Andreas Schmitt

Mathematical Sciences and STAG Research Centre
University of Southampton
Southampton SO17 1BJ, United Kingdom

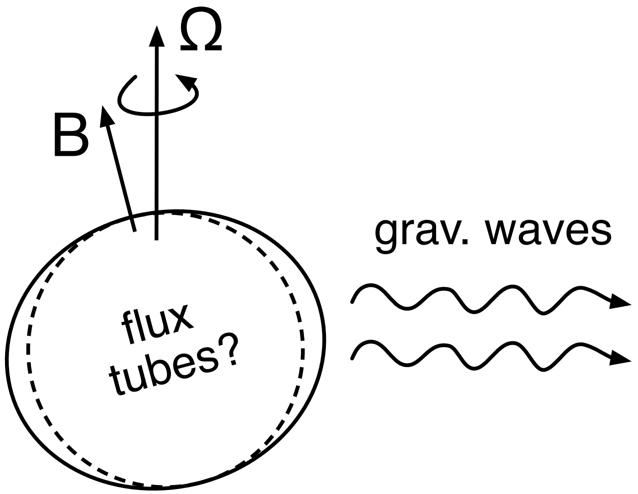
(Color-)magnetic flux tubes in dense matter

A. Haber, A. Schmitt, PRD 95, 116016 (2017); EPJ Web Conf. 137, 09003 (2017)
A. Haber, A. Schmitt, JPG 45, 065001 (2018)

Theoretical motivation

- What's the behavior of a multi-component superconductor regarding type-I/type-II?
- What (color-)flux tubes are there in a non-abelian superconductor like color-flavor locked (CFL) quark matter?

Phenomenological motivation



- Do (color-)flux tubes support ellipticities of neutron stars → gravitational waves?
  K. Glampedakis, D. I. Jones and L. Samuelsson, PRL 109, 081103 (2012)

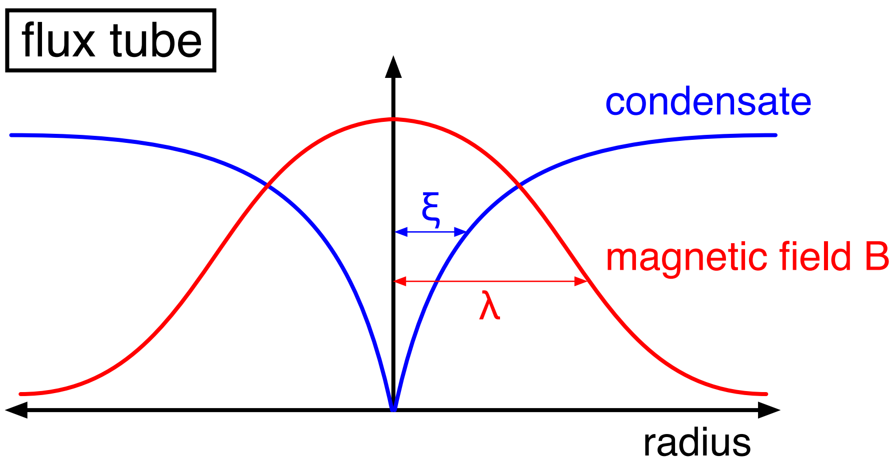
(besides magnetic forces, see also crystalline structures)

Single-component superconductor

- Ginzburg-Landau potential U for complex field ϕ with charge q coupled to gauge field A^{μ}

