Detection of gravitational waves

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IEEC

1905: Albert Einstein publishes the Special Theory of Relativity. Proposes the Principle of Relativity and Principle of Invariant Light Speed.

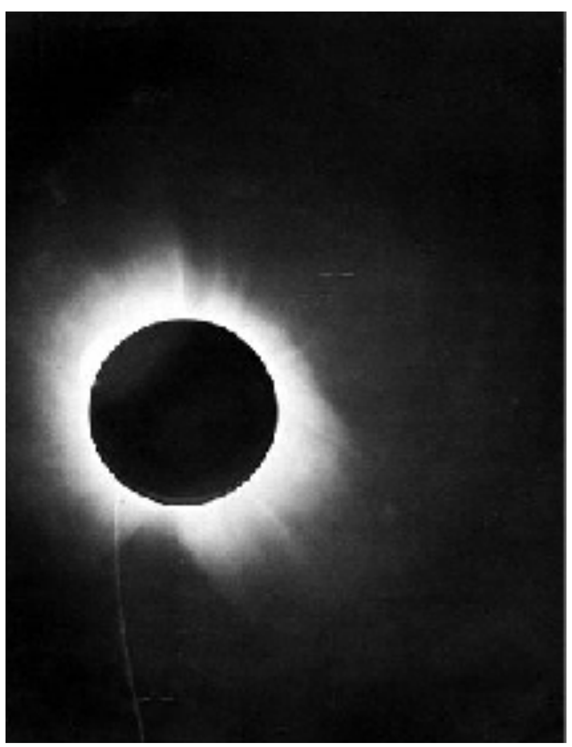
1915: Albert Einstein finishes the General Theory of Relativity

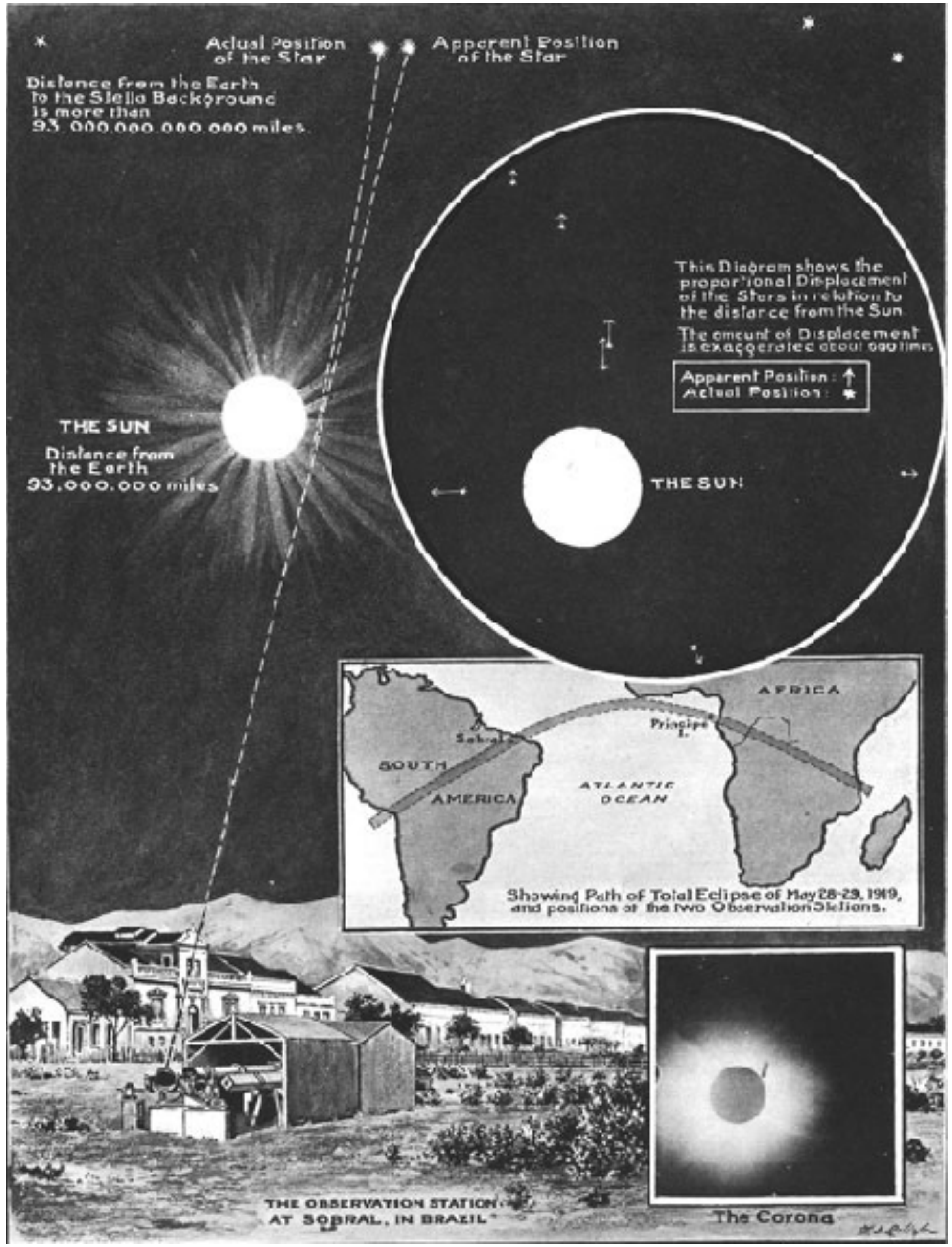
1916: Based on his General Theory of Relativity, Einstein predicts the existence of Gravitational Waves

1916: Karl Schwarzschild finds the spherically symmetric solution of Einstein's equation in vacuum

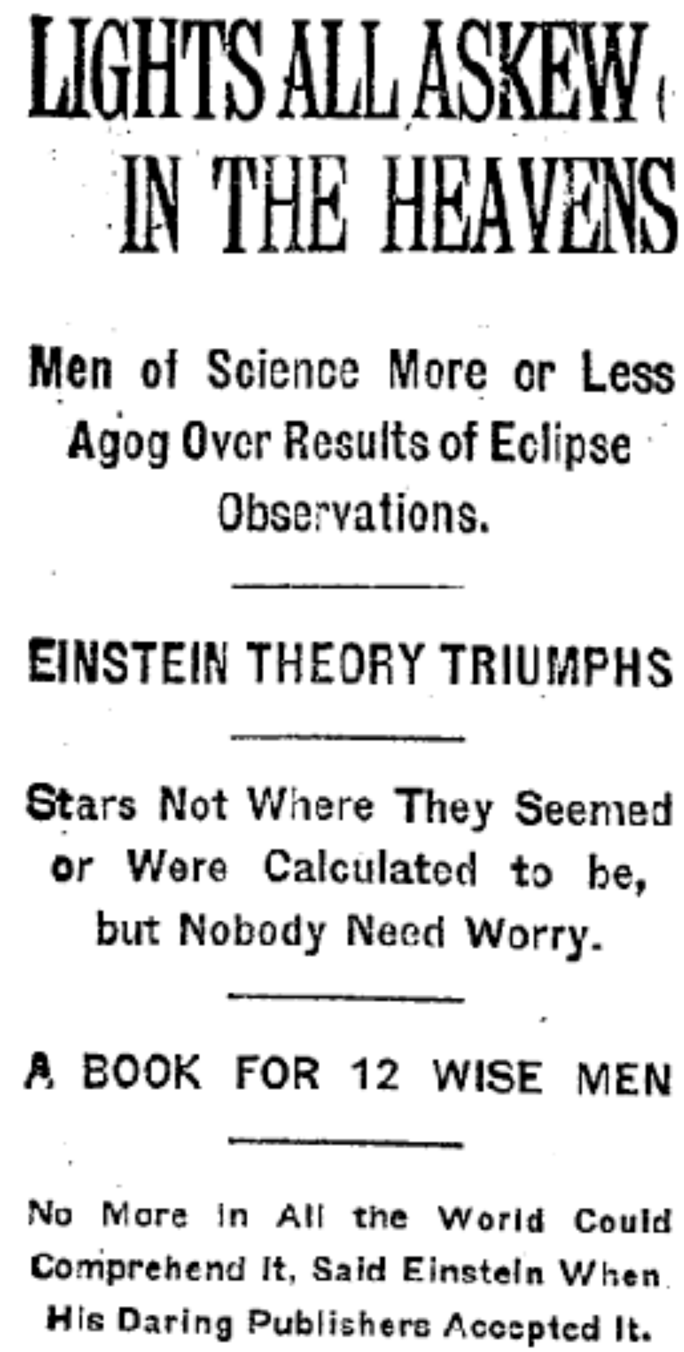
1918: Einstein continues studies on Gravitational Waves. Computes the energy lost by a system emitting gravitational waves, i.e. the Quadrupole Formula

1919: Eddington leads expedition to island of Príncipe (near Africa) to measure light deflection during solar eclipse. Confirms General Relativity prediction, major impact on newspapers all over the world.





London News, Nov. 1919



LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

New York Times, Nov. 1919

1922: Eddington scepticism about gravitational waves: ‘gravitational waves travel at the speed of thought’.

1936-38: Einstein doubts about gravitational waves being a mathematical artefact of the theory. The Einstein-Rosen paper: ‘Do Gravitational Waves exist?’

Einstein to Max Born (1936)

> Together with a young collaborator, I arrived at the interesting result that gravitational waves do not exist, though they had been assumed a certainty to the first approximation. This shows that the non-linear general relativistic field equations can tell us more or, rather, limit us more than we have believed up to now.[4]

Einstein to J.T. Tate, The Physical Review editor (1936)

> Dear Sir,
>
> We (Mr. Rosen and I) had sent you our manuscript for publication and had not authorized you to show it to specialists before it is printed. I see no reason to address the—in any case erroneous—comments of your anonymous expert. On the basis of this incident I prefer to publish the paper elsewhere.
>
> *Respectfully,*
>
> P.S. Mr. Rosen, who has left for the Soviet Union, has authorized me to represent him in this matter.

Infeld (his student) and Robertson (the referee) found a mistake in the paper. Einstein told Infeld he had independenty found the error.

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1936-38: Einstein doubts about gravitational waves being a mathematical artefact of the theory. The Einstein-Rosen paper: 'Do Gravitational Waves exist?'

