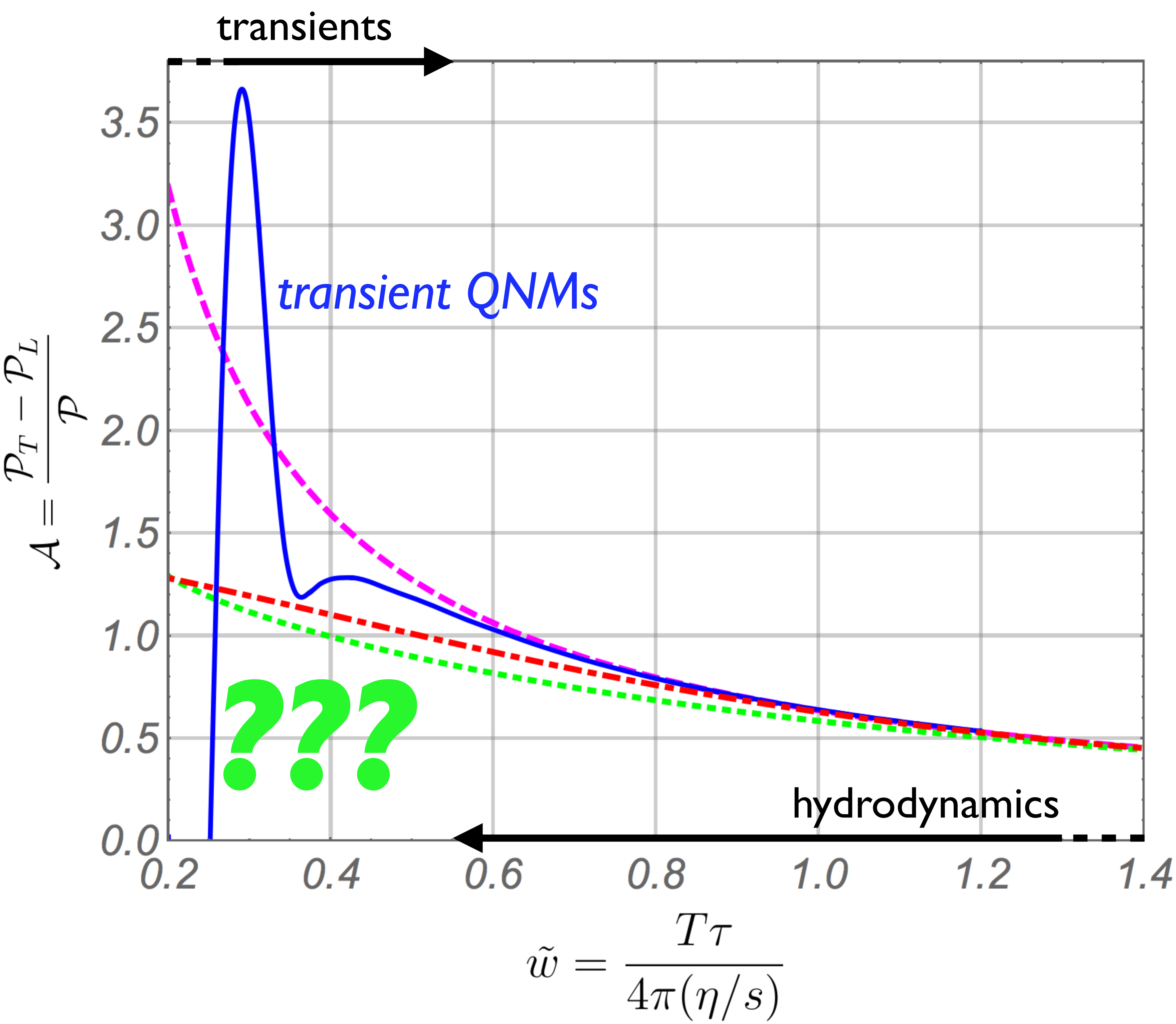
1609.04803 [nucl-th] with Kurkela, Spalinski & Svensson

1802.08225 [nucl-th] with Svensson

for a review of earlier developments, see **1707.02282 [hep-ph]** with Florkowski & Spalinski

