Gravitational Waves Data Analysis-II

Overview of continuous gravitational waves detection methods

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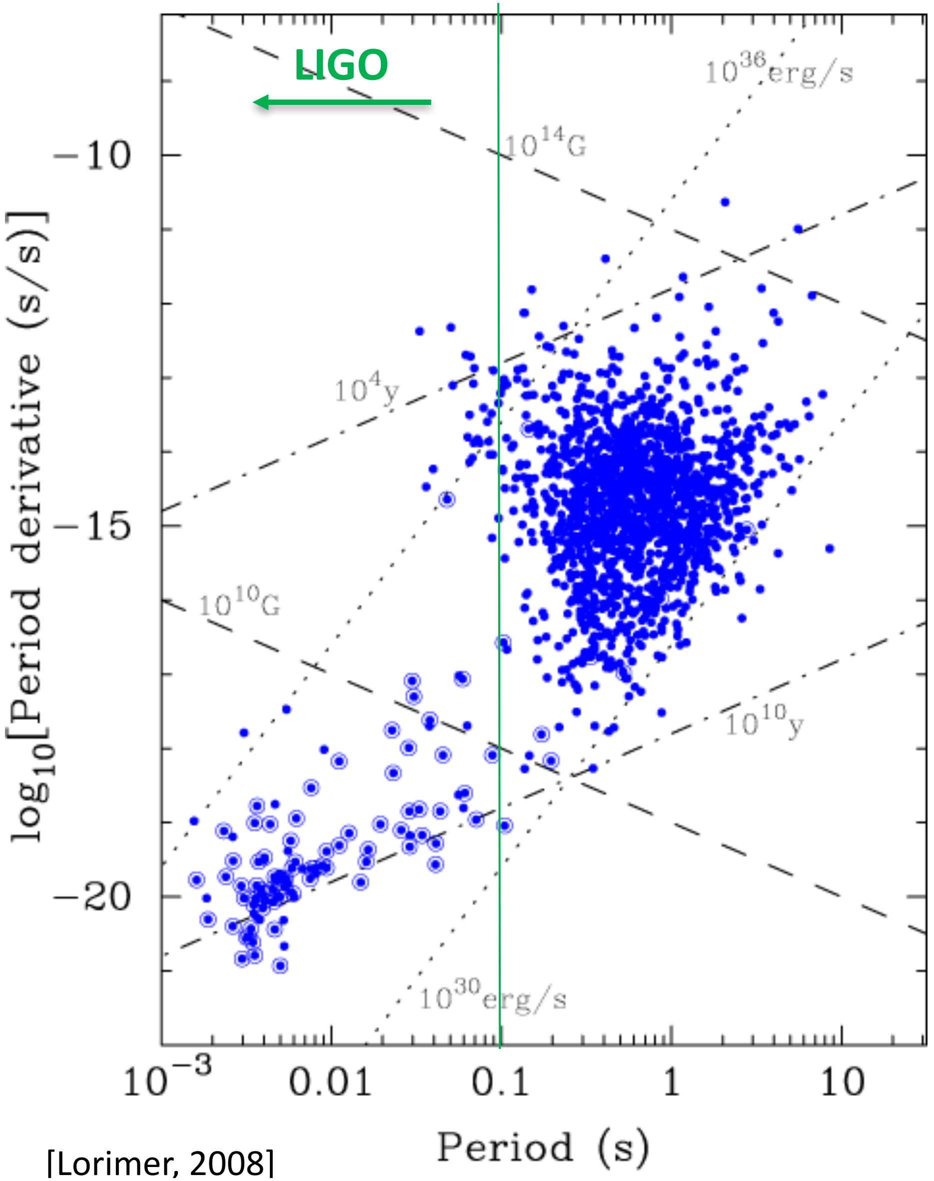
Outline-2

1. Introduction
2. LIGO and Virgo – Where do we stand?
3. Continuous Wave (CW) search assumptions
   - Emission mechanisms
   - Expected signal
   - Search categories
   - Challenges – coping with unknown source parameters
4. CW search methods
   - Sampling of methods and results to date
5. Search for signal of post-merger remnant from GW170817

No continuous gravitational waves (CW) detected… yet!

- Source emitting nearly monochromatic sinusoidal waves
- Signal is weak, but persistent over years of data taking
- Most searches begin after an observing run has ended, and all the data is in
- Still analysing LIGO O1 & O2 data
- Strict emission limits have already been set from initial detector era data on signals from known and unknown neutron star sources.

Searching CW signals



[Lorimer, 2008]

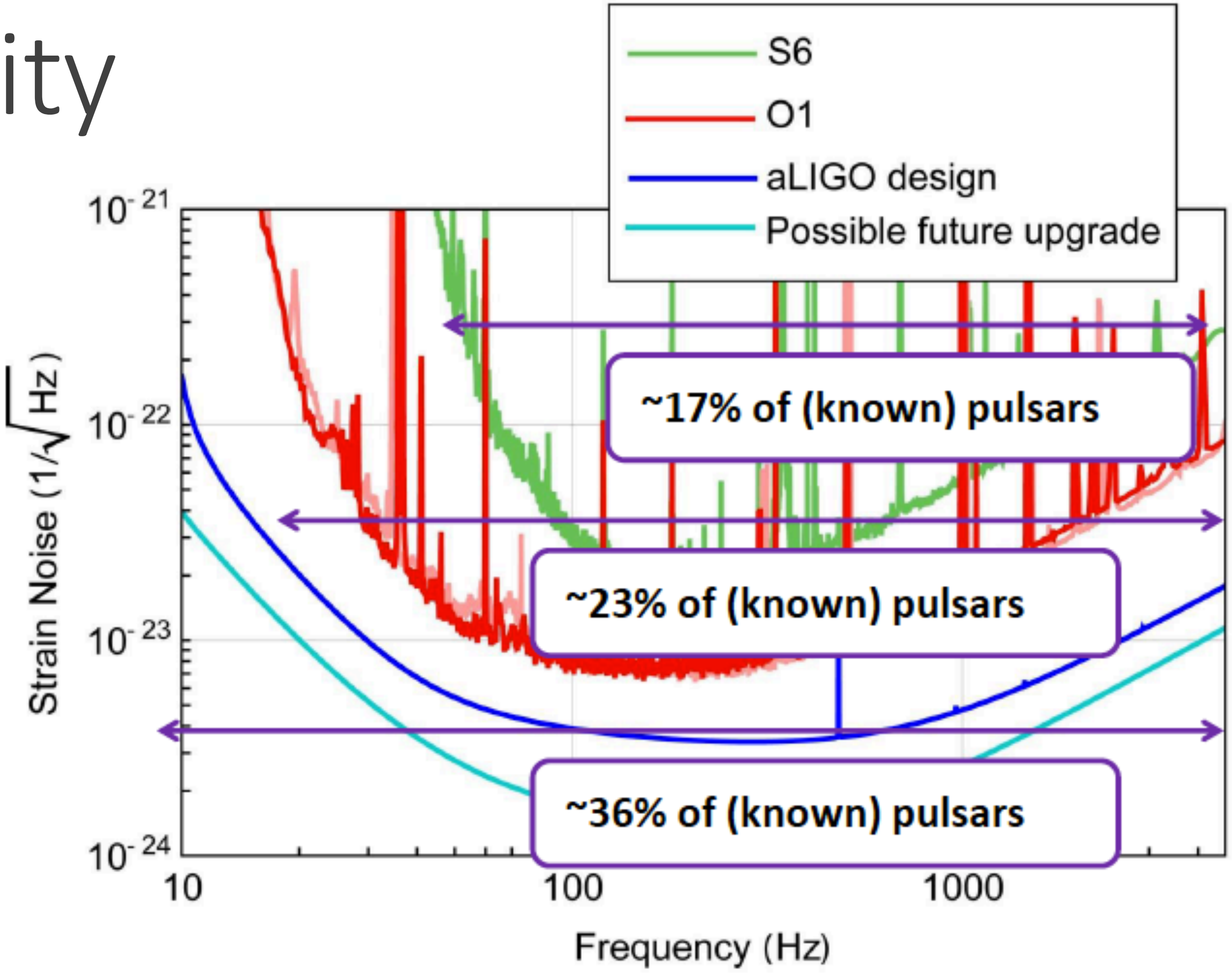
There are a few thousand known pulsars

40,000 millisecond pulsars in our galaxy
[Lorimer, Living Rev. Relativity, 11 2008]

O(10^6 – 10^7) undiscovered EM quiet NS within 5kpc [Narayan. *ApJ*, 1987]

Potential to discover off-axis pulsars or gravitars

aLIGO sensitivity



B.P. Abbott et al., *PRL* **116**:131103 (2016)

We have no idea how many signal detections to expect in first years of aLIGO/AdV.
Advocate for better modeling of neutron star deformation to enhance science case?

What can we learn?

- Great interest in detecting GW radiation: physics of such stars is poorly understood.
    - Interior properties not understood: equation of state, superfluidity, superconductivity, solid core, source of magnetic field.
    - May not even be neutron stars: could be made of strange matter!
- Constraints on:
    - NS EOS,
    - NS ellipticity,
    - NS mass

Implications for population models

Potential to discover gravastars, other compact objects

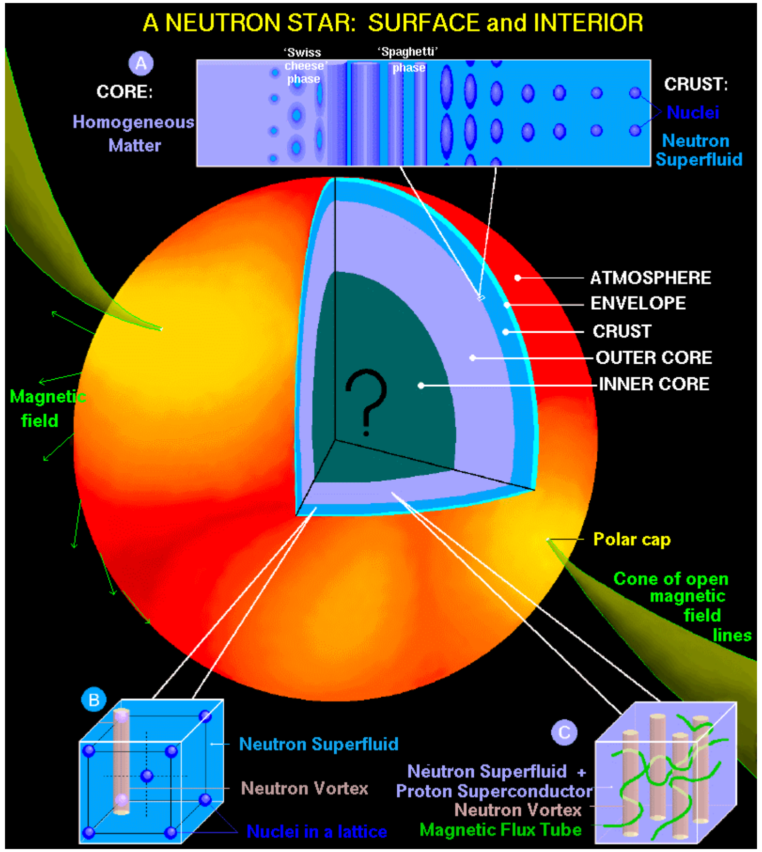


Image: Stony Brook University, J. M. Lattimer

What would we learn from a CW signal?

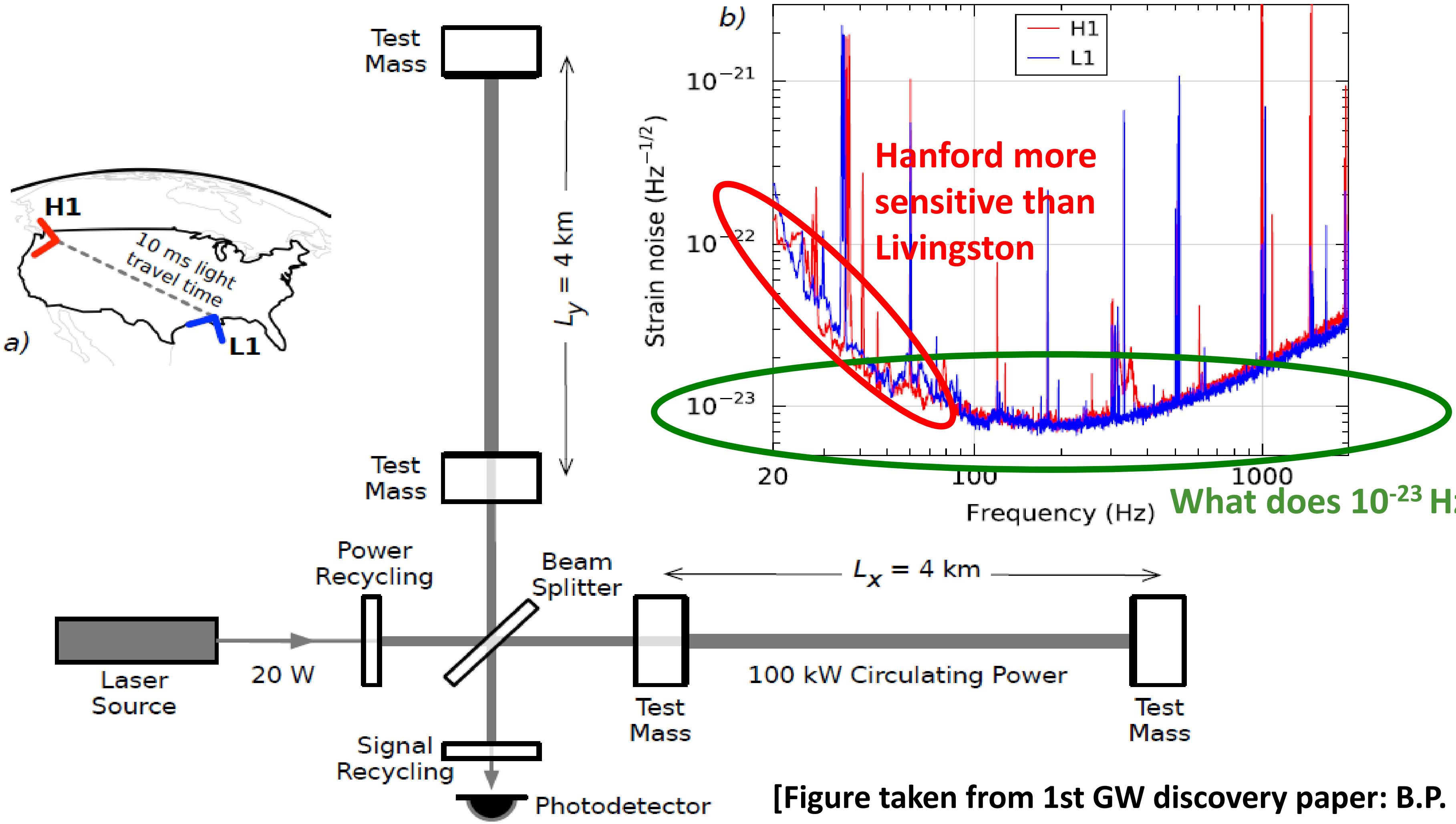
if accompanied by EM observation either from a known pulsar search or through follow-up of unknown sources

- $f_{GW} = 2f_{rot}$ star is probably a triaxial ellipsoid
- $f_{GW} \approx 2f_{rot}$ shows components producing EM and GW emission are not completely coupled (information on crust and core coupling of star?)
- $f_{GW} \approx f_{rot}$ precession plays important role in emission
- $f_{GW} \approx (4/3)\, f_{rot}$ emission from r-modes is favoured
(information on interior fluid motion of star)

Compare GW phase to EM pulse timing -- phase locking/drift?

Test General Relativity with a long-lived source.

O1 Data Run (Sept 12, 2015 – January 19, 2016)



What does 10^{-23} $Hz^{-1/2}$ mean?

[Figure taken from 1st GW discovery paper: B.P. Abbott *et al.*, PRL 116, 061102 (2016)]

Translating 1-side amplitude spectral noise density (ASD) to GW signal amplitude sensitivity:

(back of the envelope level)

Detectable signal amplitude ≈ Amplitude spectral density ÷ [Coherent observation time]$^{1/2}$ × Statistical / geometry trials factor

Examples:

BBH like GW150914:

10^{-23} Hz$^{-1/2}$ / [0.2 s]$^{1/2}$ × ~10 ≈ **several × 10^{-22}**

Actual amplitude = 10^{-21} → Very loud signal

CW signal from known pulsar measured coherently over 1 year:

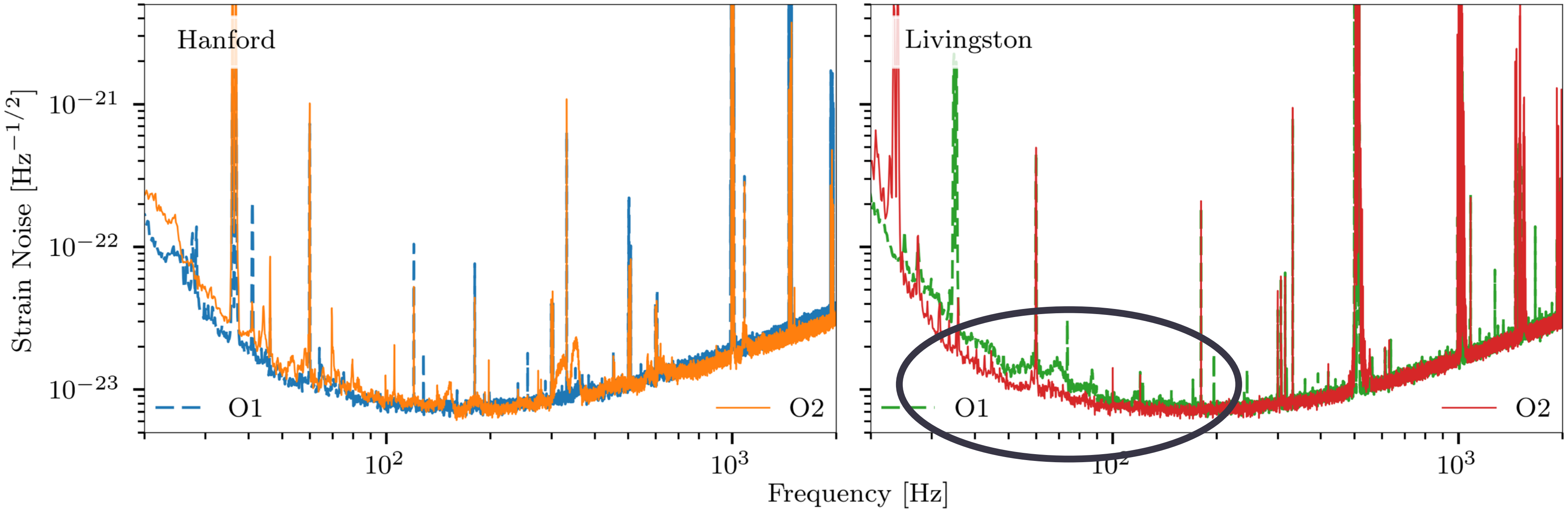
10^{-23} Hz$^{-1/2}$ / [3 × 10^{7} s]$^{1/2}$ × ~10 ≈ **2 × 10^{-26}**

In practice, most detectable CW source amplitudes lie (currently) in **10^{-25}-10^{-24}** because of large trials factors

The O2 Run (November 30, 2016 – August 25, 2017)

→Livingston more sensitive in O2 than in O1 ☺

→Hanford less sensitive ☹



B.P. Abbott *et al*, PRL **118**, 221101 (2017)

This band helpful for detecting binary black holes – and young pulsars

CW Search Assumptions

The LIGO and Virgo joint CW search group fields a diverse suite of programs to cover various source scenarios

Diversity applies to source types:

- Isolated neutron stars
- Neutron stars in binary systems
- Neutron stars undergoing accretion
- Newborn neutron stars (including BNS post-merger remnants)
- Glitching neutron stars
- Even black hole super-radiance

Diversity applies to algorithms:

- Matched filters
- Time domain heterodynes
- Semi-coherent power sums
- “Loose coherence” power sums
- Hough transforms
- Viterbi dynamical programming
- Sideband summing in orbital systems
- Double-Fourier Spectra