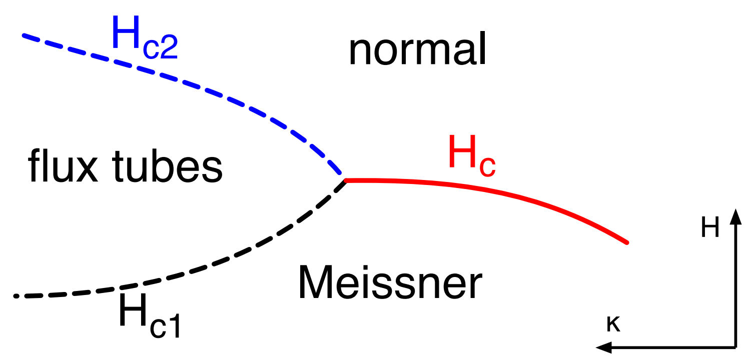
$$U = \frac{\mathbf{B}^2}{2} + |(\nabla + iq\mathbf{A})\phi|^2 - \mu^2|\phi|^2 + \lambda|\phi|^4$$

Reminder: type-I/type-II superconductivity



- Ginzburg-Landau parameter

$$\kappa = \frac{\lambda}{\xi}$$



- type-II superconductivity
  for $\kappa > 1/\sqrt{2}$: flux tube lattice
  for $H_{c1} < H < H_{c2}$
  A.A. Abrikosov, Soviet Physics JETP 5, 1174 (1957)

Multi-component superconductors (page 1/2)

- two fields with charges q_1, q_2 (neutron/proton: $q_1 = 2e$, $q_2 = 0$)

$$U = \frac{\mathbf{B}^2}{2} + \sum_{i=1,2} \left[|(\nabla + iq_i\mathbf{A})\phi_i|^2 - \mu_i^2|\phi_i|^2 + \lambda_i|\phi_i|^4 \right] + 2h|\phi_1|^2|\phi_2|^2$$

- fields are coupled indirectly via gauge field (if both $q_1, q_2 \neq 0$) and directly with coupling h (neutron/proton system: additional derivative coupling A. Haber, A. Schmitt, PRD 95, 116016 (2017))

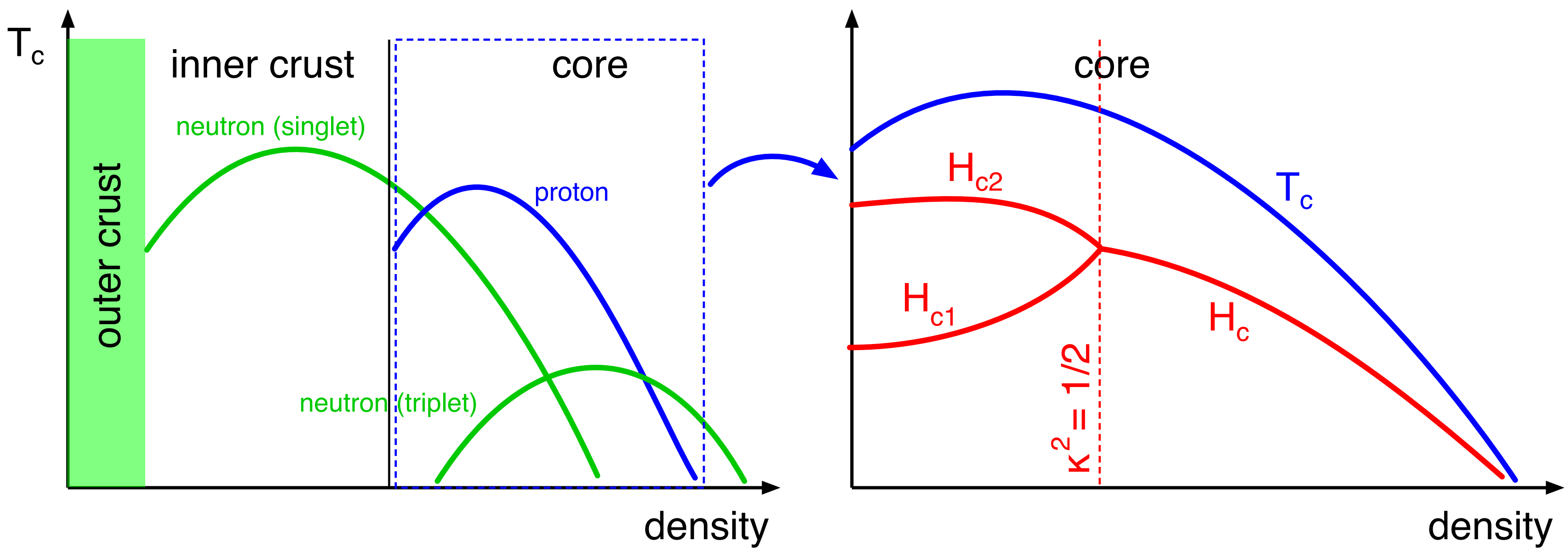
Multi-component superconductors (page 2/2)

