Scale Invariant Hard Thermal Loop resummation

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Context: QCD phase diagram/ Quark Gluon Plasma

Complete QCD phase diagram far from being confirmed:

$T \neq 0, \mu = 0$ well-established from lattice: no sharp phase transition, continuous crossover at $T_c \simeq 154 \pm 9$ MeV

Goal: more analytical approximations, ultimately in regions not much accessible on the lattice: large density (chemical potential) due to the famous "sign problem"

Tool: unconventional RG resummation of perturbative expansions

Very general: relevant both at $T = 0$ or $T \neq 0$ (and finite density too)
$\rightarrow$ in particular addresses well-known problems of unstable +badly scale-dependent $T \neq 0$ perturbative expansions

Content

- Introduction/Motivation
- Optimized (or 'screened') perturbation (OPT $\sim$ SPT)
- RG-compatible OPT $\equiv$ RGOPT
- Application: thermal nonlinear sigma model
  (many similarities with QCD but simpler)
- Application: thermal (pure gauge) QCD: hard thermal loops
- Conclusions

Introduction/Motivations

Tool: unconventional 'RG-optimized' (RGOPT) resummation of perturbative expansions
Illustrate here $T \neq 0$ nonlinear σ-model, + QCD (pure glue)

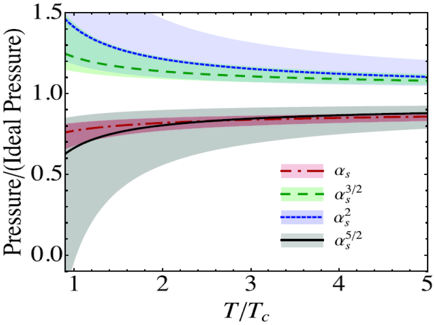
NB some previous results with our approach ($T = 0$):
estimate of the chiral symmetry breaking order parameter
$F_\pi(m_{u,d,s} = 0)/\Lambda_{\overline{MS}}^{\rm QCD}$: F_π exp input $\to \Lambda_{\overline{MS}}^{n_f=3} \to \alpha_S^{\overline{MS}}(\mu = m_Z)$.

N^3LO: $F_\pi^{m_q=0}/\Lambda_{\overline{MS}}^{n_f=3} \simeq 0.25 \pm .01 \to \alpha_S(m_Z) \simeq 0.1174 \pm .001 \pm .001$
(JLK, A.Neveu, PRD88 (2013))
(compares well with α_S lattice and world average values [PDG2016-17])

Also applied to $\langle \bar{q} q \rangle$ at N^3LO (using spectral density of Dirac operator):
$\langle \bar{q} q \rangle_{m_q=0}^{1/3}(2\,{\rm GeV}) \simeq -(0.84 \pm 0.01)\Lambda_{\overline{MS}}$ (JLK, A.Neveu, PRD 92 (2015))
Parameter free determination! (compares well with latest lattice result)

Problems of thermal perturbative expansion (QCD, $g\phi^4$, ...)

known problem: poorly convergent and very scale-dependent (ordinary) perturbative expansions:



QCD (pure glue) pressure at successive (standard) perturbation orders
shaded regions: scale-dependence for $\pi T < \mu < 4\pi T$
(illustration from Andersen, Strickland, Su '10)

2. (Variationally) Optimized Perturbation (OPT)

Trick ($T = 0$): add and subtract a mass, consider $m\,\delta$ as interaction:

$$\mathcal{L}(g, m) \to \mathcal{L}(\delta\, g, m(1-\delta)) \text{ (e.g. in QCD } g \equiv 4\pi\alpha_S)$$

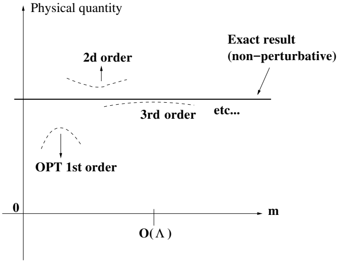
where $0 < \delta < 1$ interpolates between $\mathcal{L}_{free}$ and *massless* $\mathcal{L}_{int}$;
$\to m$: arbitrary trial parameter

