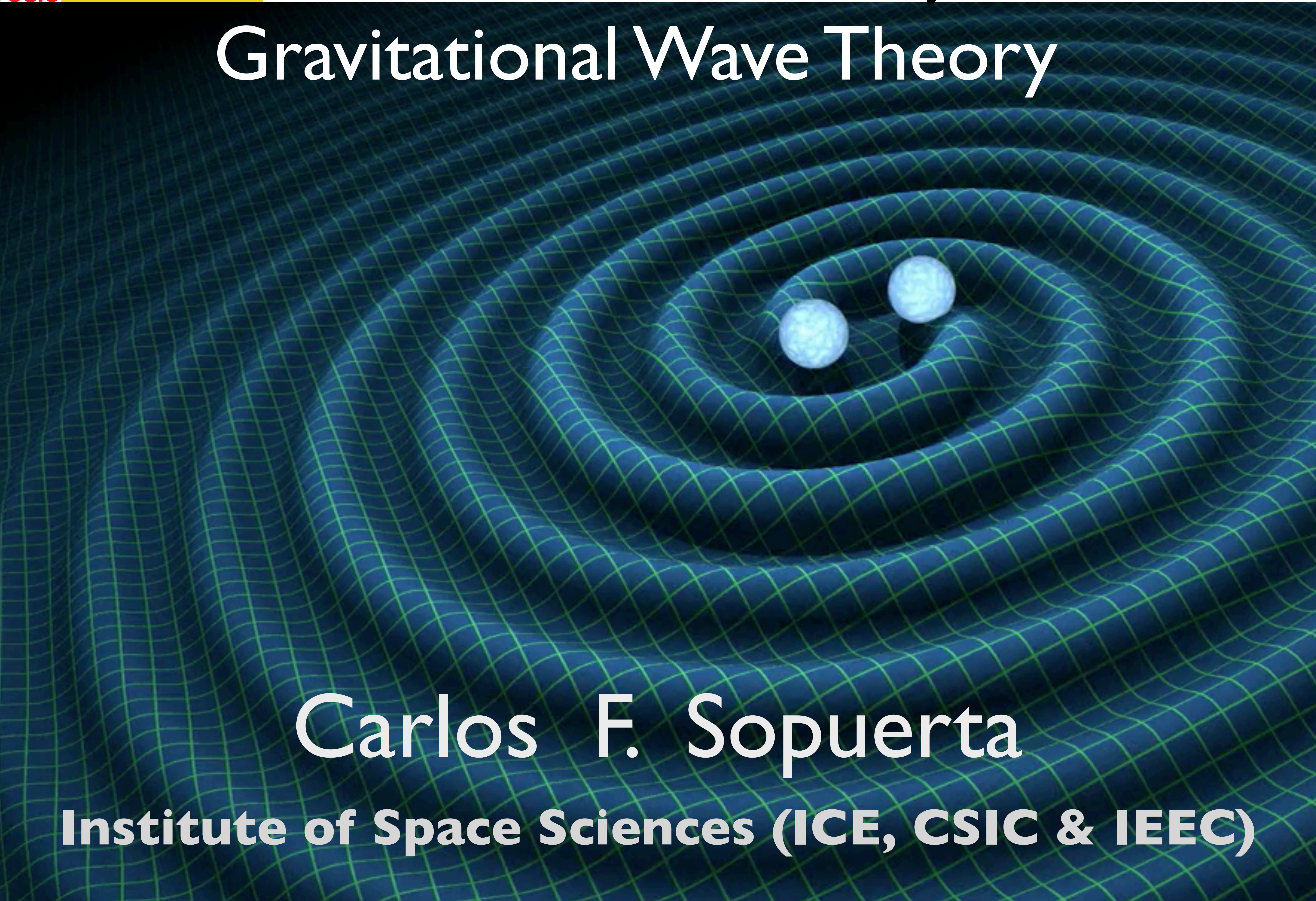
2nd ICE Summer School:
Gravitational Wave Astronomy

Gravitational Wave Theory

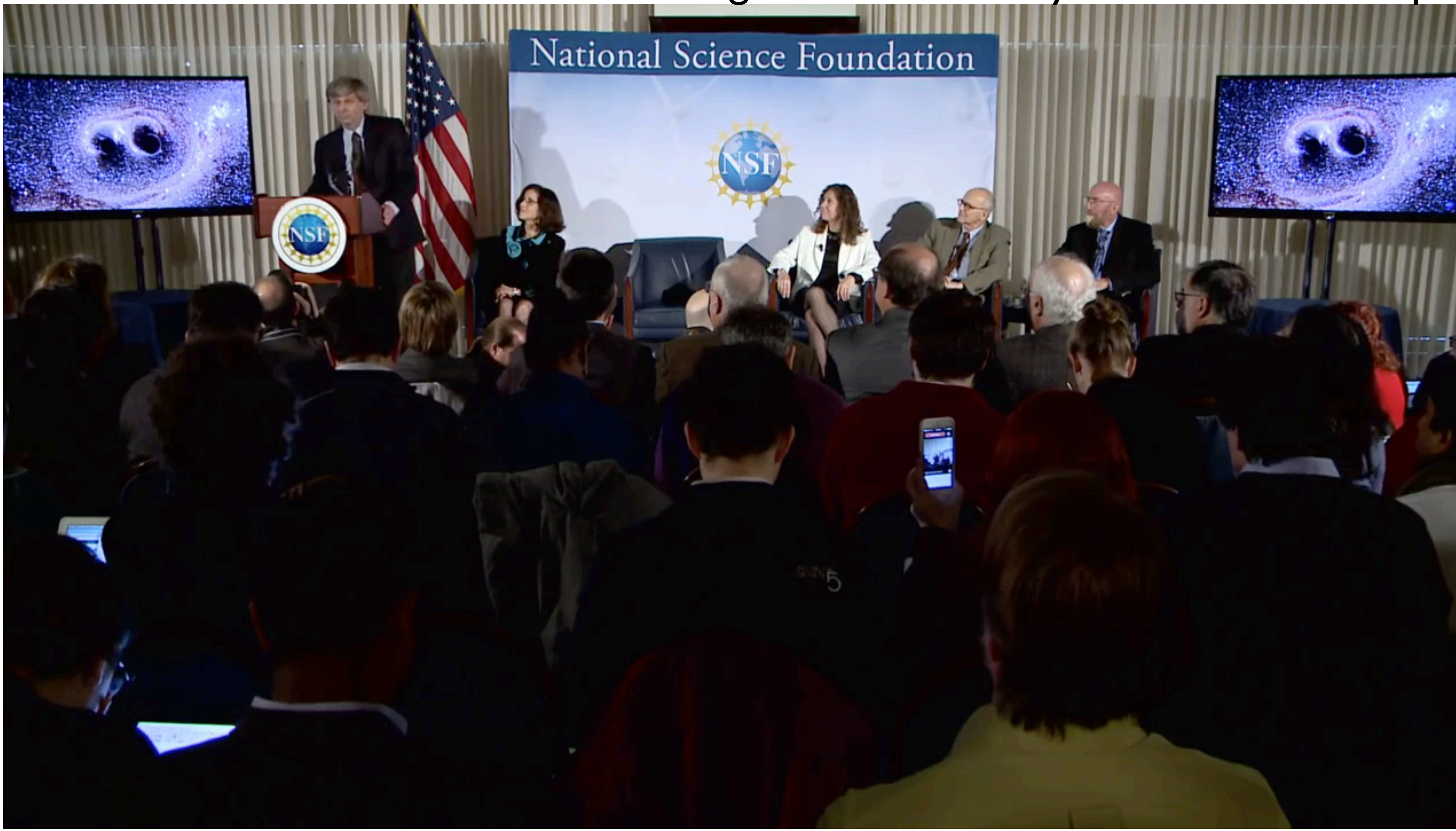


Carlos F. Sopuerta

Institute of Space Sciences (ICE, CSIC & IEEC)

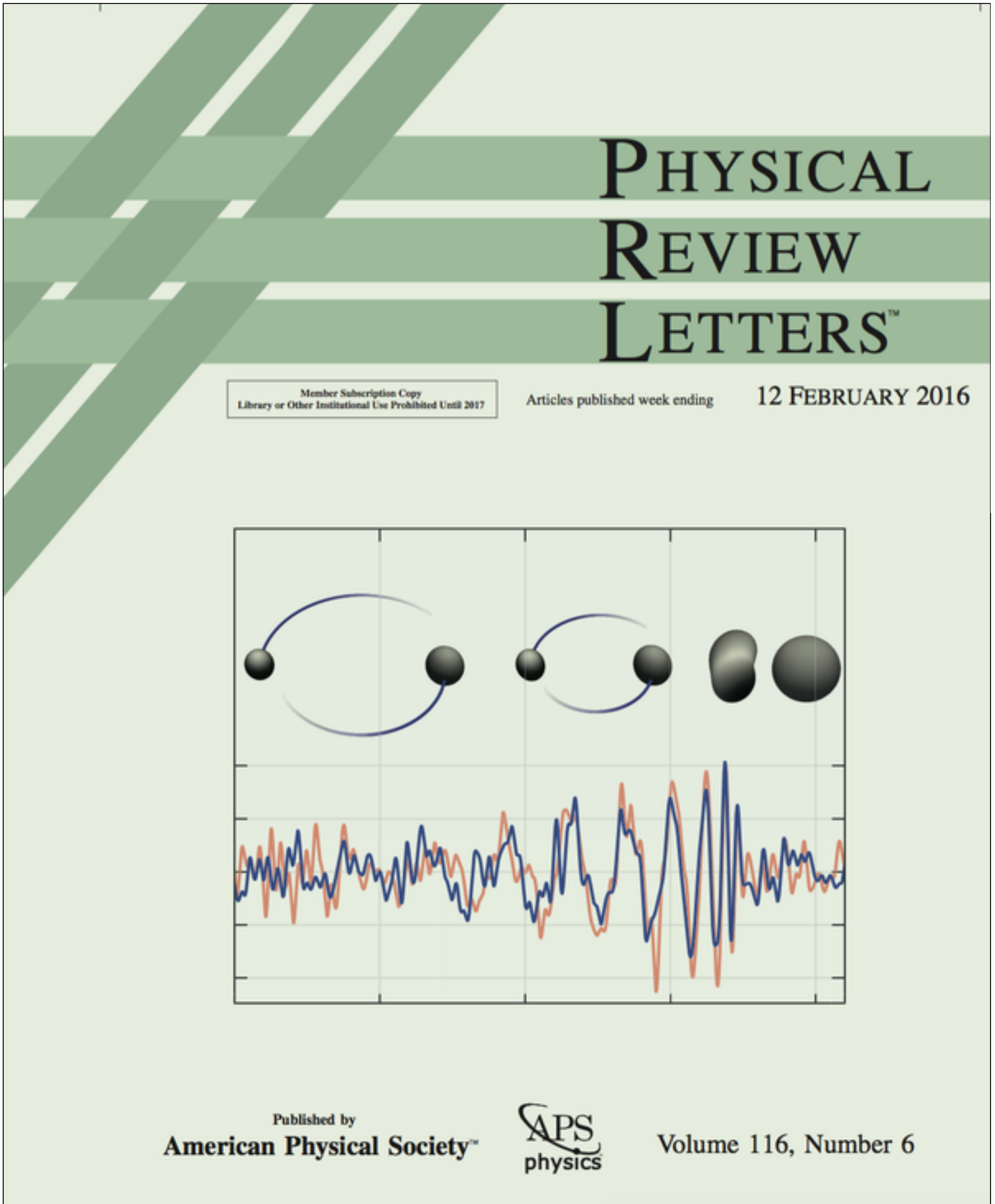
The Beginning of Gravitational Wave Astronomy

Washington DC, February 11th, 2016 at 4:30pm



David Reitze (Caltech), LIGO Executive Director

The Beginning of Gravitational Wave Astronomy



Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

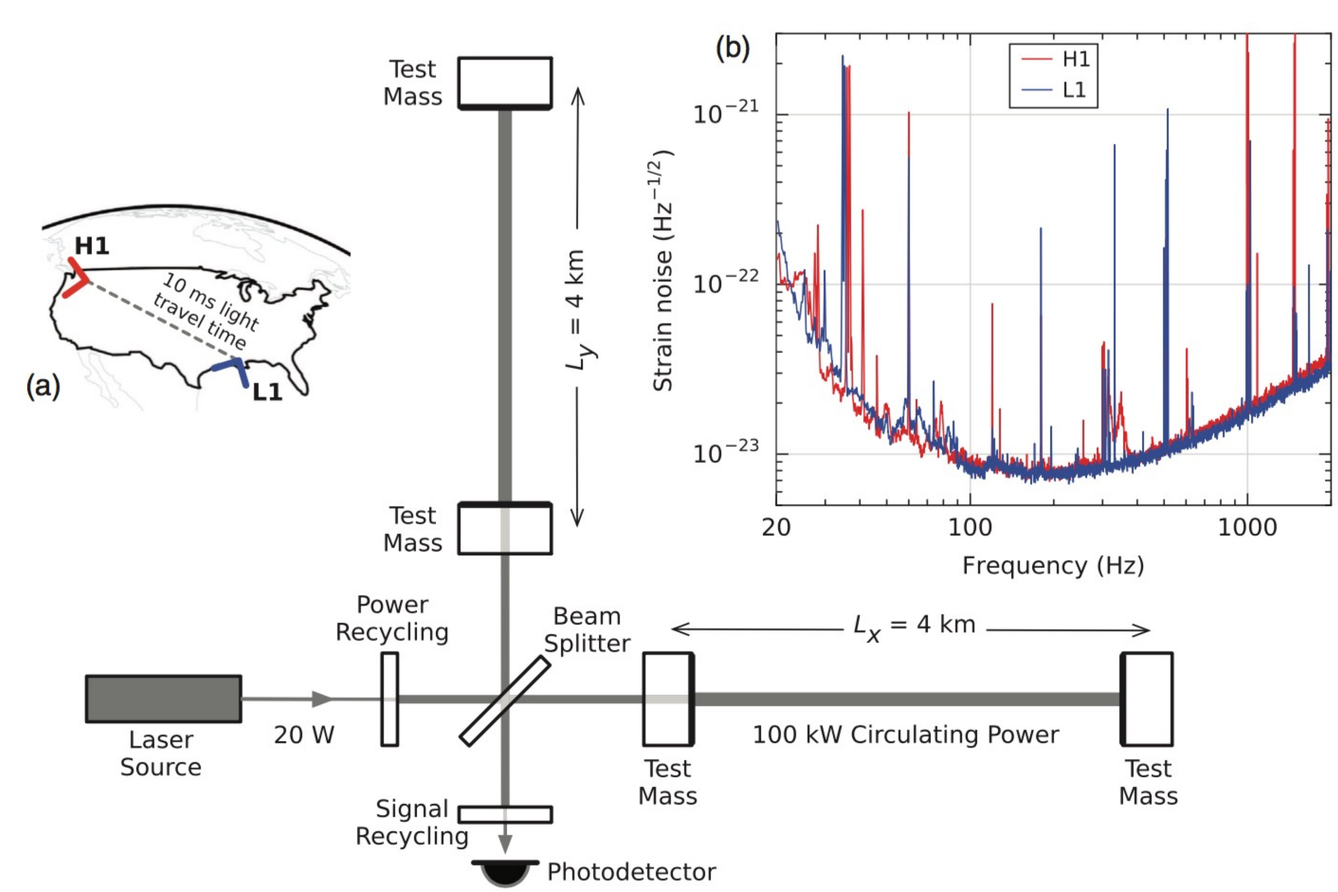
B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

DOI: 10.1103/PhysRevLett.116.061102



Current Situation of Gravitational Wave Astronomy

- Gravitational Waves exist and affect our (laser interferometric) detectors as expected (by most people...)!
- Stellar-Mass Binary Black Holes exist (first evidence), they merge and form another (bigger) black hole!
- Stellar-Mass Black Holes with masses above $30M_{\odot}$ exist!
- The LIGO-Virgo collaboration has detected the coalescence and merger of 5 Binary Black Holes and a Binary Neutron Star (with lots of electromagnetic counterparts).
- The BNS is the first known outside our galaxy.
- First direct association of a BNS merger with a short gamma-ray burst (GRB), the closest known so far.
- First measurement of the Hubble constant using gravitational waves (for the determination of the luminosity distance).

Current Situation of Gravitational Wave Astronomy

- First confirmation of the kilonova mechanism for the formation of the heaviest elements.

- The **LISA Pathfinder** mission of the European Space Agency (ESA) has just demonstrated the technology for the future LISA space-based observatory (the ESA-L3 mission), with launch in ~2030.

- Pulsar timing arrays have achieved a sensitivity in the discovery region of the expected parameter-space for GW backgrounds produced by supermassive black hole binaries

- CMB polarization experiments are improving on ground and they are sensitive to interesting values of ‘r’.   Concepts for space are being proposed.

Some Bibliography

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Outline

- Brief Introduction to General Relativity
- Gravitational Waves in Linearized Gravity
- Interaction of Gravitational Waves with Matter
- Summary of Properties of Gravitational Waves
- The Gravitational Wave Spectrum: Sources & Detectors
- Energy-Momentum Content of Gravitational Waves
- Generation of Gravitational Waves
- Inspiral of Compact Binaries

Brief Introduction to General Relativity

Newtonian Gravitation

Newtonian Dynamics

Galilean Inertial Reference Frames

Newtonian Gravity

$$\vec{F} = m_i \, \vec{a}$$

$$\vec{F} = -G \, \frac{M_1^g \cdot M_2^g}{r_{12}^2} \, \hat{r}_{12}$$

$$t' = t - t_o$$

$$m_i = M_1^g$$

$$x' = x - v \cdot t$$

Consistent!!

Electromagnetism with Newtonian Dynamics

Newtonian Dynamics

$$\vec{F} = m_i \, \vec{a}$$

Galilean Inertial Reference Frames

$$t' = t - t_o$$

$$x' = x - v \cdot t$$

Maxwellian Electromagnetism

$$\begin{aligned}
\vec{\nabla} \cdot \vec{E} &= \epsilon_o^{-1} \rho \\
\vec{\nabla} \cdot \vec{B} &= 0 \\
\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} &= \vec{0} \\
\frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} - \vec{\nabla} \times \vec{B} &= -\mu_o \vec{J}
\end{aligned}$$

Not Consistent!!

Electromagnetism with Relativistic Dynamics

Relativistic Dynamics

$$F^\mu = m_i \frac{d^2 x^\mu}{d\tau^2}$$

Lorentz Inertial Reference Frames

$$\begin{aligned} t' &= \gamma\left(t - \frac{v \cdot x}{c^2}\right) \\ x' &= \gamma\,(x - v \cdot t) \end{aligned}$$

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Maxwellian Electromagnetism

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \epsilon_o^{-1}\,\rho \\ \vec{\nabla} \cdot \vec{B} &= 0 \\ \frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} &= \vec{0} \\ \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} - \vec{\nabla} \times \vec{B} &= -\mu_o \vec{J} \end{aligned}$$

Consistent!!

Newtonian Gravitation with Relativistic Dynamics

Relativistic Dynamics

$$F^{\mu} = m_i \frac{d^2 x^{\mu}}{d\tau^2}$$

Lorentz Inertial Reference Frames

$$t' = \gamma\left(t - \frac{v \cdot x}{c^2}\right)$$

$$x' = \gamma\left(x - v \cdot t\right)$$

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Newtonian Gravity

$$\vec{F} = -G\,\frac{M_1^g \cdot M_2^g}{r_{12}^2}\,\hat{r}_{12}$$

$$m_i = M_1^g$$

Not Consistent!!

Relativistic Gravitation (General Relativity)

Einstein Dynamics

General Covariance

Einstein Gravity

$$\nabla_\mu T^{\mu\nu} = 0$$

$$\begin{aligned} t' &= f(t,x) \\ x' &= g(t,x) \end{aligned}$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Consistent!!

Both included in Einstein's Field Equations

Gravitational Waves in Linearized Gravity

Gravitational Waves in Linearized Gravity

✳ Linearized gravity is a good approximation when the local spacetime geometry deviates slightly from flat spacetime:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \qquad |h_{\mu\nu}| \ll 1 \qquad (\mu,\nu,\ldots = 0-3;\, \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1))$$

✳ Some relevant formulae:

$$h^{\mu}{}_{\nu} = \eta^{\mu\rho}h_{\rho\nu}\,, \qquad h^{\mu\nu} = \eta^{\mu\rho}\eta^{\nu\sigma}h_{\rho\sigma}\,, \qquad h = \eta^{\mu\nu}h_{\mu\nu}$$

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + \mathcal{O}(h^2)$$

$$\Gamma^{\mu}_{\rho\sigma} = \frac{1}{2}g^{\mu\lambda}\left(\partial_\rho g_{\sigma\lambda} + \partial_\sigma g_{\rho\lambda} - \partial_\lambda g_{\rho\sigma}\right) = \frac{1}{2}\eta^{\nu\lambda}\left(\partial_\rho h_{\sigma\lambda} + \partial_\sigma h_{\rho\lambda} - \partial_\lambda h_{\rho\sigma}\right) + \mathcal{O}(h^2)$$

$$R^{\mu}{}_{\nu\rho\sigma} = \partial_\rho\Gamma^{\mu}_{\nu\sigma} - \partial_\rho\Gamma^{\mu}_{\nu\sigma} + \Gamma^{\mu}_{\lambda\rho}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\mu}_{\lambda\sigma}\Gamma^{\lambda}_{\nu\rho}$$

$$= \frac{1}{2}\left(\partial^2_{\rho\nu}h^{\mu}{}_{\sigma} + \partial_\sigma\partial^\mu h_{\nu\rho} - \partial_\rho\partial^\mu h_{\nu\sigma} - \partial^2_{\sigma\nu}h^{\mu}{}_{\rho}\right)$$

Gravitational Waves in Linearized Gravity

* Some relevant formulae:

$$R_{\mu\nu} = R^{\rho}_{\mu\rho\nu} = \frac{1}{2}\left(\partial^2_{\rho\nu}h^{\rho}{}_{\sigma} + \partial_{\mu}\partial^{\rho}h_{\nu\rho} - \Box h_{\mu\nu} - \partial^2_{\mu\nu}h\right) + \mathcal{O}(h^2)$$

$$R = g^{\mu\nu}R_{\mu\nu} = \partial^{\mu}\partial^{\nu}h_{\mu\nu} - \Box h + \mathcal{O}(h^2)$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}\left(\partial^2_{\rho\nu}h^{\rho}{}_{\sigma} + \partial_{\mu}\partial^{\rho}h_{\nu\rho} - \Box h_{\mu\nu} - \partial^2_{\mu\nu}h - \eta_{\mu\nu}\partial^{\rho}\partial^{\sigma}h_{\rho\sigma} + \eta_{\mu\nu}\Box h\right) + \mathcal{O}(h^2)$$

* Linearized Einstein Field Equations:

$$\Box\bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^{\rho}\partial^{\sigma}\bar{h}_{\rho\sigma} - \partial_{\nu}\partial^{\rho}\bar{h}_{\mu\rho} - \partial_{\mu}\partial^{\rho}\bar{h}_{\nu\rho} = -\frac{16\pi G}{c^4}T_{\mu\nu}$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

Gravitational Waves in Linearized Gravity

✳ Gauge Freedom in Linearized Gravity:

$$x^\mu \longrightarrow x^{\mu'} = x^\mu + \xi^\mu \,; \qquad |\,\xi^\mu\,| \ll 1$$

$$h_{\mu\nu} \longrightarrow h'_{\mu\nu} = h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

$$\bar{h}_{\mu\nu} \longrightarrow \bar{h}'_{\mu\nu} = \bar{h}_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu + \eta_{\mu\nu}\partial_\rho \xi^\rho$$

✳ Lorenz Gauge Condition:

$$\partial^\mu \bar{h}_{\mu\nu} = 0$$

✳ Remaining Freedom in the choice of Gauge:

$$\partial^\mu \bar{h}'_{\mu\nu} = \partial^\mu \bar{h}_{\mu\nu} - \Box \xi_\nu \,.\ \mathrm{Then:}\quad \partial^\mu \bar{h}'_{\mu\nu} = 0 \quad \Longrightarrow \quad \Box\, \xi_\nu = \partial^\mu \bar{h}_{\mu\nu}$$

Gravitational Waves in Linearized Gravity

* Then, if initially we are in the Lorenz Gauge, to stay in this family of gauges the transformation vector has to satisfy:

$$\partial^\mu \bar{h}_{\mu\nu} = 0 \quad \Longrightarrow \quad \Box\, \xi_\nu = 0$$

* Linearized Einstein Field Equations in the Lorenz Gauge:

$$\Box\, \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

That is, the metric perturbation propagate exactly as waves, at the speed of light, in the Lorenz Gauge. From now, we will assume we are in vacuum.

Gravitational Waves in Linearized Gravity

✳ Traceless-Transverse (TT) Gauge: By using the remaining freedom in the choice of the Lorenz Gauge we can impose (in vacuum):

$$h_{tt} = h_{ti} = 0 \quad \text{(spatial character of the metric perturbations)}$$

$$h = h^{i}{}_{i} = 0 \quad \text{(traceless character)}$$

$$\partial^{\mu} h_{\mu\nu} = 0 \quad \Longrightarrow \quad \partial^{i} h_{ij} = 0 \quad \text{(transverse character)}$$

The "TT" Gauge conditions completely fix the local gauge freedom.

✳ The metric perturbations in the TT Gauge contain only physical (non-gauge) information.