- more fields and multiple (color-)gauge fields

$$U = \frac{\mathbf{B}_1^2}{2} + \frac{\mathbf{B}_2^2}{2} + \sum_{i=1}^{3} \left[\left| \left(\nabla + i q_{i1} \mathbf{A}_1 + i q_{i2} \mathbf{A}_2 \right) \phi_i \right|^2 - \mu^2 |\phi_i|^2 + \lambda |\phi_i|^4 \right]$$
$$-2h(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2)$$

- color superconductor: 3 scalar components and 3 gauge fields: 1 electromagnetic and 2 color fields

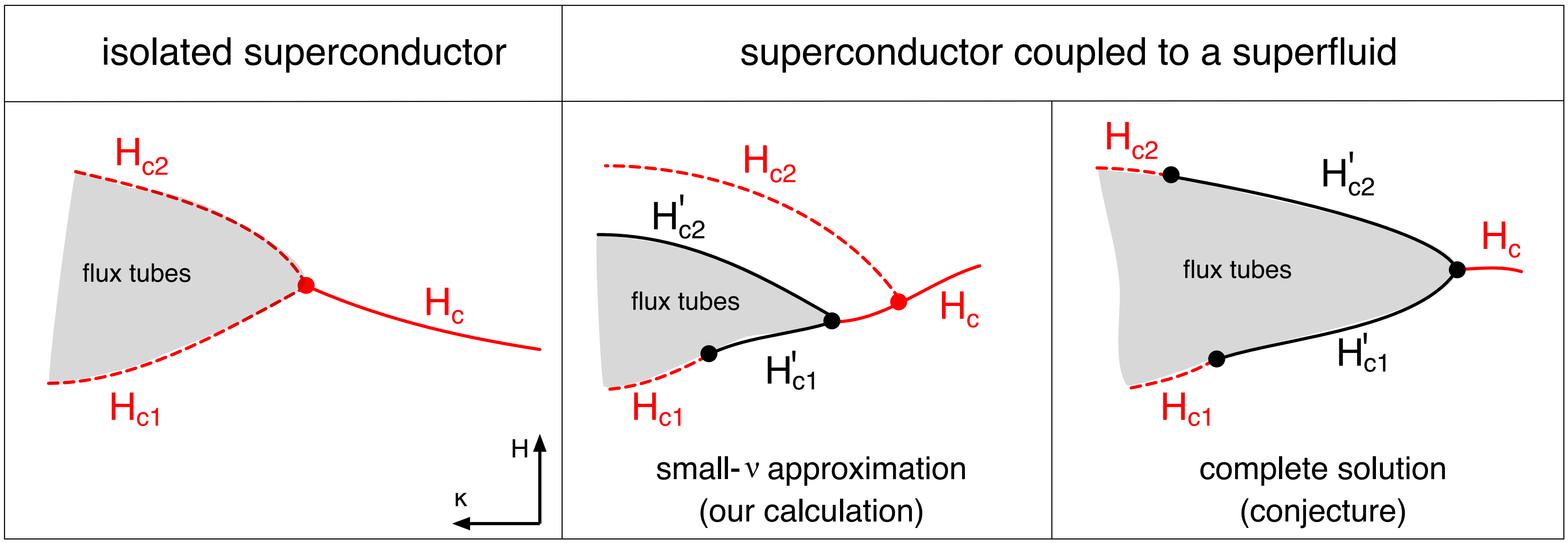
Two-component superconductors in neutron star cores



- density-dependent κ
- type-I/type-II transition in the core?
- effect of coupling to superfluid on type-I/type-II transition?