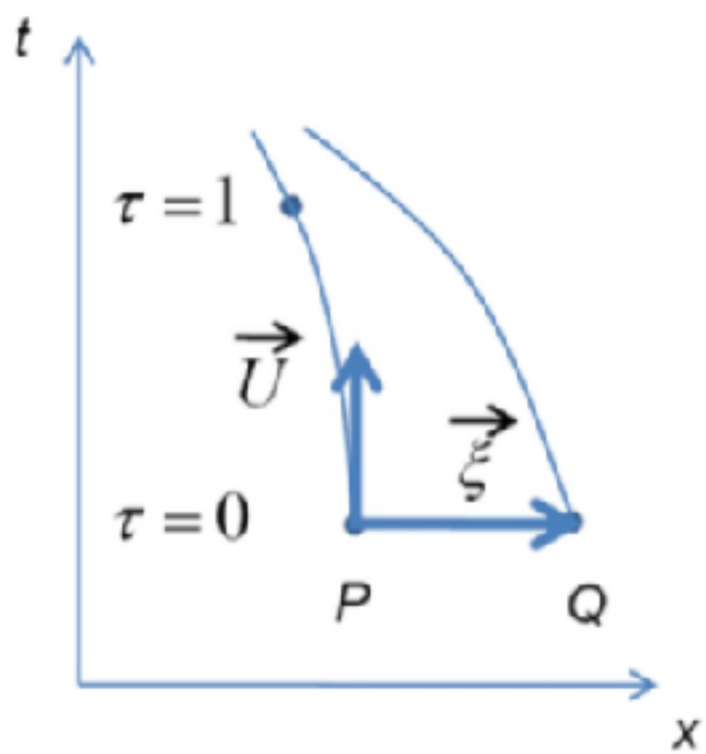
1955: Einstein dies at the age of 76 in Princeton

1955: Bern conference (Einstein *annus mirabilis* semi-centennial). Rosen reaffirms negative conclusion of 1936. Pirani/Roberson showed arguments of role of curvature tensor in producing tidal accelerations

1957: Chapel Hill Conference. Feynman's sticky bead argument

The geodesic deviation equation

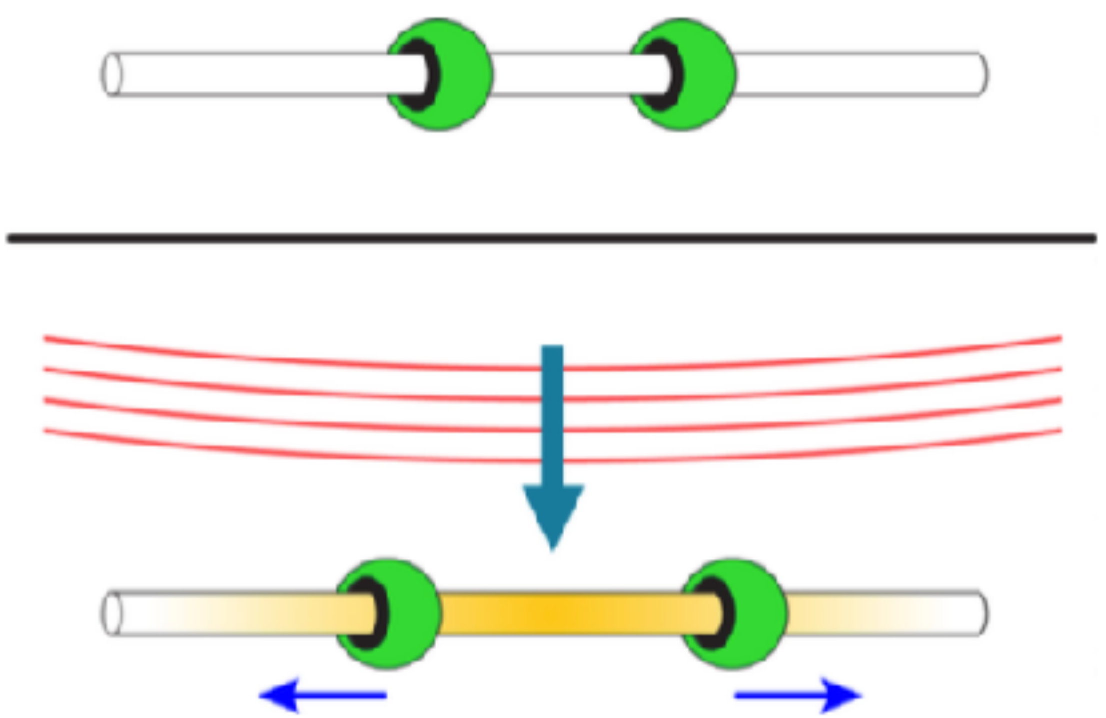
(Pirani)



$$\frac{d^2\xi^\mu}{d\tau^2} = -R^\mu{}_{\alpha\nu\beta} U^\alpha \xi^\nu U^\beta$$

The sticky bead argument

(Feynman)



1957: Bondi publishes in Nature the 'sticky bead argument'. Weber and Wheeler publish 'Reality of cylindrical waves of Einstein and Rosen' where they state '*the disturbance in question is real and not removable by any change of coordinate system*.'

1958: David Finkelstein identifies the Schwarzschild surface as an event horizon, 'a perfect unidirectional membrane: causal influences can cross it in only one direction'

1959: Weber pioneers the development of gravitational wave detectors with the resonant bars.

1963: Roy Kerr discovers the solution of Einstein's equation for spinning black holes

1969: Weber announces first detection of gravitational waves

1970s: Heinz Billings leads coincidence experiments of room-temperature resonant-mass experiments between Munich and Frascati. Results clearly refutes Weber's claim

1972: Rainer Weiss publishes 'Electromagnetically Coupled Broadband Gravitational Antennal' as MIT report. Analysis of laser interferometer as gravitational wave detector identifying noise sources and ways to deal with them

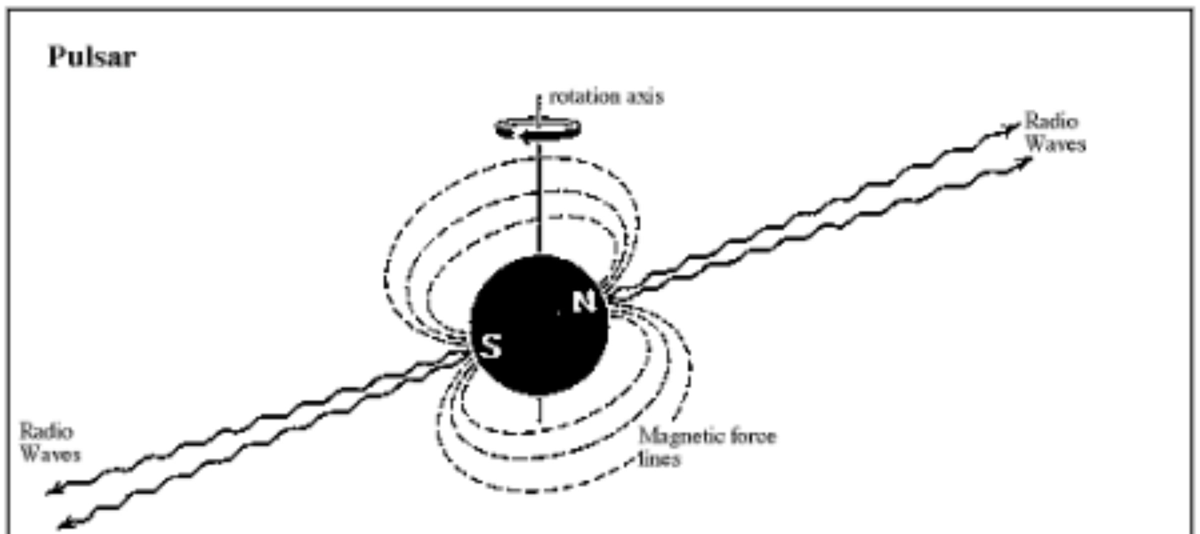
1975: Discovery of the first pulsar in a binary system (Hulse and Taylor pulsar)