Motivation 1609.04803 with Kurkela, Spalinski and Svensson



conformal RTA KT with $\eta/s = 0.624$

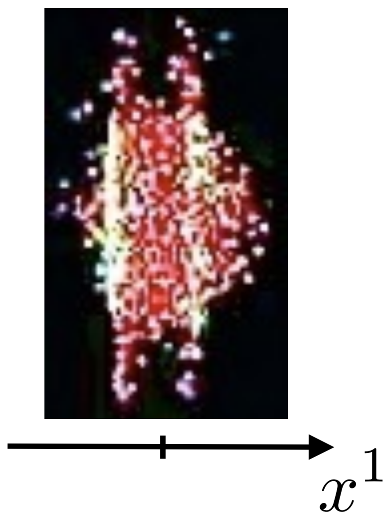
EKT with $\eta/s = 0.624$

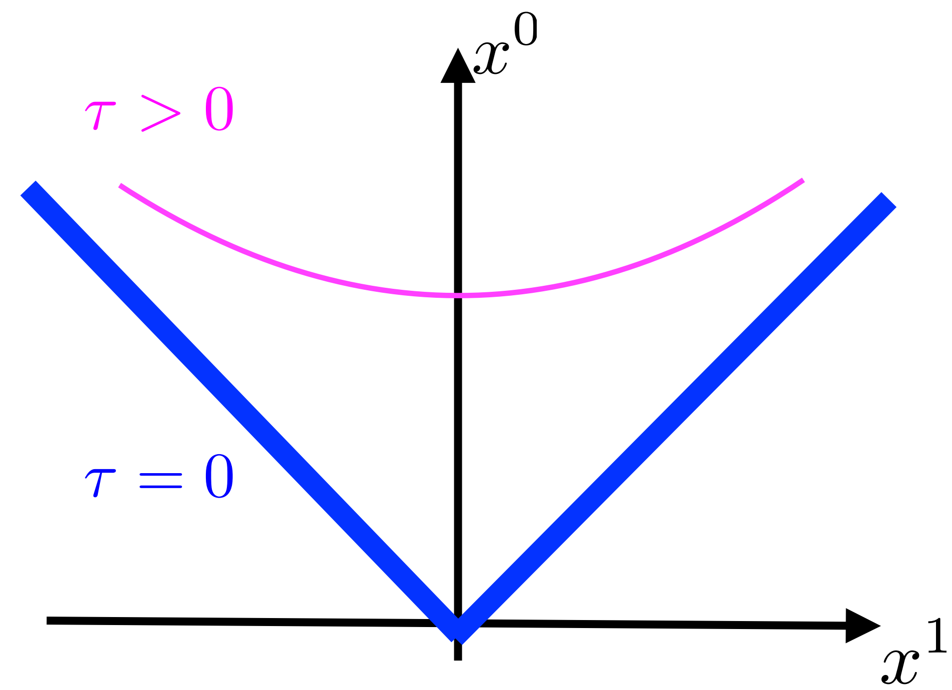
N=4 SYM (holography)

$$\frac{\Delta\mathcal{P}}{\mathcal{E}/3} = \frac{2}{\pi}\tilde{w}^{-1}$$

Boost-invariant dynamics 1982 Bjorken

const x^0 slice:





Boost-invariance: in $\left(\tau \equiv \sqrt{x_0^2 - x_1^2}, \quad y \equiv \operatorname{arctanh}\frac{x_1}{x_0}, \, x_2, \, x_3\right)$ coords no y-dep

In setups of interest, $\langle T^\mu_\nu \rangle = \operatorname{diag}\left\{ -\mathcal{E}(\tau), -\mathcal{E} - \tau\,\dot{\mathcal{E}}, \, \mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}}, \, \mathcal{E} + \frac{1}{2}\tau\,\dot{\mathcal{E}} \right\}$

Boost-invariant hydrodynamics **1103.3452** with Janik & Witaszczyk

Hydrodynamics = a class of $\langle T^{\mu\nu}\rangle := \mathcal{E}\, u^\mu u^\nu + \mathcal{P}(\mathcal{E})\,\{g^{\mu\nu} + u^\mu u^\nu\} + \pi^{\mu\nu}[\mathcal{E}, u^\alpha]$

with $\pi^{\mu\nu}[\mathcal{E}, u^\alpha] = \underbrace{-\eta\,\sigma^{\mu\nu}}_{\sim\partial} \underbrace{+\eta\,\tau_\pi \mathcal{D}\sigma^{\mu\nu} + \lambda_1\,\sigma^{\langle\mu}{}_\lambda\sigma^{\nu\rangle\lambda}}_{\sim\partial^2} + \ldots$

% see **1503.07514** with Spalinski and Romatschke's plenary talk for another take on it