Critical magnetic fields in a two-component system

- compute flux tube profiles and flux tube interaction
  $\rightarrow$ attractive long-distance interaction in type-II regime



numerical calculation supports conjecture
A. Haber, D. Müller, preliminary results

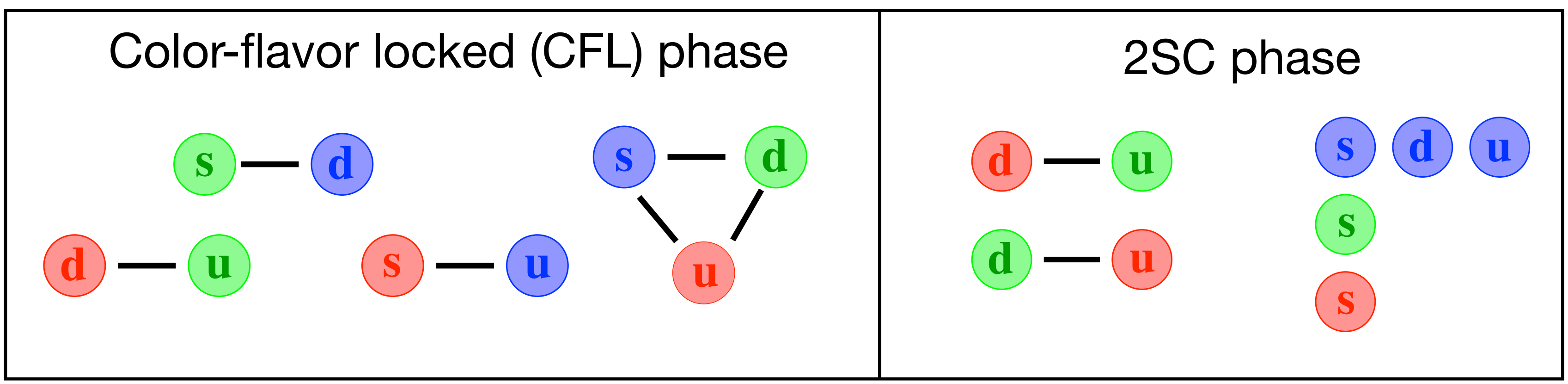
- first-order phase transition allows for flux tube clusters
  see also "type-1.5" superconductors J. Carlström, J. Garaud, E. Babaev, PRB 84, 134515 (2011)

Ginzburg-Landau potential for color superconductors

$$
\begin{aligned}
U \;=\; & -\mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] + a\mathrm{Tr}[\Phi^\dagger\Phi] + b\mathrm{Tr}[(\Phi^\dagger\Phi)^2] + c(\mathrm{Tr}[\Phi^\dagger\Phi])^2 \\
& +\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}
\end{aligned}
$$

$$
\Phi = \quad \begin{pmatrix} \phi & 0 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & \phi \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \phi \end{pmatrix} \qquad \begin{pmatrix} \phi_1(\vec{r}) & 0 & 0 \\ 0 & \phi_2(\vec{r}) & 0 \\ 0 & 0 & \phi_3(\vec{r}) \end{pmatrix}
$$

CFL 2SC ansatz for flux tubes



Mixing of gluons and photons in CFL

- symmetry breaking pattern of CFL

$$[SU(3)]_c \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset [U(1)]_Q} \times U(1)_B \to \underbrace{SU(3)_{c+L+R}}_{\supset [U(1)]_{\tilde{Q}}} \times \mathbb{Z}_2$$

- CFL is a superfluid $\to$ rotational vortices

- Meissner effect for gluons $T_1, \ldots, T_7$ ("color superconductor")