• Take any standard (renormalized) pert. series, expand in δ *after:*
$m \to m\,(1-\delta)$; $\quad g \to \delta\, g$
then $\delta \to 1$ (to recover original massless theory):

BUT a dependence in m remains at any finite δ^k-order:
fixed typically by stationarity prescription: optimization (OPT):
$\frac{\partial}{\partial m}$(physical quantity) $= 0$ for $m = \bar{m}_{opt}(\alpha_S) \neq 0$:
• $T = 0$: exhibits *dimensional transmutation*: $\bar{m}_{opt}(g) \sim \mu\, e^{-const./g}$

• $T \neq 0$ similar idea: "screened perturbation" (SPT), or *resummed* "hard thermal loop (HTLpt)" (QCD): expand around quasi-particle mass.
Does this 'cheap trick' always work? and why?

Expected behaviour (ideally)



Not quite what happens, except in simple models:
•Convergence of this procedure for $D = 1$ ϕ^4 oscillator
(cancels large pert. order factorial divergences!) Guida et al '95

particular case of 'order-dependent mapping' Seznec, Zinn-Justin '79

•QFT multi-loop calculations (specially $T \neq 0$) (very) difficult:
$\rightarrow$ empirical convergence? not clear

•Main pb at higher order: OPT: $\partial_m(...) = 0$ has multi-solutions (some complex!), how to choose right one, if no nonperturbative "insight"??

3. RG compatible OPT ($\equiv$ RGOPT)

Main additional ingredient (JLK, A. Neveu '10):

Consider a *physical* quantity (perturbatively RG invariant)
e.g. in thermal context the pressure $P(m, g, T)$):

in addition to: $\frac{\partial}{\partial m} P^{(k)}(m, g, \delta = 1)|_{m \equiv \tilde{m}} \equiv 0$, (OPT)
Require (δ-modified!) result at order δ^k to satisfy (perturbative)
Renormalization Group (RG) equation:

$$\mathrm{RG}\left(P^{(k)}(m, g, \delta = 1)\right) = 0$$

with standard RG operator :

$$\mathrm{RG} \equiv \mu \frac{d}{d\,\mu} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} - \gamma_m(g)\, m \frac{\partial}{\partial m}$$

$\beta(g) \equiv -b_0 g^2 - b_1 g^3 + \cdots, \;\; \gamma_m(g) \equiv \gamma_0 g + \gamma_1 g^2 + \cdots$

$\rightarrow$ Additional nontrivial constraint

$\rightarrow$ If combined with OPT, RG Eq. reduces to massless form:

$$\left[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right]P^{(k)}(m,g,\delta=1)=0$$

Then using OPT AND RG completely fix $m\equiv\bar{m}$ and $g\equiv\bar{g}$.

But $\Lambda_{\overline{\mathbf{MS}}}(g)$ satisfies by def.:
$[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}]\,\Lambda_{\overline{\mathbf{MS}}}\equiv 0$ (consistently at a given order for $\beta(g)$).

equivalent to:

$$\frac{\partial}{\partial\, m}\left(\frac{P^k(m,g,\delta=1)}{\Lambda_{\overline{\mathbf{MS}}}(g)}\right)=0\ ;\quad \frac{\partial}{\partial\, g}\left(\frac{P^k(m,g,\delta=1)}{\Lambda_{\overline{\mathbf{MS}}}(g)}\right)=0 \text{ for } \bar{m},\bar{g}$$

•Optimal $\bar{m}\sim\Lambda_{\overline{\mathbf{MS}}}(g)$, but true physical result from $P(\bar{m},\bar{g},T)$

•At $T=0$ reproduces at first order exact nonperturbative results in simpler models [e.g. Gross-Neveu model]

OPT + RG = RGOPT main features

•Usual OPT/Screened PT: embarrassing freedom in interpolation trick: why not $m \to m(1-\delta)^a$?
Most previous works ($T=0$, Screened PT, HTLpt $T \neq 0$) do linear interpolation ($a=1$) without deep justification
but generally (we have shown) $a=1$ spoils RG invariance!