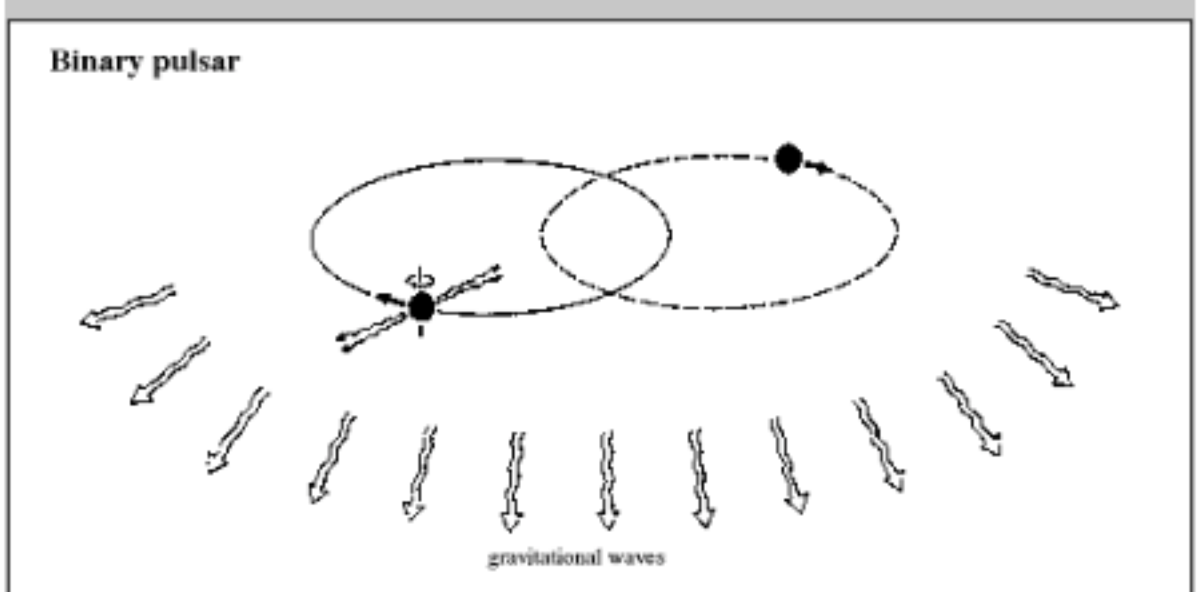
1993 Physics Nobel Prize

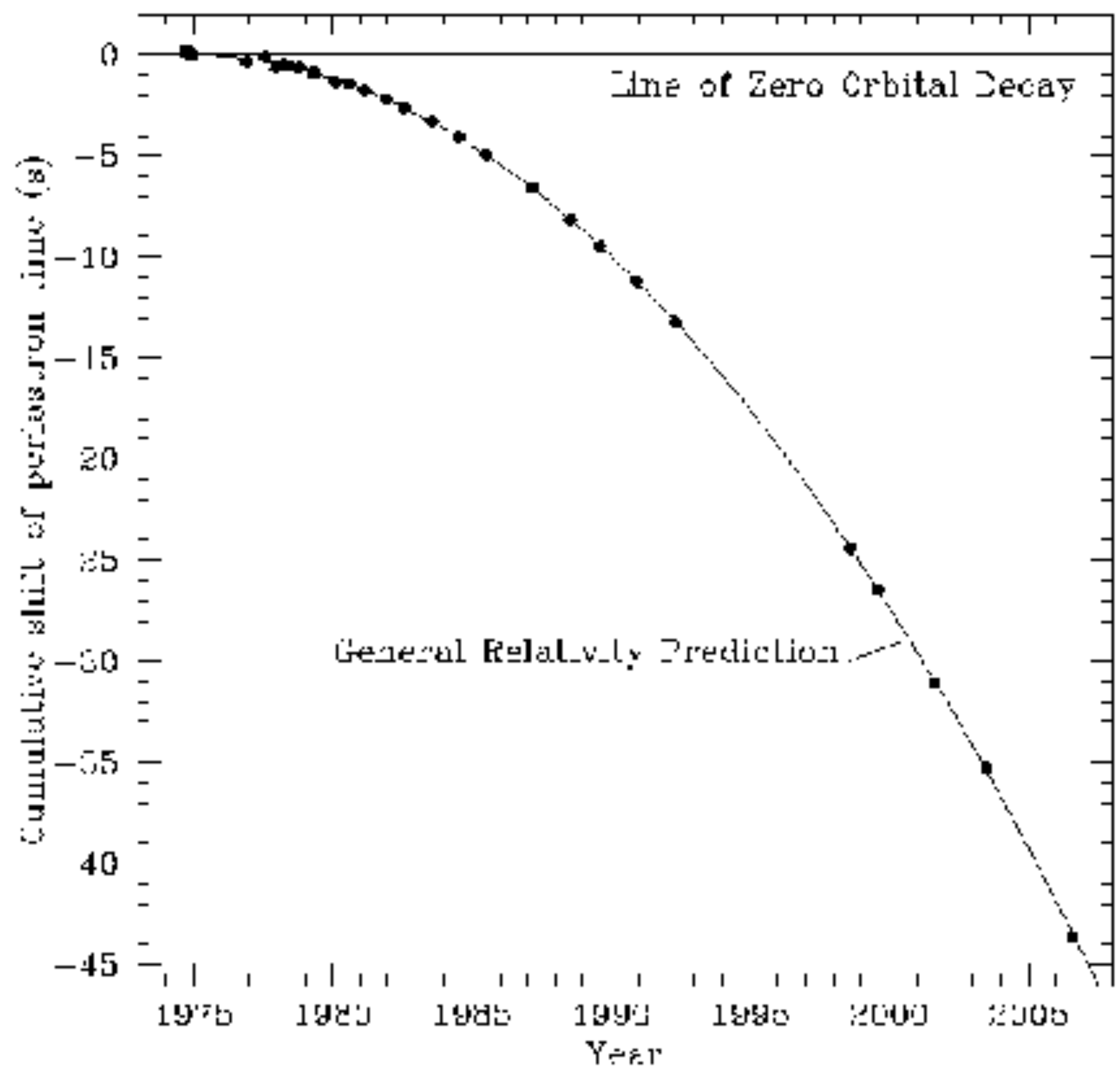
"for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation"

Russell A. Hulse

Joseph H. Taylor Jr.







Weisberg, Joel M., David J. Nice, and Joseph H. Taylor (2010)

1972: Rainer Weiss publishes ‘Electromagnetically Coupled Broadband Gravitational Antennal’ as MIT report. Analysis of laser interferometer as gravitational wave detector identifying noise sources and ways to deal with them

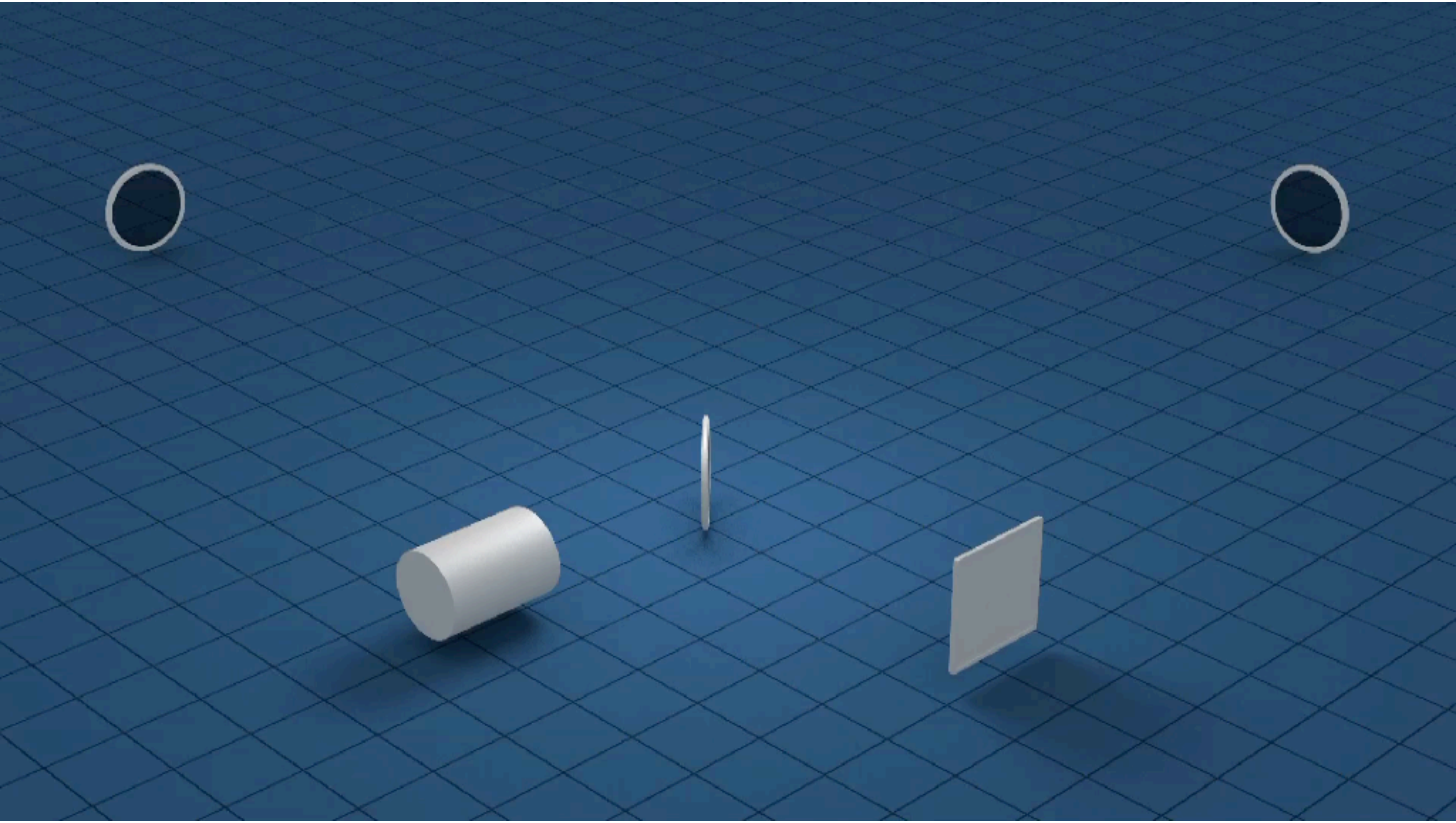
1975: Discovery of the first pulsar in a binary system (Hulse and Taylor pulsar)

Late 70s: Munich group starts (1975) construction of 3m laser interferometer prototype. Drever, in Glasgow, starts similar research (1977).

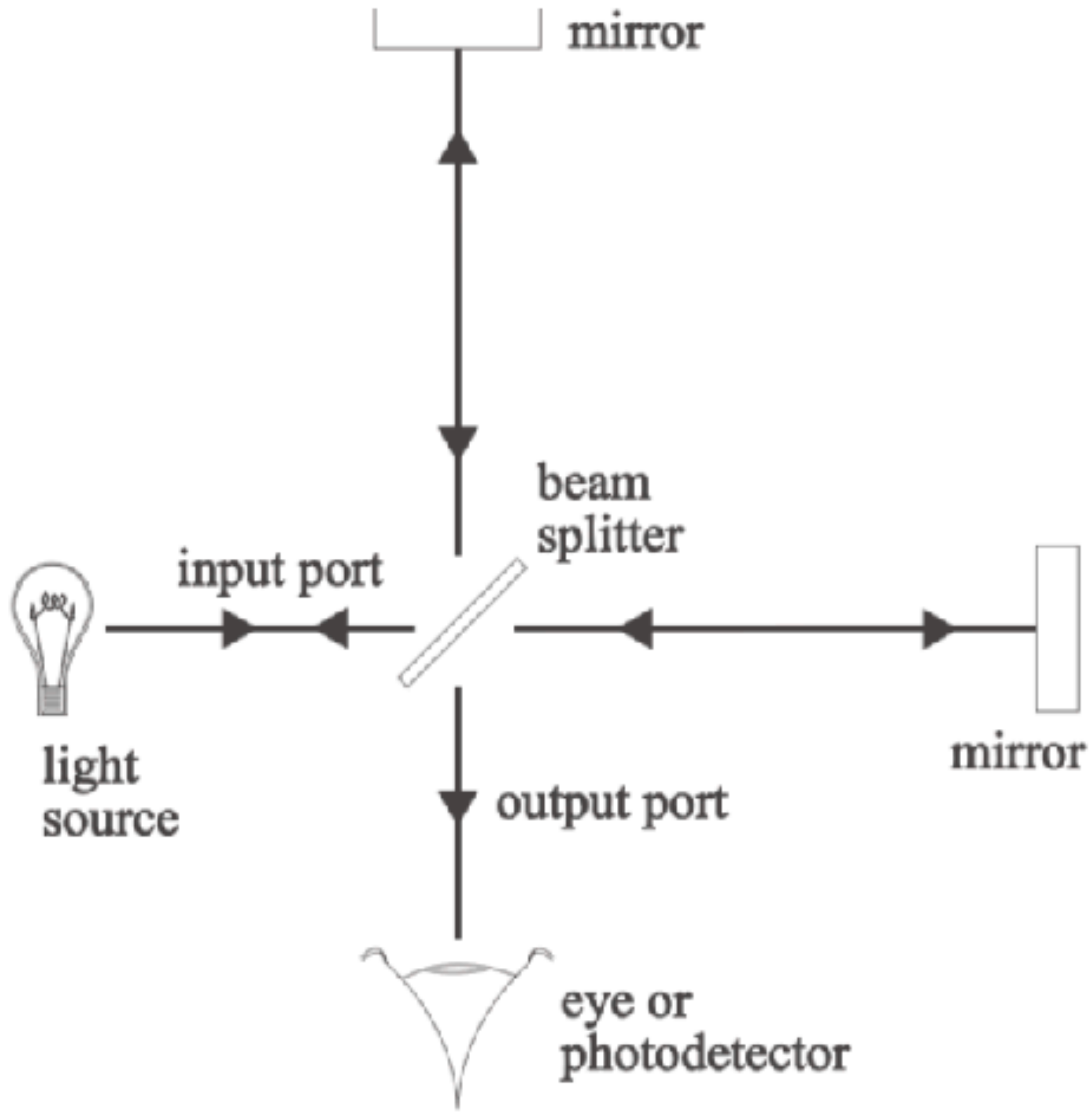
1980: Announcement of the orbital decay of the Hulse and Taylor pulsar (20% precision)

1992: Rainer Weiss, Ronald Drever and Kip Thorne founded LIGO (Laser Interferometer Gravitational Wave Observatory) as a National Science Foundation project