- all Cooper pairs neutral under $\tilde{Q} = Q + \frac{2}{\sqrt{3}} T_8$

  and (differently) charged under orthogonal combination $\tilde{T}_8$

  – $\tilde{Q}$-magnetic field penetrates CFL
  – Meissner effect for $\tilde{T}_8$-magnetic field

(Analogous to gauge field mixing in standard model, $[SU(2)] \times [U(1)] \to [U(1)]_Q$)

Ginzburg-Landau potential with rotated gauge fields

$$U = \frac{\tilde{\mathbf{B}}^2}{2} + \frac{\mathbf{B}_3^2}{2} + \frac{\tilde{\mathbf{B}}_8^2}{2} + \left|\left(\nabla + i\frac{g}{2}\mathbf{A}_3 + i\tilde{g}_8\tilde{\mathbf{A}}_8\right)\phi_1\right|^2 + \left|\left(\nabla - i\frac{g}{2}\mathbf{A}_3 + i\tilde{g}_8\tilde{\mathbf{A}}_8\right)\phi_2\right|^2$$
$$+\left|\left(\nabla - 2i\tilde{g}_8\tilde{\mathbf{A}}_8\right)\phi_3\right|^2 - \mu^2(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2) + \lambda(|\phi_1|^4 + |\phi_2|^4 + |\phi_3|^4)$$
$$-2h(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2)$$

- approximations
  - massless quarks, $m_s = m_d = m_u = 0$
  - purely bosonic approach → neglect effects of magnetic field on Cooper pair constituents
  - Ginzburg-Landau parameters μ, λ, h: (mostly) use perturbative results and extrapolate down in density (= to large couplings)

Ginzburg-Landau potential with rotated gauge fields

$$U = \frac{\tilde{\mathbf{B}}^2}{2} + \frac{\mathbf{B}_3^2}{2} + \frac{\tilde{\mathbf{B}}_8^2}{2} + \left|\left(\nabla + i\frac{g}{2}\mathbf{A}_3 + i\tilde{g}_8\tilde{\mathbf{A}}_8\right)\phi_1\right|^2 + \left|\left(\nabla - i\frac{g}{2}\mathbf{A}_3 + i\tilde{g}_8\tilde{\mathbf{A}}_8\right)\phi_2\right|^2$$
$$+ \left|\left(\nabla - 2i\tilde{g}_8\tilde{\mathbf{A}}_8\right)\phi_3\right|^2 - \mu^2(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2) + \lambda(|\phi_1|^4 + |\phi_2|^4 + |\phi_3|^4)$$
$$-2h(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2)$$

- ansatz for flux tube solutions

$$\phi_i(r,\theta) = \rho_i(r)\, e^{in_i\theta}$$

  with winding numbers n_1, n_2, n_3

- solve equations of motion for ρ_1, ρ_2, ρ_3, $\mathbf{A}_3$, $\tilde{\mathbf{A}}_8$
- boundary conditions: homogeneous CFL far away from flux tube, $\rho_i = 0$ in center if winding n_i is nonzero

- usually: vortex $\rightarrow$ baryon circulation $\Gamma = \oint d\boldsymbol{\ell}\cdot\mathbf{v}$

  flux tube $\rightarrow$ magnetic flux $\Phi = \oint d\boldsymbol{\ell}\cdot\mathbf{A}$

- CFL line defects can have both!

$$\Gamma \propto n_1 + n_2 + n_3\,, \qquad \Phi_3 \propto n_1 - n_2\,, \qquad \tilde{\Phi}_8 \propto n_1 + n_2 - 2n_3$$

CFL line defects	(n_1, n_2, n_3)	$\Gamma\ [\pi/3\mu_q]$	$\Phi_3\ [\pi/g]$	$\tilde{\Phi}_8\ [\pi/\tilde{g}_8]$
Global vortex Forbes, Zhitnitsky (2002)	(n, n, n)	$-n$	0	0
"Semi-superfluid" vortex Balachandran, Digal, Matsuura (2006)	$(0, 0, n)$	$-\frac{n}{3}$	0	$\frac{2n}{3}$
Magnetic flux tube T_{112} Iida (2005)	$(n, n, -2n)$	0	0	$-2n$
Magnetic flux tube T_{101} Haber, Schmitt (2018)	$(n, 0, -n)$	0	$-n$	$-n$