•OPT gives multiple $\bar{m}(g,T)$ solutions at increasing δ^k-orders

→ Our approach restores RG, +requires optimal solution to match perturbation (i.e. Asymptotic Freedom for QCD ($T=0$)):
$\alpha_S \to 0$ ($\mu \to \infty$): $\bar{g}(\mu) \sim \frac{1}{2b_0 \ln \frac{\mu}{\bar{m}}} + \cdots$, $\bar{m} \sim \Lambda_{QCD}$

→ At successive orders AF-compatible optimal solution (often unique) *only* appears for universal critical a:
$m \to m(1-\delta)^{\frac{\gamma_0}{b_0}}$ (in general $\frac{\gamma_0}{b_0} \neq 1$)

→ RG consistency goes beyond simple "add and subtract" trick

and removes any spurious solutions incompatible with AF

•But does not always avoid complex OPT $\bar{m}$ solutions
(if these occur, possibly cured by renormalization scheme change)

Problems of thermal perturbation (QCD but generic)

Usual suspect: mix up of *hard* $p \sim T$ and *soft* $p \sim \alpha_S T$ modes.

Thermal 'Debye' screening mass $m_D^2 \sim \alpha_S T^2$ gives IR cutoff,
BUT $\Rightarrow$ perturbative expansion in $\sqrt{\alpha_S}$ in QCD
$\rightarrow$ often advocated reason for slower convergence

Yet many interesting QGP physics features happen at not that large coupling $\alpha_S(\sim 2\pi T_c) \sim .5$, ($\alpha_S(\sim 2\pi T_c) \sim 0.3$ for pure glue)

Many efforts to improve this (review e.g. Blaizot, Iancu, Rebhan '03):

Screened PT (SPT) (Karsch et al '97) $\sim$ Hard Thermal Loop (HTLpt) resummation (Andersen, Braaten, Strickland '99); Functional RG, 2PI formalism (Blaizot, Iancu, Rebhan '01; Berges, Borsanyi, Reinosa, Serreau '05)

Our RGOPT($T \neq 0$) essentially treats thermal mass 'RG consistently':
$\rightarrow$ UV divergences induce mass anomalous dimension.

(NB some qualitative connections with recently advocated "massive scheme" approach (Blaizot, Wschebor '14))

RGOPT($T \neq 0$ generic example): ϕ^4 (JLK, M.B Pinto, PRL116 '16)

- Start from usual 2-loop PT free energy $m \neq 0$, $T \neq 0$ ($\overline{\text{MS}}$ scheme):

$$(4\pi)^2 \mathcal{F}_0 = \mathcal{E}_0 - \frac{m^4}{8}\left(3 + 4\ln\frac{\mu}{m}\right) - \frac{T^4}{2} J_0\left(\frac{m}{T}\right) + \mathcal{O}(g)(\text{2-loop})$$

$$J_0\left(\frac{m}{T}\right) \sim \int_0^\infty dp \frac{1}{\sqrt{p^2+m^2}} \frac{1}{e^{\sqrt{p^2+m^2}} - 1}$$

- $\mathcal{F}(T=0)$ has LO $\ln\mu$ dependence: compensated by $\mathcal{E}_0$ *finite* T-independent 'vacuum energy' subtraction (well-known at $T=0$):
$\mathcal{E}_0(g,m) = -m^4\left(\frac{s_0}{g} + s_1 + s_2 g + \cdots\right)$
determined such that $\mu\frac{d}{d\mu}\mathcal{E}_0$ cancels the $\ln\mu$ dependence:

$$s_0 = \frac{1}{2(b_0 - 4\gamma_0)} = 8\pi^2; \quad s_1 = \frac{(b_1 - 4\gamma_1)}{8\gamma_0\,(b_0 - 4\gamma_0)} = -1, \ldots;$$