Detector principle



The effect of a gravitational wave



The effect of a gravitational wave

A light beam in space-time

$$\begin{aligned} ds^2 \quad = 0 \quad &= g_{\mu\nu} dx^\mu dx^\nu \\ &= (\eta_{\mu\nu} + h_{\mu\nu})\, dx^\mu dx^\nu \\ &= -c^2 dt^2 + (1 + h_{11}(2\pi f t - kz))\, dx^2 \end{aligned}$$

Integrate in the path-length

$$\int_0^{\tau_{out}} dt = \frac{1}{c} \int_0^L \sqrt{1 + h_{11}} dx \approx \frac{1}{c} \int_0^L \left(1 + \frac{1}{2} h_{11} \left(2\pi f t - kz\right)\right) dx$$

Taking two the path and some approximations

$$\Delta\tau(t) = h(t) \frac{2L}{c} = h(t) \tau_{rt0}$$

The effect of a gravitational wave

Getting rid of coordinate systems, we consider just the effect of the passing GW in the lab

We locate a set of rulers and observe the effect of the wave

In the lab frame, we have the Newtonian approach $$F_{gw} = \frac{1}{2} m L \frac{\partial^2 h_{11}}{\partial t^2}$$

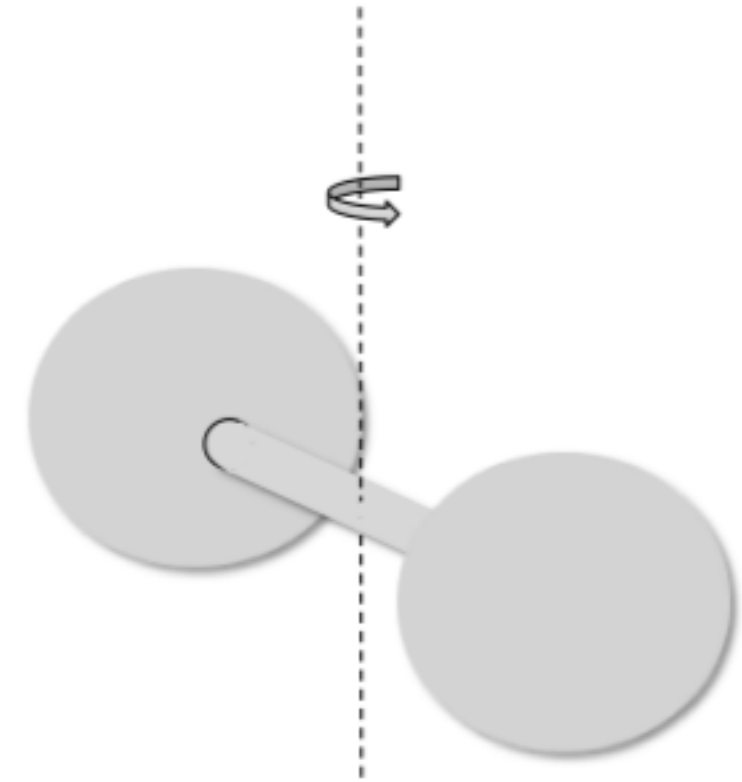
We would observe a tidal force, proportional to length $$\Delta L = hL/2$$

In this picture, we would say ‘light travel changes because test mass move’

Generation of gravitational waves

Starting from the quadruple formula

$$h_{\mu\nu} = \frac{2G}{Rc^4}\ddot{I}_{\mu\nu}$$



Notice the pre-factor is 10^{-44}

We take two 1 ton masses, rotating together at 1kHz

$$h_{lab} = 2.6 \times 10^{-33}\mathrm{m} \times \frac{1}{R}.$$

We take two 1 ton masses, rotating together at 1kHz. We need to move at leat one wavelength, 300km!

$$h_{lab} = 9 \times 10^{-39}$$

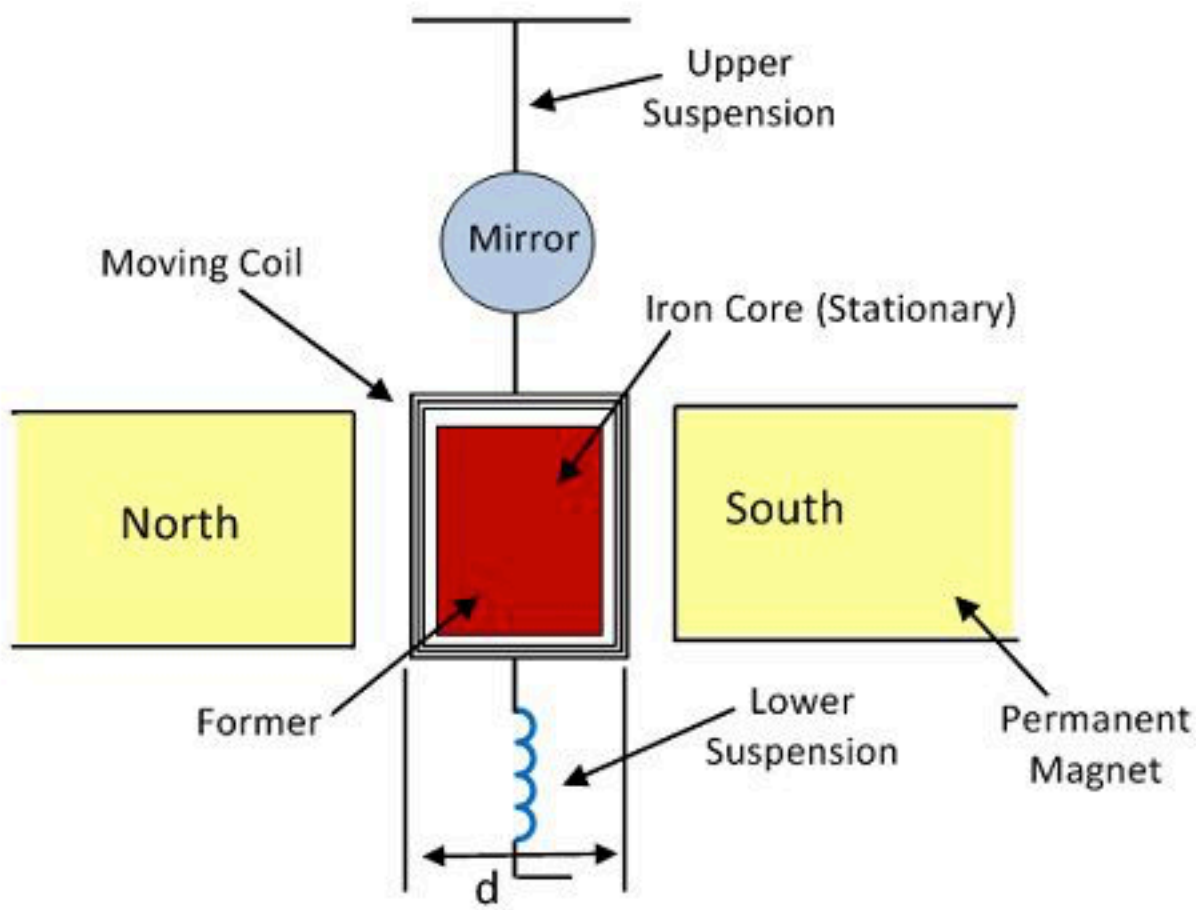
Thermal noise

- According to the Equipartition Theorem, each degree of freedom of a system in thermodynamic equilibrium at temperature T should have an energy whose expectation value is $K_b T/2$
- First measurement in the 30s with galvanometers and electrical resistance (Johnson noise)
- General dissipation-fluctuation theorem introduced by Callen and Welton (50s) although originally introduced by Nyquist to explain Johnson noise

$$S_F(\omega) = 4k_B T Re(Z)$$

- Examples:
    - mechanical viscosity - brownian
    - electrical resistance - Johnson noise

Historical digression: Galvanometer



$$\theta_{rms} = \sqrt{\frac{k_B T}{I \omega_0^2}}$$

Measuring path-length

- A interferometer detector translates GW into light power (transducer)
- If we would detect changes of 1 wavelength (10^{-6}) we would be limited to 10^{-11}, considering the total effective arm-length (100 km)
- Our ability to detect GW is therefore our ability to detect changes in light power
- Power for a interferometer will be given by

$$P_{out} = P_{in} \cos^2(k_x L_x - k_y L_y)$$

- The key to reach a sensitivity of 10^{-21} sensitivity is to resolve the path-length difference in a tiny fraction, ie. 10^{-10}

Shot-noise

- Modelling light flux as a set of discrete photons with independent arrival times: Poisson distribution

$$p(N) = \frac{\bar{N}^N e^{-\bar{N}}}{N!}$$

- Our distribution is characterised by arrival rate in a given time $\bar{N} = \bar{n}\tau$
- This leads to a fluctuation of the mean measurement and therefore of our precision given by

$$\frac{\sigma_{\bar{N}}}{\bar{N}} = \frac{\sqrt{\bar{n}\tau}}{\bar{n}\tau} = \frac{1}{\sqrt{\bar{n}\tau}}$$

Shot-noise

- Taking into account the photon energy

$$\hbar\omega = 2\pi\hbar c/\lambda$$

- The mean photon flux at the output will be

$$\bar{n} = \frac{\lambda}{2\pi\hbar c} P_{out}$$

- And the mean number of photons per interval and associated fluctuations

$$\overline{N} = (\lambda/4\pi\hbar c) P_{in}\tau \qquad \sigma_{\bar{N}}/\bar{N} = \sqrt{4\pi\hbar c/\lambda P_{in}\tau}.$$

Shot noise

- Since we are measuring power fluctuations at the output, these fluctuations are indistinguishable from mirror displacements

$$\sigma_{\delta L} = \frac{\sigma_N}{N} \Big/ \frac{1}{P_{out}} \frac{dP_{out}}{dL} = \sqrt{\frac{\hbar c \lambda}{4\pi P_{in} \tau}}.$$

- And we are using mirror displacements to measure GW as the fractional length change in one arm $\sigma_h = \sigma_{\delta L} / L$

- So brightness fluctuations are interpreted as equivalent gravitational wave noise

$$\sigma_h = \frac{1}{L} \sqrt{\frac{\hbar c \lambda}{4\pi P_{in} \tau}}.$$

Radiation pressure

- The force exerted by an electromagnetic wave of power P reflecting from a losses mirror

$$F_{rad} = \frac{P}{c}$$

- The fluctuations of the force are due to shot noise in the power

$$\sigma_F = \frac{1}{c}\sigma_P$$

- Inserting the photon energy we get to the power spectra

$$F(f) = \sqrt{\frac{2\pi\hbar P_{in}}{c\lambda}}$$

Radiation pressure

- The fluctuating force turns into a displacement in the test mass

$$x(f) = \frac{1}{m(2\pi f)^2} F(f) = \frac{1}{mf^2}\sqrt{\frac{\hbar P_{in}}{8\pi^3 c\lambda}}$$