CW Search Assumptions – Emission Mechanisms

Radiation generated by quadrupolar mass movements:

$$h_{\mu\nu} = \frac{2G}{rc^4}\frac{d^2}{dt^2}\left[I_{\mu\nu}\right]$$

No GW from axisymmetric object rotating about symmetry axis

($I_{\mu\nu}$ = quadrupole tensor, r = source distance)

Spinning neutron star with equatorial ellipticity ε_{equat}

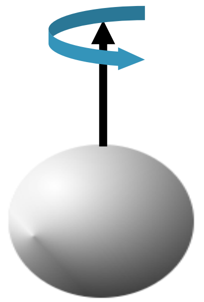
$$\varepsilon_{\mathrm{equat}} = \frac{|I_{xx} - I_{yy}|}{I_{zz}}$$

gives a strain amplitude h ($f_{GW} = 2 \bullet f_{Rot}$):

$$h = 1.1\times10^{-24}\left[\frac{kpc}{r}\right]\left[\frac{f_{GW}}{kHz}\right]^2\left[\frac{\varepsilon}{10^{-6}}\right]\left[\frac{I_{zz}}{10^{38}\ \mathrm{kg}\cdot\mathrm{m}^2}\right]$$

Courtesy: U. Liverpool

Emission mechanisms for CW



Bumpy Neutron Star

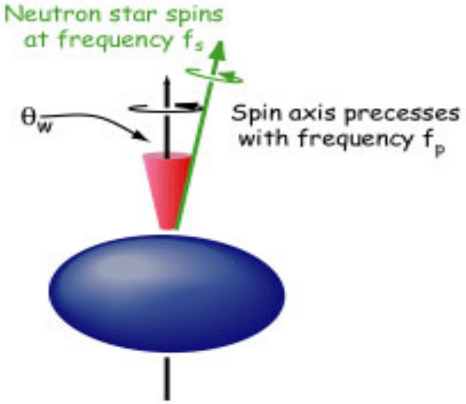
- **Equatorial ellipticity** **(e.g., – mm-high "bulge"):**

$$h \propto \varepsilon_{\text{equat}} \quad \text{with} \quad f_{GW} = 2 f_{\text{rot}}$$

- **Poloidal ellipticity** **(natural)** **+ wobble angle** **(precessing star).**

$$h \propto \varepsilon_{poloidal} \times \theta_{\text{wobble}} \quad \text{with} \quad f_{GW} = f_{\text{rot}} \pm f_{\text{precess}}$$

→ Precession due to different L and Ω axes



Wobbling Neutron Star

- **Two-component (crust+superfluid) →** $f_{\text{GW}} = f_{rot}$ and $2 f_{\text{rot}}$

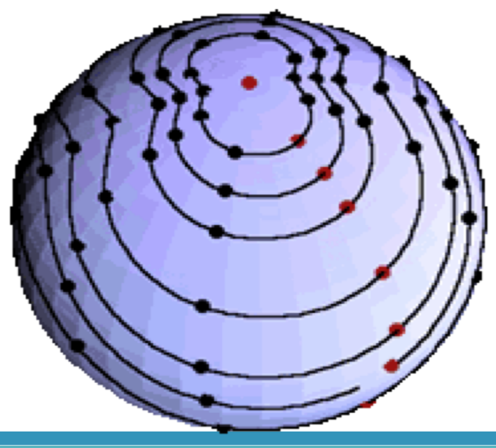
- **r modes** (rotational oscillations – CFS-driven instability)**:**

N. Andersson, ApJ 502 (1998) 708

S. Chandrasekhar, PRL 24 (1970) 611

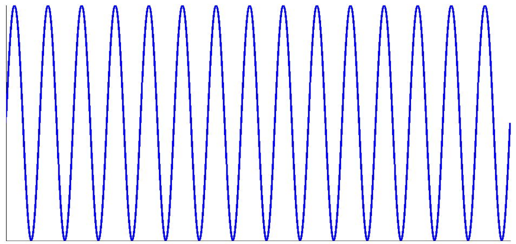
J. Friedman, B.F. Schutz, ApJ 221 (1978) 937

$$h \propto \alpha_{\text{r-mode}} \quad \text{with} \quad f_{GW} \cong \frac{4}{3} f_{\text{rot}}$$



The Signal from a NS

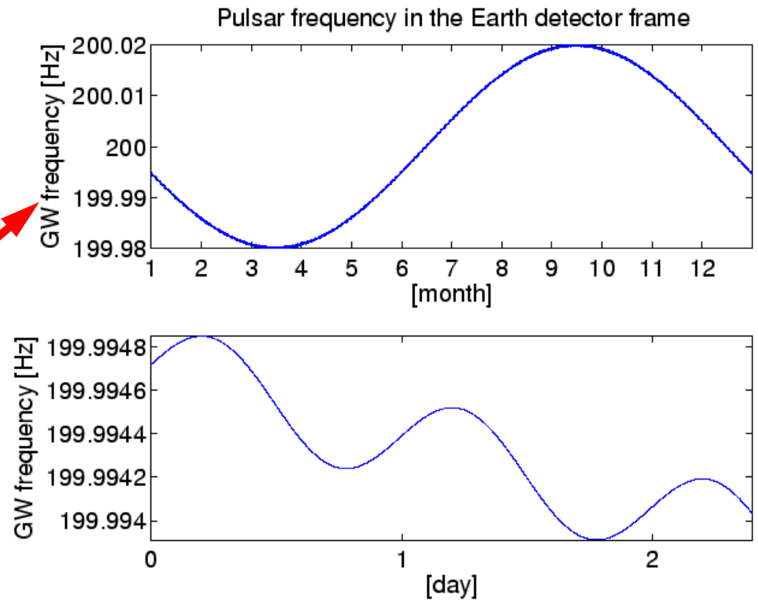
At the source



Spin-down ⇒ phase evolution

- Annual variation: up to ~10^{-4}
- Daily variation: up to ~10^{-6}

At the detector



Frequency modulation + amplitude modulation

… more complications for GW signals from pulsars in binary systems

Additional Doppler shift due to orbital motion of neutron star

Varying gravitational red-shift if orbit is elliptical

Shapiro time delay if GW passes near companion

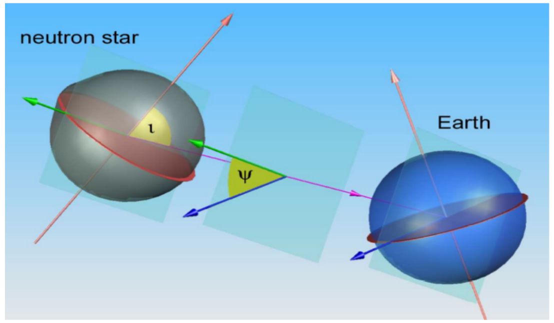
Expected Signal on Earth

For an isolated tri-axial *non-precessing* neutron star emitting at twice its rotational frequency:

$$h(\mathrm{t}) = \mathrm{F}_+(\mathrm{t};\psi)\mathrm{h}_0\left(\frac{1+\cos^2\iota}{2}\right)\cos\Phi(t) - \mathrm{F}_\times(\mathrm{t};\psi)\mathrm{h}_0\cos\iota\sin\Phi(t)$$

Signal parameters:

- ψ = polarization angle
- ι = inclination angle of source with respect to line of sight
- $\Phi(t) = \phi(t) + \phi_0$
- ϕ_0 = initial phase of pulsar
- h_0 = amplitude of the GW signal



$$h_0 = \frac{4\pi^2 G}{c^4}\frac{I_{zz}\varepsilon f_{gw}^{\ 2}}{d} \qquad \varepsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

equatorial ellipticity

Maximum deformations:

Normal NS: $\varepsilon < 10^{-5}$ [Johnson-McDaniel & Owen, 2013]

Hybrid: $\varepsilon < 10^{-3}$

Extreme quark stars: $\varepsilon < 10^{-1}$

Millisecond pulsars have spindown-implied values <u>lower than</u> 10^{-9}–10^{-6} ☹

Continuous Waves Searches

These searches have several parameters:

May be known, used in the search:

position
frequency,
frequency derivatives
orbital parameters

Not explicitly searched for:

initial phase
inclination angle
polarization
amplitude

Different algorithms have been developed, depending on what we know about the source: source parameters knowledge, waveform knowledge, isolated or binary system sources etc.

CW Search Categories (1)

Three broad categories of searches have dominated analysis:

Targeted searches for known pulsars using radio / X-ray / γ-ray ephemerides

→ Exact phase tracking – minimum trials factor (3 methods used)

→ Variation (“narrowband”) allows for EM/GW $\Delta f \sim O(10^{-3})\, f_{EM}$

Directed searches for known sources / locations (unknown / poorly known frequencies)

- Isolated and binary sources treated separately
- Fully coherent searches over days/weeks
- Semi-coherent searches over full data runs

All-sky searches for unknown sources

- Isolated and binary sources treated separately (binary esp. challenging)
- Semi-coherent searches over full data runs

➢ Methods:

Coherent methods (require accurate prediction of the phase evolution of the signal)

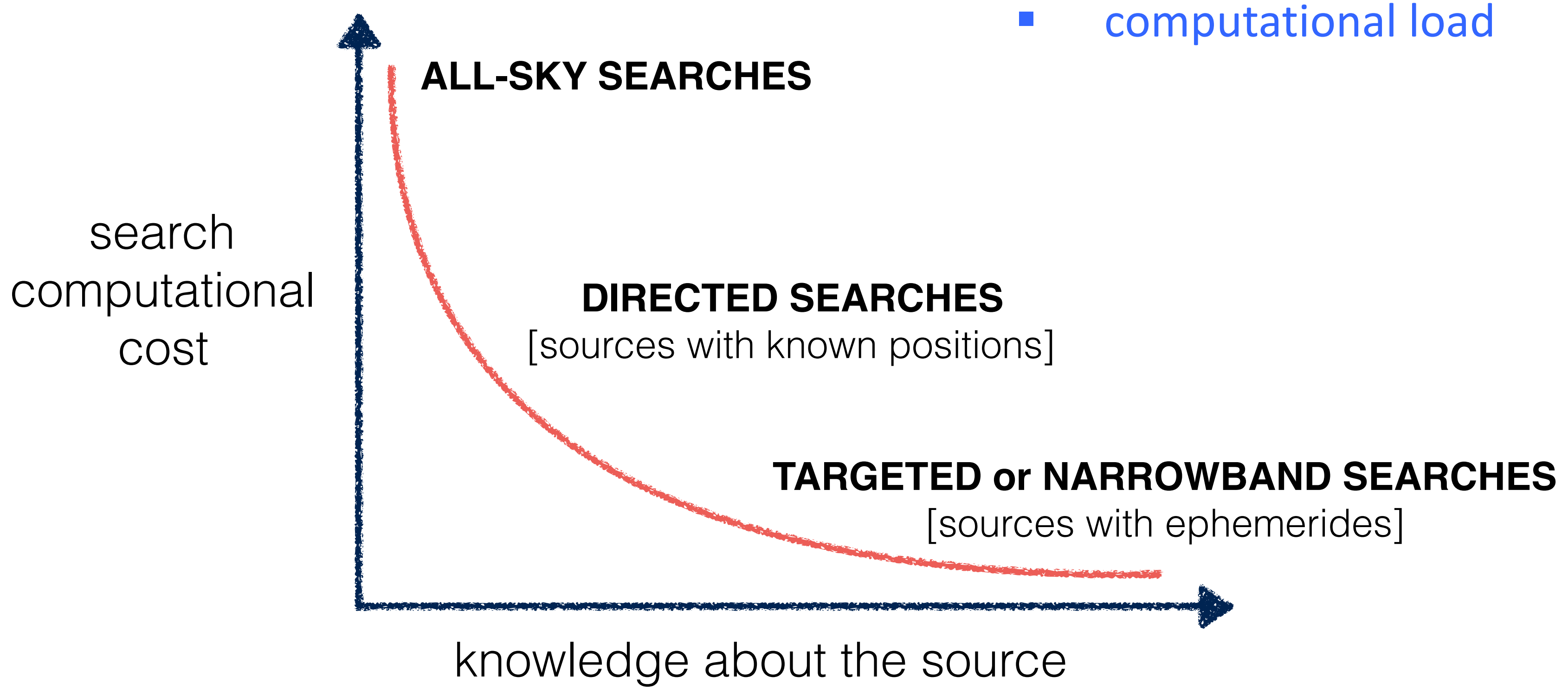
Semi-coherent methods (require prediction of the frequency evolution of the signal)

Hierarchical methods

➢ What drives the choice: The computational expense of the search

Types of CW searches

- The cornerstones of CW searches are:
  - sensitivity
  - robustness with respect to signal uncertainties and instrumental noise
  - computational load



CW Search Categories (2)

Two other categories receiving more attention:

Searches for "CW transients" (e.g., enhanced CW emission following known neutron star glitch)

→ Challenge: fewer constraints available to establish detection

Searches for newborn known neutron stars
(including short-lived post-BNS-merger remnants)

- → Challenge: rapid and poorly known frequency evolution
  (conventional Taylor expansion methods break down)
- → Advantage: turn-on of signal well defined

Computing Challenges – Directed Searches

CW searches could saturate Earth's computing – Why?

Assume a GW phase model using a Taylor expansion (source frame):

$$\Phi(t) = \phi_0 + 2\pi[f(T - T_0) + \frac{1}{2}\dot{f}(T - T_0)^2 + \frac{1}{6}\ddot{f}(T - T_0)^3 + \ldots]$$

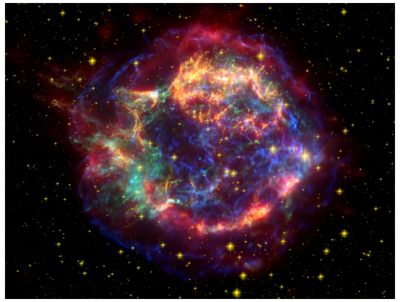
where f is the nominal signal frequency at reference time T_0 in the source frame, and T is the time of arrival of the GW signal at the solar system barycenter (SSB):

$$T = t + \delta t = t - \frac{\vec{r}_d \cdot \hat{k}}{c} + \Delta_E - \Delta_S$$

Where $\boldsymbol{r}_d$ is the detector position w.r.t. the SSB, $\boldsymbol{k}$ is the wave vector, Δ_E is the Einstein delay, and Δ_S is the Shapiro delay

Computing challenges – Directed Searches

Consider a coherent directed search for a precisely known location but unknown frequency, e.g., Cassiopeia A
(isolated X-ray source at center of supernova remnant)



To avoid cumulative phase mismatch $\Delta\Phi$ greater than, say, 0.5 radians, search templates used must have correct frequency parameters to within about

$$\Delta f \approx \frac{1}{T_{\mathrm{COH}}}, \qquad \Delta \dot{f} \approx \frac{2}{T_{\mathrm{COH}}^2}, \qquad \Delta \ddot{f} \approx \frac{6}{T_{\mathrm{COH}}^3}, \qquad \text{etc.}$$

Hence a search with long enough T_{COH} to require multiple steps in $\ddot{f}$ has a computational cost that scales as $(T_{\mathrm{COH}})^7$

→ Cost of search rises rapidly for long T_{COH} and young objects
→ Limits coherence time to O(10 days) for 300-year-old Cas A

It gets worse…

Computing challenges – All-sky Searches

Serious technical difficulty: Doppler frequency shifts

- Frequency modulation from earth's rotation (v/c ~ 10^{-6})
- Frequency modulation from earth's orbital motion (v/c ~ 10^{-4})

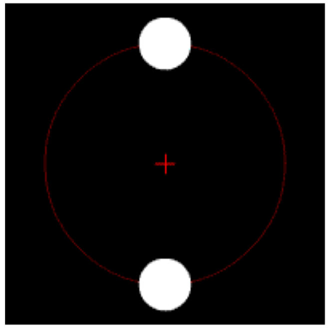
→Coherent integration of 1 year gives frequency resolution of 30 nHz

→1 kHz source spread over 6 million bins in ordinary FFT!

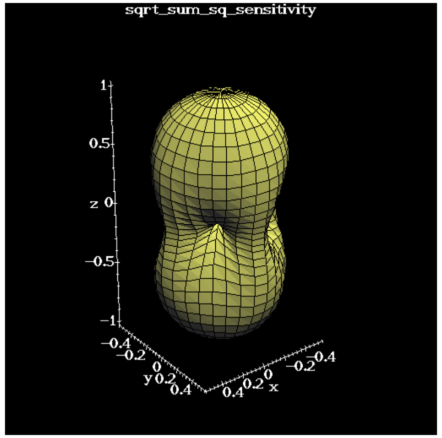
Additional, related complications:

Daily amplitude modulation of antenna pattern (polarization dependent)

Still have frequency derivatives to address...



Orbital motion of sources in binary systems



Computing challenges – All-sky Searches

Modulations / drifts complicate analysis enormously:

- Simple Fourier transform inadequate
- Every sky direction requires different demodulation
- →Number of distinct sky location scales like $f^2 T_{\mathrm{COH}}^2$
- →Computing cost scales as $(T_{\mathrm{COH}})^6$ without 2nd frequency derivative!
- All-sky survey at full sensitivity not possible (at this time)

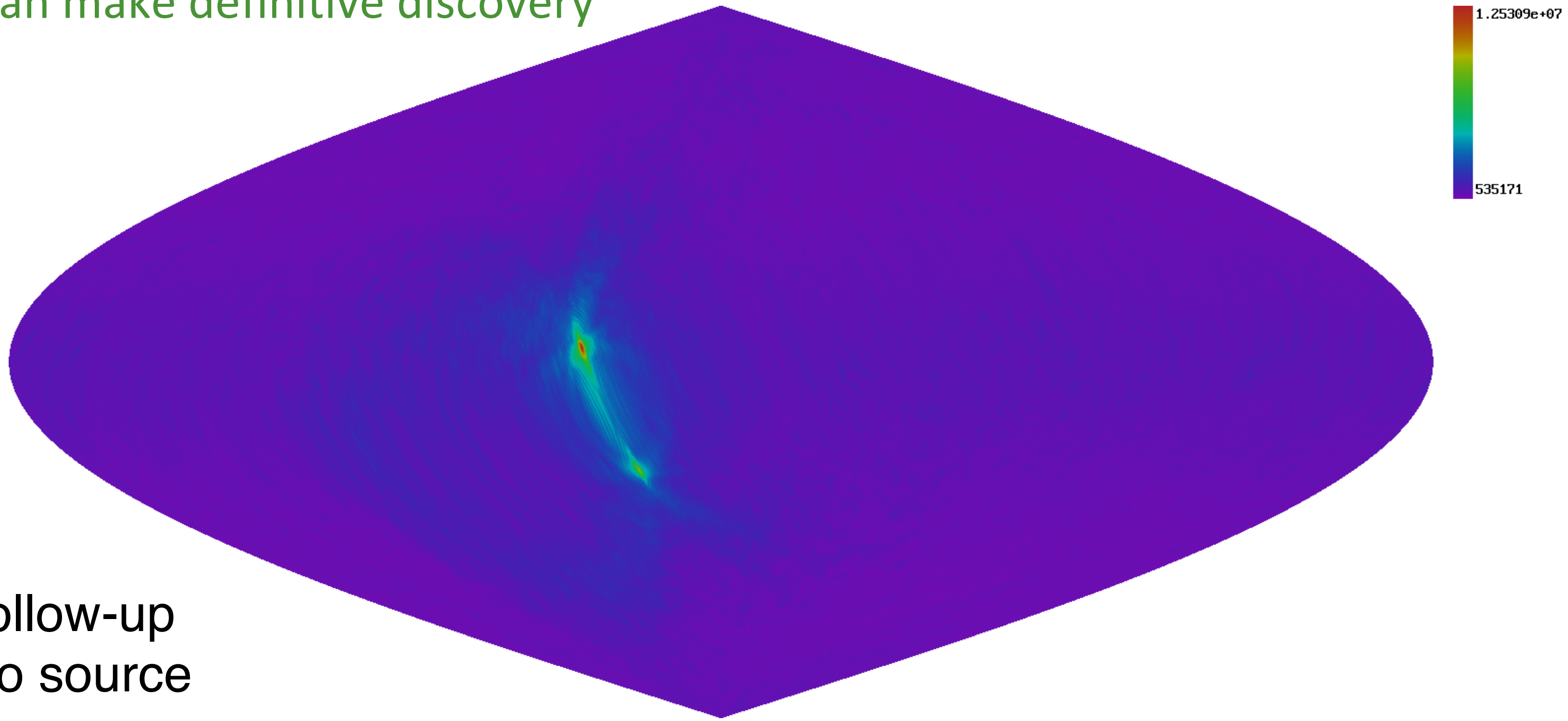
Tradeoffs to cope with astronomical computing costs:

- Restrict T_{COH} < several days for all-sky search
  → Compute demodulated spectra (e.g., "F-Statistic")
  → Exploit coincidence among different time segments
  → Sensitivity scales like $(T_{\mathrm{COH}})^{1/2}$
- Semi-coherent stacking ($N_{\mathrm{stack}} = T_{\mathrm{OBS\text{-}RUN}} / T_{\mathrm{COH}}$)
  - Raw spectral powers ($T_{\mathrm{COH}} \sim$ 0.5-2.0 hours)
  - Thresholded powers – Hough transforms
  - Demodulated spectra – F-Statistic

  → Sensitivity improves only as $(N_{\mathrm{stack}})^{1/4}$

But three substantial benefits from Doppler modulations:

- Reality of signal confirmed by need for corrections
- Corrections give precise direction of source
- Single interferometer can make definitive discovery



Can "zoom in" further with follow-up algorithms once we lock on to source

V. Dergachev, PRD 85 (2012) 062003
M. Shaltev & R. Prix, PRD 87 (2013) 084057 A. Singh *et al.*, PRD 96 (2017) 082003

Example sky map of strain power for signal injection (semi-coherent search)

Targeted Searches

Targeted searches use ephemerides from radio, X-ray, γ-ray pulsar astronomers (informal consortia of observers who co-author GW papers)

Three methods used to date:

- Time-domain heterodyne with Bayesian parameter estimation applied to 200 pulsars in O1 data
  [R. Dupuis & G. Woan, PRD 72 (2005) 102002]

- Matched-filter with marginalization over unknowns (F-Statistic & G-Statistic) applied to O(10) “high-value” known isolated pulsars
  [P. Jaranowski, A. Krolak & B. Schutz, PRD 58 (1998) 063001;
  P. Jaranowski & A. Krolak, CQG 27 (2010) 94015]

- Fourier-domain “5-vector” method exploiting carrier and amplitude-modulation sidebands applied to high-value targets
  [P. Astone et al., CQG 27 (2010) 194016;
  P. Astone et al., JPCS 363 (2012) 012038]

What is the "direct spindown limit"?