- Missed by SPT, HTLpt (QCD): explains the large scale dependence observed at higher order in those approaches! (more on this below)

- Next: expand in δ, $\delta \to 1$ after $m^2 \to m^2(1-\delta)^a$; $g \to \delta g$
RG only consistent for $a = 2\gamma_0/b_0$ ($= 1/3$ for ϕ^4 while $a = 1$ in SPT)
NB: $1/g$ in $\mathcal{E}_0$ automatically cancels in optimized energy $\mathcal{F}(\bar{m})$.

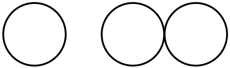
- All together lead to a much better RGOPT residual scale dependence (factor ~ 3 better at 2-loops wrt PT/SPT, much better at 3-loops)

4. Closer to QCD: 2D $O(N)$ nonlinear σ model (NLSM)

•Shares properties with QCD (asymptotic freedom, mass gap).
•At $T \neq 0$ the pressure, trace anomaly, etc have QCD-like shapes

•Nonperturbative $T \neq 0$ results available for comparison:
(lattice ($N = 3$)[Giacosa et al '12], $1/N$-expansion [Andersen et al '04])

$$\mathcal{L}_0 = \frac{1}{2}(\partial\pi_i)^2 + \frac{g(\pi_i\partial\pi_i)^2}{2(1-g\pi_i^2)} - \frac{m^2}{g}\left(1-g\pi_i^2\right)^{1/2}$$

Two-loop pressure from: 

•Advantage w.r.t. QCD: exact T-dependence at 2-loops:

$$P_{\rm pert.2loop} = -\frac{(N-1)}{2}\left[I_0(m,T) + \frac{(N-3)}{4}m^2 g I_1(m,T)^2\right] + \mathcal{E}_0,$$

$$I_0(m,T) = \frac{1}{2\pi}\left(m^2(1-\ln\frac{m}{\mu}) + 4T^2 K_0(\frac{m}{T})\right)$$

$$K_0(x) = \int_0^\infty dz \ln\left(1-e^{-\sqrt{z^2+x^2}}\right), \quad I_1(m,T) = \partial I_0(m,T)/\partial m^2$$

One-loop RGOPT for NLSM pressure

Exact T-dependent mass gap $\bar{m}(g,T)$ from $\partial_m P(m)=0$:

$$\ln\frac{\bar{m}}{\mu} = -\frac{1}{b_0\, g(\mu)} - 2K_1(\frac{\bar{m}}{T}), \qquad (b_0^{\rm nlsm} = \frac{N-2}{2\pi})$$

•Exhibits *exact* (one-loop) scale invariance:

$T=0$: $\bar{m}(T=0) = \mu e^{-\frac{1}{b_0\, g(\mu)}} = \Lambda_{\overline{MS}}^{1-{\rm loop}}$

$T \gg m$: $$\frac{\bar{m}(T)}{T} = \frac{\pi\, b_0\, g}{1-b_0\, g\, L_T}, \quad (L_T \equiv \ln\frac{\mu\, e^{\gamma_E}}{4\pi T})$$

$$P_{1L,exact}^{\rm RGOPT} = -\frac{(N-1)}{\pi} T^2 \left[K_0(\bar{x}) + \frac{\bar{x}^2}{8}\,(1+4K_1(\bar{x}))\right], \quad (\bar{x} \equiv \bar{m}/T)$$