✳ The only independent component of the Riemann tensor in the TT Gauge is (the others can be found from this one and the properties of the Riemann tensor):

$$R_{titj} = -\frac{1}{2}\partial_t^2 h_{ij}^{\mathrm{TT}}$$

Interaction of Gravitational Waves with Matter

- The solution, in the TT Gauge, for a wave travelling in the z direction, is given by:

$$\left(h_{\mu\nu}^{\rm TT}\right) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ * & h_+ & h_\times & 0 \\ * & * & -h_+ & 0 \\ * & * & * & 0 \end{pmatrix}$$

where:

$$h_{+,\times} = h_{+,\times}(t \pm z/c)$$

Interaction of Gravitational Waves with Matter

* Free falling (massive) test particles follow timelike geodesics:

$$\frac{d^2 z^\mu(\tau)}{d\tau^2} + \Gamma^\mu_{\rho\sigma}\frac{dz^\rho(\tau)}{d\tau}\frac{dz^\sigma(\tau)}{d\tau} = 0$$

where τ denotes proper time and the four-velocity satisfies

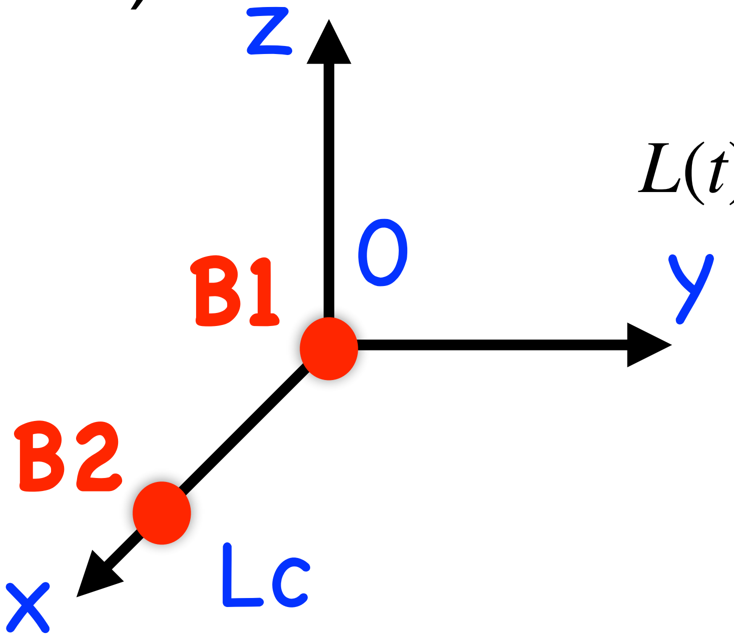
$$g_{\mu\nu}\frac{dz^\mu(\tau)}{d\tau}\frac{dz^\nu(\tau)}{d\tau} = -\,c^2$$

* From here we deduce that in the linear theory, and using the TT gauge, the coordinate location of a slowly-moving free-falling body is unaffected by passing Gravitational Waves:

$$\frac{d^2 z^i}{dt^2} = -\,c^2\Gamma^i_{tt} \approx 0$$

Interaction of Gravitational Waves with Matter

* However, the proper distance between two test bodies (B1 and B2) in free-fall oscillates as the GW passes by



$$L(t) = \int_{B1-B2\ line} \sqrt{ds^2} = \int_0^{L_c} dx \sqrt{g_{xx}(t,z)} = \int_0^{L_c} dx \sqrt{1 + h^{\rm TT}_{xx}(t - z/c)}$$

$$\approx \int_0^{L_c} dx \left(1 + \frac{1}{2} h_+ \Big|_{z=0} \right) = L_c \left(1 + \frac{1}{2} h_+ \Big|_{z=0} \right)$$

Therefore,

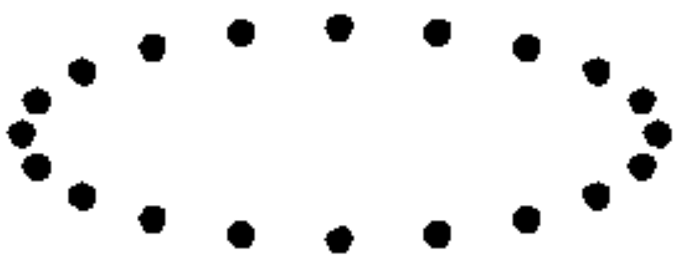
$$\frac{\delta L}{L} \approx \frac{1}{2} h^{TT}_{xx}(t, z=0) = \frac{1}{2} h_+(t, z=0)$$

Summary of Properties of Gravitational Waves

- The effect of gravitational waves on matter is to change the proper distance between the matter constituents:

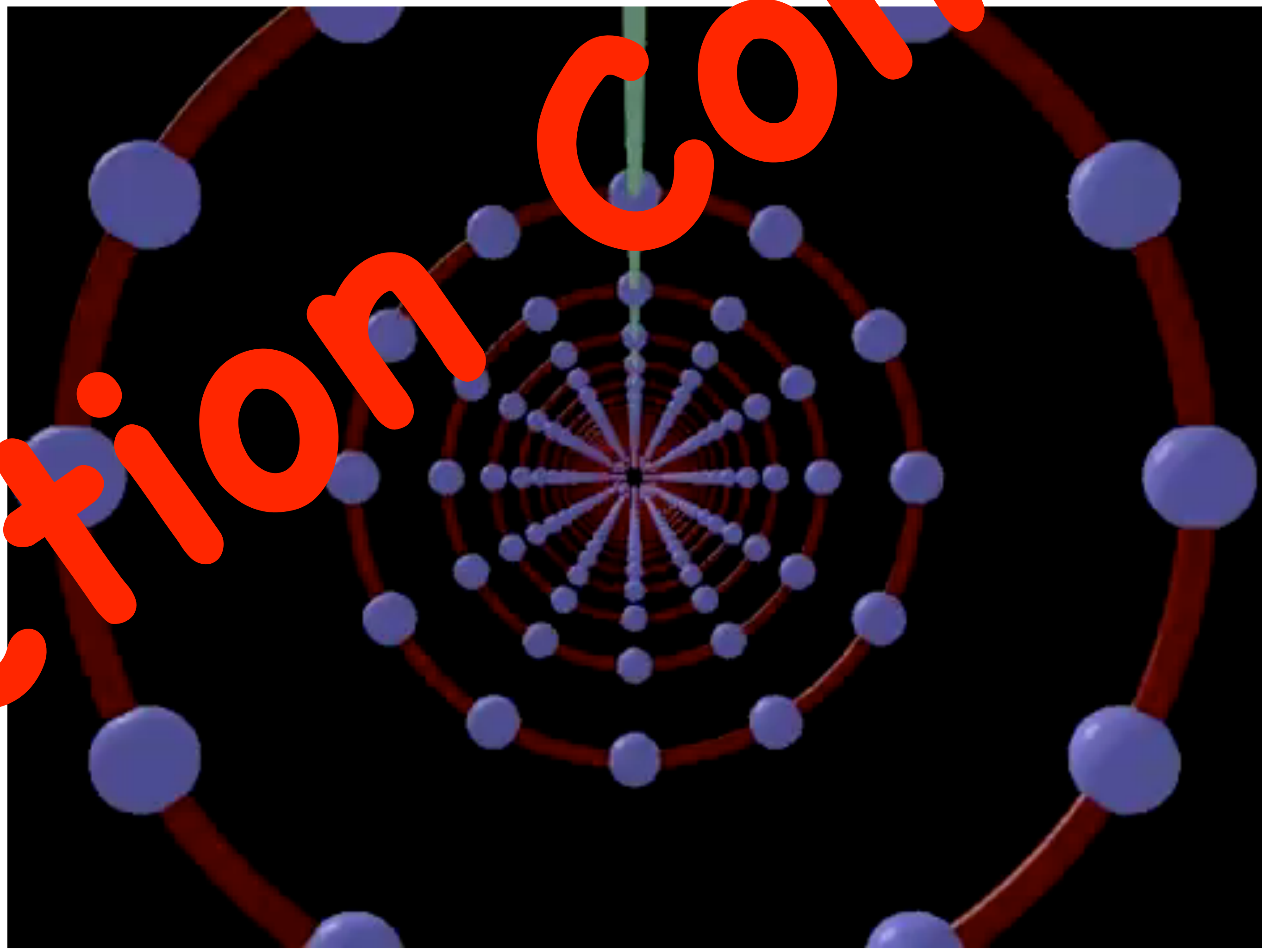
$$\Delta L \sim (\text{GW strain amplitude}) \times L$$
$$= \qquad h \qquad \times L$$

Detection concept!



+ Polarization

x Polarization



L : Proper distance between “test” bodies

GWs in GR have 2 independent polarizations!

Summary of Properties of Gravitational Waves

✳ Comparison with Electromagnetic Waves: Things in common

Both travel at the speed of light (as measured locally)

Both are transverse waves

Both have electric (polar) and magnetic (axial) components

Summary of Properties of Gravitational Waves

✴ **Comparison with Electromagnetic Waves:**

Differences

<u>EMWs</u> are generated by accelerated charges	<u>GWs</u> are generated by time-dependent distributions of energy and momentum
Dipole is the lowest order time-dependent distribution that can generate EMWs (charge conservation)	Quadrupole is the lowest order time-dependent distribution that can generate GWs (mass and linear momentum conservation)
EMWs arise from interactions of atoms, nuclei, etc. within the astrophysical source: $\lambda_{\rm EM} \ll L_{\rm source}$	GWs are generated by the bulk mass distribution of the sources: $\lambda_{\rm GW} \sim L_{\rm source}$
EMWs are good for imaging the source	GWs are not good for imaging the source. Information is extracted by means of audio-like methods

The Frequency spectra of GWs and EMWs associated with cosmic phenomena is almost complementary

It is extremely important to have a priori precise (theoretical) knowledge of the waveforms!

Summary of Properties of Gravitational Waves

✳ Comparison with Electromagnetic Waves: Differences

Detection is based on the deposition of energy (photons) in the detector:

$$L_{\mathrm{EM}} \sim \frac{1}{D_L^2}$$

The interaction with matter is important: dispersion, absorption, …

Detection based on the inference of the radiative gravitational field (h~1/r), not on the energy flux:

$$L_{\mathrm{GW}} \sim \left(\frac{dh}{dt}\right)^2 \sim \frac{1}{D_L^2}$$

Then, an enhancement in the detector sensitivity of a factor 2 increases the visible volume of the sky in a factor 8!

Very week interaction with matter: dispersion and absorption are negligible.

Summary of Properties of Gravitational Waves

✶ Some Important Facts:

- We detect GW amplitudes (h ~ 1/r), not energy fluxes (dE/dt ~ (dh/dt)x(dh/dt) ~ 1/r^2). Then, an enhancement in the detector sensitivity of a factor 2 increases the visible volume of the sky in a factor 8!

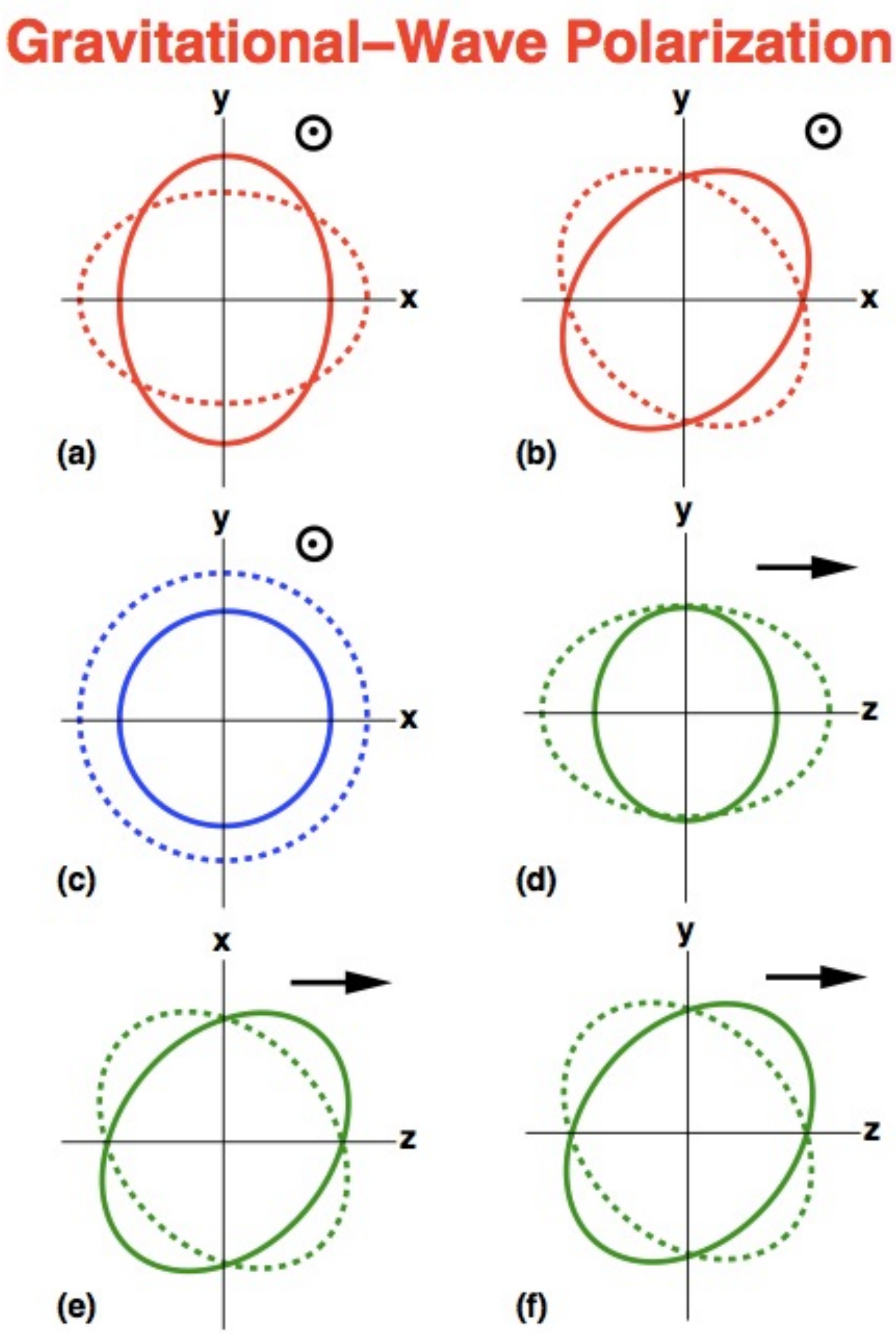
- GWs from cosmological sources at z > 1 suffer significantly from lensing). Affects Luminosity distance estimation.

- Degeneracy with redshift: M(z) = (1+z)M

- GWs are direct probes of spacetime curvature and strong gravity (radiative) regimes. They will provide observations of strong gravity regions not transparent to EM waves.

Summary of Properties of Gravitational Waves

✳ **GW Polarizations [GWs in Metric Theories of Gravity]:**



- GR has only two independent polarizations [(a) and (b) in the Figure] and corresponds to type N2 in the E(2) classification .

- An alternative theory of gravity may have up to six independent polarizations.

Eardley, Lee, Lightman, Will, Wagoner: Phys. Rev. Lett. **30**, 884 (1974)

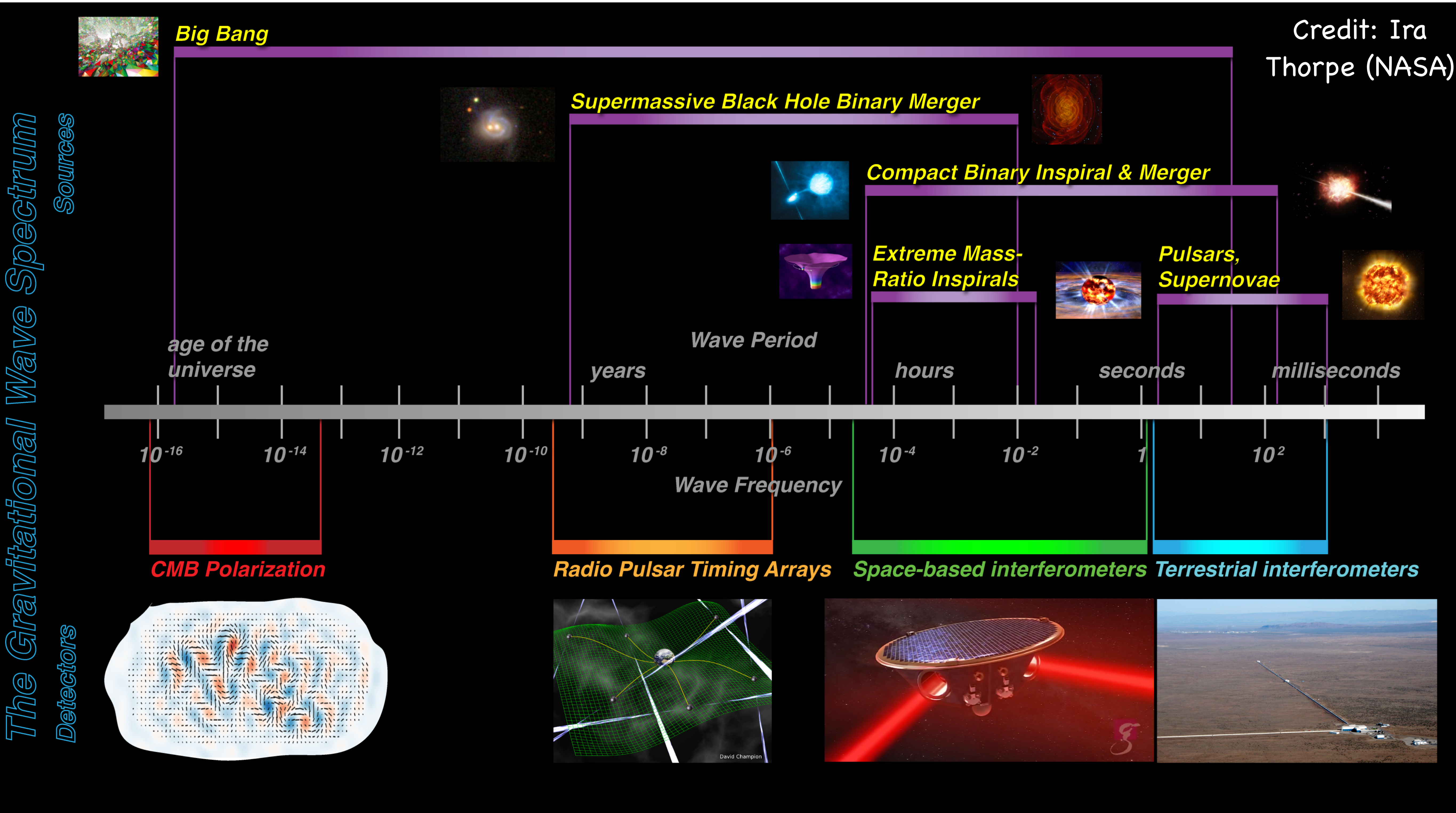
Plot from C. Will (LRR, 2006)

The Gravitational Wave Spectrum

Gravitational Wave Spectrum (with Sources & Detectors)

<- GW Detector Armlength

<- Mass/Energy



The Gravitational-Wave Spectrum

- The Ultra-low Frequency Band:

$$10^{-18}\ \mathrm{Hz} \lesssim f \lesssim 10^{-13}\ \mathrm{Hz}$$

- The very Low Frequency Band:

$$10^{-9}\ \mathrm{Hz} \lesssim f \lesssim 10^{-7}\ \mathrm{Hz}$$

- The Low Frequency Band:

$$10^{-5}\ \mathrm{Hz} \lesssim f \lesssim 1\ \mathrm{Hz}$$

- The High Frequency Band:

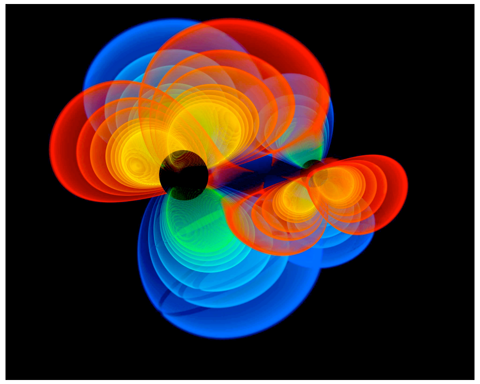
$$1\ \mathrm{Hz} \lesssim f \lesssim 10^{4}\ \mathrm{Hz}$$

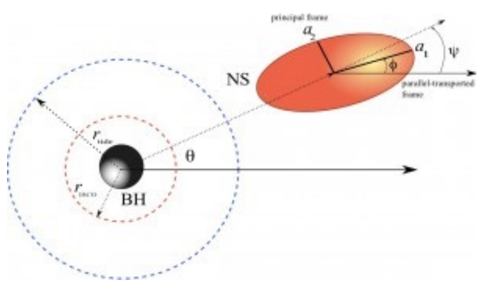
- The very High Frequency Band:

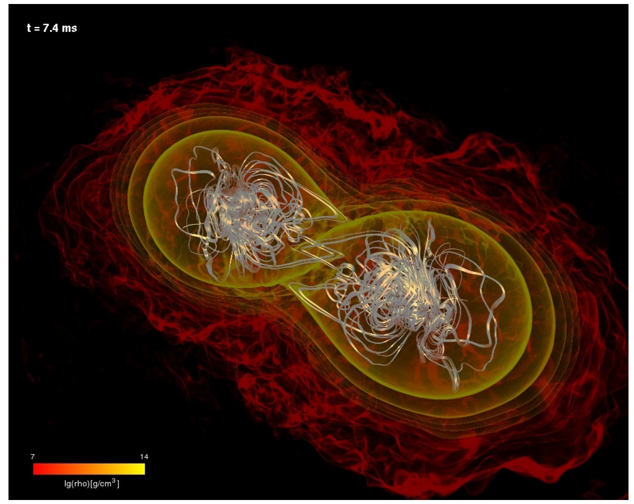
$$f > 10^{4}\ \mathrm{Hz}$$

Gravitational Wave Sources (HF Band)

Compact Binary System Coalescence

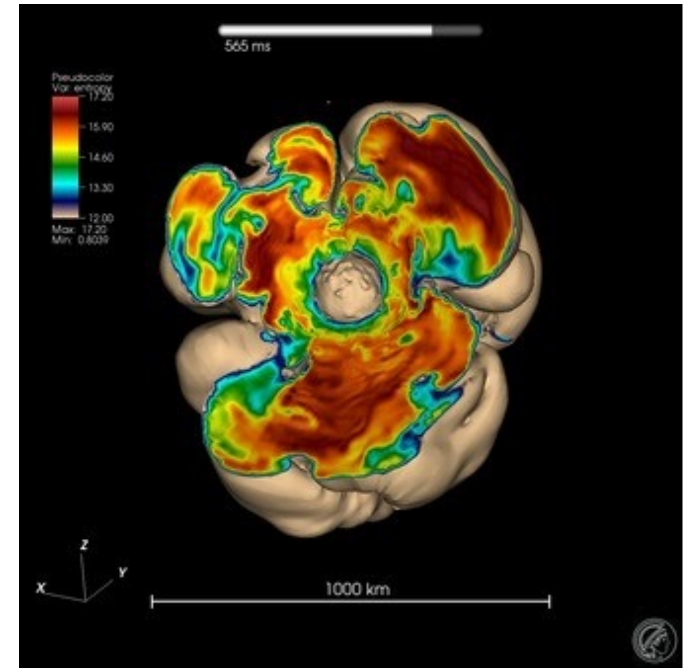




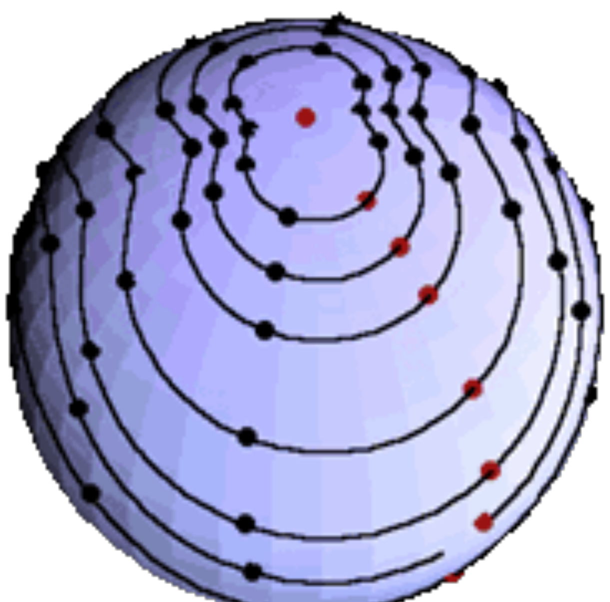


NS-NS, BH-BH, BH-NS

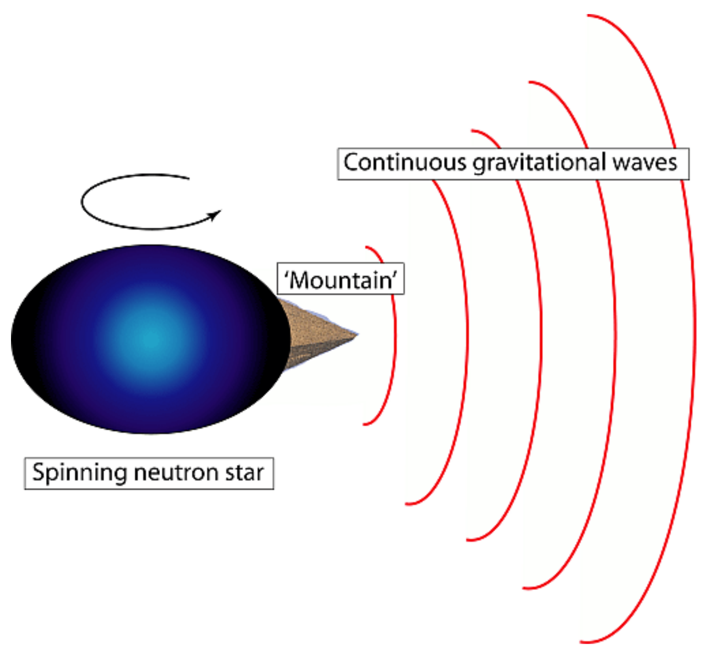
Core Collapse Supernovae



Oscillations of Relativistic Stars

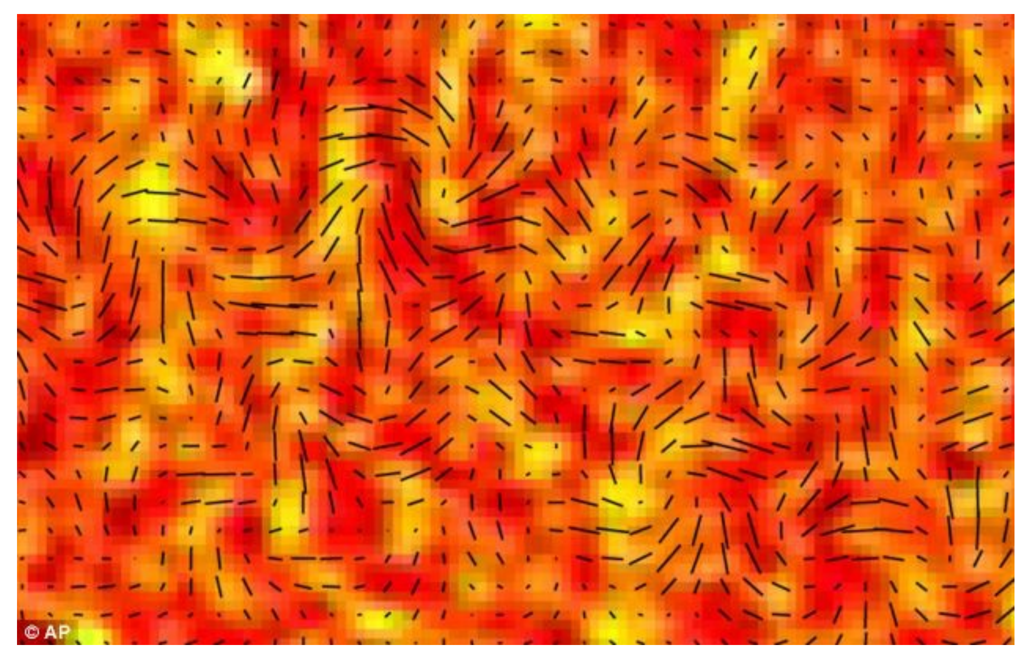


r-Modes



"mountains" in Neutron Stars

Stochastic Signals/ Gravitational Wave Backgrounds



Carlos F. Sopuerta (Institute of Space Sciences, ICE-CSIC & IEEC)

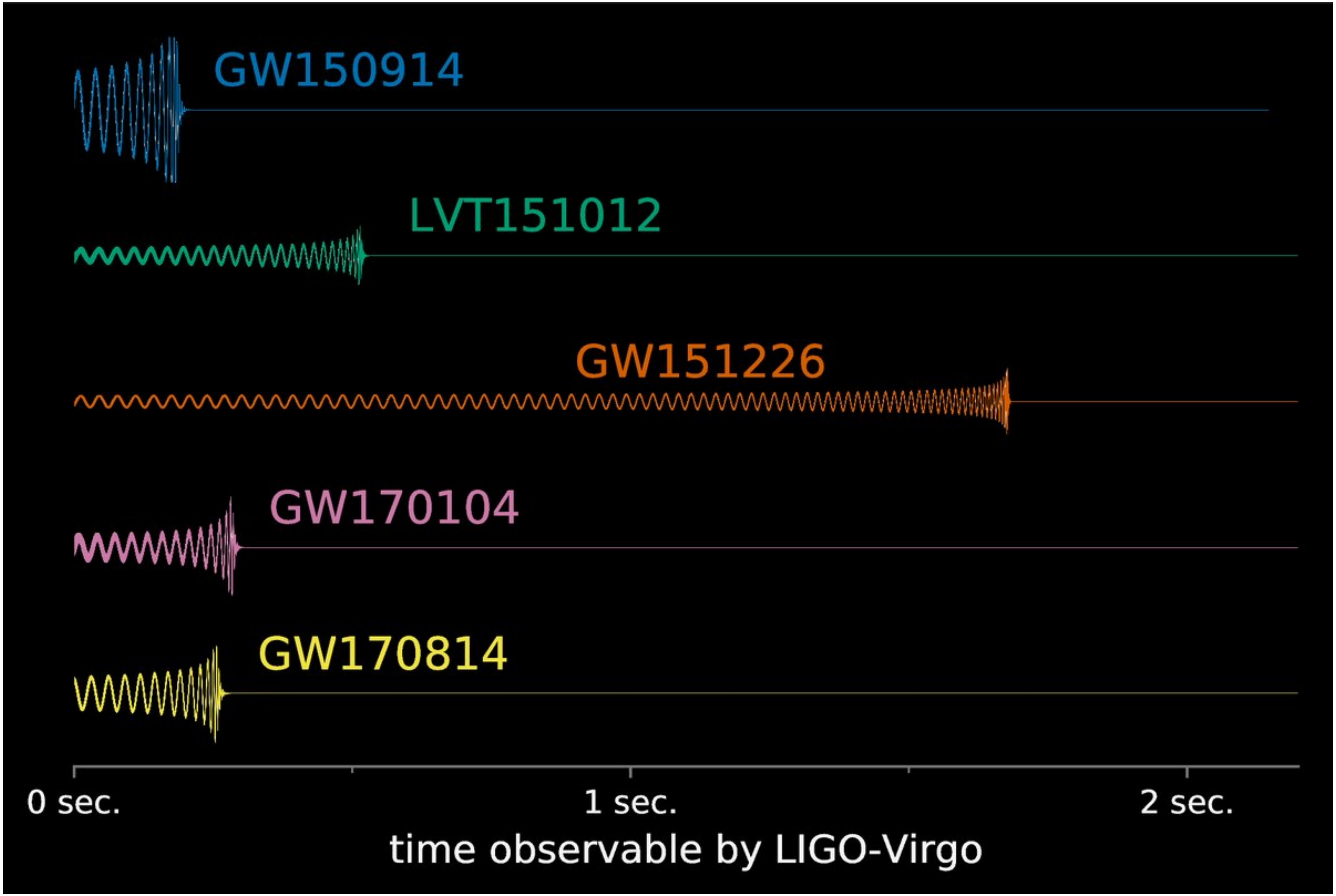
What have LIGO-Virgo found?