- vortices ($\Gamma \neq 0$): "topological" since $\pi_1[U(1)] = \mathbb{Z}$
  - global vortex decays into 3 semi-superfluid vortices
    M. G. Alford, S. K. Mallavarapu, T. Vachaspati and A. Windisch, PRC 93, 045801 (2016)
- flux tubes ($\Gamma = 0$): "non-topological" since $\pi_1[SU(3)] = 0$
  - stabilized through external magnetic field → see rest of the talk

CFL line defects	(n_1, n_2, n_3)	$\Gamma\ [\pi/3\mu_q]$	$\Phi_3\ [\pi/g]$	$\tilde{\Phi}_8\ [\pi/\tilde{g}_8]$
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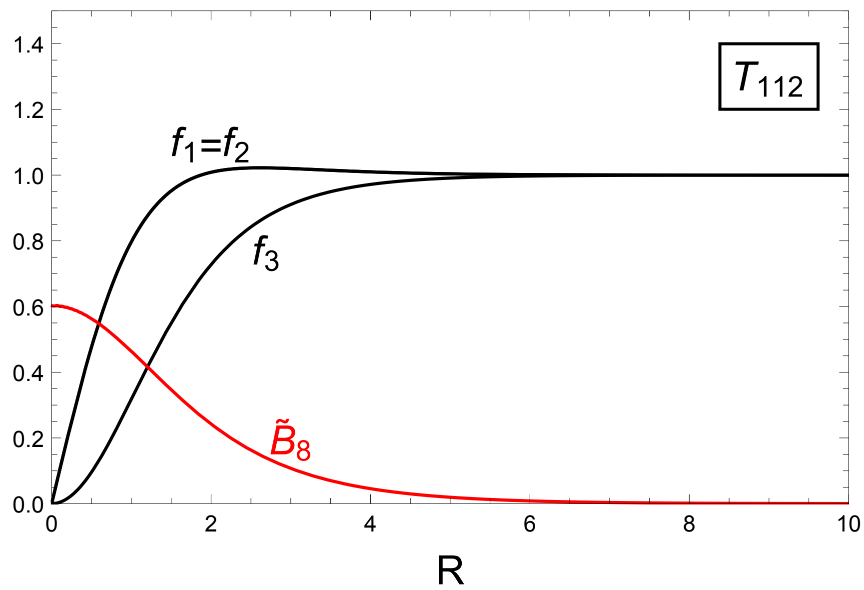
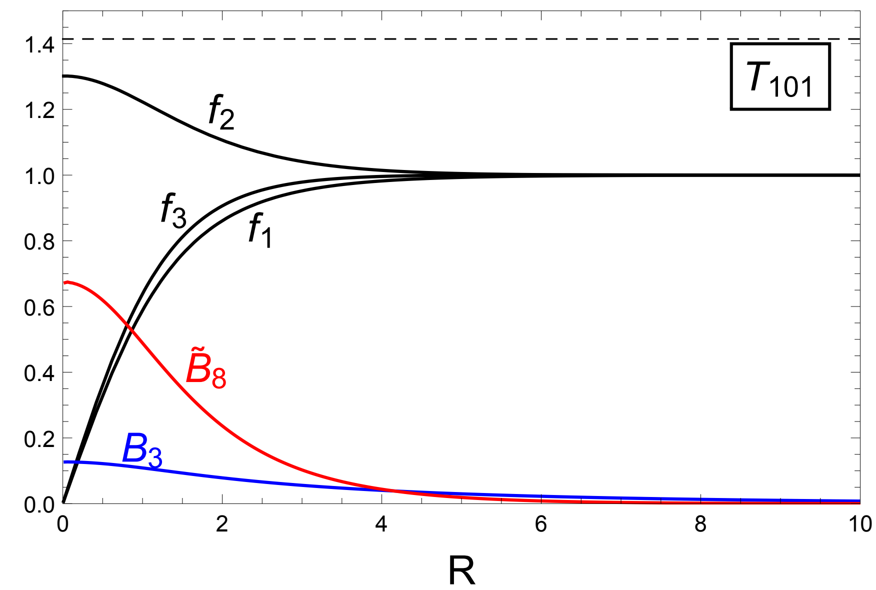
Flux tube profiles