It is useful to define the "direct spindown limit" for a known pulsar, under the assumption that it is a "gravitar", i.e., a star spinning down due to gravitational wave energy loss

Unrealistic for known stars, but serves as a useful benchmark

Equating "measured" rotational energy loss (from measured period increase and reasonable moment of inertia) to GW emission gives:

$$h_{SD} = 2.5\times10^{-25}\left[\frac{kpc}{d}\right]\sqrt{\left[\frac{1kHz}{f_{\mathrm{GW}}}\right]\left[\frac{-df_{\mathrm{GW}}/dt}{10^{-10}\,Hz/s}\right]\left[\frac{I}{10^{45}\,g\cdot cm^2}\right]}$$

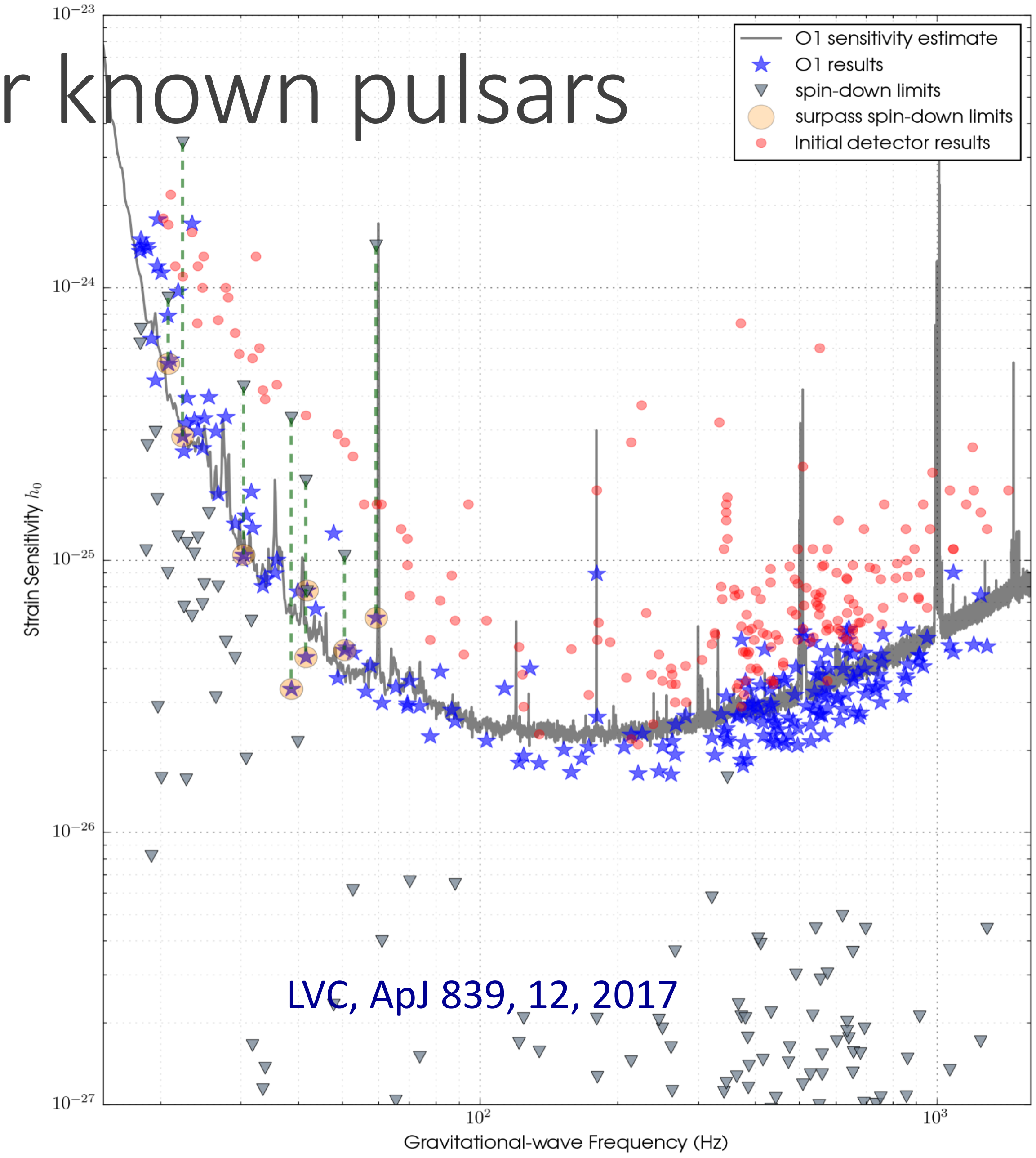
Example:

Crab → $\mathbf{h_{SD} = 1.4 \times 10^{-24}}$

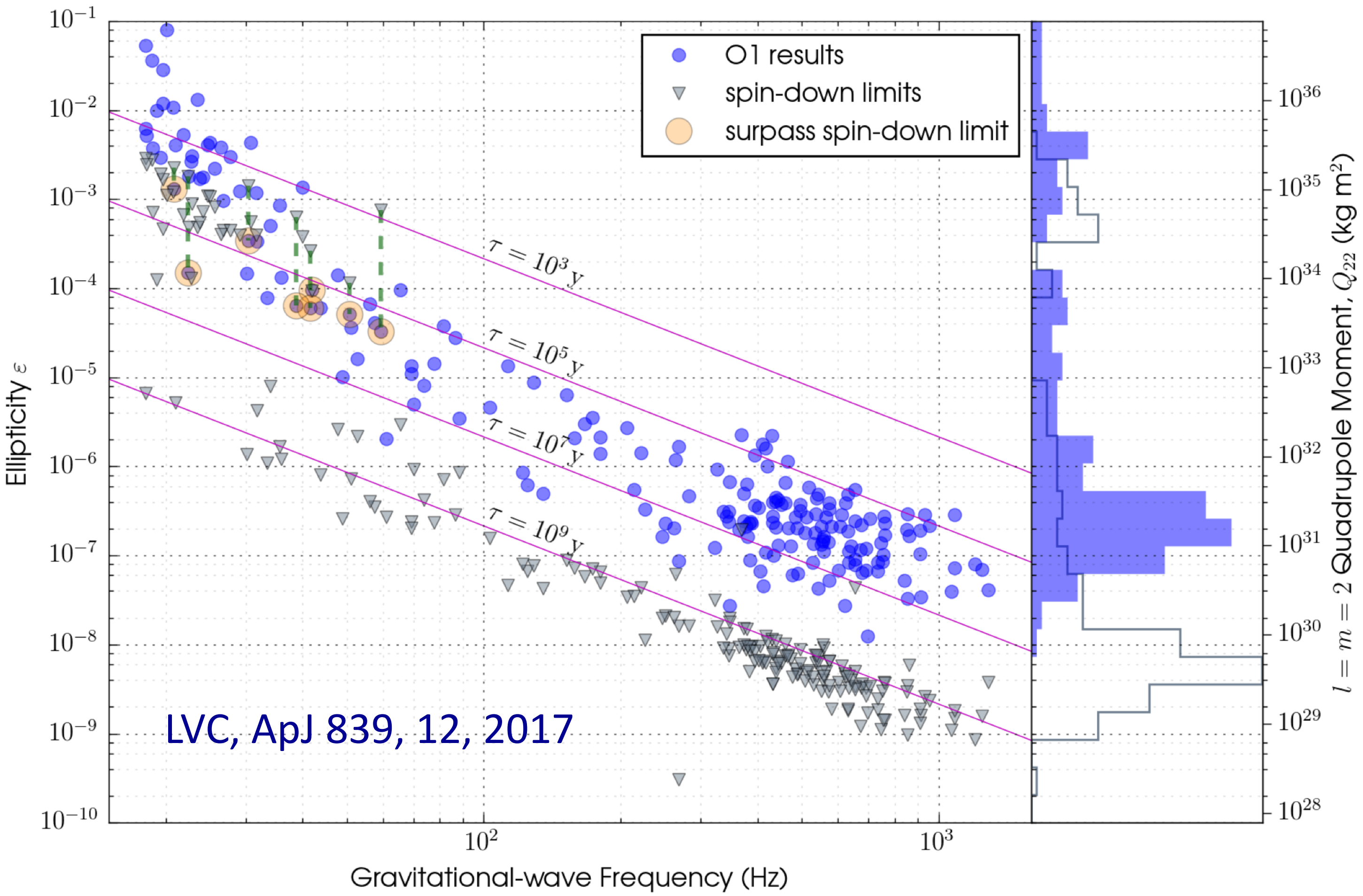
(d=2 kpc, f_{GW} = 59.5 Hz, $df_{GW}/dt = -7.4\times10^{-10}$ Hz/s)

O1 targeted search for known pulsars

- ✧About 430 known pulsars in Advanced LIGO band (~20-2000 Hz). 200 known pulsars analyzed,
- ✧11 high-value targets using 3 pipelines: TD Bayesian, TD $\mathcal{F}/\mathcal{G}$-Stat and FD 5-vector method, rest with the TD Bayesian method.
- ✧Spin-down limit beaten for 8 pulsars, including Crab & Vela:
  - ✧Crab: less than 2 x 10^{-3} $\dot{E}_{rot}$ in GW, ~10 cm deformation
  - ✧Vela: less than 10^{-2} $\dot{E}_{rot}$. *ellipticity->* ~50 cm
- ✧PSR J1918-0642: smallest UL h_0 = 1.6 x 10^{-26}.
- ✧Most constraining ellipticity is 1.3 x 10^{-8} for J0636+5129



LVC, ApJ 839, 12, 2017

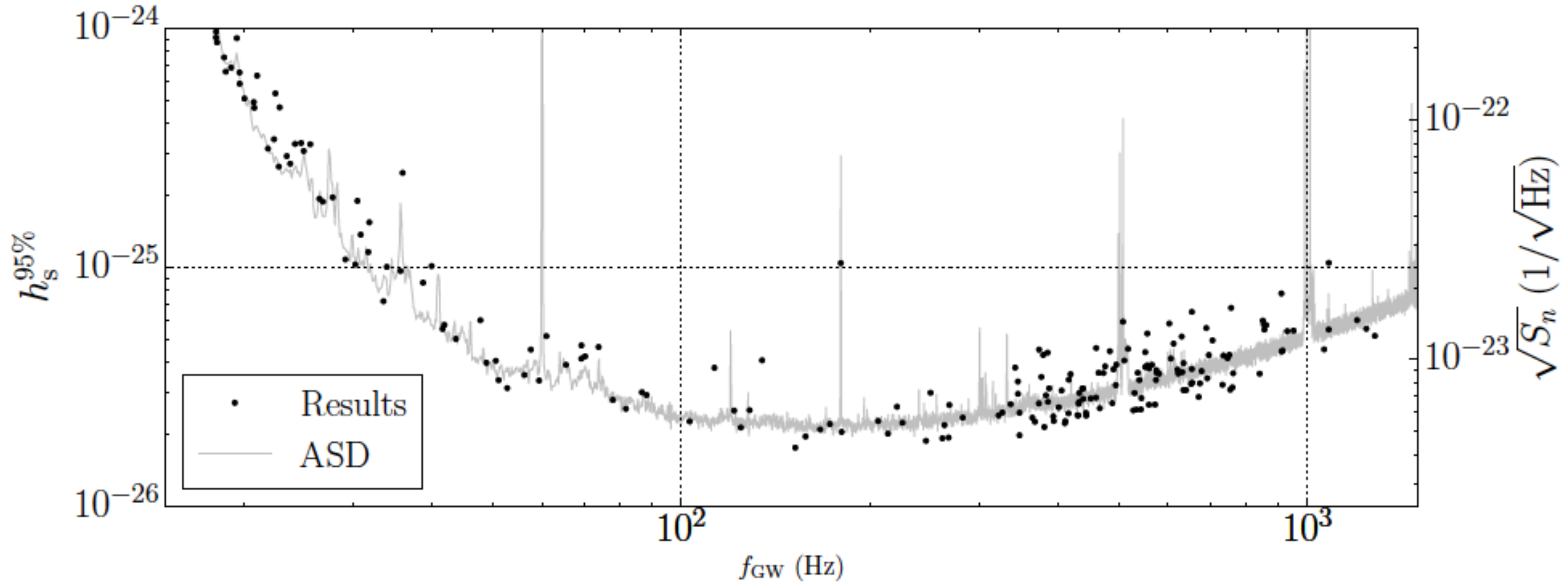


The ellipticity can roughly be converted to a "mountain" size using 25×(ε/10^{-4}) cm
The diagonal lines show the ellipticities that would be required if a star had a particular characteristic age and was losing energy purely through gravitational radiation.

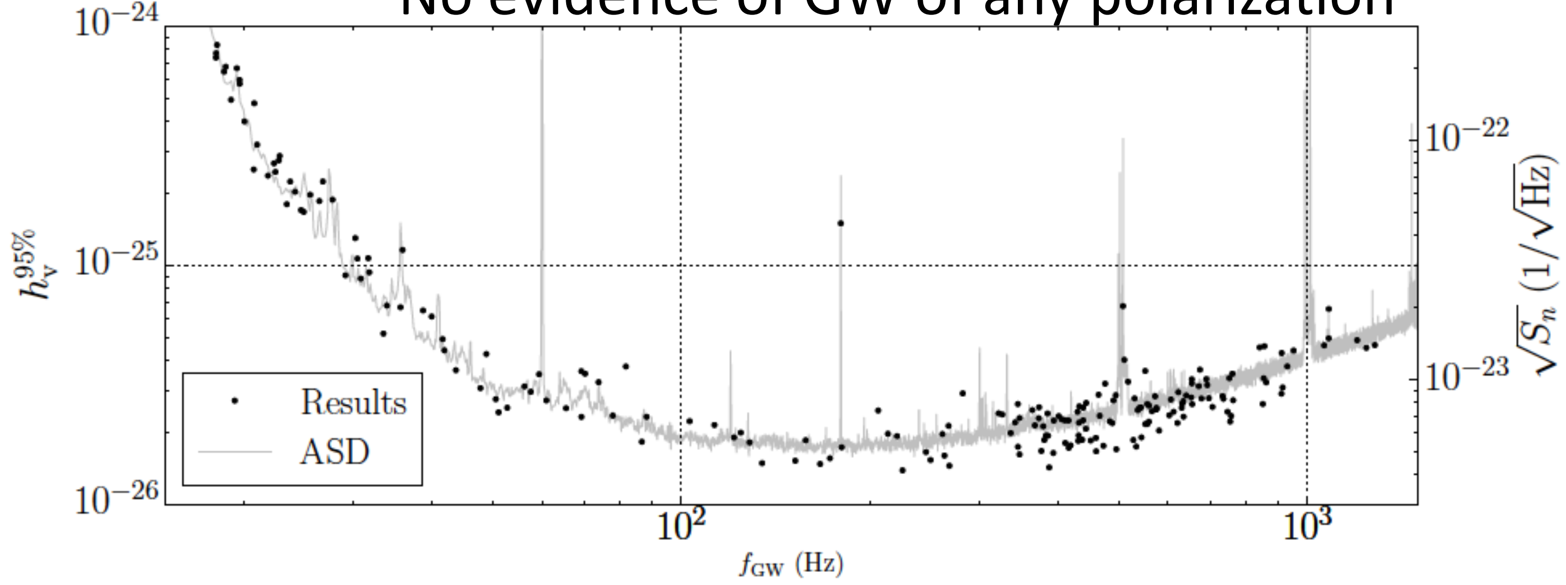
Most constraining ellipticity is 1.3 x 10^{-8} for J0636+5129

Nontensorial –

arXiv:1709.09203, PRL



No evidence of GW of any polarization



Coherent detection methods

There are essentially two types of coherent searches that are performed

Frequency domain

Conceived as a module in a hierarchical search

Matched filtering techniques. Aimed at computing a detection statistic.

These methods have been implemented in the frequency domain (although this is not necessary) and are very computationally efficient.

Best suited for large parameter space searches
(when signal characteristics are uncertain)

Frequentist approach used to cast upper limits.

Time domain

process signal to remove frequency variations due to Earth's motion around Sun and spindown

Standard Bayesian analysis,
as fast numerically but provides natural parameter estimation

Best suited to target known objects, even if phase evolution is complicated

Efficiently handless missing data

Upper limits interpretation: **Bayesian approach**

F-statistics

$$h(t;\mathcal{A},\boldsymbol{\lambda}) = \sum_{\mu=1}^{4} \mathcal{A}^{\mu}\, h_{\mu}(t;\boldsymbol{\lambda})$$

We can express *h(t)* in terms of amplitude *A* {A_+, $A_\times$, ψ, ϕ_0} and Doppler parameters λ

$$\begin{aligned} h(t;\mathcal{A},\boldsymbol{\lambda}) = {} & F_+(t;\boldsymbol{n},\psi)\, A_+ \cos\left[\phi_0 + \phi(t;\boldsymbol{\lambda})\right] \\ & + F_\times(t;\boldsymbol{n},\psi)\, A_\times \sin\left[\phi_0 + \phi(t;\boldsymbol{\lambda})\right] \end{aligned}$$

$$\begin{aligned} F_+(t;\boldsymbol{n},\psi) &= a(t;\boldsymbol{n})\, \cos 2\psi + b(t;\boldsymbol{n})\, \sin 2\psi \\ F_\times(t;\boldsymbol{n},\psi) &= b(t;\boldsymbol{n})\, \cos 2\psi - a(t;\boldsymbol{n})\, \sin 2\psi \end{aligned}$$

$$\begin{aligned} h_1(t;\boldsymbol{\lambda}) &= a(t;\boldsymbol{n})\, \cos\phi(t;\boldsymbol{\lambda}), \quad & h_2(t;\boldsymbol{\lambda}) &= b(t;\boldsymbol{n})\, \cos\phi(t;\boldsymbol{\lambda}), \\ h_3(t;\boldsymbol{\lambda}) &= a(t;\boldsymbol{n})\, \sin\phi(t;\boldsymbol{\lambda}), \quad & h_4(t;\boldsymbol{\lambda}) &= b(t;\boldsymbol{n})\, \sin\phi(t;\boldsymbol{\lambda}), \end{aligned}$$

$$\begin{aligned} \mathcal{A}^1 &= A_+\, \cos\phi_0\, \cos 2\psi - A_\times\, \sin\phi_0\, \sin 2\psi\,, \\ \mathcal{A}^2 &= A_+\, \cos\phi_0\, \sin 2\psi + A_\times\, \sin\phi_0\, \cos 2\psi\,, \\ \mathcal{A}^3 &= -A_+\, \sin\phi_0\, \cos 2\psi - A_\times\, \cos\phi_0\, \sin 2\psi\,, \\ \mathcal{A}^4 &= -A_+\, \sin\phi_0\, \sin 2\psi + A_\times\, \cos\phi_0\, \cos 2\psi\,. \end{aligned}$$

F-Statistics

Analytically maximize the likelihood over *A* $\ln \Lambda(x;h) = (x||h) - \frac{1}{2}(h||h)$

$$\ln \Lambda(x;\mathcal{A},\boldsymbol{\lambda}) = \mathcal{A}^\mu\, x_\mu - \frac{1}{2}\mathcal{A}^\mu \mathcal{A}^\nu\, \mathcal{M}_{\mu\nu}$$

$$x_\mu(\boldsymbol{\lambda}) \equiv (\boldsymbol{x}||h_\mu)\,, \quad \text{and} \quad \mathcal{M}_{\mu\nu}(\boldsymbol{\lambda}) \equiv (h_\mu||h_\nu)\,.$$

We can now maximize $\ln \Lambda$ over $\mathcal{A}^\mu$ to obtain the maximum-likelihood estimators $\mathcal{A}^\mu_{\mathrm{ML}}$ from the data $x(t)$, namely

$$\frac{\partial \ln \Lambda}{\partial \mathcal{A}^\mu} = 0 \quad \Longrightarrow \quad \mathcal{A}^\mu_{\mathrm{ML}} = \mathcal{M}^{\mu\nu}\, x_\nu\,, \qquad \text{where } \mathcal{M}^{\mu\alpha}\mathcal{M}_{\alpha\nu} = \delta^\mu_\nu$$

$$\boxed{2\mathcal{F}(x;\boldsymbol{\lambda}) = x_\mu\, \mathcal{M}^{\mu\nu}\, x_\nu}$$

Frequency domain method

The outcome of a target search is a number F^* that represents the optimal detection statistic for this search.