GRAVITATIONAL WAVE EVENT CATALOGUE

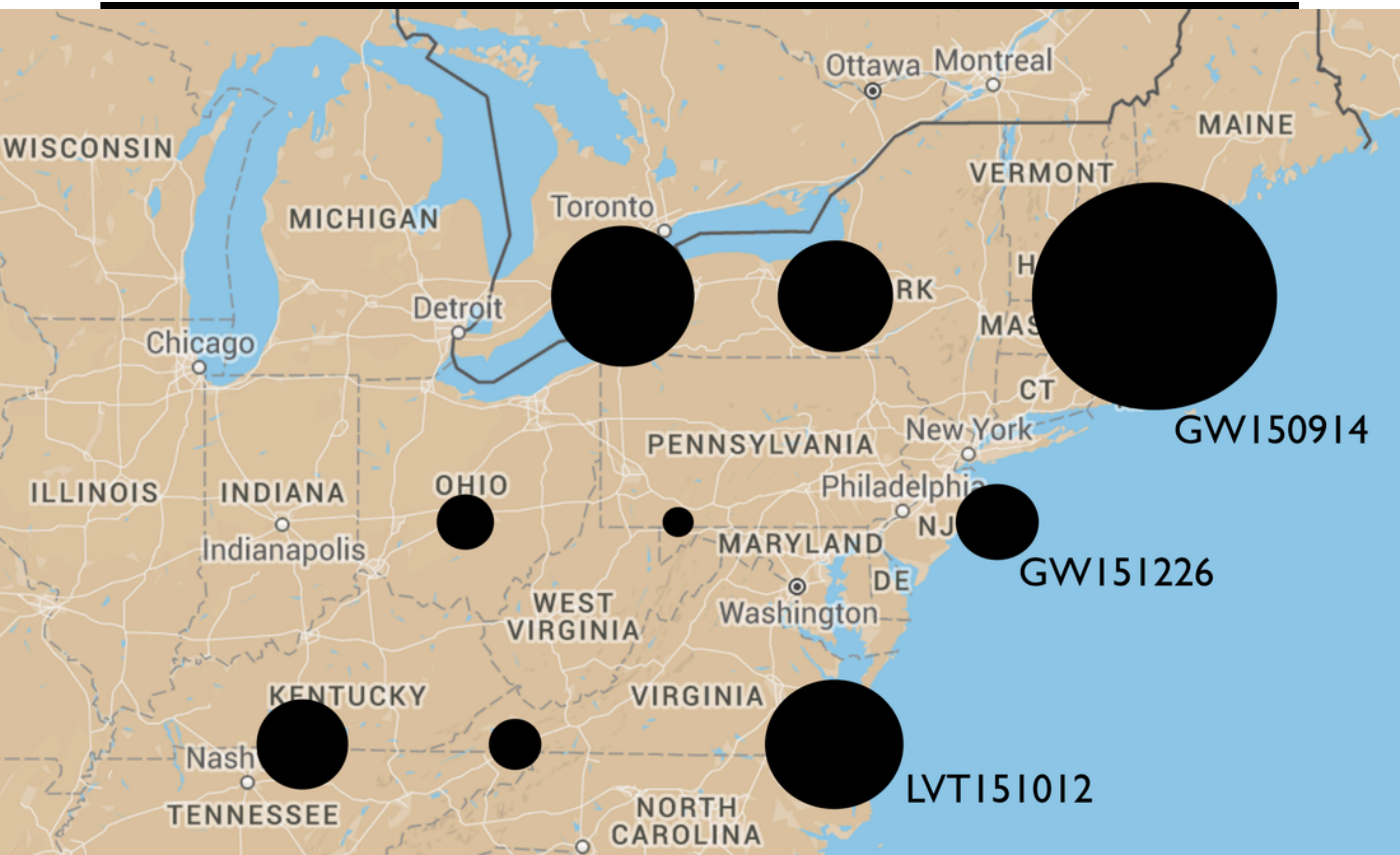
Name	GW150914	GW151226	GW170104	GW170814
Type	BH+BH	BH+BH	BH+BH	BH+BH
Mass 1 (solar mass)	36 Msun	14 Msun	31 Msun	31.5 Msun
Mass 2 (solar mass)	29 Msun	8 Msun	20 Msun	25 Msun
Final Mass	62 Msun	21 Msun	49 Msun	53 Msun
Energy Radiated	3 Msun	1.1 Msun	2 Msun	2.7 Msun
Distance (Mpc)	400 Mpc	440 Mpc	880 Mpc	540 Mpc
Duration (seconds)	~ 0.2 sec	~ 1 sec	~ 0.3 sec	~ 0.25 sec
Detectors	LIGO	LIGO	LIGO	LIGO+Virgo
	11 February 2016	15 June 2016	1 June 2017	27 September 2017

What have LIGO-Virgo found?

BBH detected waveforms



Masses of known Black Holes



What have LIGO-Virgo found?

LIGO

GW170817

Binary neutron star merger

A LIGO / Virgo gravitational wave detection with associated electromagnetic events observed by over 70 observatories.

H L V

Distance
130 million light years

Discovered
17 August 2017

Type
Neutron star merger

12:41:04 UTC
A gravitational wave from a binary neutron star merger is detected.

gravitational wave signal
Two neutron stars, each the size of a city but with at least the mass of the sun, collided with each other.

gamma ray burst
A short gamma ray burst is an intense beam of gamma ray radiation which is produced just after the merger.

+ 2 seconds
A gamma ray burst is detected.

GW170817 allows us to measure the expansion rate of the universe directly using gravitational waves for the first time,.

+10 hours 52 minutes
A new bright source of optical light is detected in a galaxy called NGC 4993, in the constellation of Hydra.

kilonova
Decaying neutron-rich material creates a glowing kilonova, producing heavy metals like gold and platinum.

Detecting gravitational waves from a neutron star merger allows us to find out more about the structure of these unusual objects.

+11 hours 36 minutes
Infrared emission observed.

+15 hours
Bright ultraviolet emission detected.

+9 days
X-ray emission detected.

This multimessenger event provides confirmation that neutron star mergers can produce short gamma ray bursts.

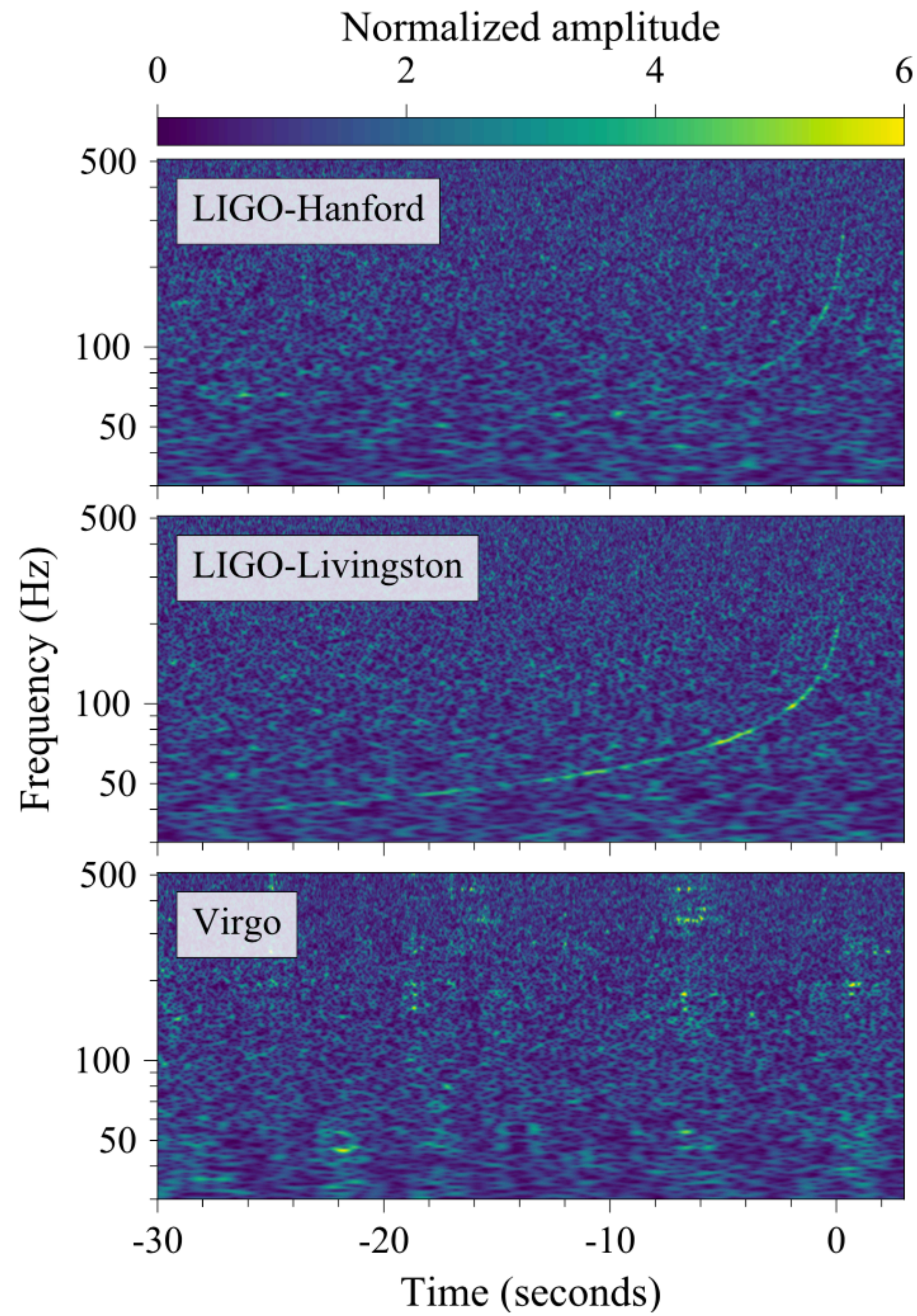
radio remnant
As material moves away from the merger it produces a shockwave in the interstellar medium - the tenuous material between stars. This produces emission which can last for years.

Au

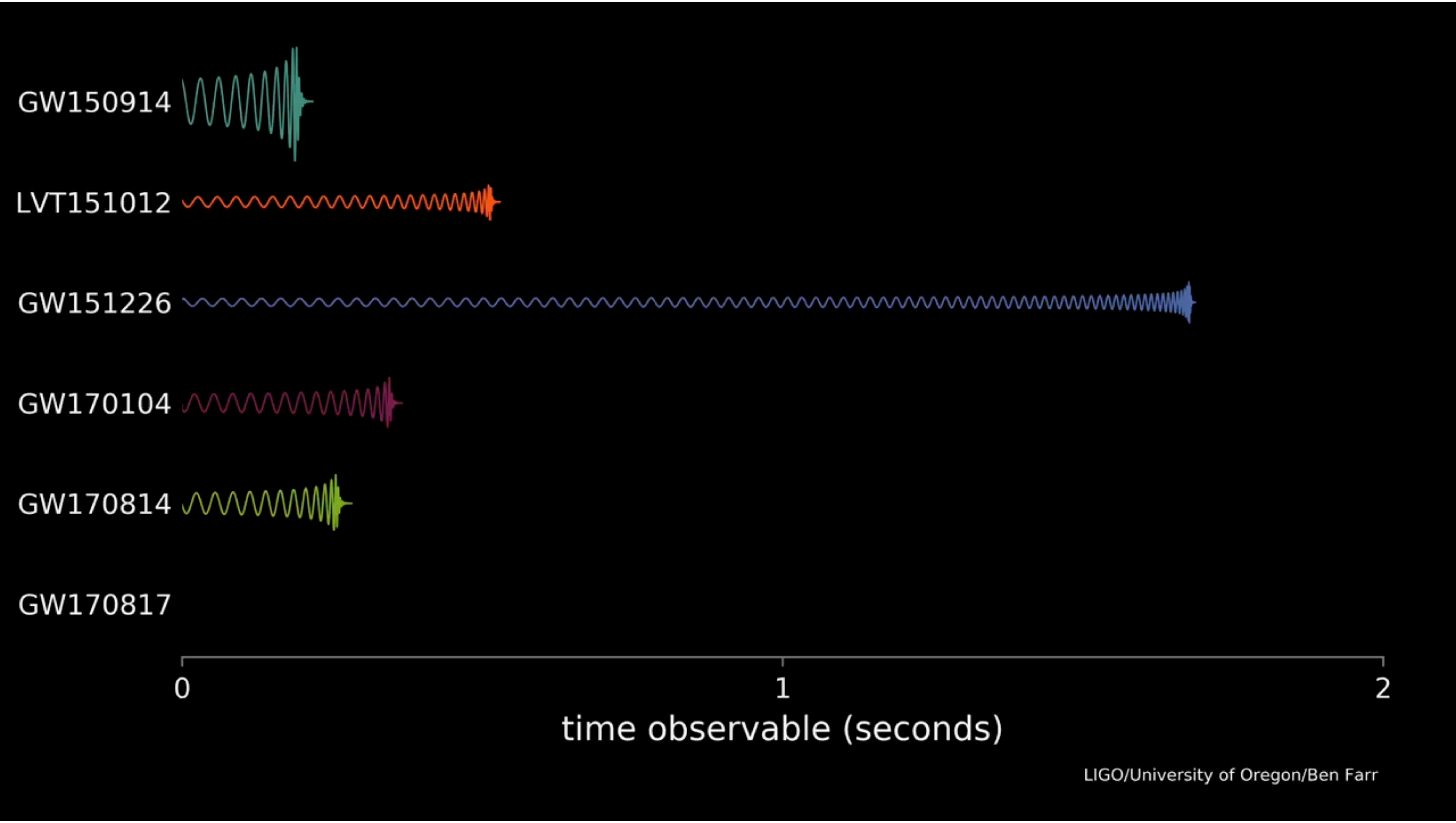
The observation of a kilonova allowed us to show that neutron star mergers could be responsible for the production most of the heavy elements, like gold, in the universe.

+16 days
Radio emission detected.

Observing both electromagnetic and gravitational waves from the event provides compelling evidence that gravitational waves travel at the same speed as light.



What have LIGO-Virgo found?



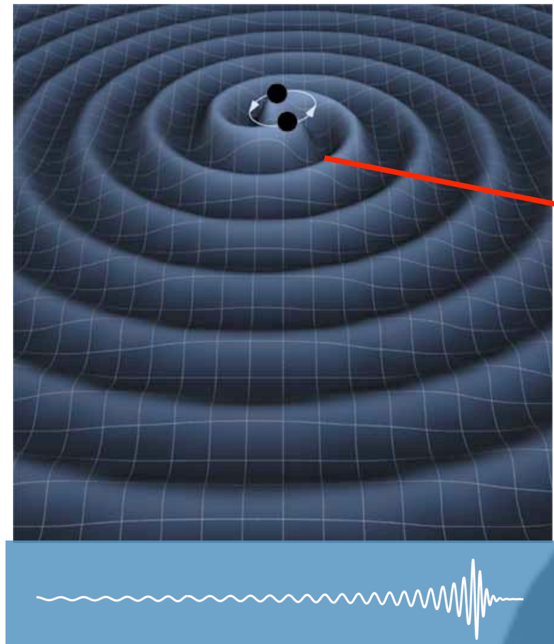
Gravitational Wave Sources (LF Band)

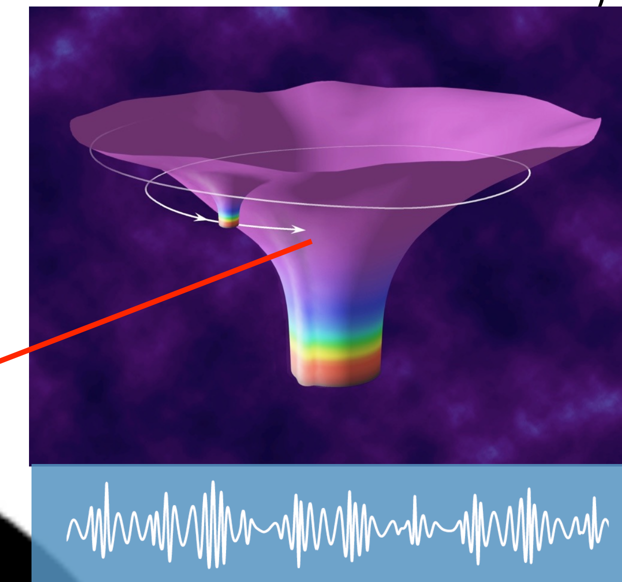
✳ The Low-Frequency Band (0.1 mHz - 1 Hz):

Extreme Mass Ratio Inspirals, EMRIs

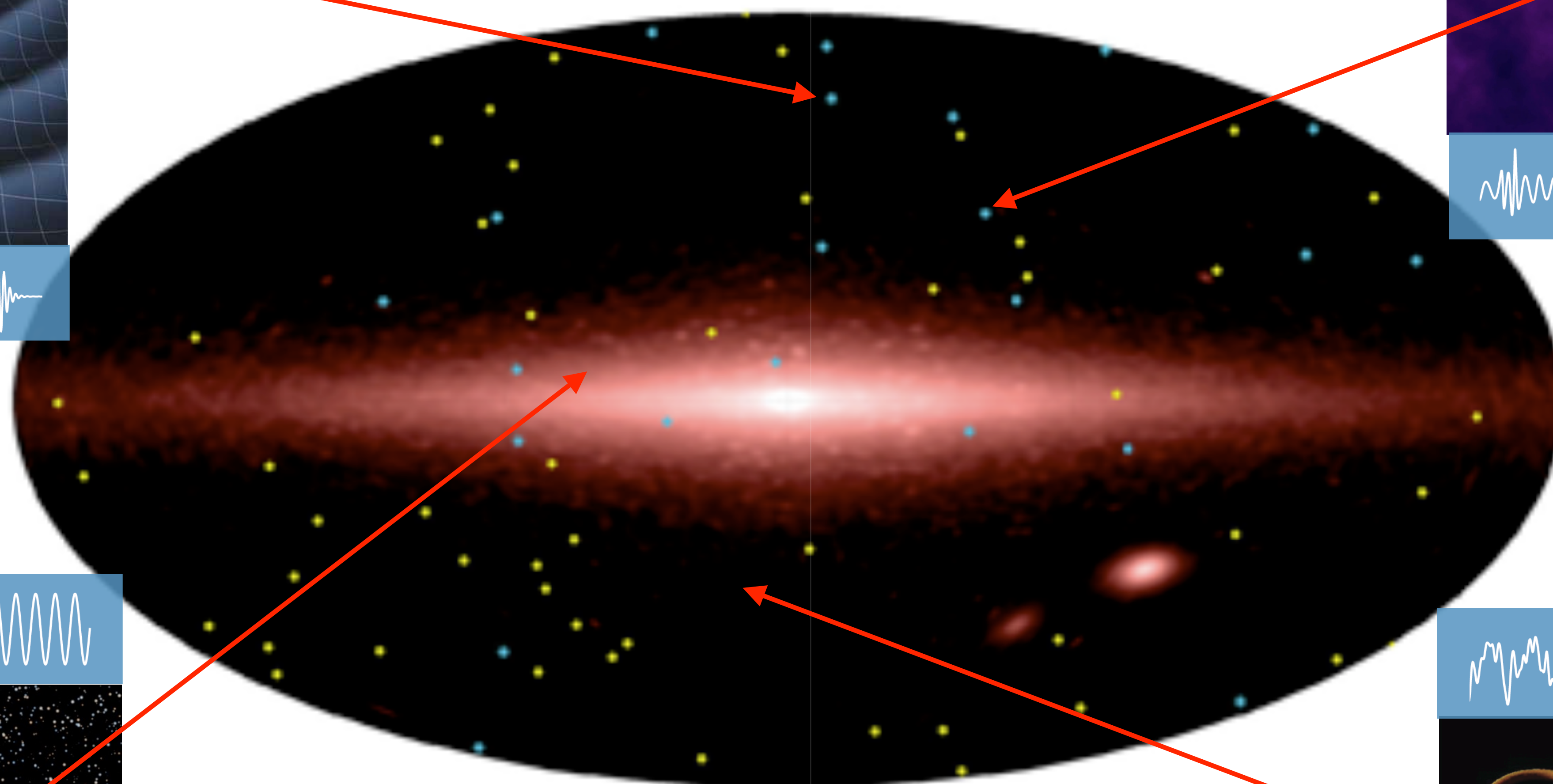
(1 to 10 $M_\odot$ into 10^4 to 5×10^6 $M_\odot$)

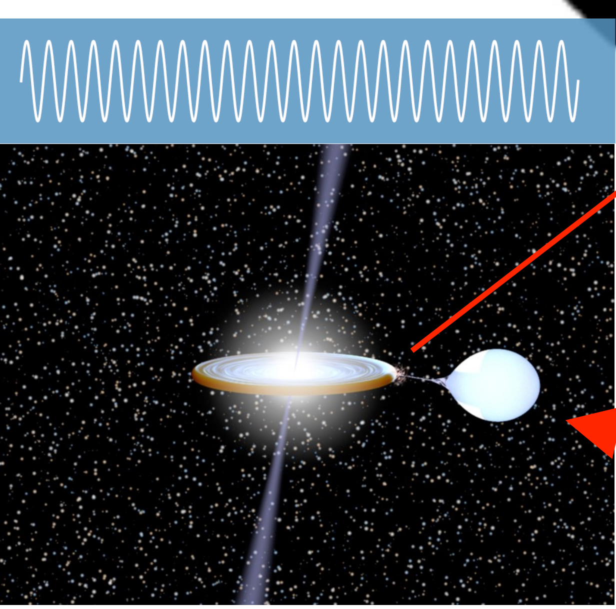
Massive Black Holes mergers (10^4 to 10^8 $M_\odot$)

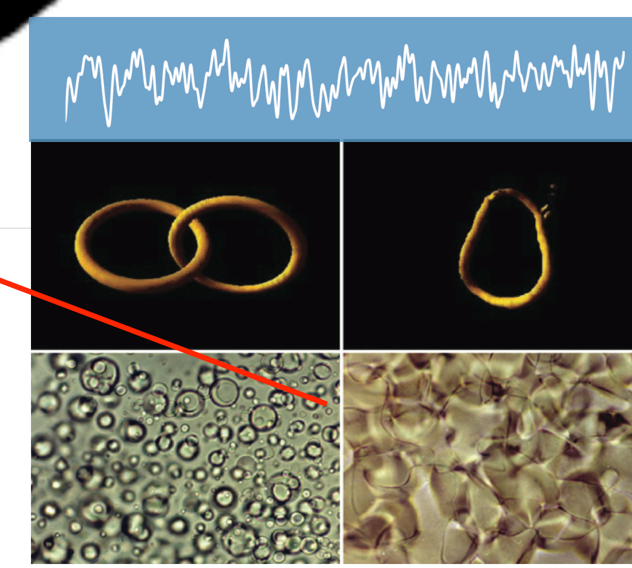




The GW Sky







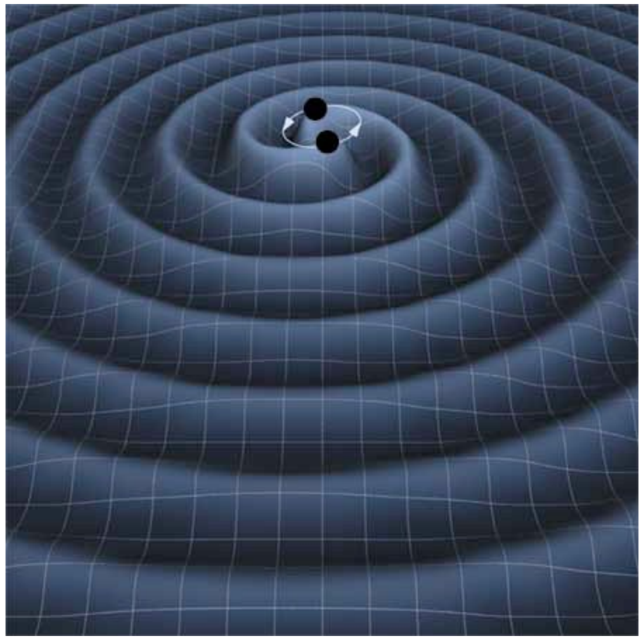
Guaranteed Sources!

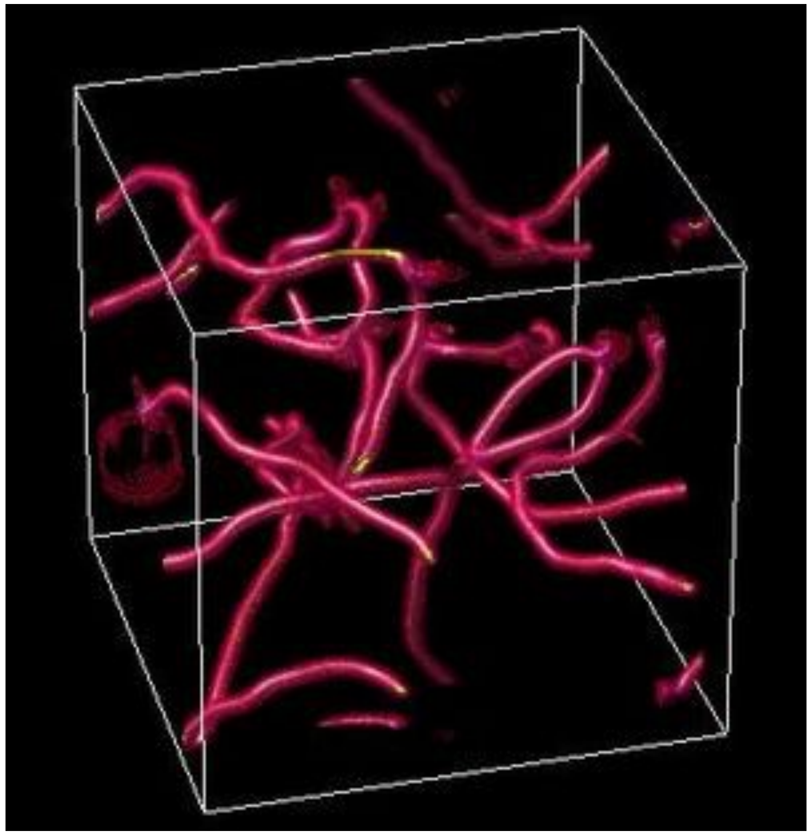
Ultra-Compact Binaries in the Milky Way

GW Stochastic Signals

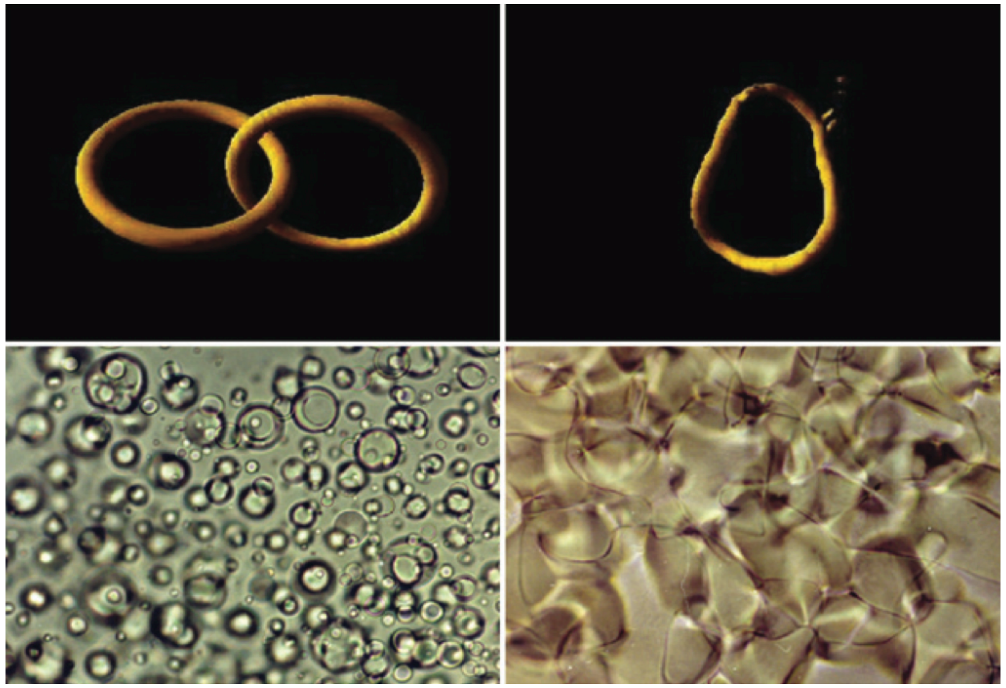
Gravitational Wave Sources (VLF Band)

Stochastic Background from Supermassive Black Holes mergers (10^8 to 10^{10} $M_\odot$)





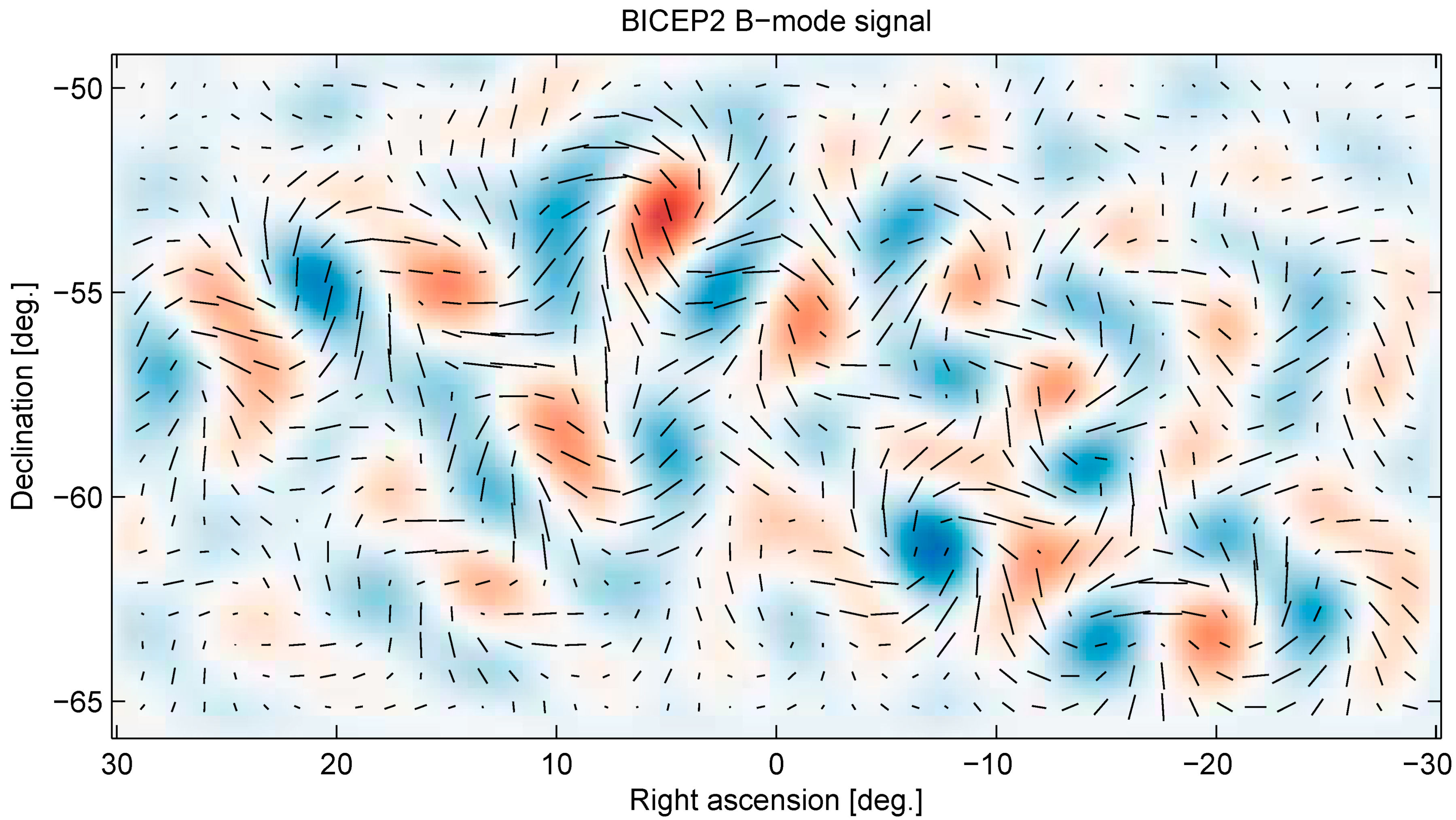
Cosmic Strings



GW Stochastic Signals

CMB Polarization Experiments and the ULF Band

$$1\ \mathrm{aHz} < f < 0.1\ \mathrm{pHz}$$



Gravitational waves from inflation generate a faint but distinctive twisting pattern in the polarization of the CMB, known as a "curl" or B-mode pattern. For the density fluctuations that generate most of the polarization of the CMB, this part of the primordial pattern is exactly zero. Shown here is the actual B-mode pattern observed with the BICEP2 telescope, with the line segments showing the polarization from different spots on the sky. The red and blue shading shows the degree of clockwise and anti-clockwise twisting of this B-mode pattern.