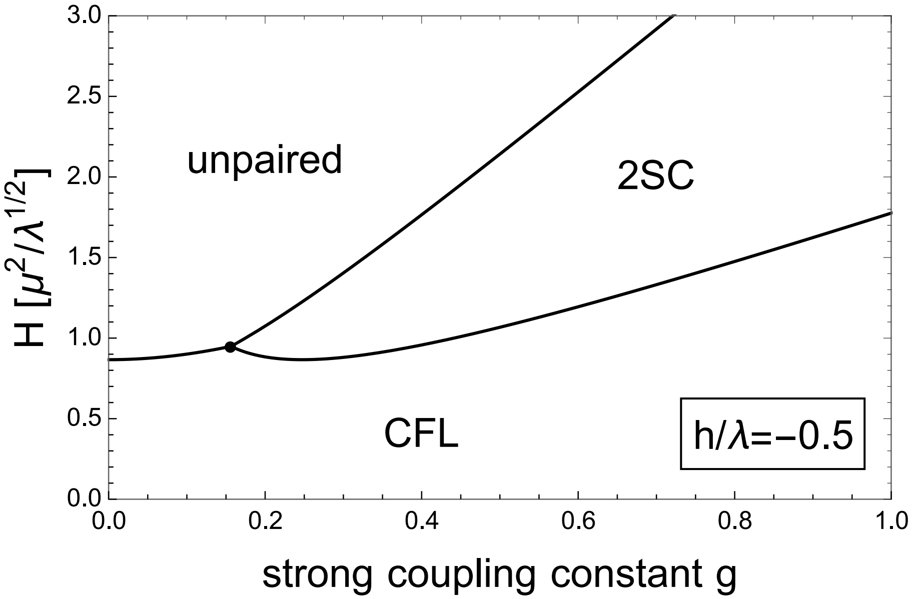




- flux tube with "unpaired core"
  K. Iida, PRD 71, 054011 (2005)

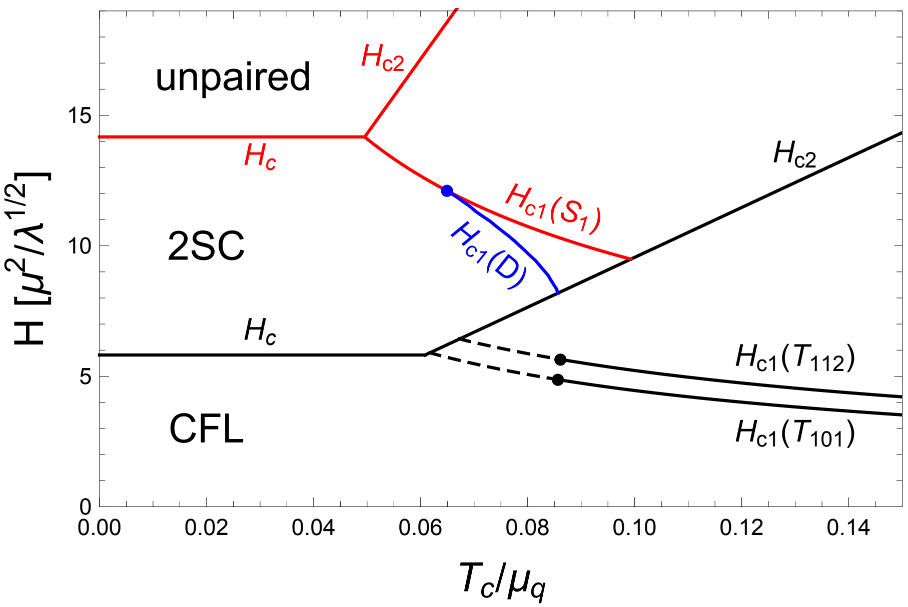
- flux tube with "2SC core"
  A. Haber, A. Schmitt, JPG 45, 065001 (2018)
- additional B_3 field
  (cost in free energy)
- non-vanishing condensate in core (gain in free energy)