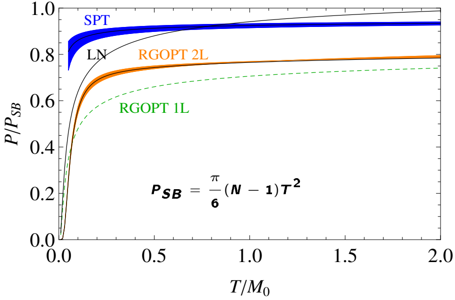
$$P_{1L}^{\rm RGOPT}(T \gg m) \simeq 1 - \frac{3}{2} b_0 g(\frac{4\pi\, T}{e^{\gamma_E}})$$

with one-loop running $g^{-1}(\mu) = g^{-1}(M_0) + b_0 \ln\frac{\mu}{M_0})$

→ nice RGOPT property: running with T emerges consistently
(for standard perturbation, SPT, HTLpt, $\mu \sim 2\pi T$ 'chosen')

NLSM pressure [G. Ferrari, JLK, M.B. Pinto, R.0 Ramos, 1709.03457,PRD]

$P/P_{SB}(N=4, g(M_0)=1)$ vs standard perturbation (PT), large N (LN), and SPT $\equiv$ ignoring RG-induced subtraction; $m^2 \to m^2(1-\delta)$:



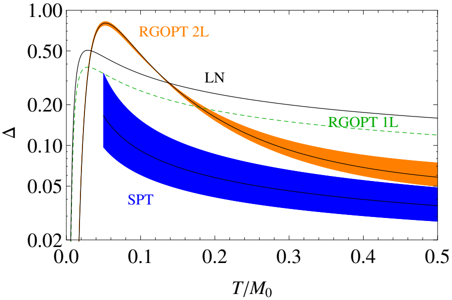
(shaded range: scale-dependence $\pi T < \mu \equiv M < 4\pi T$)

$\to$ At two-loops a moderate scale-dependence reappears, although less pronounced than 2-loop standard PT, SPT.

Higher order: RGOPT at $\mathcal{O}(g^k) \to \bar{m}(\mu)$ appears at $\mathcal{O}(g^{k+1})$ for *any* $\bar{m}$, but $\bar{m} \sim gT \to P \simeq \bar{m}^2/g + \cdots$ has leading μ-dependence at $\mathcal{O}(g^{k+2})$.

NLSM interaction measure (trace anomaly)

(normalized) $\Delta_{\text{NLSM}} \equiv (\mathcal{E} - P)/T^2 \equiv T\partial_T(\frac{P}{T^2})$



$N = 4, g(M_0) = 1$ (shaded regions: scale-dependence $\pi T < \mu = M < 4\pi T$)

•2-loop Δ_{SPT}: small, monotonic behaviour + sizeable scale dependence.

•RGOPT shape 'qualitatively' comparable to QCD, showing a peak: only obtained from interplay between $T \neq 0$ and $T = 0$ nonperturbative mass gap.

(But no phase transition in 2D NLSM (Mermin-Wagner-Coleman theorem): just reflects broken conformal invariance (mass gap)).

5. Thermal (pure glue) QCD: hard thermal loop (HTL)

QCD(glue) adaption of OPT → HTLpt [Andersen, Braaten, Strickland '99]: same trick now operates on a gluon mass term [Braaten-Pisarski '90]:

$$\mathcal{L}_{QCD}(\text{gauge}) - \frac{m^2}{2} Tr \left[G_{\mu\alpha} \langle \frac{y^\alpha y^\beta}{(y.D)^2} \rangle_y G^\mu_\beta \right], \quad D^\mu = \partial^\mu - ig\, A^\mu, \quad y^\mu = (1, \hat{\mathbf{y}})$$

(effective, explicitly gauge-invariant but nonlocal Lagrangian):

originally describes screening mass $m^2 \sim \alpha_S T^2$ + other HTL contributions [dressing gluon vertices and propagators]
But here m is arbitrary: determined by optimization in RGOPT.