Energy-Momentum Content of Gravitational Waves

Energy-Momentum Content of Gravitational Waves

* The fact that Gravitational Waves carry energy and momentum is clear from the study of their interaction with matter: We have seen how gravitational waves put in motion a circle of test particles.

* General Relativity tells us that any form of energy contributes to the curvature of spacetime. Then, we can ask ourselves whether Gravitational Waves are themselves a source of space-time curvature.

* To that end we need a framework in which to separate Gravitational Waves from a *curved* background spacetime. If we would do as in linear theory, fixing the *flat* background spacetime, we would prevent Gravitational Waves from curving the background spacetime. Instead, we need to treat the background as a *dynamical* background metric.

Energy-Momentum Content of Gravitational Waves

* Then, we start from the following separation:

$$g_{\mu\nu}(x^\rho) = \bar{g}_{\mu\nu}(x^\rho) + \delta g_{\mu\nu}(x^\rho)\,, \qquad |\,\delta g_{\mu\nu}\,| \ll 1$$

* The question is: How do we decide which part of the spacetime metric is background and which part correspond to the fluctuations induced by the gravitational waves?

* The answer is that this depends on whether we can establish a clear separation of scales. For instance, a clear separation can be done if, given a particular coordinate system, we have:

$\bar{g}_{\mu\nu}$ is a metric with a scale of variation L_B

$\delta g_{\mu\nu}$ are superimposed small amplitude perturbations characterized by a wavelength λ such that

$$\lambda/(2\pi) \ll L_B$$

Energy-Momentum Content of Gravitational Waves

- This is a separation based on spatial scales. Something similar can be done in terms of frequencies. In this case, the background is a slowly varying geometry and the perturbations are high-frequency perturbations that satisfy:

$$f \gg f_B$$

- The expansion around the background based on these separations based on scales (spatial and temporal) is called the *short-wave approximation scheme.*

- Let us start with the trace-reversed form of Einstein's equations:

$$R_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

Energy-Momentum Content of Gravitational Waves

* Let us expand now the Ricci tensor to second order:

$$R_{\mu\nu} = \bar{R}_{\mu\nu} + R^{(1)}_{\mu\nu} + R^{(2)}_{\mu\nu} + \mathcal{O}(\delta g^3)$$

* Observations:

$\bar{R}_{\mu\nu}$ only contains the background metric $\bar{g}_{\mu\nu}$ and hence,
only contains low-frequency modes

$R^{(1)}_{\mu\nu}$ is linear in $\delta g_{\mu\nu}$ and hence,
only contains high-frequency modes

$R^{(2)}_{\mu\nu}$ is quadratic in $\delta g_{\mu\nu}$ and hence,
contains both low- and high-frequency modes

Energy-Momentum Content of Gravitational Waves

* Therefore, the Einstein equations can be split into two separate equations for the low- and high-frequency parts:

$$\bar{R}_{\mu\nu} = -R^{(2),\mathrm{Low}}_{\mu\nu} + \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\mathrm{Low}}$$

$$R^{(1)}_{\mu\nu} = -R^{(2),\mathrm{High}}_{\mu\nu} + \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)^{\mathrm{High}}$$

Energy-Momentum Content of Gravitational Waves

* Scales of variation:

$$\partial \bar{g}_{\mu\nu} \sim \frac{1}{L_B}$$

$$\partial \delta g_{\mu\nu} \sim \frac{\delta g_{\mu\nu}}{\lambda}$$

$$\bar{R}_{\mu\nu} \sim \partial^2 \bar{g}_{\mu\nu} \sim \frac{1}{L_B^2}$$

Energy-Momentum Content of Gravitational Waves

* We can write the equation for the Ricci background using an averaging over the many wavelengths:

$$\bar{R}_{\mu\nu} = - < R^{(2)}_{\mu\nu} >_\lambda + \frac{8\pi G}{c^4} < T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T >_\lambda$$

* We now introduce an effective energy-momentum tensor of matter:

$$< T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T >_\lambda = \bar{T}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \bar{T}$$

Energy-Momentum Content of Gravitational Waves

* We can also introduce the following "energy-momentum" tensor:

$$t_{\mu\nu} = -\frac{c^4}{8\pi G} < R^{(2)}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{(2)} >_\lambda$$

and define its trace as follows

$$t = \bar{g}^{\mu\nu} t_{\mu\nu} = \frac{c^4}{8\pi G} < R^{(2)} >_\lambda$$

then

$$- < R^{(2)}_{\mu\nu} >_\lambda = \frac{8\pi G}{c^4} \left(t_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} t \right)$$

Energy-Momentum Content of Gravitational Waves

* Combining all these equations we can arrive at the following "coarse-grained" form of the Einstein equations:

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{8\pi G}{c^4}\left(\bar{T}_{\mu\nu} + t_{\mu\nu}\right)$$

These equations determine the dynamics of the background metric in terms of the long-wavelength of the matter energy-momentum tensor and in terms of a tensor that is quadratic in the metric (short-wave) fluctuations.

It shows the effect of the Gravitational Waves on the background curvatures, which appear to be in the form of an "effective" energy-momentum tensor.

Energy-Momentum Content of Gravitational Waves

* The Energy-Momentum Tensor of Gravitational Waves: We can identify the metric fluctuations with the gravitational waves described in the Lorenz gauge. Then, after some algebra:

$$t_{\mu\nu} = \frac{c^4}{32\pi G} < \partial_\mu h_{\alpha\beta} \partial_\nu h^{\alpha\beta} >_\lambda$$

and this tensor is gauge independent. So, it only contains the TT-modes. We can then replace it with the TT metric perturbations. The energy density is then:

$$t^{00} = \frac{c^4}{32\pi G} < \dot{h}^{\rm TT}_{ij} \dot{h}^{\rm TT}_{ij} >_\lambda$$

Energy-Momentum Content of Gravitational Waves

or in terms of the two (GR) gravitational-wave polarizations:

$$t^{00} = \frac{c^4}{16\pi G} < \dot{h}_+^2 + \dot{h}_\times^2 >_\lambda$$

✳ A consequence of the "coarse-grained" form of the Einstein equations is the following conservation law:

$$\bar{\nabla}^\mu \left(\bar{T}_{\mu\nu} + t_{\mu\nu} \right) = 0$$

and far away from the sources we can write:

$$\partial_\mu t^{\mu\nu} = 0$$

Energy-Momentum Content of Gravitational Waves

* The Energy Flux in Gravitational Waves: Using the conservation law we have just derived and integrating it over a spatial volume V bounded by a surface S we have:

$$\frac{dE}{dAdt} = ct^{00} = \frac{c^3}{32\pi G} < \dot{h}^{\rm TT}_{ij}\dot{h}^{\rm TT}_{ij} >_\lambda$$

or, using the explicit form of the area element dA:

$$dA = r^2 d\Omega \qquad \Longrightarrow \qquad \frac{dE}{dt} = \frac{c^3 r^2}{32\pi G}\int d\Omega \, < \dot{h}^{\rm TT}_{ij}\dot{h}^{\rm TT}_{ij} >_\lambda$$

Energy-Momentum Content of Gravitational Waves

in terms of the gravitational-wave polarizations:

$$\frac{dE}{dAdt} = \frac{c^3}{16\pi G} < \dot{h}_+^2 + \dot{h}_\times^2 >_\lambda$$

Therefore, the total energy radiated through dA is given by

$$\frac{dE}{dA} = \frac{c^3}{16\pi G}\int_{-\infty}^{+\infty} dt < \dot{h}_+^2 + \dot{h}_\times^2 >_\lambda = \frac{c^3}{16\pi G}\int_{-\infty}^{+\infty} dt \left(\dot{h}_+^2 + \dot{h}_\times^2\right)$$

Energy-Momentum Content of Gravitational Waves

* Some relevant explicit formulae:

$$\Gamma^{\mu}_{\rho\sigma} = \bar{\Gamma}^{\mu}_{\rho\sigma} + \delta_1\Gamma^{\mu}_{\rho\sigma} + \delta_2\Gamma^{\mu}_{\rho\sigma} + \mathcal{O}(\delta g^3)$$

$$\delta_1\Gamma^{\mu}_{\rho\sigma} = \frac{1}{2}\delta g^{\mu\lambda}\left(\bar{\nabla}_{\rho}\delta g_{\sigma\lambda} + \bar{\nabla}_{\sigma}\delta g_{\rho\lambda} - \bar{\nabla}_{\lambda}\delta g_{\rho\sigma}\right)$$

Energy-Momentum Content of Gravitational Waves

* Some relevant explicit formulae:

$$R^{(1)}_{\rho\sigma} = -\frac{1}{2}\bar{\Box}\,\delta g_{\mu\nu} - \frac{1}{2}\bar{\nabla}_{\nu}\bar{\nabla}_{\mu}\delta g + \bar{\nabla}_{\rho}\bar{\nabla}_{(\mu}\delta g^{\rho}{}_{\nu)}$$

$$R^{(2)}_{\rho\sigma} = \frac{1}{2}\bar{g}^{\rho\sigma}\bar{g}^{\alpha\beta}\left[\frac{1}{2}\left(\bar{\nabla}_{\mu}\delta g_{\rho\alpha}\right)\left(\bar{\nabla}_{\nu}\delta g_{\sigma\beta}\right) + \left(\bar{\nabla}_{\rho}\delta g_{\nu\alpha}\right)\left(\bar{\nabla}_{\sigma}\delta g_{\mu\beta} - \bar{\nabla}_{\beta}\delta g_{\mu\sigma}\right)\right.$$
$$+\delta g_{\rho\alpha}\left(\bar{\nabla}_{\nu}\bar{\nabla}_{\mu}\delta g_{\sigma\beta} + \bar{\nabla}_{\beta}\bar{\nabla}_{\sigma}\delta g_{\mu\nu} - \bar{\nabla}_{\beta}\bar{\nabla}_{\nu}\delta g_{\mu\sigma} - \bar{\nabla}_{\beta}\bar{\nabla}_{\mu}\delta g_{\nu\sigma}\right)$$
$$\left.+\left(\frac{1}{2}\bar{\nabla}_{\alpha}\delta g_{\rho\sigma} - \bar{\nabla}_{\rho}\delta g_{\alpha\sigma}\right)\left(\bar{\nabla}_{\nu}\delta g_{\mu\beta} + \bar{\nabla}_{\mu}\delta g_{\nu\beta} - \bar{\nabla}_{\beta}\delta g_{\mu\nu}\right)\right]$$

Generation of Gravitational Waves

Generation of Gravitational Waves

✳ Weak-field sources with arbitrary velocity: The starting point are the Linearized Einstein Field Equations in the Lorenz Gauge:

$$\Box\,\bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

with:

$$\partial^{\mu}\bar{h}_{\mu\nu} = \partial^{\mu} T_{\mu\nu} = 0$$

we can solve this using the Green function method:

$$\bar{h}_{\mu\nu}(x) = -\frac{16\pi G}{c^4}\int d^4x'\, G(x-x')\, T_{\mu\nu}(x')$$

Generation of Gravitational Waves

The corresponding Green's function is the retarded one:

$$G(x - x') = -\frac{1}{4\pi|\vec{x} - \vec{x}'|}\delta(x^0_{\rm ret} - x'^0)$$

where:

$$x'^0 = ct' , \quad x^0_{\rm ret} = ct_{\rm ret} , \quad t_{\rm ret} = t - \frac{|\vec{x} - \vec{x}'|}{c}$$

Then, the solution is:

$$\bar{h}_{\mu\nu}(t, \vec{x}) = \frac{4G}{c^4}\int d^3x' \frac{1}{|\vec{x} - \vec{x}'|} T_{\mu\nu}\left(t_{\rm ret}, \vec{x}'\right)$$

Generation of Gravitational Waves

We can find the solution in the TT gauge by using the TT projector:

$$h_{ij}^{\mathrm{TT}} = \Lambda_{ij,kl}\bar{h}_{kl} = \Lambda_{ij,kl}h_{kl}$$

where:

$$\Lambda_{ij,kl}(\hat{n}) = P_{ik}(\hat{n})P_{jl}(\hat{n}) - \frac{1}{2}P_{ij}(\hat{n})P_{kl}(\hat{n})$$

and:

$$P_{ij}(\hat{n}) = \delta_{ij} - n_i n_j$$

with:

$$\hat{n} = \hat{x} = \frac{\overrightarrow{x}}{|\overrightarrow{x}|}$$

Generation of Gravitational Waves

Then, the solution in the TT gauge is:

$$h_{ij}^{\mathrm{TT}}(t,\overrightarrow{x}) = \frac{4G}{c^4}\Lambda_{ij,kl}(\hat{n})\int d^3x'\,\frac{1}{|\overrightarrow{x}-\overrightarrow{x}'|}\,T_{kl}\left(t-\frac{|\overrightarrow{x}-\overrightarrow{x}'|}{c},\overrightarrow{x}'\right)$$

Generation of Gravitational Waves

∗ Low-velocity expansion: Things simplify a lot when we assume that the typical velocities within the gravitational-wave source are small as compared to the speed of light. Near spatial infinity we have:

$$h_{ij}^{\mathrm{TT}}(t, \vec{x}) = \frac{4G}{r\,c^4}\Lambda_{ij,kl}(\hat{n}) \int d^3x'\, T_{kl}\left(t - \frac{r}{c} + \frac{\vec{x}\,' \cdot \hat{n}}{c}, \vec{x}\,'\right)$$

where:

$$|\,\vec{x} - \vec{x}\,'| = |\,\vec{x}\,| - \vec{x}\,' \cdot \hat{n} + \mathcal{O}(d^2/r) = r - \vec{x}\,' \cdot \hat{n} + \mathcal{O}(d^2/r), \quad \text{and} \quad r \gg d$$

Then:

$$T_{kl}\left(t - \frac{r}{c} + \frac{\vec{x}\,' \cdot \hat{n}}{c}, \vec{x}\,'\right) \approx \left[T_{kl} + \frac{x'^i n^i}{c}\partial_t T_{kl} + \frac{x'^i x'^j n^i n^j}{2c^2}\partial_t^2 T_{kl} + \ldots\right]\left(t - \frac{r}{c}, \vec{x}\,'\right)$$

Generation of Gravitational Waves

And now the TT gauge metric perturbation has the form:

$$h_{ij}^{\mathrm{TT}}(t,\overrightarrow{x}) = \frac{4G}{r\,c^4}\Lambda_{ij,kl}(\hat{n})\left[S^{kl} + \frac{n_m}{c}\dot{S}^{kl,m} + \frac{n_m n_p}{2c^2}\ddot{S}^{kl,mp} + \ldots\right]\left(t - \frac{r}{c}\right)$$

where:

$$S^{ij}(t) = \int d^3x\, T^{ij}(t,\overrightarrow{x})\,,$$

$$S^{ij,k}(t) = \int d^3x\, T^{ij}(t,\overrightarrow{x})x^k\,,$$

$$S^{ij,kl}(t) = \int d^3x\, T^{ij}(t,\overrightarrow{x})x^k x^l\,,$$

Generation of Gravitational Waves

In order understand the physical meaning of this multipolar expansion it is more convenient to use the momenta associated with the energy density and momentum density:

$$M(t) = \frac{1}{c^2}\int d^3x\, T^{00}(t,\overrightarrow{x})\,,$$

$$M^i(t) = \frac{1}{c^2}\int d^3x\, T^{00}(t,\overrightarrow{x})x^i\,,$$

$$M^{ij}(t) = \frac{1}{c^2}\int d^3x\, T^{00}(t,\overrightarrow{x})x^i x^j\,,$$

$$M^{ijk}(t) = \frac{1}{c^2}\int d^3x\, T^{00}(t,\overrightarrow{x})x^i x^j x^k\,,$$

$$P^i(t) = \frac{1}{c}\int d^3x\, T^{0i}(t,\overrightarrow{x})\,,$$

$$P^{i,j}(t) = \frac{1}{c}\int d^3x\, T^{0i}(t,\overrightarrow{x})x^j\,,$$

$$P^{i,jk}(t) = \frac{1}{c}\int d^3x\, T^{0i}(t,\overrightarrow{x})x^j x^k\,,$$

Generation of Gravitational Waves

All these multipole moments can be related via the energy-momentum conservation equations:

$$\partial^\mu T_{\mu\nu} = 0$$

For instance:

$$\partial_\mu T^{\mu 0} = 0 \Rightarrow \partial_0 T^{00} = -\partial_i T^{i0} \Rightarrow c\dot{M} = \int_V d^3x \partial_0 T^{00} = -\int_V d^3x \partial_i T^{i0} = -\int_{\partial V} dS^i T^{0i} = 0$$

Then:

$$\dot{M} = 0\,,$$

$$\dot{M}^i = P^i\,,$$

$$\dot{M}^{ij} = P^{i,j} + P^{j,i}$$

Generation of Gravitational Waves

Similarly:

$$\dot{P}^{i} = 0\,,$$
$$\dot{P}^{i,j} = S^{ij}\,,$$
$$\dot{P}^{i,jk} = S^{ij,k} + S^{ik,j}$$

Combining these relations we can write:

$$\ddot{M}^{ij} = \dot{P}^{i,j} + \dot{P}^{j,i} = S^{ij} + S^{ji} = 2\,S^{jk} \quad\Longrightarrow\quad S^{ij} = \frac{1}{2}\ddot{M}^{ij}$$

Generation of Gravitational Waves

- Mass Quadrupole Radiation:

$$\left[h_{ij}^{\rm TT}(t,\overrightarrow{x})\right]_{\rm quad} = \frac{2G}{r\,c^4}\Lambda_{ij,kl}(\hat{n})\,\ddot{M}^{kl}(t-r/c)$$

We can now decompose the mass quadrupole into its irreducible parts:

$$M^{ij} = \left(M^{ij}-\frac{1}{3}\delta^{ij}M^{kk}\right)+\frac{1}{3}\delta^{ij}M^{kk} \equiv Q^{ij}+\frac{1}{3}\delta^{ij}M^{kk}$$

where:

$$Q^{ij} = \int d^3x\,\rho(t,\overrightarrow{x})\left(x^ix^j-\frac{1}{3}r^2\delta^{ij}\right)\,, \qquad \rho = \frac{1}{c^2}T^{00}$$

Generation of Gravitational Waves

Using the properties of the TT projector (its traceless character) we finally have:

$$\left[h_{ij}^{\rm TT}(t,\vec{x})\right]_{\rm quad} = \frac{2G}{r\,c^4}\Lambda_{ij,kl}(\hat{n})\,\ddot{Q}^{kl}(t-r/c) \equiv \frac{2G}{r\,c^4}\ddot{Q}_{ij}^{\rm TT}(t-r/c)$$

✳ Radiated Energy: The Quadrupole Formula:

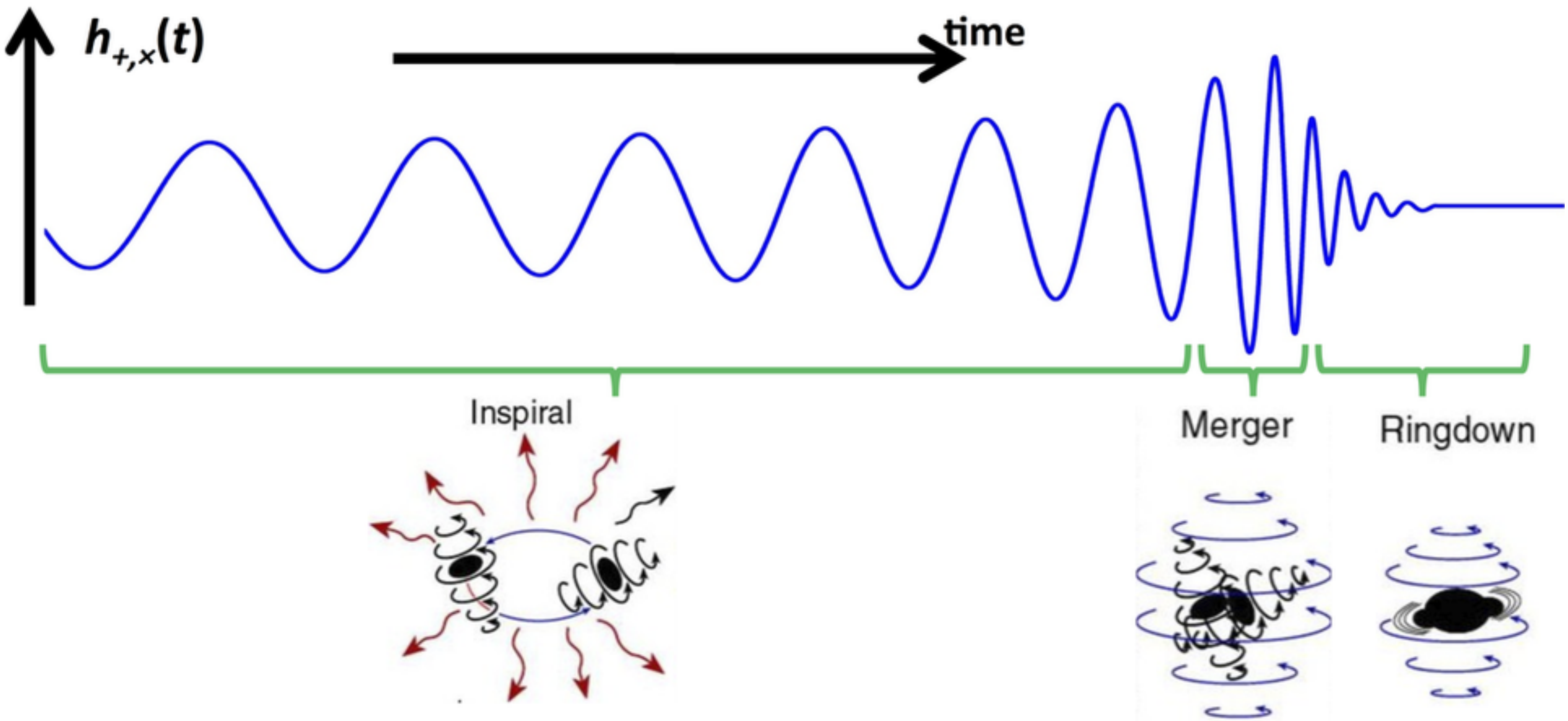
$$\frac{dE}{dAdt} = \frac{c^3}{32\pi G} < \dot{h}_{ij}^{\rm TT}\dot{h}_{ij}^{\rm TT} >_\lambda \quad \Rightarrow \quad \frac{dP}{d\Omega} = \frac{G}{8\pi c^5}\Lambda_{ij,kl}(\hat{n}) < \dddot{Q}_{ij}^{\rm TT}\dddot{Q}_{ij}^{\rm TT} >_\lambda$$

and from here the total power is:

$$P_{\rm quad} = \frac{G}{5c^5} < \dddot{Q}_{ij}^{\rm TT}\dddot{Q}_{ij}^{\rm TT} >_\lambda$$