$2F^*$ is a random variable: For *Gaussian stationary noise*, follows a χ^2 distribution with 4 degrees of freedom with a non-centrality parameter $\lambda \propto (h|h)$. Fixing ι, ψ *and* ϕ_0 , for every h_0, we can obtain a pdf curve: $p(2F|h_0)$

The frequentist approach says the data will contain a signal with amplitude $\geq h_0$, with confidence C, if in repeated experiments, some fraction of trials C would yield a value of the detection statistics $\geq F^*$

$$C(h_0) = \int_{2F^*}^{\infty} p(2F \mid h_0)\mathrm{d}(2F)$$

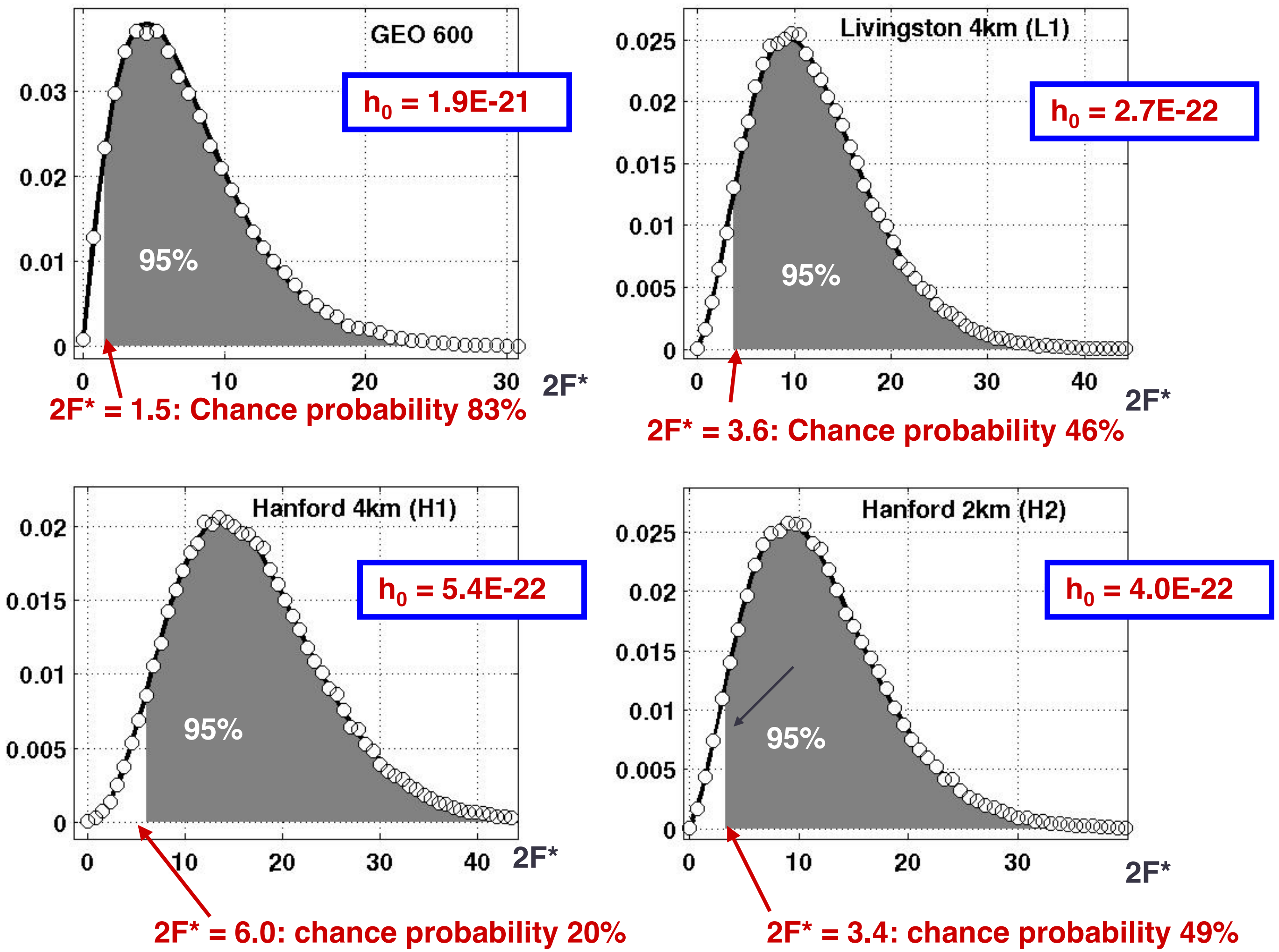
Use signal injection Monte Carlos to measure Probability Distribution Function (PDF) of F

Measured PDFs for the F statistic

with fake injected *worst-case* signals at nearby frequencies

S1 Example

Note: hundreds of thousands of injections were needed to get such nice clean statistics!



Time Domain Search for GWs from Known Pulsars

Sky position and spin frequency known accurately

Method: heterodyne time-domain data using the known spin phase of the pulsar

- Requires precise timing data from radio or X-ray observations
- Include binary systems in search when orbits known accurately
- Exclude pulsars with significant timing uncertainties
- Special treatment for the Crab and other pulsars with glitches, timing noise

Heterodyne, i.e. multiply by: $e^{-i\phi(t)}$

Known phase evolution of the signal: account for both spin-down rate and Doppler shift.

so that the expected demodulated signal is then:

$$y(t_k;\mathbf{a}) = \frac{1}{4} F_+(t_k;\psi) h_0 \left(1+\cos^2\iota\right) e^{i\phi_0} - \frac{i}{2} F_\times(t_k;\psi) h_0 (\cos\iota) e^{i\phi_0}$$

The only remaining time dependence is that of the antenna pattern

Here, $a = a(h_0, \psi, \iota, \phi_0)$, a vector of the signal parameters.

Search for Gravitational Waves from Known Pulsars

Heterodyne IFO data at the instantaneous GW frequency (2*radio rotation frequency)

- Coarse stage (fixed frequency) 16384 $\Rightarrow$ 4 samples/sec
- Fine stage (Doppler & spin-down correction) $\Rightarrow$ 1 samples/min $\Rightarrow$ B_k

- Low-pass filter the data at each step. E.g., at 0.5Hz
- Down-sample from 16384Hz to 1/60Hz, or 1/minute $\Rightarrow$ B_k
  - The data is down-sampled via averaging, yielding one value B_k of the complex time series, every 60s

$$y(t;\mathbf{a}) = \tfrac{1}{4} h_0 F_+(t,\psi)(1+\cos^2\iota)e^{2i\phi_0} - \tfrac{1}{2} i h_0 F_\times(t,\psi)\cos\iota e^{2i\phi_0}$$

Noise level is estimated from the variance of the data over each minute to account for non-stationarity $\Rightarrow$ σ_k

Additional parameters a = a(h_0, ϕ_0 , ψ, ι), are inferred from their Bayesian posterior probability distribution, assuming Gaussian noise

Time domain: Bayesian approach

We take a Bayesian approach, and determine the joint posterior distribution of the probability of our unknown parameters, using uniform priors on h_0 ,cos ι, ψ and ϕ_0 over their accessible values, i.e.

$$\underbrace{p(\mathbf{a} \mid \{B_k\})}_{\text{posterior}} \propto \underbrace{p(\mathbf{a})}_{\text{prior}} \cdot \underbrace{p(\{B_k\} \mid \mathbf{a})}_{\text{likelihood}}$$

The likelihood $\propto$ **exp(-χ^2/2), where**

$$\chi^2(\mathbf{a}) = \sum_k \left| \frac{B_k - y(t;\mathbf{a})}{\sigma_k} \right|^2$$

To get the posterior PDF for h_0, marginalizing with respect to the nuisance parameters cos ι, ψ and ϕ_0 given the data B_k

$$p(h_0 \mid \{B_k\}) \propto \iiint e^{-\chi^2/2} \mathrm{d}\phi_0 \mathrm{d}\psi \mathrm{d}\cos\iota$$

The Bayesian Approach

Bayesian statistics: quantifying the degree of certainty (or "degree of belief") of a statement being true

$$\underbrace{p\left(\vec{a} \mid \{B_k\}\right)}_{\text{posterior}} \propto \underbrace{p\left(\vec{a}\right)}_{\text{prior}} \underbrace{p\left(\{B_k\} \mid \vec{a}\right)}_{\text{likelihood}}$$

Assume uniform priors on all parameters

Combine posterior pdfs from different detectors

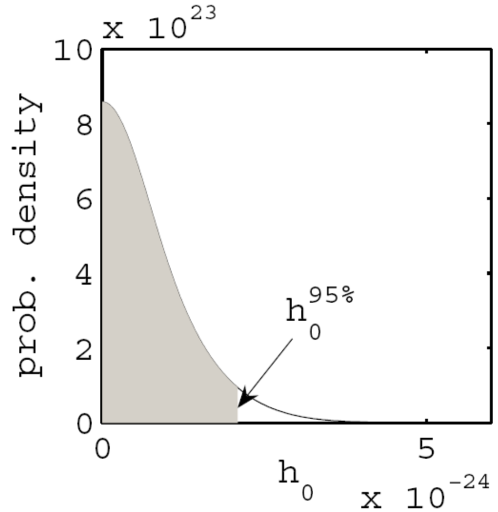
$$p(B_k|\mathbf{a})_{\mathrm{Joint}} = p(B_k|\mathbf{a})_{\mathrm{H1}} \cdot p(B_k|\mathbf{a})_{\mathrm{H2}} \cdot p(B_k|\mathbf{a})_{\mathrm{L1}}$$

Marginalize the posterior over *nuisance* parameters (ϕ_0, ψ, ι)

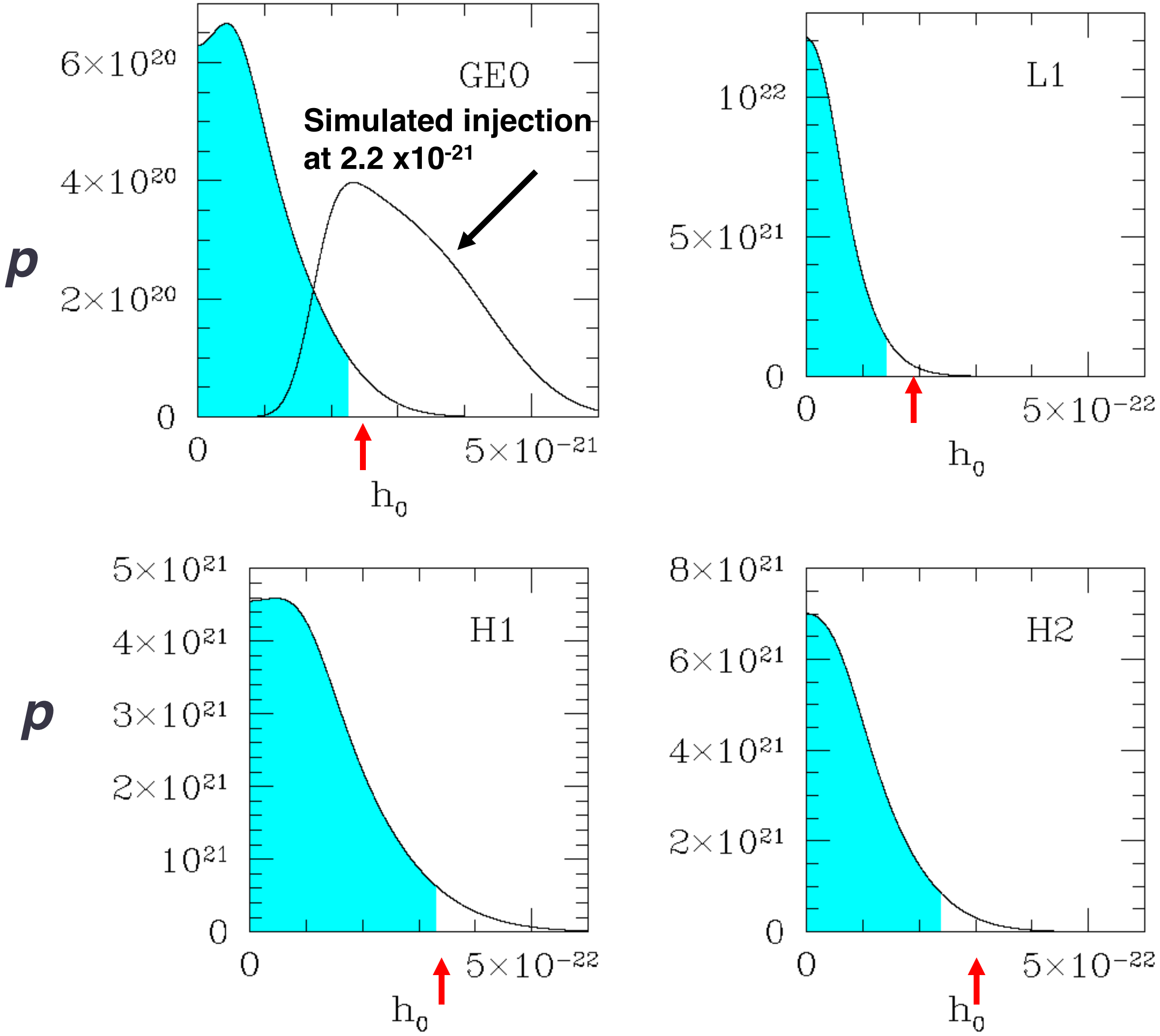
(i.e. integrate over possible values)

Set 95% upper limits on GW amplitude emitted by each pulsar solving:

$$0.95 = \int_0^{h_0^{95\%}} p(h_0|\{B_k\}) dh_0$$



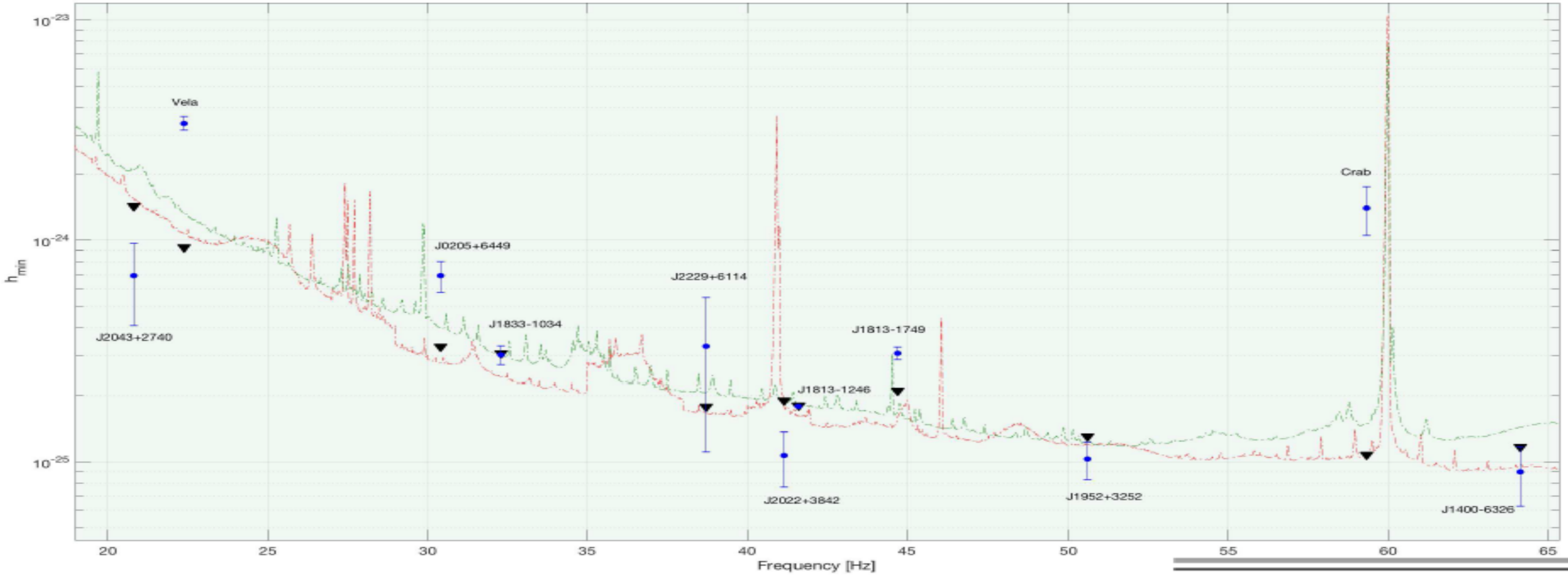
Posterior PDFs for CW time domain analyses



S1 Example

shaded area = 95% of total area

Narrow band search



LVC, PRD 96, 122006, 2017

Name	$h_{ul} \cdot 10^{-25}$	ϵ_{ul}	h_{ul}/h_{sd}
J0205+6449	3.8	$1.2 \cdot 10^{-3}$	0.88
J0534+2200 (Crab)	1.1	$5.8 \cdot 10^{-5}$	0.07
J0835-4510 (Vela)	9.3	$5.3 \cdot 10^{-4}$	0.27
J1400-6326	1.2	$1.9 \cdot 10^{-4}$	0.89
J1813-1246	1.8	$2.5 \cdot 10^{-4}$	0.88
J1813-1749	1.9	$4.8 \cdot 10^{-4}$	0.64
J1833-1034	3.1	$1.3 \cdot 10^{-3}$	0.99
J1952+3252	1.3	$1.4 \cdot 10^{-4}$	1.31
J2022+3842	1.9	$8.5 \cdot 10^{-4}$	1.38
J2043+2740	14.4	$3.5 \cdot 10^{-3}$	1.52
J2229+6114	1.8	$3.4 \cdot 10^{-4}$	0.54

Search over narrow frequency band.
Allows EM and GW phases evolution to not be completely phase-locked.
Ephemeris, doesn’t need to be up to date.