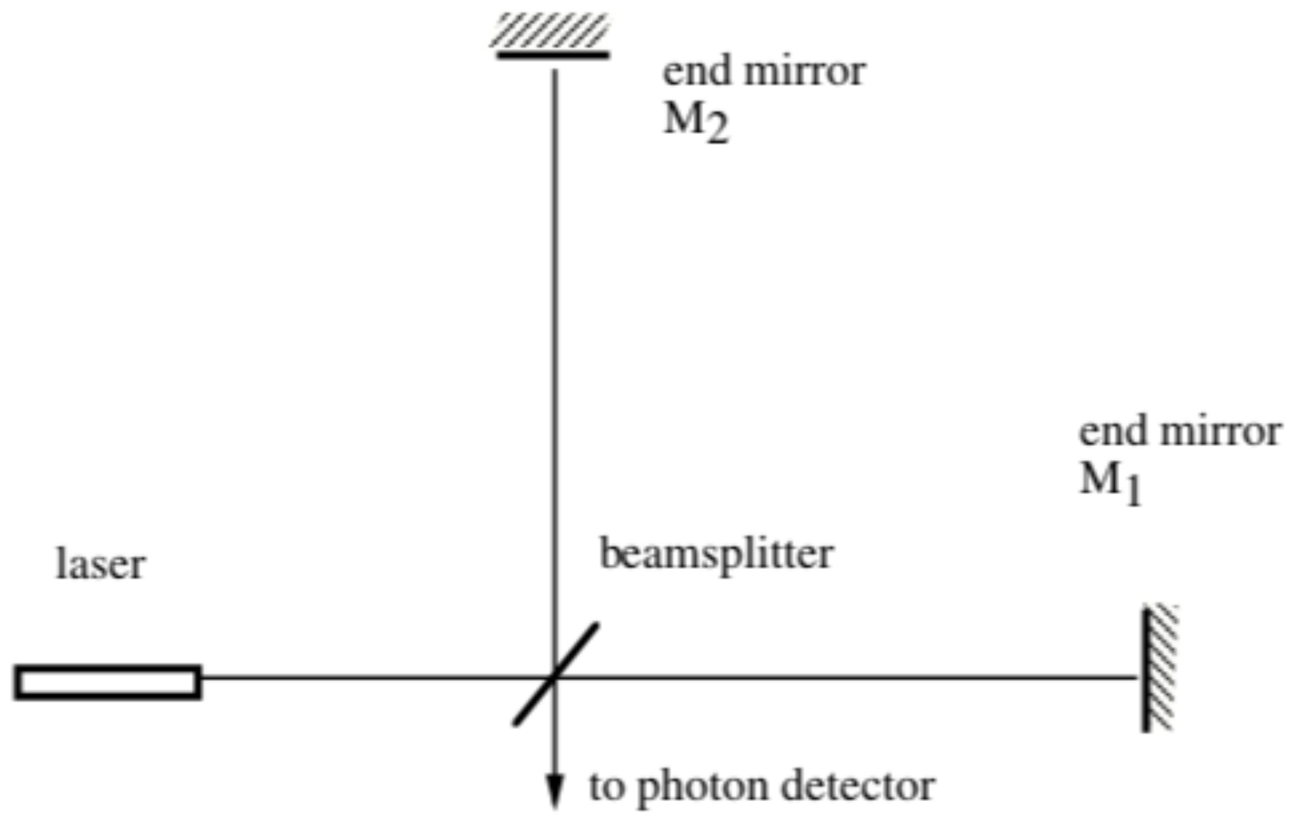
- which can be expressed, as in the previous case, as an equivalent gravitational wave noise

$$h_{rp}(f) = \frac{2}{L}x(f) = \frac{1}{mf^2 L}\sqrt{\frac{\hbar P_{in}}{2\pi^3 c\lambda}}$$

- Radiation pressure and shot noise are competing effects, what would be the noise if we minimise this two contributions, $h_{rp}(f,P) = h_{shot}(f,P)$

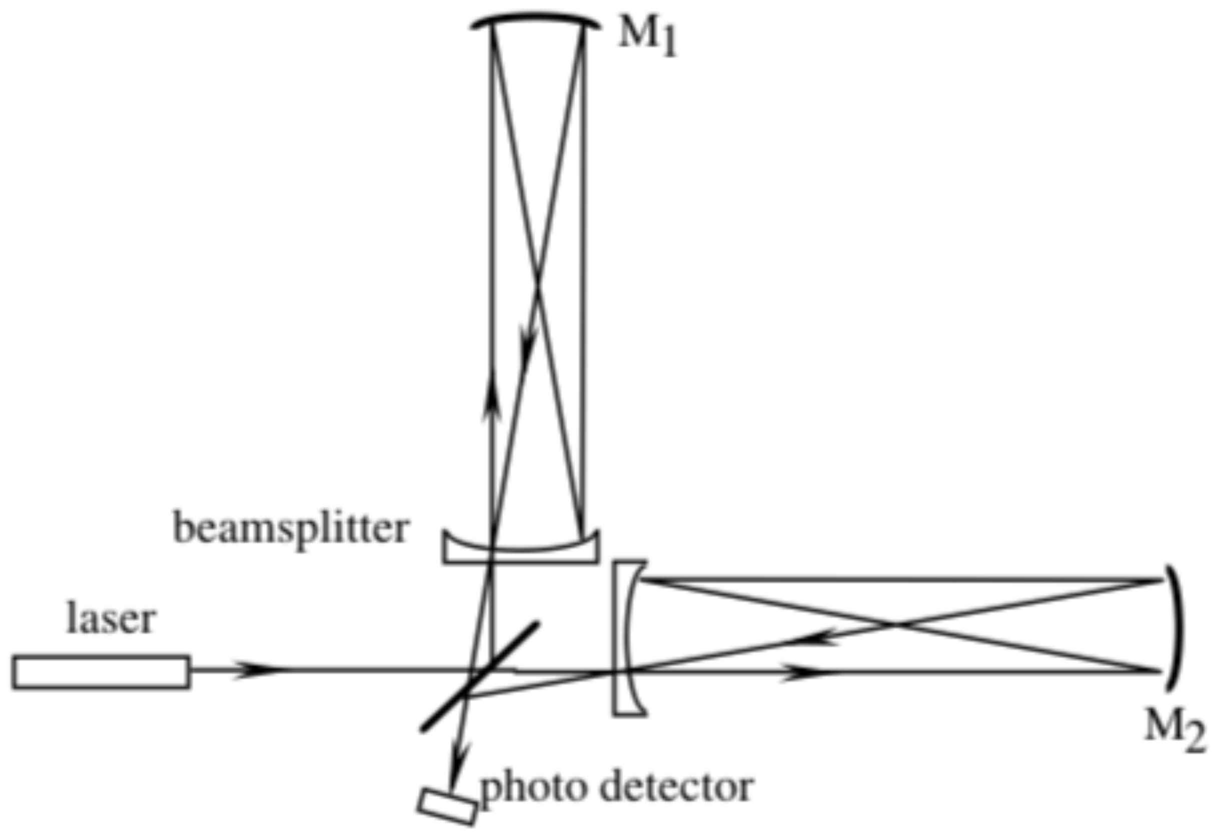
$$h_{QL}(f) = \frac{1}{\pi f L}\sqrt{\frac{\hbar}{m}}.$$

Michelson interferometer



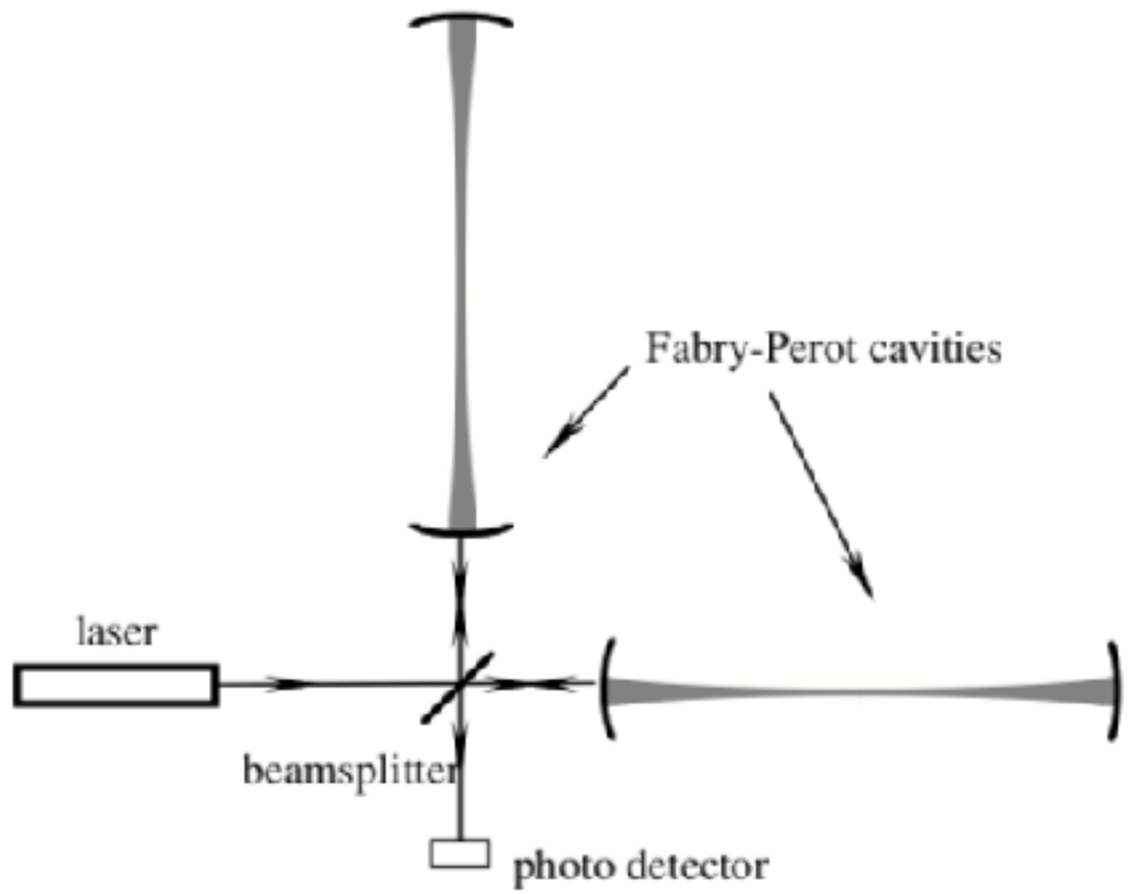
- Three free test masses
- Working at the 'dark fringe' (180º out of phase), reducing shot noise and power fluctuations
- There is an optimal length, e.g. f =1kHz , L = c/2f = 150km