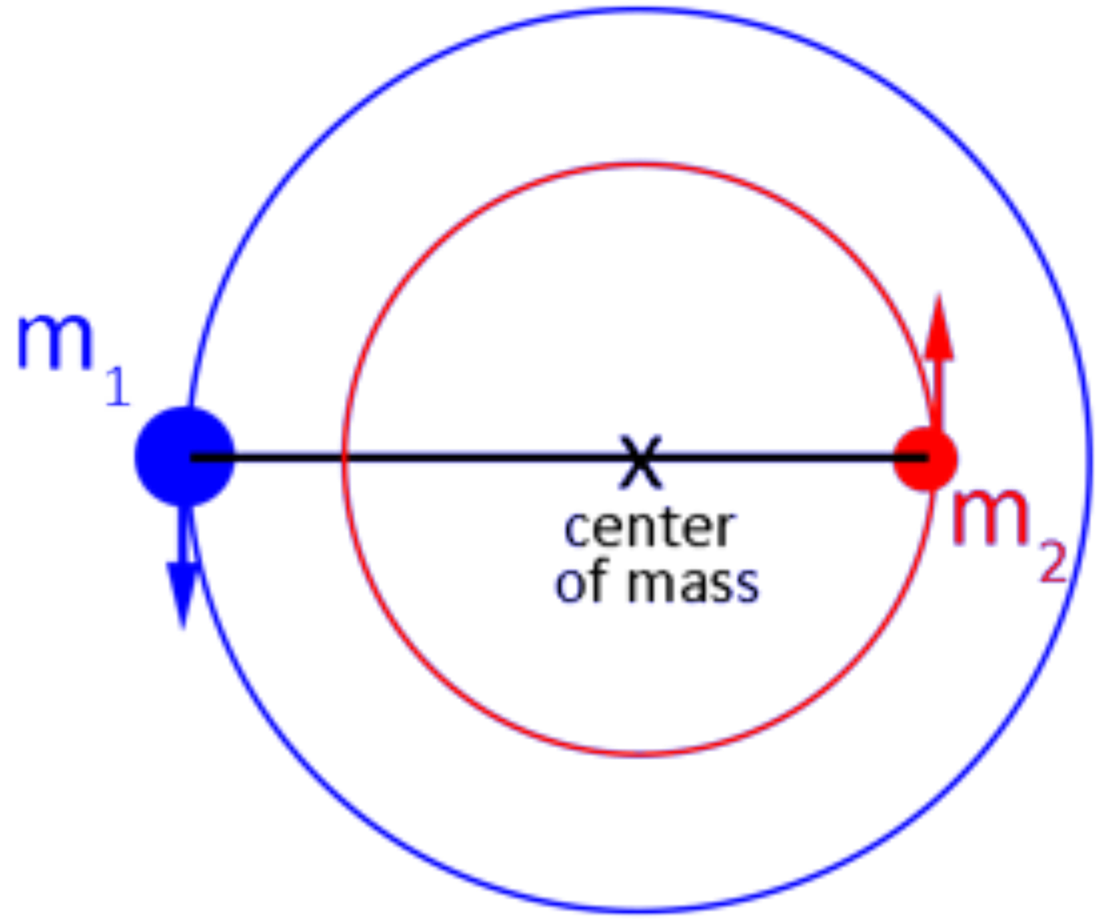
Inspiral of Compact Binaries

Inspiral of Compact Binaries

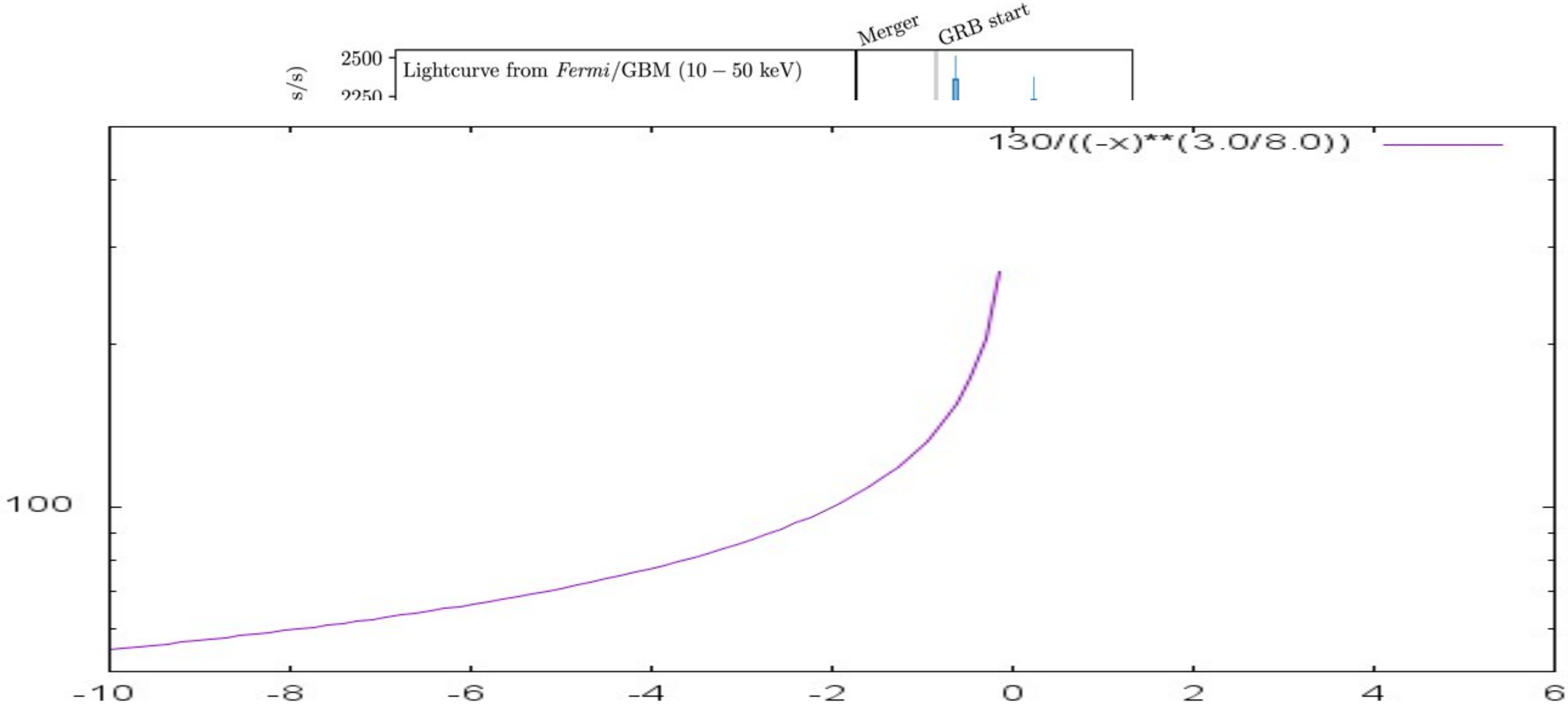


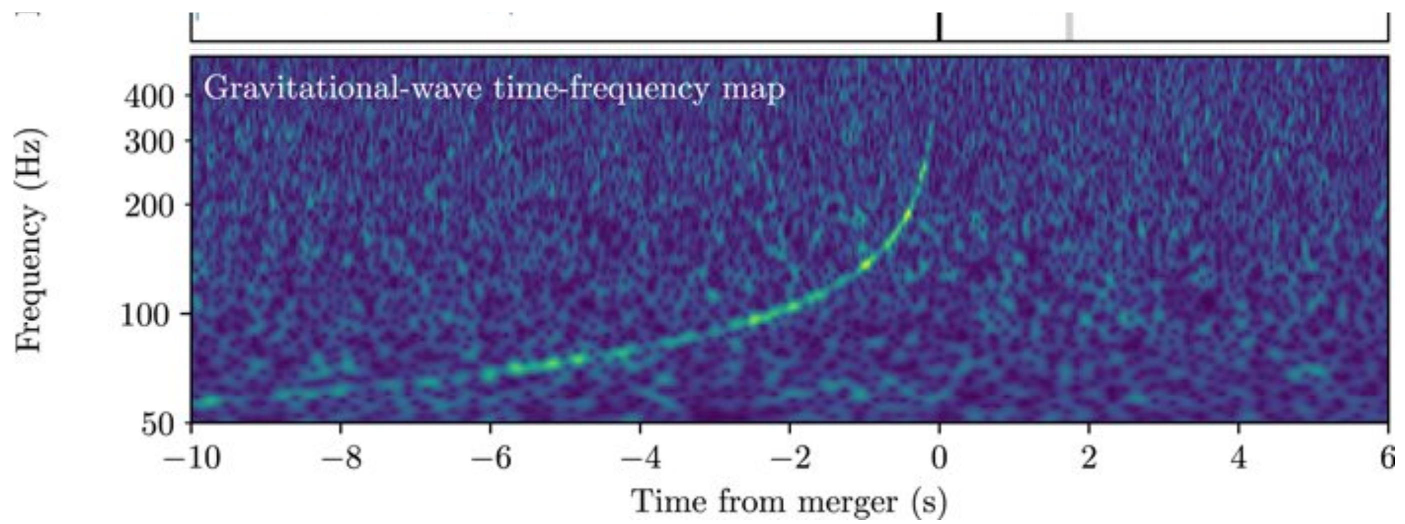
Inspiral of Compact Binaries

* Newtonian Binary:



Inspiral of Compact Binaries





Tests of General Relativity

Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariabeln gegenüber kovariant sind.

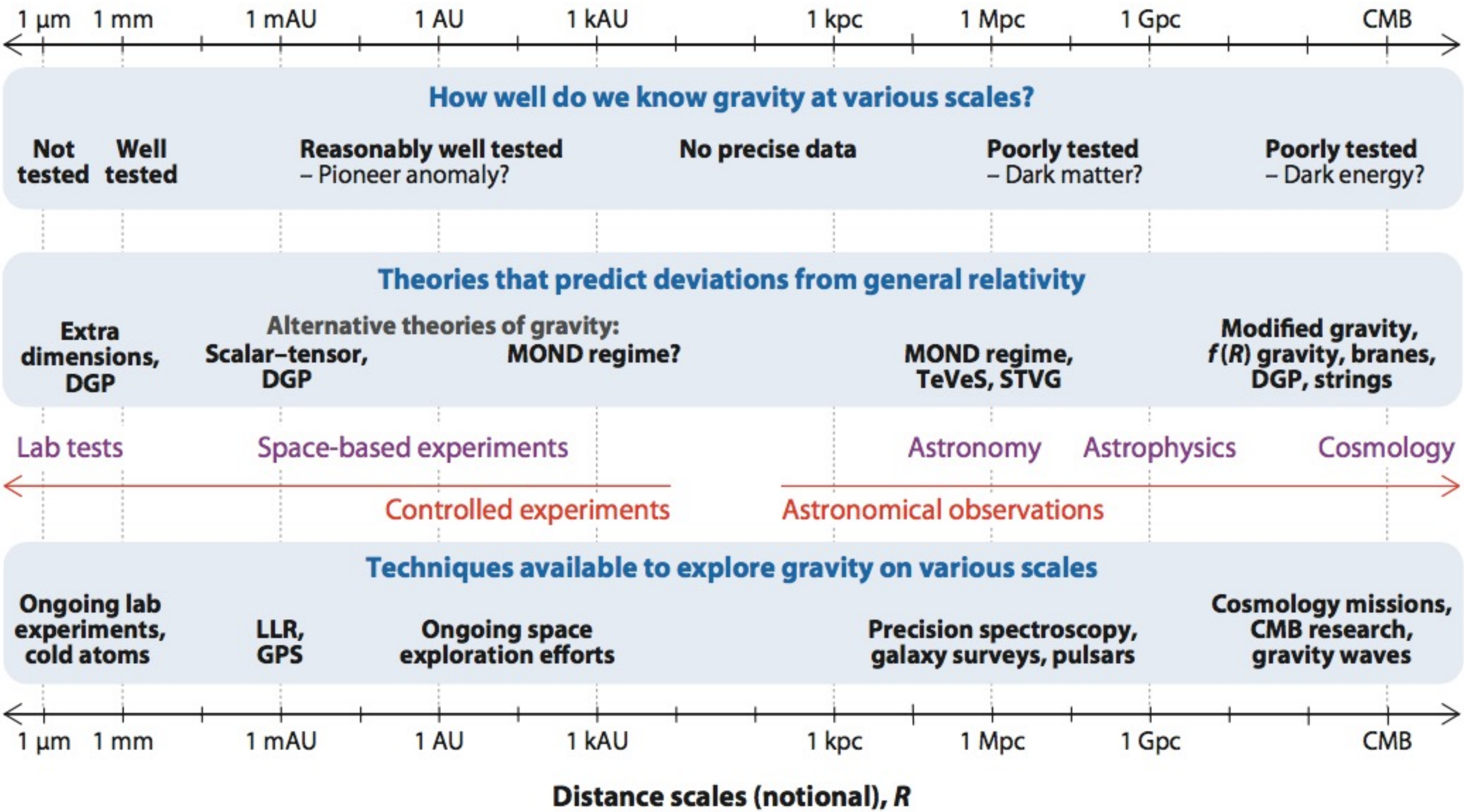
Der Entwicklungsgang war dabei folgender. Zunächst fand ich Gleichungen, welche die Newtonsche Theorie als Näherung enthalten

"I have just completed the most splendid work of my life..."

-- to his son Hans Albert, 1915

First of all, I found equations containing the Newtonian theory as an approximation...

Tests of General Relativity



S.G. Turyshev (2008)

Tests of General Relativity

- In General Relativity, Gravity is described by a metric tensor that couples in a universal way with the matter fields of the Standard Model (Einstein Equivalence Principle).

- This universal coupling has a number of implications for experiments and observations..

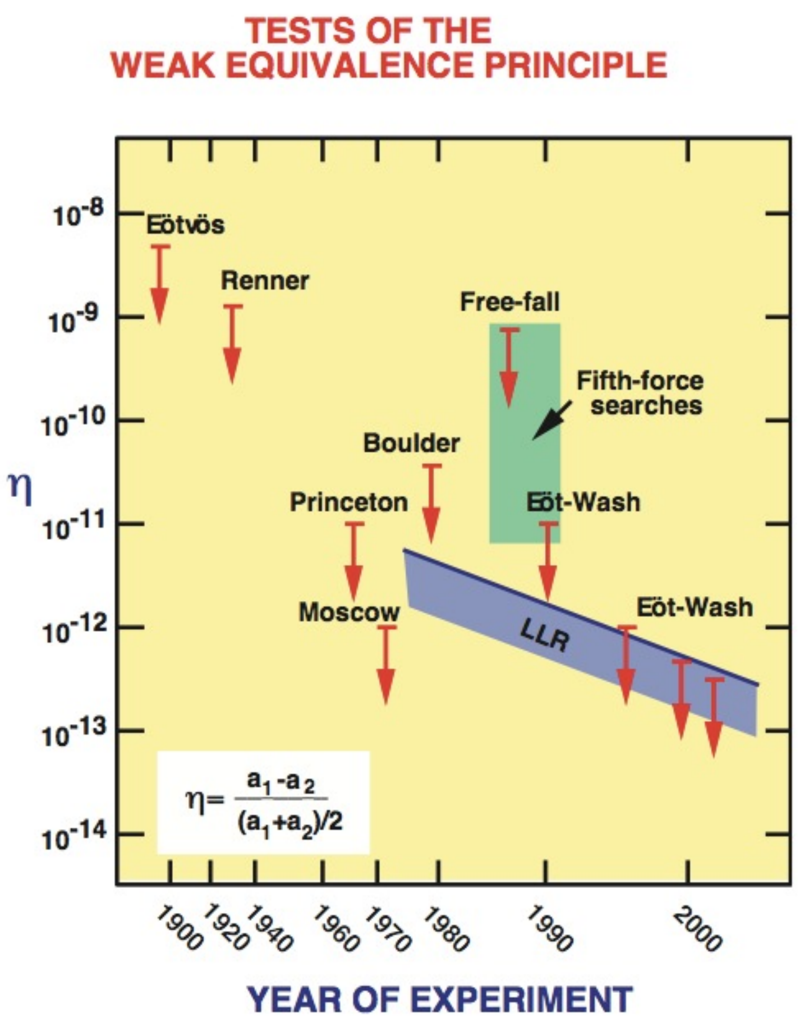
Tests of General Relativity

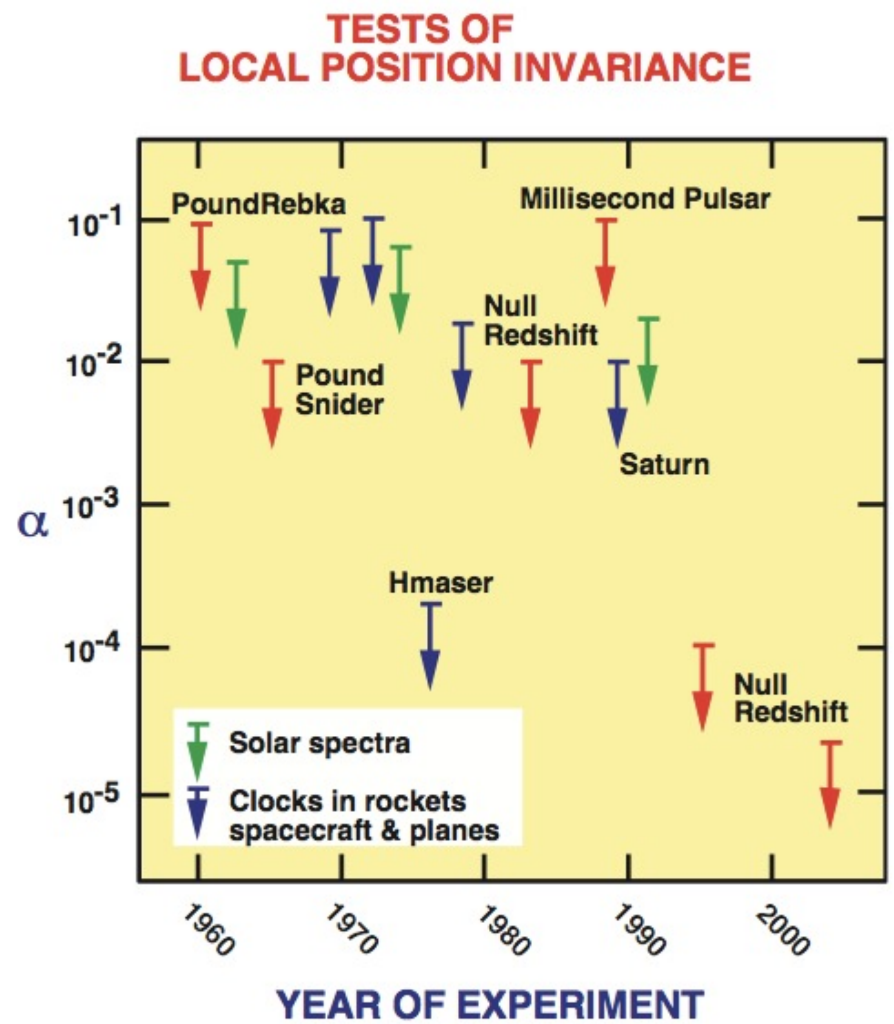
• The outcome of local non-gravitational experiments, referred to local standards, does not depend on where, when, and in which locally inertial frame, the experiment is performed.

• This means that local experiments should neither feel the cosmological evolution of the Universe (constancy of the *constants*), nor exhibit preferred directions in spacetime (isotropy of space, local Lorentz invariance).

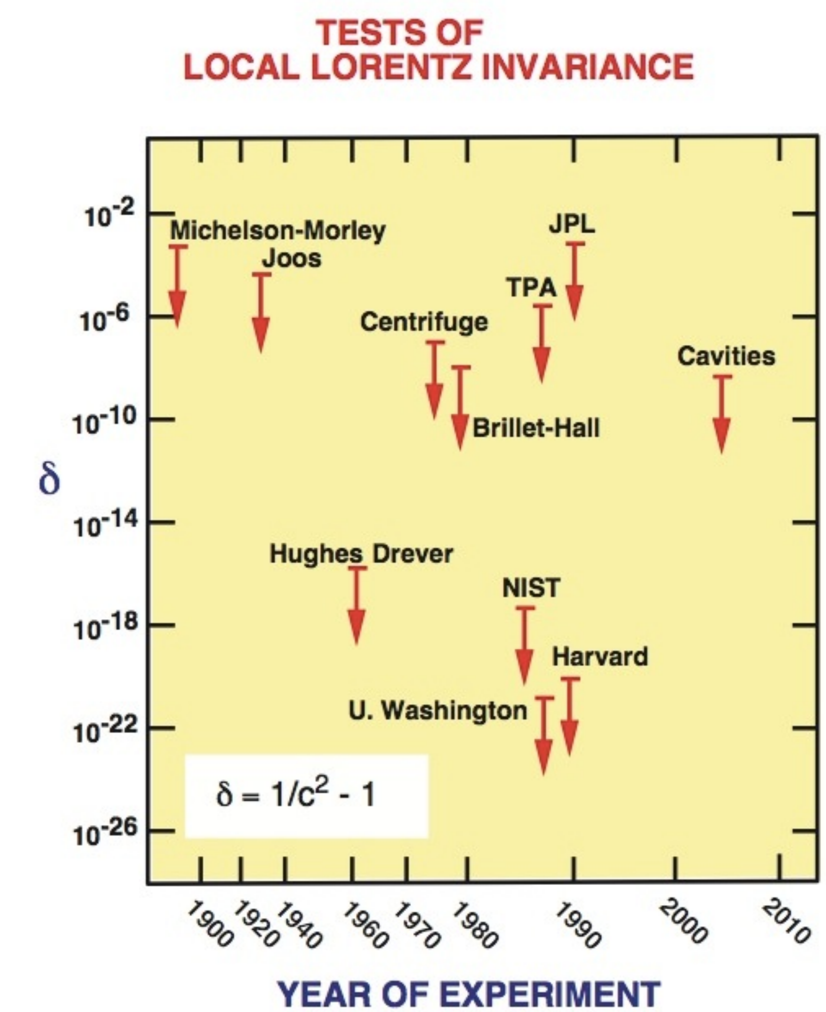
• In addition, we have the Weak Equivalence principle, which has been tested many times.

Tests of General Relativity





$$\Delta\nu/\nu = (1+\alpha)\Delta U/c^2$$



C.M.Will (2014)

Tests of General Relativity

Constant k	Limit on $\dot{k}/k$ (yr^{-1})	Redshift	Method
Fine structure constant ($\alpha_{\mathrm{EM}} = e^2/\hbar c$)	$< 30 \times 10^{-16}$	0	Clock comparisons
	$< 0.5 \times 10^{-16}$	0.15	Oklo Natural Reactor
	$< 3.4 \times 10^{-16}$	0.45	^{187}Re decay in meteorites
	$(6.4 \pm 1.4) \times 10^{-16}$	0.2 – 3.7	Spectra in distant quasars
	$< 1.2 \times 10^{-16}$	0.4 – 2.3	Spectra in distant quasars
Weak interaction constant ($\alpha_{\mathrm{W}} = G_{\mathrm{f}} m_{\mathrm{p}}^2 c/\hbar^3$)	$< 1 \times 10^{-11}$	0.15	Oklo Natural Reactor
	$< 5 \times 10^{-12}$	10^9	Big Bang nucleosynthesis
e-p mass ratio	$< 3 \times 10^{-15}$	2.6 – 3.0	Spectra in distant quasars

Bounds on changes of non-gravitational constants C.M.Will (2014)

Tests of General Relativity

Method	$\dot{G}/G$ (10^{-13} yr^{-1})
Lunar laser ranging	4 ± 9
Binary pulsar $1913+16$	40 ± 50
Helioseismology	0 ± 16
Big Bang nucleosynthesis	0 ± 4

C.M. Will (2014)

Tests of General Relativity

C.M.Will (2014)

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
α_3	Violation of conservation of total momentum?	0	0	0
ζ_1		0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2$$
$$+2(1+\zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij}$$
$$+(2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3),$$

$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U$$
$$-\alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}),$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).$$

Parametrized Post-Newtonian (PPN) Formalism

Tests of General Relativity

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	2×10^{-4}	VLBI
$\beta - 1$	perihelion shift	8×10^{-5}	$J_{2\odot} = (2.2 \pm 0.1) \times 10^{-7}$
	Nordtvedt effect	2.3×10^{-4}	$\eta_{\mathrm{N}} = 4\beta - \gamma - 3$ assumed
ξ	spin precession	4×10^{-9}	millisecond pulsars
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		4×10^{-5}	PSR J1738+0333
α_2	spin precession	2×10^{-9}	millisecond pulsars
α_3	pulsar acceleration	4×10^{-20}	pulsar $\dot{P}$ statistics
ζ_1	—	2×10^{-2}	combined PPN bounds
ζ_2	binary acceleration	4×10^{-5}	$\ddot{P}_{\mathrm{p}}$ for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	lunar acceleration
ζ_4	—	—	not independent

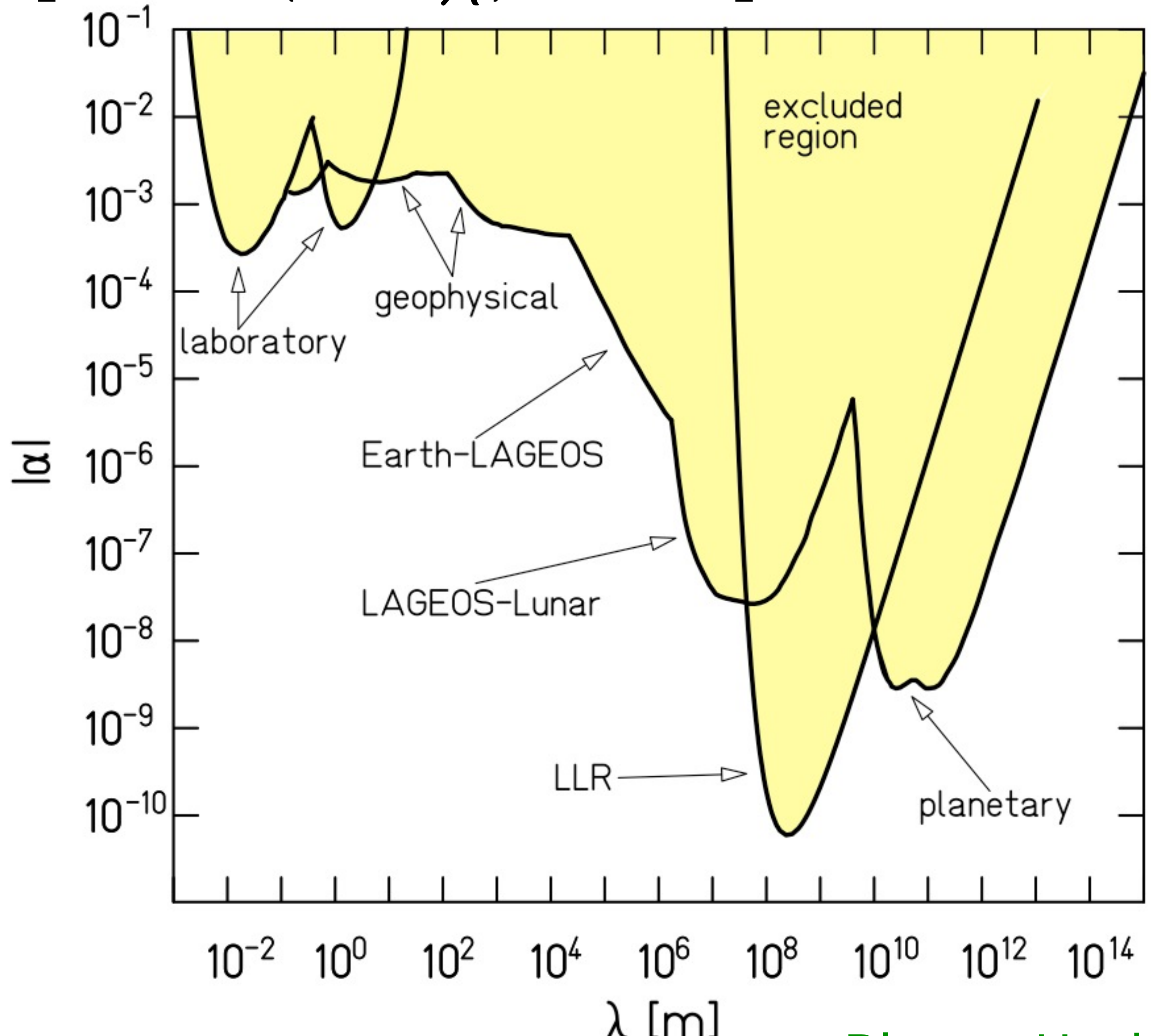
C.M. Will (2014)

Tests of General Relativity

- Modifications of Newtonian Gravity

$$F = G\frac{m_1 m_2}{r^2}\left[1 + \alpha\left(1 + \frac{r}{\lambda}\right)e^{-r/\lambda}\right]$$

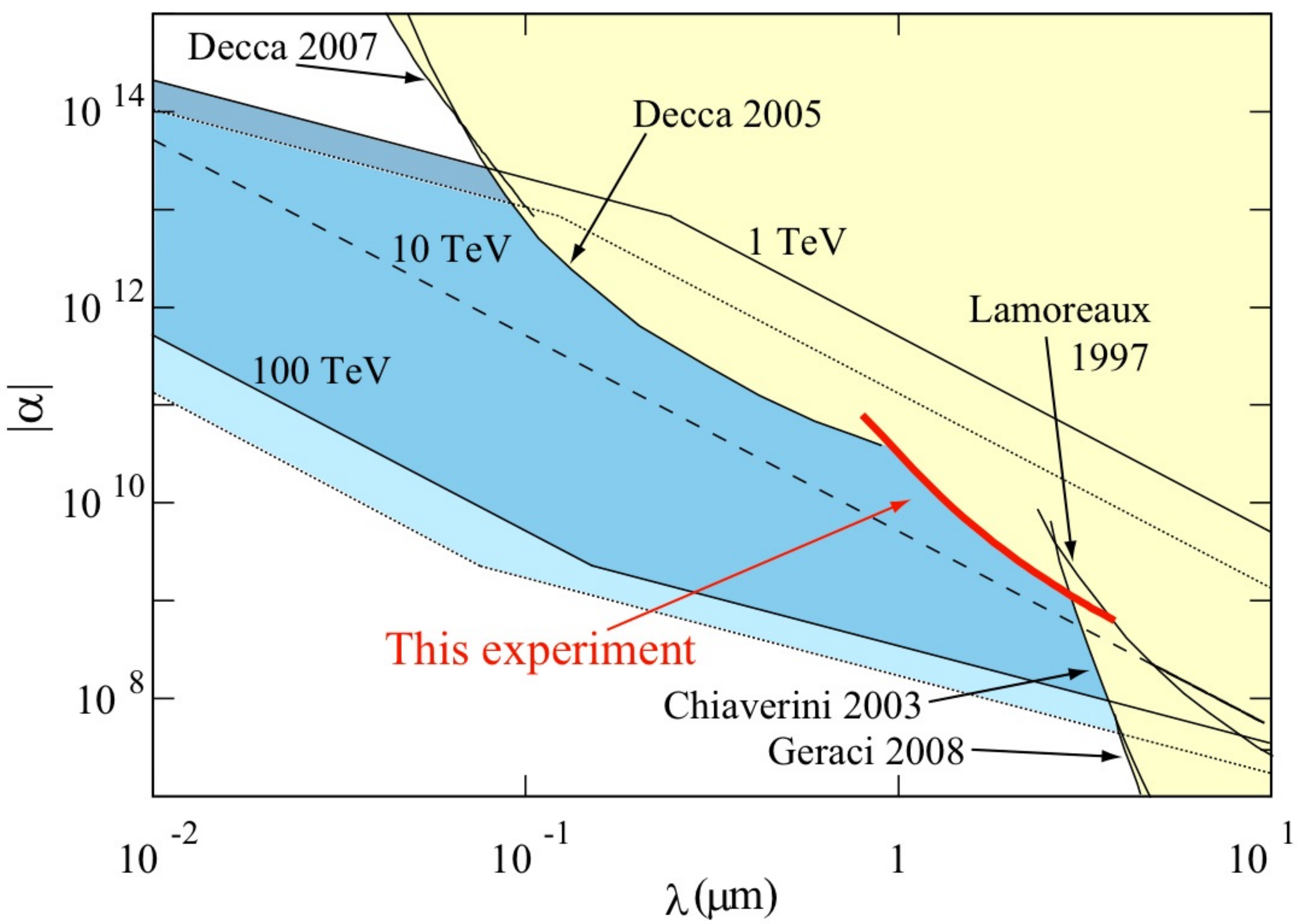
This includes gauge bosons; extra-dimensional models; dark energy scale probes.