Semi-Coherent Methods

split data of length T into N_{seg} segments of length T_{seg}
+ combine coherent results by summing:

Fully Coherent Search

Coherent + Coherent + Coherent + Coherent + Coherent

Sum = Semicoherent Search

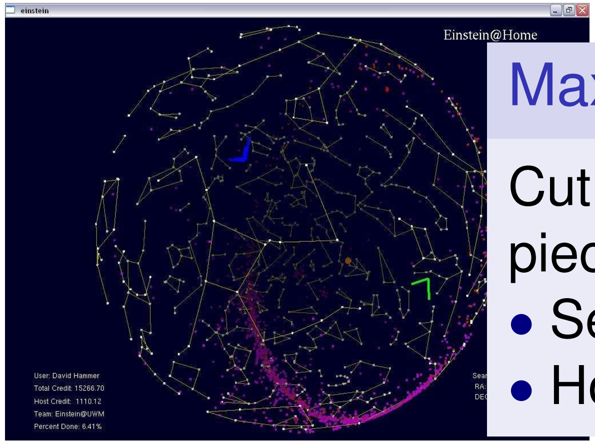
less sensitive for same amount of data, but much faster!

☞ allows analysing all the data

☞ more sensitive at fixed computing cost if $\Delta(\mathrm{sky}, f, \dot{f}, \ldots) > 0$

[+] maximize sensitivity (template banks, N_{seg}, T_{seg}) at fixed cost

[+] maximize computing power: clusters, GPUs, Einstein@Home



Maximize available computing power

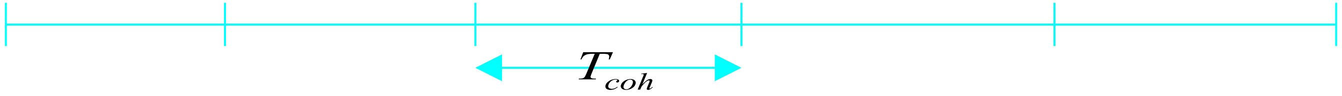
Cut parameter-space $\Delta(\text{sky}, f, \dot{f}, \ldots)$ into small pieces

- Send these "workunits" to participating hosts
- Hosts return finished work and request next

- Public distributed computing project, launched Feb. 2005
- $\sim$100,000 participants, $\sim$ 5 PFlop/s (24x7)
- All-sky and directed CW searches
- Workunits run $\sim 6-12\,\mathrm{h}$ on hosts
- Typical search runs $\sim 3-12$ months on E@H

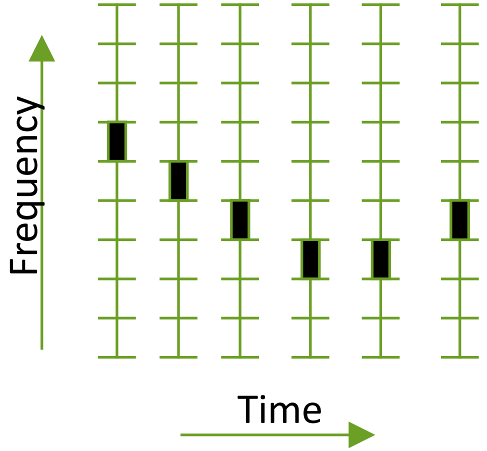
☞ You can sign up and help! `https://einsteinathome.org`

Semi-coherent power-sum methods



The idea is to perform a search over the total observation time using a *semi-coherent* (sub-optimal) method.

Several methods have been developed to search for cumulative excess power from a hypothetical periodic gravitational wave signal:



Stacked power spectra

- Based on FT each segment (T_{coh} ~ 0.5-2.0 hours) from h(t),
- Examining successive spectral estimates, tracking the frequency drifts due to Doppler modulations and df/dt as the incoherent step.
  - **Raw spectral powers:**
    Stack-slide (Radon transform),
    Power-flux, Loose coherence
  - **Thresholded powers** (Robust pattern detection technique)
    Frequency Hough transform ,
    Sky Hough transform

$$\mathrm{SNR} \propto \frac{h_o}{\sqrt{S_n}} \mathrm{T_{coh}}^{\frac{1}{2}} \mathrm{N_{seg}}^{\frac{1}{4w}},$$

$w(\mathrm{N_{seg}}, p\mathrm{FA})$ range $[1, \approx 3.5]$

[Prix & Shaltev, PRD85, 2012]

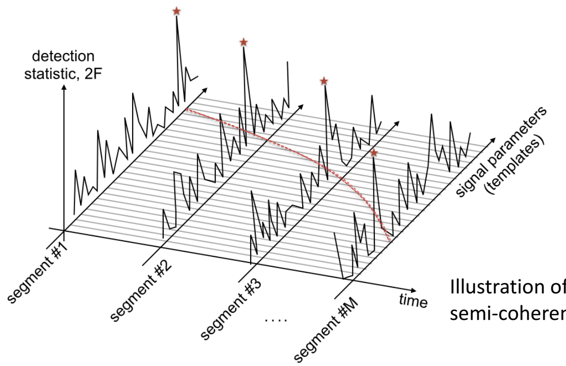


Illustration of stack-slide semi-coherent method

Multi-segment Demodulated spectra – F-Statistic
(T_{coh} ~ 1-few days)

- Coincident *F*-Statistic
- Stacked *F*-Statistic (Einstein@Home)

The Hough Transform

Robust pattern detection technique.

We use the Hough Transform to find the pattern produced by the Doppler modulation (due to the relative motion of the detector with respect to the source) and spin-down of a GW signal in the time – frequency plane of our data:

$$\boxed{f(t) - \hat{f}(t) = \hat{f}(t)\,\frac{\vec{v}(t)\cdot\vec{n}}{c}}$$

Detector RF

SSB

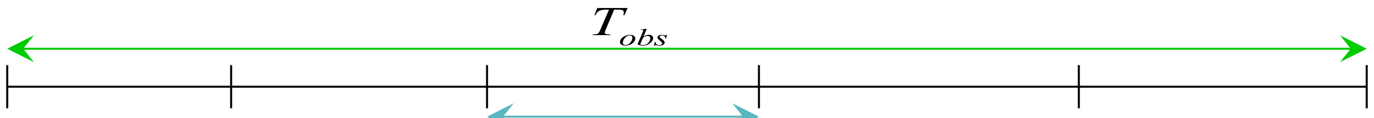
$$\hat{f}(t) = \hat{f}_0 + \dot{f}(t - t_0) + \ldots$$

For isolated NS the expected pattern depends on the parameters: $\{\alpha,\ \delta,\ f,\ \dot{f},\ldots\}$

The Hough Transform

Procedure:

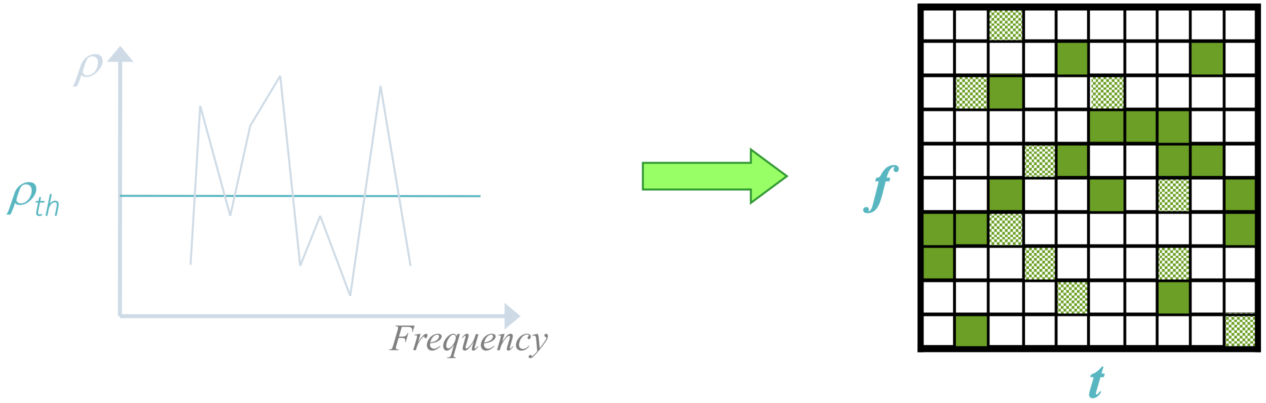
1. Break up data (*x(t) vs t*) into segments



$$\Delta T = \frac{T_{obs}}{N}$$

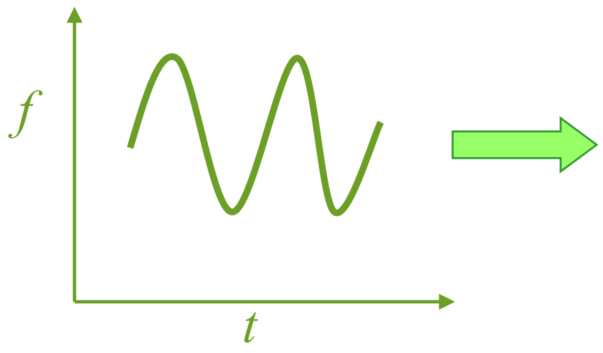
2. Take the FT of each segment and calculate the corresponding normalized power in each case (ρ_k)

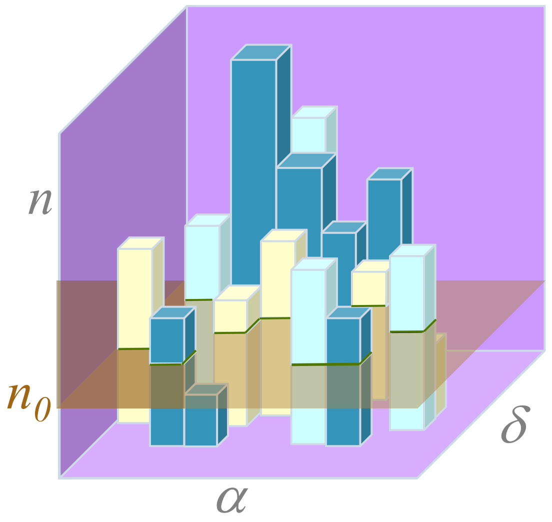
3. Select just those that are over a certain threshold $\rho_{th.}$

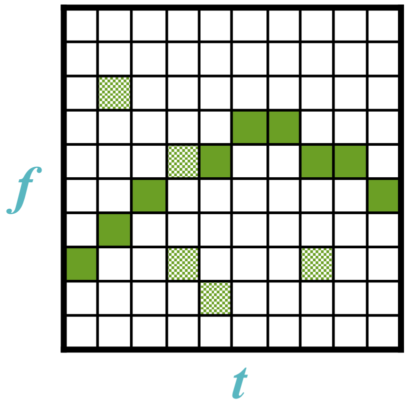


The Hough Transform

Procedure:







Hough Transform Statistics

The probability for any pixel on the *time - frequency* plane of being selected is:

$$p = \begin{cases} \text{Signal absent} & q = e^{\rho_{th}} \\ \text{Signal present} & \eta = e^{\rho_{th}}\left\{1 + \dfrac{\rho_{th}}{2}\lambda_k + O\left(\lambda_k^2\right)\right\} \end{cases}$$

SNR for a single SFT $\lambda_k = \dfrac{4\left|\tilde{h}(f_k)\right|^2}{T_{coh}\, S_n(f_k)}$

After performing the Hough Transform N SFTs, the probability that the pixel $\{\alpha, \delta, f_0, \dot{f}\}$ has a number count n is given by

Without Weights

Number Count $n = \sum_{i=1}^{N} n_i$

$$p(n) = \binom{N}{n} p^n (1-p)^{N-n}$$

	Signal absent	Signal present
Mean	$\langle n \rangle = N q$	$\langle n \rangle = N\eta$
Variance	$\sigma^2 = N q (1-q)$	$\sigma^2 = N\eta(1-\eta)$

With Weights

Number Count $n = \sum_{i=1}^{N} \omega_i n_i$ $\quad \sum_{i=1}^{N} \omega_i = N$

$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\langle n \rangle)^2}{2\sigma^2}}$$

	Signal absent	Signal present
Mean	$\langle n \rangle = N q$	$\langle n \rangle = qN + \dfrac{q\rho_{th}}{2}\sum_{i=1}^{N}\omega_i\lambda_i$
Variance	$\sigma^2 = \sum_{i=1}^{N}\omega_i^2\, q(1-q)$	$\sigma^2 = \sum_{i=1}^{N}\omega_i^2\eta_i(1-\eta_i)$

Frequentist upper limit

Perform the Hough transform for a set of points in parameter space $\lambda=\{\alpha,\delta,f_0,f_i\}\in \mathbf{S}$, given the data:

$$\mathbf{HT: S} \rightarrow \mathbf{N}$$

$$\lambda \rightarrow n(\lambda)$$

Determine the maximum number count n*

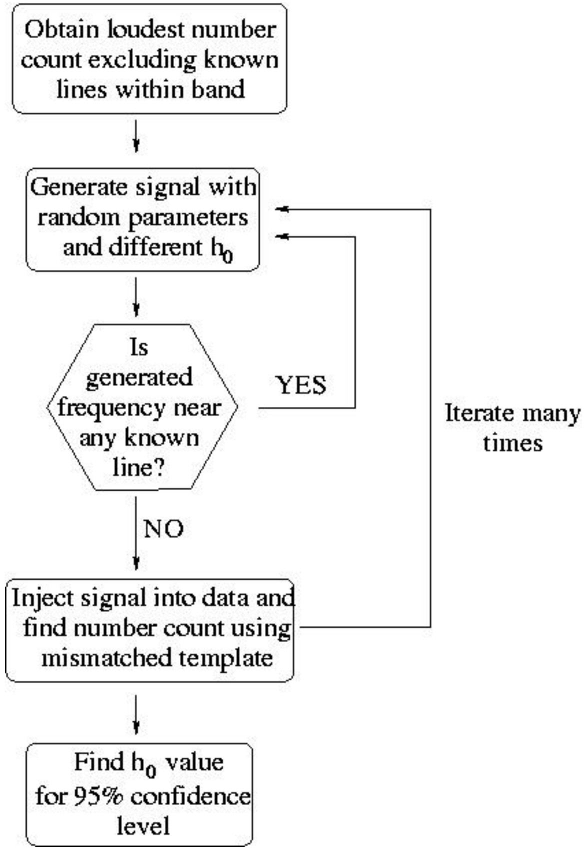
$$n^* = \max(n(\lambda)) : \lambda \in \mathbf{S}$$

Determine the probability distribution $p(n|h_0)$ for a range of h_0

The 95% frequentist upper limit $h_0^{95\%}$ is the value such that for repeated trials with a signal $h_0 \geq h_0^{95\%}$, we would obtain $n \geq n^*$ more than 95% of the time

$$0.95 = \sum_{n=n^*}^{N} p\left(n/h_0^{95\%}\right)$$

Compute $p(n|h_0)$ via Monte Carlo signal injection, using $\lambda \in \mathbf{S}$, and $\phi_0 \in [0,2\pi]$, $\psi \in [-\pi/4,\pi/4]$, $\cos\iota \in [-1,1]$.

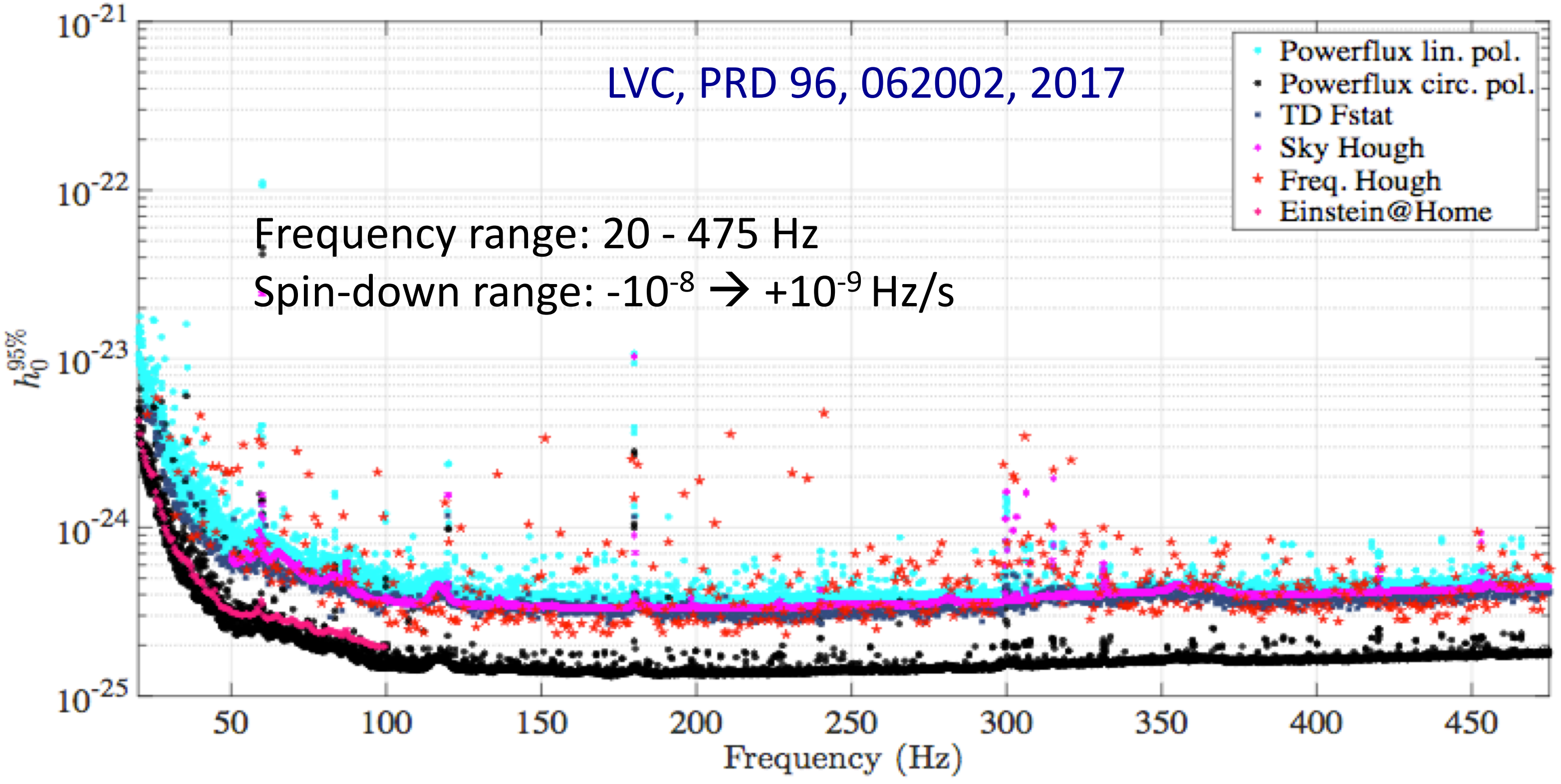


All-sky Searches for Unknown Isolated Stars

Search methods used in Initial LIGO data:

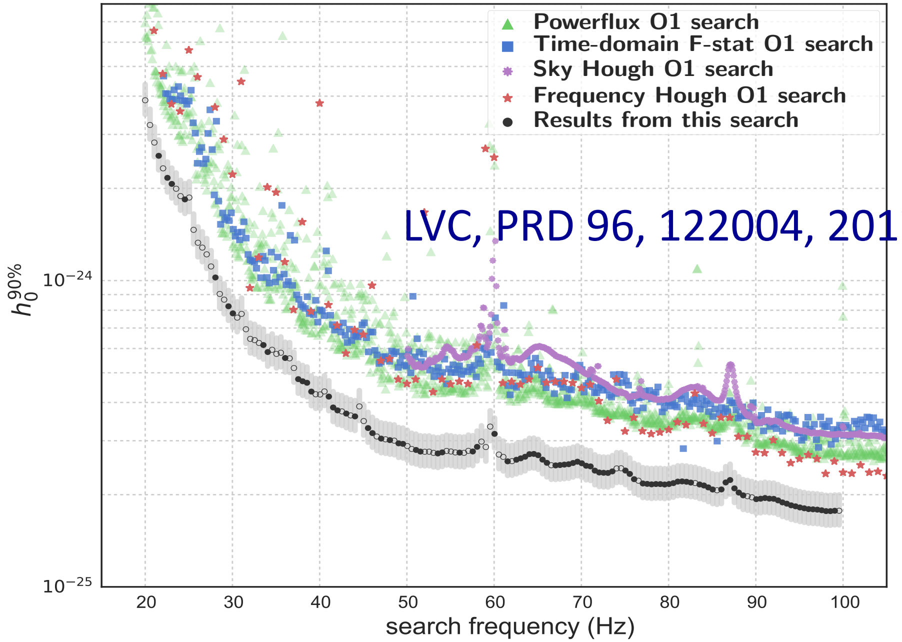
- Coherent *F*-Statistic (T_{COH} = 10 hours)
  [P. Jaranowski, A. Krolak & B. Schutz, PRD 58 (1998) 063001; B. Abbott *et al*., PRD 76 (2007) 082001]
- Stacked power spectra ("stack-slide" & variants) (Tcoh = 0.5-2.0 hours]
  - Stack-slide [P. Brady *et al*., PRD 57 (1998) 2101; P. Brady & T. Creighton, PRD 61 (2000) 082001; B. Abbott *et al*, PRD 77 (2008) 022001]
  - Sky Hough transform [A.M. Sintes & B. Krishnan, JPCS 32 (2006) 206]
  - PowerFlux / Loose coherence [B. Abbott *et al*, PRD 77 (2008) 022001; V. Dergachev, CQG 27 (2010) 205017]
  - Frequency Hough transform [P. Astone *et al*., PRD 90 (2014) 042002]
- Multi-segment *F*-Statistic (T_{COH} ~ 1-few days)
  - Coincident *F*-Statistic
    [P. Astone *et al*., PRD 82 (2010) 022005; J. Aasi *et al*., CQG 31 (2014) 165014]
  - Stacked *F*-Statistic (Einstein@Home)
    [R. Prix, PRD 75 (2007) 023004; H.J. Pletsch & B. Allen, PRL 103 (2009) 181102;
    K. Wette & R, Prix, PRD 88 (2013) 123005; D. Keitel, PRD 93 (2016) 084024;
    M. Shaltev *et al*., PRD 89 (2014) 124030; M.A. Papa *et al*., PRD 94 (2016) 122006]