A delay line Michelson interferometer



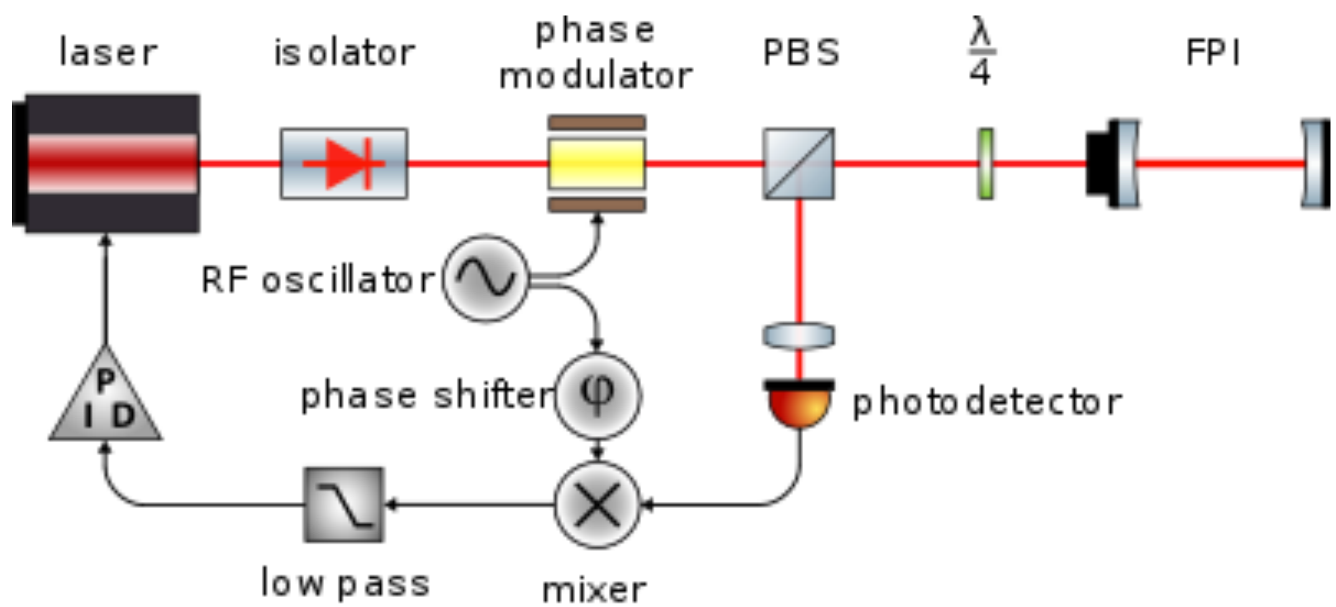
- 3km x 50 bounces = 150km
- Number of bounces limited by reflection losses
- A problem: scattered light phase α = 2π f ΔL/c (laser stab., laser modulation)
- Garching 30m prototype

Fabry-Perot interferometer



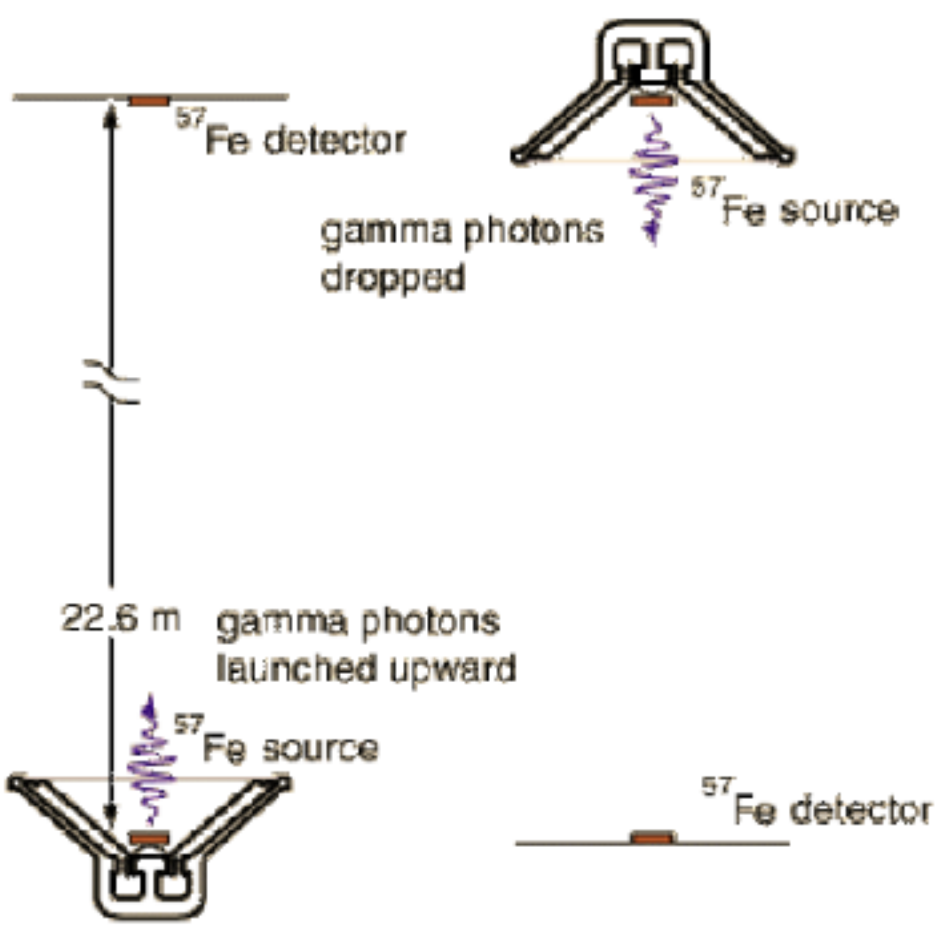
- Add two mirrors to form a cavity
- Measure differential phase change between cavities (differential because laser freq. noise)
- A problem: scattered light phase $\alpha = 2\pi f \Delta L/c$ (laser stab., laser modulation)
- Option: lock laser in wavelength to one cavity and then lock second laser to wavelength

The Pound-Drever-Hall technique



Historical digression: The Pound-Rebka experiment

- Pound and Rebka measured for the first time the gravitational redshift (1960)
- Based on the recently discovered Mössbauer effect (1958, Nobel prize 1961),14.4 keV gamma from ^{57}Fe
- Gravitational redshift $\Delta\nu \sim 10^{-15}$