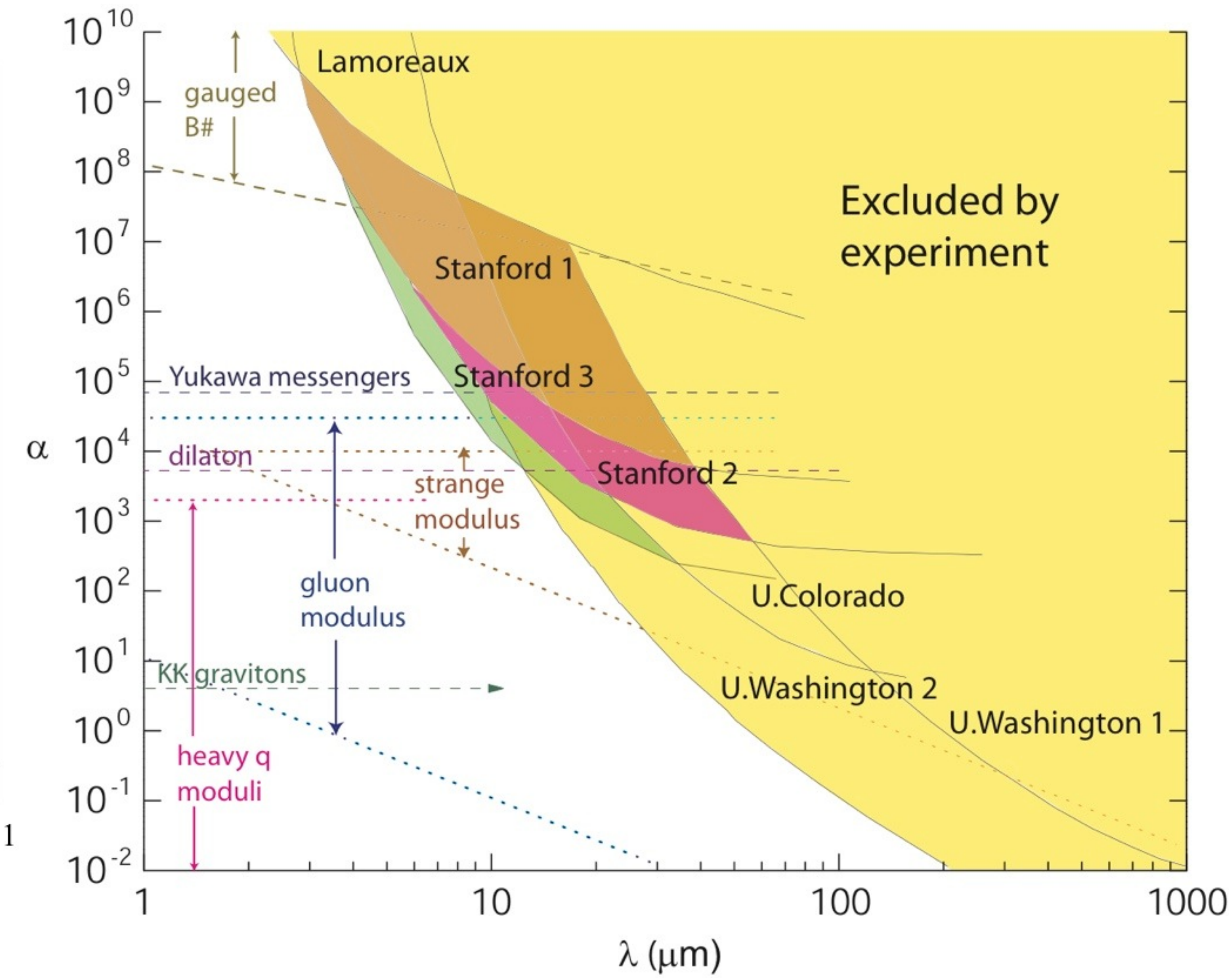
Blayne Heckel

Tests of General Relativity

- Short-range modifications of Newtonian gravity.



M. Masuda, M. Sasaki: *Limits on Nonstandard Forces in the Submicrometer Range*. Phys. Rev. Lett. **102** (2009) 171101



A.A. Geraci et al: *Improved constraints on non-Newtonian forces at 10 microns.* Phys. Rev. D**78** (2008) 022002

The Hulse-Taylor binary pulsar

Meas... eters for B1913+16 System

1993 Nobel Prize in Physics

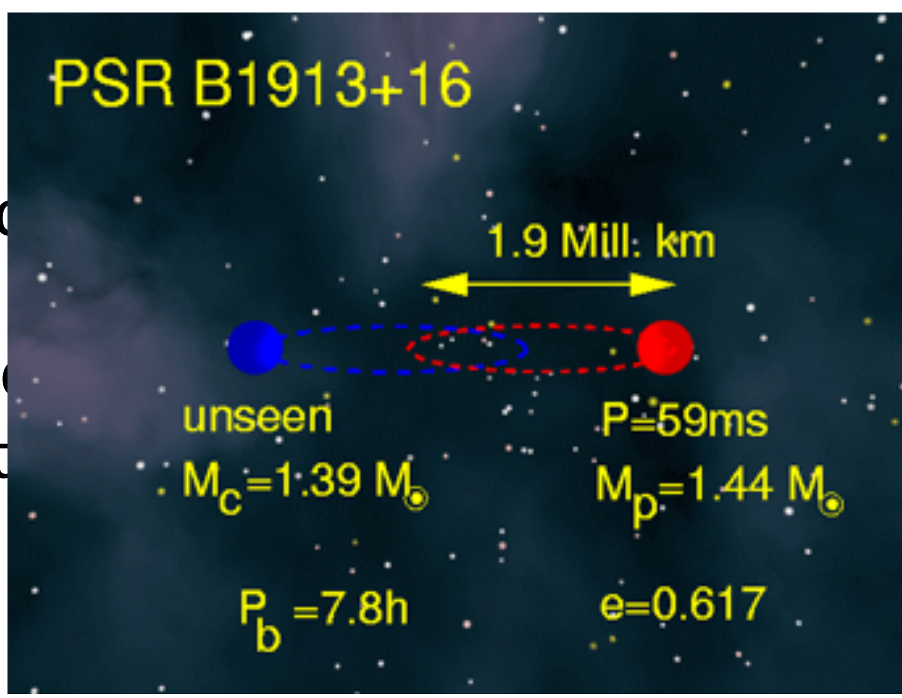
KUNGL. VETENSKAPSAKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES

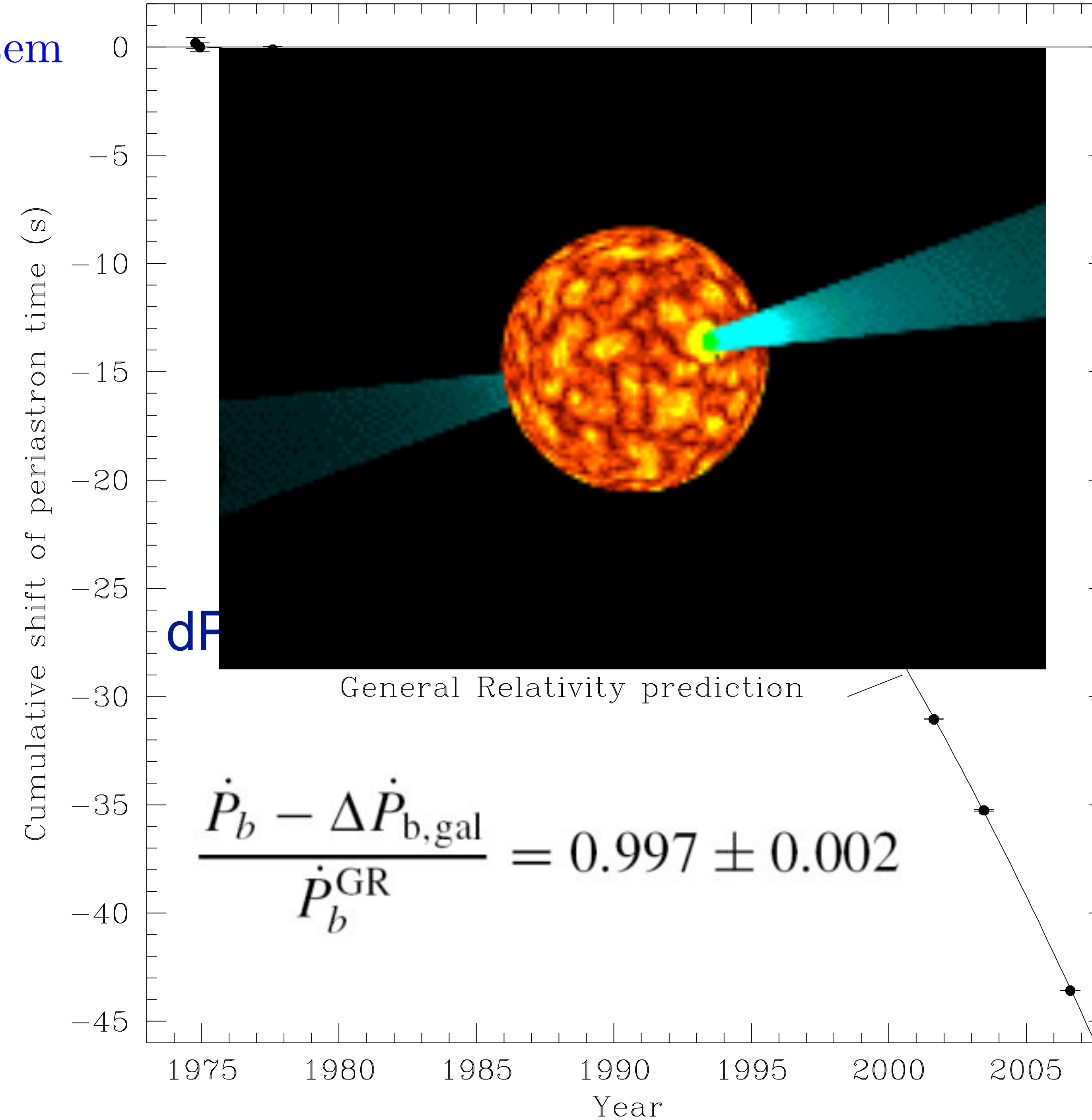
	Value
$a_p \sin i$ (s)	2.341782 (3)
......	0.[illegible]
......	52[illegible] (4)
......	0.[illegible] (4)
......	29[illegible]
......	4.[illegible]
......	0.[illegible]
s)...	−[illegible]

Russell A. Hulse

Joseph H. Taylor Jr.

... The ... orbital period ... from the e... ation, accor... within about

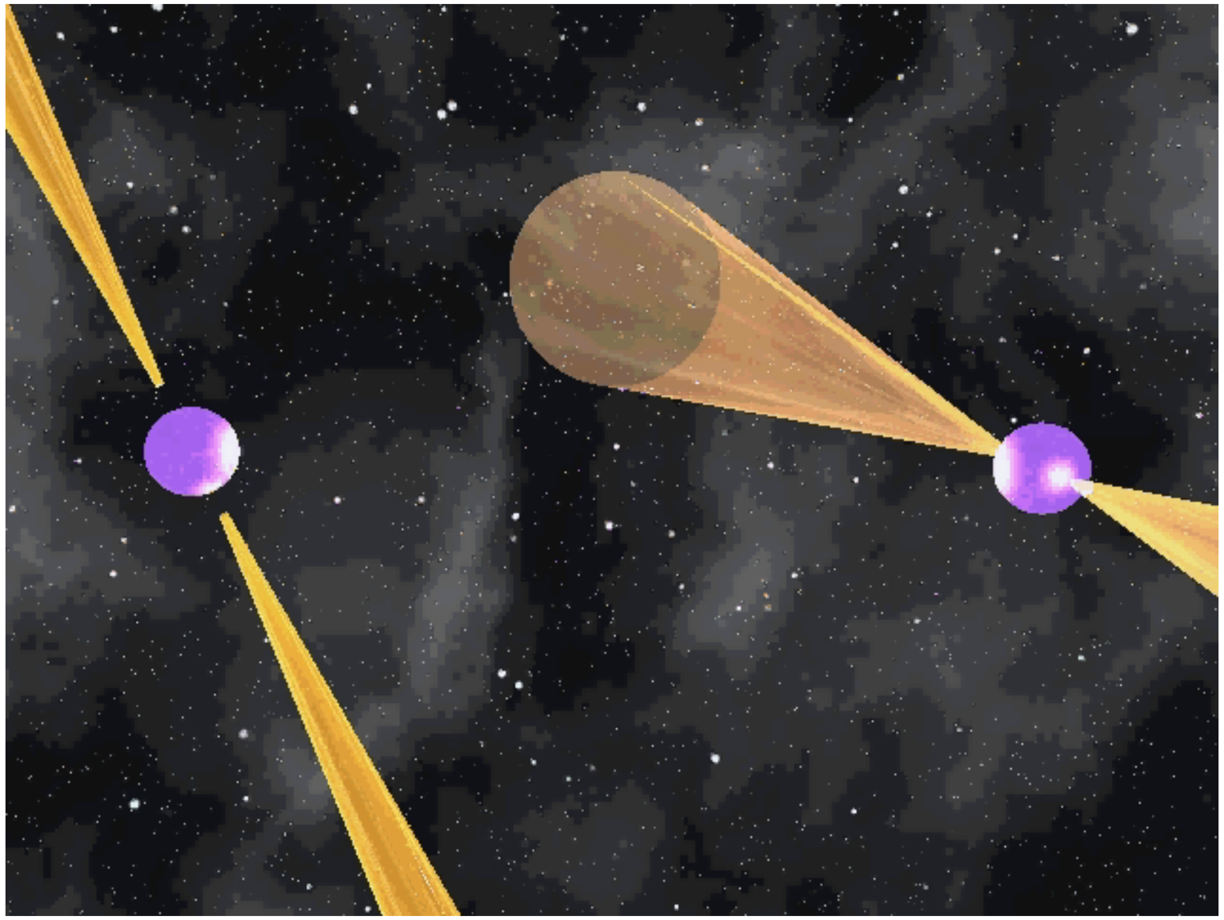




$$\frac{\dot{P}_b - \Delta\dot{P}_{\rm b,gal}}{\dot{P}_b^{\rm GR}} = 0.997 \pm 0.002$$

Weisberg, Nice & Taylor (1011.0718)
Astrophysical Journal, **722**, 1030-1034 (2010)

Other (recent) Pulsars



The Double Pulsar: PSR J0737-3039A/B

$dP_b/dt = (-1.248 \pm 0.001) \times 10^{-12}$

Agreement with General Relativity to 0.1%!

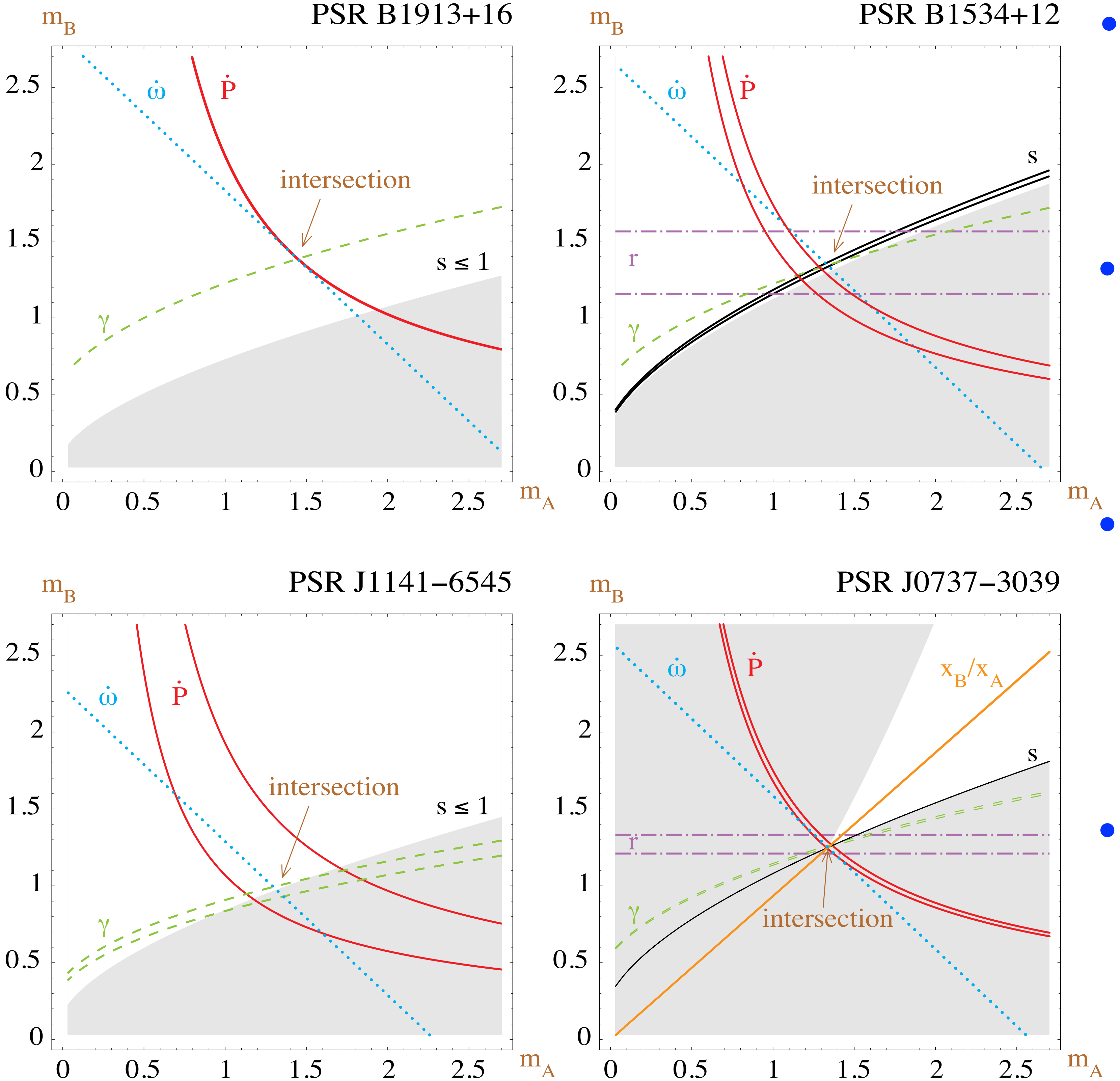
PSR J0348+0432: A binary system with a pulsar and a white dwarf

$$\dot{P}_{\rm b}^{\rm GR} = (-2.58^{+0.07}_{-0.11}) \times 10^{-13} \text{ s s}^{-1}$$

$$\dot{P}_{\rm b}/\dot{P}_{\rm b}^{\rm GR} = 1.05 \pm 0.18$$

Non-linear Regime, but Weak Field!

Tests of General Relativity with Pulsars

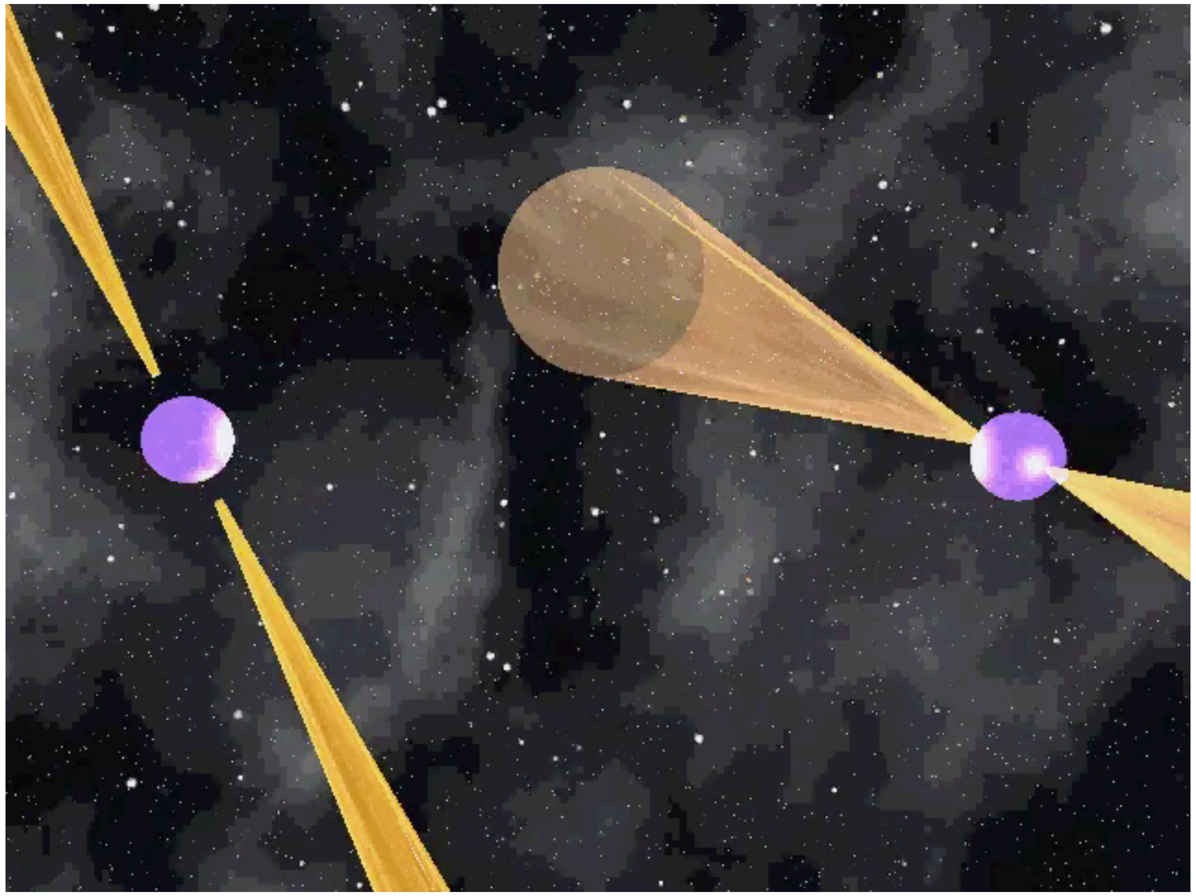


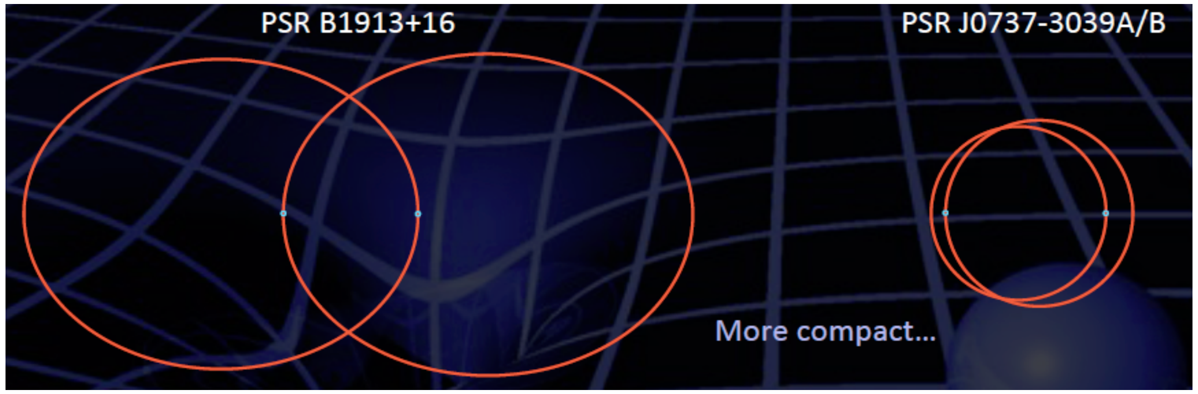
- Binary pulsars contain strong gravity domains. Therefore, they allow for tests of the strong-field regime of gravitational theories.
- This can be done in a phenomelogical way by using a set of Keplerian and post-Keplerian parameters for the dynamics of the binary pulsar.
- In General Relativity (and also in scalar-tensor theories) the post-Keplerian parameters can be written in terms of the Keplerian ones and the two masses of the binary.
- The measurement of a post-Keplerian parameter produces a line (band) in the mass plane. Then, measuring N+3 post-Keplerian parameters yields N+1 tests of gravity.

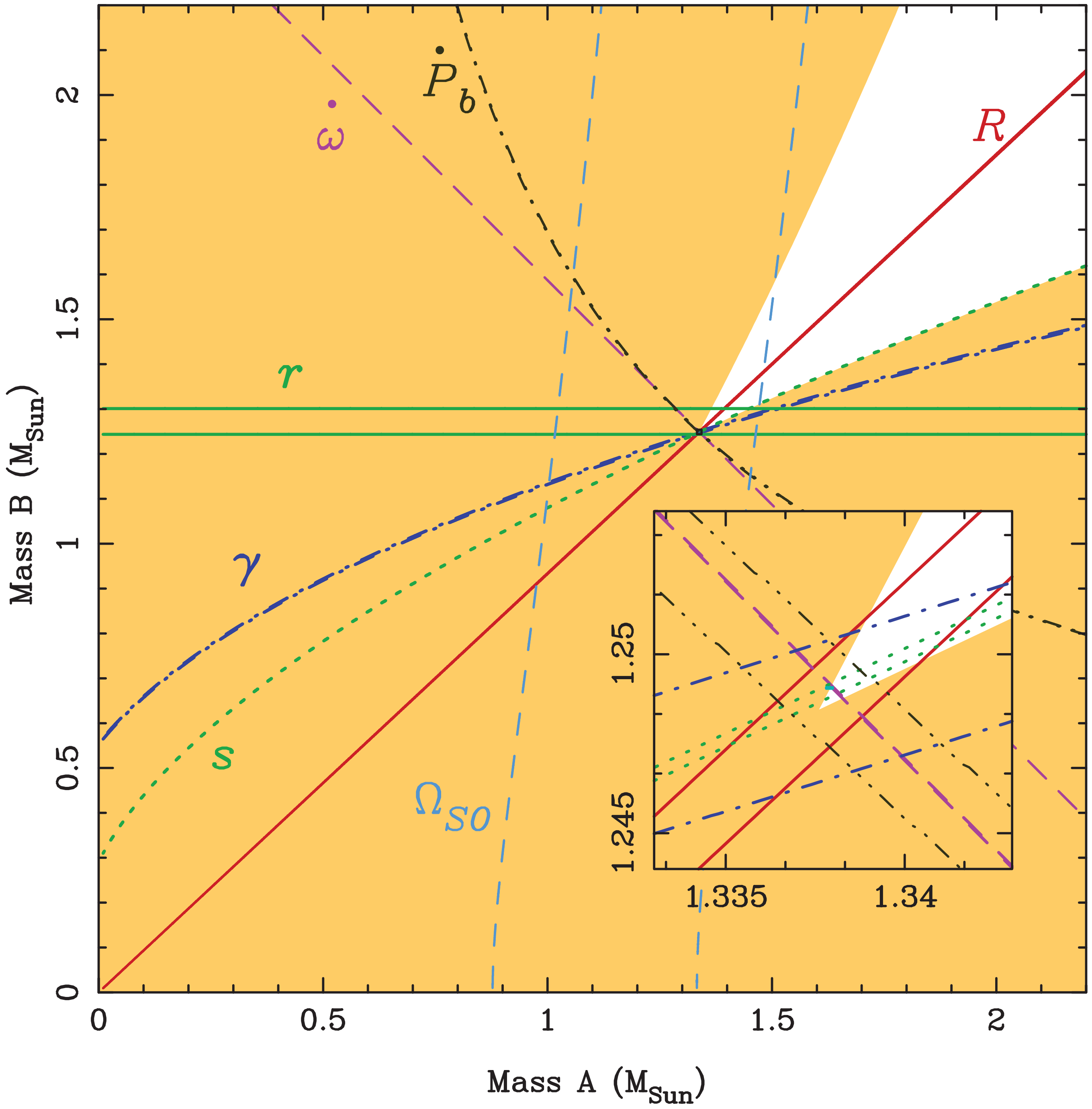
[From: Damour, arXiv:0705.3109]

Tests of General Relativity with Pulsars

- Tests with pulsars: PSR J0737-3039A/B, the only known double pulsar. Tests of GR to the 0.05% level.