O1 all-sky fast turnaround searches

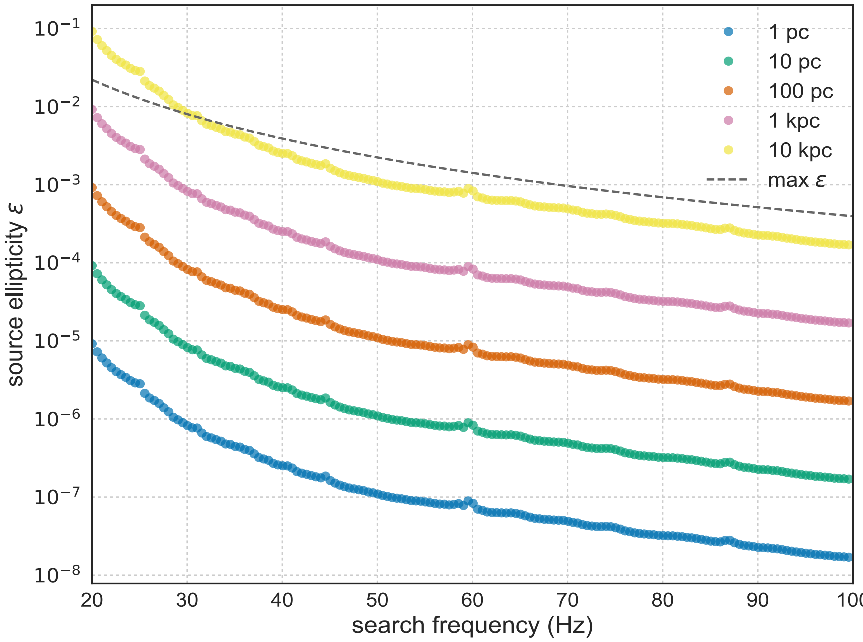


Four pipelines: PowerFlux, time-domain F-statistic, Sky Hough and Frequency Hough
No candidate survived the follow-up.
Significant improvement in the ULs with respect to past analyses (> 3x)
Best limits on h_0 of 1.5×10^{-25} (optimal source spin orientation) near 170 Hz
At 475 Hz sensitive to NS with ellipticity $\varepsilon > 8 \times 10^{-7}$ as far as 1kpc, for optimal spin orientation

O1 all-sky Einstein@Home search



LVC, PRD 96, 122004, 2017



Search ranges: f : [20, 100] Hz, spin-down: [-2.6, 0.3] x 10^{-9} Hz/s

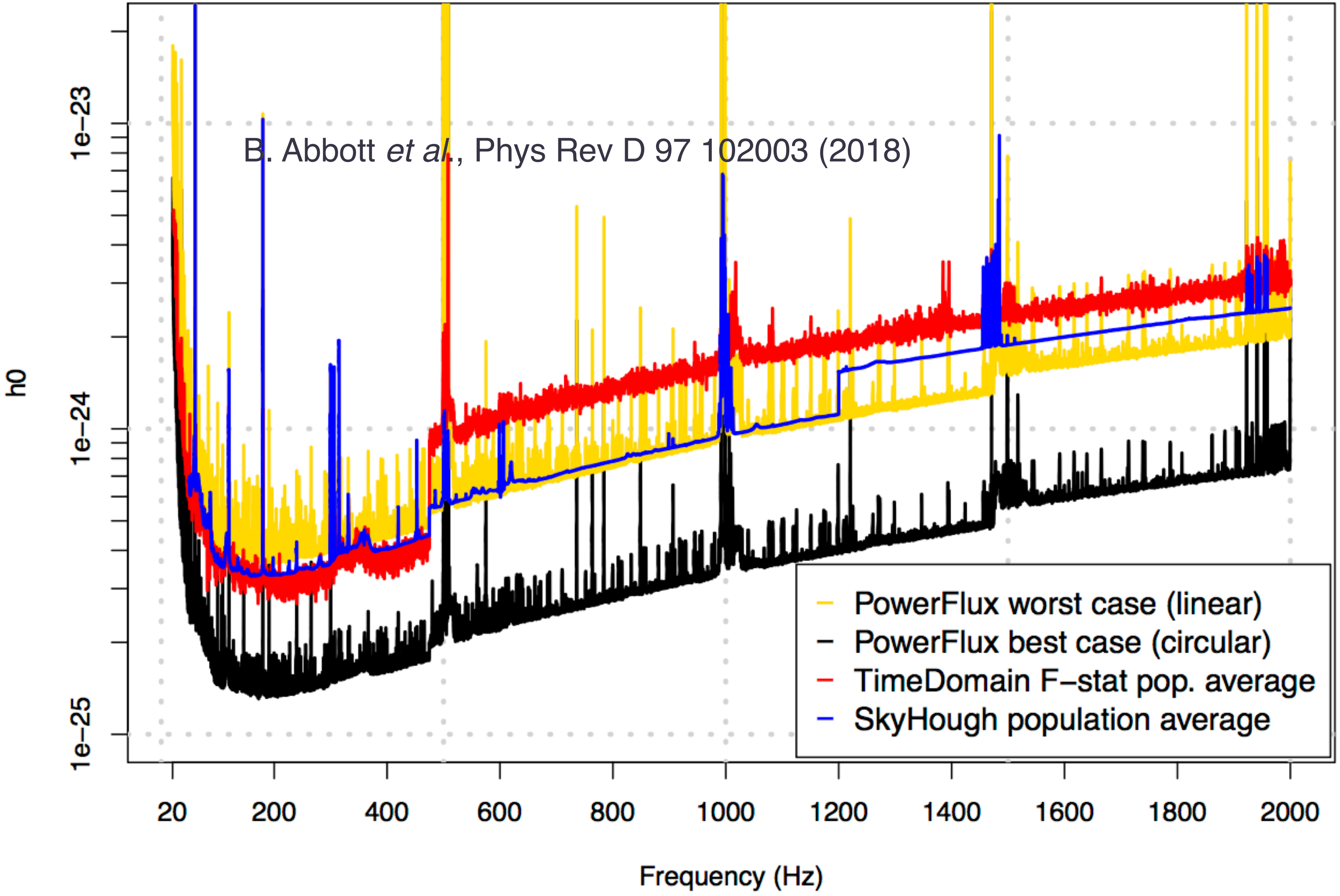
N_{seg}=12, T_{coh}= 210h= 8.8 d ☞ 3x10^{17} templates

Best limits on h_0 of 1.8 x 10^{-25} (marginalised) near 100 Hz

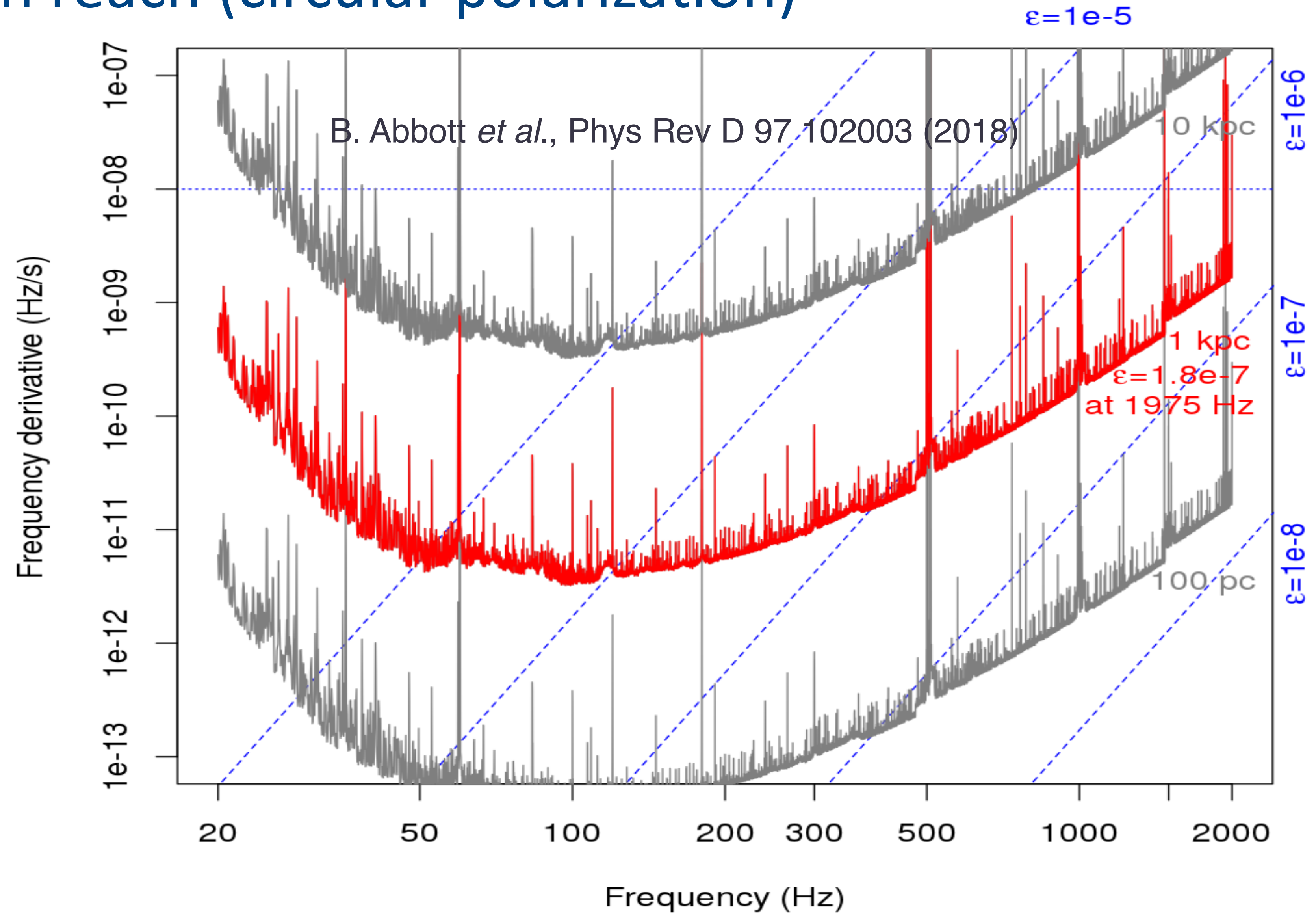
Sensitivity Depth *D*~49 Hz^{-1}

At 55Hz, exclude sources with ellipticity above 10^{-5} within 100pc

O1 full band all-sky search



Search reach (circular polarization)



Sensitive to NS with ellipticity > 1.8 x 10^{-7} as far as 1kpc, for optimal spin orientation

Directed Searches for Isolated Stars

Coherent directed searches for known sources / locations with unknown or poorly known frequencies carried out in initial and advanced LIGO data using the F-Statistic with optimized templating
[K. Wette *et al.*, CQG 25 (2008) 235011]

Applied in final initial LIGO S6 and advanced LIGO O1* runs to nine supernova remnants [J. Aasi *et al.*, ApJ 813 (2015) 39] and with barycenter-resampled *F*-Statistic (quicker) to core of globular cluster NGC 6544
[B. Abbott *et al.*, PRD 95 (2017) 082005]

Semi-coherent directed searches using stacked *F*-Statistic carried out in initial LIGO data for galactic center [J. Aasi *et al.*, PRD 88 (2013) 102022] and for Cas A [S.J. Zhu *et al.*, PRD 94 (2016) 082008]

→ Semi-coherent searches using full data set now carried out on Einstein@Home
→ Achieve O(2) sensitivity improvement over quicker coherent searches

*Result from O1 under review

What is the "age-based spindown limit"?

If a star's age is known (e.g., historical SNR), but its spin is unknown, one can still define an indirect spindown upper limit by assuming gravitar behavior has dominated its lifetime:

$$\tau = \frac{f}{4\,(df\,/\,dt)}$$

And substitute into h_{SD} to obtain
[K. Wette *et al*., CQG 25 (2008) 235011]

$$h_{ISD} = 2.2\times10^{-24}\left[\frac{kpc}{d}\right]\sqrt{\left[\frac{1000\,yr}{\tau}\right]\left[\frac{I}{10^{45}\,g\cdot cm^2}\right]}$$

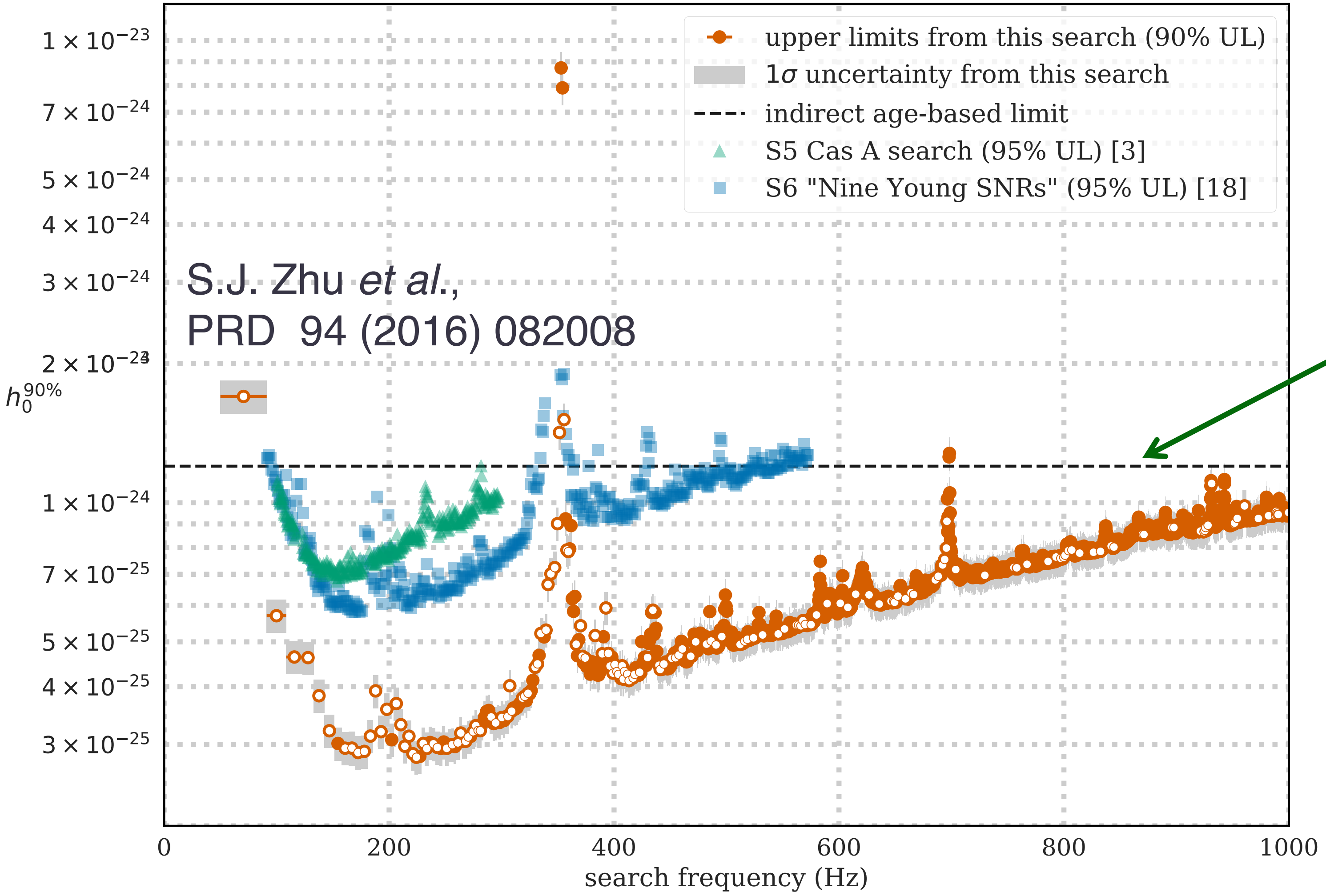
Example:

Cassiopeia A → **$h_{ISD} \approx 1.1 \times 10^{-24}$**

(d ~ 3.4 kpc, τ ~ 340 yr)

Initial LIGO results – Directed Search

Search for Cassiopeia A – Young age (~300 years) requires search over 2nd derivative



indirect upper limit

(based on age, distance)

Directed Searches for Binary Stars

The prospect of detecting Scorpius X-1 (X-ray-bright LMXB) in Advanced LIGO / Virgo data has spurred renewed algorithm development in recent years to detect CW sources in binary systems.