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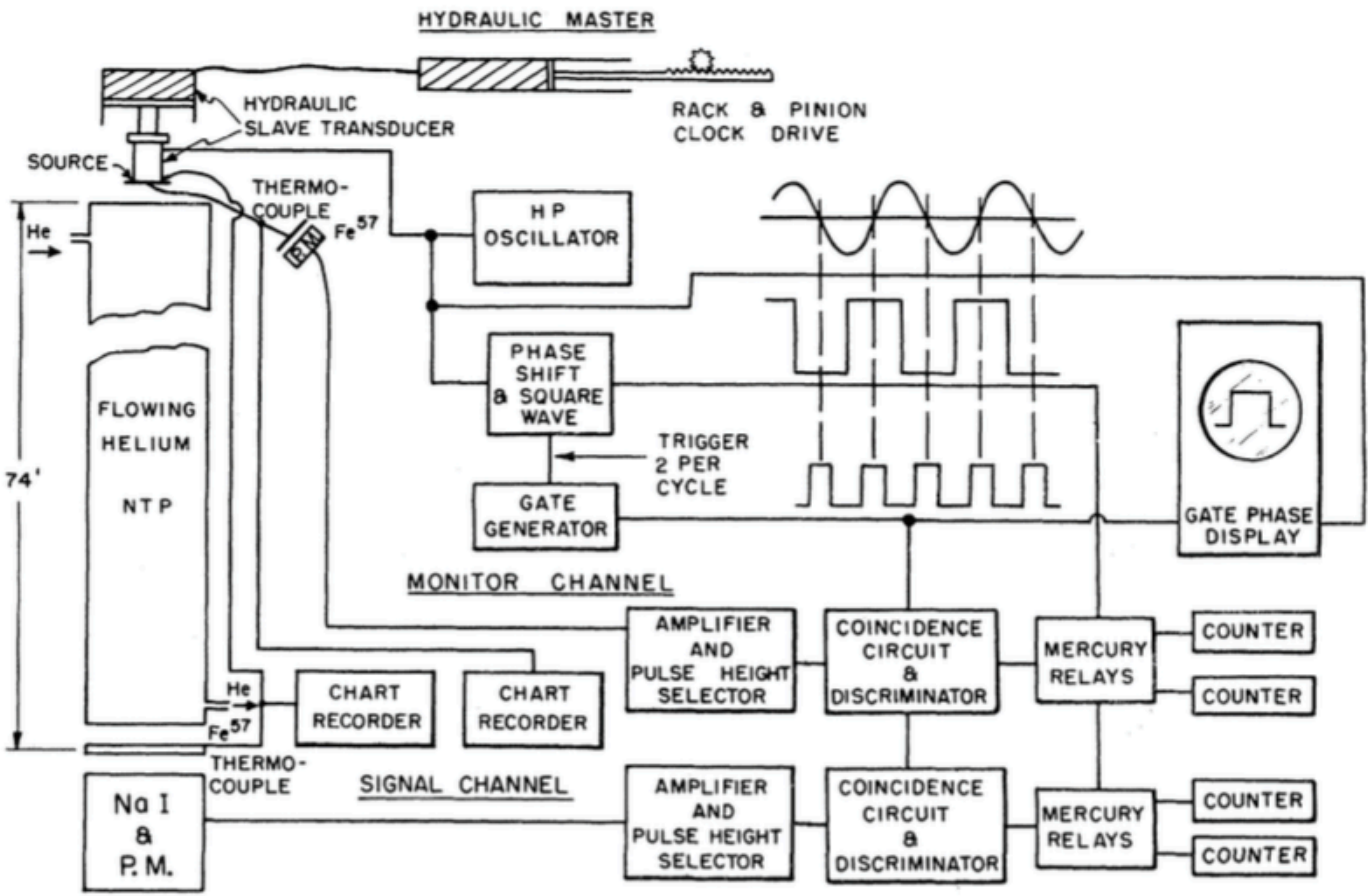
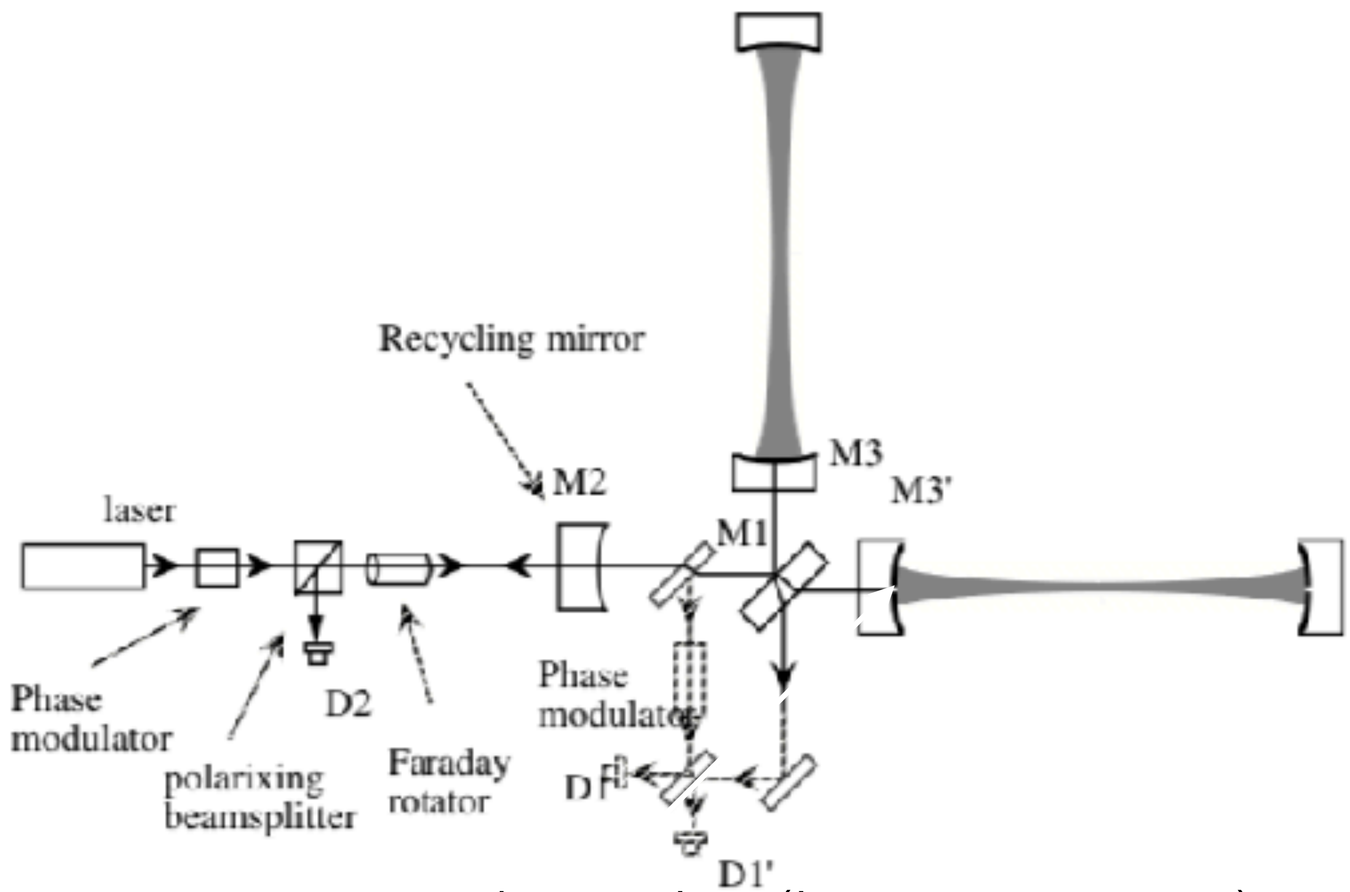
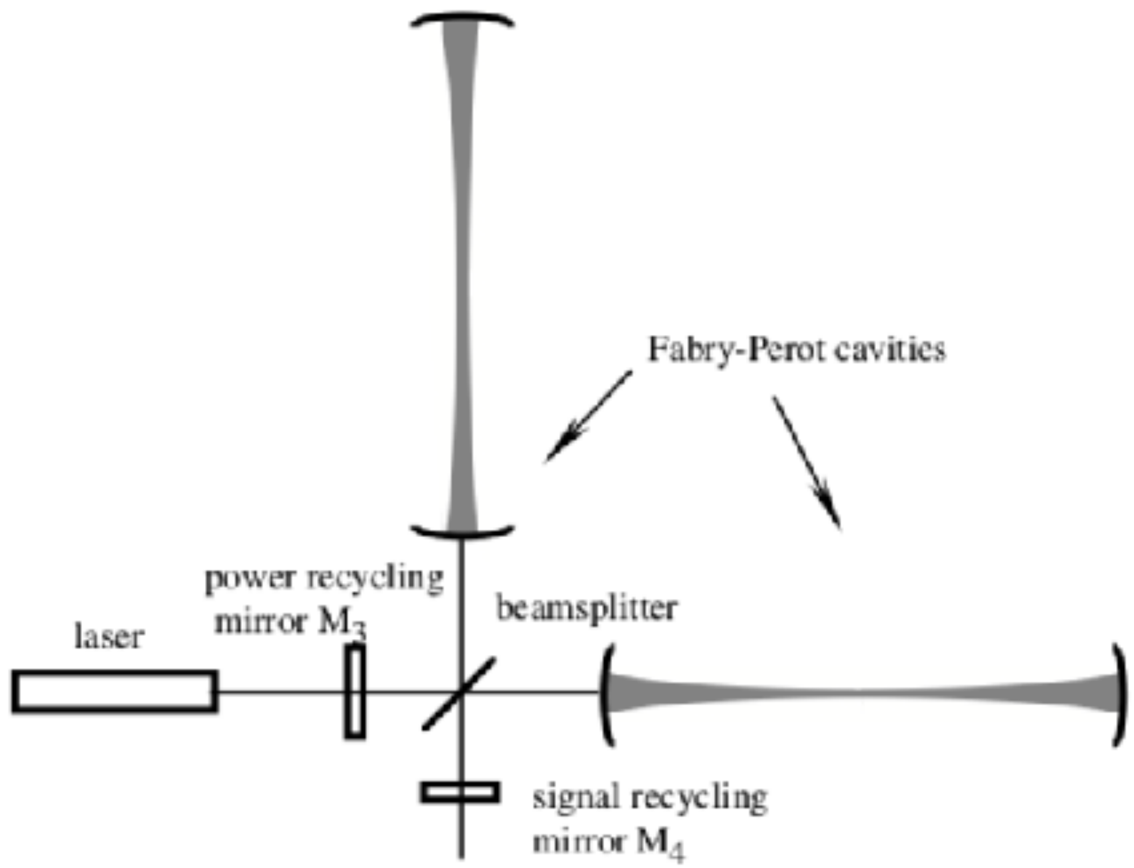


Power recycling



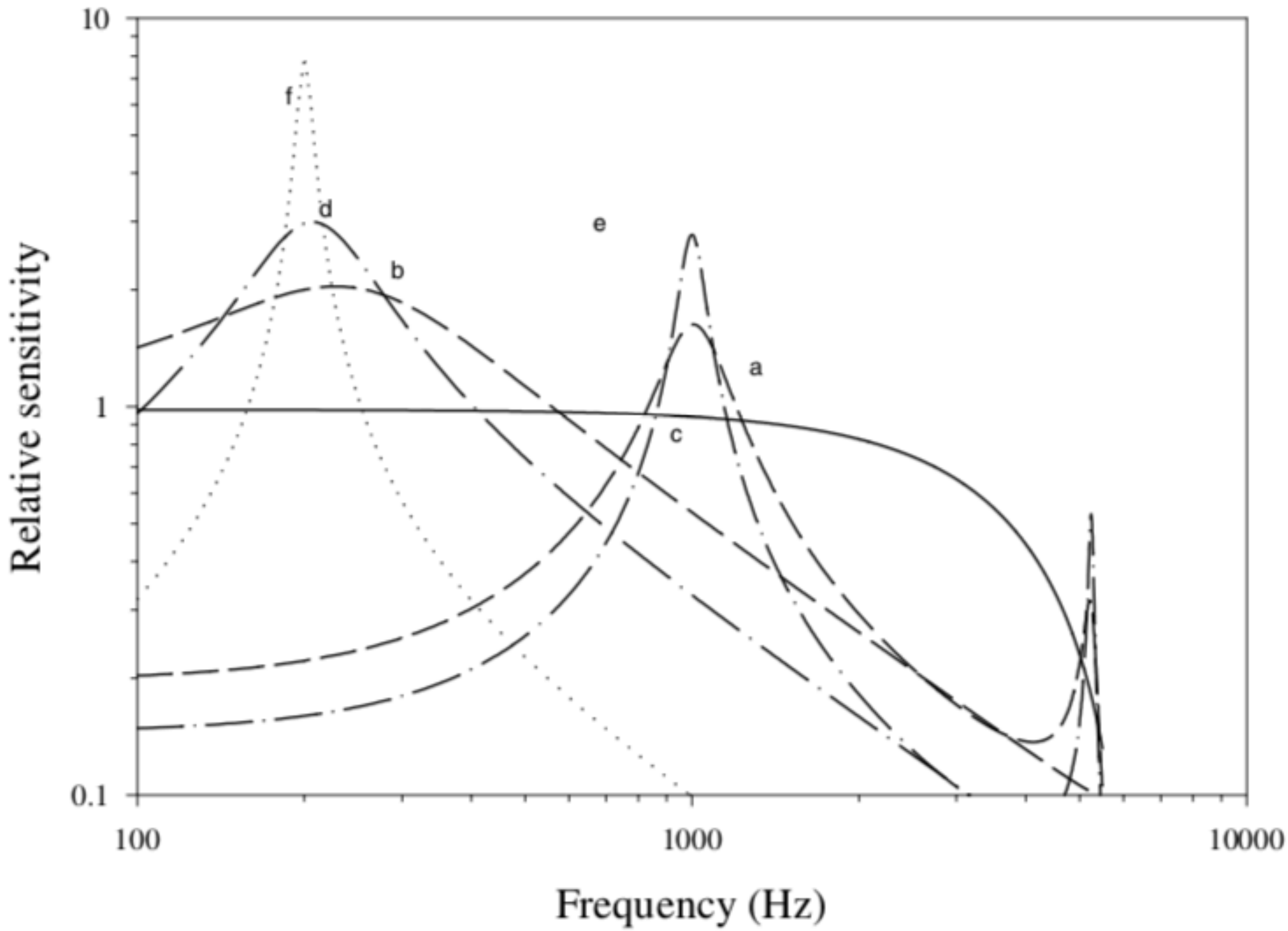
- Two competing error sources: shot noise (increase power) and radiation pressure (reduce power)
- An optimal power: $P_{\rm opt} = \dfrac{\lambda mc}{8\pi\tau^2\sqrt{\eta}}$
- For a LIGO-like experiment: P_{opt} = 60MW ie. the detector will be shot noise limited (working at ‘dark fringe’ -> most of light lost)
- Recycling mirror: to recover light, carefully located to form a resonator together with cavity and beam-splitter