Boost-invariant conformal hydrodynamics:

$$\pi^2{}_2 - \pi^y{}_y \equiv \mathcal{P}_T - \mathcal{P}_L = \frac{\mathcal{E}}{3}\left(\overset{\sim\eta/s}{a_1}\,\underbrace{\frac{1}{\tau\,T}}_{\equiv w^{-1}} + a_2\,\underbrace{\frac{1}{(\tau\,T)^2}}_{\equiv w^{-2}} + \ldots\right)$$

% gradient expansion becomes a series in $\frac{1}{w}$; $a_1,\ a_2, \ldots$ are the same for all states

How does relativistic kinetic theory remember about initial conditions? 1802.08225 with Svensson

In kinetic theory, the fundamental variable is $f(x,p)$ solving the Boltzmann equation

$$p^\mu \partial_\mu f = \mathcal{C}[f] \longleftarrow \text{ interactions}$$

For the boost-invariant flow, f is a function of τ and 2 components of p^μ

What kind of corrections to $\mathcal{A}_h \equiv a_1\, w^{-1} + a_2\, w^{-2} + \ldots$ defined using

$$\langle T^{\mu\nu} \rangle = \int \mathrm{d}P\, f(x,p)\, p^\mu p^\nu$$

carry the vast amount of information about an initial state, $f(\tau_0, p)$, to late times w?

The relaxation time approximation 1802.08225 with Svensson

We focus on $\mathcal{C}$ in the relaxation time approximation

$$\mathcal{C} \sim -\frac{1}{\tau_{rel}}\left(f - f_{eq}[f]\right)$$

We take $\tau_{rel} \sim \frac{1}{T^{\Delta}} \sim \frac{1}{\mathcal{E}^{\Delta/4}}$ with the conformal case $\Delta = 1$; Now $w \equiv \frac{\tau}{\tau_{rel}}$.

This theory can be reformulated as an integral equation for $\mathcal{E}(\tau)$ 1984 Baym

represents $f(\tau_0, p)$

$$\mathcal{E}(\tau)\exp\left(\int_{\tau_0}^{\tau}\frac{\mathrm{d}\tau'}{\tau_{rel}(\tau')}\right) = \mathcal{E}_0(\tau) + \int_{\tau_0}^{\tau}\frac{\mathrm{d}\tau'}{\tau_{rel}(\tau')}\, H\left(\frac{\tau'}{\tau}\right)\mathcal{E}(\tau')\exp\left(\int_{\tau_0}^{\tau'}\frac{\mathrm{d}\tau''}{\tau_{rel}(\tau'')}\right)$$

$= \frac{\tau'^2}{2\,\tau^2} + \frac{\arctan\sqrt{\frac{\tau^2}{\tau'^2}-1}}{2\sqrt{\frac{\tau^2}{\tau'^2}-1}}$ and comes from $f_{eq}[f]$

The gradient expansion

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Integration by parts of $\int_{\tau_0}^{\tau} \frac{\mathrm{d}\tau'}{\tau_{\mathrm{rel}}(\tau')} H\left(\frac{\tau'}{\tau}\right) \mathcal{E}(\tau') \exp\left(\int_{\tau_0}^{\tau'} \frac{\mathrm{d}\tau''}{\tau_{\mathrm{rel}}(\tau'')}\right)$ gives hydrodynamics:

$$\sum_{j=1}^{\infty} \left(-\tau_{rel}(\tau')\,\frac{\mathrm{d}}{\mathrm{d}\tau'}\right)^{j} H\left(\frac{\tau'}{\tau}\right) \mathcal{E}(\tau')\Big|_{\tau'=\tau} = 0$$

rewriting in terms of $\frac{\mathcal{P}_T - \mathcal{P}_L}{\mathcal{E}/3} \equiv \mathcal{A}\left(w \sim \frac{\tau}{\mathcal{E}^{-\Delta/4}} \sim \tau^{1-\Delta/3} + \dots\right)$ at large w:

$$\mathcal{A}_h = \frac{8}{5}\,w^{-1} + \left(\frac{88}{105} - \frac{8}{15}\Delta\right) w^{-2} + \dots$$

% for $\Delta = 0$ we went to 1000 derivatives; for $\Delta = 1$ up to 425, otherwise up to ~100

Lessons:

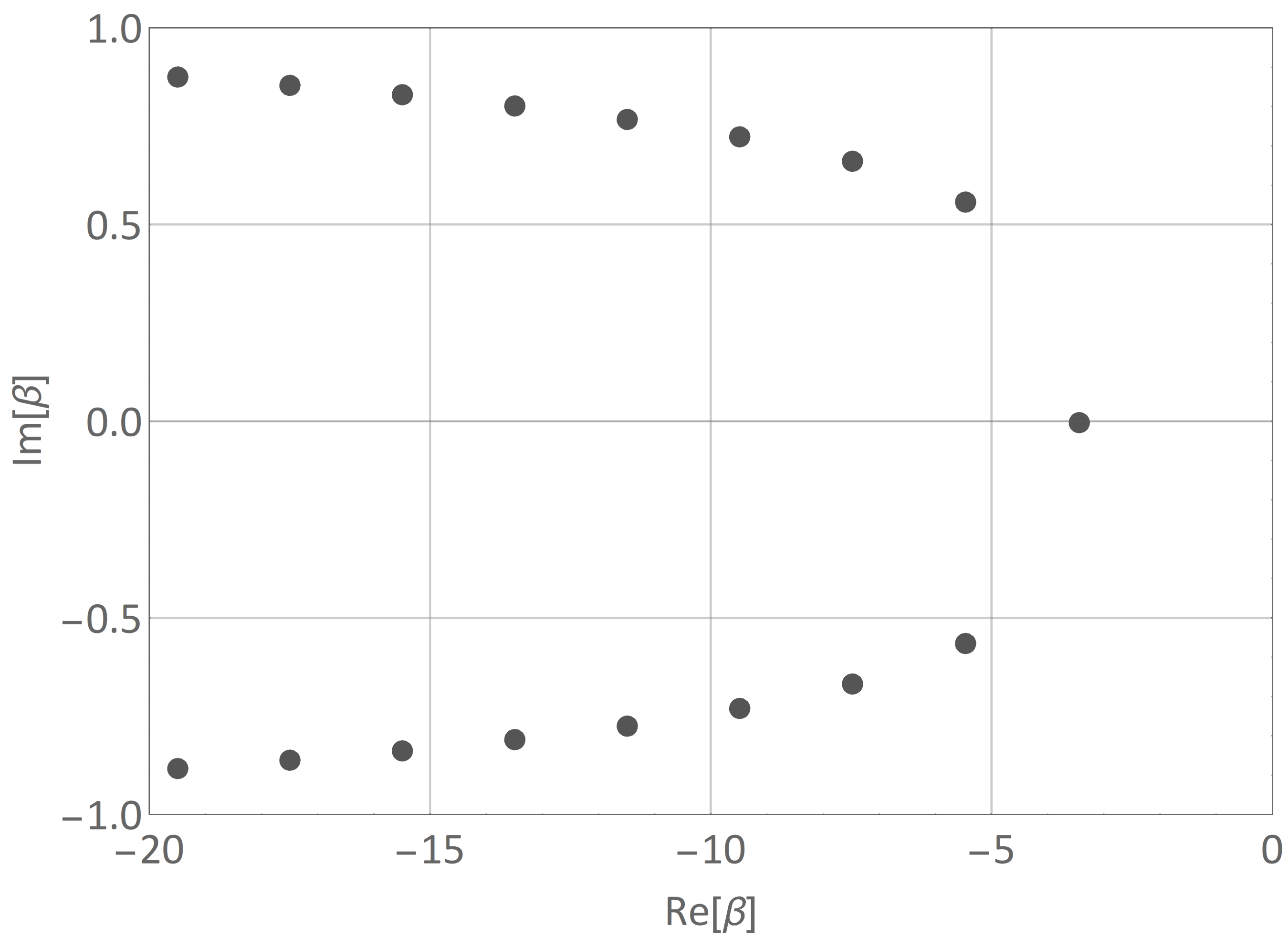
- gradient expansion diverges, $a_n \sim n!$ for $n \gg 1$, as for N=4, BRSSS, …
  see **1707.02282** with Florkowski & Spalinski
- for $\Delta \geq 3$ c is too small to counter expansion - no hydrodynamics at late τ

The transient modes 1802.08225 with Svensson

$$\mathcal{E}(\tau)\exp\left(\int_{\tau_0}^{\tau}\frac{\mathrm{d}\tau'}{\tau_{\mathrm{rel}}(\tau')}\right)=\mathcal{E}_0(\tau)+\int_{\tau_0}^{\tau}\frac{\mathrm{d}\tau'}{\tau_{\mathrm{rel}}(\tau')}H\left(\frac{\tau'}{\tau}\right)\mathcal{E}(\tau')\exp\left(\int_{\tau_0}^{\tau'}\frac{\mathrm{d}\tau''}{\tau_{\mathrm{rel}}(\tau'')}\right)$$