Sco X-1's spin frequency unknown, but

- Orbital period known precisely, time of ascending nodes known well
- Projected semi-major axis bounded

Search methods used in Initial LIGO data:

- Coherent *F*-Statistic (T_{COH} = 6 hours)
  [P. Jaranowski, A. Krolak & B. Schutz, PRD 58 (1998) 063001;
  B. Abbott *et al*., PRD 76 (2007) 082001]

- Radiometer stochastic radiation search
  [S.W. Ballmer, CQG 23 (2006) S179; B. Abbott *et al*., PRL 118 (2017) 121102]

- *F*-Statistic sideband summing (T_{COH} = 10 days)
  [C. Messenger & G. Woan, CQG 24 (2007) S469; L. Sammut *et al.*, PRD 89 (2014) 043001;
  B. Abbott *et al*. PRD 91 (2015) 062008]

- Double Fourier spectra
  [G.D. Meadors, E. Goetz & K. Riles, CQG 33 (2016) 105017;
  G.D. Meadors *et al*., PRD 95 (2017) 042005]

Directed Searches for Binary Stars

Mock data challenge carried out comparing these algorithms and new one based on cross-correlation with signal frequency demodulation
→ **Clear winner was demodulated cross-correlation**
[C. Messenger *et al.*, PRD 92 (2015) 023006]

Cross-correlation allows tuning of performance via effective coherence length as short Fourier transforms are combined from different interferometers and observation intervals
[S. Dhurandhar *et al.*, PRD 77 (2008) 082001; J.T. Whelan *et al.*, PRD 91 (2015) 102005;
B. Abbott *et al.*, ApJ 841 (2017) 47]

Recent improvement to F-Statistic sideband summing: combining multiple 10-day segments with Viterbi tracking
[S. Suvorova *et al.*, PRD 93 (2016) 123009; B. Abbott *et al.*, PRD 95 (2017) 122003]

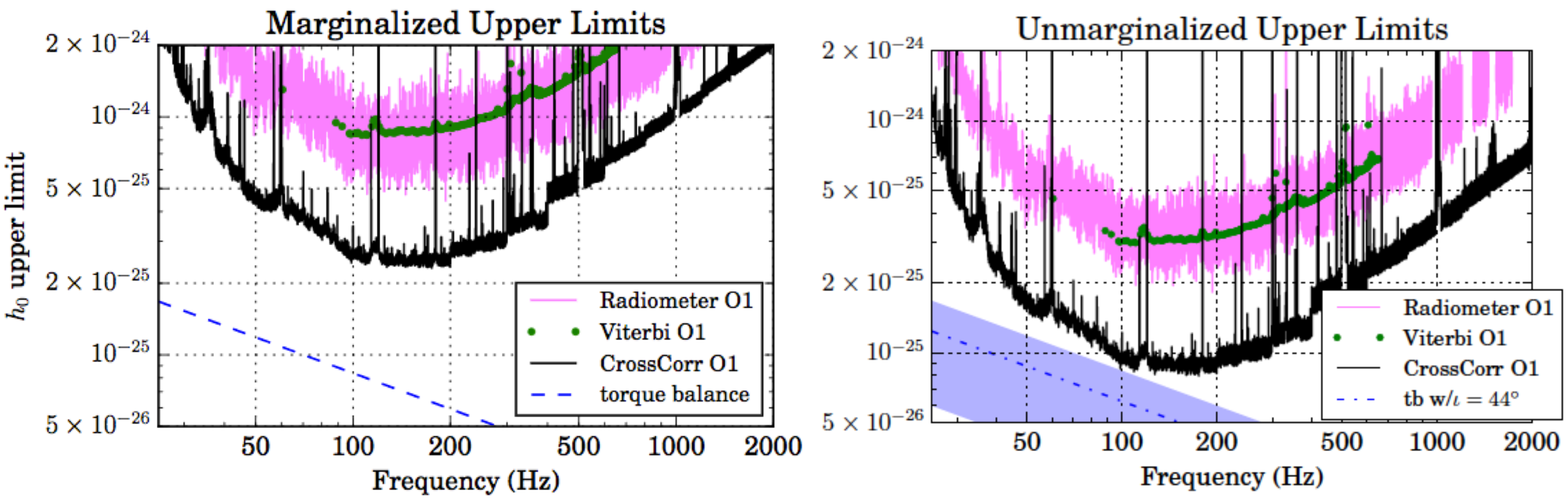
More algorithms on near horizon:
- Sideband with orbital phase tracking [S. Suvorova et al., PRD 96 (2017) 102006]
- Stacked *F*-Statistic [P. Leaci & R. Prix, PRD 91 (2015) 102003]

Next mock data challenge will include stochastic spin-wandering
[A. Mukherjee, C. Messenger & K. Riles, PRD 97, 043016 (2018)]

O1 directed Scorpius X-1 searches

LMXBs: Accretion can generate an asymmetric mass or current quadrupole

[LVC ApJ 847 (2017) 47, LVC PRD 95 (2017) 122003, LVC PRL 118 (2017) 121102]



No evidence for standard CW signal in the range 25 Hz-2kHz

95% Upper limits improvements of a factor 7 over initial-LIGO

Marginalised limits within 3.4 of torque limit at 100Hz. For circular polarisation, factor of 1.2 above torque limit

The torque balance line is an optimistic expected signal strength inferred from the X-ray flux from Sco X-1

$$h_{tb} \approx 3 \cdot 10^{-27} \left(\frac{F_X}{10^{-8}\, erg \cdot cm^2 \cdot s^{-1}} \right) \left(\frac{f_{rot}}{300\,Hz} \right)^{-1/2}$$

All-sky Searches for Unknown Binary Stars

Search method used in Initial LIGO data:

Double Fourier spectra ("TwoSpect") – similar to semicoherent PowerFlux
[E. Goetz & K. Riles, CQG 28 (2011) 215006;
J. Aasi *et al*., PRD 90 (2014) 062010]

→ Sensitivity tradeoff to cover enormous parameter space is severe

Other algorithms on near horizon or under development

- Radiometer method with sidereal-folded data
  [E. Thrane *et al*., PRD 91 (2017) 124012]

- "Polynomial" method using short-T_{COH} filters and coincidence
  [S. van der Putten *et al*., JPCS 228 (2010) 012005]

- Autocorrelation of spectograms
  [A. Vicere & M. Yvert, CQG 33 (2016) 165006]

Sensitivity Depth

Different search algorithms use different fractions of available data from a run and have different scalings of sensitivity with observation time [e.g., $(T_{OBS})^{1/2}$, $(T_{OBS})^{1/4}$]

Detector sensitivities vary by orders of magnitude over detection band

To make rule-of-thumb comparisons across different searches easier, the CW search group has adopted the notion (R. Prix) of "sensitivity depth"

→ Gives "bottom line" sensitivity of search over given data set w.r.t. noise floor at given frequency:

$$\text{Depth} = [\, h_0 \text{ (95\% CL)} / \text{ASD}(f)\,]^{-1} \qquad \text{(Units: Hz}^{1/2}\text{)}$$

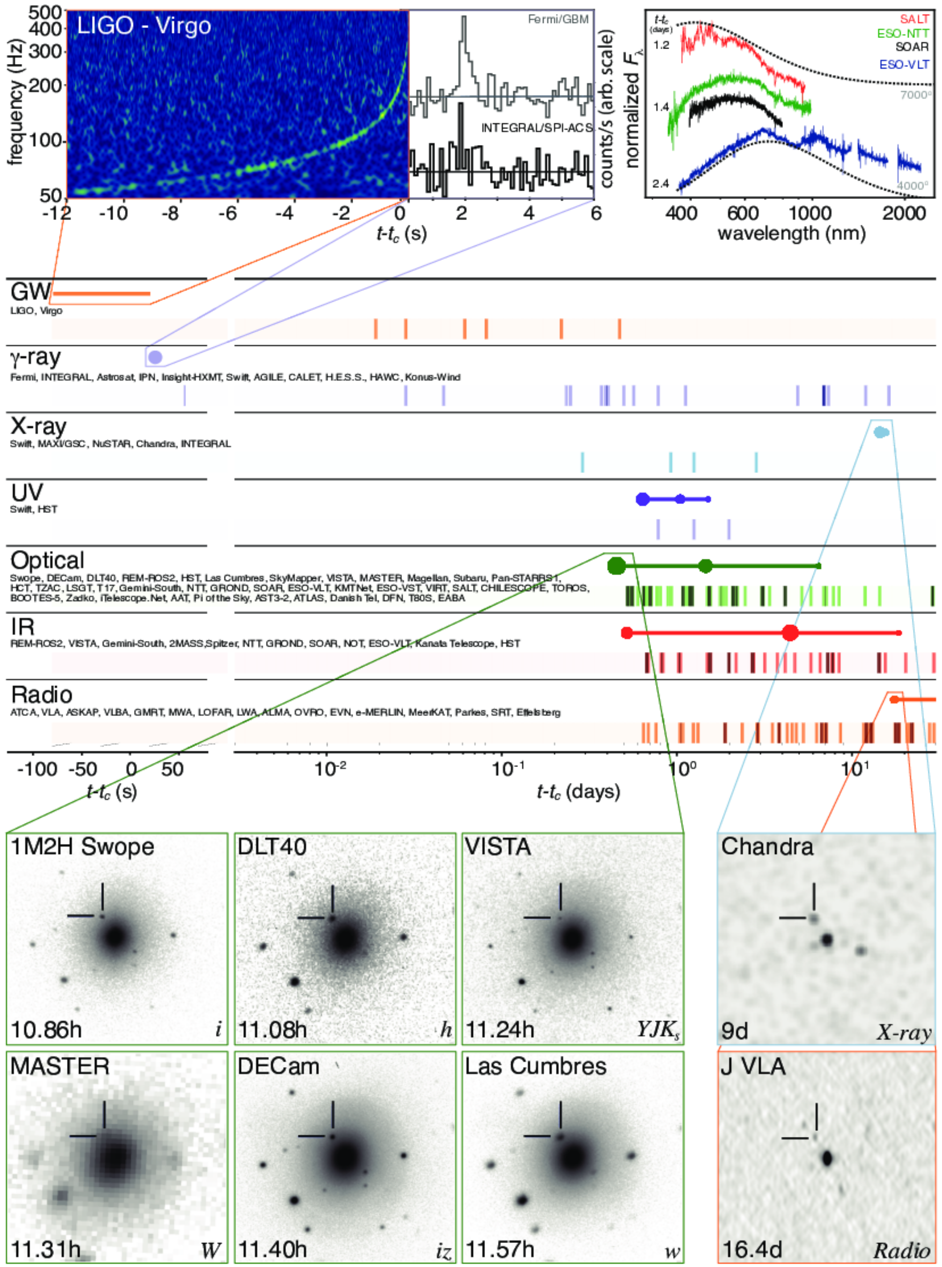
→ Sensitive searches have greater depth

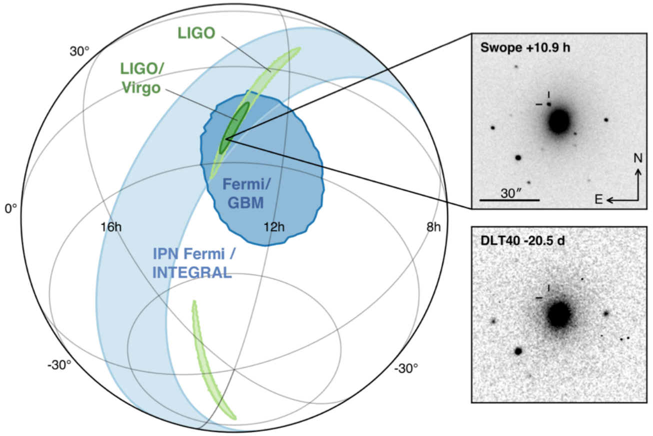
Sensitivity Depth

Examples: (current algorithms)

• Targeted search over 1 year:	~500
• Narrowband search over 4 months:	~100-150
• Directed isolated search over 10 days:	~30
• Semi-coherent directed isolated search over 1 year:	~60
• Cross-correlation directed binary search over 4 months:	~80
• All-sky isolated search over 4 months:	~25
• All-sky binary search over 1 year:	~3

These depths too affect how we choose to focus resources on searches (computational, human)





Post-merger remnant from GW170817

Challenges for longer-duration search:

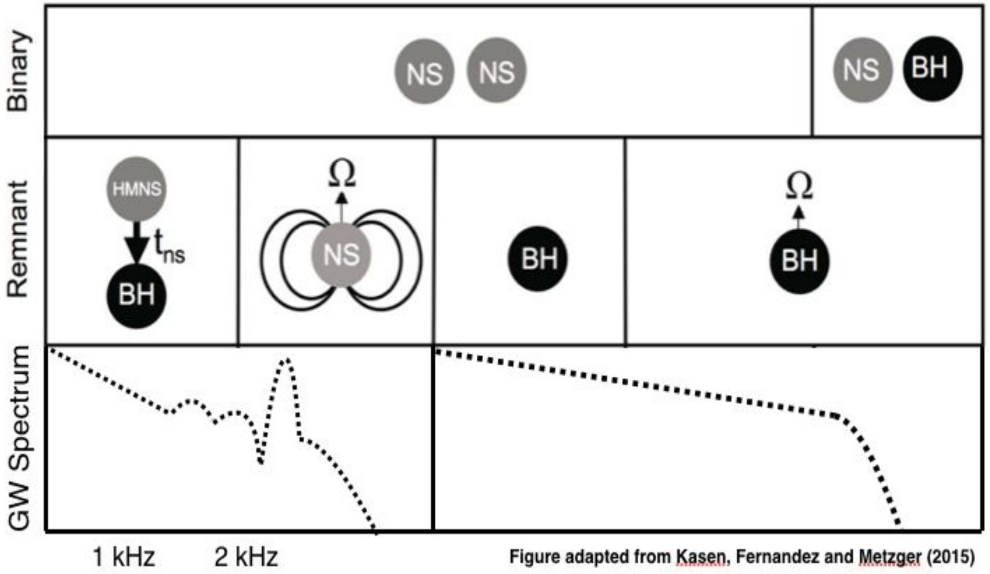
- Unknown GW emission frequency and (rapid) signal evolution
- Distance is far for traditional CW searches

$$d = 40^{+8}_{-14}\ \mathrm{Mpc}$$

- Detection unlikely, but worth developing the searches, verify expectations & determine future prospects.

Post-merger Scenarios

Theoretically, the end-product of a BNS merger can be (depending on the remnant mass and EOS):



- Prompt collapse to a BH (BH ringing >6 kHz)
- Unstable hypermassive NS, supported by differential rotation and thermal pressure
  → Collapse after 10-100 ms
- Supramassive, metastable NS, supported by centrifugal force
  → Collapse after O(10-10000 s)
- Stable remnant NS (t_{GW}~100 ms, minutes-weeks+)

GW emission from the remnant

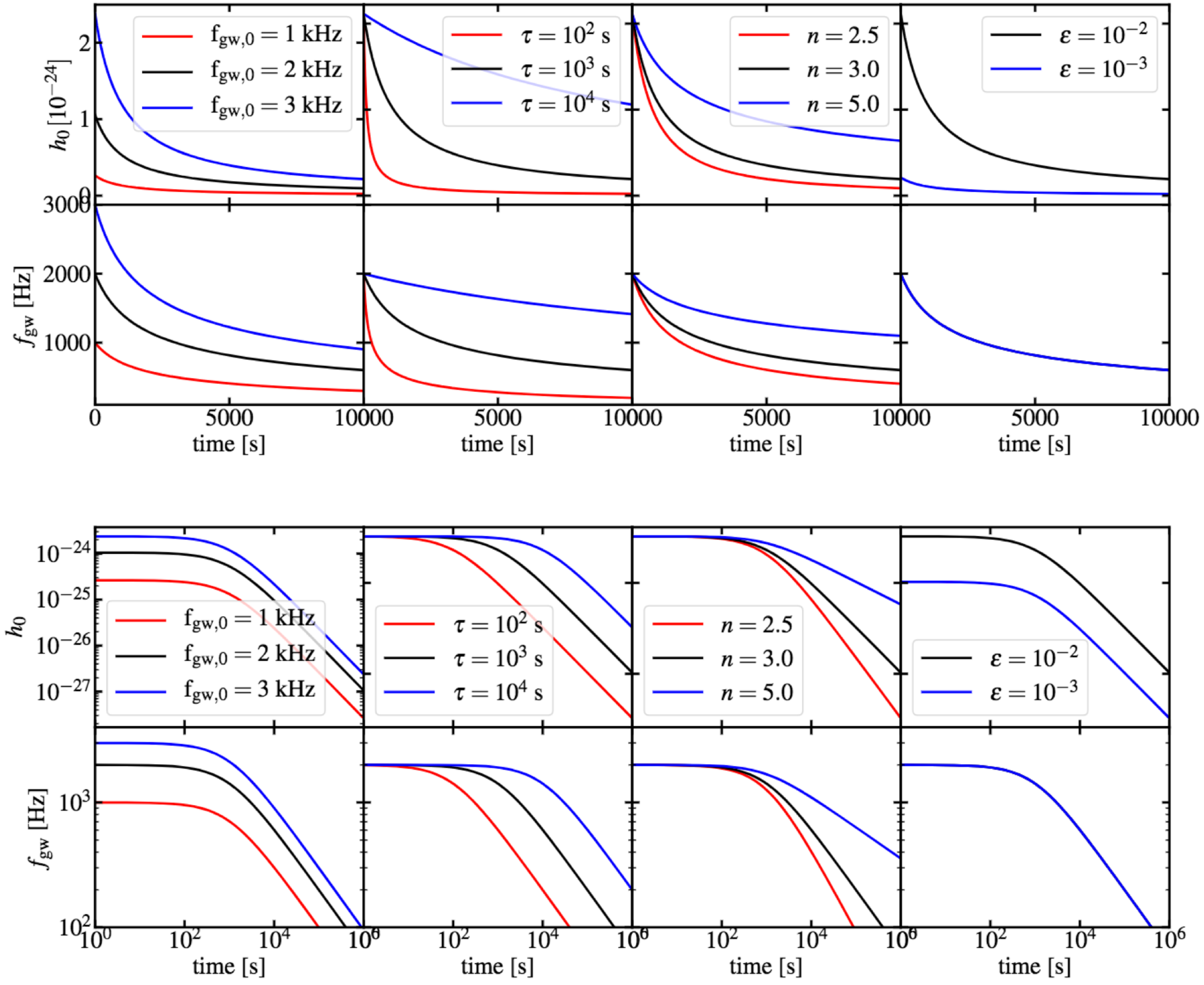
The newborn NS may emit a “transient” GW signals due to:

- the formation of a millisecond magnetar
(e.g. Giacomazzo+, 2013, 2014; Dall’Osso+, 2012; Lü+, 2015; Rowlinson+, 2013; Gompertz+, 2013, 2014)

- the development of dynamical or secular instabilities, like r-modes or bar-modes
(e.g. Andersson, 1998; Lindblom, 1998; Corsi+, 2009; Passamonti+, 2013; Doneva+, 2015)

- ?

Magnetar-like emission waveform examples



Periodic signal with power law frequency evolution

$$\dot{\Omega} = -k\Omega^n$$

depending on the braking mechanisms

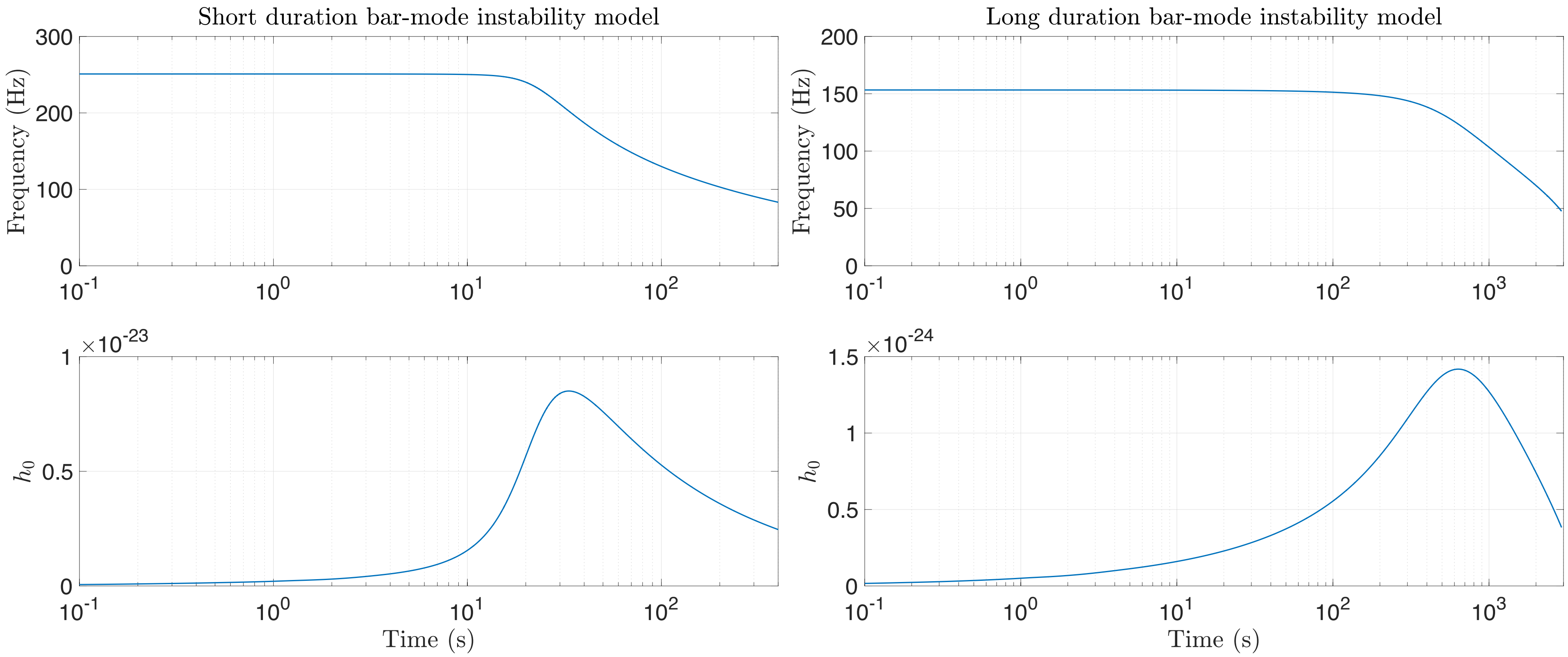
GW contribution to the spin-down cannot be excluded

Due to fast spin-down the GW signal frequency may become too small after hours-days

Lasky, et al., LIGO-T1700408

Bar-mode instability model waveform examples

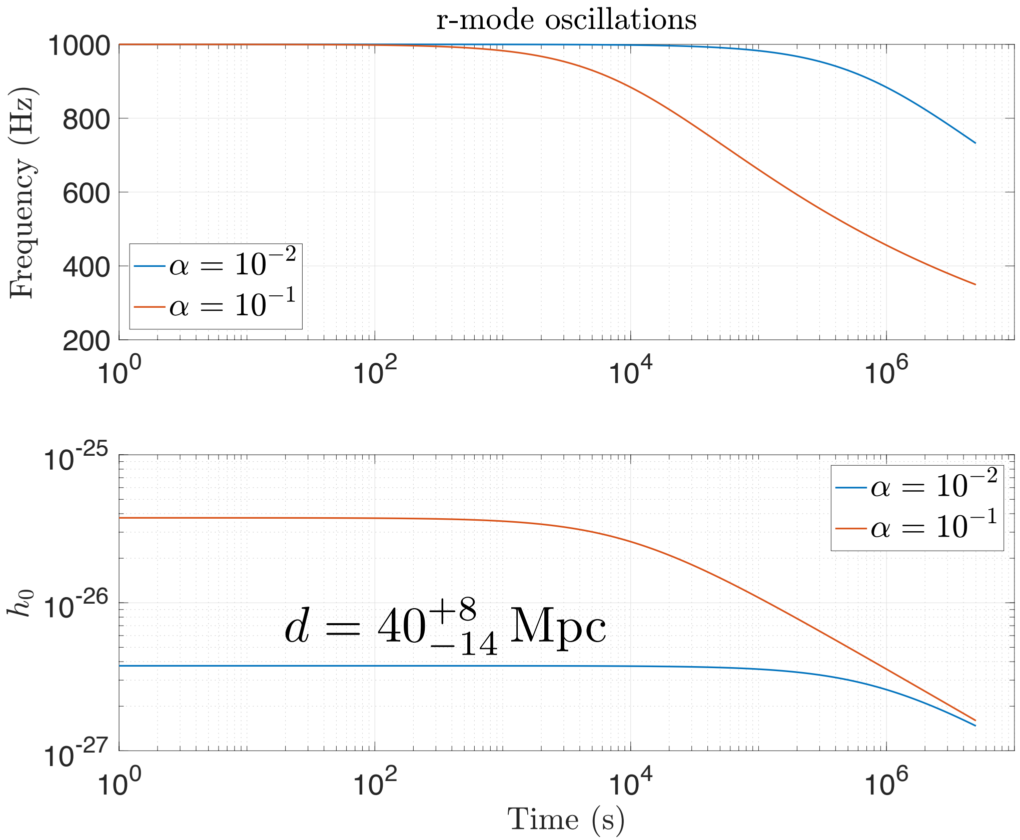
Dynamical and secular, with different time scales, depending on the ratio between kinetic and gravitational binding energy



“Typical” GRB magnetar: 1.4 Msun, 20 km radius, 1e14 G, n = 1

Corsi & Meszaros 2009 ; Coyne et al. 2016

R-mode instability (e.g. Haskell, 2016 for a review)



Mytidis, et al. 2015 ApJ 210 27

GW frequency ~4/3 f_{rot}.