Dual recycling

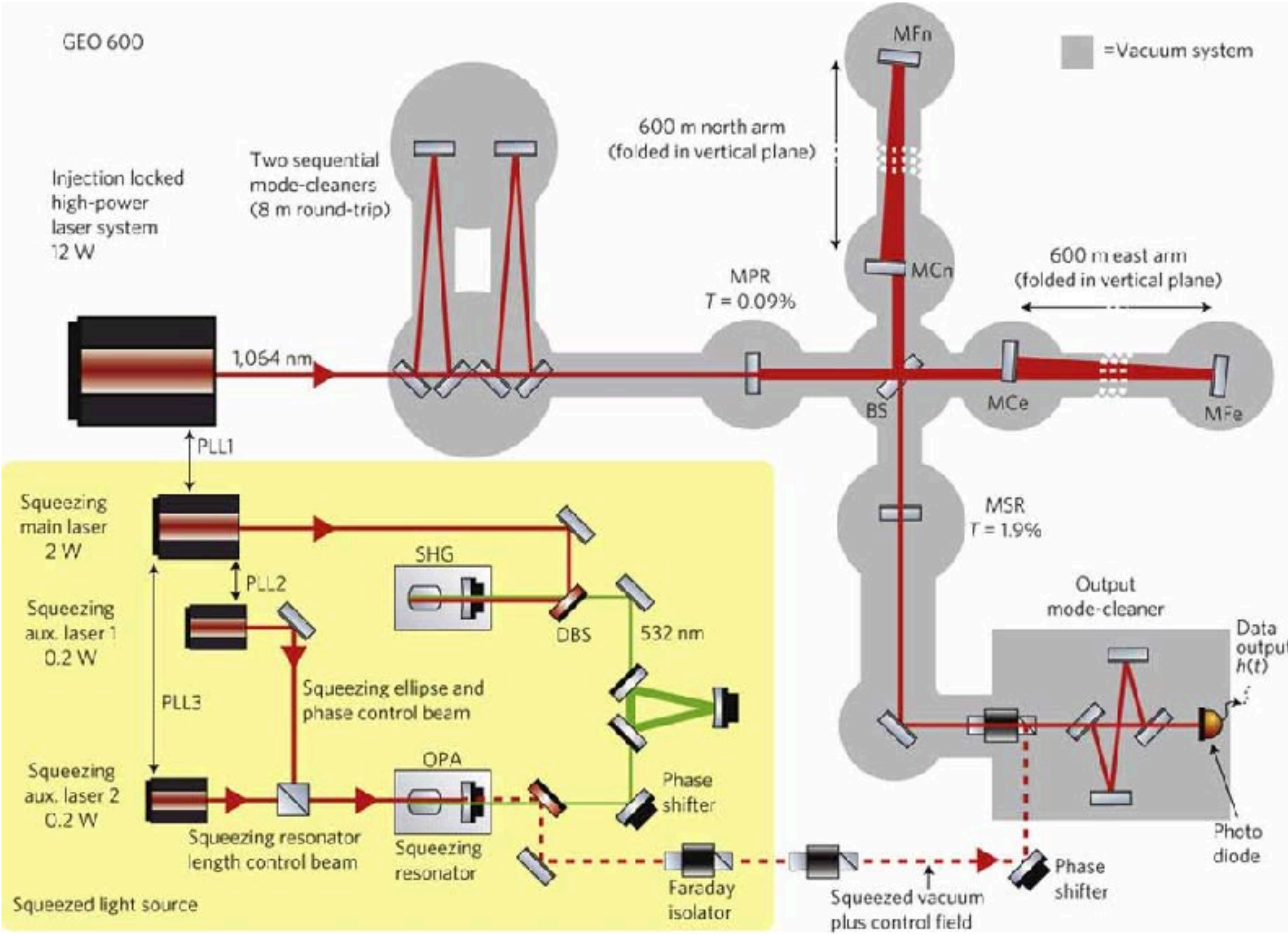


- A M4 mirror located at the detector output
- If perfectly matched, no light reaches M4
- When GW signal reaches the detector, it produces sidebands that leak to M4 which can then ‘recycle’ this signal
- The relative position of the M4 determines the tuning frequency of the dual recycling

Dual recycling



Squeezed light



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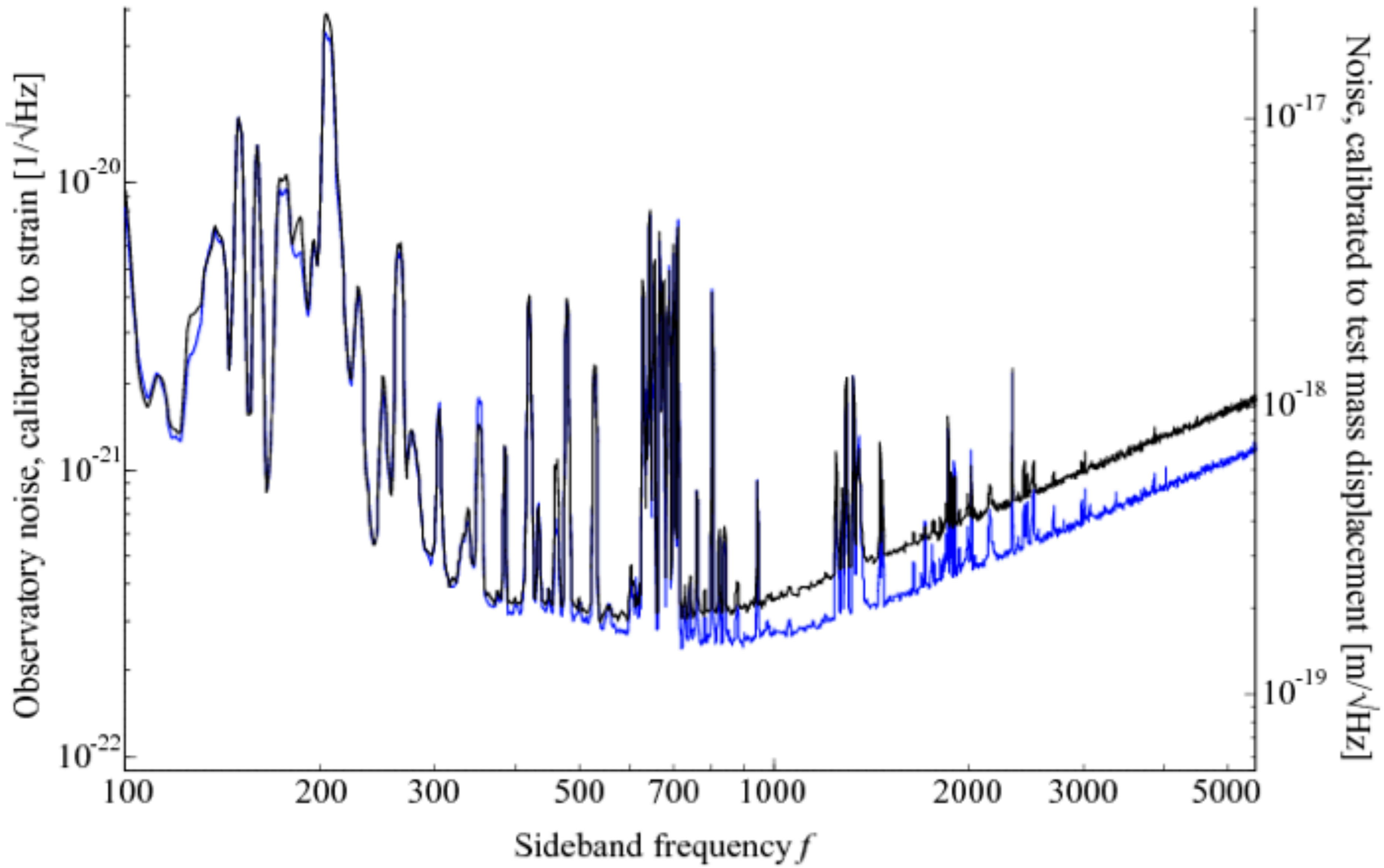
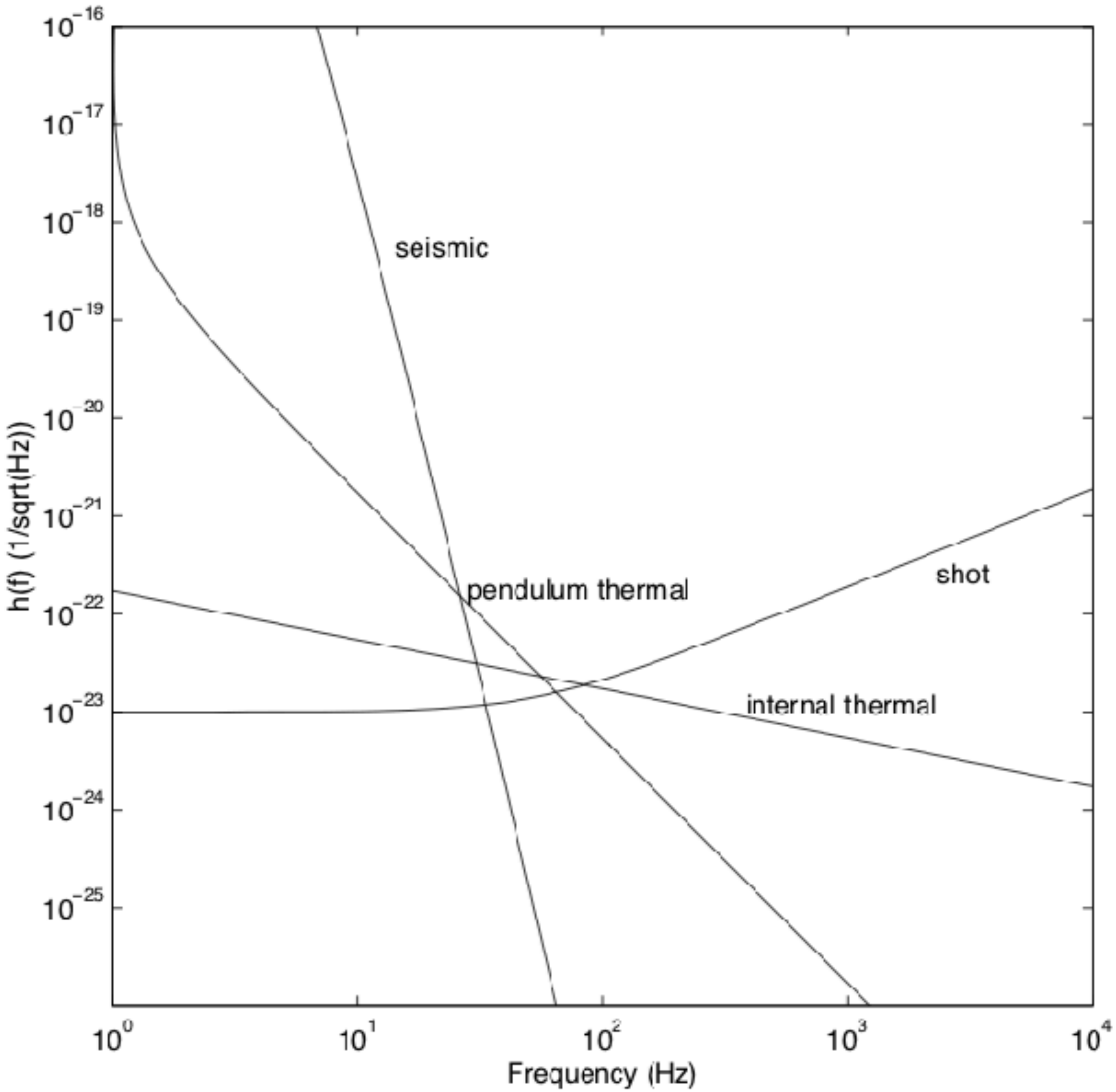


Summary



seismic
pendulum thermal
shot
internal thermal
h(f) (1/sqrt(Hz))
Frequency (Hz)
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}