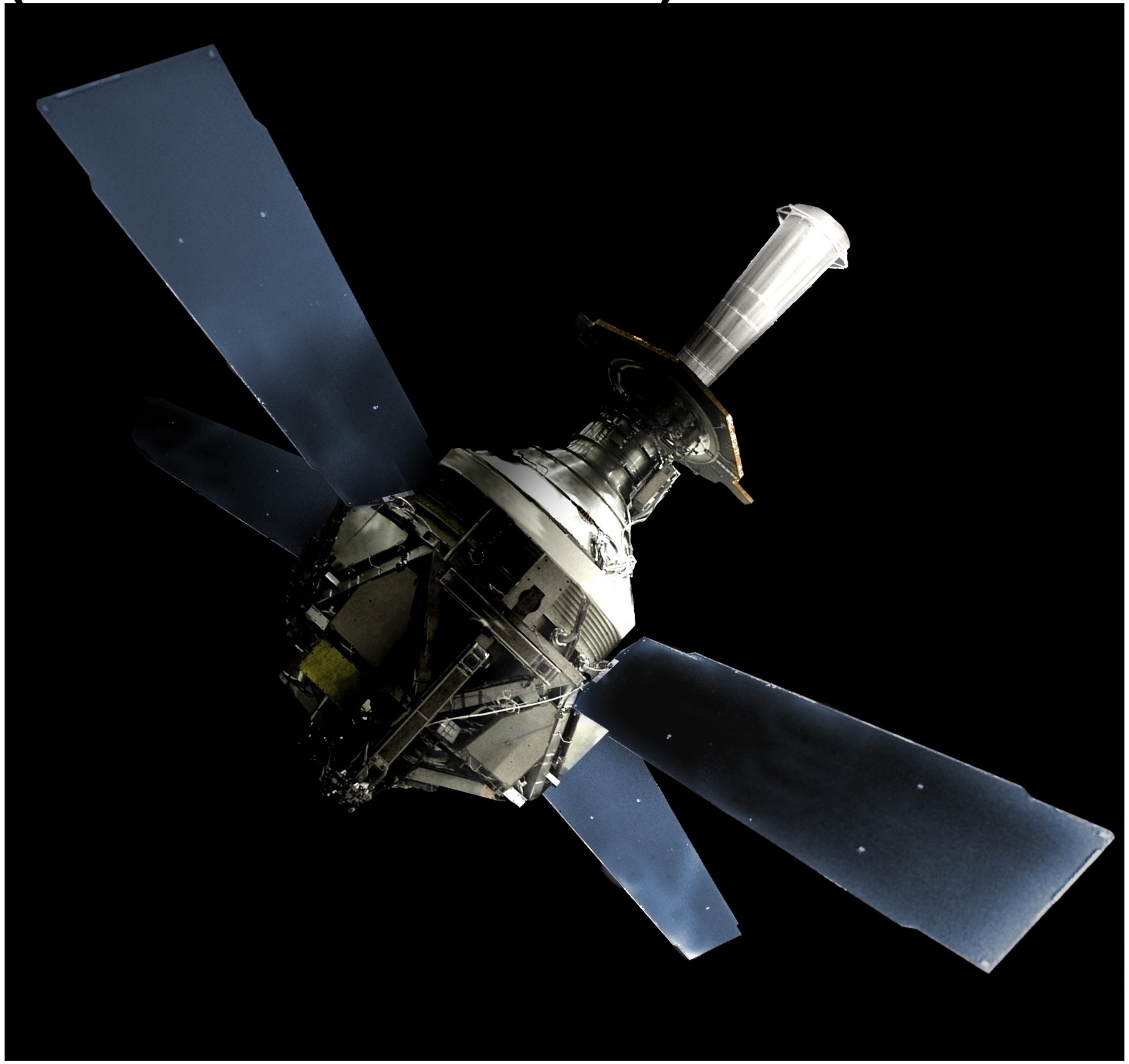






Tests of General Relativity

- GravitoMagnetism: **Gravity Probe B** (NASA+Stanford)

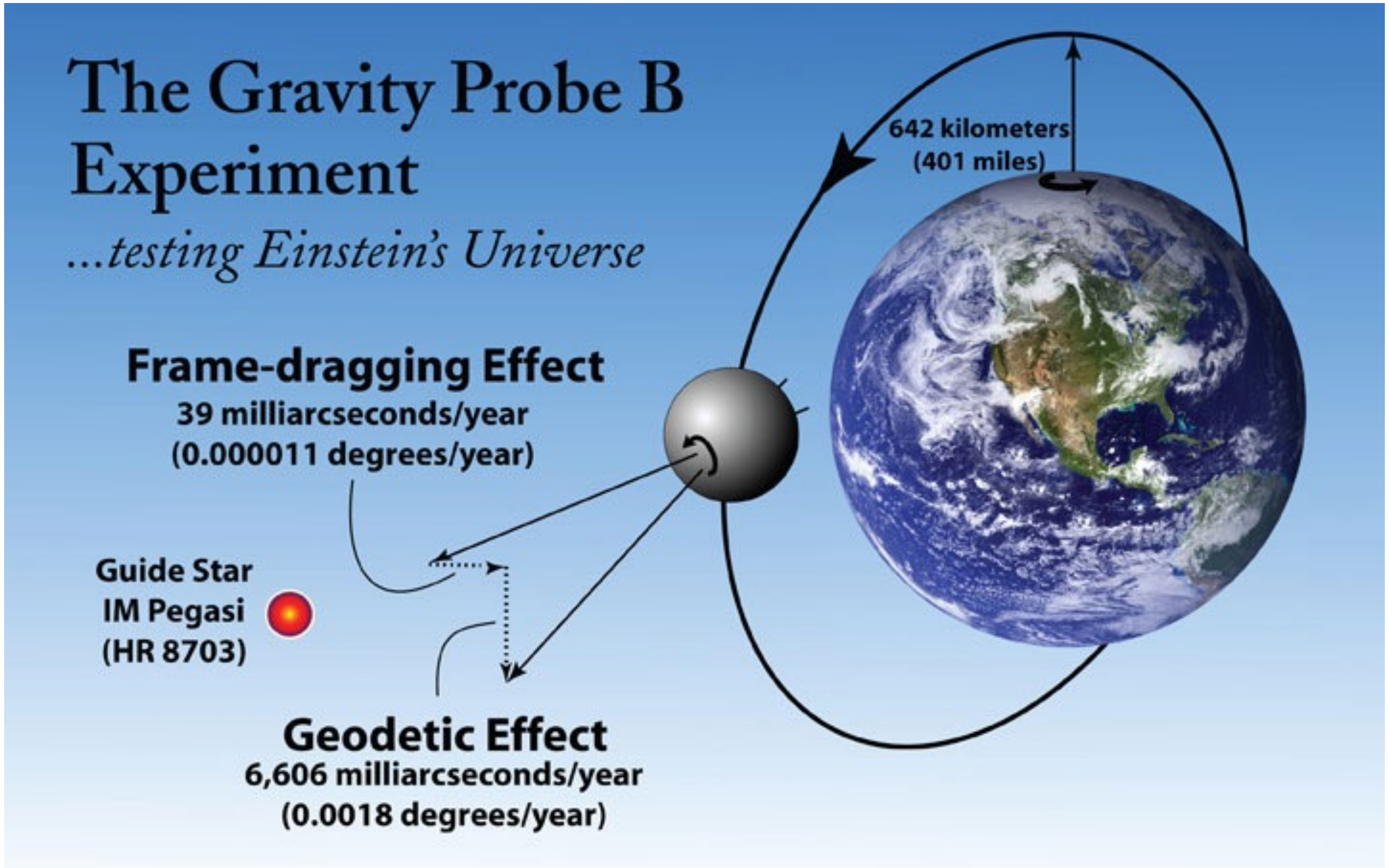


- In General Relativity, not only mass produces gravitational fields, but also the linear and angular momentum of matter. Those gravitational fields produce effects similar to magnetic fields: Gravito-magnetism.
- In particular, rotating matter drags inertial reference systems (Lense-Thirring effect) around it.
- In addition, if the reference system is also rotating (spin), there will be also precession of the spin (*geodetic precesion*).

Tests of General Relativity

- GravitoMagnetism: **Gravity Probe B**

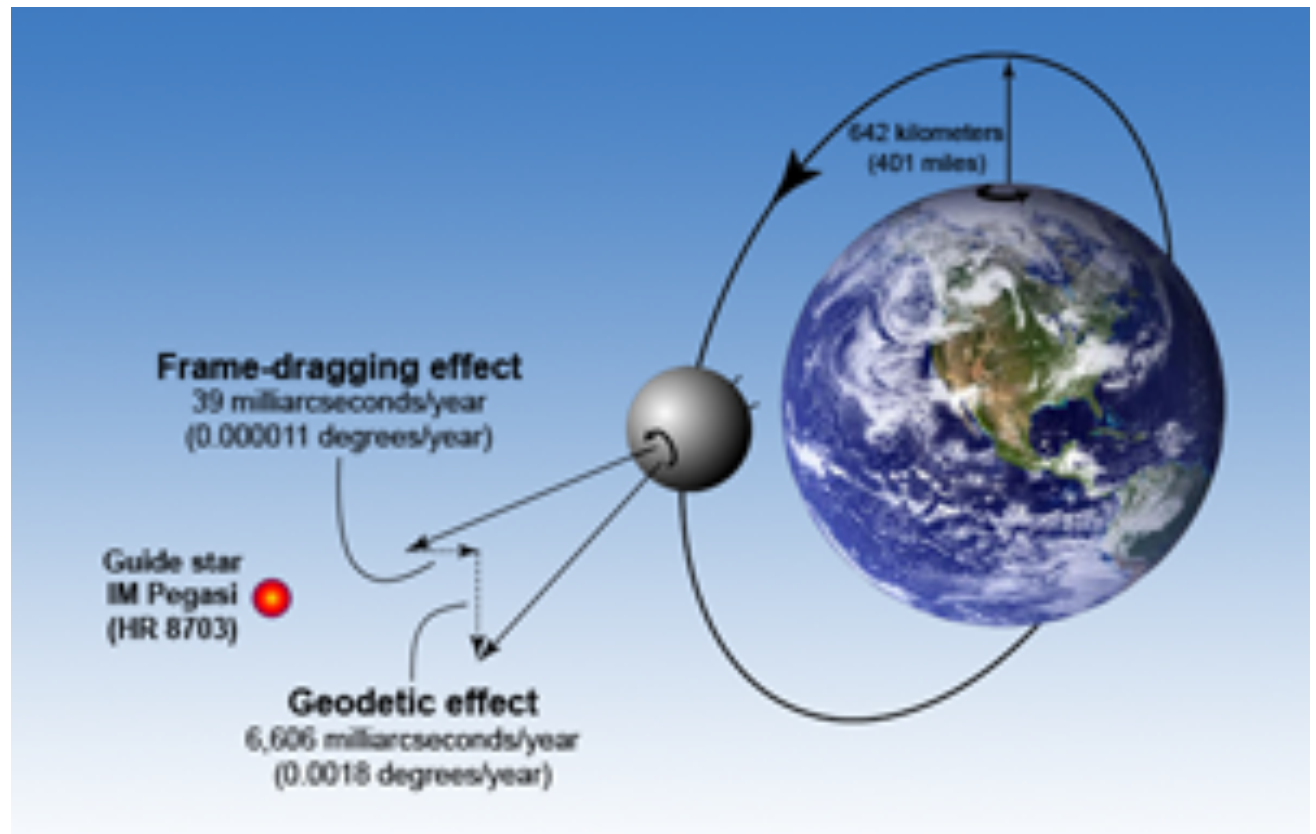
- Gravity Probe B (NASA+Stanford) consisted in an experiment (2004-2005) to measure these two effects (Lense-Thirring and *geodetic precession*)



Tests of General Relativity

- GravitoMagnetism: **Gravity Probe B**

- After many difficulties, the results obtained agree with General Relativity with a precision of 0.3% (*geodetic precession*) and 20% (Lense-Thirring).



	Measured	Predicted
Geodetic precession (mas)	6602 ± 18	6606
Frame-dragging (mas)	37.2 ± 7.2	39.2

Towards Extreme Gravity

- What is Extreme Gravity? Let us use two figures of merit

- Newtonian Potential (dimensionless):

$$\varepsilon \equiv \frac{\phi_{\rm Newtonian}}{c^2} \sim \frac{GM}{Rc^2}$$

- Spacetime curvature associated with a particular physical phenomena:

$$\xi \equiv \frac{GM}{R^3c^2} \sim \frac{1}{\ell^2}$$

Towards Extreme Gravity

- Let us use the first one (Newtonian Potencial)
  - Gravity in the Solar System (perihelion advance, frame dragging, time delays, etc.):

$$\frac{\phi_{Newtonian}}{c^2} = \frac{GM_\odot}{c^2\ 1\mathrm{AU}} \sim 10^{-8}$$

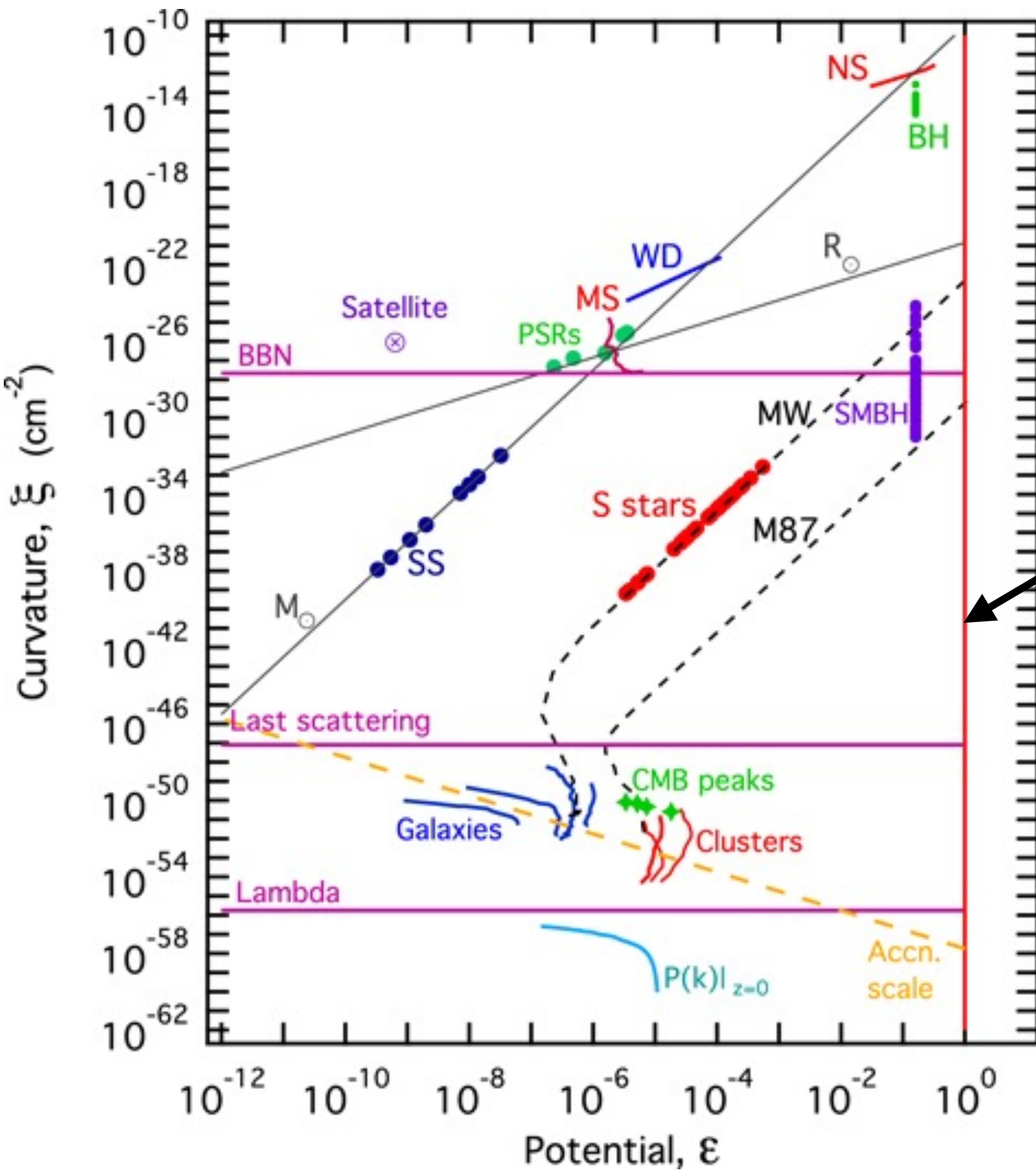
  - Gravity with Pulsars:

$$\frac{\phi^{\mathrm{NS}}_{Newtonian}}{c^2} \sim \frac{GM_{\mathrm{NS}}}{c^2\ r_{\mathrm{NS}}} \sim 10^{-1} \qquad \frac{\phi^{\mathrm{Binary}}_{Newtonian}}{c^2} \sim \frac{GM_\odot}{c^2\ r^{\mathrm{periastron}}_{\mathrm{Hulse-Taylor}}} \sim 10^{-6}$$

  - Gravity with neutron stars and black holes:

$$\frac{\phi_{Newtonian}}{c^2} \sim \frac{GM_{\mathrm{MBH}}}{c^2\ (\mathrm{a\ few}\ r_{\mathrm{Horizon}})} \sim 10^{-1} - 1$$

Towards Extreme Gravity

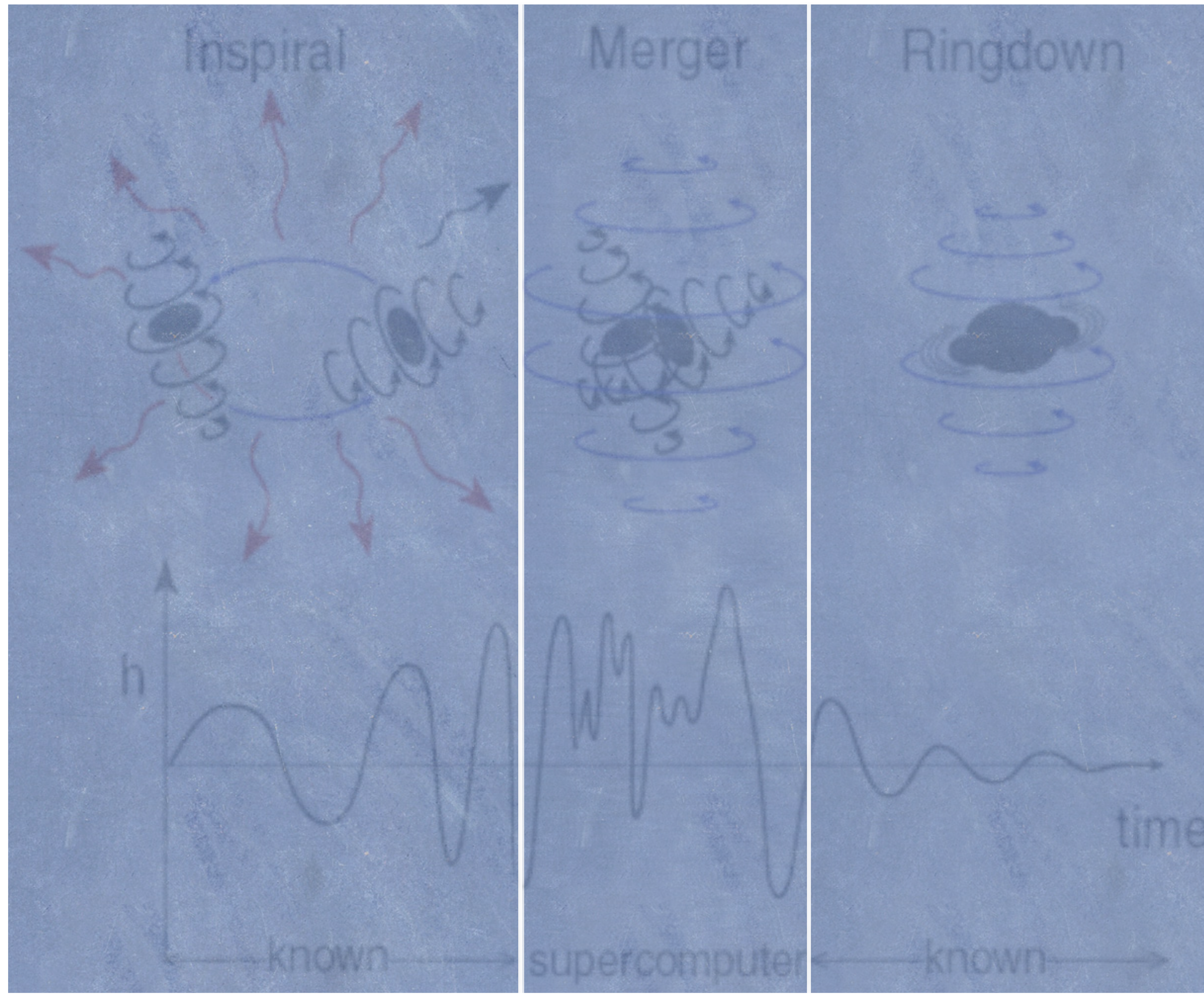


Black Hole Horizon

Baker, Psaltis, and Skordis (2015)

Fundamental Physics with Massive Black Hole Mergers

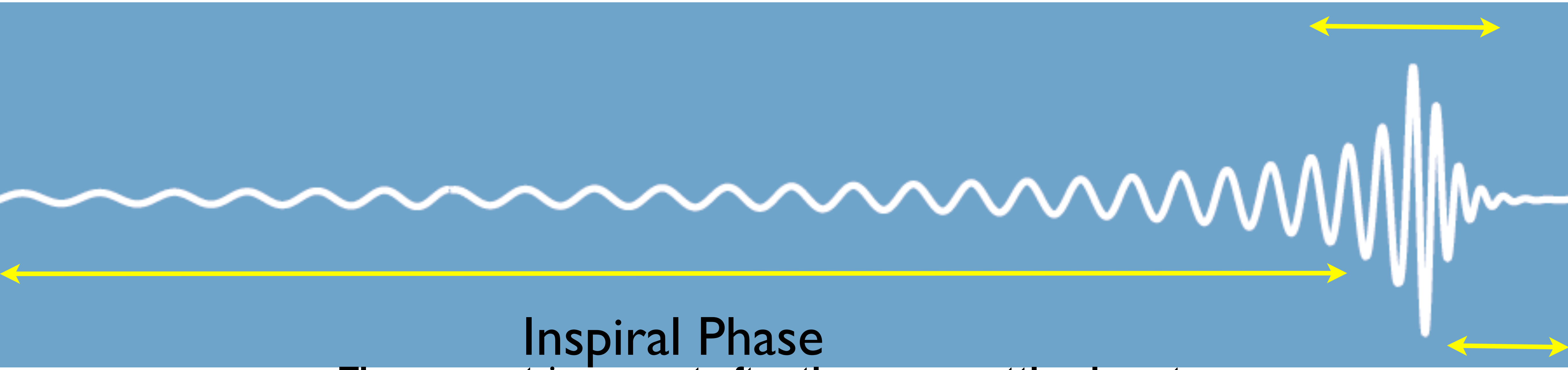
The system here resembles a perturbed single Black Hole. The evolution can be followed using BH perturbation theory (evolution of damped sinusoids, i.e. Quasi-normal modes).



From: Kip Thorne

Fundamental Physics with Massive Black Hole Mergers

- High precision measurements of Strong Gravity



Thousands of cycles at SNRs 50-1000.

The phase carries information about the propagation of the Gravitational Waves

Can test theories of gravity that predict massive gravitations, improving present bounds.

Short bursts of Gravitational Waves in the non-linear regime of General Relativity.

It is a very energetic event with a power corresponding to ~ 10^{22} times the power of the Sun.

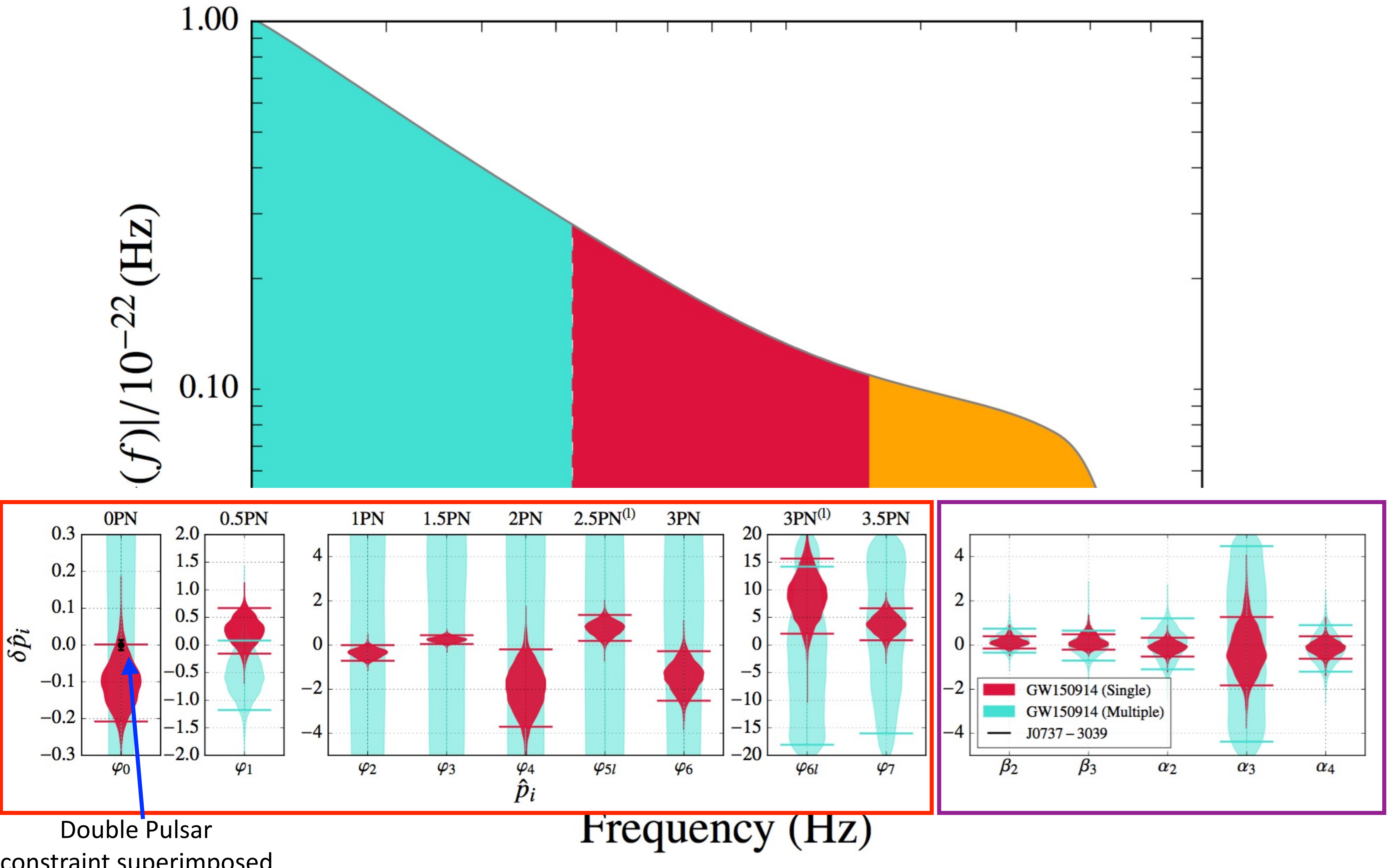
The asymmetric remnant after the merger settles down to a single (Kerr) Black Hole. In this “relaxation” process the system emits Gravitational Waves that are combinations of the QuasiNormal Modes (QNMs) of the final Black Hole.

The QNMs, according to General Relativity, only depend on the Mass and Spin of the Black Hole (*no hair* conjecture).

The identification of two QNMs provides a test of the geometry of Black Holes (are they really Kerr Black Holes?). The QNM spectrum is sufficiently rich to allow for distinction of different object. eLISA will measure several QNMs with sufficient SNR to carry out these tests.

Carlos F. Sopuerta (Institute of Space Sciences, ICE-CSIC & IEEC)

Tests of General Relativity with GW150914



Tests of General Relativity with GW150914

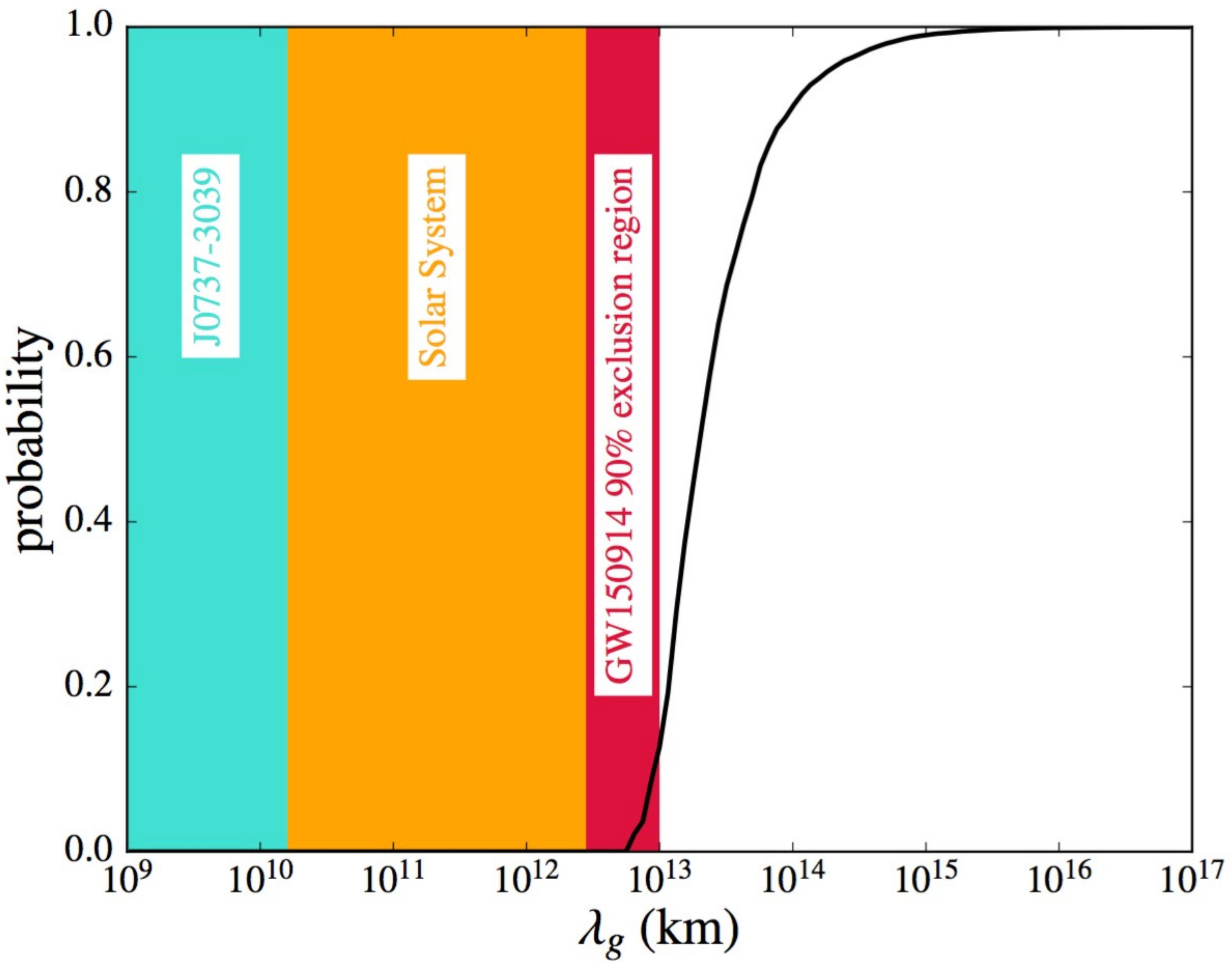


FIG. 8. Cumulative posterior probability distribution for λ_g (black curve) and exclusion regions for the graviton Compton wavelength λ_g from GW150914. The shaded areas show exclusion regions from the double pulsar observations (turquoise), the static Solar System bound (orange) and the 90% (crimson) region from GW150914.