Frequency evolution described by a power law with braking index n=7;

r-mode saturation amplitude depends on NS EOS

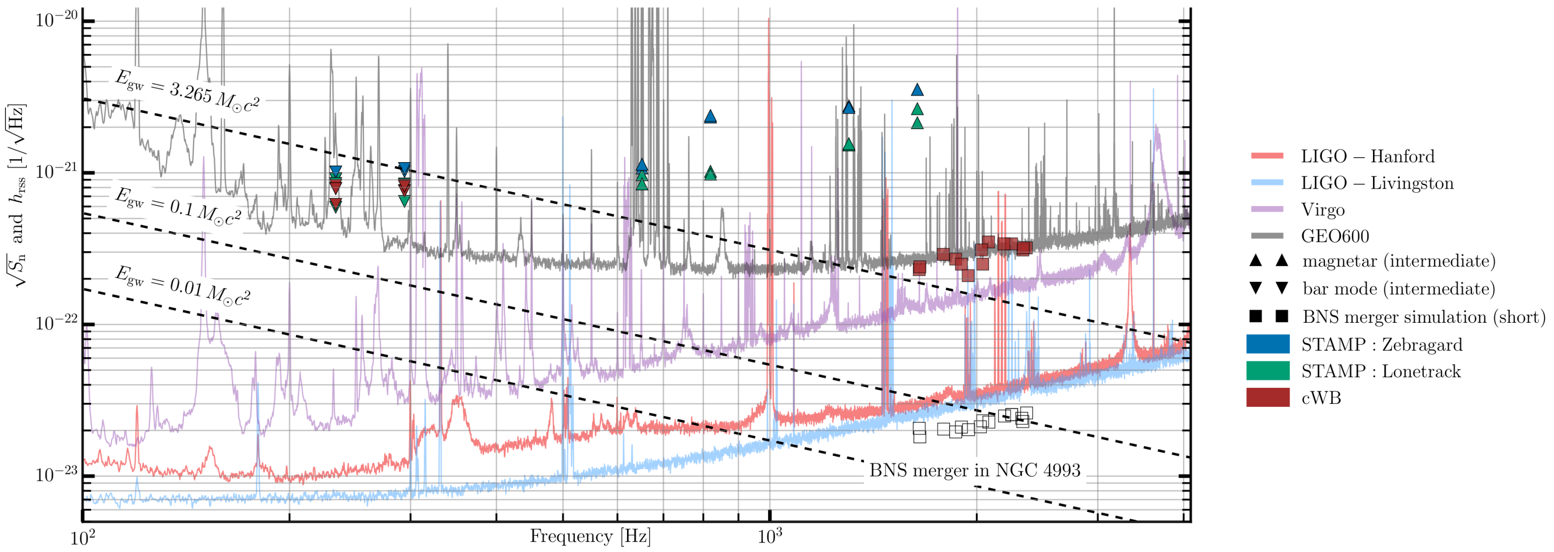
GW signal time scale O(hours-days)

Short-duration remnant searches

LVC, ApJL 851 (2017) sets upper limit for short (<1 s) and intermediate (<500 s) duration post-merger signals for GW170817

Two pipelines (cWB, STAMP), based on excess power in time-frequency:

- cWB burst analysis ($\lesssim$ 1s 1–4 kHz HL and $\lesssim$ 1000s 24–2048 Hz HLV)
- STAMP directional stochastic analysis (500 s maps, 24–4000 Hz HL)



- CW pipelines not involved
- At least a factor of 10 above model predictions

Long-duration remnant searches

Predictions rule of thumb: "Signals detectable up to ~20-25 Mpc in Advanced LIGO-Virgo detectors, **assuming matched filtering**".

In general, of course, we cannot use matched filtering as we do not know signal parameters in advance.

We must rely on semi-coherent or un-modeled methods, which are less sensitive.

EM observations crucial to reduce parameter space and to improve detection significance

$t \gg 500$ s fall within CW + stochastic groups' remit

Available data: from GW170817 coalescence to end of O2: 8.5 days

Main physical model: power law spindown ('ms magnetar')

$$\dot{\Omega} = -k\Omega^n$$

$$f_{\mathrm{gw}}(t) = f_{\mathrm{gw},0}\left(1 + \frac{t}{\tau}\right)^{\frac{1}{1-n}} \qquad \tau = \frac{-\Omega_0^{1-n}}{k(1-n)}$$

Long-duration remnant searches

Broadly speaking, two kinds of searches are being considered for the search of very long transient signals from the remnant of GW170817:

Unmodelled search (10^2 to 10^4s intermediate duration)

- **STAMP-VLT** (stochastic search) based on excess power in spectrograms (Thrane+, 2013)
- hidden-Markov **Viterbi** tracking , previously used for Sco X-1, young SNRs [Suvorova et al PRD 96 (2016) 102006, Sun et al, PRD 97 (2018) 043013]
  - →More robust against fluctuations and transient instrumental glitches
    Require short tracking step size (1 s)
    Optimal observing time ~ spin-down time-scale

Model-based searches (hours/days duration)

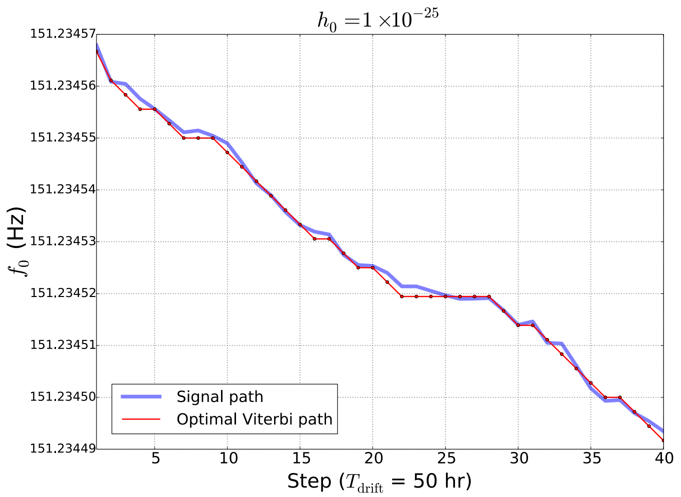
- based on variations of methods used for standard continuous wave searches: **SkyHough** and **FrequencyHough:** previously used for all-sky searches [LVC, PRD 96 (2017) 062002]
  - → Allow to estimate signal parameters (in case of detection)

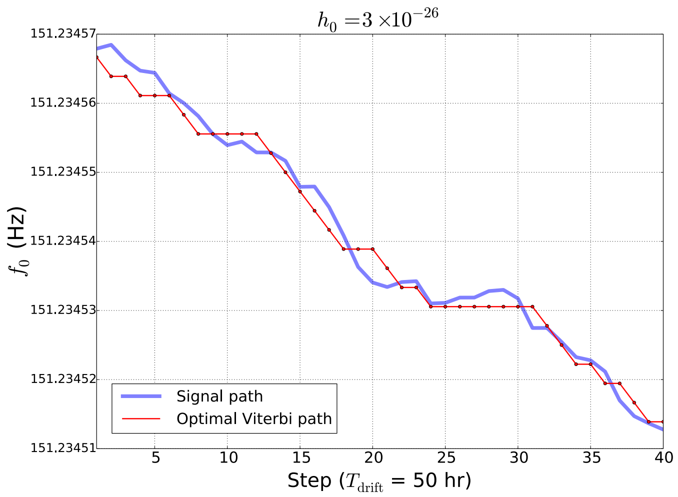
4 pipelines: 2 'unmodelled', 2 semi-coherent with grid over parameter space

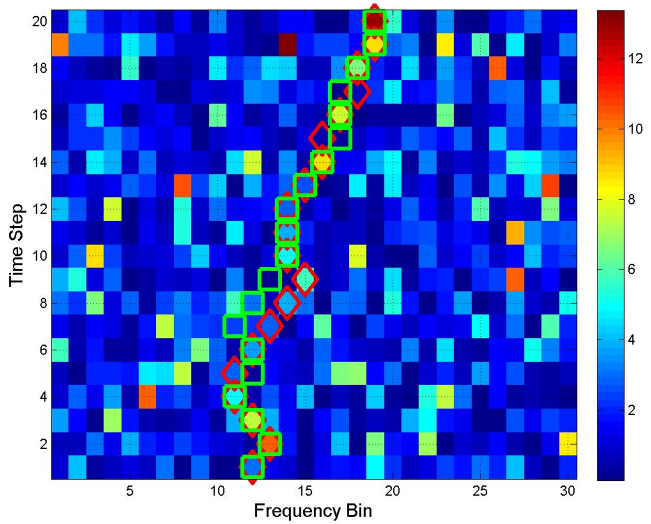
Hidden Markov model and Viterbi

Tracking rapidly spin-down CW signals with timing noise

- $\dot{f}_{0\mathrm{inj}} \sim 10^{-11}\,\mathrm{Hz\,s^{-1}}$, $\tau_{\mathrm{spin-down}} \sim \tau_{\mathrm{timing-noise}}$
- $\cos\iota \sim 0.7$, $\sqrt{S_h} = 4 \times 10^{-24}\ \mathrm{Hz}^{-1/2}$
- $h_0 = 1 \times 10^{-25}$ (left) and $h_0 = 3 \times 10^{-26}$ (right)



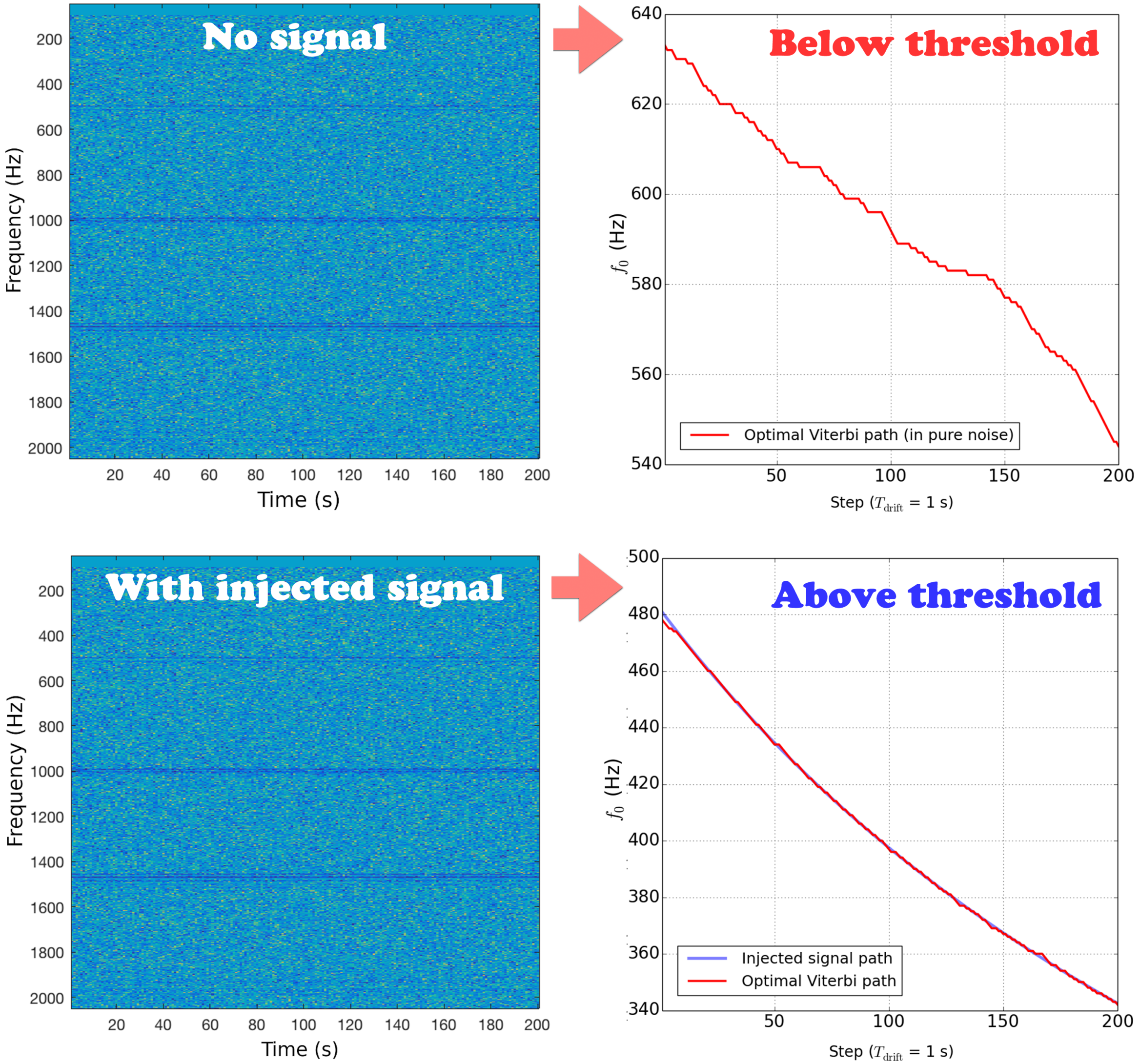




Sun et al., PRD 97, 043013 (2018)

Viterbi

Example: Tracking a millisecond magnetar signal



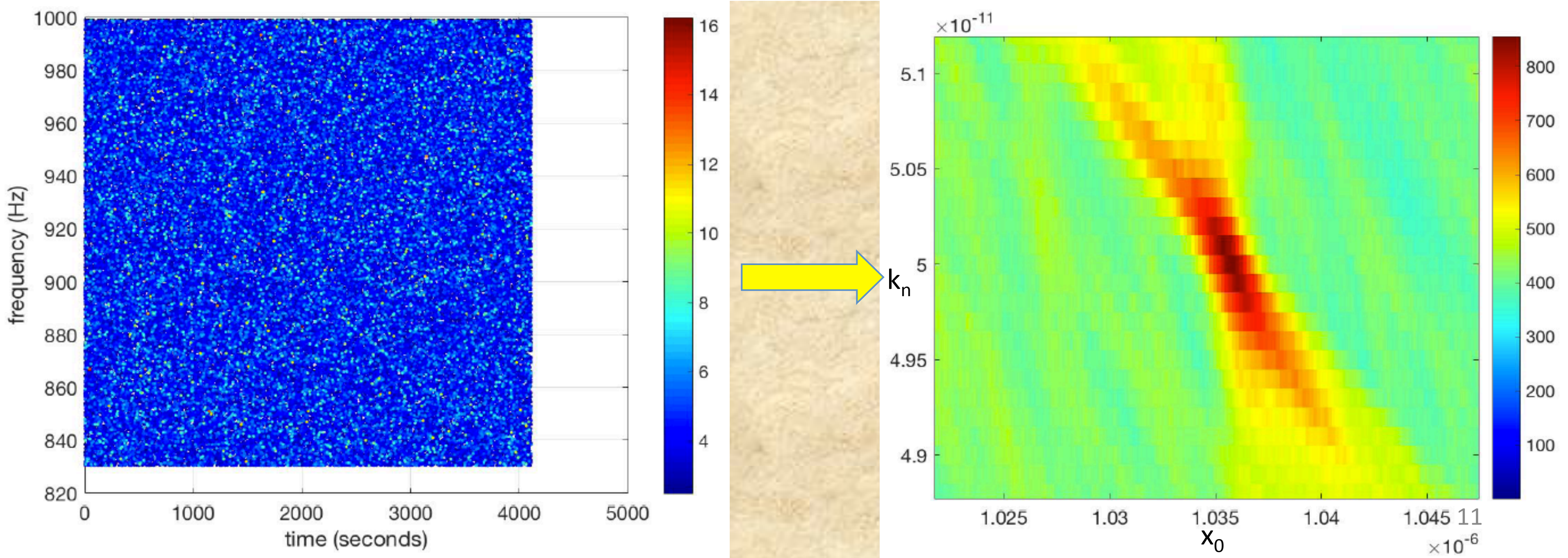
Generalizing the FrequencyHough

Assumes a power law for the frequency evolution:
Peaks in time-frequency plane are mapped into straight lines in the transformed x_0/k_n plane

$$f(t)=f_0\left(1+(n-1)\frac{\dot{f}_0}{f_0}(t-t_0)\right)^{-1/(n-1)}$$

$$\left(x_0=f_0^{1-n},\ k_n=\dot{f}_0/f_0\right)$$

The search is done over a grid built on the parameter space: f_0, k_n, n



Magnetar case (n = 3)

Search plan

- Grid over braking index $n \rightarrow$ we search over all possibilities of emission mechanisms (and combinations)
- 1 Hough per braking index
- f_0: [500, 2000] Hz
- n: [2.5, 7]
- $\dot{f}$: $[-\frac{1}{2^2}, -\frac{1}{16^2}]$ Hz/s
- Looking for sources lasting at most 1 day
- Beginning the search about 1 hour after the merger
  - Signal immediately after merger is very complicated
  - Less frequency variation later because spindown is smaller
  - Improve sensitivity because we use longer FFTs
- Search with varying T_{FFT} (2,4,8 s) in different portions of the parameter space to maximize sensitivity
- Computation cost: O(days)

Detection of long-transient signals from GW170817 is unlikely, but it is important to setup analysis methods.

Detection of GWs from the post-merger would provide a wealth of information on the remnant.

Conclusions

- LIGO and Virgo Collaborations have set forth a robust program to detect continuous gravitational waves
- Detecting one source would provide rich laboratory – including post-merger remnant searches!
- Critically important: improved detectors, sensitive, robust and faster algorithms, and continued collaboration with EM partners
- No guaranteed future CW detections, but . . .

. . . we are entering very interesting territory!