Phase structure without flux tubes



- in weak coupling: $h/\lambda = -0.5$
- CFL superseded by 2SC except for small values of strong coupling constant g (even though here $m_s \simeq 0$!)
- in neutron stars $\mu_q \simeq 400\,\mathrm{MeV} \Rightarrow g \simeq 3.5$

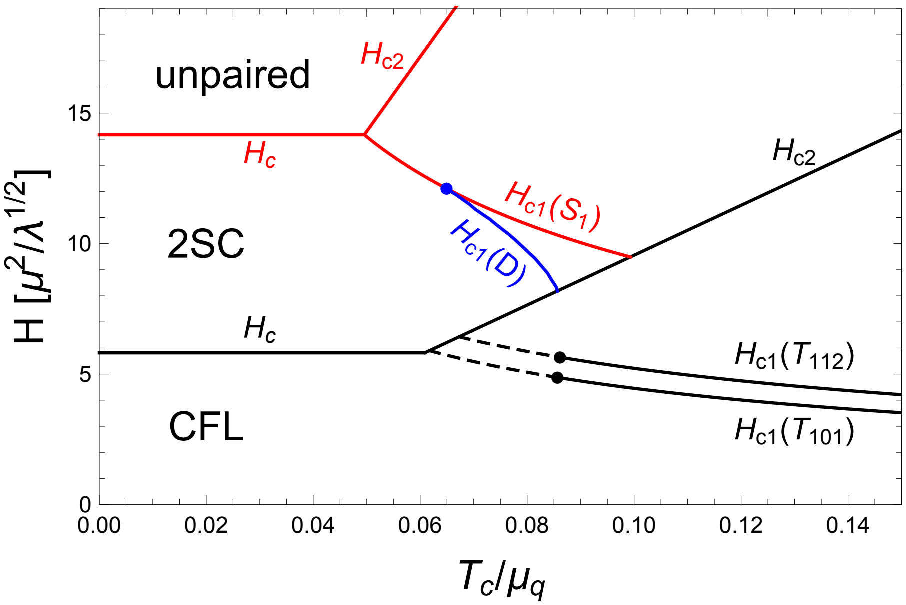
Phase diagram including flux tubes



- type-II regime for sufficiently large T_c/μ_q
  (neutron stars: $T_c/\mu_q \sim (10-100)/(400-500) \sim 0.1$)

- type-I/type-II transition complicated (multi-component structure!)

Phase diagram including flux tubes



- CFL flux tubes with 2SC core (T_{101}) preferred
- critical fields $H \sim 10^{19}\,\mathrm{G} \gg H_{\mathrm{NS}}$, however:
  creation of flux tubes through cooling into superconducting phase?
- 2SC domain walls (D) preferred over ordinary 2SC flux tubes (T_1)
  for sufficiently large T_c/μ_q

Summary

- multi-component superconductors have a nontrivial type-I/type-II transition
- dense quark matter is a multi-component superconductor and has various possible line defects
- CFL flux tubes (without baryon circulation) are not protected by topology, but can be stabilized by a magnetic field
- we have found new magnetic defects in CFL and 2SC:
  CFL tubes with 2SC core and 2SC domain walls
- defects in superconducting/superfluid nuclear and quark matter are relevant for neutron star observables, e.g., gravitational waves