- However, this is worse than limits from (model dependent) galaxy cluster and weak-lensing measurements

Extreme-Mass-Ratio Inspirals (EMRIs)

- In the present Universe star formation rate and AGN activity are declining. As a consequence, we observe a population of quiescent massive black holes. The current census comprises about 75 massive black holes out to $z < 0.03$ (e.g., Sgr* A in the Milky Way).

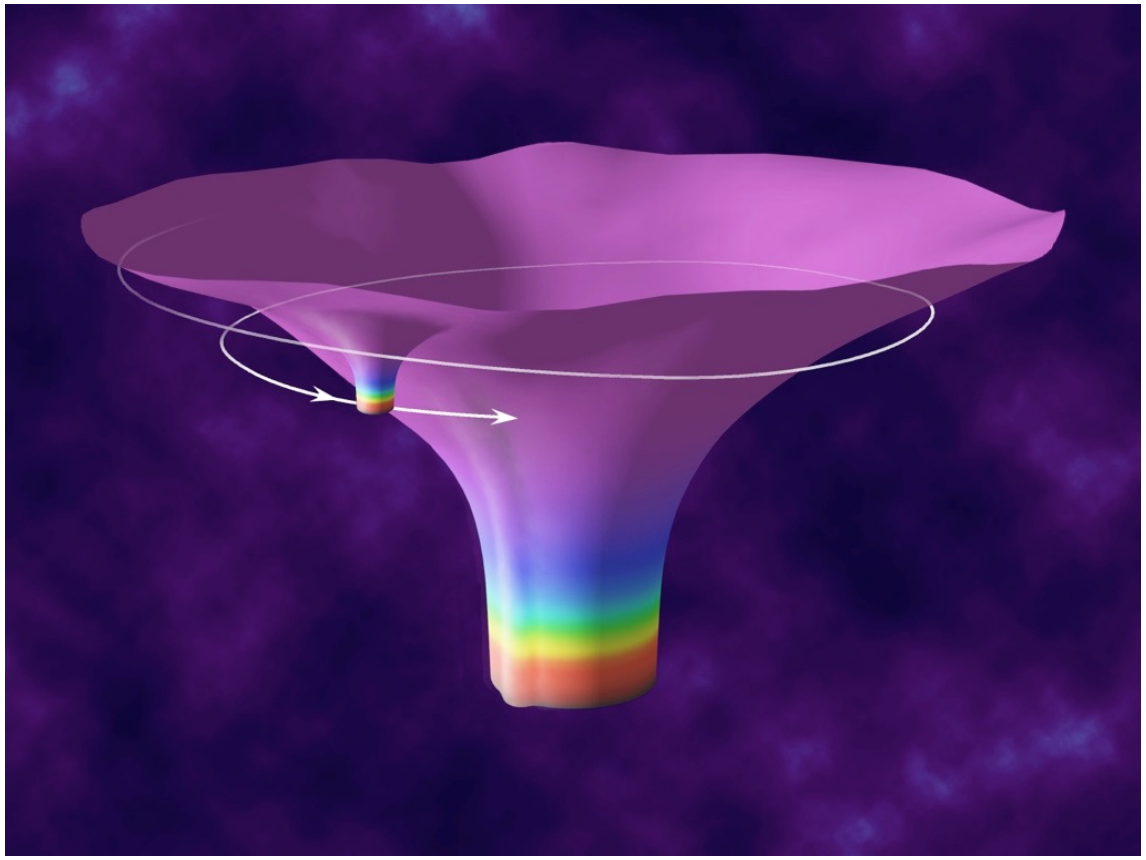
- The Milky Way Black Hole has a close young stellar population contrary to our expectations, as star-forming clouds are expected to be tidally disrupted.

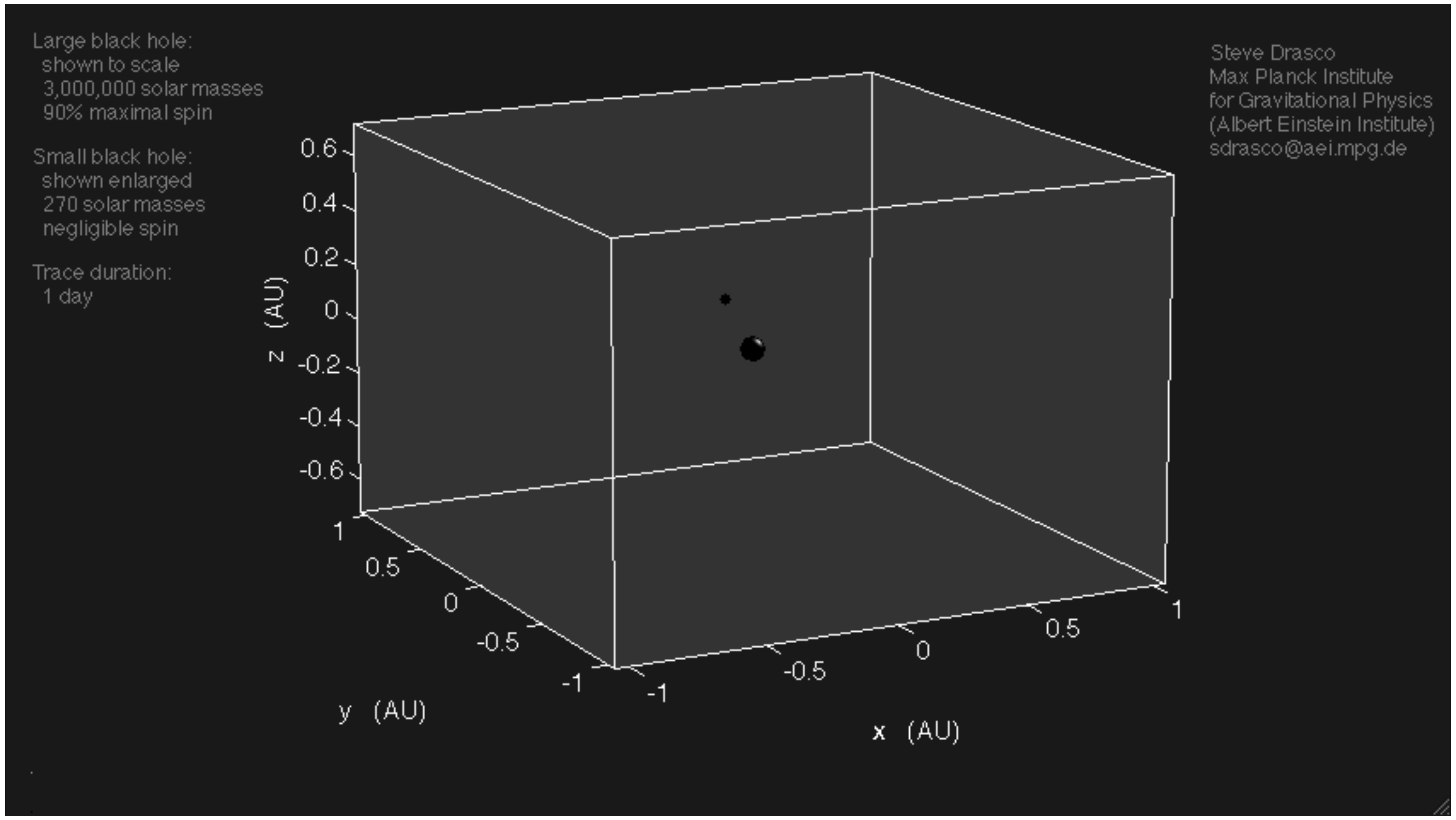
- LISA will probe the neighborhood of quiescent black holes using EMRIs: A compact star (a neutron star or a stellar mass black hole) captured in a highly relativistic orbit around the massive black hole and spiralling through the strongest field regions a few Schwarzschild radii from the event horizon before plunging into it

Extreme-Mass-Ratio Inspirals (EMRIs)

- LISA will detect EMRIs with an SNR > 20 in the mass interval for the central black hole between $10^4 < M/M_\odot < 5 \times 10^6$ out to redshift $z \sim 0.7$, covering a co-moving volume of 70 Gpc^3, a much larger volume than current observations of dormant galactic nuclei.

- The signals are long lasting (1-2 yrs) so that the SNR is built up as the contribution of many cycles ($\sim 10^5$ cycles during the last year before plunging into the central black hole).





Extreme-Mass-Ratio Inspirals (EMRIs)

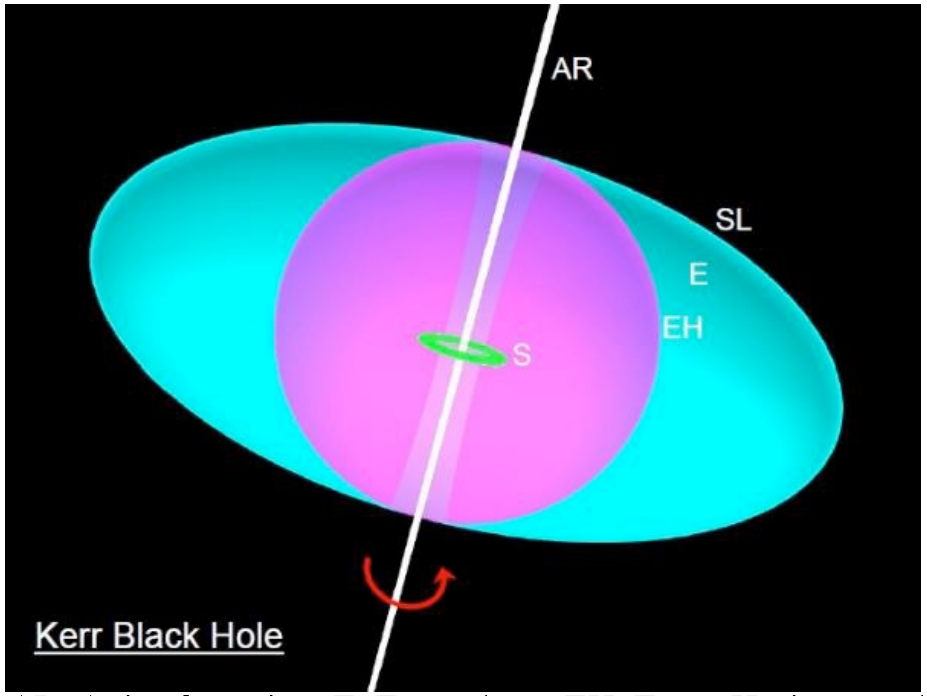
- Long and complex signal that carries very precise information about the gravitational multipole moments of the central Black Hole.

- LISA measurement of EMRI signals will provide the best tests of deviations from the Kerr geometry of General Relativistic Black Holes (boson stars, gravastars, etc.).

LISA measurement of EMRI signals will also test alternative theories of gravity like Scalar-Tensor theories, Chern-Simons, higher-dimensional models (brane-worlds), etc.

Testing Strong Gravity within General Relativity

- In General Relativity, provided the no-hair conjecture is true, black holes are described by the Kerr gravitational field/geometry, which is fully determined by the mass and spin (in the astrophysical scenario).



AR: Axis of rotation; E: Ergosphere; EH: Event Horizon, and SL: Static Limit; S: Singularity.

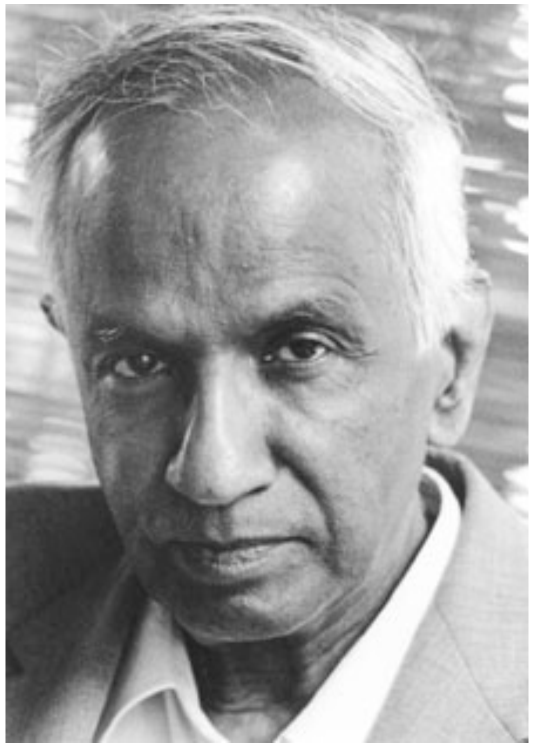
$$ds^2 \equiv g_{\mu\nu}\, dx^\mu\, dx^\nu = -\left(1 - \frac{2Mr}{\rho^2}\right)dt^2 + \rho^2\, d\theta^2 + \left(\frac{\rho^2}{\Delta}\right)dr^2 - \frac{4Mra}{\rho^2}\sin^2\theta\, d\phi\, dt + \left[(r^2 + a^2) + \frac{2Mra^2}{\rho^2}\sin^2\theta\right]\sin^2\theta\, d\phi^2$$

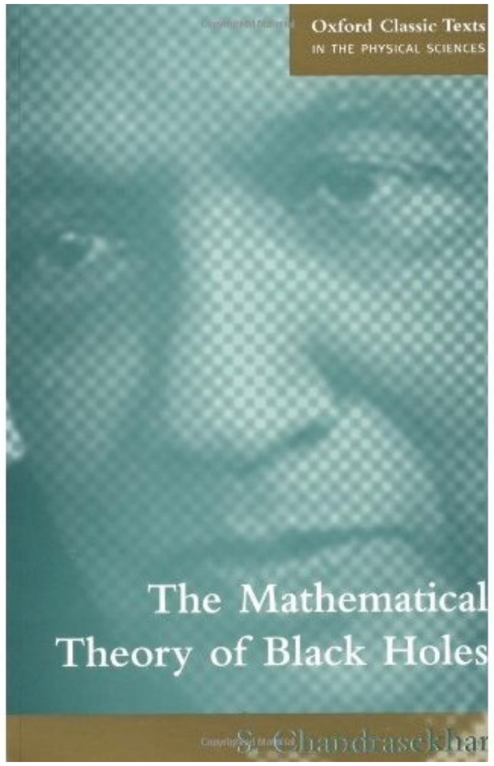
- The gravitational wave emission and the inspiral dynamics is fully determined by solutions of Einstein Equations.

$$G_{\alpha\beta} = \frac{8\pi G}{c^4}\, T_{\alpha\beta}$$

Testing Strong Gravity within General Relativity

Subrahmanyan Chandrasekhar (Physics Nobel Prize 1983):





"The black holes of nature are the most perfect macroscopic objects there are in the universe. They are the most perfect almost by definition since the only elements in their construction are our concepts of space and time. And since the general theory of relativity provides only a single unique family of solutions for their description, they are the simplest objects as well"

Testing Strong Gravity within General Relativity

- The orbital motion is locally characterized by three fundamental frequencies:

f_r : Frequency of Radial Motion (from apoapsis to periapsis and back)

f_θ : Frequency of Polar Motion (Oscillations of the orbital plane)

f_φ : Frequency of motion around the spin axis

- These frequencies evolve in time due to the gravitational backreaction (a.k.a. radiation reaction)

$$h_{+,\times}(t) = \sum_{k,l,m} h^{(k,l,m)}_{+,\times} e^{2\pi i \left[k f_r(t) + l f_\theta(t) + m f_\varphi(t)\right]}$$

Testing Strong Gravity within General Relativity

- With all these assumptions we can to detect EMRIs and determine their physical parameters. The parameter error estimations are [Barack & Cutler, PRD, **69**, 082005 (2004)]

$$\Delta(\ln M_\bullet)\,, \quad \Delta(\ln m_\star)\,, \quad \Delta\left(\frac{S_\bullet}{M_\bullet^2}\right) \sim 10^{-4}$$

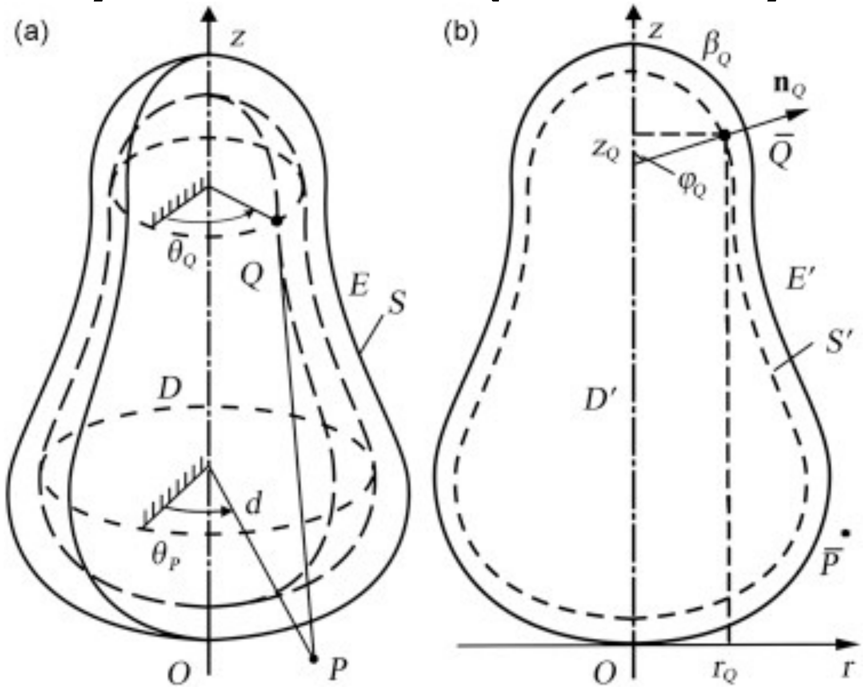
$$\Delta e_0 \sim 10^{-(3-4)}\,, \quad \Delta\left(\ln\frac{m_\star}{D_\mathrm{L}}\right) \sim 10^{-(1-2)}$$

- For EMRIs with:

$$M_\bullet = 10^6 M_\odot\,, \quad m_\star = 10 M_\odot\,, \quad \mathrm{SNR} \sim 30$$

Testing Strong Gravity within General Relativity

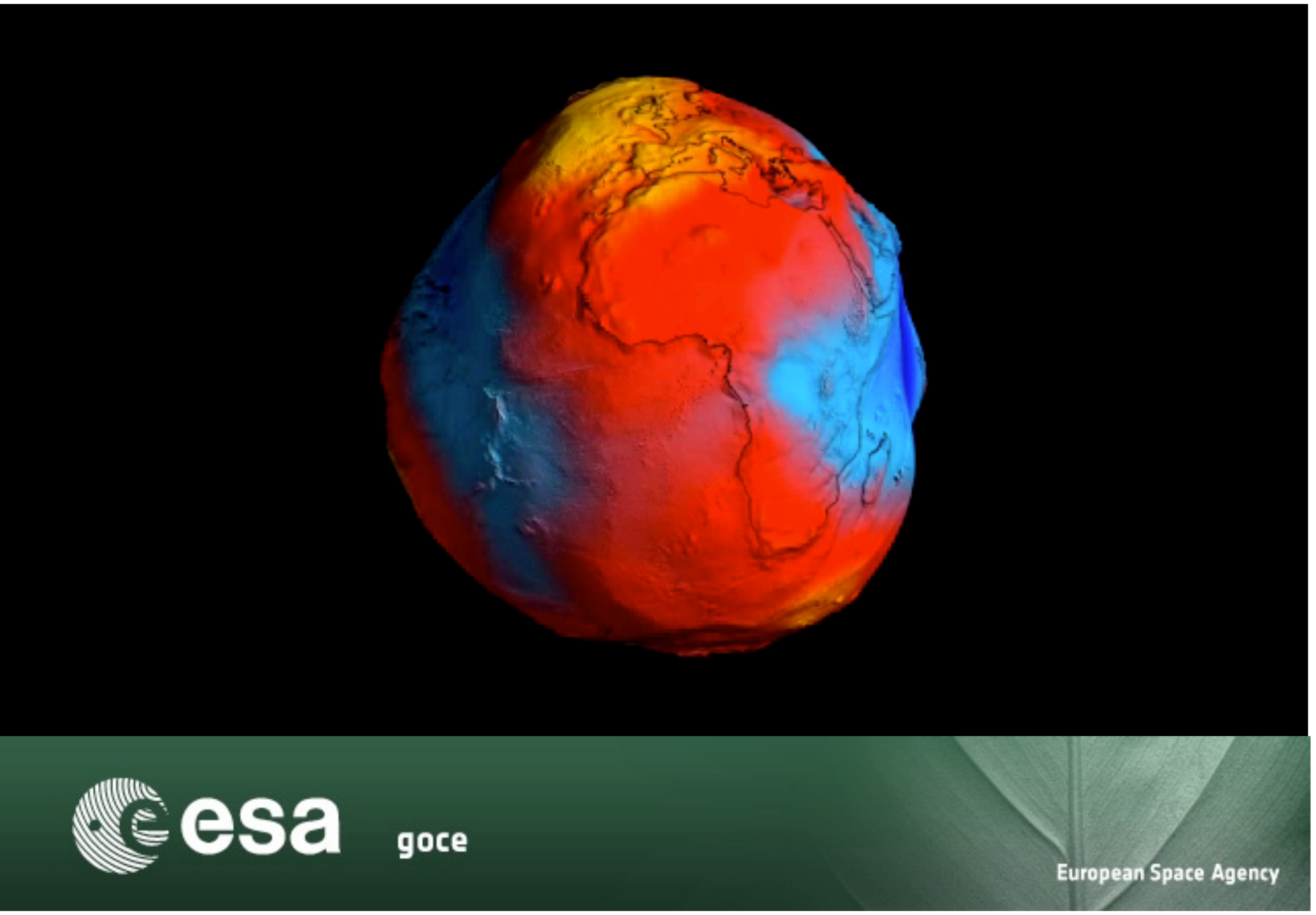
- Within General Relativity we can test the Kerr Hypothesis: "The dark, compact and very massive objects sitting at the galactic centers are stationary BHs described by the Kerr solution of General Relativity".
- But to perform such tests, we need to consider models that consider alternative geometries to Kerr.
- A quite general approach is to consider stationary and axisymmetric (and asymptotically flat) geometries.



- The exterior gravitational field of these objects is completely determined by a set of multipole moments.

Testing Strong Gravity within General Relativity

- For Earth's gravitational field:



$$V(\vec{r}) = -G \sum_{\ell, m} \frac{M_{\ell m}}{r^{\ell+1}} \, Y_{\ell m}(\theta, \varphi)$$

$M_{\ell m}$: Multipole moments

GOCE can measure up to

$\ell_{\text{MAX}} \sim 200$

- For a Kerr BH in GR: There are two sets of moments:

$$M_\ell + i\, J_\ell = M_\bullet \left(i \frac{S_\bullet}{M_\bullet c} \right)^\ell$$

- Different measurements of multipole moments provide different tests of the Kerr hypothesis.

Kerr Hypothesis: "The exterior gravitational field of the dark, compact and very massive objects sitting at the galactic centers can be well described by the vacuum, stationary, and axisymmetric solutions of General Relativity whose multipole moments satisfy the Kerr relations".

Testing Strong Gravity within General Relativity

- This program was pioneered by Ryan [Ryan, PRD, **52**, 5707 (1995); **56**, 1845 (1997)]

TABLE IV. The error $\delta\theta^i$ for each parameter θ^i, when fitting up to the $l_{\rm max}$th moment, using LISA. We use the abbreviation $L(\cdots)\equiv\log_{10}(\cdots)$. We assume $\mu=10M_\odot$, $M=10^5M_\odot$, $r=3M$, and $S/N=100$.

$l_{\rm max}$	$L(\delta t_*/\text{sec})$	$L(\delta\phi_*)$	$L(\delta\mu/\mu)$	$L(\delta M/M)$	$L(\delta s_1)$	$L(\delta m_2)$	$L(\delta s_3)$	$L(\delta m_4)$	$L(\delta s_5)$	$L(\delta m_6)$	$L(\delta s_7)$	$L(\delta m_8)$	$L(\delta s_9)$	$L(\delta m_{10})$
0	0.74	−0.25	−5.90	−5.80										
1	1.71	0.33	−4.92	−4.70	−4.53									
2	2.37	1.41	−4.17	−4.01	−3.89	−2.82								
3	2.73	2.52	−3.49	−3.28	−2.94	−2.27	−1.66							
4	4.54	3.80	−2.40	−2.23	−1.97	−0.81	0.07	0.72						
5	5.99	4.74	−0.73	−0.55	−1.12	0.47	0.59	0.95	2.35					
6	6.05	4.87	−0.68	−0.50	−1.00	0.54	0.94	1.53	2.35	2.58				
7	6.07	4.88	−0.66	−0.48	−0.99	0.56	0.94	1.53	2.35	2.81	2.68			
8	6.07	4.88	−0.65	−0.47	−0.98	0.57	0.96	1.56	2.35	2.81	3.16	3.68		
9	6.08	4.91	−0.65	−0.47	−0.96	0.58	1.04	1.67	2.35	2.82	3.20	3.74	4.10	
10	6.08	4.92	−0.64	−0.46	−0.95	0.58	1.05	1.69	2.35	2.82	3.20	3.75	4.17	4.70

- This uses quasi-circular and quasi-equatorial orbits. The conclusion is that a LISA-like detector may be able to estimate 3-5 moments (1-3 tests of the Kerr hypothesis).

Testing Strong Gravity within General Relativity

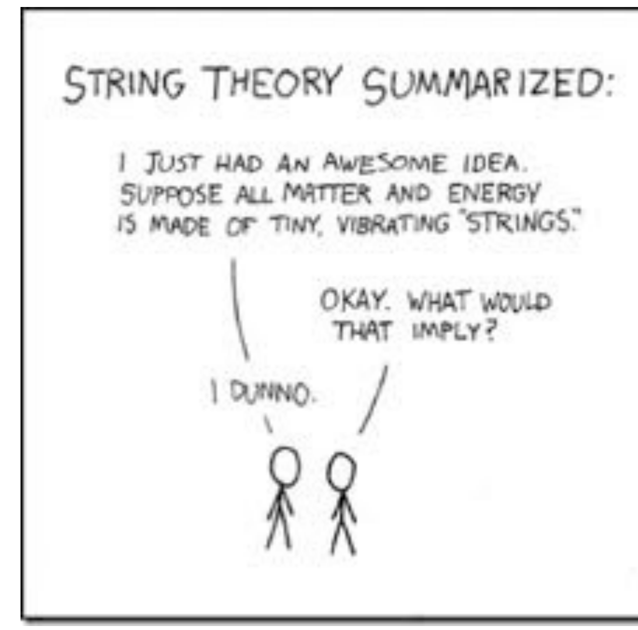
- Barack & Cutler extended their study [PRD, **75**, 042003 (2007)] to consider a central object with a mass quadrupole. The error estimations for this parameter (using generic orbits) are in the range:

$$\Delta\left(\frac{M_2}{M_0^3}\right) \sim 10^{-(2-4)}$$

which is a considerably better error estimate than Ryan's estimate.

Testing Strong Gravity beyond General Relativity

- Again, to test General Relativity, we must use models that consider non-GR dynamics for EMRIs.
- The Landscape of Theories of Gravity is very rich...



- Not all theories are suitable to consistently study EMRIs.

Testing Strong Gravity beyond General Relativity

- EMRIs in Scalar-Tensor theories [Yunes, Pani & Cardoso, arXiv:1112.3351]: Radiation of the scalar field can produce significant changes with respect to GR (floating orbits).

- In [Gair & Yunes, PRD, **84**, 064106 (2011)] EMRI waveforms a la Barack & Cutler for EMRIs with bumpy metric and modified gravity theories have been constructed.

Conclusions

Conclusions

- LIGO has inaugurated the Era of Gravitational Wave Astronomy in the High-frequency Band.

- There are great expectations for the direct detection of Gravitational Waves within the present decade (Ground Based detectors and PTAs).

- The L3 mission of ESA will be devoted to Gravitational Wave Astronomy in the Low-frequency Band. LISA Pathfinder has demonstrated the technology.

Many Thanks for your attention!