$$P^{HTL,exact}_{1\text{-loop}} = (N_c^2 - 1) \times$$
$$\left\{ \frac{m^4}{64\pi^2}(C_{11} - \ln\frac{m}{\mu}) + \int_0^\infty \frac{d\omega}{(2\pi^3} \frac{1}{e^{\frac{\omega}{T}} - 1} \int_\omega^\infty dk\, k^2 (2\phi_T - \phi_L) \right.$$
$$\left. - \frac{T}{2\pi^2} \int_0^\infty dk\, k^2 \left[2\ln(1 - e^{-\frac{\omega_T}{T}}) + \ln(1 - e^{-\frac{\omega_L}{T}}) \right] - \frac{\pi^2 T^4}{90} \right\}$$

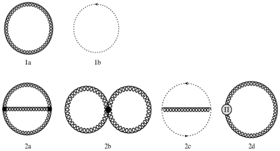
where $k^2 + m^2 \left[1 - \frac{\omega_L}{2k} \ln(\frac{\omega_L + k}{\omega_L - k}) \right] = 0$; $f(\omega_T) = 0$; ϕ_L, ϕ_T: complicated.

- Exact 2-loop? daunting task...

NB possibly simpler effective gluon mass models/prescriptions exist...
[e.g. Reinosa, Serreau, Tissier, Weschbor '15]

...But HTLpt advantage: calculated up to 3-loops α_S^2 (NNLO)

at two-loops:



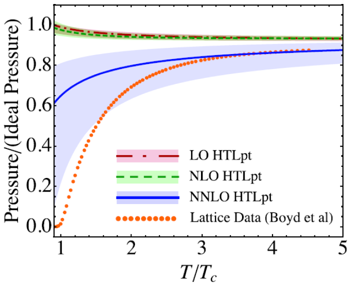
[Andersen et al '99-'15] BUT only as m/T expansions
Drawback: HTLpt $\equiv$ high-T approximation (by definition)

$$P^{HTLpt}_{\text{1-loop},\overline{\text{MS}}} = P_{\text{ideal}} \times$$
$$\left[1 - \frac{15}{2}\hat{m}^2 + 30\hat{m}^3 + \frac{45}{4}\hat{m}^4(\ln\frac{\mu}{4\pi T} + \gamma_E - \frac{7}{2} + \frac{\pi^2}{3})\right]$$

$\hat{m} \equiv \frac{m}{2\pi T}$ $\qquad$ $P_{\text{ideal}} = (N_c^2 - 1)\pi^2 \frac{T^4}{45}$

Standard HTLpt results

Pure gauge at NNLO (3-loops) [Andersen, Strickland, Su '10]:



Reasonable agreement with lattice results (down to $T \sim 2 - 3T_c$) only emerges at NNLO (3-loop) for low scale $\mu \sim \pi T - 2\pi T$.

NB even better agreement with lattice (for central scale choice) when including quarks.

Main HTLpt issue: drastically increasing scale dependence at NNLO order
Moreover HTLpt mass prescription: $\bar{m} \to m_D^{pert}(\alpha_S)$ [rather than optimizing $\partial_m P(m)$, to avoid complex optimized solutions]:
but OPT generally captures "more nonperturbative" information.

RGOPT adaptation of HTLpt =RGOHTL

Main changes:

• Crucial RG invariance-restoring subtractions in Free energy (pressure):
$P_{HTLpt} \to P_{HTLpt} - m^4(\frac{s_0}{\alpha_S} + s_1 + \cdots)$: reflects its anomalous dimension.

• Interpolate with $m^2(1-\delta)^{\frac{\gamma_0}{b_0}}$, where gluon mass anomalous dimension defined from (available) counterterm.

•Scale dependence improves at higher orders since RG invariance maintained at all stages:
-from subtraction terms (prior to interpolation)
-from interpolation keeping RG invariance.