Linearization: $\mathcal{E}(\tau)-\mathcal{E}_h(\tau)\sim\exp\left(-\hat{\alpha}\int_{\tau_0}^{\tau}\frac{\mathrm{d}\tau'}{\tau_{rel}(\tau')}\right)w^{\beta}+\ldots$

$\hat{\alpha}=1$: exponential decay over τ_{rel} with β's given by $\int_0^1 dx\,H(x)\,z^{\beta(1-\Delta/3)-\Delta/3}=0$. Solutions:



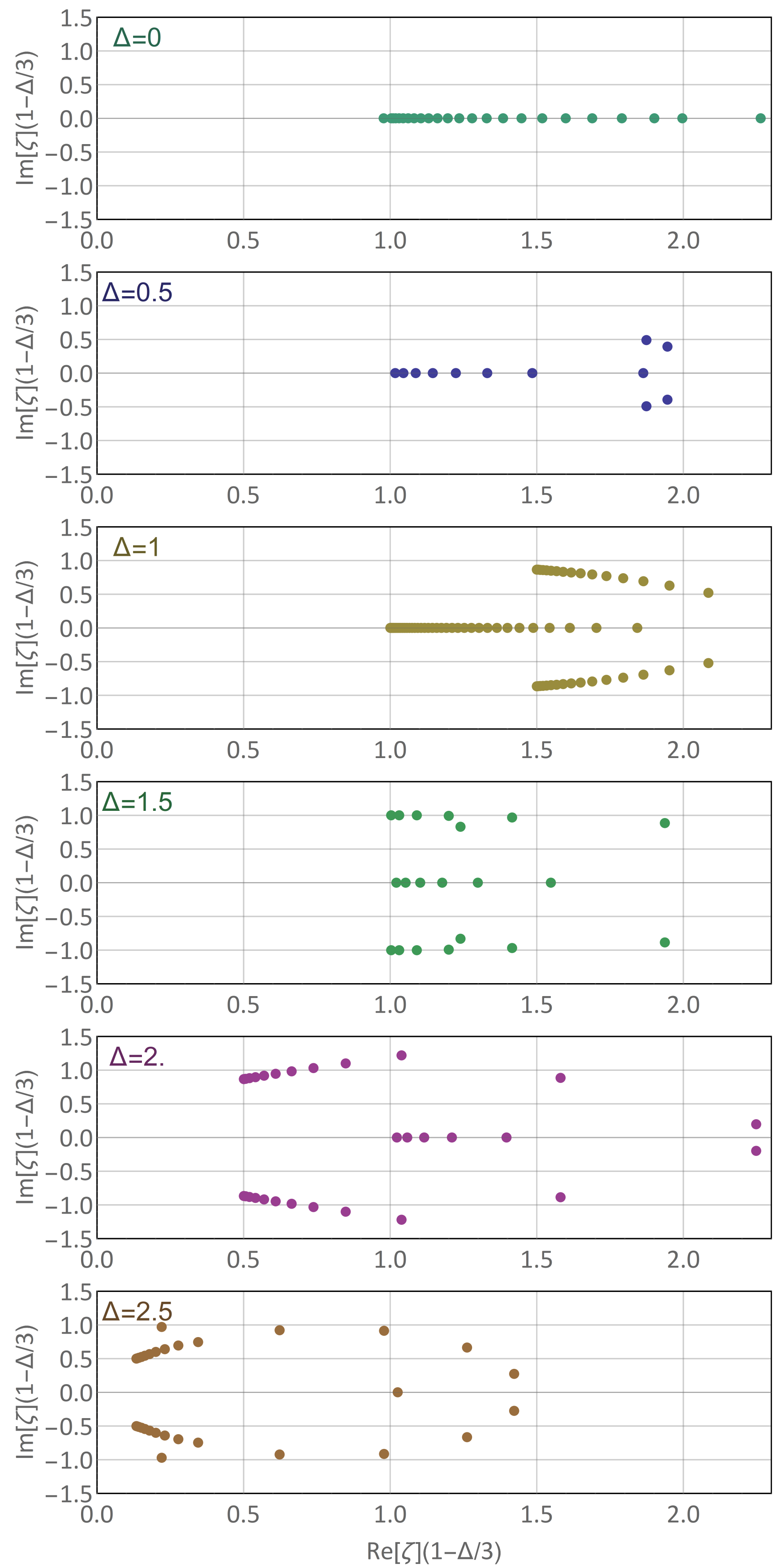
∞-many transients: $\delta A^{(j)}(w)\sim e^{-\frac{1}{1-\Delta/3}w}\,w^{\Re[\beta_j]+\frac{(5-\Delta)(21-5\Delta)}{5\,(3-\Delta)^2}}\cos\left(\Im[\beta_j]\,\log w+\phi_j\right)+\ldots$