•SPT,HTLpt,... do not fulfill this:
yet harmless (scale dependence moderate) up to 2-loops,
because the (leading order) RG-unmatched term $\mathcal{O}(m^4 \ln \mu)$ is perturbatively '3-loop' $\mathcal{O}(\alpha_S^2)$ from $m^2 \sim \alpha_S T^2$.

→ But explains why HTLpt scale dependence dramatically resurfaces at 3-loops!

Preliminary RGO(HTL) results (1- and 2-loop, pure glue)

One-loop: exactly scale-invariant pressure (like ϕ^4 and NLSM):

$$\frac{P(T \gg m)}{P_{ideal}} = 1 - \frac{15}{4}\hat{m}^2 + \frac{15}{2}\hat{m}^3 + \mathcal{O}(\hat{m}^6)$$

$$\hat{m}(OPT) = G\,(1 + \sqrt{1 - \frac{1}{3G}}),\ G \text{ 'coupling'} = (\ln \frac{4\pi T}{\Lambda_{\overline{MS}}} + const.)^{-1}$$

•Once accepting arbitrary m in $\mathcal{L}_{HTL}$, like in NLSM RGOPT includes nontrivial $P(T=0) \simeq -const\,\Lambda^4_{\overline{MS}}$

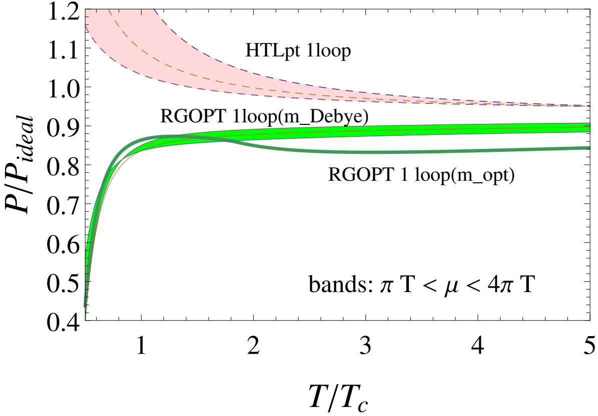
•Pb however: this 'exact' OPT $\hat{m}$ becomes complex for small enough G: essentially an artefact of $\overline{MS}$-scheme +high-T approx (gives small $Im[P]/Re[P]$)

• Our attitude: one-loop approximation is not final stage:
Pragmatic: at one-loop we thus take $Re[P(g)]$ in relevant T/T_c range.

• Yet consistent with Stefan-Boltzmann limit: $P(g \to 0) \to P_{ideal}$

Alternatively evade this pb if adopting HTLpt prescription: $m \to m^{PT}_{Debye}$
(consistent with standard PT: $P/P_{SB}\ 1 - 15/4(\alpha_S/\pi) + \cdots$)
but loose exact 1-loop scale invariance
• 2-loops: RGOPT gives a real unique solution.

RGOPT vs HTLpt: one-loop pressure



NB: bending of P_{RGOPT} for small T essentially due to $-P_{RGOPT}(T=0) \neq 0$.

2-loop RGOHTL: need new calculations...

Crucial RG-consistent 2-loop subtractions determined by $\alpha_S m^4 \ln \mu$ term (non-logarithmic terms relevant too).

But these are $\mathcal{O}(\alpha_S T^4 \frac{m^4}{T^4})$ of 2-loop HTL contributions, not available in literature (to best of our knowledge):

give $\sim$ 30 independent integrals, half being (very) complicated, e.g.

$$\sum\!\!\!\!\!\!\!\int_{P,Q} \frac{T_P T_Q (p+q)^2}{p^2 P^2 q^2 Q^2 (P+Q)^2}; \quad T_P \equiv \int_0^1 dc \frac{P_0^2}{P_0^2 + c^2 p^2}$$

$(P^2 = P_0^2 + p^2)$, $c \equiv$HTL angle (averaging).

Present status: work under good progress but need to be checked, specially the difficult non-logarithmic parts (i.e. finite parts in dim.reg.)