Gradient expansion vs. transient modes

1802.08225 with Svensson

$$B\mathcal{A}_h = \sum_{n=1}^{\infty} \frac{a_n}{n!}\xi^n \approx$$

low-order Padé approximant:



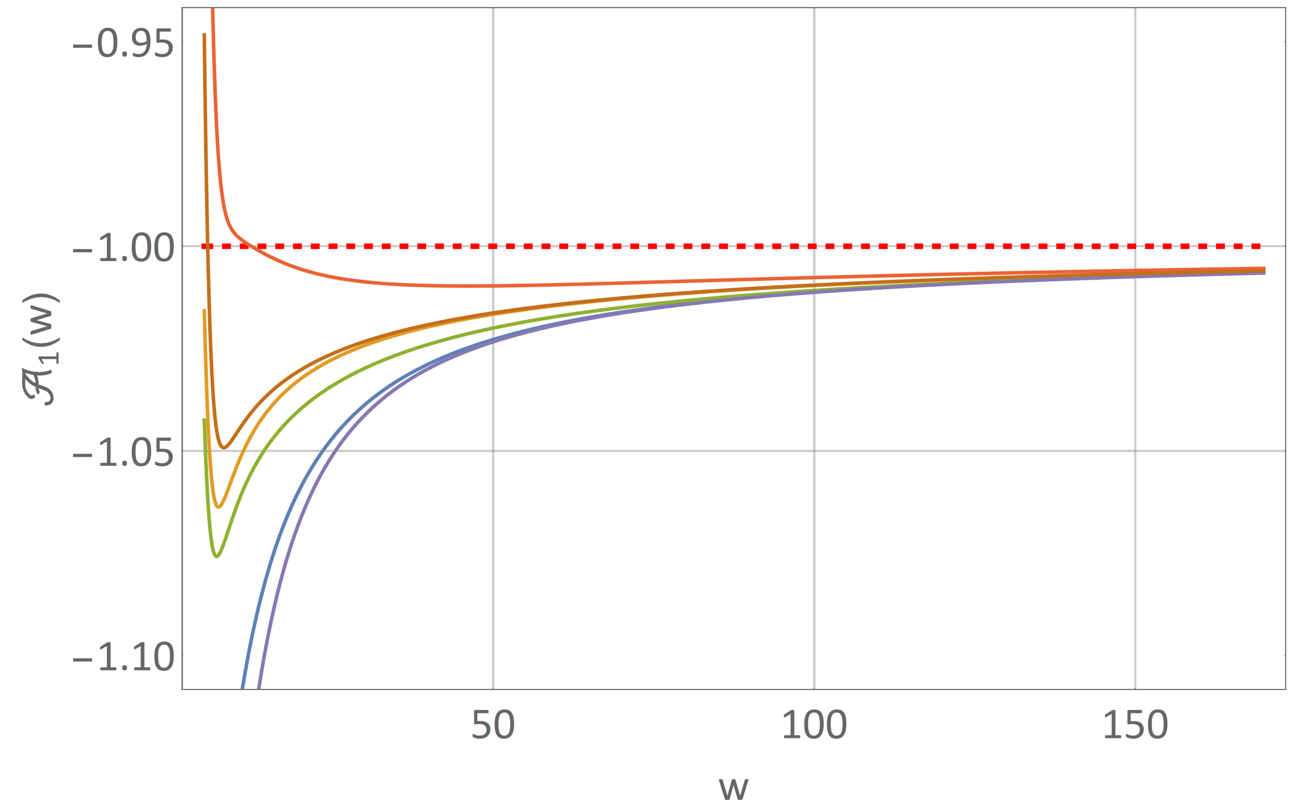
real axis singularities consistent with transients

off-real axis singularities related to unphysical contour deformations of

$$\int_{\tau_0}^{\tau} \mathrm{d}\tau' \, H\left(\frac{\tau'}{\tau}\right) \ldots$$

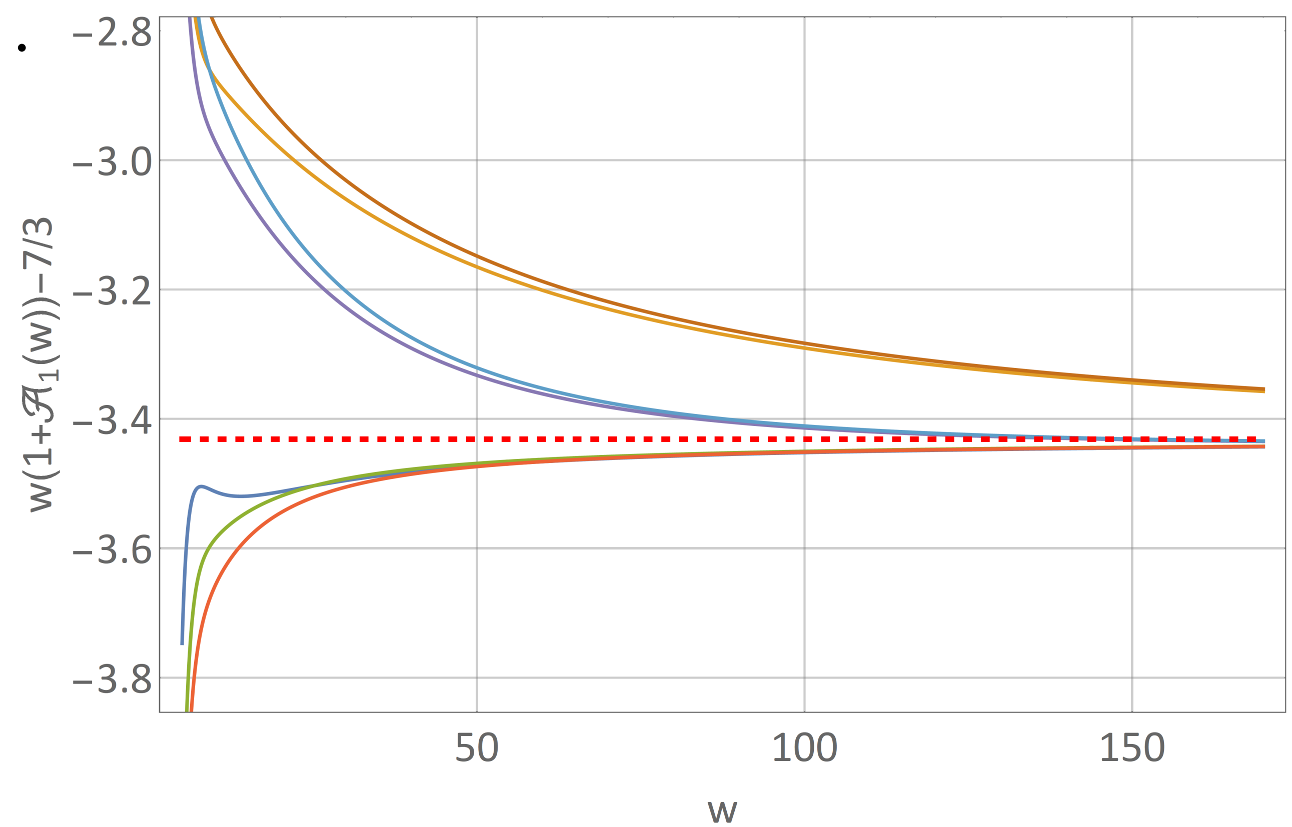
For $\Delta > 2$ they dominate large orders hydrodynamics

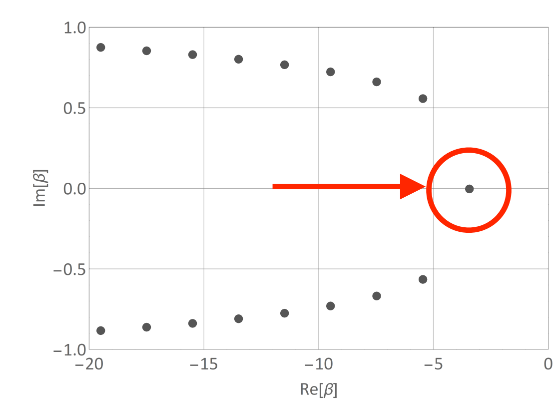
Seeing transients for $\Delta = 0$ 1802.08225 with Svensson



$$\mathcal{A}_1 = \frac{\mathrm{d}}{\mathrm{d}w} \log \left(\mathcal{A} - \mathcal{A}' \right)$$