NB high-$T \leftrightarrow T = 0$ correspondance:

$C_{20} \ln^2 \frac{\mu}{T} + C_{21} \ln \frac{\mu}{T} + C_{22}^{(T \gg m)} \leftrightarrow C_{20} \ln^2 \frac{\mu}{m} + C_{21} \ln \frac{\mu}{m} + C_{22}^{(T=0)}$

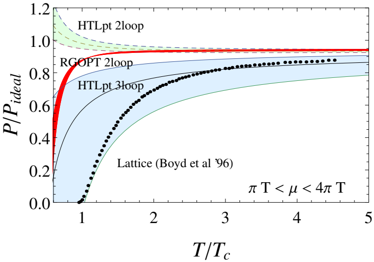
•However Leading Log (LL) C_{20} determined straightforwardly from RG from one-loop.

•Similarly the 2-loop (perturbative) scale invariance is guaranteed by $s_1 = f[C_{10}]$ independently of precise C_{10} value!

(Preliminary!) RGO(HTL) results (2-loop, pure glue)

2-loop illustrated here for simplest LL approx.: $C_{21} = C_{22} = 0$
but RG-subtraction $s_1(C_{21} = 0)$ consistent

Moderate scale-dependence reappears at 2-loops
but sensible improvement wrt HTLpt



[JLK, M.B Pinto, to appear soon]

NB scale dependence should further improve at 3-loops, generically:

RGOPT at $\mathcal{O}(\alpha_S^k) \to \bar{m}(\mu)$ appears at $\mathcal{O}(\alpha_S^{k+1})$ for *any* $\bar{m}$, but
$\bar{m}^2 \sim \alpha_S T^2 \to P \simeq \bar{m}_G^4/\alpha_S + \cdots$: leading μ-dependence at $\mathcal{O}(\alpha_S^{k+2})$.

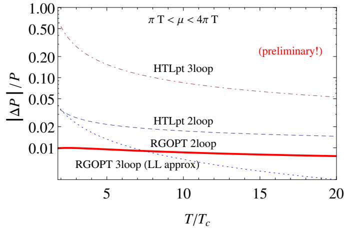
- Warning: low $T \sim T_c$ genuine $P(T)$ shape sensitive to true C_{21}, C_{22} (crucially needed before possibly comparing RGOPT vs lattice).

(Very) preliminary RGO(HTL) approximate 3-loop results

3-loops: exact $m^4\alpha_S^2$ terms need extra complicated calculations, but
$P^{3l}_{RGOHTL} \sim P^{2l}_{RGOHTL} + m^4\alpha_S^2(C_{30}\ln^3\frac{\mu}{2\pi T} + C_{31}\ln^2\frac{\mu}{2\pi T} + C_{32}\ln\frac{\mu}{2\pi T} + C_{33})$:

leading logarithms (LL) and next-to-leading (NLL) C_{30}, C_{31} fully determined from lower orders from RG invariance

Within this LL, NLL approximation and in $T/T_c \gtrsim 2$ range where it is more trustable:



We assume/expect true coefficients will not spoil this improved scale dependence.

Summary and Outlook

•RGOPT includes 2 major differences w.r.t. previous OPT/SPT/HTLpt... approaches:

1) OPT +/or RG optimizations fix optimal $\bar{m}$ *and possibly* $\bar{g} = 4\pi\bar{\alpha}_S$

2) Maintaining RG invariance uniquely fixes the basic interpolation $m \to m(1-\delta)^{\gamma_0/b_0}$: discards spurious solutions and accelerates convergence.

• At $T \neq 0$, exhibits improved stability + drastically improved scale dependence (with respect to standard PT, but also w.r.t. HTLpt)

•Paves the way to extend such RG-compatible methods to full QCD thermodynamics (work in progress, starting with $T \neq 0$ pure gluodynamics) specially for exploring also finite density