$$= -1 + \frac{\beta_1 + 7/3}{w} + \dots$$

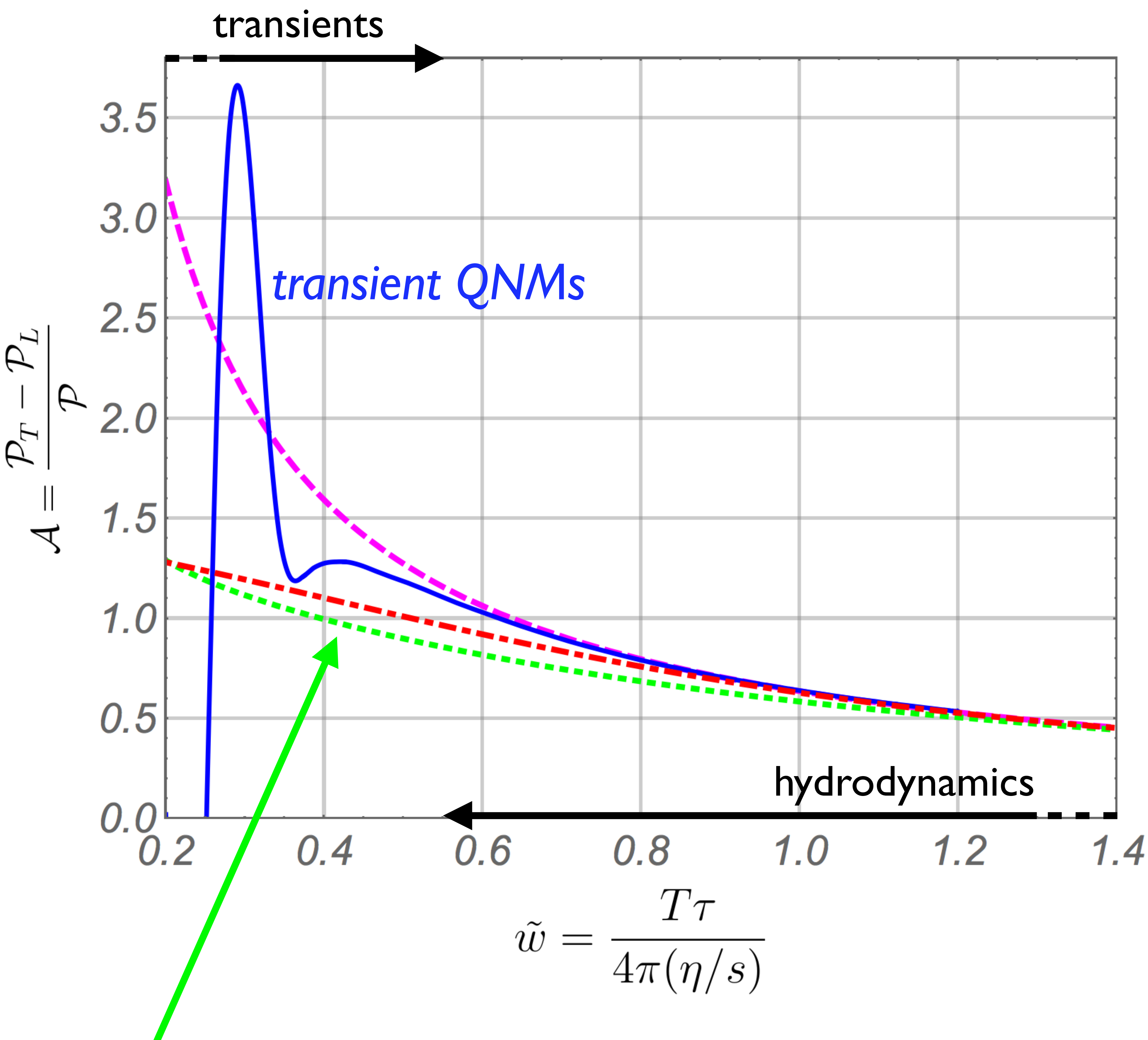




Outlook

1609.04803 with Kurkela, Spalinski & Svensson; **1802.08225** with Svensson

see also **1707.02282** with Florkowski & Spalinski for a viewpoint



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N=4 SYM (holography)

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∞-many transients: $\delta A^{(j)}(w) \sim e^{-\frac{3}{2}w} w^{\Re[\beta_j]+\frac{16}{5}} \cos\left(\Im[\beta_j]\log w + \phi_j\right) + \ldots$